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Separating equilibria, under-pricing and security design*

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Abstract

Classical security design papers equate competitive capital markets to securities being fairly priced in expectation. We revisit Nachman and Noe (1994)'s adverse selection setting, modeling capital-market competition as free entry of investors, and allowing firms to propose prices of securities, as happens in private securities placements and bank lending. We show that separating equilibria exist in which high types issue *under-priced* debt, while low types issue more informationally-sensitive securities (e.g., equity). We also uncover pooling equilibria in which firms issue under-priced debt. These results provide foundations for the pecking-order theory of external finance, and positive profits for uninformed lenders.

Keywords: Adverse selection, strictly positive profits, security design

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1 Introduction

Which securities should be issued to finance investments by firms with information that investors lack? And will these securities be fairly priced? Seminal papers by Myers (1984) and Myers and Majluf (1984) posed these questions, developing a 'pecking-order theory' according to which debt is preferred to equity as it minimizes the under-pricing of the securities issued by the best firms. Nachman and Noe (1994) (henceforth, NN) provides foundations for this theory, showing that within the class of monotonic securities that satisfy limited liability *and zero investor profits*, issuing risky debt is uniquely optimal.

This result has two caveats. First, NN predict that *all* firms would pool on the same debt contract. Thus, as Leary and Roberts (2010) observe, 'strictly speaking, the pecking order does not allow for any savings behavior or equity issuances'. Therefore, NN cannot be used to micro-found empirical tests based on firms' heterogenous financing choices.[1] Second, the assumption that this pooling debt contract *must* be fairly priced in expectation conflicts with the evidence that many securities issued by firms appear to be under-priced.[2]

This papers relaxes the assumption that investor profits must be zero, and shows how adverse selection generates both a pecking-order theory, and under-pricing of securities despite free and costless entry of investors. Specifically, we prove that separating equilibria exist in NN's environment in which the best types issue under-priced debt, while the worst types issue an alternative, fairly-priced security (e.g., equity) that is more informationally-sensitive than debt (DeMarzo, Kremer and Skrzypacz (2005)). In a separating equilibrium, competitive investors make strictly positive profits on the debt issued by best types. This is because this debt is priced so that investors would make zero profits if it were issued by the worst type. We then prove that pooling equilibria exist in

[1]Common tests ask whether 'better firms' rely relatively more on debt (e.g., Frank and Goyal (2003), Fama and French (2005)). These tests are inconsistent with NN's pooling equilibrium. As Leary and Roberts (2010) note, they test a 'modified pecking-order theory' that lacks solid foundations.

[2]See, for example, the literature on the *credit spreads puzzle* (Bai, Goldstein and Yang (2020)) and on the cost of bank loans (Chodorow-Reich, Darmouni, Luck and Plosser (2020)).

which firms pool on under-priced debt, i.e., debt priced below its expected pooling value.

We obtain two testable predictions. First, worse firms issue less debt, which holds strictly in any separating equilibrium, and trivially under pooling. Second, better firms issue under-priced debt, which is a *generic* feature of equilibrium—i.e., it holds at every separating equilibrium and every pooling equilibrium *except* NN's. The contrast with NN's result reflects the fact that, when non-negative investor profits are allowed, it is no longer without loss to constrain firms to only issue *unpriced* securities, as NN implicitly do, committing to selling the securities at some auction price. Indeed, firms routinely offer *priced securities*, for example negotiating loans with banks at proposed interest rates, or seeking to privately place a certain number of shares with an investor.

To understand what sustains a separating equilibrium with positive profits despite free entry of investors, one needs to start from the three properties that must hold in any 'reasonable' separating equilibrium of classic security design settings, such as NN. First, low-type firms are indifferent between mimicking or not the equilibrium security choice of high types—i.e., their incentive constraint binds. Second, securities issued by low types are fairly priced. Third, zero-profit curves across types do not intersect. Together these properties imply that investors must make profits on high-type firms in any separating equilibrium. Equilibria in which investors expect zero profits must be pooling (Innes (1993), Nachman and Noe (1994), DeMarzo and Duffie (1999)).

Unlike these papers, we do not impose zero investor profits. To see why 'reasonable' equilibria with strictly positive investor profits exist, consider a separating equilibrium in which high-type firms issue a *priced debt* security—e.g., a bank loan—that just covers external financing needs, and low types issue equity that also just meets their financing needs. The properties above imply that investors must break even on the debt were it issued by *low-type* firms. Thus, any deviation that could benefit high-type firms would also appeal to low-type firms. This includes offering off-equilibrium an unpriced debt contract, such as a public bond, as in NN. Therefore, even the strong D1 refinement of Cho and Kreps (1987) has no bite, and pessimistic off-equilibrium beliefs sustain a 'reasonable' separating equilibrium with positive profits.

Turning to pooling equilibria, we first confirm that an equilibrium exists in which all firms pool on issuing an unpriced debt contract, as in NN, and investors break even. Then, we complete this characterization by showing that this is the sole equilibrium allocation that features zero investor profits, while a continuum of equilibria exists in which firms pool on issuing under-priced debt. The most severe under-pricing corresponds to the case in which debt is priced so that investor profits would be zero if the debt was issued by low-type firms, as under separation. Thus, *all firms are worse off under separation than pooling*, even though equilibrium contracts are optimal and renegotiation-proof.

