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The pitfalls of pledgeable cash flows: soft budget constraints, zombie lending and under-investment *

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Abstract

We show that when borrowers are privately informed about their creditworthiness and lenders have a soft budget constraint, efficient investment requires a limit on the fraction of a firm's cash flows that can be pledged to outsiders. That is, pledgeability should neither be too low nor too high. An increase in pledgeability, or, more broadly, creditor rights, can either promote re-investment in zombie firms, which increases other firms' cost of capital, or it can lead to inefficient under-investment, depending on the composition of equilibrium credit demand. Thus, greater pledgeability can reduce net social surplus, and even trigger a Pareto loss.

Key words: Pledgeability, Investment efficiency, Soft budget constraint, Asymmetric information, Collateral, Zombie lending, Under-investment

JEL classification: G21, G32, G33, G38

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1 Introduction

Recent regulatory interventions have notably facilitated the pledging of collateral for firms. For instance, in 2006 France derogated 'possessory asset ownership'. This allowed a firm to retain some control over the assets it pledged to creditors (Aretz et al. (2020)). So, too, recent US court rulings have enabled firms to pledge particular intangible assets, such as patents, as collateral (Mann (2018)). The potential benefits of greater cash flow pledgeability in terms of relaxing a firm's financing constraint are well-understood (Rampini and Viswanathan (2013)). However, the potential costs are not.[1]

In this paper, we argue that one cost associated with high pledgeability of cash flows is that it can *soften* a lender's budget constraint, making it impossible for it to credibly commit *not* to re-invest in the future, in case the quality of the investment was revealed to be bad.[2] We identify two types of inefficiency that can emerge with high pledgeability. First, it might promote borrowing by negative net present value firms, as these firms anticipate that their initial lender will re-invest in the future, even in cases in which the market would be unwilling to extend further credit to them. That is, high pledgeability can foster lending to zombie firms, implying that stronger creditor rights can actually *increase* the cost of raising capital for good firms. Second, high pledgeability of cash flows can also make it impossible for good firms to receive financing, if there are enough negative net present value borrowers in the economy, leading to inefficient under-investment.

Our baseline model features firms with investment projects—possibly positive or negative net present value—that require financing from deep-pocketed lenders. We first consider a monopolist lender, and then show that similar qualitative insights emerge in the polar-opposite case of perfect competition, modeled as free entry of potential lenders. At the initial financing stage, a firm privately knows the quality of its project, which de-

[1]According to Donaldson, Gromb and Piacentino (2020), greaterpledgeability may give rise to commitment problems for borrowers, due to non-exclusive contracting and the possibility of diluting the initial lenders. However, Bernhardt, Koufopoulos and Trigilia (2020a) prove that such limited commitment alone is not enough: even in a generalized version of Donaldson et al. (2020), greater pledgeability can only benefit a firm, as it can always issue enough secured debt to reduce the amount of pledgeable cash available to repay new (more junior) creditors, solving any non-exclusivity problem.

[2]Kornai (1986) coined the term 'soft budget constraint' while studying creative destruction in socialist economies. Dewatripont and Maskin (1995) formalized it as a commitment problem for lenders.

termines its cash flow distribution, where the high-quality project's cash flow distribution first-order-stochastically-dominates the low-quality one's. After a firm receives financing and invests, lenders learn the firm's type and subsequently face a re-investment decision, to which they cannot credibly commit ex ante. In contrast to Dewatripont and Maskin (1995) who assume full pledgeability of cash flows, we allow pledgeable cash flows to be limited, and show how equilibrium outcomes hinge on the degree of pledgeability.

When pledgeability of cash flows is sufficiently low, we obtain the well known result that the credit market breaks down—i.e., no firm receives financing, regardless of its quality. Thus, with very limited pledgeability, increasing creditor rights can only relax a firm's financing constraint. Specifically, we find that when good projects are sufficiently likely to succeed, if one increases pledgeability further, then a threshold is reached at which investment is efficient in equilibrium. That is, high-quality firms borrow and invest, because pledgeable funds are sufficient in expectation to cover the investment costs of lenders, while low-quality firms prefer to stay out of the market, as they anticipate no future re-investment at the interim stage when their quality will be revealed to be low.

Next, and perhaps surprisingly, we show that this efficient-investment equilibrium *requires* pledgeability to be limited: once the fraction of a firm's pledgeable cash flows exceeds a second critical threshold, efficient investment can no longer be sustained. This is because, when pledgeability is sufficiently high, lenders anticipate that they will also want to re-invest in low-quality firms in the future, rendering it impossible to screen firm qualities ex ante. Thus, our model highlights that, in the presence of asymmetric information, having more pledgeable assets that can be used as collateral can actually harm a firm.

As pledgeability rises above the threshold required for the efficient-investment equilibrium to exist, two possible scenarios arise, depending on the share of high-quality firms in the population. If this share is large, then higher rates of pledgeability trigger a zombie-lending equilibrium in which all firms anticipate re-investment in the future—even if their quality is low—and, as a consequence, they choose to borrow ex ante. The rise of zombie firms in recent years, especially in southern-European countries with relatively strong creditor rights, points to the relevance of this channel (McGowan et al. (2017)).[3]

[3] Recent work on zombie (or forbearance) lending includes Storz et al. (2017), Banerjee and Hofmann

If, instead, the share of good firms is smaller, then lenders can no longer extend credit and make non-negative profits. In this case, greater pledgeable cash flows give rise to a market breakdown in which no credit is extended ex ante, even to good firms. As this result applies to populations of borrowers with relatively few unicorns, and many bad ideas, its predictions naturally apply to startup firms. Indeed, in this context our model can reconcile the findings in Ersahin et al. (2020) that strengthening creditor rights in the US reduced startup entry, especially among riskier firms. Moreover, it can provide an alternative, non-behavioral, explanation for findings in Vig (2013) that higher creditor rights triggered lower borrowing by firms in India, without resorting to any liquidation bias.

Notably, if greater pledgeability can lead the economy from an equilibrium at which investment is efficient to another equilibrium in which the credit market breaks down, then higher creditor rights not only can reduce the net social surplus, but they can also induce a Pareto loss. In particular, they can harm both high-quality firms and lenders without making low-quality firms better off. While such statements might appear paradoxical once they have been extrapolated outside of its context, we argue that they actually make perfect sense in a credit market in which lenders have a soft budget constraint problem.

The second part of our paper introduces competition among lenders, both initially and at the re-investment stage. We first show that, in terms of real investment by firms, nothing changes when we modify the market structure so radically. That is, when pledgeability is low there is still no investment; when it is intermediate there is still efficient investment; and when it is high, there is still either under- or over-investment. Further, the same thresholds separate these regions of parameters under monopoly and under competition.

Allowing for capital market competition lets us uncover the relation between pledgeability and a firm's cost of capital, which cannot be meaningfully studied with a monopolistic lender. In particular, we identify a hump-shaped relationship between creditor rights and a firm's cost of capital. When pledgeability is so low that there is a credit market breakdown, the cost of capital is effectively infinite, and increasing creditor rights to a point that enables firms to invest must reduce their funding costs. The lowest pos-

(2018), Bonfim et al. (2020), Acharya et al. (2020) and Schivardi et al. (2020). The Economist dedicated two columns to zombie lending in its 09/26/2020 edition.

sible cost of capital for a good firm occurs at intermediate pledgeability, when creditor rights are low enough to make a lender's budget constraint hard, thus precluding entry by negative-NPV firms. In this region, good firms can finance themselves at the full-information rate. With further increases in creditor rights, the cost of capital shoots up, reaching a peak at the threshold that makes negative-NPV firms indifferent between staying in or out of the market. At this threshold, firms anticipate zombie lending at future dates, so that good firms now have to subsidize entry by negative-NPV firms. As pledgeability improves further, the cost of capital for good firms falls again, but it never reaches the minimum, because low-quality firms remain in the investment pool.

