

**The Race between Population and Technology:
Real Wages in the First Industrial Revolution**

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The Race between Population and Technology: Real Wages in the First Industrial Revolution

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Abstract

We investigate a structural model of demographic-economic interactions for England during 1570 to 1850. We estimate that the annual rate of population growth consistent with constant real wages was 0.4 per cent before 1760 but 1.5 per cent thereafter. We find that exogenous shocks increased population growth dramatically in the early decades of the Industrial Revolution. Simulations of our model show that if these demographic shocks had occurred before the Industrial Revolution the impact on real wages would have been catastrophic and that these shocks were largely responsible for very slow growth of real wages during the Industrial Revolution.

Keywords: epidemic disease; Industrial Revolution; Malthusian checks; nuptiality; population growth; real wages; technological progress.

JEL Classification: N13; N33.

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In recent years, pessimism has tended to dominate the long-running debate over standards of living during the Industrial Revolution in Britain. Real wage growth is seen as very slow to accelerate and failing to keep up with labour productivity growth. This episode has attracted renewed interest from economists in the context of fears about the impact of artificial intelligence and robotics on the labour market; they see the long wait for the innovations of the industrial revolution to benefit workers as a precursor for a similar experience today.

Yet in a longer-term demographic context this experience can be seen in a different light. Tony Wrigley sees real wage growth of around 0.5 per cent as a ‘truly remarkable achievement’ at a time of unprecedented population growth, which a century earlier would have precipitated large falls in real wages.¹ He argues that in pre-industrial England population growth at 0.5 per cent per year was the ‘speed limit’ – anything faster than this meant that real wages would decrease.

In this paper, we scrutinize Wrigley’s claim by estimating a structural model of economic-demographic interactions for post-1570 England. This is in the tradition of Ronald Lee and Michael Anderson, but we use it not only to examine the role of Malthusian preventive and positive checks to population growth but also to quantify the ‘rate of absorption’ of the economy, i.e., the rate at which population could grow consistent with constant real wages, both before and during the Industrial revolution.²

Our analysis is based on a new series for real annual earnings derived from estimates made by Jane Humphries and Jacob Weisdorf linked to those made by Charles Feinstein.³ As we show, unlike series for real day wages commonly used in earlier work, this series exhibits trend growth in real earnings of 0.45 per cent per year after 1660 while the English population increased from 5.3 million in 1660 to 7.2 million in 1780. This suggests that the economy might be described as already being ‘post-Malthusian’.

Our estimates of the structural model deliver several important results, some of which are significantly different from those obtained in earlier research.

First, we find that preventive checks were stronger than usually supposed and, although weaker in later decades, lasted through the early nineteenth century. We estimate an elasticity for births with respect to real earnings of 0.90 during 1570 to 1760 and 0.54 during 1761 to 1850.

Second, we find that positive checks, while weaker than preventive checks as expected, were also stronger than previously estimated prior to the mid-eighteenth century, after which they were no longer statistically significant. We estimate an elasticity for deaths with respect to real earnings of -0.42 during 1570 to 1760.

Third, exogenous shocks to both fertility and mortality were a prominent feature of the period, notably in the late-eighteenth century when they account for a dramatic rise in population growth. We estimate that shocks associated with changes in nuptiality and epidemic disease raised the crude birth rate and lowered the crude death rate by about 6 per 1000 during the late eighteenth- and early-nineteenth century. An important implication is that the real earnings growth enjoyed by ‘post-Malthusian England’ was quite fragile.

Fourth, we estimate that population growth greater than 0.4 per cent per year could not be absorbed with constant real wages during 1570 to 1760, but in the industrializing economy with stronger technological progress this rose to 1.5 per cent per year in 1761 to 1850. This implies that the late-

¹ Wrigley, “Coping with Rapid Population Growth”, p. 31.

² Lee and Anderson, “Malthus in State Space.”

³ Humphries and Weisdorf, “Unreal Wages”; Feinstein, “Pessimism Perpetuated.”

eighteenth and early-nineteenth century surge in population growth, which peaked at 1.5 per cent per year at the end of the Napoleonic Wars, was way above the 0.4 per cent per year speed limit of the pre-industrial-revolution economy.

Fifth, when we simulate our model for 1761 to 1850 with the speed limit kept at the 0.4 per cent per year level of the pre-1760 period, counterfactual wages are completely undermined by rapid population growth and are projected to plummet to 12 per cent of the actual 1780 level by 1840. Even with post-1760 demographic shocks eliminated, real wages would have fallen to 57 per cent of the 1780 level. These simulations are, of course, not realistic because collapsing real wages would no doubt have triggered drastic demographic reactions: what they do illustrate is the magnitude of the demographic threat that the industrial revolution had to contend with.

Sixth, when we simulate our model for 1761 to 1850 (which has the higher speed limit of 1.5 per cent per year) with the late-eighteenth century demographic shocks eliminated, we find that it predicts real wage growth of 1.15 per cent per year between 1780 and 1840 compared with the actual outcome of 0.15 per cent per year.

These estimates strongly support Wrigley's claim that the modest real wage growth observed during the Industrial Revolution was a remarkable achievement in the face of considerable demographic pressure. In the race between population growth and technological progress the latter won narrowly. Put differently, they say that demography, rather than labour-saving technological progress or Marxist exploitation, seriously undermined the scope for the Industrial Revolution to raise real wages.

It must be emphasized that our model does not take account of possible feedbacks from population growth to technological progress. In theory at least, a larger population with greater spatial competition, as Klaus Desmet, Avner Greif and Stephen Parente propose, or cheaper labour, as Robert Allen would predict, could have had (opposite) implications for the rate of innovation.⁴ Perhaps our simulations are better regarded as reference scenarios rather than confident predictions of the counterfactual outcome in the absence of demographic shocks.

LITERATURE REVIEW

The classic account of population growth in England before the Industrial Revolution is that of Wrigley and Roger Schofield.⁵ They painted a picture of a Malthusian model with weak homeostasis, i.e., with large shocks and a slow return to an equilibrium real wage which remained constant until 1800, in which population growth could be sustained at about 0.5 per cent per year. They argued that there was clear evidence of the operation of a preventive check which dominated the system overall, but that the positive check had disappeared by the 17th century. Their analysis used a real wage series based on the work of Henry Phelps-Brown and Sheila Hopkins with missing observations interpolated.⁶

Wrigley and Schofield relied on informal inference using graphs, but since then several papers whose results are summarized in Table 1 have employed time-series econometric techniques to investigate their dataset. Most authors have preferred to use alternative indices of real day wages constructed by Allen and Gregory Clark.⁷ In the end, however, it makes relatively little difference which of these real wage series are used. These studies have at best found evidence of very weak positive checks but somewhat stronger evidence in favour of moderate preventive checks. On the other hand, analysis

⁴ Desmet and Parente, "Evolution of Markets"; Allen, *British Industrial Revolution*.

⁵ Wrigley and Schofield, *Population History*.

⁶ Phelps-Brown and Hopkins, "Seven Centuries."

⁷ Allen, "Great Divergence; Clark, "Macroeconomic Aggregates."

of parish-level data by Morgan Kelly and Cormac O'Grada, also summarized in Table 1, finds evidence of quite strong preventive and positive checks, especially up to the mid-eighteenth century.⁸

It is now possible to revisit these issues using the series of real annual earnings constructed by Humphries and Weisdorf.⁹ They find that the number of days worked per year was rising from the late 16th to the early 19th century such that real annual earnings were growing even though day wages were stagnant. This suggests that the economy was already 'post-Malthusian' and could also make a difference to estimates of the responses of fertility and mortality to changes in workers' real incomes.

The structural models estimated by Lee and Anderson and by Nicholas Crafts and Terence Mills provide estimates of the maximum rate of population growth compatible with constant real wages.¹⁰ Their estimates are that this rate was 0.47 per cent per year for 1600 to 1795 and 0.42 for 1541 to 1799, respectively, which are similar to the inference made by Wrigley and Schofield. Both studies find that subsequently this maximum sustainable population growth rate rose markedly to 2.01 and 2.32 per cent per year, respectively.

