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Sonia R. Bhalotra, Manuel Fernández, and Fan Wang†
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Abstract
We develop an equilibrium model of the labor market to investigate the joint evolution of gender gaps in labor force participation and wages. We do this overall and by task-based occupation and skill, which allows us to study distributional effects. We structurally estimate the model using data from Mexico over a period during which women’s participation increased by fifty percent. We provide new evidence that male and female labor are closer substitutes in high-paying analytical task-intensive occupations than in lower-paying manual and routine task-intensive occupations. We find that demand trends favored women, especially college-educated women. Consistent with these results, we see a widening of the gender wage gap at the lower end of the distribution, alongside a narrowing at the top. On the supply side, we find that increased appliance availability was the key driver of increases in the participation of unskilled women, and fertility decline a key driver for skilled women. The growth of appliances acted to widen the gender wage gap and the decline of fertility to narrow it. We also trace equilibrium impacts of growth in college attainment, which was more rapid among women, and of emigration, which was dominated by unskilled men. Our counterfactual estimates demonstrate that ignoring the countervailing effects of equilibrium wage adjustments on labor supplies, as is commonly done in the literature, can be misleading.

JEL classifications: J16, J21, J24, J31, O33
Keywords: Female labor force participation, gender wage gap, technological change, supply-demand framework, task-based approach, wage distribution, wage inequality

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†Bhalotra: University of Warwick, sonia.bhalotra@warwick.ac.uk; Fernández: Universidad de los Andes, man-fern@uniandes.edu.co; Wang: University of Houston, fwang26@uh.edu
1 Introduction

A secular increase in the labor force participation of women (FLFP) is one of the most salient features of the labor market over the last century (Killingsworth and Heckman 1987; Costa 2000; Goldin 2006; Fogli and Veldkamp 2011; Fernández 2013; Goldin and Olivetti 2013). Nevertheless, there is limited evidence of how this massive change in the size and composition of the labor force has altered the wage distribution. Economic theory suggests that, as long as men and women are imperfect substitutes in production, increases in women’s labor supply will create greater downward pressure on the wages of women than on the wages of men, and hence widen the gender wage gap. The size of this effect will depend upon the elasticity of substitution between male and female labor. We argue that this elasticity is likely to depend on the task content of the occupation. If occupations are ordered across the wage distribution, the impacts of a rise in women’s labor supply on the gender wage gap (and on wage inequality within gender) will vary across the wage distribution.

We structurally estimate an equilibrium model that extends the canonical labor demand-supply model discussed in Katz and Autor (1999) (also see Katz and Murphy (1992) and Murphy and Welch (1992) and Card and Lemieux (2001)), allowing male and female labor to be imperfect substitutes, with the degree of substitution varying with occupational task content (and also with skill level). Our model provides a unified framework in which four key channels through which FLFP and the wage structure are related are studied simultaneously. In addition to imperfect substitutability between types of labor, this includes gender- and skill-biased technical change (that shifts relative demand), trends in marriage, fertility, uptake of home appliances, and legislative protection of women’s economic rights (non-wage variables that shift relative labor force participation), and skill-upgrading and emigration (changes in demographic composition that shift potential relative labor supplies). In contrast to much of the literature, which provides partial equilibrium estimates, we provide general equilibrium estimates, allowing that labor supplies respond to changes in the equilibrium wage structure, see Section 2. Ours would appear to be the first attempt to analyze the range of demand and supply channels simultaneously, quantifying their relative importance and their distributional consequences, within and between genders and across the wage distribution.1

Section 1.1 provides a more detailed comparison and more comprehensive references to the related literature but given its density we briefly indicate here the four key dimensions in which our modeling approach departs from most existing work. One, ours is the first attempt to model an elasticity of substitution between male and female labor that varies by task-based occupation, and by skill. Following Autor, Levy, and Murnane (2003), we categorize occupations as intensive in analyt-

1. When we refer to relative demand or supply in this paper, we refer to male relative to female labor demand or supply.
ical, routine, or manual tasks. We demonstrate that the synthesis of the traditional labor demand-supply model with the task-based approach is a useful way to analyze distributional effects.

