WARWICK

ECONOMICS

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January 2021 No: 1328

Warwick Economics Research Papers

ISSN 2059-4283 (online)

ISSN 0083-7350 (print)

Profiting from the poor in competitive lending markets with adverse selection*

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January 7, 2021

Abstract

We provide theoretical foundations for positive lender profits in competitive credit markets with asymmetric information, where potential borrowers have scarce collateralizable assets. Strikingly, when some borrowers have negative net present value projects, an equilibrium always exists in which lenders make positive profits, despite their lack of 'soft' information and free entry of competitors. We then establish that greater access to collateral for borrowers reduces lender profits, and we relate our findings to the empirical evidence on micro-credit, payday lending, and, more broadly, retail and small business financing.

Keywords: Adverse selection, positive profits, collateral, free entry, market breakdown, credit markets

JEL classification: D82, D86

*We thank Swarnava Biswas, Yixin Chen, Ramona Dagostino, Michael Gofman, Ron Kaniel, Jorge Lemus, Jeanine Miklos-Thal, Guillem Ordonez-Calafi, Christian Opp, Dmitry Orlov, Xuyuanda Qi, Michael Raith, Heikki Rantakari, Amir Sufi, Pavel Zryumov for helpful suggestions.

1 Introduction

Extending credit to high-risk, unwealthy individuals or firms tends to be very profitable for lenders, despite the significant risks involved. Examples include credit card companies, which profit disproportionately from low FICO score borrowers (Agarwal, Chomsisengphet, Mahoney and Stroebel (2015)), as well as payday lenders (Melzer (2011)) and micro-credit institutions (Karlan and Zinman (2010)).[1] How and why is this so? Providing a rational explanation for these lender profits remains an open challenge. First, as retail credit markets tend to be quite competitive, these profits cannot be dismissed as just oligopolistic rents.[2] Second, many retail lenders rely almost exclusively on credit scoring—a practice that has been reinforced by algorithms used by 'fintech' companies (Di Maggio and Yao (2020))—and so they do not seem to incur substantial costs related to the acquisition of soft information about individual borrowers that could reconcile their profits. Finally, even where soft information acquisition may be essential, such as in small business lending, its cost cannot fully account for the wide credit spreads that lenders charge small and medium enterprises (Schwert (2020), Chodorow-Reich et al. (2020)).

In this paper, we propose a parsimonious, yet robust, rational theory of lender profits in competitive credit markets plagued by adverse selection and limited collateralizable assets for borrowers. Our theory is built around the observation that, when many potential borrowers have negative net present value investment projects, credit flows require either a screening device that separates good borrowers from bad, or else it requires interest rates to be sufficiently high. Collateral can provide such a screening device when it is abundant (Besanko and Thakor (1987), Bester (1985)), but the evidence suggests that it is scarce in many settings. We show how, when borrowers have limited collateral, lenders must charge interest rates that are high enough to keep bad borrowers out; and at these rates, they make strictly positive profits on the pool of investing borrowers. Competing lenders

[1] See Stegman (2007) and Skiba and Tobacman (2019) for further evidence on payday lenders. According to Chuck Waterfield, founder of the no profit Micro-Finance Transparency, in micro-credit 'it isn't uncommon to find return on equity consistently in excess of 25%' (Wall Street Journal, August 11, 2015).

[2] Some may posit behavioral explanations, for example, that consumers do not observe or understand hidden fees from which lender profits originate. But, then competitive lenders should compete for the profits associated with hidden fees by reducing rates on observable dimensions.

cannot benefit from undercutting with lower interest (or collateral) rates, lest they draw bad borrowers in. We then derive the testable implications that follow from this insight.

We consider a credit market in which borrowers can have either positive or negative net present value investment projects, and seek financing from deep-pocketed lenders. Capital supply is competitive, in that there is free and costless entry of potential lenders. However, in contrast to most of the existing literature, we do not **assume** that capital market competition implies that lenders must make zero profits. Instead, we just assume that lenders are rational, which means that they must make non-negative profits, and we let the level of equilibrium profits be an outcome of the analysis, rather than an assumption. Consistent with the evidence, we also suppose that potential borrowers have very limited access to collateralizable assets, which means that a lender's recovery rate upon default is expected to be small. This rules out the possibility that good borrowers can credibly signal their identity by pledging enough collateral to their lenders.

We begin our analysis from the benchmark case in which borrowers have no access to collateral at all. This case is not realistic, as individuals and firms typically possess at least some durable assets that can be liquidated at a positive, perhaps small, price—e.g., vehicles, tools, land or dwellings. However, it helps to develop the economic intuition behind our results. In this special case, it is well known that the absence of collateral might trigger a market breakdown equilibrium, at which no credit is extended, where lender profits are zero (Tirole (2010), chapter 6). Our first observation is that this market breakdown can be a unique equilibrium, as argued in the literature, *if and only if* lender profits are exogenously *constrained* to be zero. Relaxing this constraint, we show that—absent collateral—a very different equilibrium always exists, in which low types stay out of the market, while high types invest, promising to repay all of the cash flows generated by the investment to lenders. In this equilibrium, which satisfies standard refinements, all borrowers get the same payoff as with a market breakdown. However, lenders are better off, as they extract the full surplus from the investment opportunity. Thus, the equilibrium allocation Pareto dominates the market-breakdown equilibrium and maximizes net social surplus.

Next, we turn our attention to the realistic case in which potential borrowers have access to limited collateralizable assets; qualitatively similar outcomes obtain when borrowers have a few dollars that can be used for a downpayment. From a theoretical standpoint, the model with limited collateral shares many similarities with its limit case of no collateral: just as in the no-collateral case, with limited collateralizable assets the zero-profit curves for lenders corresponding to different types of borrowers do not intersect. As a result, there does not exist a collateralized credit contract at which: (i) borrowers raise funds at their full information interest rate; and (ii) lender profits are zero on every borrower type. Thus, a natural question to ask is: which equilibrium is robust to the introduction of collateral? Is it the market breakdown or the positive-profit separating equilibrium?

Strikingly, we prove that when borrowers have access to some collateral the credit market never breaks down in equilibrium. This is because low types are less productive than high types, and so their participation constraint binds at a lower interest rate. Even when little collateral is available to borrowers, the no-investment allocation can still be broken by a high type offering a feasible interest rate just high enough to keep low types out. This deviation would make low types worse off relative to the status quo, at which they get their outside option payoff and do not invest. In contrast, the deviation is strictly profitable for high types, if lenders assign probability one that they are behind it, and hence accept the offer. It follows that the credit market never breaks down when borrowers have access to **any** amount of collateral, no matter how small.

In contrast, in our baseline two-borrower type setting with collateral, there always exists a unique separating equilibrium that satisfies the Intuitive Criterion. In this equilibrium, the high type pledges all its collateral, and it borrows at an interest rate just high enough to leave low types indifferent between mimicking or not. It follows that lenders make strictly positive profits from financing high types. Deviations to lower interest rates and/or reduced collateral cannot be profitable, as they would make low types strictly better off whenever there is a chance that lenders would accept the offer, and lenders would expect to lose money in such a circumstance. Thus, the Intuitive Criterion has no bite, and the separating equilibrium similarly survives stronger refinements, such as D1.

An important implication of this analysis is that, as access to collateral for borrowers improves, the equilibrium interest rate—which is the rate needed to keep low types out—falls, because low types fear losing a more valuable collateral. This implies that *increased* *access to collateral reduces lender profits*, a novel testable prediction that is consistent with the high profits earned by credit card companies on low FICO score borrowers, even though they make close to no profit on high FICO score borrowers, for whom there arguably is less of an adverse selection problem, and higher expected recovery rates on the collateral (Agarwal et al. (2015)). Of note, at this positive-profit equilibrium, the interest rate does *not* depend on the fraction of each type in the population. Therefore, the equilibrium is robust to the introduction of heterogeneous priors across agents over the distribution of types, or even ambiguity aversion over the fractions of types.