A notable feature of the estimated fertility and mortality equations estimated by both Lee and Anderson and by Crafts and Mills is that the intercepts vary considerably over time: in other words, the economy was subject to significant demographic shocks. In both papers, substantial shocks which raised fertility and lowered mortality are reported for the late eighteenth/early-nineteenth century period.¹¹ These findings resonate with work in historical demography which highlights substantial changes in marriage behaviour and in mortality from epidemic disease at this time. Jack Goldstone describes the period as one of 'demographic revolution' as age at marriage fell appreciably, while Romola Davenport stresses the much-reduced mortality from typhus and smallpox, the latter being largely eliminated by vaccination.¹²

The rate of growth of real wages during the Industrial Revolution has long been, and continues to be, controversial. Recent estimates by Clark and by Feinstein produce quite different estimates of the increase in real wages between 1780 and 1840 of 46.5 and 12 per cent, respectively.¹³ It is fair to say that Feinstein's relatively pessimistic real wage index has been widely accepted.¹⁴ It has also been incorporated into the influential paper by Allen, which argues that labor-augmenting technological change acted to stifle real wage growth during the classic Industrial Revolution period.¹⁵

Those who adopt the Feinstein real wage index as the best available, including Feinstein himself, generally give the impression that real wage growth during the Industrial Revolution was a very disappointing outcome. It is frequently noted that rapid population growth was not conducive to strong wage growth and this was quantified by Allen using a calibrated growth model. He found that

⁸ Kelly and O'Grada, "Preventive Check" and "Living Standards".

⁹ Humphries and Weisdorf, "Unreal Wages."

¹⁰ Crafts and Mills, "From Malthus to Solow"; Lee and Anderson, "Malthus in State Space".

¹¹ In fact, the behaviour of the intercepts is very similar in each case; compare Figure 7 of Lee and Anderson, "Malthus in State Space", and Figure 14 of Crafts and Mills, "From Malthus to Solow."

¹² Goldstone, "Demographic Revolution"; Davenport, "First Stages".

¹³ Clark, "Macroeconomic Aggregates"; Feinstein, "Pessimism Perpetuated." The difference between the two estimates comes from the price indices used to deflate money wages; a review by Allen, "Pessimism Preserved", concluded that Feinstein's estimates are to be preferred.

¹⁴ See, for example, the textbook accounts by Voth, "Living Standards", and by Pamuk and van Zanden, "Standards of Living". Feinstein's estimate is also the preferred series in the compilation by Thomas and Dimsdale of macroeconomic data for the Bank of England, "Millennium of UK Data" (Table A48, column B).

¹⁵ Allen, "Engels' Pause."

if population had not increased at all real wages would have grown *pari-passu* with real GDP per worker and would have been 46 per cent above the 1780 level in 1840.¹⁶

However, the counterfactual of what would have happened to real wages faced with a large demographic shock in the absence of the Industrial Revolution, the issue raised by Wrigley, is rarely confronted. Had Feinstein explicitly addressed this point, it seems unlikely that he would have written that it was almost a century before the majority of the working class obtained any economic benefits from the Industrial Revolution.¹⁷

One person who did contemplate this counterfactual is Hans-Joachim Voth; he conjectured that real income per head might have fallen by 8 to 18 per cent, but did not provide an explicit model to support this hypothesis.¹⁸ A way forward, which has not so far been exploited, is to investigate the implications of the population surge for real wages using the structural models of pre-industrial economic-demographic interactions developed to examine the homeostatic equilibrium proposed by Wrigley and Schofield.

DATA

In the light of the discussion above we address three topics with new quantitative analysis. First, we consider the issue of the stationarity of real wages prior to the Industrial Revolution using estimates of annual earnings. Second, we re-examine Malthusian preventive and positive checks in terms of responses to annual earnings rather than day wages. Third, we turn to the standard of living debate and investigate counterfactual real wages during the Industrial Revolution using a structural model that highlights the implications of demographic shocks and limits to growth.

The data that we require are obtained as follows. Annual demographic estimates for England (crude birth rate, crude death rate and population size) are available for 1570 to 1850 in Wrigley and Schofield.¹⁹ In common with the earlier papers listed in Table 1, these are the basis for our analysis. We break new ground by relating these to estimates of real annual earnings which are better suited to the analysis of demographic-economic interactions than the day wage series used by previous authors since days worked per year varied considerably over time. For 1570 to 1770 we use the series constructed by Humphries and Weisdorf.²⁰ We splice their earnings series to that of Feinstein at 1770 and use his real annual earnings estimates for the years to 1850. Where appropriate we compare results obtained with this series for real earnings with those that are generated by analysis of the day wages series of Allen and Clark.²¹

TREND GROWTH OF REAL WAGES

Figure 1 shows the logarithms of three real wage series for 1570 to 1850: (a) the Humphries and Weisdorf series spliced with Feinstein's series at 1770 (denoted HWF); (b) Allen's real day wages; and (c) Clark's real day wages. For HWF a segmented trend has been superimposed with a break point at 1660: pre-break the trend is flat while post-1660 trend growth is estimated at 0.45 per cent per year. While this segmented trend is best interpreted as an exploratory device, it is worth noting that pre-

¹⁶ Allen, "Engels' Pause."

¹⁷ Feinstein, "Pessimism Perpetuated", p. 652.

¹⁸ Voth, "Living Standards", p. 294.

¹⁹ Wrigley and Schofield, *Population History*, pp. 531-535.

²⁰ Humphries and Weisdorf, "Unreal Wages", Table A4.

²¹ Allen, "Great Divergence"; Clark, "Macroeconomic Aggregates."

break a standard unit root test conclusively rejects this hypothesis in favour of stationarity around a constant mean, with a test statistic of -5.26 compared to the 1% critical value of -3.50 , while post-break a unit root test also conclusively rejects a unit root, this time in favour of stationarity around a linear trend, with a test statistic of -7.14 compared to a 1% critical value of -4.01 .

The HWF real earnings series indicates that England can be described as a post-Malthusian economy prior to the Industrial Revolution for the period from the mid-seventeenth century. Trend growth of real earnings accompanied a steady rise in the population from 5.3 million in 1660 to 7.2 million in 1780. As Humphries and Weisdorf point out, their earnings data contrast with the picture of a Malthusian plateau of stagnant incomes derived from previous series based on day wage rates, but echo the national income estimates of Stephen Broadberry et al.²² An econometric analysis of that series by Crafts and Mills found trend growth of real GDP per person from 1663.²³

By contrast, Allen's day wage series shows no trend for the entire period 1570 to 1850: the 1570 value is 7.45, the 1850 value is 8.17 and the sample mean is 6.82. More formally, a unit root test conclusively rejects this hypothesis in favour of stationarity around a constant mean, with a test statistic of -5.44 as compared to a 1% critical value of -3.45 . Clark's day wage series is essentially trendless until 1800, with the 1570 value of the index being 52.8 and the 1799 value being just 51.2, only possessing a linear trend during the 19th century when trend growth up to 1850 was 1.26 per cent per annum.

This result for the Clark series is entirely consistent with the author's view that the Industrial Revolution marked the end of the Malthusian era and delivered a sustained and significant growth in real wages. The possibly surprising result that the Allen series is stationary through to 1850 is explained by the series being for London builders, who are well known not to have shared in the increasing prosperity of the North during the Industrial Revolution.²⁴

Given that the HWF real annual earnings series exhibits different properties from the real day wage series used hitherto to analyse economic-demographic interactions in England prior to the Industrial Revolution, it is important to revisit the estimation of structural models for that period.

A STRUCTURAL MODEL

As noted above, a similar structural model of economic-demographic interactions using Wrigley and Schofield's population estimates for early-modern England was previously estimated by both Lee and Anderson and Crafts and Mills.²⁵ We return to this project for three reasons. First, new and improved estimates of real earnings have become available from the work of Humphries and Weisdorf.²⁶ Second, we believe that the estimation methods used by these earlier papers can be improved upon. Third, and most importantly, we have in mind a different purpose, namely to compare the ability of the economy to withstand demographic pressure before and during the Industrial Revolution in order to develop a different perspective on real wage growth in the latter period from that which is generally adopted in the literature.

²² Broadberry et al., *British Economic Growth*.

²³ Crafts and Mills, "Six Centuries".

²⁴ Slow growth of average real wages during the Industrial Revolution is an outcome resulting from stagnation in the South and significant growth in the North of England. On the real wages of London building craftsmen, see Schwarz, "Standard of Living."

²⁵ Lee and Anderson, "Malthus in State Space"; Crafts and Mills, "From Malthus to Solow."

²⁶ Humphries and Weisdorf, "Unreal Wages."