Two, we allow demand trends to vary by gender- and skill, across occupations. Relative demand trends may reflect, inter alia, the impacts of automation, the rise of teamwork, the growing importance of social skills in the workplace, or declining discrimination against women. Previous work allows for one or the other but not all of this heterogeneity, but we find that the greater flexibility is of substantive relevance in accounting for the distributional patterns in the data. Our approach contrasts with a large literature on women’s labor supply that takes demand as given, see Keane, Todd, and Wolpin (2011) for a survey.

Among studies that estimate demand parameters using the supply-demand framework (Katz and Autor 1999), most assume inelastic short-run labor supply, consistent with their (implicit) focus on men, see Section 1.1. Our third contribution lies in endogenizing labor force participation and occupation-specific labor supplies. More specifically, we allow labor supply to respond to endogenously determined equilibrium wages, to gender and skill biased changes in the potential workforce, and to non-wage shifters of participation that have been identified in the literature—fertility, marriage, household appliances, and legislative progress in protecting women’s economic rights. Most previous work studies these factors in isolation, estimates their impacts in a partial equilibrium setting, and does not consistently distinguish female and male labor by skill. To assess the empirical relevance of this in our setting, we provide counterfactuals under partial equilibrium (PE) and under general equilibrium (GE).

Our fourth innovation is that, by mapping our model to observed time-series data on wages and labor quantities, we develop equilibrium solution and estimation methods to jointly identify supply- and demand-side parameters. We detail a model-based approach to identification with potentially wide applicability, see Section 5.3. We discuss the difficulty of using supply-shifter instruments to tackle the bias arising from relative demand shocks when labor supply is elastic to wages. We also show that estimating the model using only demand-side relative optimality conditions, regressing observed relative wages on observed relative quantities can lead to bias in the presence of measurement error and/or relative demand shocks. Rather than taking observables as regressors, we endogenously solve for both equilibrium wages and equilibrium labor quantities and match them against observables.\footnote{2}

We apply this framework to investigate the joint evolution of women’s labor force participation and the wage structure in Mexico. Starting from about 1990, Mexico has experienced one of the largest increases in FLFP in the world during the

\footnote{2: We have created solution and estimation algorithms that improve equilibrium market-clearing precision, re-usability, and scalability of the framework. The solution and estimation code is available on our project website.}

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last quarter century (Ñopo 2012; The World Bank 2012). FLFP among women aged 25-55 increased 50%, from close to 40 percent in 1990 to close to 60 percent in 2013, rising from 4.7 to 14.7 million. A motivation for the analysis in this paper is that changes in the gender wage gap varied dramatically across the wage distribution. The unconditional wage gap widened by more than 30 percentage points at the 5th percentile of the wage distribution, while narrowing by 18 percentage points at the 95th percentile of the distribution. This cannot be explained away by compositional change- we conduct a decomposition of the gender wage gap across percentiles of the distribution (following Firpo, Fortin, and Lemieux (2007, 2009)), which suggests that changes in the gap in the sample period are primarily wage structure changes.

Our structural estimates are able to explain the distributional patterns in the data, and they illuminate some classic considerations in labor economics. Our first result is that male and female labor are closer substitutes in high-wage analytical task-intensive occupations (elasticity of 2.94) than in lower-wage manual or routine task-intensive occupations (elasticities of 1.09 and 1.28, respectively). This contributes to explaining why the increase in women’s LFP exerted greater downward pressure on wages at the lower than at the upper end of the distribution. We also find that the elasticity of substitution between skilled and unskilled labor is smaller in analytical and routine tasks (1.43) than in manual tasks (elasticity of 3.82). On its own, this implies an erosion of the college premium although primarily for men because our estimates indicate that changes in relative demand favored women.3

Indeed, our finding that demand trends favored women is possibly the most striking pattern we observe. Of all the factors we consider, this had the largest impact on the gender wage gap. Although evident across occupation and skill groups, the increase in demand for female labor is skill-biased and thus has significant distributional effects, explaining the contraction of the gender wage gap at the top of the wage distribution.4

Our finding of skill-biased technical change is in line with findings elsewhere including in the US, where it has been argued to explain rising income inequality (Katz and Autor 1999; Acemoglu and Autor 2011). However, in contrast to the US and other OECD countries, Mexico experienced a compression of wage inequality among men, with educational (i.e., skill) upgrading more than offsetting the increasing demand for skill. Our analysis contributes two new insights to the literature.