Our setting is particularly relevant for developing economies, where property rights are weak and poorly enforced, and where a large share of the population is financially constrained. In this context, it has been proposed that establishing stronger property rights—often referred to as titling—could relax financing constraints (De Soto (1989, 2000)). The empirical evidence suggests that titling, when properly enforced, does improve access to credit and reduces the cost of borrowing, cæteris paribus (see Paulson and Townsend (2004) and Woodruff (2001)). As our model predicts that *lender profits should decrease with titling*, a test of this hypothesis could shed light on the debate between proponents and detractors of micro-finance. If, indeed, lender profits fall, then this could suggest that the high profits made by micro-financiers are at least partially due to the presence of informational asymmetries, as many proponents of micro-finance have argued.

In the rest of the paper, we establish that our equilibrium characterizations extend robustly when we enrich our baseline model. To show that the restriction to a two-type setting does not affect our qualitative results, we also consider the polar-opposite case of a continuum of borrower types. In this case, as is well known, the Intuitive criterion has no bite, so we use the D1 criterion (Cho and Kreps (1987)) to refine the set of equilibria.

We find that, as in the baseline setting, lenders make profits in equilibrium (generically), borrowers offer as much collateral as possible, and all equilibria have the threshold property that the interest rate leaves the marginal unfunded type indifferent between raising financing and staying out of the credit market. In the continuous type setting, the analogue of the separating equilibrium with two types is a set of partial-pooling equilibria. This set is fully characterized by two cutoff types. The upper cutoff consists in the borrower type on which lenders would make zero profit under full information. In every equilibrium, all types higher than this cutoff (i.e., all positive-NPV borrowers) must raise financing. The lower cutoff is the type such that lenders make zero profit on the pool of all borrowers that invest. In every equilibrium, all borrowers less productive than this lower cutoff—who all have negative-NPV projects—never raise financing.

The reasoning above yields that an investment threshold can be supported in equilibrium only if it lies within these two cutoff-types. We next show that every investment threshold within the two cutoffs can be supported in equilibrium. Moreover, lender profits are strictly positive for any equilibrium threshold above the lower cutoff, while they are zero at the lower cutoff. This confirms that the positive lender profits in our two-type setting are not knife-edged, but instead emerge naturally not only with a finite number of types, but even with a continuum. Lender profits increase with the investment cutoff, and are highest at the efficient equilibrium in which only positive-NPV borrowers invest. Finally, just as in the two-type setting, lender profits in this efficient equilibrium decrease with a borrower's access to collateralizable assets.

We then return to the two-type setting to enrich the contracting space. Consistent with most of the literature, our baseline model rules out menus of contracts and stochastic mechanisms. While it is well known that stochastic mechanisms can never be part of an equilibrium in adverse selection models with negative-NPV types (Tirole (2010)),[3] in these settings there is scope for borrowers to offer menus of contracts that give high types incentives to invest and pay low types to stay out of the market. Such menus are rarely—if ever—observed empirically, leading Tirole (2010) to outline an argument for ruling them out. Tirole notes that menus may attract "fake entrepreneurs, who do not even have a project". He conjectures that, as a result, the fraction of low-quality firms "could quickly become very close to one, leading to a market breakdown".

This leads us to extend our model by introducing menus of contracts and allowing for entry of 'fake entrepreneurs', drawn by the possibility of collecting a payment from

[3]The intuition for why stochastic mechanisms do not help is straightforward: starting from an equilibrium in which high types invest with probability one and low types stay out, it is immediate that any deviation to a lower (higher) investment probability for high (low) borrower types can only reduce the surplus generated, without helping with incentive compatibility.

lenders in return for not investing.[4] We obtain three main results. First, regardless of the extent of entry by 'fake entrepreneurs', markets never break down when borrowers have some collateral. Second, the separating equilibrium with positive profits exists even when borrowers can deviate to menus of contracts. Third, with unlimited entry of 'fake entrepreneurs', the unique incentive-compatible menu is equivalent to the single contract that sustains the separating equilibrium with positive lender profits.

Related literature Our benchmark model without collateral is a canonical setting in which competitive lenders finance risky investments under ex-ante adverse selection, such as the one reviewed in Tirole (2010), chapter 6. In similar environments, seminal papers have argued that if borrowers have *enough* collateral, then they can credibly signal their quality to investors, which must therefore make zero profits in equilibrium (Besanko and Thakor (1987), Bester (1985)). In contrast, we consider borrowers with limited collateral, and highlight that the robust equilibrium entails positive lender profits.

Alternatively, researchers have argued that good borrowers may signal their types by relying on other costly signals such as high interim cash flows (Padilla and Pagano (1997)), borrowing from 'informed lenders' (Rochet and Tirole (1996)), selecting a tougher certifier (Lerner and Tirole (2006)), borrowing short-term (Diamond (1991)), retaining risky tranches of their investment portfolios (Leland and Pyle (1977)), or underpricing at the IPO, to subsequently obtain re-investment at cheaper terms (Welch (1989), Allen and Faulhaber (1989)). Our paper shows that positive lender profits must arise in equilibrium in the absence of these signaling opportunities—a case that seems empirically relevant when thinking about retail lending to unwealthy borrowers. Moreover, our model has no deadweight losses, implying that our positive-profits equilibrium is renegotiation proof, unlike settings where the use of costly signals induces deadweight losses in equilibrium.

More broadly, our model fits into a large stream of work that analyzes equilibrium properties of collateralized debt contracts (e.g., Rampini and Viswanathan (2013), Li, Whited and Wu (2016), Hartman-Glaser and Mann (2017), Calomiris, Larrain, Liberti and Sturgess (2017), DeMarzo (2019), Donaldson, Gromb and Piacentino (2020), Bern-

[4] Dávila and Hébert (2019) similarly consider entry by 'fake entrepreneurs', albeit in a different context.

hardt, Koufopoulos and Trigilia (2020)). This literature largely focuses on ex post moral-hazard problems with limited commitment, in which lender profits are zero in equilibrium. In contrast, we find that with adverse selection and scarce collateral, lender profits should be positive; and that they should fall with a borrower's access to collateralizable assets.

As our main application is retail lending to risky, unwealthy borrowers, it is natural to relate our work to that on micro-credit in developing countries. Theoretical papers in this literature do not typically allow for adverse selection, but rather build on moral-hazard models (e.g., versions of the Innes (1990) model—see the survey by Banerjee (2003)). This means that their models cannot deliver positive lender profits under competition, and hence—to fit the data, where micro-lenders tend to make large profits—they must introduce barriers to competition (e.g., Besley, Burchardi and Ghatak (2012)). Relative to this literature, we show that positive profits may arise not only because of limited competition among lenders, but also due to the limited information that lenders have about borrowers.

The paper unfolds as follows. Section 2 develops and solves the baseline model with two borrower types. Section 3 considers the case of a continuum of types. Section 4 introduces menus of contracts and 'fake entrepreneurs'. Section 5 concludes.

2 Baseline model

Our two date ($t = 0, 1$) economy features two types of borrowers that must obtain external credit to finance a project. A type $\theta \in \{\theta_H, \theta_L\}$ borrower has a project that requires an investment of \$1 at $t = 0$ to generate cash flows at $t = 1$ that equal x with probability θ, and are zero otherwise. A fraction $p \in (0, 1)$ of borrowers are high types with $\theta = \theta_H$. Thus, the mean borrower type is $\theta_0 := p\theta_H + (1 - p)\theta_L$. Each borrower type has access to some amount of *collateral* $c \geq 0$. Alternatively, the parameter c could capture the availability of cash for downpayment. Despite some formal differences, the model with collateral and that with cash are qualitatively similar. While we assume that the available collateral is the same for both types, our analysis trivially extends to encompass asymmetric collateral, and qualitative results are unchanged.[5] Importantly, we assume:

Assumption 1. *Only the high type has a positive NPV project:* $\theta_H x > 1 > \theta_L x$.

Assumption 2. *The average type is negative-NPV for lenders:* $\theta_0 x + (1 - \theta_0)c < 1$.

Borrowers can raise funds from competitive lenders. The risk-free rate is normalized to zero, and all agents are risk neutral. Thus, under full information, competitive lenders can lend to high types at a rate $R_H = \frac{1}{\theta_H}$, while low types are denied credit and don't invest. Note that our baseline model makes two simplifying restrictions on the contract space: (i) borrowers cannot offer a menu of contracts to lenders; and (ii) contracts are not stochastic—i.e., the probability of investment if a contract is accepted by a lender is one. The restriction to a deterministic mechanism is without loss of generality, because stochastic mechanisms do not survive standard refinements in models where the average borrower is negative-NPV, as our Assumption 2 imposes (see, e.g., the discussion in Tirole (2010), chapter 6). However, menus of contracts can be issued in a 'reasonable' equilibrium. Accordingly, we later explicitly allow for menus and show that our results continue to hold.