We conclude by investigating how regulation can affect both zombie lending and under-investment, focusing on the role played by nominal interest rates, as currently discussed in policy circles (see, e.g., Banerjee and Hofmann (2018)).[4] Understanding this interest rate channel is especially important as rates have recently been very low relative to historical standards. We show that, while a low interest rate can relax a firm's financing constraint when pledgeability is very limited, it can exacerbate the soft budget constraint problem when pledgeability is higher, insofar as it reduces the required rate of return for lenders to re-invest. This leads either to inefficient over-investment and zombie lending, or to inefficient under-investment. As creditor rights tend to be strong in advanced economies, especially in south-European countries where zombie lending is pervasive (Djankov et al. (2007)), our analysis suggests that the positive effects of increasing interest rates in these economies could be a first-order consideration.

Relatedly, the presence of zombie lending implies that the credit spread for good firms—i.e., the difference between the interest charged to them by lenders and the risk-free rate—can be *larger* when interest rates are low than when they are high. This is because, as interest rates fall, the quality of the pool of borrowers deteriorates, which increases the cost of corporate borrowing relative to a higher interest rate scenario. This

[4]This topic is receiving lots of press coverage. See, for instance, the recent Wall Street Journal (WSJ) article "A Nation of Zombie Borrowers Isn't Inevitable—Even With More Debt" (May, 2020). It is worth noting that there is some disagreement among central bankers regarding the quantitative effect of low interest rates on zombie lending. For example, the ECB's board member Philip R. Lane recently declared that "when you quantify it, we don't think that low interest rates as such are a big source of zombie dynamics." ("WSJ Interview With ECB's Philip R. Lane" (October, 2020)).

result is another novel implication of our analysis, one that can reconcile the empirical finding that the pass-through of monetary policy to credit spreads depends on the economic conditions, and it looks quite different from that predicted by frictionless macro-finance models (see, e.g., Gilchrist and Zakrajšek (2012) or Gertler and Karadi (2015)).

Importantly, one can empirically distinguish our channel from the competing story of time-varying risk aversion. To see this, observe that time-varying risk aversion not only predicts a negative correlation between interest rates and credit spreads due to recessions and credit busts, but it also implies that—because high credit spreads are supposedly due to high discount rates—they should be associated with higher expected returns moving forward. In contrast, our story involves cash flows, not discount rates, and therefore expected returns moving forward are unaffected. In other words, low interest rates should not predict higher returns, which is consistent with the evidence.

The paper unfolds as follows. Section 2 reviews the related literature; Section 3 introduces our baseline model; Section 4 characterizes equilibrium in the baseline model; Section 5 uncovers the pitfalls associated to greater pledgeability; Section 6 introduces competitive capital markets in the baseline model; Section 7 discusses the relation between pledgeability and the cost of capital; Section 8 discusses the effect of changing nominal interest rates on zombie lending and real investment by firms; Section 9 concludes.

2 Literature review

From a theoretical standpoint, our model builds on the seminal insight of Dewatripont and Maskin (1995) that a soft budget constraint emerges as the outcome of adverse selection and the impossibility for lenders to commit *not* to re-invest in the future, once a project is revealed to be of low quality (see also Qian (1994) and Berglof and Roland (1998)). In the context of this classical literature, our key insight is that limited pledgeability can toughen the budget constraints of lenders, making it *cheaper* for firms to raise funding. Paradoxically, limited creditor rights can relax a firm's financing constraint.

There has been a recent emphasis in the literature on the relation between limited commitment and creditor rights (see, e.g., DeMarzo (2019)). Specifically, Donaldson, Gromb and Piacentino (2020) posit that greater pledgeability might backfire in the presence of non-exclusive contracting, because it can enable excessive dilution. However, Bernhardt, Koufopoulos and Trigilia (2020b) show that, even in a generalized version of Donaldson et al. (2020)'s model, the firm can always pledge enough collateral to undo any negative effect of greater pledgeability. Thus, with non-exclusivity alone, a firm is always weakly better off with greater creditor rights. In contrast, our paper considers a commitment problem on the lender's side, not the borrower's, and it introduces asymmetries of information to derive real (both private and social) costs associated with high creditor rights.

More broadly, this topic is part of a vast literature on the real effects of limited commitment, which mostly focuses on commitment problems for borrowers (e.g., non-exclusivity) and includes the contributions of Bizer and DeMarzo (1992), Kahn and Mookherjee (1998), Parlour and Rajan (2001), and, more recently, Bennardo, Pagano and Piccolo (2015) and van Boxtel, Castiglionesi and Feriozzi (2020). Relative to these papers, we consider the interaction between ex-post moral hazard (i.e., limited pledgeability), and a commitment problem that is driven by the presence of ex-ante adverse selection.

Our paper contributes to the literature on pledgeability, collateral and corporate investment (see, e.g., Kiyotaki and Moore (1997), Rampini and Viswanathan (2013, 2019) and Li, Whited and Wu (2016)). While these papers largely emphasize the positive role of collateral in relaxing a firm's financing constraint, the declining reliance on secured debt by US firms in recent decades (Benmelech, Kumar and Rajan (2020)) suggests that greater effort should be devoted to understanding the costs of greater pledgeability for firms (see, e.g., Rampini and Viswanathan (2020)). We contribute to this debate by providing micro-foundations for the costs of high pledgeability, in terms of softening a lender's budget constraint and making it harder for credit to be allocated efficiently.

Our analysis also contributes to a growing theoretical literature on zombie lending. Bruche and Llobet (2014) consider the problem of a regulator that faces banks with incentives to hide losses and gamble for resurrection. They design an optimal scheme to make banks reveal their losses to a regulator, considerations that are absent from our model. Hu and Varas (2020), instead, consider a relationship lender who develops private, soft information over time, and shows that the initial lender may have an incentive to extend bad loans in order to keep the reputations of its borrowers artificially high. In contrast, our model has no soft information and no reputational concerns, and it focuses on the interaction between ex-ante adverse selection and ex-post moral hazard. Another related paper is van Boxtel, Castiglionesi and Feriozzi (2020), which considers a setting where non-exclusivity interacts with a moral-hazard problem à la Holmstrom and Tirole (1998), to argue that non-exclusive contracting—and the possibility it introduces of diluting the initial creditors of a firm—can limit the firm's access to long-term funds, making the firm less profitable at the outset and, under some conditions, turning it into a zombie.

Finally, our results speak to multiple strands of empirical work. One strand is the growing empirical literature on the causes and consequences of zombie lending (see Storz, Koetter, Setzer and Westphal (2017), Bonfim, Cerqueiro, Degryse and Ongena (2020), Acharya, Crosignani, Eisert and Eufinger (2020) and Schivardi, Sette and Tabellini (2020)). In this context, we highlight a new channel (creditor rights) that can cause more pervasive zombie lending, and we reconcile this channel with the 'monetary policy' channel suggested by the empirical literature, according to which zombie lending arises especially when interest rates are low (see, e.g., Banerjee and Hofmann (2018)).

A second strand of empirical work studies the consequences of changing the degree of creditor rights and/or the ease with which a borrower's assets can be pledged as collateral (in our model these notions correspond). While there is a large literature on this topic, one that spans multiple decades and mostly documents positive consequences of high creditor rights (see, e.g., LaPorta, de Silanes, Shleifer and Vishny (1997) or Djankov, McLiesh and Shleifer (2007)), recent work highlights potential downsides. Vig (2013) shows that high creditor rights led to lower borrowing in India, and interprets this as evidence of lenders' liquidation bias. Acharya, Amihud and Litov (2011) find that, across countries, high creditor rights are associated with low risk-taking. Ersahin, Irani and Waldock (2020) find that higher creditor's rights in the US have been associated with lower startup entry, especially among riskier firms. Consistent with these results, we highlight that increased creditor rights can entail costs, not just benefits.