Comparing pooling and separating equilibria reveals a fundamental contrast between our framework and costly signaling models. Consider, for instance, the literature on under-pricing in IPOs (e.g., Welch (1989), Grinblatt and Hwang (1989) or Allen and Faulhaber (1989)). In these settings, high types credibly signal their identity by leaving money on the table in the IPO market, in order to enjoy reduced financing costs in future SEOs (Seasoned Equity Offerings). Thus, accounting for SEOs, high types *always* benefit from separating. In contrast, high types in our model *never* benefit from separation, because they issue under-priced debt once, and there are no subsequent financing rounds.

To our knowledge, it is a novel feature of our model that competitive investors can make positive profits despite the 'single crossing' of indifference (iso-profit) curves. To see why, it helps to contrast our model and the security design problem under adverse selection studied in Brennan and Kraus (1987), in which firms start with existing assets and exogenous amounts of outstanding debt and equity. Because firms can issue 'negative equity' (by repurchasing existing shares), in that setting there exists a feasible mix of securities that firms can issue at which a specific iso-profit curve—the zero-profit curve—intersects across types. It follows that investors must make zero profits in equilibrium. In contrast, in classic security-design models, such as NN, firms have no securities to repurchase, so zero-profit curves *never* intersect at some feasible securities bundle. As a result, separating equilibria exist in which uninformed investors make positive profits on high-type firms.[3]

To underscore that positive-profit equilibria arise naturally in financing problems under adverse selection, we note that they also emerge in models where some firm types have negative net present value projects (e.g., Dewatripont and Maskin (1995) and Tirole (2010), chapter 6)). In such settings, previous work that restricted attention to zero investor profits predicted a market breakdown instead. However, Bernhardt, Koufopoulos and Trigilia (2020) show that a market breakdown can be a unique equilibrium *only if* investors are exogenously constrained to make zero profits. Otherwise, a separating equilibrium always exists in which investors make profits on high-type firms. Moreover, if firms have access to any collateral at all, then the separating equilibrium becomes unique.

Our extension of NN is empirically relevant. NN's mechanism in which firms choose securities and then investors bid in auctions can describe public bond issuance by large corporations, or even IPOs and SEOs. However, it does not capture private securities placements to investors, or bank lending and syndication, where firms seek to raise a fixed amount with a loan at a given interest rate. Private financing instruments represent a common funding channel for small and medium enterprises that invest under severe informational asymmetries. In contrast, public issuances typically come from larger, established corporations—entities where informational asymmetries are far lower.[4] Our generalization of NN's security design problem extends it to many new economic environments.

2 Example

To clarify economic intuition, consider a simple example. A firm needs \$1 to finance a project. A high-type firm's project yields \$6 with probability $\frac{1}{2}$, and zero otherwise. A

[3]Of note, these separating equilibria are robust to ambiguity aversion over the population share of each firm type, or their cash flow distributions. This reflects that each security offered is priced so that investor profits would be zero if issued by low types. This result is relevant for the growing literature on security design under ambiguity aversion (Carroll (2015), Lee and Rajan (2018) and Malenko and Tsoy (2020)).

[4]Our setting is particularly relevant for bank lending, where a large literature relates bank profits with an informational advantage of banks (e.g., Dell'Ariccia and Marquez (2004)). In our model, such profits may obtain even though banks lack informational advantages over the market on their borrowers.

low-type firm's project yields \$4 with probability $\frac{1}{3}$, and zero otherwise. Each type is equally likely. There is free entry of risk-neutral investors and the risk-free rate is zero. Suppose firms can only issue debt, equity, or a combination.[5] Debt offers bondholders a payment of $\min\{x, D\}$ for $x \in \{0, 4, 6\}$, where D is the face value of debt. Equity, which is junior to debt, gives shareholders a dividend of $\max\{\alpha(x - D), 0\}$, where $\alpha \in [0, 1]$ is the share sold. Thus, a mix of debt and equity is worth $\frac{1}{2}[\max\{\alpha(6 - D), 0\}] + \frac{1}{2}[\min\{6, D\}]$ if issued by high types; $\frac{1}{3}[\max\{\alpha(4 - D), 0\}] + \frac{1}{3}[\min\{4, D\}]$ if issued by low types; and it is worth the average of these two expressions if firms pool on the same mix.