The game. We analyze a two-stage signaling game. In the first stage, a borrower proposes a *collateralized debt contract* $D = (R, C)$ to the competitive lenders, which consists of an interest rate R and an amount of collateral C. The lenders then form a belief $p'(D) := \Pr.[\theta = H|D]$ about the type of borrower that offered the contract. In the second stage, depending on this belief, lenders accept or reject the contract. If lenders accept, then investment, which is observable and verifiable, occurs and payoffs realize. If, instead, lenders reject the proposed funding terms, then the borrower consumes its endowment, which we normalize to zero. The expected payoff of a type θ borrower that offers a contract D is

$$U_\theta(D) := c + i[\theta(x - R) - (1 - \theta)C], \tag{1}$$

[5] Unlike in the literature that follows Akerlof (1970), such as Einav, Finkelstein and Cullen (2010), in our model a breakdown cannot be restored by resorting to heterogenous collateralizable assets across types. To see this, note that if the high type has more collateral, it could use it to signal its type and invest at its full-information rate. In contrast, if the low type has more collateral, then, to conceal its identity, it would never be optimal for it to pledge more than what the high type can pledge. Thus, in a credit market model with collateral, the average cost curve can never lie above the demand curve for all types, and so the type of breakdown that can arise in Akerlof-style settings never obtains here.

where $i \in \{0, 1\}$ denotes the decision of a lender. When $i = 1$, the lender funds the project. When $i = 0$, there is no financing. We focus on Perfect Bayesian Equilibria that satisfy the Intuitive Criterion of Cho and Kreps (1987):

Definition 1. *A Perfect Bayesian Equilibrium of the game that survives the Intuitive Criterion must satisfy the following properties:*

1. *Sequential rationality: borrower types propose contracts D_H^* and D_L^* optimally, given the lenders' beliefs and their associated optimal acceptance decision function $i^*(D)$;*
2. *Belief consistency: $p'(D_H^*)$ and $p'(D_L^*)$ are derived from Bayes' Rule;*
3. *Intuitive criterion: there does not exist a $D \notin \{D_H^*, D_L^*\}$ such that: $U_L(D|p') < U_L(D_L^*)$ for any $p' \in [0, 1]$ and $U_H(D|p' = 1) > U_H(D_H^*)$; or $U_H(D|p') < U_H(D_H^*)$ for any $p' \in [0, 1]$ and $U_L(D|p' = 0) > U_L(D_L^*)$.*

Whenever an equilibrium satisfies the conditions detailed in Definition 1, we refer to it as a *'reasonable' equilibrium*, as property (3) represents an equilibrium refinement.

2.1 Benchmark case: no collateral ($c = 0$)

With no collateral, it is easy to see that a separating equilibrium in which lenders earn zero profits and high types obtain funding at the full-information rate $R_H = \frac{1}{\theta_H}$ does not exist. This is because the zero-profit curves across types do not intersect when $c = 0$: the expected payoff to a low type from mimicking the high type and borrowing at R_H is $\theta_L(x - \frac{1}{\theta_H}) = \frac{\theta_L}{\theta_H}(\theta_H x - 1) \geq 0$, where the inequality follows from Assumption 1 that the high type has a positive net present value project. Thus, low types mimic and the contract becomes loss making for lenders.

Similarly, pooling equilibria with investment, in which both types of borrowers propose the same interest rate and lenders fund the project do not exist. By Assumption 2, the average borrower has a negative-NPV project. Therefore, the expected project proceeds cannot cover the unit investment required by a lender, and hence a lender cannot breakeven on the pool. It follows that, when $\theta_0 x < 1$, the celebrated market breakdown result obtains: there exists an equilibrium with no investment because at any feasible rate $R \leq x$ both types would enter the market, causing expected losses for lenders.

Our first observation is that this equilibrium is unique only if one *imposes exogenously* that lenders must make zero profits, and, in particular, that lenders cannot earn strictly positive profits in equilibrium. This restriction is not innocuous: Individual-rationality only requires that investors make non-negative profits, and whether or not free entry drives profits to zero should be an endogenous outcome of the model. Proposition 1 yields:

Proposition 1 (No collateral)**.** *When $c = 0$, the 'reasonable' Perfect Bayesian Equilibria of the two-stage signaling game are as follows:*

Separating: High types borrow at $R = x$, while low types do not invest. Lenders make strictly positive profits.

Breakdown: There is also an equilibrium in which no credit is extended.

Once one allows for non-negative lender profits, an equilibrium emerges in which the high type proposes $R = x$ and invests, while low types stay out of the market.[6] That is, a separating equilibrium always exists in which the low types are kept out through their participation constraint. In a model such as ours, this is the only form of separation that can obtain. With no collateral, both low and high-type borrowers make zero expected returns regardless of whether they offer to borrow at $R = x$ or not, leaving them indifferent between pursuing the investment and staying out. As a result, there is an equilibrium in which only high types enter, lenders make strictly positive profits of $\theta_H x - 1 > 0$, and net social surplus is maximized.

The co-existence of the market-breakdown equilibrium with this separating equilibrium underscores a key observation. Lenders can make strictly positive profits in equilibrium even though they have neither market power, nor an informational advantage. Interestingly, our analysis reveals that, in environments with ex-ante adverse selection, it is precisely the lenders' informational *dis*advantage that drives lender profits above zero.

We next extend the analysis to the more realistic setting of positive collateral, i.e., $c > 0$, and show that only a positive-profit separating equilibrium survives whenever the amount of collateralizable assets a borrower has access to is sufficiently scarce.

[6]Equivalently, in equilibrium, a low type may offer any $R < x$, which would then be rejected by lenders. A low type would earn strictly positive profits were that offer accepted so that any feasible $R < x$ could be offered by a low type on-the-equilibrium path.

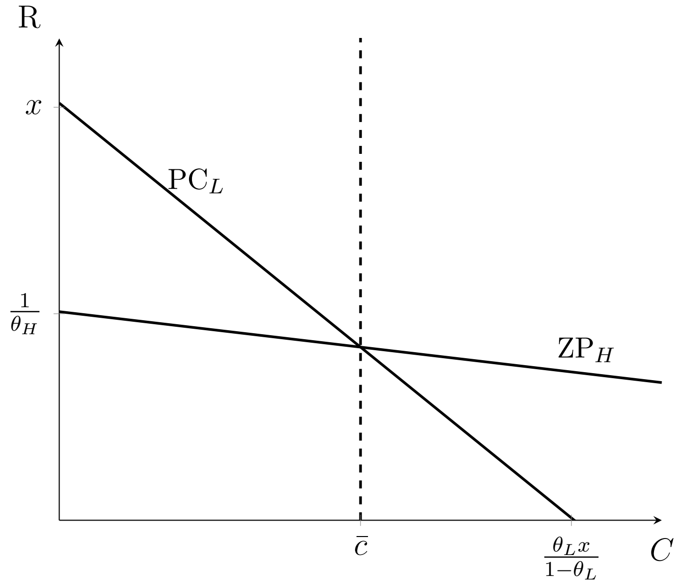
2.2 Positive collateral

When collateral abundant, the full-information contract becomes feasible, making the problem uninteresting. This occurs whenever c is so high that a low type prefers to stay out rather than mimic a high type and pledge enough of its collateral to obtain financing at the full-information rate $R_H = \frac{1-(1-\theta_H)c}{\theta_H}$. This happens whenever

$$c \geq \bar{c} \equiv \frac{\theta_L}{\theta_H - \theta_L}\left(\theta_H x - 1\right) > 0. \tag{2}$$

The last inequality holds as the high type has a positive NPV project (Assumption 1). We henceforth restrict attention to the interesting range of collateral $c \in (0, \bar{c})$. As Figure 1 illustrates, for such collateral levels, the zero-profit curve for a high type (denoted by ZP_H in the figure) never intersects the participation constraint for a low type (denoted by PC_L in the figure), so the qualitative driving forces of the model do not differ from those of the $c = 0$ case. And yet, despite these similarities, our first result shows that whenever collateralizable assets are positive, markets never break down in equilibrium.