3 Baseline model

To establish the effect of greater pledgeability on real investment by firms, we first consider a monopolistic lender. We then show that similar qualitative outcomes arise when the credit market is perfectly competitive, and there is free entry of lenders.

There are three dates $t = 0, 1, 2$, a continuum of mass one of firms and a monopolistic lender. At $t = 0$, each firm privately observes the quality of its investment project $\theta \in \{L, H\}$. A firm of quality θ has the choice between investing or not at $t = 0$. If the firm invests, it requires 1 unit of external financing at *both* date zero *and* date one in order to generate a cash flow at date two of $x > 2$ with probability p_θ. Cash flows are zero otherwise. The firm faces a (perhaps tiny) opportunity cost of investment in the form of an outside opportunity with value $y > 0$ that is foregone if it pursues the investment.[5] If there is no re-investment at $t = 1$, then the firm is liquidated at a value that we normalize to zero. We assume that $1 > p_H > p_L > 0$ and denote the fraction of H firms in the population by $\alpha \in (0, 1)$. Finally, we assume that only a fraction $\lambda \in (0, 1)$ of the realized cash flow x is pledgeable to outsiders.

The lender has a deep pocket at both $t = 0$ and $t = 1$, and even though it learns θ perfectly before $t = 1$, it cannot commit to a future re-investing plan ex ante, at date $t = 0$. Rather, this decision is taken at $t = 1$ and it needs to be ex post optimal—i.e., subgame perfect. We denote the (gross) risk-free rate of return by r. To make the problem interesting, we make the following assumptions on parameter values:

A1. $p_H > \frac{r(r+1)}{x}$: project H has a positive net present value at $t = 0$.

A2. $p_L < \frac{r(r+1)}{x}$: project L has a negative net present value at $t = 0$.

A3. $p_L > \frac{r}{x}$: project L has a positive net present value at $t = 1$.

A4. $y < p_L(1 - \lambda)x$: a firm always prefers to invest at $t = 0$ when it anticipates that there will be re-investment in its project at $t = 1$, regardless of its type θ.

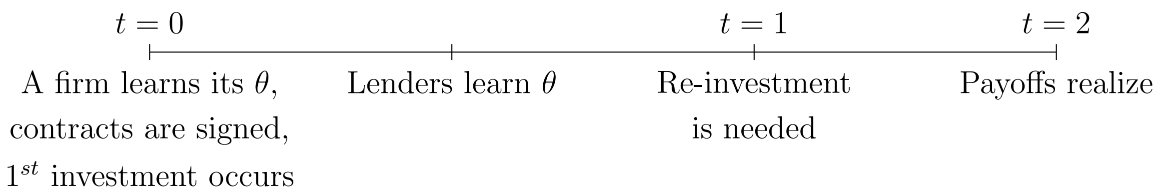
[5] Identical outcomes would emerge if firm's manager/owner derives a disutility if its firm is liquidated, capturing future reputational losses that could be triggered when a borrower is revealed to be an L type, as in Dewatripont and Maskin (1995). The two models are mathematically isomorphic.

Reversing **A1** trivially leads to no financing, as then all borrowers have negative net present value projects. Reversing **A2**, instead, leads to the uninteresting outcome in which all projects are financed as they all have a positive net present value, ex ante. Reversing **A3** implies that the low-quality project would never receive re-investment at $t = 1$. Thus, borrowers with low-quality projects never seek financing at $t = 0$, as they anticipate getting a payoff of zero, while they can secure a payoff of $y > 0$ by not investing. Finally, reversing **A4** leads to the uninteresting outcome in which low-quality firms do not want to invest, aligning their interests with those of the lender.

One could introduce to our model a private benefit $B \geq 0$ that the lender would receive from completion of a project it funded at $t = 0$. Such a private benefit could capture a bank manager's reputational concerns. However, because we find that, with high pledgeability, investment is inefficient even without private benefits, when $B = 0$, it follows that our qualitative results would hold a fortiori when $B > 0$.

At $t = 0$, the lender chooses whether to lend 1 unit of capital to the borrower in exchange for a future repayment at $t = 2$. As the lender has all bargaining power, it will extract the full pledgeable monetary return of the project, λx, leaving to the borrower only the non-pledgeable cash flow $(1 - \lambda)x$. Under **A4**, the expected non-pledgeable cash flow exceeds the borrower's outside option. Figure 1 summarizes the timing of the game.

Figure 1: Timeline



4 Equilibrium characterization

Given limited commitment, one must solve the problem recursively, starting at the re-investment stage at $t = 1$ and then proceeding to the $t = 0$ contracting problem. However, without further analysis, one immediate result that obtains in models with limited pledgeability is that if pledgeability λ is too low, then no financing can occur at $t = 0$:

Proposition 1 (Low pledgeability: $\lambda < \underline{\lambda}$)**.** *If the pledgeability of cash flows λ is less than $\frac{r(r+1)}{p_H x} \equiv \underline{\lambda}$, then there is no financing in equilibrium. As a result, there is inefficient under-investment in the H project.*

Proof. All proofs are in the Appendix. □

Proposition 1 states that, if pledgeability is low, then no firm can borrow at $t = 0$, regardless of whether it has a positive or negative net present value project. It follows that there is inefficient under-investment in high-quality projects, and that increasing cash-flow pledgeability in this region can only weakly benefit borrowers and lenders. Thus, for sufficiently low pledgeability λ, the usual negative relation between a firm's pledgeable cash flows and the tightness of its financing constraint arises.

The next lemma characterizes how a lender's optimal re-investing decision at $t = 1$ depends on a project's quality θ and the extent λ of cash-flow pledgeability. Once a lender learns that a firm's quality θ, re-investing yields the lender an expected payoff of $p_\theta \lambda x - r$, which is non-negative only if $\lambda \geq \frac{r}{p_\theta x}$. Thus, we have

Lemma 1. *At $t = 1$, the lender re-invests in a firm of quality θ if and only if $\lambda \geq \frac{r}{p_\theta x}$.*

Proposition 1 states that there can be investment at date zero only if $\lambda \geq \underline{\lambda} = \frac{r(r+1)}{p_H x}$. Lemma 1 implies that if $\theta = H$, then there is re-investment only if $\lambda \geq \frac{r}{p_H x}$. Because $\underline{\lambda} = \frac{r(r+1)}{p_H x} > \frac{r}{p_H x}$, it follows that a high-quality project always receives re-investment, whenever it is financed. As for a low-quality project, it only receives re-investment if its cash flow is sufficiently pledgeable. To see this, define the threshold $\overline{\lambda}$ at which a low-quality firm's expected pledgeable cash flow $p_L \lambda x$ just covers the re-investment costs at $t = 1$ of r: $\overline{\lambda} \equiv \frac{r}{p_L x}$. Whenever $\frac{p_H - p_L}{p_L} > r$, we have that $\overline{\lambda} > \underline{\lambda}$. The next Proposition summarizes.

Proposition 2 (Re-investment at $t = 1$)**.** *When $\lambda \geq \underline{\lambda}$, a high-quality project always receives re-investment at $t = 1$. A low-quality project receives re-investment if and only if $\lambda \geq \max\{\underline{\lambda}, \overline{\lambda}\}$. Further, $\overline{\lambda} > \underline{\lambda}$ if and only if $\frac{p_H - p_L}{p_L} > r$.*

These results begin to hint at the tensions associated with the level of cash flow pledgeability. If pledgeability is too low—that is, if $\lambda < \underline{\lambda}$—then Proposition 1 shows that it is impossible to re-invest in any project at $t = 1$, which precludes the financing of H projects.