3. The skill premium for women was quite stable over the period under the offsetting influences of appliance ownership (which drew unskilled women into the workforce and pushed it up) and fertility decline (which drew skilled women in and pushed it down). The buoyancy of the skill premium for women relative to men was also a reflection of demand trends that favored skilled women.

4. At the top of the wage distribution, increased demand for skilled female labor outpaced the impact of increases in the relative supply of female labor, aided by a high degree of substitutability between male and female labor in the analytic task intensive occupations that appear in this region. In the lower regions of the wage distribution, the demand for unskilled female labor increased more sluggishly and the increased supply of female labor exerted greater downward pressure on female relative to male wages because of the more limited substitutability of men and women in the routine and manual task occupations that populate this region of the distribution.
First, that the increase in participation of skilled women was a driver of the compression of male inequality (a decline in the college premium). Second, that there was no marked change in wage inequality among women despite their rapid skill acquisition.\(^5\)

Turning to the supply-side, we confirm the finding in earlier research that the aggregate wage elasticity of labor supply is higher among women than men (especially among the unskilled), and that female (but not male) aggregate wage elasticities have decreased over time, in line with women’s growing labor market attachment. However the aggregate wage elasticity of labor supply, which is more often reported, is only empirically relevant if wages across occupations jointly shift up or down. In fact, forces such as trade or technological change will tend to move relative wages across occupations. We provide new evidence showing that these relative wage movements spark occupational mobility. We document considerable heterogeneity in the wage elasticity by occupation, skill, and gender, accounting for which is relevant to understanding equilibrium outcomes.\(^6\)

Our fourth set of findings pertains to non-wage determinants of participation. Our GE estimates indicate that marriage, fertility, household appliances, and the increasing scope of legislation designed to facilitate the economic participation of women jointly explain about a third of the reduction in the gender LFP gap. However our PE estimates put this figure at 85%. This suggests that previous estimates of the role of factors like appliances or fertility emerging from PE models are likely to be biased upwards. The estimates differ markedly by skill. An increased penetration of household appliances is the main driver of participation among unskilled women, essentially because there was much more room for growth in uptake of appliances in the unskilled group. Fertility declined in both groups, and is the strongest predictor of participation of skilled women. Our counterfactual analysis shows that increased appliance availability hastened the divergence of the gender gap at the bottom of the wage distribution (among unskilled workers), and that fertility decline muted convergence of the gender wage gap at the top of the wage distribution (among skilled workers). The decline of marriage and progressive realization of women’s economic rights (which will include impacts of declining employer discrimination on

\(^5\) These conclusions emerge from the underlying structure which, as discussed, reveals that female and male labor are better substitutes in analytic occupations (which attract a larger share of skilled workers); that demand trends favored women (especially within the skilled worker group); and that skilled male and female labor are similarly responsive to the analytic-task wage. Our analysis of counterfactuals illustrates how the relatively flexible structure illuminates the pathways to equilibrium outcomes.

\(^6\) The aggregate labor force participation of unskilled women and men is most responsive to the wage in manual task-intensive occupations, with men more responsive than women. Conversely, participation of skilled workers is most responsive to the analytical task wage, and the elasticity is now similar between men and women. However women are more sensitive to the routine-task wage, appearing to move between routine and analytical task intensive occupations as a function of the occupational wage.
labor supply) had smaller though, in cases, notable effects.\footnote{Although this is not always analyzed, we find non-negligible responses of male labor to the studied supply shifters.}

Our final set of findings investigates the role of demographic factors that have shifted the size and distribution of potential labor supply. We consider two trends that changed the gender-skill composition of the labor force in our analysis period, and that characterize contemporary trends or recent history in many countries. These are emigration, which was disproportionately of unskilled men, and educational upgrading, which occurred more rapidly among women. As emigration was male-dominated, it reduced the gender participation gap. Since female labor supply is more wage elastic than male labor supply and because male and female labor are imperfect substitutes, emigration led to a widening of the gender wage gap.