Figure 1: Profit curves and feasible allocations



Lemma 1. *When $c > 0$, the market can never break down.*

Proof. All proofs are in the Appendix. □

The intuition behind Lemma 1 is that, when $c > 0$, there is always a deviation that a high type can make from the market-breakdown status quo that credibly signals its type. The deviation consists in offering a strictly positive amount of collateral $C \in (0, c]$ to lenders, in exchange for credit at an interest rate high enough that (i) low types strictly prefer to stay out of the market, as they lose their collateral and are not sufficiently likely to succeed in their investment; (ii) lender profits are strictly positive. The proof shows that such a deviation exists no matter how small c is. Thus, the market breakdown equilibrium cannot be 'reasonable' —it fails to satisfy the conditions detailed in Definition 1.

Lemma 1 highlights the fragility of the no-trade equilibrium. Not only is it *one* of multiple equilibria when $c = 0$, but it *only exists in this case.* In contrast, we now show that a positive-profits separating equilibrium exists regardless of the amount of collateral $c \in [0, \bar{c})$, proving that it is the only robust equilibrium of our model.

Lemma 2. *For every collateral $c < \bar{c}$, there exists a unique separating equilibrium in which high types propose a contract $D_B = (R_B, C_B)$ where $R_B := \frac{\theta_L x - (1-\theta_L)c}{\theta_L}$ and $C_B := c$. Low types either propose $D \neq D_B$ or do not seek funding. Lenders fund projects if and only if D_B is proposed, earning strictly positive profits.*

Lemma 2 shows that there always exists a unique separating equilibrium, and in this equilibrium investment is efficient—i.e., low types stay out of the credit market. The contract offered by high types leaves low types indifferent between raising financing or not. Because the interest rate reflects the break-even condition for low types, but is paid by high types, it follows that lenders earn strictly positive profits. Lenders will reject contracts with lower interest rates, as low types would profit from such contracts, which would make them unprofitable for lenders. In the unique equilibrium, high types pledge all of their available collateral c. Doing so reduces the rents that lenders extract from high types, as collateral is more costly to pledge for low types, because they are more likely to fail. Thus, an equilibrium with less collateral does not survive the Intuitive Criterion.

Of note, in this unique separating equilibrium both the interest rate and the collateral offered do not depend on the fraction of each type in the population. Thus, the equilibrium is robust to the introduction of heterogeneous priors across agents or even ambiguity aversion over the fractions of types. Moreover, lender profits are sustained in equilibrium even though they have no informational advantage over other investors, and despite free and costless entry of competitors. Finally, note that the pricing of loans according to R_B implies that lender profits are unaffected by small changes in the risk-free rate.

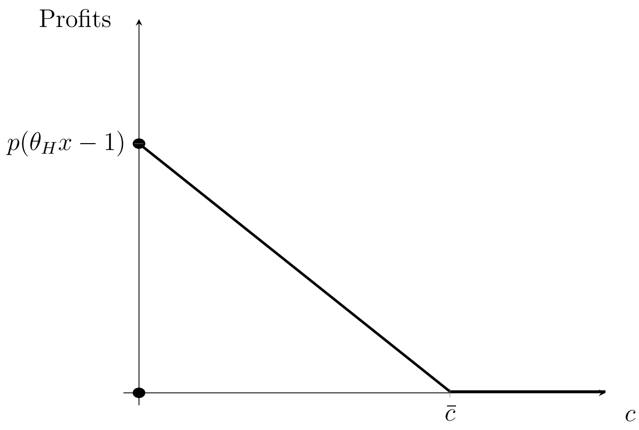
Proposition 2 summarizes the set of Perfect Bayesian Equilibria of the game.

Proposition 2 (Positive colletaral)**.** *When $c \in (0, \bar{c})$, there exists a unique 'reasonable' Perfect Bayesian Equilibrium of the two-stage signaling game. This equilibrium is separating: high types borrow at $R_B = \frac{\theta_L x-(1-\theta_L)c}{\theta_L}$ and set collateral $C_B = c$, while low types do not invest. Lenders make strictly positive expected profits from funding high types.*

The analysis thus far implies that starting from a market breakdown equilibrium and increasing the available collateral from zero to some tiny $\epsilon > 0$ leads to a discontinuous jump in expected lender profits from 0 to $p(\theta_H x - 1) - p\frac{(\theta_H-\theta_L)}{\theta_L}\epsilon > 0$. In sharp contrast, starting at some $c > 0$, the effect of increasing c on lender profits is continuous and decreasing. Figure 2 illustrates this result. When $c = 0$ there can be two levels of lender profits in equilibrium: zero or the full net present value of the high-type project. However, strikingly, once one increases the available collateral from zero to any small $\epsilon > 0$, the zero-profit, market-breakdown equilibrium vanishes, and the unique equilibrium features strictly positive lender profits. Such equilibrium can only be obtained by departing from the exogenous assumption that competitive lenders must make zero profits, as we do.

It is useful to compare the evolution of competitive lenders' *expected profits* for different degrees of borrower collateral in our model, depicted in Figure 2, with the evolution of *realized profits* by US credit card lenders across FICO scores found by Agarwal et al. (2015). Indeed, not only do Agarwal et al. (2015) find that 'the data suggest that credit cards were a very profitable segment of the banking industry, even at the height of the financial crisis', as 'across all FICO scores, profits average 1.6% of ADB', but they also found that most of these profits came from borrowers with FICO scores under 640—i.e., from riskier borrowers (see their Figure 3)—who also tend to have the least amount of collateralizable assets. Realized profits reached double digits only for borrowers between 520 and 600. While realized and expected profits should not be confounded, as the former do not account for risk,

Figure 2: Competitive lender profits in equilibrium



the evidence points to riskier borrowers as a major source of profits for lenders. A formal test of our model would compare lender profits before and after a shock to the value of borrowers' collateral, predicting a decline in profits when collateral become more available. A similar effect would be associated to a successful titling program, à la De Soto (2000).

3 Continuous types

We now establish that our main findings do not critically depend on the assumption that there are only two types of borrowers. To make this point forcefully, we consider the polar-opposite case of a continuum of types $\theta \in [\theta_L, \theta_H]$, distributed according to some strictly increasing cumulative distribution function $G(\theta)$. To maintain consistency with our baseline model, we modify Assumption 2 as follows:

Assumption 2'. *The average borrower type is negative-NPV for lenders:*

$$\int_{\theta_L}^{\theta_H} [\theta x + (1-\theta)c] dG(\theta) < 1.$$

To characterize the set of 'reasonable' equilibria with a continuum of types, we strengthen the refinement from the Intuitive criterion to the D1 criterion of Cho and Kreps (1987).

Definition 2. *A Perfect Bayesian Equilibrium of the game that survives the D1 refinement must satisfy the following properties:*

1. *Sequential rationality: borrower types propose contracts D^*_θ optimally for every θ, given the lenders' beliefs and their optimal acceptance decision function $i^*(D)$;*
2. *Belief consistency: $p'(D^*_\theta)$ is derived from Bayes' Rule, for every θ;*
3. *D1: for every (θ, θ'), there does not exist a $D \neq D^*_\theta$ such that $\{p' | U_{\theta'}(D|p') \geq U_{\theta'}(D^*_{\theta'})\} \subseteq \{p' | U_\theta(D|p') > U_\theta(D^*_\theta)\}$ and $U_\theta(D|\text{Full Information}) > U_\theta(D^*_\theta)$.*

Thus, in this section a 'reasonable' equilibrium consists of any PBE that satisfies D1.

Lemma 3. *Any 'reasonable' equilibrium consists of a partition of types into two pools, with a threshold $\hat{\theta} \in (\theta_L, \theta_H)$ such that each type $\theta < \hat{\theta}$ stays out of the market, while each type $\theta > \hat{\theta}$ borrows and invests using the same collateralized debt contract $D = (R, C)$.*

Lemma 3 shows that the only type of 'reasonable' equilibrium that can arise with a continuum of types features partial-pooling, with just two pools of types: a pool that invests at the same equilibrium contract, and a pool that stays out of the credit market. In such an equilibrium, a type θ invests only if $\theta(x - R) - (1 - \theta)C \geq 0$. Because the left-hand-side of this inequality increases in θ, an equilibrium is characterized by a single threshold $\hat{\theta} \in [\theta_L, \theta_H]$ such that all types $\theta \geq \hat{\theta}$ invest, and all other types stay out.