If, instead, pledgeability is too high—that is, if $\lambda \geq \overline{\lambda}$—then Proposition 2 shows that the lender cannot credibly commit to terminating bad projects at $t = 1$. In this case, the lender will 'throw good money after bad', re-investing in all projects once the initial investment is sunk. This *soft budget constraint* problem makes it attractive for borrowers with low-quality projects to raise financing at the outset, anticipating that they will not be terminated at the interim stage. As we will show, the consequence of having borrowers with low-quality projects who seek funding is either inefficient over-investment in low-quality projects at $t = 0$, or a complete market breakdown, depending on the share of high-quality projects in the population α.

In what follows, we restrict attention to the setting in which the interest rate satisfies $r < \frac{p_H - p_L}{p_L}$, so that $\overline{\lambda} > \underline{\lambda}$. We characterize how the equilibrium depends on creditor rights, starting at intermediate levels of cash flow pledgeability $\lambda \in [\underline{\lambda}, \overline{\lambda})$.

Proposition 3 (Intermediate pledgeability: $\lambda \in [\underline{\lambda}, \overline{\lambda})$)**.** *If $\lambda \in [\underline{\lambda}, \overline{\lambda})$, then investment is efficient in equilibrium: only high-quality firms choose to borrow at $t = 0$, and they receive efficient re-investment at $t = 1$. Low-quality firms stay out of the market.*

Proposition 3 shows that intermediate degrees of pledgeability may lead to efficient investment at both $t = 0$ and $t = 1$, generating a large economic surplus. With intermediate pledgeability levels, borrowers with low-quality projects understand that they will not receive re-investment at $t = 1$. As a result, they prefer not to borrow at the outset, as their opportunity cost is strictly positive. That is, the limited creditor rights discourage low-quality firms from the credit market, as they anticipate that lenders will not re-invest in them. Thus, whenever pledgeability is intermediate, so that $\lambda \in [\underline{\lambda}, \overline{\lambda})$, firms with high quality projects receive financing, even if they comprise a small share of the population.

This analysis implies that if one starts from $\lambda \in [\underline{\lambda}, \overline{\lambda})$ and increases pledgeability, then this can never increase net social surplus. We now show show that further increasing pledgeability eventually *reduces* net social surplus. That is, there is a sense in which, beyond a certain point, strengthening creditor rights harms investment efficiency.

Proposition 4 (High pledgeability: $\lambda \geq \overline{\lambda}$)**.** *When pledgeability is at least $\overline{\lambda}$, equilibrium investment is inefficient, failing to maximize net social surplus. We have two cases:*

1. *If $r \leq \alpha \frac{p_H - p_L}{p_L}$, then all projects get funded both at $t = 0$ and at $t = 1$. Thus, there is inefficient over-investment in the L project.*
2. *If $r > \alpha \frac{p_H - p_L}{p_L}$, then there exists a threshold $\hat{\lambda} \equiv \frac{r(r+1)}{x(\alpha(p_H - p_L) + p_L)}$ such that:*
   - *(a) If $\lambda \in [\overline{\lambda}, \hat{\lambda})$, then there is no financing—i.e., the market breaks down.*
   - *(b) If $\lambda \geq \hat{\lambda}$, then there is inefficient over-investment in the L project at $t = 0, 1$.*

Proposition 4 reveals the cost associated with high pledgeable cash flows. Specifically, it shows that investment is always inefficient in equilibrium. If the population share of H projects is sufficiently high that $\alpha \frac{p_H - p_L}{p_L} \geq r$, then over-investment problems arise: all firms are funded at $t = 0$, even those with low-quality projects, and all firms receive re-investment at the interim stage. The intuition is that the availability of pledgeable cash flows interacts with the lender's limited commitment problem, making it desirable for it to refinance a bad project at $t = 1$: once the initial investment is sunk, the project turns to having a positive net present value for the lender.

If, instead, the population share of high-quality projects is lower so that $\alpha \frac{p_H - p_L}{p_L} < r$, then outcomes hinge on the degree of pledgeability. With relatively low pledgeability, credit markets break down and no borrower obtains financing at $t = 0$. This is because the lender expects to lose on low-quality projects, and gains on high-quality projects would fail to cover these losses. However, once $\lambda > \hat{\lambda}$, the lender more than breaks even in expectation from funding all projects. Thus, investment is feasible, but the lender's inability to commit not to re-invest in low-quality projects leads to over-investment.

5 The pitfalls of pledgeable cash flows

Now that we have characterized equilibrium outcomes, we next present some basic properties of how increased pledgeability, or creditor rights, affect equilibrium allocations and real investment. Corollary 1 emphasizes the first implication of Propositions 3 and 4: net social surplus is maximized by intermediate pledgeability.

Corollary 1 (Net social surplus)**.** *If pledgeability is such that $\lambda \in [\underline{\lambda}, \overline{\lambda})$, then investment is efficient and net social surplus is maximized: only H projects are financed at $t = 0$, and these projects receive efficiently re-investment at $t = 1$ whenever needed. Otherwise, there is either inefficient over-investment by L types at $t = 0$, and re-investment in L projects at $t = 1$, or credit markets break down completely and there is no investment.*

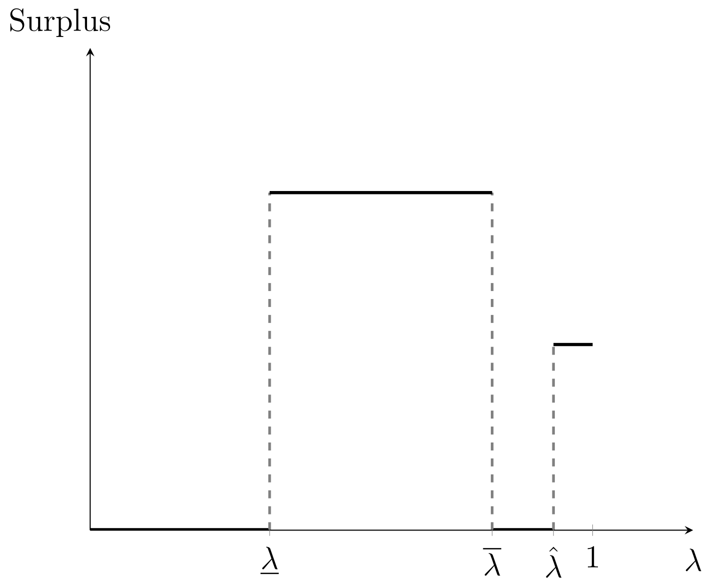
Corollary 1 highlights that, in terms of net social surplus, the most efficient allocation requires an intermediate degree of pledgeability, high enough that a high-quality project can be financed at $t = 0$, but low enough so that a borrower with a low quality project would never never receive re-investment at $t = 1$, if initiated. Increasing pledgeability has a dramatic effect on surplus when the population share of high-quality projects is small: anticipating re-investment at $t = 1$ due to the increased pledgeability of cash flows, low-quality firms enter the market, rendering it impossible for the monopolist lender to make profits, leading to a complete credit market breakdown.

If the population share of high-quality projects is larger, then greater pledgeability leads to inefficient over-investment, which benefits low types at the expense of the lender. Figure 2 plots net social surplus as a function of pledgeability λ when $\frac{p_H - p_L}{p_L} > r > \alpha \frac{p_H - p_L}{p_L}$ so that all of the regions described in Propositions 3 and 4 are non-empty.

The possibility that increasing the pledgeability of cash flows can shift equilibrium outcomes from the efficient level of investment to a complete market breakdown means that making cash flows more pledgeable can even induce Pareto losses. Corollary 2 highlights that, in the presence of a soft budget constraint problem, increasing a firm's pledgeable cash flows can harm efficiency in the strongest possible sense, making both lenders and borrowers with high-quality projects strictly worse off, without making potential borrowers with low-quality projects better off:

Corollary 2 (Pareto inefficiency)**.** *An increase in pledgeability from $\lambda \in [\underline{\lambda}, \overline{\lambda})$ to $\lambda' \geq \overline{\lambda}$ triggers a Pareto loss if and only if: (i) the share of H projects is sufficiently low, $r > \alpha \frac{p_H - p_L}{p_L}$; and (ii) the new level of pledgeability λ' is low enough that $\lambda' < \hat{\lambda}$.*

Figure 2: The pitfall of pledgeable cash flows



Parameters: $p_H = 0.7$, $p_L = 0.25$, $x = 5$, $y = 1$, $\alpha = 0.3$

6 Competitive capital markets

Our analysis has considered a monopolist lender who can extract the entire pledgeable monetary return from a project. We now consider the opposite scenario with free entry of lenders at both dates zero and one. This allows us to derive novel predictions concerning the effect of pledgeability on a firm's cost of capital.