The increasing share of skilled women (women with a college degree) among potential workers acted to widen the gender labor force participation gap, and narrow the gender wage gap. In our discussion of this counterfactual, we explain the mechanisms by which allowing GE effects reverse the PE effect on the gender participation gap (the PE effect being a narrowing), and that it magnifies (about threefold) the PE effect on the gender wage gap. Our discussion illustrates nicely the role that the substitutability of male and female labor, the substitutability of skilled and unskilled labor, and the gender- and skill-specific wage elasticities of labor supply play in propagation of feedback from the equilibrium wage structure. This highlights the relevance of allowing variation in these different elasticities by task-based occupation and skill.

The rest of the paper is organized as follows: Section 1.1 positions this paper more clearly relative to related strands of literature. Section 2 provides a brief overview of the model structure, designed to profile the main forces at work. Section 3 discusses the data, and presents the stylized facts on changes in the wage and occupational structure over the last quarter century in Mexico. Section 4 presents a decomposition of changes over time in the gender wage gap across the wage distribution, showing that these are driven by wage structure changes. In Section 5, we formulate an equilibrium model of the labor market and describe the empirical strategy used to estimate its parameters. We discuss model fit, estimates of demand- and supply-side parameters, and wage elasticities in Section 6. In Section 7, through counterfactual exercises, we investigate the relative contribution of non-wage determinants of labor supply, demographics, and gender- and skill-biased technological changes to the evolution of gender wage and participation gaps under both PE and GE assumptions. Robustness exercises using alternative specifications of the model and different measures of labor supply are in Section 8. Section 9 concludes.
1.1 Related Literature

In this section, we position our work in relation to existing work, delineating the nature of our contributions.

**Variation in substitutability of labor types by task-based occupation.** The idea that women may have better social skills (Deming 2017) or, more broadly, that men and women have different skill-sets is captured by our allowing imperfect substitutability of female and male workers. Deming (2017) does not attempt to model the labor market equilibrium and many related studies, including Hsieh et al. (2019), assume male and female labor are perfect substitutes. We contribute to the literature on the task-based approach (Autor, Levy, and Murnane 2003; Autor, Katz, and Kearney 2006; Dorn 2009; Acemoglu and Autor 2011; Autor and Dorn 2013; Altonji, Kahn, and Speer 2014; Goos, Manning, and Salomons 2014; Michaels, Natraj, and Van Reenen 2014) in being the first to introduce imperfect substitutability between male and female labor, with the degree of substitution varying by task-based occupation. We also provide task-specific estimates of the elasticity of substitution between skilled and unskilled labor.

The existing literature has emphasized that the degree of substitutability or complementarity between factors of production is determined by the tasks they are employed to perform, but it has typically looked at how the arrival of new technology or capital substitutes for labor in different occupations. We adapt the framework to focus on how the arrival of new female labor substitutes for male labor in different occupations. This contributes to explaining differences in the evolution of the gender wage gap across the wage distribution, and the evolution of within-gender inequality.

The importance of occupation is underlined in Vella and Moscarini (2004), Kranz (2006), and Kambourov and Manovskii (2008, 2009a, 2009b), among others. They argue that occupation is a better measure of skill than education, and that occupational demand shifts are crucial for understanding changes in the wage structure. In line with this, we differentiate (male and female, and skilled and unskilled) labor by occupation, and we also allow for occupational demand shifts. Our model does well in predicting changes in the occupational wage structure.