Lee and Anderson specify a structural model linking real wages, fertility and mortality via population.²⁷ Abstracting from net immigration, there is an identity linking the change in population in year t to crude birth and death rates. If p_t is the logarithm of population and b_t and d_t are the crude birth and death rates, then

$$p_t = p_{t-1} + b_t - d_t \quad (1)$$

i.e., since $p_t - p_{t-1}$ is population growth, this must equal the crude birth rate minus the crude death rate. The structural model is centred on the real wage equation

$$x_t = \alpha_t - \beta p_t + s_t \quad (2)$$

where x_t is the logarithm of the real wage and β is the elasticity of wages with respect to labour. The disturbance term s_t represents the impact of short-term shocks on wages such as weather and harvest conditions and is modelled as a random walk

$$s_t = s_{t-1} + \varepsilon_t \quad (3)$$

where ε_t is zero mean white noise. The time-varying intercept in (2), α_t , is the demand for labour, and is assumed to follow a random walk whose *change*, c_t , is also assumed to follow a random walk:

$$\alpha_t = \alpha_{t-1} + c_t \quad (4)$$

$$c_t = c_{t-1} + v_t \quad (5)$$

where v_t is white noise. Why this particular specification? Over the period 1570 to 1850 the logarithm of population, p_t , is an $I(2)$ process, as argued by Roy Bailey and Marcus Chambers and confirmed by unit root tests, while the logarithm of HWF real wages, x_t , while shown above to be segmented trend stationary, may nevertheless be approximated quite well by an $I(1)$ process.²⁸ Thus equation (2) can only be ‘balanced’, and hence amenable to statistical analysis, if α_t is also $I(2)$ and cointegrated with p_t . The specification of equations (4) and (5) ensures that α_t is indeed $I(2)$. Since α_t is unobservable, we can only assume that it is cointegrated with p_t , so that the linear combination $\alpha_t - \beta p_t$ is $I(1)$. With the random walk specification (3) for s_t , all variables in (2) are then $I(1)$ and the equation is balanced. In terms of its evolution, α_t is expected to increase over time: as industrialisation becomes important this would accelerate its rate of increase through an upward drift or shift in c_t , which can be characterised as the rate of technological progress.²⁹

Figure 2 shows the crude death and birth rates. The death rate is modelled as a function of lagged real wages and lagged death rates plus a time varying intercept (two lags on each variable are chosen as this setting was found to capture adequately the dynamics of the relationship):

$$d_t = m_t + \sum_{i=0}^2 \delta_i \nabla x_{t-i} + \sum_{i=1}^2 \lambda_i d_{t-i} + u_t \quad (6)$$

The positive check implies that $\sum \delta_i < 0$, while the presence of the lagged death rates allows for shocks to mortality caused by environmental fluctuations such as weather and disease prevalence. If we define the positive check as the dynamic multiplier $po = \sum \delta_i / (1 - \sum \lambda_i)$ then the (time varying)

²⁷ Lee and Anderson, “Malthus in State Space”.

²⁸ Bailey and Chambers, “Long Term Demographic Interactions”.

²⁹ Strictly speaking, it is anything that shifts the demand curve for labour outwards. However, we follow general usage in using the term technological progress to describe c .

elasticity is defined as $\epsilon_{d,t} = po/d_t$, although the (constant) mean elasticity $\bar{\epsilon}_d = po/\bar{d}$ is typically reported. The intercept m_t reflects long-term exogenous influences on mortality, such as international exchange of diseases and cultural practices, and is assumed to follow a random walk

$$m_t = m_{t-1} + \varphi_t \quad (7)$$

where φ_t is assumed to be white noise, as is the error in (6), u_t .

The crude birth rate is modelled in a similar fashion as

$$b_t = n_t + \sum_{i=0}^2 \mu_i \nabla x_{t-i} + \kappa_i \sum_{i=1}^2 b_{t-i} + r_t \quad (8)$$

$$n_t = n_{t-1} + \xi_t \quad (9)$$

The preventive check implies $\sum \mu_i > 0$ and r_t and ξ_t are white noise. The inclusion of the lagged birth rate captures transient influences on fertility such as variations in weather, health and morbidity, while the random walk structure of n_t models longer term effects such as occupational structure and changes in the rules and customs that regulate marriage. Defining the preventive check as $pr = \sum \mu_i / (1 - \sum \kappa_i)$ leads to the elasticities $\epsilon_{b,t} = pr/b_t$ and $\bar{\epsilon}_b = pr/\bar{b}$.

The model given by equations (2)–(9) may be cast in state space form and estimated by maximum likelihood using Kalman filter technology. We estimate the model for 1570-1760 and for 1761 to 1850. These periods capture the economy before and during the Industrial revolution, respectively. It is conventional to see the onset of the Industrial Revolution in the third quarter of the eighteenth century marked by famous inventions such as James Watt's steam engine in 1769 and, more generally, with a notable increase in patenting from the late 1750s.³⁰ We take 1760 as our dividing line following Moller and Sharp, but also report estimates using 1750 or 1770 as alternatives.³¹ Our main focus is on results obtained using the HWF real earnings series but we also report estimates based on the Allen and Clark real wages series in Tables 2 and 3.

When estimating equation (2) across these sub-periods, it was found that the estimate of the variance of the innovation to the rate of technological progress c_t was always extremely small. If the variance of v_t is indeed zero, then α_t collapses to a deterministic linear trend, since now $c_t = c_{t-1} = c$, so that technological progress grows at a constant rate and (4) becomes

$$\alpha_t = \alpha_{t-1} + c = \alpha_0 + ct$$

On noting that $\nabla s_t = \varepsilon_t$, (2) can then be written as

$$x_t = \alpha_0 + ct - \beta p_t + \varepsilon_t / \nabla$$

i.e.,

$$\nabla x_t = c - \beta \nabla p_t + \varepsilon_t \quad (10)$$

which is simply a regression in the first differences ∇x_t and ∇p_t .

³⁰ Nuvolari and Tartari, "Bennet Woodcroft."

³¹ Moller and Sharp, "Malthus in Cointegration Space." Wrigley and Schofield's demographic data start in 1541 but we omit the early years, which saw extraordinary mortality with period life expectancy at birth as low as 22 for the 1556 quinquennium.

On assuming that the innovation to c_t is indeed zero, so that technological progress grows at the constant rate c , the sub-period estimates of β , c and the 'rate of absorption' c/β from equation (2) are shown in Appendix Tables A1 and A2. Although the elasticity β is never precisely estimated whatever the sub-period, the most precise estimates are found for the sub-periods starting at 1570, where they take values of approximately 3. The rate of technological progress c is found to be significant for these sub-periods, where it is estimated to be around 1.3 per cent per year. It is also significant for those sub-periods ending in 1850, with these giving estimates of between 2 and 4 per cent per year. The rate of absorption c/β is thus estimated to be around 0.4 per cent per year for the sub-periods beginning in 1570 and around 1.4 for those sub-periods ending in 1850.

Table 2 reports estimates of equation (2) for 1570 to 1760 and 1761 to 1850 for HWF and, for comparison, for the Allen and Clark real day wage series. For 1570–1760, the series give rather similar estimates of equation (2), with β estimated to be between 3 and 3.5, c around 1.2 and c/β around 0.4. These estimates for the maximum rate of population growth compatible with constant real wages in the period 1570 to 1760 are not much different from the earlier estimates of Crafts and Mills, Lee and Anderson, and Wrigley and Schofield. They suggest that the use of real annual earnings rather than real day wages does not make much difference in this period. In common with the first two of those earlier studies, using HWF real earnings the rate of absorption is estimated to have increased subsequently as the Industrial Revolution took hold.³²

The sub-period estimates of the positive and preventive checks and associated elasticities from the death and birth rate equations (6) and (8) are shown in Appendix Tables A3 and A4. Both checks and elasticities are always significant for the sub-periods starting at 1570. Death rate elasticities are estimated to be around 0.4, while birth rate elasticities are estimated to be around 0.8. For sub-periods ending in 1850, death rate elasticities are typically around 0.2, albeit insignificant, while birth rate elasticities remain significant at around 0.5.

Table 3 reports estimates of the death and birth rate equations (6) and (8) using HWF real earnings for 1570 to 1760 and 1761 to 1850 with estimates based on the Allen and Clark day wage series for comparison. For 1570–1760, estimates of the positive check and associated elasticity are significant for all three series, the elasticities being around -0.4 for HWF and Allen and -0.6 for Clark. For 1761–1850 both sets of estimates are substantially reduced in size and significance, with elasticities only around -0.16 . The preventive check and associated elasticity are also significant for all three series before 1760, the elasticities being estimated to be 0.8, 1.0 and 1.44 for HWF, Allen and Clark, respectively. After 1760 these estimates are reduced substantially, although remaining significant, the elasticities being between 0.5 and 0.6.