Without loss of generality, we consider debt contracts that promise to repay the date-zero lenders $\min\{z_0, D_0\}$, for some $D_0 \geq 0$ and $z_0 \in \{0, x\}$, while they promise to repay the date-one lenders $\min\{z_1, D_1\}$, for $D_1 \geq 0$ and $z_1 \in \{0, x - D_0\}$. Indeed, because the state is binary, the only security design aspect that matters for allocations is to establish priority of the date zero creditors, relative to any subsequent creditor. As in Bernhardt, Koufopoulos and Trigilia (2020a), secured debt contracts serve the purpose of establishing priority, and only in this sense are they preferred to other, more junior securities, such as unsecured bonds or equity. The timing of the game is as follows:

$t = 0$:

Stage 1: Lenders offer to lend 1 unit of capital to firms in exchange for a debt contract with face value D_0 and maturity $t = 2$.[6]

Stage 2: A firm of type θ either accepts or rejects. If it rejects, it receives the outside option payoff $y > 0$. If it accepts, investment takes place.

$t = 1$: Lenders learn all firms' types, and can offer to lend 1 unit of capital to firms in exchange for a debt contract with face value D_1 and maturity $t = 2$. If a firm rejects, its project is terminated; otherwise, we proceed to $t = 2$.

$t = 2$: Cash flows realize and are distributed according to the contracts signed at $t = 0, 1$.

We start from the low-pledgeability case, where credit markets break down even with a monopolist lender. The same logic yields that they also shut down with competition among lenders.

Proposition 5 (Low pledgeability $\lambda < \underline{\lambda}$). *When pledgeability is low, so that $\lambda < \underline{\lambda} \equiv \frac{r(r+1)}{p_H x}$, the unique equilibrium of the competitive model features a credit market breakdown, in which, regardless of its project quality, no firm raises financing at $t = 0$.*

We now consider what happens at the re-investment stage, $t = 1$. Because there is full information at this point, there are two separate markets for high and low projects. We refer to the date-zero lender as the *insider*, while other potential lenders are *outsiders*.

Lemma 2 (Re-investment at $t = 1$). *At the re-investment subgame, equilibrium outcomes as a function of the date-zero face value of debt D_0 and project quality θ are as follows:*

1. *If $\lambda \geq \frac{r + p_\theta D_0}{p_\theta x}$, then both outsiders and insiders are willing to re-invest at date one, and the face value of the $t = 1$ debt is $D_1^* = \frac{r}{p_\theta}$.*
2. *If $\lambda < \frac{r + p_\theta D_0}{p_\theta x}$, then there are two cases:*
   - *(a) If $\lambda \geq \frac{r}{p_\theta x}$, then there is zombie-lending—i.e., re-investment is negative NPV for outsiders, while the insider finances the firm at a face value $D_1^* = \lambda x - D_0$.*
   - *(b) Otherwise, there is no re-investment in project θ.*

[6] It is immediate to see that the possibility of committing extra funds from the outset can only make the adverse selection problem worse, which justifies our choice to abstract way from it.

Because $D_0 > 0$, it follows from the inequalities in Lemma 2 that whenever outsiders are willing to invest in the firm at date one, the insider is also willing to re-invest. The converse is not true: there is a range of pledgeability of cash flows for which zombie lending occurs. On this range, investing in the firm is negative-NPV for outsiders, but it is positive-NPV for the inside lender, as it has 'skin in the game' due to its past investment.

Therefore, to check whether a project will be terminated or not it suffices to answer the question: would the inside lender re-invest? If the answer is negative, then the firm is terminated at date one. First observe that when $\theta = H$ there can never be termination at date one because $\frac{r}{p_H x} < \frac{r(r+1)}{p_H x} = \underline{\lambda}$. In contrast, when $\theta = L$ there is re-investment at $t = 1$ if and only if $\lambda \geq \overline{\lambda}$. This has two important implications. First, it implies that our optimal contract, and the allocation it implements, are renegotiation proof. Second, it implies that investment is the same with competition and with a monopolist:

Corollary 3. *At date one, the investment regions under competition are identical to those for a monopolistic lender: if $\lambda \geq \overline{\lambda}$ then all projects receive re-investment; but if $\lambda < \underline{\lambda}$, then no project is funded. If $\lambda \in [\underline{\lambda}, \overline{\lambda})$, then only type H projects receive re-investment, and L projects are terminated.*

It remains to derive the equilibrium investment at date zero and the associated face value of debt D_0 when $\lambda \geq \underline{\lambda}$. With intermediate pledgeability, we have:

Proposition 6 (Intermediate pledgeability, $\lambda \in [\underline{\lambda}, \overline{\lambda})$)**.** *When pledgeability is intermediate and $\lambda \in [\underline{\lambda}, \overline{\lambda})$, the unique equilibrium under competition is as follows: L firms do not invest at $t = 0$, while H firms invest by borrowing at the full-information face values of debt $D_0^* = \frac{r^2}{p_H}$ at date zero, and $D_1^* = \frac{r}{p_H}$ at date one. Lender profits are zero.*

Proposition 6 establishes that with intermediate levels of pledgeability, investment is efficient. That is, high-quality firms borrow at all dates, while low-quality firms stay out of the market. Moreover, competition among lenders for high-quality firms results in zero expected lender profits at both dates $t = 0$ and $t = 1$, because $p_H D_0^* = r^2$ and $p_H D_1^* = r$. Importantly, comparing Propositions 3 and 6 reveals that the rate of investment with intermediate pledgeability is unaffected by the degree of competition among lenders, as it is identical under monopoly and free entry. We now characterize equilibrium outcomes with high levels of cash flow pledgeability.

Proposition 7 (High pledgeability $\lambda \geq \overline{\lambda}$)**.** *When pledgeability is at least $\overline{\lambda}$, the unique equilibrium under competition hinges on the degree of pledgeability as follows:*

1. *If $r \leq \alpha \frac{p_H - p_L}{p_L}$, then there is inefficient over-investment, and all projects get funded at $t = 0, 1$. Further, there exists a threshold $\tilde{\lambda} \equiv \overline{\lambda} + \frac{r^2}{(\alpha p_H + (1-\alpha) p_L) x}$ such that:*
   (a) *If $\lambda \in [\overline{\lambda}, \tilde{\lambda})$, then the $t = 0$ pooling face value of debt is given by $D_0^* = \frac{r^2 - (1-\alpha)(p_L \lambda x - r)}{\alpha p_H}$. The date-zero lenders break even anticipating that:*
      - *With an H project, there will be competition for re-investment from outsiders at $t = 1$, so that the $t = 1$ face value of debt will be $D_1^* = \frac{r}{p_H}$.*
      - *With an L project, there will be zombie-lending: there is no competition from outsiders at $t = 1$, while insiders re-invest at $D_1^* = \lambda x - D_0$.*