The literature on the wage structure has tended to be descriptive or partial

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8. Any job requires, to a greater or lesser degree, cognitive, manual, physical, socio-emotional, and interpersonal skills. The relative importance of any subset of skills is then a function of the specific activities that workers are performing. As long as there is some difference in the bundle of skills that men and women supply to the labor market, the substitutability of male and female labor will tend to vary across occupations. By allowing imperfect substitutability of male and female labor, we allow, for example, that women have stronger social skills and that the demand for social skills is greater in certain occupations, and increasing over time; further discussion of this is below.

9. Card and Lemieux (2001), Card and DiNardo (2002), and Eckstein and Nagypal (2004), among others, argue that failing to account for the different dimensions of labor including education, gender, and occupation may compromise our understanding of the drivers of changes in the wage structure.
equilibrium in nature (see the discussion in Johnson and Keane (2013)). Among the first equilibrium models, Heckman, Lochner, and Taber (1998a, 1998b) distinguish labor by skill, Lee (2005) differentiates labor by skill and occupation (defining occupation as white- vs. blue-collar), and Lee and Wolpin (2006, 2010) differentiate labor by skill, occupation, gender, and age, but they assume they are perfect substitutes in production. Johnson and Keane (2013) similarly differentiate different types of labor, and allow them to be imperfect substitutes. However they do not estimate substitution elasticities by occupation. Our approach is also related to Hsieh et al. (2019) and Burstein et al. (2020), recent papers which, like us, study the labor market equilibrium over time in a setting with CES-based demand aggregation and occupational choice. Unlike our paper, these papers do not focus on gender and also do not allow occupation-specific elasticities by gender and skill groups.

Absent occupational heterogeneity, only a few previous studies have investigated the substitutability of female for male labor when considering the impacts of female labor supply on changes in the wage structure. Topel (1994) examined whether the rise in female labor supply contributed to rising inequality in the U.S. during the 1970s and 1980s, concluding that it did, by depressing the wages of low-skilled male workers. Juhn and Kim (1999) challenged this result, arguing that it was dissipated by accounting for changes in relative demand. Notably, we estimate an equilibrium model that accounts for changes in relative supply and demand. They argued that college-educated women are close substitutes for college-educated men (a result that we formalize, but distinguishing education and task-based occupation), so that their entry into the labor market may have tempered the growth in male wage inequality in the 1980s. This resonates with similar debates in the immigration literature.10 Our modeling approach can be adapted to analyze immigration.

Only two studies appear to have attempted to directly estimate the elasticity of substitution between male and female labor, both on US data. Exploiting state level variation in U.S. military mobilizations for World War II, Acemoglu, Autor, and Lyle (2004) report estimates of around 3, and Johnson and Keane (2013) report an elasticity between 1.85 and 2.2 during 1968–1996. Neither of these studies (or any other) allows the elasticity of substitution between male and female labor to vary by task-based occupation. The range of the elasticities we estimate is consistent with existing estimates, but the heterogeneity we find by occupation and skill is quantitatively important and has significant distributional consequences.

**Occupation-specific relative demand trends.** The question of how substitutable female and male labor are is relevant to recent research emphasizing changes in the demand for skills. First, there is evidence of growing demand for (non-cognitive) social skills (Deming 2017), and some evidence that women have

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10. For instance, Card (2009) argues that the effects of immigrants on US wages are small, whereas Aydemir and Borjas (2007) argue that recent immigration has reduced US wages, particularly for low-skilled natives.
stronger social skills (Cortes, Jaimovich, and Siu 2018). Second, the importance of manual (brawn-intensive) skills in which men have a biologically-rooted comparative advantage has declined (Galor and Weil 1996; Blau and Kahn 1997; Weinberg 2000; Rendall 2017; Black and Spitz-Oener 2010; Pitt, Rosenzweig, and Hassan 2012; Aguayo-Tellez et al. 2013; Rendall 2013). Third, the marketization of home production has contributed to the growth of service industries including child care and catering (Lup Tick and Oaxaca 2010; Akbulut 2011; Olivetti and Petrongolo 2014; Ngai and Petrongolo 2017). All of these factors suggest increases in the relative demand for female labor that are likely to differ across the task distribution.