Figure 1: Equilibria in the example

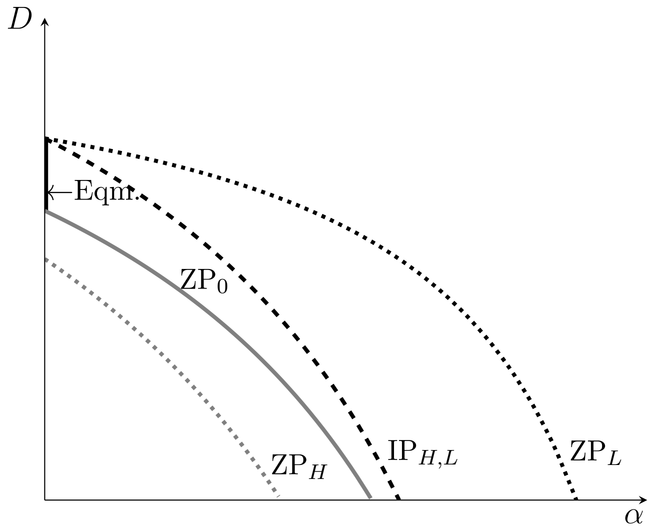

Figure 1 plots the four relevant iso-profit curves in (α, D) space. ZP_H is the zero-profit line for high types, described by $\frac{1}{2}[\max\{\alpha(6 - D), 0\}] + \frac{1}{2}[\min\{6, D\}] = 1$; ZP_L is the zero-profit line for low types, described by $\frac{1}{3}[\max\{\alpha(4 - D), 0\}] + \frac{1}{3}[\min\{4, D\}] = 1$; and ZP_0 is the zero profit line for the pool, $\frac{1}{2}\left[\frac{1}{2}\alpha(6 - D) + \frac{1}{2}D\right] + \frac{1}{2}\left[\frac{1}{3}\alpha(4 - D) + \frac{1}{3}D\right] = 1$. Finally, $\text{IP}_{H,L}$ is the iso-profit curve for high types that offer only debt ($\alpha = 0$) on ZP_L, and so is parallel to ZP_H. To derive $\text{IP}_{H,L}$, solve ZP_L for D when $\alpha = 0$ to obtain $D = 3$. Next, compute the value of $(\alpha, D) = (0, 3)$ if issued by high types by

[5] In this example, one can trivially separate the types by exploiting the non-overlapping support. Our main model assumes a common support and shows that the economics is unaffected.

substituting into the left-hand side of ZP_H, to get $\frac{3}{2}$. Thus, $IP_{H,L}$ is characterized by $\frac{1}{2}[\max\{\alpha(6-D), 0\}] + \frac{1}{2}[\min\{6, D\}] = \frac{3}{2}$.

NN show that if investors must make zero profits, then both types pool on issuing debt, fairly priced for the average type. To see this, start from any mix of debt and equity (α, D). First, because $ZP_H < ZP_0 < ZP_L$, investors cannot make zero profits on all firms in a separating equilibrium.[6] Therefore, the equilibrium contract must be on the zero-profit pooling line, ZP_0. Second, if firms pool on anything other than debt, then, because ZP_H is steeper than ZP_L, high types could deviate to issuing debt that is slightly under-priced from the average firm type's perspective. The deviation would be unprofitable for bad types, but strictly profitable for high types if accepted by investors, violating D1. Therefore, issuing equity cannot be part of a 'reasonable' equilibrium.

Turning to our positive-profits equilibria, suppose high types offer only debt on ZP_L, while low types offer only equity on ZP_L. Then, debt is under-priced because it is issued by high types, but equity is fairly priced. The high type's iso-profit line to which this contract belongs is $IP_{H,L}$, which lies strictly below ZP_L whenever $\alpha > 0$. Therefore, deviating to any contract that could benefit high types, also benefits low types. Thus, common refinements including D1 have no bite: with sufficiently pessimistic beliefs, this deviation would be unprofitable, sustaining the positive-profits equilibrium. Indeed, separating equilibria exist in which low types issue any mix of debt and equity such that $\alpha > 0$ on ZP_L, while high types only issue debt on ZP_L. Moreover, pooling equilibria in which all firms issue under-priced debt exist: all debt contracts along the y-axis in Figure 1, between ZP_0 and ZP_L—as highlighted by the solid segment—are equilibrium securities.

The example illustrates clear testable empirical implications, despite equilibrium multiplicity: (i) *high types issue relatively more debt than low types*; and (ii) *high types' equilibrium securities are under-priced*. We now show that these results extend to richer settings.

[6] Abusing notation, we write $ZP_H < ZP_0$ to indicate that the cost of financing under full-information for high types is strictly below that for the fifty-fifty pool. Thus, the iso-profit curve for high types has a strictly lower α for any given D than the pooling iso-profit curve.

3 Model

Our two date model features two types of risk-neutral firms that must obtain external credit to finance a project. Firms can raise funds from a continuum of competitive, risk-neutral investors.[7] We normalize the risk-free rate to zero. Projects require investment of \$1 at date $t = 0$ to generate random cash flows $\tilde{x}$ at $t = 1$. A firm's type $\theta \in \{\theta_H, \theta_L\}$ corresponds to its expected cash flows, where $\theta_H > \theta_L > 1$.[8] Thus, both firm types have strictly positive NPV projects, so it is efficient to fund them. A fraction $p \in (0, 1)$ of firms are high types with $\theta = \theta_H$. The mean firm type is $\theta_0 := p\theta_H + (1 - p)\theta_L$.

We denote a realization of $\tilde{x}$ by x, and the probability density function over x for a type-θ firm by $f_\theta(x)$, which we assume is strictly positive on a common, finite support $[0, \bar{x}]$. The random cash flows associated with the two firm types are ordered by the strict Monotone Likelihood Ratio Property (MLRP): $\frac{\partial}{\partial x}\frac{f_H(x)}{f_L(x)} > 0$ for every $x \in [0, \bar{x}]$.[9] Thus, the probability that a cash flow x comes from high types increases in x.

Contracts. As in NN, firms can sell *securities* $s(x) : [0, \bar{x}] \to \mathbb{R}$ that specifiy payouts to investors as a function of the realized cash flows x. NN also require the price $Q(s)$ of any equilibrium security equal its expected value given investor beliefs about the firm type offering it. In contrast, we enlarge the contract space to let firms propose selling prices, as occurs in private security placements. Definition 1 formalizes this distinction.

Definition 1. *Denote the contract offered by a type θ firm by c_θ. A contract consists of either a* priced security, *i.e.* $c_\theta = (s_\theta, Q_\theta(s_\theta))$, *which competitive investors can accept or reject to buy,*[10] *or an* unpriced security, $c_\theta = s_\theta$, *whose price $Q(s)$ is determined by Bertrand competition among the competitive investors.*

[7] If instead of a continuum of investors we had any number $N \geq 2$, our results would hold a fortiori, as positive-profits equilibria are easier to sustain with Betrand competition than free entry.