It should be noted that the estimates of the checks and elasticities for 1570-1760 are rather higher than those of the earlier structural model-based studies reported in Table 1. It should be emphasised that this is not a consequence of using a different earnings series to those used in previous studies, as the estimates in Tables 2 and 3 are reasonably close for all three series. Rather, it is because we have set up equations (6) and (8) to contain lagged death and birth rates as regressors to deal with dynamics, rather than using an autoregressive error specification. Consequently, the positive and preventive checks are to be interpreted as *long-run* multipliers, with the elasticities thus being long-

³² For 1761–1850, c and β become small and insignificant for the Allen and Clark series, so using the superior series constructed by Feinstein for the post-1770 years does make a difference in this period.

run as well, as opposed to earlier elasticity estimates obtained using structural models in which the dynamics are limited to lags of real wages, as is implied by imposing an autoregressive error specification, and which are therefore more short-run in nature. The comparison is analogous to the argument made by David Hendry and Grayham Mizon, who show that an autoregressive error specification arises as a consequence of imposing a set of ‘common factor’ restrictions on a general dynamic model containing lagged dependent variables. Since these restrictions may not be empirically acceptable, they will typically lead to biased estimates of multipliers and elasticities if they are erroneously imposed.³³ A likelihood ratio test of jointly imposing single common autoregressive factors on equations (6) and (8) does indeed reject their imposition at less than the 5% significance level, so providing empirical support for the above argument.³⁴

Our estimates support the argument of Kelly and O’Grada that studies using aggregate data have underestimated the strength and longevity of Malthusian checks in England.³⁵ With regard to the positive check, our estimate is close to the average of their estimated elasticities for 1539 to 1750. However, we find evidence of even stronger preventive checks than they did in early-modern England and a short-run impact of (somewhat weaker) changes in earnings on births on into the nineteenth century. This is not entirely unexpected given the emphasis of an older literature on the impact of the business cycle on the timing of marriages and births even in the late nineteenth century.³⁶

Figure 3 shows the time varying intercepts from equations (6) and (8), m_t and n_t , estimated using the HWF series. The mortality shocks show a generally increasing evolution up to 1760 but then follow a declining path. The fertility shocks undergo a decline during the middle of the 17th century and a noticeable increase in the early part of the 19th century. These shocks, which can be thought of as shifts in the fertility and mortality schedules, are large, especially in the second sub-period. This is perhaps to be expected as similar findings are reported by both Lee and Anderson and Crafts and Mills.³⁷

The fertility intercept rose by almost 6 per 1000 between 1770 and 1815. There are several possible reasons. These include a decline in stillbirths and an increase in illegitimate fertility, but the main contributor was a reduction in age at first marriage and in the proportion who never married.³⁸ The reasons for changes in nuptiality are disputed but they include increases in proletarianization of the labour force, better protection from the Poor Law against unemployment, including through payment of children’s allowances, and changes in cultural norms.³⁹

The mortality intercept fell by about 6 per 1000 between 1780 and 1830. This was an exogenous shock due to improved ways of combating epidemic disease rather than a consequence of improved nutrition. By the mid-nineteenth century mortality from lethal diseases such as malaria, typhus and,

³³Hendry and Mizon, “Serial Correlation as a Convenient Simplification.”

³⁴The test is constructed along the lines of that proposed by Sargan, “Some Tests”. The test statistic is 10.70, which is distributed as chi square with 4 degrees of freedom.

³⁵ Kelly and O’Grada. “Preventive Check” and “Living Standards.”

³⁶ Silver, “Births, Marriages and Income Fluctuations”; Yule, “Changes in the Marriage- and Birth-Rates.”

³⁷ Lee and Anderson, “Malthus in State Space”; Crafts and Mills, “From Malthus to Solow.”

³⁸ Wrigley, “British Population”, pp. 71-76.

³⁹ Goldstone, “Demographic Revolution”; Boyer, “Malthus Was Right”; Griffin, “A Conundrum Resolved?”

especially, smallpox, had largely been overcome through policies such as quarantine and, for the last-named, vaccination.⁴⁰

Taken together, these demographic shocks imply an impact of over 1 percentage point per year on the population growth rate before any response from Malthusian checks, which, in any case, had become considerably weaker by the late eighteenth century. Using the formula $(Pr - Po)\beta$ for the rate of convergence to equilibrium given by Lee, during 1761 to 1850 the half-life of an exogenous shock is 26 years.⁴¹ This implies that the growth of annual real earnings enjoyed by early modern England was quite fragile. While the combination of an increasing population and real earnings growth can be labelled as 'post-Malthusian', this should not be regarded as modern economic growth in the sense of an economy in which technological progress could be relied upon to deliver rising living standards and the threat of growth reversals had been completely overcome.⁴²

SIMULATIONS

We now proceed to simulate the structural model to investigate the implications of demographic shocks for the growth of real earnings. First, we consider what would have happened if the exogenous changes in demography of the late eighteenth century had occurred earlier while the economy still had only the rate of absorption, c/β , of the early-modern economy. Second, we look at what would have happened during the Industrial Revolution in the absence of the changes in nuptiality and mortality from epidemic disease as growth potential increased.

For the first pair of these simulations we impose the 1570–1760 estimates reported in Table 2 for HWF real earnings, which we take to represent the economy prior to the Industrial Revolution with $\beta = 3.176$ and $c = 1.28$ per cent per annum (rather than their 1761–1850 estimates of 1.107 and 1.65), and use the estimated death and birth rate equations (6) and (8) for 1761 to 1850 reported in Table 3, i.e., $Po = -4.09$ and $Pr = 20.41$. In other words, we remove the positive effect of the Industrial Revolution on the absorption rate and examine the impact of demography on real earnings in the counterfactual situation. Figure 4 shows the logarithm of the actual HWF series for the years 1760 to 1850 along with the two simulated real earnings series and Table 4 reports the estimates in index-number form for selected years.

The first simulation fixes the intercepts of the death and birth rate equations at their 1760 values of $m = 26.574$ and $n = 35.255$, i.e., it removes the demographic shocks of the late eighteenth and early nineteenth centuries. For the second simulation the estimated time-varying intercepts m_t and n_t are used instead so that the economy is faced with the actual demographic shocks. Both simulations show a large fall in real earnings during the Industrial Revolution period rather than the slow increase actually observed and in the second simulation there is a precipitous decline in real earnings. As is reported in Table 4, in the first case counterfactual real earnings are 57 per cent of the actual 1780 level in 1840 and in the second case only 12 per cent. These simulations highlight the vulnerability of the 'post-Malthusian' economy to demographic pressure but are not realistic counterfactuals. They

⁴⁰ Davenport, "First Stages."

⁴¹ Lee, "Accidental and Systematic Change". This calculation takes the point estimate of the positive check disregarding its statistical insignificance.

⁴² Pace Humphries and Weisdorf, "Unreal Wages", p. 2867, who suggest that their earnings series implies that the take-off into modern economic growth should be pushed back from the nineteenth century to the late sixteenth century.

go well outside the range of observations of real earnings on which the model was estimated and no doubt vigorous responses would have been provoked – in the worst-case scenario through a dramatic positive check, as in the nightmare envisaged by Thomas Ashton.⁴³

Figure 5 again shows the logarithm of the actual HWF real earnings series for 1760 to 1850 and four further simulated real earnings series. These simulations now use the estimates for the rate of absorption from the 1761-1850 sub-period, i.e., $\beta = 1.107$ and $c = 1.65$ percent per annum. Table 5 reports selected values of actual HWF and the four simulations in index number form. The first simulation reports the predicted values of HWF earnings using the estimates of equations (6) and (8) for the 1761 to 1850 sub-periods. These track actual HWF quite well except for 1800, when real earnings were adversely affected by a major spike in grain prices. The predicted value for 1841 of 113.6 compares well with an actual value of 111.6.

The remaining three simulations eliminate one or both of the demographic shocks by fixing the intercepts of the death and birth rates at their 1760 levels. Removing the post-1760 mortality shock but retaining the fertility shock delivers a modest increase in counterfactual compared with actual real earnings of about 21 per cent in 1840. Removing the fertility shock while retaining the mortality shock has an appreciably larger positive impact on real earnings: the counterfactual value in 1840 is about 61 per cent above actual real earnings. With both the fertility and mortality shocks suppressed, real earnings in 1840 are predicted to have been 93 per cent higher: in that case growth of real earnings would have averaged 1.15 per cent per year, i.e., about 1 percentage point higher than the actual growth rate.