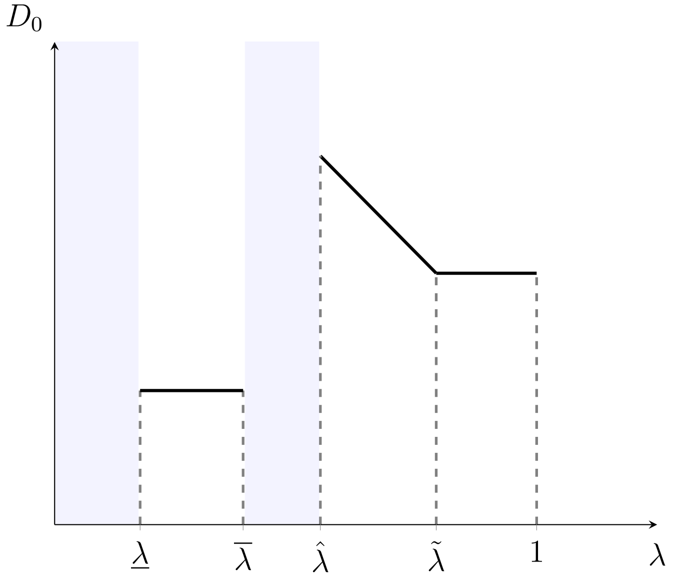
   (b) *If $\lambda \geq \tilde{\lambda}$, then all firms invest at date zero, at a pooling face value of debt $D_0^* = \frac{r^2}{(\alpha p_H + (1-\alpha) p_L)}$. Date-zero lenders break even anticipating that they will face competition at date one regardless of the project's quality θ.*
2. *If $r > \alpha \frac{p_H - p_L}{p_L}$, then there exists a region $\lambda \in [\overline{\lambda}, \hat{\lambda})$ in which there is a market breakdown and no firm receives credit at $t = 0$.*

Proposition 7 clarifies that, when pledgeability is high, the equilibrium investment is always inefficient, as there is either under- or over-investment. Specifically, when the population share of low types is sufficiently large, and pledgeability is limited, then markets breakdown and no borrower invests. Otherwise, the familiar over-investment equilibrium arises, in which all borrowers invest at date zero and receive re-investment at $t = 1$. Proposition 7 also helps uncover the effect of greater pledgeability on a firm's cost of capital and the possibility of zombie-lending. We next explore these predictions.

7 Pledgeability and a firm's cost of capital

We first explore the relation between creditor rights and the cost of capital for a type H firm at $t = 0$. When $\frac{p_H - p_L}{p_L} > r > \alpha \frac{p_H - p_L}{p_L}$, there exist two regions in which credit markets break down. The first region corresponds to the familiar low-pledgeability case $\lambda < \underline{\lambda}$, while the second corresponds to the intermediate pledgeability case $\lambda \in [\overline{\lambda}, \hat{\lambda})$. In contrast, when pledgeability λ is between $\underline{\lambda}$ and $\overline{\lambda}$, investment is efficient in equilibrium, with high-quality firms borrowing at the full information rate, and low-quality firms staying out of the market. It follows immediately that, as Figure 3 illustrates, if $\frac{p_H - p_L}{p_L} > r > \alpha \frac{p_H - p_L}{p_L}$, then a firm's cost of capital is non-monotone with respect to creditor rights.

Figure 3: Pledgeability and an H firm's cost of capital



Parameters: $p_H = 0.9$, $p_L = 0.3$, $x = 6.5$, $\alpha = 0.3$, $r = 1$
The shaded regions correspond to market breakdowns

It turns out that this non-monotonicity is a more general result, which holds whenever $\hat{\lambda} < 1$. To see this, note that in Figure 3 the highest cost of capital *conditional on the firm receiving financing at* $t = 0$ arises at $\hat{\lambda}$. That is, when $\lambda < \hat{\lambda}$, then either the firm does not receive funding at $t = 0$, or it can borrow at the full-information rate (if $\theta = H$ and $\lambda \in [\underline{\lambda}, \overline{\lambda})$). When $\lambda > \hat{\lambda}$, instead, an H firm always receives financing at $t = 0$, but it is charged a pooling rate at which it subsidizes investment by L firms.

In this case, as Figure 3 shows, starting at $\lambda \geq \hat{\lambda}$, the cost of capital falls with creditor rights. As the mechanism that drives the negative relation between creditor rights and a firm's cost of capital in this region is a novel prediction,we explain it in detail. The first important piece is the pricing of debt at the re-investment date, depending on the quality of a firm θ. When $\theta = H$, this pricing is competitive, and equal to the full-information rate for high-quality firms $D_1 = \frac{r}{p_H}$. This is because outside lenders are always willing to invest in the firm, and therefore they compete with the insider.

In contrast, lending to a $\theta = L$ firm is a negative-NPV investment for outsiders (i.e., the firm is a zombie). As a result, when pledgeability is close to $\hat{\lambda}$, the funds given at $t = 1$ to a low-quality firm will only be provided by the inside lender, as it already has skin in the game. Thus, the insider will extract the full non-pledgeable cash flow λx from the type $\theta = L$ firm. It follows that, as λ rises, the cost of capital for good firms falls, as they are required to pay a lower subsidy to low-quality firms in equilibrium.

Finally, at $\lambda = \tilde{\lambda}$, creditor rights become high enough that there is now competition from outside lenders at $t = 1$ or low-quality firms, which again makes the face value of debt at date zero insensitive to pledgeability.

Summing up, increasing the fraction of firm cash flows that is pledgeable can tighten a good firm's financing constraint, in two distinct ways. When enough firms are low quality, higher pledgeability can impede access to external financing to good firms—that is, it can make a good firm's cost of capital effectively infinite. Alternatively, when there are fewer low-quality firms, greater pledgeability can increase a good firm's cost of capital, because now the firm subsidizes low-quality firms that, with greater pledgeability, strictly prefer to borrow ex ante, anticipating future re-investing by lenders, while these low-quality firms preferred to stay out of the credit market when pledgeability was more limited.

8 Interest rates and zombie lending

While the rise of zombie firms in recent years has been documented empirically (e.g., Storz et al. (2017), Banerjee and Hofmann (2018), Bonfim et al. (2020), Acharya et al. (2020) and Schivardi et al. (2020)), it is less clear what a regulator can do to deal with this issue. One tool that has been discussed is the use of monetary policy in the form of nominal interest rates. In this section, we analyze the effects of changing nominal interest rates on zombie lending and real investment by firms through the lens of our model.

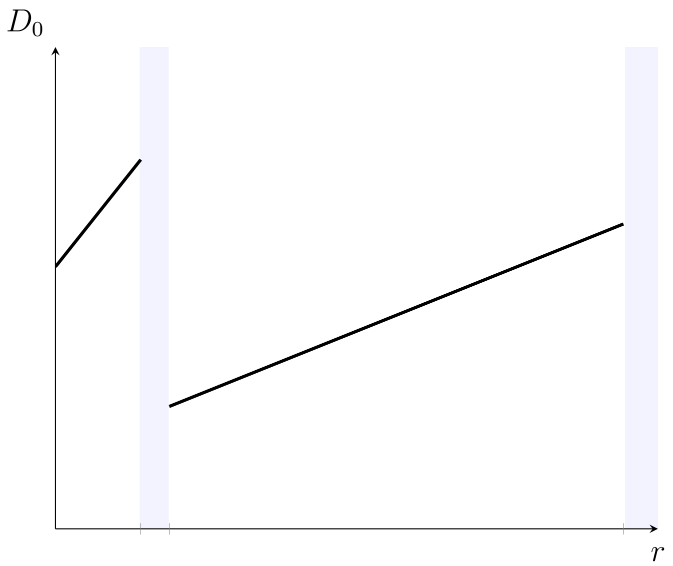
To uncover the relation between creditor rights and the extent of zombie lending that occurs in equilibrium, one first needs a precise definition of zombie lending in our model. Consistent with the empirical literature, we say that a firm is a *zombie* when the present value of its assets net of the value of any outstanding liability previously contracted by the firm is negative. Thus, a firm of quality θ is a zombie firm at $t = 1$ if $p_\theta \lambda(x - D_0) < r$.