[8] NN do not assume that a type coincides with expected cash flows, but this economizes on notation and the models are isomorphic.

[9] Technically, NN impose Conditional Stochastic Dominance (CSD), a weaker condition than MLRP. The subsequent literature mostly works with MLRP (e.g., DeMarzo et al. (2005)), as it is more tractable and does not alter economic intuition.

[10] With multiple bidders, a lottery allocates the security to one bidder.

Most private securities placements involve *priced* securities, as firms often seek to raise a fixed amount that equals their external financing needs for a budgeted project. A bank loan of \$1 is a priced debt security where $s = \min\{x, D\}$ for some $D \in [0, \bar{x}]$ and $Q(s) =$ \$1. A private equity issuance for \$1 of capital is a priced equity security where $s = \alpha x$ for some $\alpha \in [0, 1]$ and $Q(s) =$ \$1. The expected value of such securities for investors need not be \$1. In contrast, a public bond issuance is an unpriced security $s = \min\{x, D\}$ for some $D \in [0, \bar{x}]$, where the amount raised $Q(s)$ is the outcome of an auction.

Definition 1 clarifies the difference between our environment and NN's. NN restrict attention to *unpriced* securities, while we allow firms to issue *both* priced and unpriced securities, at their discretion. As is standard in the literature, we assume that a feasible security s satisfies two-sided limited liability and monotonicity:[11]

Definition 2. *A contract c is* feasible *if it offers investors a feasible security s. A security s is* feasible *if it satisfies:*

(LL) Limited Liability: $s(x) \in [0, x]$ for every x.

(M) Monotonicity: for every $(x, x' < x) \in [0, \bar{x}]^2$, $s(x) \geq s(x')$ and $x - s(x) \geq x' - s(x')$.

Limited Liability (LL) implies that securities regulate how to split cash flows between firms (which receive $x - s_\theta(x)$) and investors (who receive $s_\theta(x)$). Monotonicity (M) implies that feasible securities are continuous functions of realized cash flows x.

Signaling game. At stage 1, a type-θ firm offers a feasible contract $c_\theta \in \{(s_\theta, Q_\theta(s_\theta)), s_\theta\}$, which consists of either a priced security $(s_\theta, Q_\theta(s_\theta))$, or an unpriced security s_θ. After a contract c is issued, investors form beliefs $p'(c) := \Pr.[\theta = H|c]$ about the type offering it. At stage 2, given these beliefs, investors bid for the contract. For priced securities, bids consist of acceptance/rejection decisions. If $Q \geq 1$ is accepted, then (observable, verifiable) investment occurs. Otherwise, the firm consumes its endowment, which we

[11]Limited liability guarantees that an equilibrium exists, and is needed to derive most results in the security design literature. Monotonicity ensures that the optimal security for θ_H types is debt, but is not needed for existence of separating equilibria. We maintain this assumption to simplify exposition and comparisons with NN.

normalize to zero. If $Q > 1$, we follow NN and assume that the difference $Q - 1$ is paid to the firm's shareholders as a dividend at $t = 0$.

We focus on Perfect Bayesian Equilibria (PBE) that satisfy the D1 refinement. The expected payoff to a type θ firm that offers security s and invests is given by

$$U_\theta(c) := Q(s) - 1 + \theta - \int_0^{\bar{x}} s_\theta(x) dF_\theta(x). \quad (1)$$

$Q(s)$ is the price paid by investors for security s; \$1 is the required investment; θ is the full-information expected value of a type-θ project; and $\int_0^{\bar{x}} s_\theta(x) dF_\theta(x)$ is the security's full-information expected value. We focus on 'reasonable' Perfect Bayesian equilibria, i.e., PBE that satisfy the D1 refinement (Cho and Kreps (1987)):

Definition 3. *A 'reasonable' PBE is a pair of feasible contracts* (c_L^*, c_H^*)*, and investor beliefs* $p'(c)$ *for every feasible contract* c *that satisfy:*

- *SR Sequential rationality: firms propose contracts* c_H^* *and* c_L^* *optimally, given investor beliefs* $p'(c)$ *and the associated optimal bidding decision;*
- *BC Belief consistency:* $p'(c_H^*)$ *and* $p'(c_L^*)$ *are derived from Bayes' Rule.*
- *D1 Refinement: there does not exist a pair of types* θ, θ' *and a feasible* $c \neq c_\theta^*$ *such that:* $\{p' | U_{\theta'}(c|p') \geq U_{\theta'}(c_{\theta'}^*)\} \subset \{p' | U_\theta(c|p') > U_\theta(c_\theta^*)\}$.[12]

The D1 refinement is standard in this literature. It rules out equilibria sustained by off-equilibrium beliefs that assign positive probability to types that weakly benefit from the deviation 'in strictly less cases' than other types, in a set-inclusion sense. In such cases, D1 forces off-equilibrium beliefs to assign probability one to the firm type that benefits 'in strictly more cases' than any other type.

[12] We use the standard definition of the D1 refinement (Cho and Kreps (1987)), which ranks sets of off-equilibrium posteriors. NN's alternative definition ranks off-equilibrium prices. The definitions are equivalent: to any price $Q \geq 1$ offered off-equilibrium there is a corresponding off-equilibrium posterior, and vice versa.

4 Results

4.1 Separating equilibria

We first analyze separating equilibria. It is well known that separating equilibria cannot exist *if investors must make zero profits* on every security offered in equilibrium. Lemma 1 summarizes this.