These simulations suggest that demographic shocks which raised population growth to a new high during the Industrial Revolution undermined its potential to raise real wages. They strongly support Wrigley's assessment that the modest real wage growth of the early nineteenth century was 'a truly remarkable achievement.'⁴⁴ The threat to real earnings from earlier marriage and lower mortality from epidemic disease was substantial. The economic benefits of the Industrial Revolution for the working class were realised straightaway through the avoidance of big reductions in living standards. Technology narrowly won the race with population.

That said, this victory had a strong regional dimension which illustrates this conclusion very well. While real wages in the industrializing north rose in the face of rapid population growth, real wages in the agricultural south fell as population increased more modestly but at rates above the pre-1760 'speed limit'. For example, between 1770 and 1840 population in Lancashire rose by about 370 per cent and real wages increased over the same period by 23 per cent while in Dorset population rose by only 80 per cent but real wages fell by 25 per cent.⁴⁵

⁴³ 'The central problem of the age was how to feed and clothe and employ generations of children outnumbering by far those of any earlier time. Ireland was faced by the same problem. Failing to solve it, she lost in the forties about a fifth of her people by emigration or starvation and disease. If England had remained a nation of cultivators and craftsmen, she could hardly have escaped the same fate.' Ashton, *Industrial Revolution*, p. 129.

⁴⁴ Wrigley, "Coping with Rapid Population Growth", p.31.

⁴⁵ Population estimates are taken from Mitchell, *British Historical Statistics* and Wrigley, "English County Populations"; money wage data are for agricultural labourers as reported by Hunt, "Industrialization." There are no county-level cost-of-living indices so the estimates for real wages in the text have used Feinstein's national index as a deflator. They are admittedly rough and ready but the basic point is undoubtedly correct.

DISCUSSION

The analysis of the previous section applies most readily if population growth and technological progress are independent of one another. In that case, the race between population and technology is uncomplicated except that the winner was perhaps not obvious *ex ante* – the demographic shocks might have come along a bit sooner and/or the acceleration in technological progress might have arrived somewhat later, in which case the Industrial Revolution would not have prevented a serious fall in real wages.

Joel Mokyr's interpretation is consistent with the technology side of these assumptions and has empirical support as at least part of the explanation for a move to faster and sustained technological progress.⁴⁶ For Mokyr, the Enlightenment is the key underpinning of the Industrial Revolution. This entailed a new method of invention based on systematic empiricism and experimentation that established what worked and which accumulated and made accessible useful knowledge which could promote further technological advance. Central to this accumulation and diffusion of an evidence-based method of invention were 'knowledge access institutions' (KAIs), organizations whose objectives were to produce and disseminate scientific and technological knowledge, of which the first was the Royal Society (1660), and which included the Lunar Society (1765), the Manchester Literary and Philosophical Society (1781), the British Association for the Advancement of Science (1831) and various Mechanics Institutes from 1823. The total number of KAIs was only 3 in 1761 but had risen to 1014 by 1851. KAIs had a strong causal impact on patenting in their localities.⁴⁷

If, however, technological progress is taken to be endogenous and influenced by population growth, as several recent papers suggest, then our model would need to be re-specified and the findings from the simulations would be modified. In the formulation of the calibrated model of Nico Voigtländer and Voth, fertility restraint is central to the achievement of a high enough income level to promote a substantial manufacturing sector, whose capital formation and externalities deliver an acceleration of TFP.⁴⁸ If the demographic shocks, with their severe implications for real wages, had come earlier to thwart this process, the consequences might have been indefinite postponement of the Industrial Revolution.

Some authors see population growth as having direct effects on the rate of technological progress, but their size and direction are very difficult to estimate and remain contentious. Endogenous or semi-endogenous growth models with scale effects will suggest that population growth has a positive impact on productivity growth, but this would probably not be enough to make a big difference to the results of the simulation which considered demographic shocks in the context of the pre-1760 rate of absorption.⁴⁹

Perhaps a more interesting way of arguing for positive effects of population growth is an approach that embodies an effect of market size on incentives to innovate. Desmet et al. suggested that this may be relevant for England, given that the demographic expansion entailed an intensification of

⁴⁶ Mokyr, *Enlightened Economy*

⁴⁷ Dowey, "Mind Over Matter".

⁴⁸ Voigtländer and Voth, 'Demographic factors.'

⁴⁹ Based on Jones, "Sources of U.S. Economic Growth", an impact on labour productivity growth of 0.05 times the increase in the population growth rate might be a plausible order of magnitude.

spatial competition, and they provide a calibrated model in support of this claim.⁵⁰ On the other hand, Allen believes in the favourable impact of a high-wage environment on the labor-saving innovations that were central to the Industrial Revolution. Demographic shocks could have removed this configuration and there is evidence that the most important patents were often for labor-saving machinery.⁵¹ What the net effect of these offsetting arguments might be on the outcome of a simulation of demographic shocks in the pre-1760 economy is unclear.

In a well-known paper which also incorporates a market-size effect, Daron Acemoglu argued that during the Industrial Revolution an abundance of unskilled labor resulting from rapid population growth stimulated directed technical change which delivered technologies, such as the self-acting mule, that were unskilled-labor augmenting and displaced skilled labor through mechanization.⁵² This might be reflected in the much-expanded use of child labor and a rise in the share of jobs that were unskilled from about 30 per cent in 1700-49 to 33 per cent in 1750 to 1799 and about 40 per cent in 1800 to 1849.⁵³ At present, however, this remains an untested hypothesis.

The interpretation of our simulation results that we offered in the previous section was that workers received immediate benefits from the Industrial Revolution because it pre-empted the falls in real earnings that demographic shocks threatened to deliver. If, however, rather than being exogenous, the shifts in fertility and mortality schedules were entirely a consequence of the Industrial Revolution, then a different inference might be made, namely, that real wages in the early nineteenth century would have been much the same with or without the Industrial Revolution.

It is unlikely that the mortality shock was promoted by improvements in nutrition associated with the growth of real earnings or, more specifically, can be attributed to the Industrial Revolution. Recent reviews of the evidence on food supplies per person have concluded that they probably decreased somewhat in the later eighteenth and early nineteenth centuries and recent surveys of the evidence on nutritional status, as reflected in declining heights, have come to similar conclusions.⁵⁴ Insofar as industrialization per se had an impact on mortality, it was a negative one through the adverse impact of urbanization.⁵⁵ As noted earlier, the decline in mortality from epidemic disease in the late eighteenth and early nineteenth century is better regarded as exogenous.

It is much less clear to what extent the fertility shock was a result of the Industrial Revolution. Changes in nuptiality were central to rising birth rates, but they were supplemented by increased illegitimacy and fewer stillbirths. It is marriage behaviour which has been most closely linked to the Industrial Revolution, notably by Goldstone, who argued that it changed as a result of better employment opportunities for the proletarianized labour force.⁵⁶ This argument has not, however, met with general approval. George Boyer found evidence that payment of children's allowances under the Old Poor Law accounted for much of the increase in fertility between 1780 and 1820, while Wrigley and

⁵⁰ Desmet et al., "Spatial Competition." This model has a tipping point which triggers 'take-off', rather than a linear relationship between population growth and productivity growth.

⁵¹ Allen, *British Industrial Revolution*; Nuvolari et al., "Patterns of Innovation."

⁵² Acemoglu, "Directed Technical Change."

⁵³ On child labor, see Humphries, "Childhood" and de Pleijt and Weisdorf, "Human Capital Formation" for estimates of the share of unskilled jobs

⁵⁴ Kelly and O'Grada, "Numerare Est Errare"; Meredith and Oxley, "Food and Fodder"; Cinnirella, "Optimists or Pessimists"; Komlos and Küchenhoff, "Diminution."

⁵⁵ Williamson, *Coping with City Growth*; Woods, *Demography*.

⁵⁶ Goldstone, "Demographic Revolution."

his co-authors stressed that the downward trend in age at marriage was very similar across diverse economic and geographical areas.⁵⁷ Emma Griffin has suggested that the evidence relating changes in nuptiality to economic factors is far from robust and argued that changing cultural and social norms played a significant role.⁵⁸ On balance, the evidence suggests that the fertility shock probably was not primarily a response to the Industrial Revolution but the jury is still out.

Overall, the literature provides reasons why our simulations might be misleading but no compelling evidence that this is the case. It is theoretically possible that real wages were protected from demographic shocks because a big increase in population growth would trigger the Industrial Revolution. It is also possible to construct an argument that the demographic implications of the Industrial Revolution largely precluded real wage growth for several decades. But there is no compelling evidence to believe either of these propositions.