We next establish that equilibria with a single threshold for funding must feature full collateralization of a borrower's assets—that is, $C = c$.

Lemma 4. *Any 'reasonable' equilibrium features full collateralization of a borrower's asset, i.e., $C = c$.*

To see this result, totally differentiate a borrower's payoff with respect to R and C. This yields $-\theta dR - (1 - \theta)dC = 0$ if and only if $\frac{dR}{dC} = -\frac{(1-\theta)}{\theta}$, which is strictly increasing in θ. Therefore, starting from a candidate equilibrium with $C < c$, higher types can always find a contract (R', C') with $C' > C$ that (i) makes them better off if accepted by lenders, (ii) makes lower types worse off regardless of the lender's off-equilibrium beliefs, (iii) generates strictly positive profits for lenders on higher types.

We now establish that the cutoff-type $\hat{\theta}$ that is indifferent between borrowing or not in equilibrium can be neither *too low*, nor *too high*. Lemma 3 implies that the threshold type $\hat{\theta}$ must be indifferent between entering or not, so $\hat{\theta}(x-R)-(1-\hat{\theta})c=0$, or equivalently, $R(\hat{\theta})=\frac{\hat{\theta}x-(1-\hat{\theta})c}{\hat{\theta}}$. This is the same expression as R_B, but with $\hat{\theta}$ instead of θ_L. Let $\overline{\theta}$ be the zero-NPV borrower type, $1=\overline{\theta}x$, and let $\underline{\theta}$ be the lowest cutoff type at which lender profits are non-negative—that is, it solves $\mathbb{E}[\theta|\theta\geq\underline{\theta}]R(\underline{\theta})+(1-\mathbb{E}[\theta|\theta\geq\underline{\theta}])c=1$. Because the L project has a negative net present value, and G is strictly increasing and continuous, we have $\underline{\theta}\in(\theta_L,\theta_H)$.

Lemma 5. *In any 'reasonable' equilibrium, $\hat{\theta}\geq\underline{\theta}$, and all types $\theta\in[\overline{\theta},\theta_H]$ raise funding.*

To see the intuition behind Lemma 5, suppose, by contradiction, that there is an equilibrium in which the threshold $\hat{\theta}$ type exceeds $\overline{\theta}$. Then high types have a profitable deviation: they can offer a slightly lower interest rate to the lenders such that all borrowers who would want to invest at this rate still have a positive-NPV project. Thus, the deviation is profitable and such a high threshold cannot be part of an equilibrium.

Having established that investment thresholds can neither be below $\underline{\theta}$ nor exceed $\overline{\theta}$, we now show that any other threshold $\hat{\theta}\in[\underline{\theta},\overline{\theta}]$ can be sustained in a 'reasonable' equilibrium:

Proposition 3. *With a continuum of types, there exist a continuum of 'reasonable' equilibria, characterized by thresholds $\hat{\theta}\in[\underline{\theta},\overline{\theta}]$. Every type $\theta\geq\hat{\theta}$ offers a contract $D=(R(\hat{\theta}),c)$ and invests, while every type $\theta<\hat{\theta}$ stays out of the credit market. Lender profits are strictly positive whenever $\hat{\theta}>\underline{\theta}$, and zero otherwise.*

Proposition 3 clarifies the sense in which the equilibrium set depends on the cardinality of the type space. As in the two-type case, any equilibrium features investment only by sufficiently productive types. Moreover, generically, lenders make strictly positive profits in equilibrium. The main difference is that while now almost all equilibria feature positive lender profits, investment is almost always inefficient. Specifically, efficient investment occurs only at the *highest* possible level of equilibrium profits for competitive lenders:

Corollary 1. *With a continuum of types, in the unique efficient equilibrium $\hat{\theta}=\overline{\theta}$. Any other equilibrium features inefficient over-investment. Lender profits are strictly positive in every equilibrium with $\hat{\theta}>\underline{\theta}$, and they are zero in the non-generic case of $\hat{\theta}=\underline{\theta}$.*

Moreover, the highest level of lender profits—which obtains at the efficient-investment equilibrium—monotonically decreases with the extent of inefficient over-investment.

The first part of the Corollary follows from the definition of $\overline{\theta}$, which is the threshold-type with a zero-NPV project under full information—i.e., it solves $\overline{\theta}x = 1$. The comparative static on lender profits across equilibria follows because profits must monotonically increase with $\hat{\theta}$ on the relevant range $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$. Our analysis reveals that regardless of the number of borrower types, our simple asymmetric information framework provides a foundation for lender profits even in a competitive setting where lenders do not produce any soft information about borrowers, and borrowers are fully rational. The key ingredient that distinguishes our model from other settings in which lender profits are zero is that lenders face severe adverse selection, in the sense that they must concern themselves with the possibility that their borrowers have negative-NPV investment project.

Corollary 1 also highlights another important implication of our analysis, which is that interest rates can be 'too low' due to asymmetric information. This contradicts the common intuition that adverse selection reduces the prices of the securities issued by high-quality firms in credit markets, and hence lower interest rates are 'good'. In sharp contrast, we find that a planner seeking to maximize net social surplus would find it optimal to impose a *floor* on interest rates of $R(\overline{\theta})$. With such a policy, lender profits in equilibrium will be strictly positive, but the regulator can then tax them and redistribute the proceeds without distorting the allocation.

A second implication of Proposition 3 is that lender profits continue to depend negatively on the availability of collateral for borrowers. In the two-type case this prediction was immediate, due to the unique equilibrium. In the continuous-type case, there are multiple equilibria for every level of c. Nonetheless, the highest amount of profits that lenders can obtain in equilibrium falls monotonically with the availability of collateral:

Corollary 2. *The highest level of lender profits possible in equilibrium decreases with the level of collateral c available to borrowers.*

4 Menus and 'fake entrepreneurs'

Our baseline model presumes that borrowers cannot offer menus of contracts. However, as the literature has established (see, e.g., Tirole (2010), chapter 6), the planner's solution might involve all borrowers offering a pooling menu of contracts in which lenders pay low types not to invest. We now return to our two-type setting in order to extend the model to allow for menus of contracts, and allow for the possible entry of 'fake entrepreneurs' who are drawn by the possibility of collecting payments from lenders in return for not investing.

In full generality, borrowers can offer menus of contracts to lenders that consist of a tuple $\kappa = \{R_\theta, C_\theta, q_\theta, y_\theta\}_{\theta\in\{H,L\}}$, where R_θ is the interest rate paid by a type-θ borrower that invests, C_θ is its posted collateral, y_θ is a payment from the lender to a type θ that does not invest, and $q_\theta \in \{0,1\}$ denotes whether or not investment occurs. In the first stage of the game, borrowers offer a menu. In the second stage, lenders either accept or reject given their beliefs. If a lender accepts, then the borrower selects its preferred option from the menu: either the H- or L- tuple (i.e., κ_H or κ_L). By the revelation principle, the equilibrium menu can be designed so that L-types prefer the L option, and H-types prefer the H option. Thus, the following incentive compatibility conditions must hold:

$$\underbrace{q_H[\theta_H(x-R_H)-(1-\theta_H)C_H]+(1-q_H)y_H}_{\equiv U_H(\kappa_H)} \geq \underbrace{q_L[\theta_H(x-R_L)-(1-\theta_H)C_L]+(1-q_L)y_L}_{\equiv U_H(\kappa_L)} \quad (IC_H)$$

$$\underbrace{q_L[\theta_L(x-R_L)-(1-\theta_L)C_L]+(1-q_L)y_L}_{\equiv U_L(\kappa_L)} \geq \underbrace{q_H[\theta_L(x-R_H)-(1-\theta_L)C_H]+(1-q_H)y_H}_{\equiv U_L(\kappa_H)}. \quad (IC_L)$$

We also allow for the possibility that if lenders offer to pay low types to stay out, then this might attract *'fake entrepreneurs'*, effectively increasing the share of low types in the population. We model this in the simplest possible way, by assuming that whenever $y_L > 0$, the proportion of high types in the population falls to γp, for some $\gamma \in [0,1]$. This nests our baseline model as the special case of $\gamma = 1$, and it allows for free entry of bad types as the special case of $\gamma = 0$, as suggested by Tirole (2010).