Lemma 1. *There is no separating PBE in which all securities issued are fairly priced: there is no equilibrium with $s_H \neq s_L$ and $\mathbb{E}_\theta[s_\theta(x)] = Q(s_\theta)$, $\forall\theta$.*

Proof. If $\mathbb{E}_\theta[s_\theta(x)] = Q(s_\theta)$, $\forall\theta$, then $U_\theta = \theta - 1$ for every θ in equilibrium. The incentive constraint for low-type firms not to mimic high-types reads: $\theta_L - 1 \geq Q(s_H) - 1 + \theta_L - \mathbb{E}_L[s_H(x)] \iff \mathbb{E}_L[s_H] \geq Q(s_H)$. Because $\mathbb{E}_H[s_H] = Q(s_H)$, incentive compatibility requires $\mathbb{E}_H[s_H] \leq \mathbb{E}_L[s_H] \iff \int_0^{\bar{x}} s_H(f_H - f_L)dx \leq 0$. Integrating by parts yields $\int_0^{\bar{x}} \frac{ds_H}{dx}(F_L - F_H)dx \leq 0$. Monotonicity of security payments in cash flows (Condition M), yields $ds_H \geq 0$ and since s_H is a continuous function, it is differentiable almost everywhere. By continuity and because investment is risky, we have $ds_H > 0$ for a positive measure of cash flows. Strict MLRP implies that $F_H(x) < F_L(x)$, for every $x \in [0, \bar{x})$. Therefore, the incentive compatibility constraint cannot hold. □

Lemma 1 has two implications. First, separating equilibria in which firms issue unpriced securities cannot exist. This corresponds to NN's Proposition 1. For our analysis, it follows that we can restrict attention to separating equilibria in which at least one type issues a priced security. Second, Lemma 1 implies that a separating equilibrium must feature strictly positive profits for competitive investors on at least one firm type.

We now highlight properties of separating equilibria with positive investor profits. First, in any 'reasonable' separating equilibrium, low-type firms must be indifferent between offering their equilibrium contract c_L and mimicking high-type firms by offering $c_H \neq c_L$. If not, high-type firms would have deviations that would not attract low types, but make high types better off, violating D1. Thus,

Lemma 2. *A separating PBE satisfies D1 only if the incentive constraint for low-type firms binds: $U_L(c_L) = U_L(c_H)$.*

Proof. Suppose, by contradiction, that a separating equilibrium satisfying D1 exists at which $U_L(c_L) > U_L(c_H) \iff \mathbb{E}_L[s_H] - Q(s_H) > \mathbb{E}_L[s_L] - Q(s_L)$. Investors are individually rational so $\mathbb{E}_L[s_L] - Q(s_L) \geq 0$. Thus, $\mathbb{E}_L[s_H] - Q(s_H) > 0$. By strict MLRP, $\mathbb{E}_L[s] < \mathbb{E}_H[s]$ for any strictly monotonic security s. Therefore, $\mathbb{E}_H[s_H] - Q(s_H) > \mathbb{E}_L[s_H] - Q(s_H) > 0$: the participation constraint for investors upon observing the high-type equilibrium security s_H is slack.

Consider a deviation by high types to a priced security $s'_H = \alpha s_H$, for α in $(\frac{Q(s_H)-Q(s_L)+\mathbb{E}_L[s_L]}{\mathbb{E}_L[s_H]}, 1)$, such that $Q(s'_H) = Q(s_H)$. This interval is non-empty, as $0 < \frac{Q(s_H)-Q(s_L)+\mathbb{E}_L[s_L]}{\mathbb{E}_L[s_H]} < 1$, because $\mathbb{E}_L[s_H] - Q(s_H) > \mathbb{E}_L[s_L] - Q(s_L)$, and $\mathbb{E}_L[s_L] - Q(s_L) \geq 0$.

Deviating to s'_H would be unprofitable for low types, regardless of investor beliefs. To see this, notice that at the lowest possible α, where $\alpha = \frac{Q(s_H)-Q(s_L)+\mathbb{E}_L[s_L]}{\mathbb{E}_L[s_H]}$, incentive compatibility for low types requires $\mathbb{E}_L[s_L] - Q(s_L) \leq \mathbb{E}_L[s'_H] - Q(s'_H) = \mathbb{E}_L[\alpha s_H] - Q(s_H) = \mathbb{E}_L[s_L] - Q(s_L)$. In contrast, the deviation is profitable for high-type firms because (i) it costs less, $\mathbb{E}_H[s'_H] < \mathbb{E}_H[s_H]$; and (ii) it would be accepted by investors. Thus, a contradiction obtains. □

The intuition behind Lemma 2 is straightforward. However, it has the important implication that the incentive constraint for high-type firms never binds in separating equilibria. To see this, start from a pair (s_L, s_H) with $s_H \neq s_L$ such that $\mathbb{E}_L[s_L] = \mathbb{E}_L[s_H]$. Consider the effect of a change of measure on both sides of the equation, from F_L to F_H. When the securities are not identical, the effect must be heterogeneous. Moreover, the incentive constraint must hold. Therefore, high-type firms must issue *less informationally-sensitive securities* (DeMarzo et al. (2005)) than low-type firms at a separating equilibrium.

Definition 4. *A security s is more* informationally sensitive *than security s' if its payoff crosses that of s' from below, in the space of cash-flow realizations $[0, \bar{x}]$. Formally, if*

$\mathbb{E}_\mu[s] = \mathbb{E}_\mu[s']$ *then* $\mathbb{E}_\mu[\partial s/\partial x] > \mathbb{E}_\mu[\partial s'/\partial x]$, *where* μ *denotes any probability distribution over the set of types* $\theta \in \{\theta_L, \theta_H\}$.