It follows from this definition that there are two possible date-one outcomes for zombie firms: either they receive reinvestment from their initial lenders, in which case there is *zombie lending*, or they do not receive reinvestment, in which case they are liquidated. From a date zero perspective, high-quality firms would like lenders to commit not to lend to zombies at date one, but this type of commitment is not possible. As a result, in equilibrium there is zombie lending in the intermediate pledgeability region where $\lambda \in [\max\{\overline{\lambda}, \hat{\lambda}\}, \tilde{\lambda})$. In this case, insiders face a soft budget constraint problem that leads them to re-invest in firms whose assets have a negative net present value at $t = 1$.

We can now ask: what happens in the model as nominal interest rates increase? Figure 4 uncovers the relation between interest rates and a firm's cost of capital. Through this relation, it also reveals the real effects of interest rates in terms of corporate investment. At the zero-lower bound, where the gross risk-free rate is 1, the cost of capital for good firms is relatively high, because they subsidize low-quality firms that enter anticipating a soft budget constraint problem for lenders at $t = 1$, and the consequent zombie lending. Locally, increasing interest rates makes things worse, as it does not affect the composition of the pool of investing firms. Increasing the interest rate further may also lead to the impossibility for lenders to make profits on firms at $t = 0$, which leads to the first shaded region, where a market breakdown occurs.

However, a central bank can set interest rates high enough to alleviate zombie lending. That is, there exist rates to the right of the first shaded region in Figure 4, where, paradoxically, a firm's cost of capital is *lower* than what it would be at very low interest rates (i.e., close to the y-axis of Figure 4). This is because, at such a rate, lenders can effectively commit not to lend in the future to zombie firms, and this keeps zombies out of the credit market at $t = 0$. Once the threshold is reached, increasing interest rates further can only raise capital costs for firms, up to the point where the second shaded region is reached, on the right of the plot in Figure 4, where a second market breakdown occurs, as the financing constraint for good firms is too tight.

Figure 4: The two sides of changing interest rates



Parameters: $p_H = 0.9$, $p_L = 0.3$, $x = 6.5$, $\alpha = 0.3$, $\lambda = 0.9$
The shaded regions correspond to market breakdowns

9 Conclusions

This paper provides micro-foundations for the costs of high creditor rights and pledgeable cash flows. Specifically, it identifies plausible conditions under which, when firms are privately informed about their quality and only some of them has positive net present value projects, investment is efficient in equilibrium if and only if creditor rights are limited. High creditor rights introduce a soft budget constraint problem for lenders, which results in one of two possible inefficiencies, depending on the population share of 'good' firms. If this share is large, then high creditor rights trigger a zombie lending equilibrium in which good firms subsidize other, negative-NPV firms, and therefore face a high cost of raising capital. If, instead, the share of good firms is limited, then high creditor rights can cause a market breakdown, in which even good firms go unfunded.

We also show that a regulator considering the use of monetary policy to address these inefficiencies should bear in mind that interest rates are a double-edged sward. On the one hand, increasing interest rates alleviates the soft budget constraint for lenders when pledgeability is relatively high, as at sufficiently high interest rates they can effectively commit not to re-invest in low-quality firms in the future. This reduces the incentives for low-quality firms to borrow at the outset, *reducing* the cost of financing for good firms. On the other hand, interest rates can tighten a firm's financing constraint, inducing more under-investment when pledgeability is relatively low, as lenders might not be able to break-even given the limited pledgeable cash flow a firm is anticipated to generate.

Assessing which effect is likely to be first-order is beyond the scope of our analysis. However, the existing empirical evidence suggests that the beneficial effect of increasing interest rates may be substantial in advanced economies, as they are currently characterized by: (i) very low nominal interest rates relative to historical standards; and (ii) relatively high degrees of creditor rights.

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Appendix

Proof of Lemma 1

Proof. The highest possible quality of a pool of firms from the lender's perspective occurs when it is solely composed of H projects. At $t = 0$, the expected pledgeable cash flow of an H project is $\lambda p_H x$, which exceeds its funding cost if only if $\lambda p_H x \geq r(r+1)$ or, equivalently, if $\lambda \geq \frac{r(r+1)}{p_H x} \equiv \underline{\lambda}$. Otherwise, there must be no financing in equilibrium. □

Proof of Proposition 2

Proof. If the lender learns that $\theta = H$, then by re-investing it expects a payoff equal to $p_H \lambda x - r$, which is positive only if $\lambda \geq \frac{r}{p_H x}$. Given that $\underline{\lambda} = \frac{r(r+1)}{p_H x} > \frac{r}{p_H x}$, whenever $\lambda \geq \underline{\lambda}$ the H project would receive (efficient) re-investment in at $t = 1$.

If, instead, the lender learns that $\theta = L$, then, from re-investing, it expects a payoff of $p_L \lambda x - r$, which is positive only if $\lambda \geq \frac{r}{p_L x} \equiv \overline{\lambda}$. Further, direct calculations yield that $\overline{\lambda} > \underline{\lambda}$, i.e., $\frac{r}{p_L x} > \frac{r(r+1)}{p_H x}$, if and only if $r < \frac{p_H - p_L}{p_L}$. □

Proof of Proposition 3

Proof. From Lemma 1, only type H projects receive re-investment at $t = 1$ when $\lambda < \overline{\lambda}$. Anticipating that L firms will be terminated at the interim stage, an L firm strictly prefers not to raise financing at $t = 0$ to obtain $y > 0$. In contrast, anticipating that H projects will receive re-investment at the interim stage, a borrower with a type H project strictly prefers to raise financing at $t = 0$ because its expected payoff from investment equals $(1 - \lambda)p_H x > (1 - \lambda)p_L x > y$, where the last inequality follows from **A4**. Thus, the lender can infer that only H projects are in the pool at $t = 0$. Therefore, it finances the project at date zero as $\lambda p_H x \geq r(r+1)$ if and only if $\lambda \geq \frac{r(r+1)}{p_H x} \equiv \underline{\lambda}$. □

Proof of Proposition 4

Proof. From Lemma 1, when $\lambda \geq \overline{\lambda}$, all borrowers seek financing at $t = 0$, for every θ, anticipating that they always receive re-investment at $t = 1$. This is because, although the lender has all bargaining power and can extract the full pledgeable return of a project at $t = 2$, a borrower keeps the non-pledgeable part of the cash flow. The lender is willing to provide financing to the pool of borrowers at $t = 0$ only if $(\alpha p_H + (1-\alpha)p_L)\lambda x \geq r(r+1)$, or, equivalently, only if $\lambda \geq \frac{r(r+1)}{x(\alpha(p_H - p_L)+p_L)} \equiv \hat{\lambda}$. Algebra yields that $\hat{\lambda} > \overline{\lambda}$ if and only if $\alpha\frac{p_H - p_L}{p_L} < r$. Thus, if $\alpha\frac{p_H - p_L}{p_L} \geq r$, then there is inefficient over-investment in the L project at both $t = 0$ and $t = 1$. If $\alpha\frac{p_H - p_L}{p_L} < r$, instead, then there are two sub-cases: (i) when $\lambda \in [\underline{\lambda}, \hat{\lambda})$, there is no financing in equilibrium; (ii) when $\lambda \geq \hat{\lambda}$, instead, there is inefficient over-investment in the L project at both $t = 0$ and $t = 1$ (i.e., there is a zombie-lending equilibrium). Thus, an inefficiency always arises for all α. □

Proof of Corollary 1

Proof. There are three possible equilibrium outcomes. First, there could be *Over investment* (OI), where all projects are financed at $t = 0$. In this case, there must be inefficient re-investing of bad projects at $t = 1$, else L borrowers would stay out of the market. Thus, the net social surplus is $S_{OI} \equiv (\alpha p_H + (1-\alpha)p_L)x - r(r+1) - y$. Second, there may be *Under Investment* (UI), in which case the net surplus is $S_{UI} \equiv y$. Finally, there may be *Efficient investment* (EI), in which case the net surplus is $S_{EI} \equiv \alpha(p_H x - r(r+1)) + (1-\alpha)y$.