Because high types are more productive than low types, we always have $U_H(\kappa_H) >$

$U_L(\kappa_H)$. Therefore, there exists a continuum of menus that satisfy both ICs. In what follows, when we refer to a menu we mean a *non-degenerate* menu—i.e., a menu with two distinct options—as a degenerate menu is equivalent to a single contract. We next detail properties of any non-degenerate equilibrium menu that satisfies the Intuitive Criterion:

Lemma 6. *For an equilibrium in which all types offer a non-degenerate menu of contracts to exist and satisfy the Intuitive Criterion: (i) IC_L must bind, (ii) low types must not invest ($q_L = 0$), (iii) the high type's contract is fully collateralized ($C_H = c$), and (iv) high types invest ($q_H = 1$).*

The intuition behind Lemma 6 is as follows. First, having less than full collateralization cannot be optimal for high types, as they have a lower cost of pledging them relative to low types. Second, having a positive probability of low types investing ($q_L > 0$), or high types not investing ($q_H < 1$) decreases the resources generated by the menu, which can be redistributed among the agents. Thus, again, it cannot be part of an optimal menu.

In light of Lemma 6, without loss of generality, we can set $y_H = 0$. IC_L, which binds, reads $y_L = \theta_L(x - R_H) - (1 - \theta_L)c$. At such a menu, lender profits are $\gamma p[\theta_H R_H + (1 - \theta_H)c] - (1 - \gamma p)y_L$ or, substituting IC_L inside, $\gamma p\theta_H R_H - (1 - \gamma p)(\theta_L(x - R_H) - (1 - \theta_L)c)$. Thus, profits are non-negative if:

$$R_H \geq \frac{(1 - \gamma p)[\theta_L x - (1 - \theta_L)c]}{\gamma p\theta_H + (1 - \gamma p)\theta_L} \equiv R_M. \tag{3}$$

The next Lemma establishes that with the measure one of 'fake entrepreneurs' suggested by Tirole (2010), the only incentive compatible menu offers a subsidy to low types of $y_L = 0$ and an interest rate for high types of $R_H = R_B$. That is, the unique incentive compatible menu is equivalent to the unique separating equilibrium with positive profits.

Lemma 7. *When $\gamma = 0$, the only incentive compatible menu is equivalent to the separating contract with positive profits: high types borrow at R_B, low types stay out and $y_L = 0$.*

This result follows because, from equation (3), $R_M(\gamma = 0) = R_B$. At any $R < R_B$, low types would prefer the H-option within the menu, violating incentive compatibility.

When, instead, $\gamma > 0$, equation (3) implies that $R_M < R_B$. It follows that there exists a continuum of equilibria in which menus are offered, indexed by the interest rate $R_H \in [R_M, R_B]$ received by high types and the associated positive subsidy $y_L = \theta_L(x - R_H) - (1 - \theta_L)c$ to low types that satisfy the Intuitive Criterion.

Proposition 4. *If one allows for menus, the separating allocation described in Proposition 2 remains a 'reasonable' equilibrium.*

In addition, when $\gamma > 0$ there exist 'reasonable' equilibria in which all types offer a menu with collateral $C_H = c$, interest rate $R_H \geq R_M$ and subsidy $y_L = \theta_L(x - R_H) - (1 - \theta_L)c$, where $R_H \leq R_B$. Lenders make non-negative profits and only high types invest.

As the measure of 'fake entrepreneurs' increases, R_B is unchanged, while $\partial R_M / \partial \gamma < 0$. Therefore, the range of possible menu equilibria shrinks.

5 Conclusions

Our paper considers markets plagued by severe adverse selection, where the average borrower has a negative-NPV investment project and limited access to collateralizable assets. We show that, instead of a market breakdown, the robust equilibrium features strictly positive lender profits, even though (i) lenders are competitive, in that there is free and costless entry; and (ii) lenders do not generate any soft information about borrowers. This equilibrium is robust to the introduction of menus of contracts, and lender profits can be taxed up to 100% and redistributed, without introducing any market distortion. The qualitative features of positive lender profits hold even with a continuum of borrower types, in which case lender profits are highest at the efficient equilibrium where only borrowers with a positive-NPV project invest, and they decrease with the extent of over-investment by negative-NPV types. Thus, our asymmetric information model reconciles the high lender profits found in many competitive, retail credit markets.

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Appendix: Proofs

Proof of Lemma 1

Suppose the market breaks down in equilibrium. Consider a deviation by the high type in which it offers a contract D that yields the low type a negative payoff from mimicking, i.e., offering a pair $(R, C = c)$ that satisfies $\theta_L(x - R) - (1 - \theta_L)c = -\epsilon$, for a small $\epsilon \in (0, \frac{\theta_H - \theta_L}{\theta_L}c)$. Solving this for R yields $R = x - \frac{(1-\theta_L)c-\epsilon}{\theta_L}$. If such a contract is accepted by lenders, low types would be strictly worse off than at the market breakdown. As for high types, their payoff if lenders accept is

$$\theta_H(x - R) - (1 - \theta_H)c = \frac{\theta_H - \theta_L}{\theta_L}c - \frac{\theta_H}{\theta_L}\epsilon > 0,$$

where the last inequality follows from $\epsilon < \frac{\theta_H - \theta_L}{\theta_L}c$. Lender profits at the deviant contract, conditional on the offer coming from a high type, read $\theta_H R + (1 - \theta_H)c = \theta_H x + \frac{\theta_L - \theta_H}{\theta_L}c + \frac{\theta_H}{\theta_L}\epsilon \geq 1 \iff \frac{\theta_H - \theta_L}{\theta_L}c \leq \theta_H x - 1 + \frac{\theta_H}{\theta_L}\epsilon$. Because $\epsilon > 0$, and since $c < \bar{c} = \frac{\theta_L}{\theta_H - \theta_L}(\theta_H x - 1)$, the inequality always holds. Thus, by the Intuitive Criterion we must have $p'(D) = 1$ and the market does not break down.

Proof of Lemma 2

Existence. Suppose that high types offer a contract D_B that makes the participation constraint of low types bind:

$$\theta_L(x - R_B) - (1 - \theta_L)c = 0 \iff R_B = \frac{\theta_L x - (1 - \theta_L)c}{\theta_L}. \tag{4}$$

This contract can be sustained in equilibrium by the belief that D_B is offered by a high type, while any other D observed off-equilibrium is believed to be offered by a low type. A deviation to a contract with a lower interest rate and/or collateralization can only benefit a type if lenders accept it with positive probability. A low type would always deviate if this acceptance probability is positive, and a high type would only deviate if the acceptance probability is high enough.[7] Thus, this separating equilibrium with positive profits satisfies the Intuitive Criterion.

Uniqueness. A separating equilibrium exists only if $R \geq R_B$. Were $R < R_B$, a low type would also pool on R, as it could earn strictly positive profits if funded. First, fix D to be such that $(R \in (R_B, x), C = c)$. In such an equilibrium, a low type would stay out, but a high type would be willing to propose it. Low types would be made strictly worse off by deviating to $R' \in (R_B, R)$ were the offer accepted because their expected payoff from offering this rate would be strictly negative, and thus below their status quo payoff from staying out. In contrast, high types would be strictly better off offering an $R' \in (R_B, R)$ so lenders must believe that $p'(D) = 1$; and with this belief, the lenders would accept

[7] Indeed, if the low type proposed an interest rate $R \in (0, R_B)$, it would earn strictly positive profits if accepted, so such an offer can also be observed on the equilibrium path.

because the high type's project has a strictly positive net present value. Thus, any separating equilibrium with $C = c$ and $R > R_B$ does not satisfy the Intuitive Criterion.