Debt is the least informationally-sensitive monotone security: fixing a family of securities with the same expected value under some measure μ over types, the payoff of any monotonic non-debt security must cross that of debt from below. We use this to conclude that low-type firms must issue more informationally-sensitive securities in equilibrium.

Lemma 3. *A separating PBE satisfies D1 only if the incentive constraint for high-type firms is slack:* $U_H(c_H) > U_H(c_L)$. *Therefore, the security issued by low-type firms,* s_L, *is more informationally sensitive than that issued by high-type firms,* s_H.

Proof. Suppose, by contradiction, there exist s_H, s_L with $s_L \neq s_H$ and $U_H(c_H) = U_H(c_L)$. Then $U_L(c_H) = U_L(c_L)$ from Lemma 2. Combining these equations yields $\mathbb{E}_L[s_L] - \mathbb{E}_H[s_L] = \mathbb{E}_L[s_H] - \mathbb{E}_H[s_H]$, or, equivalently: $\int_0^{\bar{x}}(s_L - s_H)d(F_L(x) - F_H(x)) = 0$. From strict MLRP, this requires $s_L = s_H$, a contradiction. Lemma 2 then implies that $\mathbb{E}_H[s_H] - \mathbb{E}_L[s_H] < \mathbb{E}_H[s_L] - \mathbb{E}_L[s_L]$. Lemma 2 implies that we need to consider the family of securities with the same expected value under the measure that puts probability one on low-type firms. Strict MRLP implies that, within this set of securities, high-type firms strictly prefer the least informationally-sensitive one, so s_H must be strictly less informationally sensitive than s_L. □

Lemma 1 rules out separating equilibria with zero expected profits on all securities. We next establish that in any separating equilibrium, competitive investors never make strictly positive profits on low-type firms. Therefore, investors must make strictly positive expected profits on high types in any separating equilibrium.

Lemma 4. *A separating PBE satisfies D1 only if investors do not make positive profits on low-type firms:* $\mathbb{E}_L[s_L] = Q(s_L)$.

Proof. Suppose a separating equilibrium exists with $\mathbb{E}_L[s_L] > Q(s_L)$. From Lemma 3, low-type firms can deviate to a security $s'_L = \beta s_L$ for $\beta \in (\frac{Q(s_L)-Q(s_H)+\mathbb{E}_H[s_H]}{\mathbb{E}_H[s_L]}, 1)$ at which all incentive constraints hold: the security is constructed so that it is never a profitable deviation for high types. The proof logic from Lemma 2 guarantees that s'_L is feasible. Moreover, for $\beta \in (\frac{Q(s_L)}{\mathbb{E}_L[s_L]}, Q(s_L))$, which is a non-empty interval given $\mathbb{E}_L[s_L] > Q(s_L)$, the participation constraint for investors holds. Therefore, deviating to s'_L is profitable when $\theta = \theta_L$, but not when $\theta = \theta_H$, violating D1. □

Our last preliminary lemma establishes that in any 'reasonable' separating equilibrium, high-type firms must offer a priced security that exactly covers their financing needs. Intuitively, all firms value cash in the same way, but high types expect to pay more to lenders for any security, so reducing cash and security differentially appeals to high types.

Lemma 5. *A separating PBE satisfies D1 only if high-type firms offer a priced security that raises the exact amount needed by the project:* $Q(s_H) = 1$.

Proof. Suppose a 'reasonable' PBE exists with $Q(s_H) > 1$. $\mathbb{E}_L[s_H] - Q(s_H) = \mathbb{E}_L[s_L] - Q(s_L) = 0$ from Lemmas 2 and 4; and $\mathbb{E}_H[s_H] - Q(s_H) < \mathbb{E}_H[s_L] - Q(s_L)$ from Lemma 3. Consider the deviation for high-type firms to another priced security $(s'_H, Q(s'_H))$ such that $Q(s'_H) = Q(s_H) - \epsilon s'_H = \alpha s_H$ for some $\alpha \in (0, 1 - \frac{\epsilon}{\mathbb{E}_H[s_H]})$. The proof logic from Lemma 2 guarantees that s'_H is feasible. If accepted, s'_H makes high types strictly better off relative to s_H. Further, low types do not mimic since $\mathbb{E}_L[s'_H] - Q(s'_H) > 0 \iff \alpha\mathbb{E}_L[s_H] - Q(s_H) + \epsilon > 0 \iff (1-\alpha)\mathbb{E}_L[s_H] - \epsilon < 0 \iff \left(\frac{\mathbb{E}_L[s_H]}{\mathbb{E}_H[s_H]} - 1\right)\epsilon < 0 \iff \mathbb{E}_L[s_H] < \mathbb{E}_H[s_H]$, where to proceed from $(1-\alpha)\mathbb{E}_L[s_H] - \epsilon$ to $\left(\frac{\mathbb{E}_L[s_H]}{\mathbb{E}_H[s_H]} - 1\right)\epsilon$ simply plug the highest possible value for α inside the first expression. $\mathbb{E}_L[s_H] < \mathbb{E}_H[s_H]$ by strict MLRP, so the deviation is profitable, violating D1. □

Proposition 1 builds on these lemmas to establish that separating equilibria exist, and that they share the same qualitative structure. Specifically, in every separating equilibrium, high types always issue debt; while low types issue more informationally-sensitive securities such as equity or a non-degenerate mix of debt and equity.