CONCLUSIONS

We have estimated a structural model of demographic-economic interactions for England before and during the Industrial Revolution with a view to examining the implications of population growth for real earnings. We find that there was modest trend growth of real earnings after 1660 but that this was vulnerable to demographic shocks.

In the fifty years or so after 1780, exogenous shocks which lowered mortality and raised fertility raised the rate of population growth substantially. If these shocks had occurred a few decades previously, they would have had a seriously adverse effect on real wages. In the absence of these demographic shocks, real wage growth during the Industrial Revolution would have been considerably faster.

In this context, the modest growth in real wages during the early decades of the Industrial Revolution was indeed 'a remarkable achievement'. In the race between population growth and technological progress, technology won but, like the Battle of Waterloo, it was 'a damn close-run thing'.

⁵⁷ Boyer, "Malthus Was Right"; Wrigley et al., *English Population History*.

⁵⁸ Griffin, "A Conundrum Resolved?"

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Table 1. Positive and Preventive Checks: Previous Estimates

	Method	Real Wage Data	Period	Preventive Check Elasticity	Positive Check Elasticity
Lee & Anderson	State Space	WS	1541-1870	0.12	-0.08
Nicolini	Stationary VAR	Allen	1541-1640	0.03	-0.11
			1641-1740	0.11	0.05
			1741-1840	-0.07	0.03
Crafts & Mills	State Space	Clark	1541-1645	0.09	-0.03
			1646-1799	0.23	0.04
Moller & Sharp	Cointegrated VAR	Allen	1564-1760	0.32	-0.10
		Clark	1564-1760	0.21	-0.22
Kelly & O'Grada	Multi-Level	Clark	1539-1600	0.40	-0.68
	Regression		1601-1650	0.50	-0.28
			1651-1700	0.38	0.08
			1701-1750	0.51	-0.59
			1751-1800	0.23	0.11

Note: all studies use the aggregate demographic data constructed by Wrigley and Schofield except those by Kelly and O'Grada, which are based on 404 parishes in the Cambridge Group database. Bold type indicates significant at 10%. When using logarithms of real wages, elasticities are computed at the sample means of birth rates (for preventive checks) and death rates (for positive checks).

Sources: Lee and Anderson, "Malthus in State Space"; Crafts and Mills, "From Malthus to Solow", Kelly and O'Grada, "Preventive Check", Kelly and O'Grada, "Living Standards", Moller and Sharp, "Malthus in Cointegration Space", Nicolini, "Was Malthus Right?"

Table 2. Estimates of Real Wage Equation (2)**1570–1760**

x	$\hat{\beta}$	c (%)	c/β (%)
HWF	3.176*(1.864)	1.28*(0.74)	0.40 (0.33)
Allen	3.249*(1.848)	1.13*(0.73)	0.34 (0.29)
Clark	3.470*(0.767)	1.24*(0.39)	0.36*(0.14)

1761–1850

x	$\hat{\beta}$	c (%)	c/β (%)
HWF	1.107 (2.333)	1.65*(0.68)	1.49*(0.84)
Allen	0.161 (2.311)	0.11 (0.87)	0.67 (5.69)
Clark	0.393 (3.609)	0.53 (0.52)	1.36 (12.55)

Note: Standard errors shown in parentheses; * denotes significance at 10%.

Table 3. Estimates of death rate equation (6) and birth rate equation (8)**1570-1760**

x	Positive check, Po	Elasticity, $\bar{\epsilon}_d$	Preventive check, Pr	Elasticity, $\bar{\epsilon}_b$
HWF	-11.36*(3.80)	-0.42*(0.14)	25.72*(5.72)	0.80*(0.18)
Allen	-11.87*(3.96)	-0.43*(0.14)	31.91*(7.13)	1.00*(0.22)
Clark	-16.69*(7.24)	-0.61*(0.26)	46.09*(7.94)	1.44*(0.25)

1761-1850

x	Positive check, Po	Elasticity, $\bar{\epsilon}_d$	Preventive check, Pr	Elasticity, $\bar{\epsilon}_b$
HWF	-4.09 (3.13)	-0.16 (0.12)	20.41*(4.71)	0.54*(0.13)
Allen	-4.24*(2.03)	-0.17*(0.08)	19.01*(6.40)	0.50*(0.17)
Clark	-4.10 (2.96)	-0.16 (0.12)	22.55*(4.61)	0.60*(0.12)

Note: Standard errors shown in parentheses; * denotes significance at 10%.

Table 4 Actual and simulated HWF real earnings using 1570–1760 estimates of β and c .

	Actual	Simulated: Intercepts fixed	Simulated: Intercepts variable
1780	100.0	77.7	63.3
1800	81.6	70.0	41.7
1820	106.9	63.1	21.6
1840	111.5	56.9	11.7

Note: intercepts fixed at 1760 values.

Table 5 Actual and simulated HWF real earnings using 1761–1850 estimates of θ and c

	Actual	Simulated: Intercepts variable	Death rate Intercept fixed	Birth rate Intercept fixed	Simulated: Intercepts both fixed
1780	100.0	103.6	103.3	108.9	108.2
1800	81.6	113.5	114.8	134.9	136.2
1820	106.9	112.5	119.7	160.5	171.1
1840	111.5	113.5	135.2	179.9	214.8

Note: intercepts fixed at 1760 values

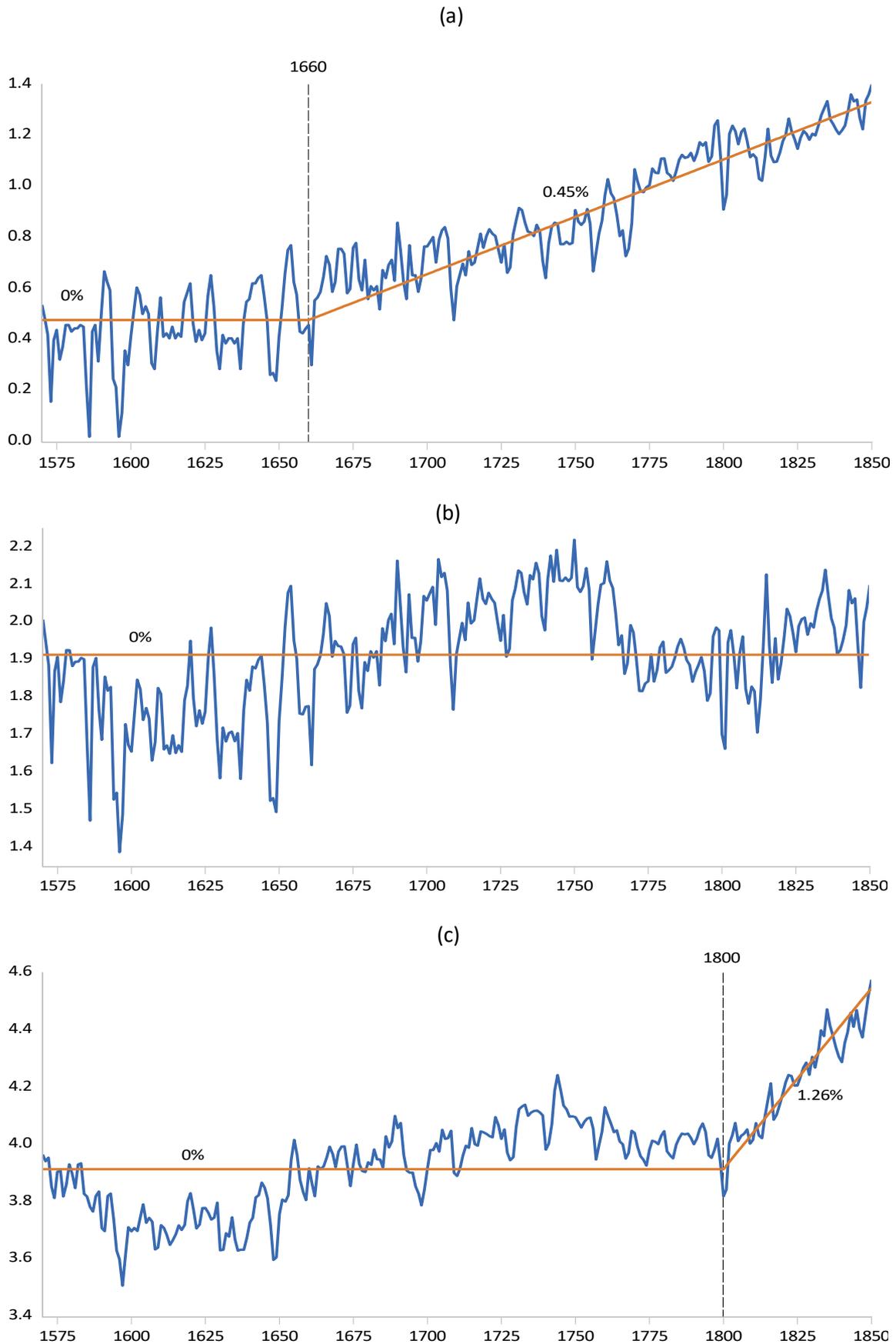


Figure 1 Logarithms of real earnings, 1570–1850: (a) HWF; (b) Allen; (c) Clark.