It remains to show that there cannot be a 'reasonable' separating equilibrium with $C < c$. Suppose, by contradiction, that there exists a 'reasonable' PBE at which the high type offers $D' = (R', C' < c)$ such that lender profits are non-negative. Moreover, suppose that at D' the participation constraint for a low type binds—if it holds with an inequality, the conclusion will hold a fortiori. It follows that $\theta_L(x - R') - (1 - \theta_L)C' = 0 \iff R' = \frac{\theta_L x - (1-\theta_L)C'}{\theta_L}$. The high type's expected payoff at D' reads $\theta_H(x - R') - (1 - \theta_H)C' = \theta_H(x - \frac{\theta_L x - (1-\theta_L)C'}{\theta_L}) - (1 - \theta_H)C' = \frac{\theta_H - \theta_L}{\theta_L}C'$. Consider now the deviation to another contract $D = (R = R_B + \epsilon, C = c)$, for some small $\epsilon > 0$. The deviant contract can never make a low type better off, as the same contract with $R = R_B$ makes the low type's participation constraint binding. Moreover, the contract makes strictly positive profits for lenders if $p'(D) = 1$. It remains to check whether high types are strictly better off at D relative to D', conditional on $p'(D) = 1$. The expected payoff for a high type at D conditional on $p'(D) = 1$ is $\theta_H(x - R_B - \epsilon) - (1 - \theta_H)c = \theta_H(x - \frac{\theta_L x - (1-\theta_L)c}{\theta_L} - \epsilon) - (1 - \theta_H)c = \frac{\theta_H - \theta_L}{\theta_L}c - \theta_H \epsilon$. This is greater than the payoff at D' if and only if: $\frac{\theta_H - \theta_L}{\theta_L}c - \theta_H \epsilon > \frac{\theta_H - \theta_L}{\theta_L}C'$, or, equivalently, $\frac{\theta_H - \theta_L}{\theta_L}(c - C') > \theta_H \epsilon$. Because $c > C'$, there always exists an $\epsilon > 0$ small enough so that the inequality holds. Thus, there is no separating equilibrium where the collateral C is less than c.

Proof of Lemma 3

Suppose, by contradiction, that there exists a multi-threshold equilibrium, in which at least two contracts are offered. By monotonicity, there must exist at least two threshold types in this case. We index them from the highest to the lowest by $i = 1, ..., N$, so that $\hat{\theta}_1 > ... > \hat{\theta}_N$. By construction, types $\theta < \hat{\theta}_N$ do not raise financing. Denote the equilibrium contract offered by a type $\hat{\theta}_i$ by $D_i = (R_i, C_i)$.

Claim 1: *For $i < N$, the threshold-types $\hat{\theta}_i$ must be such that $\hat{\theta}_i x > 1$.*

Proof. Suppose not, and $\hat{\theta}_i x \leq 1$ for some $i < N$. Then, contract D_i must be loss making for lenders, contradicting the premise that every type above $\hat{\theta}_N$ invests in equilibrium. □

Claim 2: *In equilibrium, $C_1 = c$.*

Proof. Suppose not, and $C_1 < c$. Because we must have $R_1 > 0$ for lenders to be willing to invest, we can consider the deviation to $D' = (R' = R_1 - \epsilon, C' = C_1 + \delta)$ for $(\epsilon, \delta) \gg 0$. A type θ is better off if the deviant contract contract is accepted by lenders when $\theta(x - R_1) - (1-\theta)C_1 \leq \theta(x - R') - (1-\theta)C' \iff \theta \geq \frac{\delta}{\epsilon+\delta} := \tilde{\theta}$, where $\tilde{\theta}$ is the investment threshold at the deviation. The highest types strictly benefit from the deviation if and only if $\theta_H > \frac{\delta}{\epsilon+\delta} \iff \delta < \frac{\theta_H \epsilon}{1-\theta_H} \equiv \delta_1$. Moreover, the new threshold $\tilde{\theta}$ is a positive-NPV borrower if and only if $\overline{\theta} \leq \tilde{\theta} = \frac{\delta}{\epsilon+\delta} \iff \delta \geq \frac{\epsilon}{x-1} \equiv \delta_2 > 0$, where $x > 1$ by Assumption 1. Finally, the lowest possible value of the deviation for a lender consists in it being offered by the threshold type $\tilde{\theta}$ with probability one, in which case lender profits are

$$\tilde{\theta}R' + (1-\tilde{\theta})C' - 1 = \underbrace{\tilde{\theta}R_1 + (1-\tilde{\theta})C_1 - 1}_{\geq 1 \text{ as } D_1 \text{ was accepted, and } \tilde{\theta} > \overline{\theta}} + \underbrace{(1-\tilde{\theta})\delta - \tilde{\theta}\epsilon}_{\geq 0 \iff \delta > \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \equiv \delta_3} \geq 0.$$

It remains to verify that the set of $\delta \in [\max\{\delta_2, \delta_3\}, \delta_1)$ is non-empty. To see that this is so, observe first that $\delta_2 \leq \delta_3 \iff \frac{\epsilon}{x-1} \leq \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \iff \tilde{\theta}x - 1 \geq 0$, which always holds because $\tilde{\theta}$ is a positive-NPV borrower. Moreover, $\delta_1 \geq \delta_3 \iff \frac{\theta_H \epsilon}{1-\theta_H} \geq \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \iff \tilde{\theta} \leq \theta_H$, which always holds. Thus, there exists a pair (ϵ, δ) that makes the deviation profitable for high types, contradicting the optimality of D_1. □

It is immediate from Claim 1 that $D_2 = (R_2 > R_1, C_2 < C_1 = c)$. This is because: (i) by feasibility, we cannot have $C_2 > c$; (ii) if $R_2 \leq R_1$, then every type above $\hat{\theta}_1$ would prefer contract 2; and (iii) the only other possibility is $D_1 = D_2$, which contradicts the multi-threshold presumption. Therefore, $D_2 = (R_2 > R_1, C_2 < c)$.

Claim 3: *In equilibrium, $C_2 = c$.*

Proof. Suppose not, and $C_2 < c$. Because we must have $R_2 > 0$ for lenders to be willing to invest, we can consider the deviation to $D' = (R' = R_2 - \epsilon, C' = C_2 + \delta)$ for $(\epsilon, \delta) \gg 0$. A type θ is better off if the deviant contract contract is accepted by lenders when $\theta(x - R_2) - (1-\theta)C_2 \leq \theta(x - R') - (1-\theta)C' \iff \theta \geq \frac{\delta}{\epsilon+\delta} := \tilde{\theta}$, where $\tilde{\theta}$ is the investment threshold at the deviation. The threshold type strictly benefit from the deviation if and only if $\hat{\theta}_1 > \frac{\delta}{\epsilon+\delta} \iff \delta < \frac{\hat{\theta}_1\epsilon}{1-\hat{\theta}_1} \equiv \delta_1$. Moreover, the new threshold $\tilde{\theta}$ consists in a positive-NPV borrower if and only if $\overline{\theta} \leq \tilde{\theta} = \frac{\delta}{\epsilon+\delta} \iff \delta \geq \frac{\epsilon}{x-1} \equiv \delta_2 > 0$, where $x > 1$ by Assumption 1. Finally, the lowest possible value of the deviation for a lender consists in it being offered by the threshold type $\tilde{\theta}$ with probability one. As shown in Claim 2, there exists a pair (ϵ, δ) that makes the deviation profitable for high types, contradicting the optimality of D_2. □

It follows that one must have $R_1 = R_2$, contradicting the multi-threshold presumption.

Proof of Lemma 4

Suppose, by contradiction, that there exists an equilibrium that satisfies D1 at which $D = (R, C < c)$. At the equilibrium, a type invests only if $\theta(x - R) - (1 - \theta)C \geq 0$, and so the threshold type that invests is such that $\hat{\theta}(x - R) - (1 - \hat{\theta})C = 0$. To satisfy the participation constraint of lenders, we must have that the interest rate is strictly positive: $R > 0$. We establish our Lemma by proving two claims.