Proposition 1. *There exists a continuum of separating equilibria that satisfy D1. Every separating equilibrium features the following common properties:*

1. *High-type firms issue a standard debt security:* $s_H = \min\{x, D_H\}$*;*
2. *The security* s_H *is under-priced: the face value* D_H *sets* $\mathbb{E}_L[\min\{x, D_H\}] = 1$*;*
3. *Investors make strictly positive profits on high-type firms:* $\mathbb{E}_H[\min\{x, D_H\}] > 1$*.*
4. *Low-type firms issue any security different from debt (i.e.,*$s_L \neq \min\{x, D\}$ *for any* $D \in [0, \bar{x}]$*), such that* $\mathbb{E}_L[s_L] = Q(s_L) \geq 1$*.*

Proof. Lemma 3 ensures that low types do not issue debt. First suppose that property (1) fails, and both firm types issue non-debt securities: $s_\theta \neq \min\{x, D_\theta\}$ for $D_\theta \in [0, \bar{x}]$, for each θ. Consider the deviation by high-type firms to a debt security $s'_H = \min\{x, D_H\}$ for some $D_H \in [0, \bar{x}]$, where D_H is such that $\mathbb{E}_L[\min\{x, D_H\}] = 1+\epsilon$, for $\epsilon > 0$, small. By Lemma 2, this deviation is unprofitable for low-type firms, because low types are indifferent between mimicking or not when $\epsilon = 0$. High-type firms compare $\mathbb{E}_H[\min\{x, D_H\}]$ with the expected payout at the posited equilibrium $\mathbb{E}_H[s_H]$. Strict MLRP implies that if D is the face value of debt such that $\mathbb{E}_L[\min\{x, D\}] = 1$, then $\mathbb{E}_H[\min\{x, D\}] < \mathbb{E}_H[s_H]$, where the inequality follows because the payoff of any feasible (monotonic) non-debt security crosses that of debt from below. Thus, one can increase slightly the face value of debt from D to D_H so that $\mathbb{E}_H[\min\{x, D_H\}] < \mathbb{E}_H[s_H]$, and $\mathbb{E}_L[\min\{x, D_H\}] = 1 + \epsilon$.

Now, let high-type firms issue debt: $s_H = \min\{x, D_H\}$ for the D_H such that $\mathbb{E}_L[\min\{x, D_H\}] = 1$. The only deviation that could lower the payout for high types is to a security s' such that $\mathbb{E}_L[s'] < 1$. However, this would be profitable for low types whenever it is profitable for high types (i.e., when the investors' off-equilibrium belief is such that the offer is accepted). Thus, D1 has no bite in this case.

Property (4) follows immediately from the previous analysis, because if low-type firms issue debt, then high-type firms cannot find a security design that separates them without attracting low types. Thus, if s_L is debt, there must be pooling. Property (2) follows from Lemmas 2 and 4, which imply that $\mathbb{E}_L[\min\{x, D_H\}] = \mathbb{E}_L[s_L] = 1$. Property (3) follows from the strict MLRP assumption. □

Proposition 1 is novel in the context of the security design literature, providing insights into both the theoretical and empirical literature on capital structure and security design. Theoretically, our results show that separating equilibria exist in which investors make strictly positive profits on high-type firms. These separating equilibria are often ruled out by imposing zero-profits constraints on investors in equilibrium (see, e.g., NN's 'competitive rationality' condition for equilibrium). With this added constraint, consistent with our characterization, the unique equilibrium that satisfies D1 has all firms pooling on the same debt contract, which is fairly priced for the average firm type.

Empirically, our results provide foundations to tests of the 'pecking-order' hypothesis. Such tests typically ask whether in some sample, the set of 'better firms' (high types) relies more on debt than the set of 'worse firms' (low types). But as we just argued, security design under asymmetric information *and* zero profits necessarily delivers a pooling equilibrium in which every firm relies on debt, contrary to the pecking-order premise. Our paper shows that once one allows for positive investor profits—in a competitive economy with free entry of investors—then a solid foundation for these tests of the pecking-order obtains. That is, in our separating equilibria, high types issue *only* debt, while low-types must issue other securities such as equity.

Our micro-foundations for empirical tests of whether high-type firms issue debt and low-types a combination of debt and equity do *not* rely on the concept of a firm's *debt capacity*, which is not micro-founded in this adverse selection setting (Leary and Roberts (2010)). Instead, in our model, low types have an intrinsic preference relative to high types for more informationally-sensitive securities (e.g., equity rather than debt). That is, in the context of our example, the iso-profit curve of low types is flatter (in equity-debt space) than that of high types (see Figure 1).

4.2 Pooling equilibria

We conclude by characterizing all pooling equilibria, in which $s_L = s_H \equiv s_P$, for some feasible s_P. Given a zero-profit restriction for investors, NN identify the unique pooling equilibrium that satisfies D1. In this equilibrium all firm types pool on a debt contract that is fairly priced for the average firm type. We relax this restriction, letting investors earn non-negative profits on the equilibrium security offered by the pool of firms.

Investors make non-negative profits in a pooling equilibrium if and only if $Q(s_P) \leq \mathbb{E}_0[s_P] := p\mathbb{E}_H[s_p] + (1-p)\mathbb{E}_L[s_p]$. We first establish that in any pooling equilibrium with non-negative profits, the security s_P on which the two types pool must be debt.