It is easy to see that, whenever over-investment can be implemented in equilibrium, $S_{OI} > 0$. First, $S_{OI} > 0$ if and only if $(\alpha p_H + (1-\alpha)p_L)x > r(r+1) + y$. Second, in the over-investment region, $\lambda \geq \hat{\lambda}$ if and only if $(\alpha p_H + (1-\alpha)p_L)x \geq \frac{r(r+1)}{\lambda}$. The left-hand side of both inequalities is the same. Therefore, $\frac{r(r+1)}{\lambda} > r(r+1)+y$ implies that $S_{OI} > 0$. Note that $y < p_L x(1-\lambda)$ by Assumption **A4**. Thus, $r(r+1)+y < r(r+1)+p_L x(1-\lambda)$, and $r(r+1)+p_L x(1-\lambda) < \frac{r(r+1)}{\lambda}$ if and only if $r(r+1) > \lambda p_L x$, which holds by Assumption **A2**.

It is immediate that $S_{EI} > 0$, while $S_{EI} < S_{OI}$ if and only if $(1-\alpha)(p_L x - r(r+1) - y) > y$, which cannot hold because: (i) $p_L x < r(r+1)$ by Assumption **A2**; (ii) this, together with $y > 0$, implies that the left-hand side of the inequality must be negative, while the right-hand side is strictly positive. Thus, the net surplus can be ranked: $S_{EI} > S_{OI} > S_{UI}$. The corollary follows. $\square$

Proof of Corollary 2

Proof. When $\lambda \in [\underline{\lambda}, \overline{\lambda})$, lender expected profits are $\alpha(p_H x - r(r+1)) > 0$, where the inequality follows from Assumption **A1** that H projects are positive NPV. An H firm's expected profits are $(1-\lambda)p_H x > 0$ and an L firm gets y by staying out of the market.

Now, increase pledgeability to $\lambda' \in (\overline{\lambda}, \hat{\lambda})$, which requires that $\alpha \frac{p_H - p_L}{p_L} < r$. In this case, there is a market breakdown, which leaves the L firm indifferent to the increase in pledgeability, but makes the H firm worse off (as it now gets $y < (1-\lambda)p_H x$, by **A4**), and the lender worse off, as it does not capture any fraction of the H project's positive NPV.

When, instead, $\alpha \frac{p_H - p_L}{p_L} \geq r$, the increase in pledgeability to $\lambda' > \overline{\lambda}$ does not result in a Pareto loss, because a bad firm is better off, as it now receives funding. However, it does not amount to a Pareto improvement, because the lender is strictly worse off. $\square$

Proof of Lemma 2

Proof. A type θ-quality project generates a maximum expected payout of $p_\theta(\lambda x - D_0)$ at $t = 1$. An outsider is willing to invest only if $p_\theta(\lambda x - D_0) \geq r$. If this inequality holds, then re-investment is positive NPV. Therefore, competition drives the face value at $t = 1$ to be such that $p_\theta D_1 = r$, or, equivalently, $D_1^* = \frac{r}{p_\theta}$.

When $\lambda < \frac{r + p_\theta D_0}{p_\theta x}$, there are two cases. If $p_\theta D_0 + p_\theta(\lambda x - D_0) \geq r$ or, equivalently, if $p_\theta \lambda x \geq r$, then the inside lender wants to re-invest in the firm, i.e., we have *zombie lending*. Further, as the inside lender is a monopolist at $t = 1$, the face value will equal the full pledgeable cash, i.e., $D_1^* = \lambda x - D_0$. If, instead, $p_\theta \lambda x < r$, then re-investment by the inside lender is unprofitable, so projects of quality θ are terminated. $\square$

Proof of Proposition 6

Proof. If $\lambda \in [\underline{\lambda}, \overline{\lambda})$, then it is anticipated that low-quality projects will always be terminated at $t = 1$. As firms face a strictly positive opportunity cost of investment, only high-quality firms borrow and invest. One possibility is for the firm to be financed by outsiders at $t = 1$, which requires $\lambda \geq \frac{r+p_H D_0}{p_H x}$. In this case, with competition at $t = 0$, the initial creditors—who anticipate competition also at the re-investment date—break even by offering a face value of $D_0^* = \frac{r^2}{p_H}$. Therefore, $\lambda \geq \frac{r+p_H D_0}{p_H x} = \frac{r(r+1)}{p_H x} = \underline{\lambda}$, which always holds in this region. It follows that whenever $\lambda \in [\underline{\lambda}, \overline{\lambda})$, firms can borrow at $t = 1$ at the competitive rate $D_1^* = \frac{r}{p_H}$. $\square$

Proof of Proposition 7

Proof. If $\lambda \in [\overline{\lambda}, \hat{\lambda})$, then, as shown in Proposition 4, there is a market breakdown even with a monopolist lender. It follows that there cannot be investment with competition.

If $\lambda \geq \overline{\lambda}$, it is anticipated that all projects will receive re-investment at date one. There are two cases. First, if $\lambda < \frac{r+p_L D_0}{p_L x}$, then outsiders are not willing to provide funds at $t = 1$. In this case, at date zero it is anticipated that (i) the insider will be a monopolist if its project turns out to be of L quality; and (ii) the insider will provide funds at $t = 1$ as well. Thus, the zero profit condition for lenders at $t = 0$ reads $\alpha(p_H D_0 - r^2) + (1-\alpha)(p_L \lambda x - r(r+1)) = 0$, or, equivalently, $D_0 = \frac{r^2-(1-\alpha)(p_L\lambda x - r)}{\alpha p_H}$. As we hypothesized that $\lambda < \frac{r+p_L D_0}{p_L x}$, it must be that $\lambda < \frac{r}{p_L x} + \frac{r^2-(1-\alpha)(p_L\lambda x-r)}{\alpha p_H x}$. Solving the expression for λ yields $\lambda < \frac{r}{p_L x} + \frac{r^2}{(\alpha p_H+(1-\alpha)p_L)x} = \overline{\lambda} + \frac{r^2}{(\alpha p_H+(1-\alpha)p_L)x} \equiv \tilde{\lambda}$. Clearly, $\tilde{\lambda} > \overline{\lambda}$.

Alternatively, it could be that $\lambda \geq \frac{r+p_L D_0}{p_L x}$, in which case it is anticipated that at $t = 1$ there will be competition both for high and for low quality projects. Aa a result, competition at $t = 0$ implies that $(\alpha p_H + (1-\alpha)p_L)D_0 = r^2$, or, equivalently, that $D_0 = \frac{r^2}{(\alpha p_H+(1-\alpha)p_L)}$. As we hypothesized that $\lambda \geq \frac{r+p_L D_0}{p_L x}$, we have $\lambda \geq \overline{\lambda} + \frac{r^2}{(\alpha p_H+(1-\alpha)p_L)x} = \tilde{\lambda}$. Further, $\tilde{\lambda} > \hat{\lambda}$ if and only if $\alpha p_H + (1-\alpha)p_L > p_L$, which always holds. Finally, because $\partial\tilde{\lambda}/\partial\alpha < 0$ and $\tilde{\lambda} = 1 \iff \alpha\frac{p_H-p_L}{p_L} = \frac{r(r+1)-p_L x}{p_L x - r} > 0$ (where the last inequality follows from Assumptions **A2** and **A3**), we conclude that whenever $\alpha\frac{p_H-p_L}{p_L} < \frac{r(r+1)-p_L x}{p_L x-r}$ we have $\tilde{\lambda} > 1$, while when $\alpha\frac{p_H-p_L}{p_L} \geq \frac{r(r+1)-p_L x}{p_L x-r}$ we have $\tilde{\lambda} \leq 1$. $\square$