Claim 1: *The threshold-type $\hat{\theta}$ must be such that $\hat{\theta}x \leq 1$.*

Proof. Suppose not, and $\hat{\theta}x > 1$. Consider the deviation to an alternative contract $D' = (R' = R - \epsilon, C' = C)$, for some $\epsilon > 0$. The new marginal type is $\hat{\theta}'(x - R') - (1 - \hat{\theta}')C = 0 = \hat{\theta}(x - R) - (1 - \hat{\theta})C = 0 \iff (\hat{\theta}' - \hat{\theta})(x - R + C) + \hat{\theta}'\epsilon = 0 \iff \hat{\theta}' = \hat{\theta} - \frac{\hat{\theta}'\epsilon}{x - R + C} < \hat{\theta}$. If ϵ is sufficiently small, we have $\hat{\theta}' > \overline{\theta} := \frac{1}{x}$. It follows that, regardless of a lender's off-equilibrium beliefs,the lender must always accept the deviant contract. As a consequence, any type $\theta > \hat{\theta}'$ strictly prefers the deviation, contradicting the premise that the contract D can be sustained in equilibrium. □

Claim 2: *If $\hat{\theta}x \leq 1$, then the contract D cannot satisfy D1.*

Proof. Consider the deviation to an alternative contract $D' = (R' = R - \epsilon, C' = C + \delta)$, for $(\epsilon, \delta) \gg 0$. A type θ is better off if the deviant contract contract is accepted by lenders when $\theta(x - R) - (1 - \theta)C \leq \theta(x - R') - (1 - \theta)C' \iff \theta \geq \frac{\delta}{\epsilon + \delta} := \tilde{\theta}$, where $\tilde{\theta}$ is the investment threshold at the deviation. The highest types strictly benefit from the deviation if and only if $\theta_H > \frac{\delta}{\epsilon + \delta} \iff \delta < \frac{\theta_H \epsilon}{1 - \theta_H} \equiv \delta_1$. Moreover, the new threshold $\tilde{\theta}$ consists in a positive-NPV borrower if and only if $\overline{\theta} \leq \tilde{\theta} = \frac{\delta}{\epsilon + \delta} \iff \delta \geq \frac{\epsilon}{x - 1} \equiv \delta_2 > 0$, where $x > 1$ by

Assumption 1. Finally, the lowest possible value of the deviation for a lender is when it is being offered by the threshold type $\tilde{\theta}$ with probability one, in which case lender profits are

$$\tilde{\theta}R' + (1-\tilde{\theta})C' - 1 = \underbrace{\tilde{\theta}R + (1-\tilde{\theta})C - 1}_{\geq 1 \text{ as } D \text{ was accepted, and } \tilde{\theta} > \hat{\theta}} + \underbrace{(1-\tilde{\theta})\delta - \tilde{\theta}\epsilon}_{\geq 0 \iff \delta > \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \equiv \delta_3} \geq 0.$$

It remains to verify that the set of $\delta \in [\max\{\delta_2, \delta_3\}, \delta_1)$ is non-empty. To see this, observe first that $\delta_2 \leq \delta_3 \iff \frac{\epsilon}{x-1} \leq \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \iff \tilde{\theta}x - 1 \geq 0$, which always holds because $\tilde{\theta}$ is a positive-NPV borrower. Moreover, $\delta_1 \geq \delta_3 \iff \frac{\theta_H \epsilon}{1-\theta_H} \geq \frac{\tilde{\theta}\epsilon}{1-\tilde{\theta}} \iff \tilde{\theta} \leq \theta_H$, which always holds. Thus, there exists a pair (ϵ, δ) that make the deviation profitable for high types, contradicting the optimality of the contract D. □

Proof of Lemma 5

Suppose by contradiction that there is an equilibrium with investment threshold $\theta' > \overline{\theta}$. Consider a deviation by some high type $\theta \geq \theta'$ to a contract $D = (R^-, c)$, where $R^- := R(\theta') - \epsilon$, for some small $\epsilon > 0$. Because $\theta' > \underline{\theta}$, we know that at $R(\theta')$ lenders make strictly positive profits. Is the deviation profitable? To answer, we need to figure out what inference can be made off-equilibrium by lenders about the type of borrower behind the deviation. First, it is easy to rule out that the deviation comes from any type for whom the participation constraint at R^- is violated. That is, a lender can put positive probability only on types such that $\theta(x - R^-) - (1-\theta)c \geq 0 \iff \frac{\theta' - \theta}{\theta\theta'}c \leq \epsilon$. In particular, ϵ can be chosen to be small enough that the set of types for whom the deviation might be profitable only includes types $\theta > \overline{\theta}$. Once all types below $\overline{\theta}$ are excluded, then lenders make strictly positive profits on the deviation regardless of their beliefs. This contradicts that the conjectured equilibrium was ‘reasonable’.

Proof of Proposition 3

It remains to prove that every investment threshold $\hat{\theta} \in [\underline{\theta}, \overline{\theta}]$ can be sustained as a ‘reasonable’ equilibrium. To see this, consider the off-equilibrium deviation for a high type $\theta > \hat{\theta}$ to a contract $D' = (R(\hat{\theta}) - \epsilon, c)$. For such a type, who was investing at the conjectured equilibrium, the deviation is strictly profitable if accepted by lenders, and strictly worse than the equilibrium allocation if lenders reject it. In contrast, for a type $\theta = \hat{\theta}$ and for small set of types below $\hat{\theta}$, the measure of which is defined by the parameter ϵ, the deviation makes them strictly better off if accepted by lender, and indifferent otherwise. Thus, D1 forces us to conclude that the deviation comes from a set of type bounded above by $\hat{\theta}$. But because $\hat{\theta} < \overline{\theta}$, lenders expect to make losses at this deviation. It follows that the conjecture equilibrium satisfies D1.

Proof of Corollary 2

From Corollary 1, we know that, for every c, lender profits are maximized at the threshold $\hat{\theta} = \overline{\theta}$. At this threshold, lender profits are

$$
\begin{aligned}
\mathbb{E}[\theta|\theta \geq \overline{\theta}]R(\overline{\theta}) + (1 - \mathbb{E}[\theta|\theta \geq \overline{\theta}])c &= \mathbb{E}[\theta|\theta \geq \overline{\theta}]\frac{\overline{\theta}x - (1-\overline{\theta})c}{\overline{\theta}} + (1 - \mathbb{E}[\theta|\theta \geq \overline{\theta}])c \\
&= \mathbb{E}[\theta|\theta \geq \overline{\theta}]x - c\frac{\mathbb{E}[\theta|\theta \geq \overline{\theta}] - \overline{\theta}}{\overline{\theta}}
\end{aligned}
$$

Differentiating lender profits with respect to c yields $-\frac{\mathbb{E}[\theta|\theta \geq \overline{\theta}]-\overline{\theta}}{\overline{\theta}} \leq 0$, where the inequality follows from the fact that $\mathbb{E}[\theta|\theta \geq \overline{\theta}] \geq \overline{\theta}$, and is strict whenever $\overline{\theta} < \theta_H$, which, in turn occurs whenever the θ_H project has a strictly positive NPV, $\theta_H x > 1$.

Proof of Lemma 6

First, suppose that an equilibrium exists in which IC_L is slack. Then deviating to an incentive compatible, feasible menu with a lower y_L (or a higher R_L) together with a slightly lower R_H makes high types better off whenever it is accepted by the lender, but it would make low types worse off. The Intuitive Criterion then implies that the deviation must come from a high type, leading to its acceptance and making it profitable. Thus, IC_L must bind. That $C_H = c$ follows immediately from $\theta_L < \theta_H$. That an equilibrium with investment at which $q_H = 0$ cannot exist is immediate from the fact that the only positive-NPV borrower type is $\theta = \theta_H$. Finally, suppose that $q_L = 1$. It is immediate that IC_L and IC_H jointly hold if and only if $R_H = R_L$, which is equivalent to a pooling equilibrium with a single contract.

Proof of Proposition 4

First, we establish that our separating equilibrium with a single contract and positive lender profits remains an equilibrium in the presence of menus. At this equilibrium, high types offer R_B, low types stay out and, therefore, they consume their outside option which we normalized to zero. Thus, they would be weakly better off offering any incentive compatible menu κ at which $y_L \geq 0$. High types would only benefit if the menu is accepted, as $R_H \leq R_B$. Therefore, the Intuitive Criterion does not bite.

Second, to see that a continuum of menu equilibria exist, consider one such equilibrium with interest rate $R_H \in (R_M, R_B)$. Deviating to a menu with a lower interest rate $R' \in (R_M, R_H)$ benefits both types if and only if the deviation is accepted by the lenders. Thus, the Intuitive Criterion does not bite. That $\partial R_M / \partial \gamma < 0$ is immediate from (3).