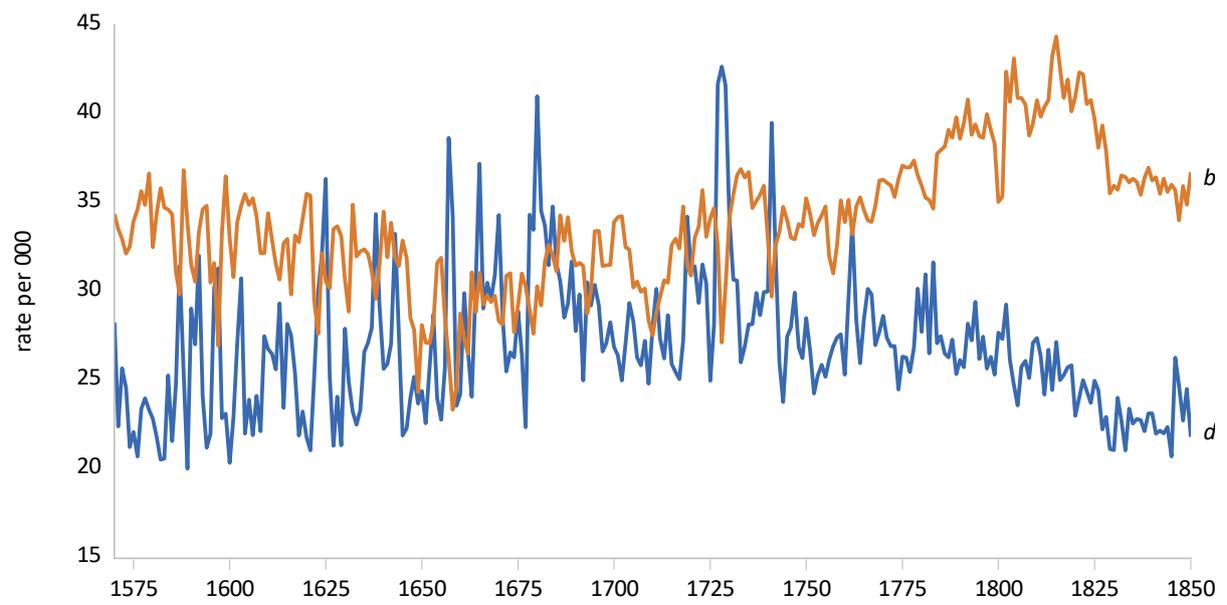


Figure 2 Crude death (*d*) and birth (*b*) rates; 1570–1850.

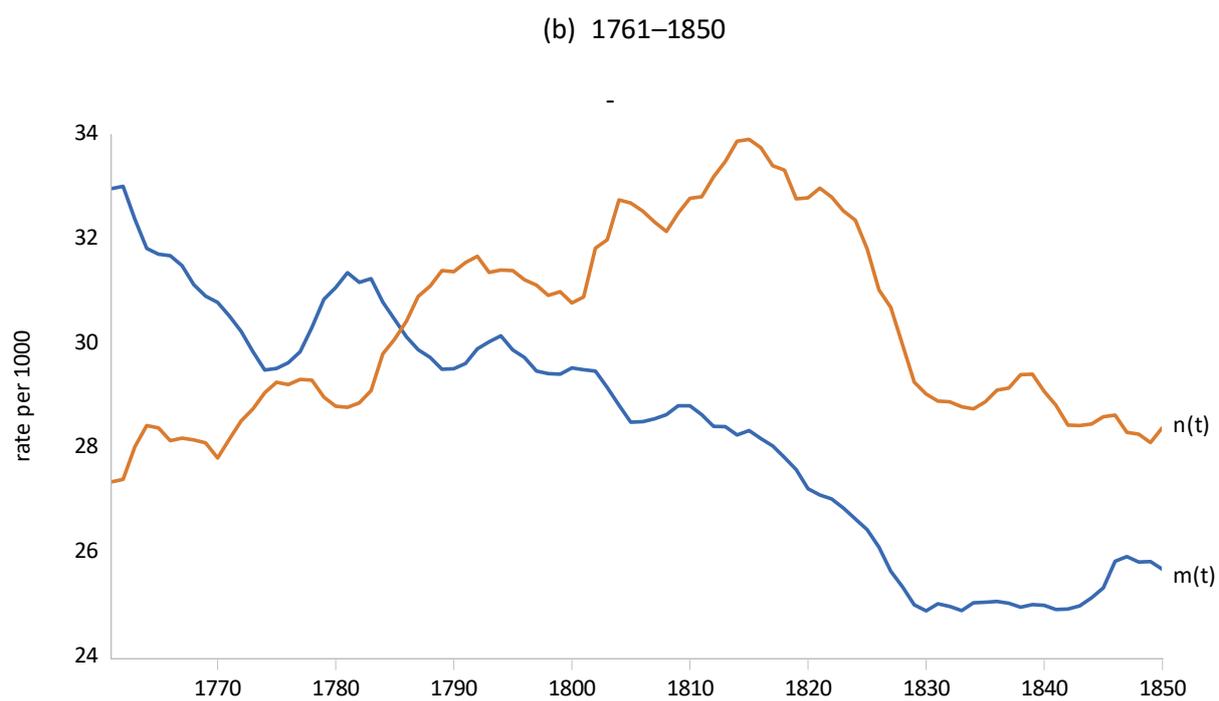
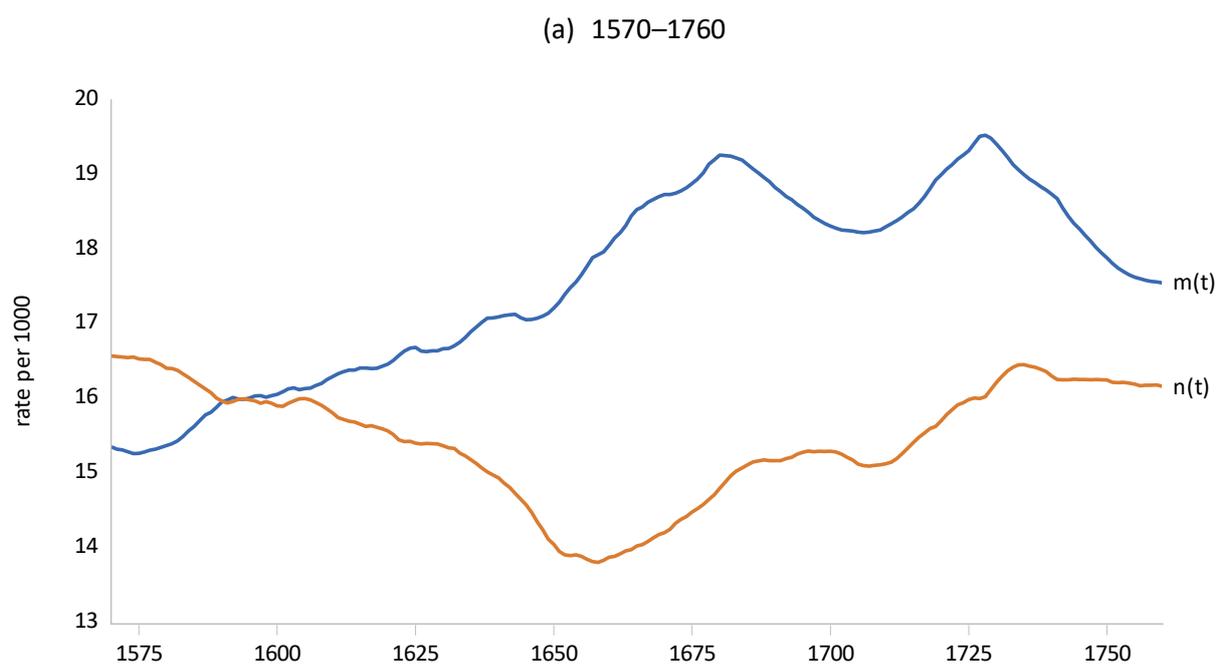


Figure 3 Time varying mortality, m_t , and fertility, n_t , intercepts: (a) 1570–1760; (b) 1761–1850.