The studies cited in this paragraph do not estimate the elasticity of substitution between male and female labor at all and, in any case, not by occupation. Moreover, they tend to analyze demand trends in partial equilibrium. We differ in simultaneously analyzing supply and demand. Our counterfactual analysis provides estimates under partial and general equilibrium that demonstrate the empirical importance of allowing for GE effects.

By allowing the estimated demand trends to vary by occupation, education, and gender, we take forward a literature in which the gender wage gap or the skill premium has been analyzed with respect to either task-biased, skill-biased, or gender-biased technical change, rather than allowing that all are at play at once. For instance, Bound and Johnson (1992) and Katz and Autor (1999) focus on skill-biased technological change, Pitt, Rosenzweig, and Hassan (2012) allow for gender-biased technical change, Goos, Manning, and Salomons (2014) allow for routine-biased technical change, but we allow for all of these. Our approach affords a clearer characterization of the role of demand in determining wage inequality between and within gender.

Analysis of labor supply. By endogenizing labor supply in a Katz and Murphy (1992) style model, we provide a modeling framework for analysis of increases in women’s LFP, which can be adapted to estimate the equilibrium effects of other substantial changes in the size or composition of the labor force, such as are created by immigration. At one end, most studies of determinants of women’s labor supply are partial equilibrium studies that take demand as given, for a survey see Keane, Todd, and Wolpin (2011). At the other end, there is a long tradition in labor economics of studying changes in the wage structure using nested-CES aggregators to model labor demand.11 In most of these models, the parameters of interest are the elasticities of substitution between workers of different types, and a common strategy to estimate them is to exploit natural experiments that shift relative labor supply but are unrelated to idiosyncratic changes in the demand-side of the econ-

11. For instance, see, among others, Katz and Murphy (1992), Katz and Autor (1999), Card and Lemieux (2001), Borjas (2003), Ottaviano and Peri (2012), and Manacorda, Manning, and Wadsworth (2012), and see discussion of debates over parameter identification in Borjas, Grogger, and Hanson (2012)
omy. However, this approach requires labor supply to be inelastic (unresponsive to wages), an assumption that is not supported by the evidence.\textsuperscript{12}

We depart from this tradition in providing estimates of how skilled and unskilled women and men change their labor supply in response to changes in occupation-, gender- and skill-specific equilibrium wages. Heckman, Lochner, and Taber (1998a) endogenize human capital accumulation, but they assume inelastic labor supply; in contrast we endogenize labor supply and not human capital, our focus being on participation in a sample of prime-age individuals who have completed their education. We do, however, allow changes in the composition of human capital (skill) of potential workers to influence equilibrium wages.\textsuperscript{13}

Existing studies have shown that women’s labor force participation is driven by declines in fertility (Katz and Goldin 2000; Costa 2000; Cruces and Galiani 2007), changes in marriage rates (Grossbard-Shechtman and Neuman 1988; Fernández and Wong 2014; Greenwood et al. 2016), improvements in technology and capital (e.g., appliances) used for home production (Costa 2000; Greenwood, Seshadri, and Yorukoglu 2005; Cavalcanti and Tavares 2008; Coen-Pirani, León, and Lugauer 2010) and attitudinal changes towards female work (Rindfuss, Brewster, and Andrew 1996; Costa 2000; Fernández, Fogli, and Olivetti 2004; Goldin 2006; Fernández 2013), which are often reflected in legislation protective of women’s economic rights (Doepke and Zilibotti 2005; Platteau and Wahhaj 2014). We provide the first attempt to analyze all of these factors within one study, showing how they affect the evolution of the wage and occupational structure, jointly with demand trends and under partial vs general equilibrium constructs.