Lemma 6. *In any pooling PBE that satisfies the D1 refinement, all firm types issue debt:* $s_L = s_H = \min\{x, D_P\}$ *for some* $D_P \in [0, \bar{x}]$.

Proof. Suppose there is a pooling equilibrium where $s_P \neq \min\{x, D_P\}$ for some $D_P \in [0, \bar{x}]$. Consider the deviation to the debt security $s' = \min\{x, D'\}$ with a face value D' such that $\mathbb{E}_0[\min\{x, D'\}] = \mathbb{E}_0[s_P]$. Evidently, s' is less informationally sensitive than s_P. Thus, low-type firms are worse off at s' than s_P, while high-type firms are better off, conditional on the offer being accepted by investors. Thus, D1 implies that investors hold the off-equilibrium belief $p'(s') = 1$, making the deviation profitable. □

We next establish that in any pooling equilibrium, all firms raise the minimum amount needed for investing. NN (Theorem 8) proved that this holds in the unique 'reasonable' equilibrium in which firms issue unpriced debt. We extend this result to priced securities.

Lemma 7. *In any pooling PBE that satisfies the D1 refinement, all firm types raise exactly the capital required to invest in the project, that is:* $Q(s_p) = 1$.

Proof. Suppose that a 'reasonable' pooling equilibrium exists with $Q(s_p) \neq 1$. We only need to consider $Q(s_p) > 1$ because $Q(s_p) < 1$ would mean a firm cannot invest, so no capital would be provided, as investors would expect to lose money. Let the high type deviate to a security s' such that $Q(s') = 1$. Because of cross-subsidization, $\int_0^{\bar{x}} s_p(x)dF_H(x) > Q(s_p) > \int_0^{\bar{x}} s_p(x)dF_L(x)$. Therefore, the deviation is only profitable for high types, if accepted, while it is never profitable for low types. □

The characterization of all pooling equilibria in Proposition 2 below follows directly from Lemmas 6-7. One equilibrium is the zero-profit debt equilibrium identified by the literature. The other equilibria involve all firm types issuing *under-priced* debt, with a degree of underpricing that spans from zero to the extreme in which debt is priced so that it would make zero profits for investors *if* issued only by low-type firms.

Proposition 2. *There exists a continuum of pooling equilibria that satisfy D1. In any such equilibrium, all types raise exactly \$1 of capital by issuing debt with a face value* $D_P \in [D_0, D_H]$*, where* D_H *solves* $\mathbb{E}_L[\min\{x, D_H\}] = 1$ *and* D_0 *solves* $\mathbb{E}_0[\min\{x, D_0\}] = 1$.

Proof. Lemma 6 implies that we can restrict attention to debt contracts offered by all types on-the-equilibrium path. Lemma 7 implies that we can restrict attention to contracts that raise exactly \$1. Any pooling debt contract with face value $D > D_H$ would violate the incentive constraint for low types. Moreover, investors would reject any pooling debt contract with face value $D < D_0$, as they would expect to lose money. Consider now a candidate equilibrium in which debt is issued with a face value $D_P \in [D_0, D_H]$. Then $\mathbb{E}_0[\min\{x, D_P\}] \geq 1$, so investors would accept the contract. We must show that it satisfies D1, regardless of the expected profits accruing to investors. To see this, observe that, starting from a debt contract, for some security s' to benefit high-type firms, it must be that $\mathbb{E}_L[s'] < \mathbb{E}_L[\min\{x, D_P\}]$. Thus, whenever the off-equilibrium belief held by investors leads to acceptance, both types benefit strictly. Therefore, D1 has no bite. □

Importantly, high-type firms are strictly better off in all pooling equilibria that have debt contracts with face value $D_P \in [D_0, D_H)$ than they are in all separating equilibria, *even though* high types cross-subsidize low types in pooling equilibria. This reflects the fact that the debt security issued by high types in separating equilibria would break-even for investors if issued solely by low types. This is the opposite of outcomes in settings where better firms separate via costly signals to minimize the subsequent mispricing of their securities.

5 Conclusion

Economic theorists typically equate unfettered competition with zero profits. Our paper reconsiders the classical security design problem of a firm that seeks finance from competitive investors for a project whose quality is private information of the firm. Nachman and Noe (1994) show that if one *imposes* that the investors earn zero profits, then all firm types necessarily pool on offering a debt contract that is fairly priced in expectation.

We highlight that, from a theoretical standpoint, NN *restrict the contract space*, not allowing firms to offer investors *priced* securities. In practice, priced securities are often offered in private placements or bank lending, where an issuer shops around, proposing and negotiating prices with different potential investors, and then selecting the best deal. We let firms issue priced or unpriced securities to competitive investors who provide capital if they can earn non-negative profits in expectation. We show that restrictions to zero profits are not driven by some un-modeled Bertrand competition, but rather by the exclusion of securities with proposed prices—and that this restriction has bite. We identify 'reasonable' separating equilibria in which low types issue fairly-priced securities that are more informationally-sensitive than debt, while high types issue debt that is fairly-priced for low types, and hence under-priced for high types. We also identify a range of pooling equilibria in which all types issue debt that is under-priced relative to the average firm type.

Our model provides theoretical foundations for empirical tests of the pecking-order theory of external finance. These tests cannot be reconciled by existing pecking-order theories, which predict that all firms pool on debt, and that equity is never issued. Our model also provides foundations for the profitability of bank loans, syndicated loans and privately-placed debt, in a setting where lenders do *not* have any informational advantage or moral hazard problem to justify their rents. Banks make strictly positive profits even though they are competitive and lack 'soft' information about firms.

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