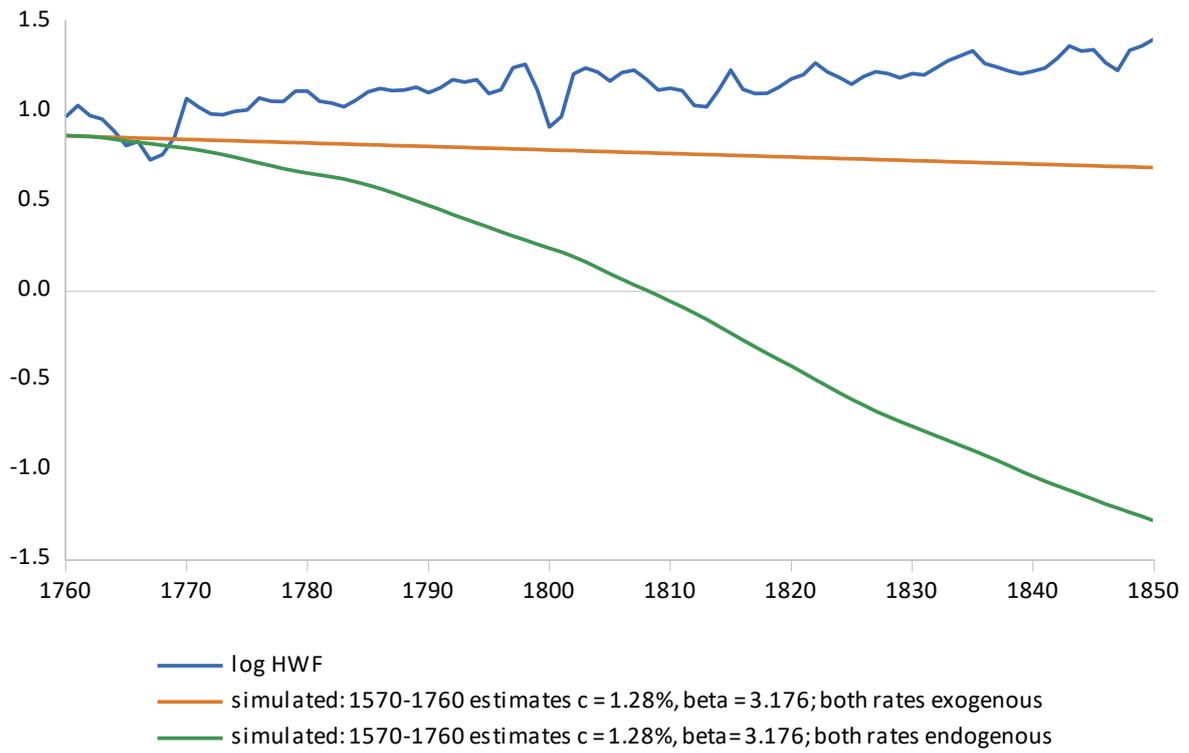


Figure 4 Simulated HWF real earnings for 1760–1850 using the structural model fitted over the period 1570–1760.

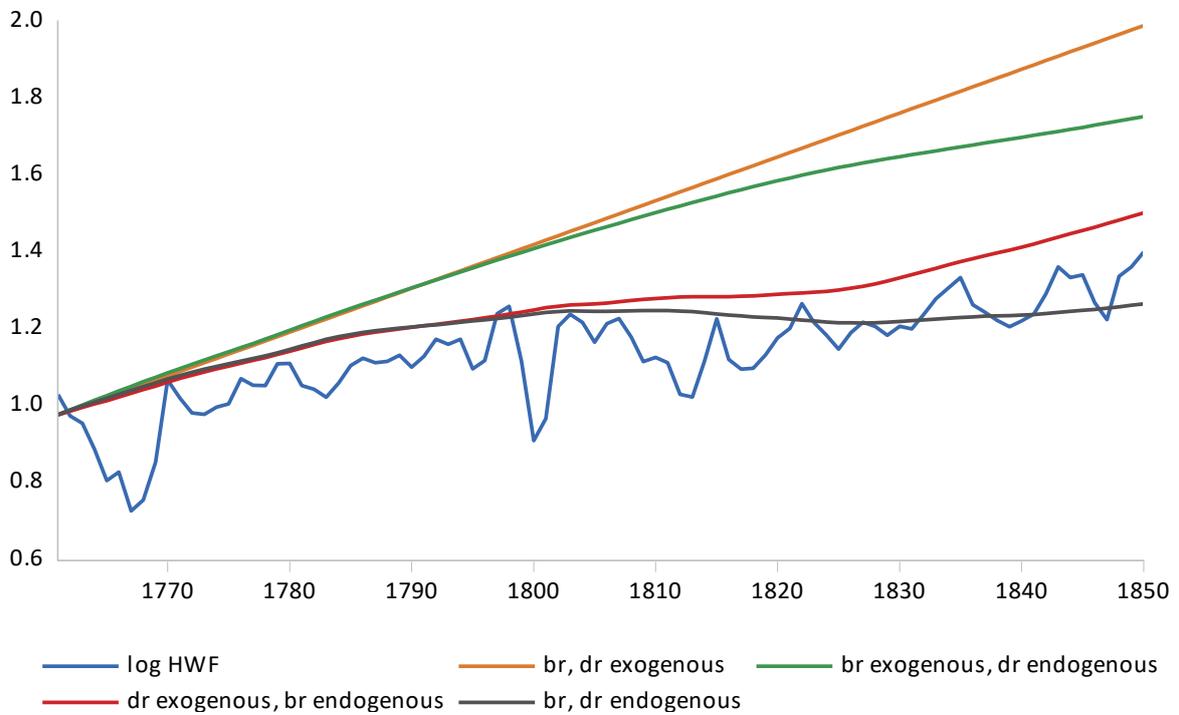


Figure 5 Simulated HWF real earnings 1760–1850 using structural model fitted over the period 1760–1850.

APPENDIX

Table A1. Sub-period Estimates of Real Wage Equation (2): 1570– t

t	$\hat{\beta}$	c (%)	c/β (%)
1750	3.125 (1.961)	1.19 (0.77)	0.38 (0.34)
1760	3.175*(1.823)	1.28*(0.74)	0.40 (0.33)
1770	2.959 (1.792)	1.27*(0.72)	0.43 (0.36)

Note: Standard errors shown in parentheses; * denotes significance at 10%.

Table A2. Sub-period Estimates of Real Wage Equation (2): $t - 1850$

t	$\hat{\beta}$	c (%)	c/β (%)
1750	1.624 (2.097)	2.21*(0.66)	1.36 (1.80)
1760	1.312 (2.085)	1.93*(0.67)	1.47 (2.39)
1770	3.003 (2.567)	3.97*(0.66)	1.32 (1.15)

Note: Standard errors shown in parentheses; * denotes significance at 10%.

Table A3. Sub-period Estimates of Death Rate Equation (6) and Birth Rate Equation (8): 1570 – t

t	Positive check, P_o	Elasticity, $\bar{\epsilon}_d$	Preventive check, P_r	Elasticity, $\bar{\epsilon}_b$
1750	-11.38*(4.01)	0.41*(0.15)	26.11*(6.16)	0.82*(0.19)
1760	-11.36*(3.80)	0.42*(0.14)	25.72*(5.72)	0.80*(0.18)
1770	-10.83*(3.67)	0.39*(0.13)	24.72*(5.44)	0.77*(0.17)

Note: Standard errors shown in parentheses; * denotes significance at 10%.

Table A4. Sub-period Estimates of Death Rate Equation (6) and Birth Rate Equation (8): $t - 1850$

t	Positive check, Po	Elasticity, $\bar{\epsilon}_d$	Preventive check, Pr	Elasticity, $\bar{\epsilon}_b$
1750	-5.50 (4.33)	0.21 (0.17)	19.93*(3.64)	0.53*(0.10)
1760	-5.28 (4.32)	0.21 (0.17)	18.14*(3.75)	0.48*(0.10)
1770	-9.22 (5.98)	0.36 (0.24)	22.14*(4.60)	0.58*(0.12)

Note: Standard errors shown in parentheses; * denotes significance at 10%.