\textbf{Equilibrium models with segmented labor markets.} As discussed previously, the literature on women’s labor supply tends to take demand (and wages) as given, and the literature that estimates labor demand parameters tends to assume inelastic supply. We depart from these studies in estimating an equilibrium model, which estimates demand parameters, endogenizes labor supply, and accounts for demographic changes that alter the size and composition of potential workers. In this section we discuss how our approach relates to the handful of papers that develop and estimate empirical equilibrium models of segmented labor markets. We consider Lee and Wolpin (2006), Johnson and Keane (2013), Hsieh et al. (2019), Burstein et al. (2020), Morchio and Moser (2021), and Cavounidis et al. (2022).

We summarize here some differences in our model on the demand- and supply-sides, and digress to discuss the conceptualization of (gender-based) discrimination in an equilibrium model. Naturally one reason that the modelling differs across these

\textsuperscript{12} See, among others, Killingsworth and Heckman (1987), Blundell and Macurdy (1999), Keane (2011), and Bargain and Peichl (2016).

\textsuperscript{13} Education and fertility are endogenized in some partial equilibrium labor supply models. These models can handle greater choice complexity and dynamics in effect because only an “inner-loop” for the dynamic life-cycle problem needs to be solved, but there is no need to worry about a potential multi-dimensional “outer-loop” of market clearing conditions.
studies is that they pursue different research questions.\footnote{Lee and Wolpin (2006) study equilibrium occupational selection with demand aggregation over blue, white, and pink collar jobs in a dynamic life-cycle setting. Johnson and Keane (2013) study the changing patterns of wages and participation along fine occupational cells in the U.S., allowing for demand aggregation by gender, education, and occupation cells in an overlapping generations labor supply framework. Hsieh et al. (2019) explore the effect of labor misallocation due to taste discrimination in a static labor supply framework where human capital and occupational choices are jointly determined, and they use a single level of occupation-based CES-aggregation. Burstein et al. (2020) study the effects of immigration on local labor markets, considering demand aggregation over tradable labor tasks across regions and over immigrants and non-immigrants in a static labor supply framework. In a random search framework, Morchio and Moser (2021) develop a model that endogenously generates a continuous distribution of gender wage gaps that are explained by occupation- and ability-specific Beckerian taste-based discrimination parameters. In order to decompose changes in relative demand for jobs and changes in the relative productivity of skills, Cavounidis et al. (2022) build an equilibrium labor market model in which final outputs aggregate over occupation-specific outputs, which are determined by endogenously-chosen levels of analytical, routine, and manual skills of workers. They map the solution of a decentralized planning problem to observable changes in skill composition by occupations, without seeking to model equilibrium wages and labor supplies.}

Similar to these papers, we rely on CES demand aggregation. Similar to Burstein et al. (2020) and in contrast to the other papers, we allow for aggregation by task content of occupation, additionally in gender and skill cells. Our labor supply structure follows Lee and Wolpin (2006) and Johnson and Keane (2013) in assuming that occupational selection is based on individual- and occupation-specific preference shocks, rather than productivity draws as in Hsieh et al. (2019) and Burstein et al. (2020). We explain changes in labor force participation over time, leveraging trends in the range of observables that previous (partial equilibrium) studies have analyzed. In contrast, Lee and Wolpin (2006) rely on age trends and Johnson and Keane (2013) rely on indirect utility trends.

Similar to us, Hsieh et al. (2019) and Burstein et al. (2020), for instance, study labor market changes over several decades, making it likely that there are shifts in both demand and supply curves that jointly determine equilibrium, making it challenging to use instrument-based identification. A contribution of our framework relative to these studies is that we provide detailed equilibrium solution, identification, and estimation discussions/algorithms that are scalable in alternative settings. Also, while our model follows this largely macro/calibration-focused literature, we build up the model with empirical-micro based evidence through quantile regressions first. We specify a more general model that allows for heterogeneous elasticities for subgroups by occupations or task-content of occupation.

In contrast to the dynamic labor supply structures in Lee and Wolpin (2006) and Johnson and Keane (2013) that allow for human capital and labor market choices, we focus on labor-market decisions of prime-age individuals (age 25 to 55). The dynamic labor supply problems in Lee and Wolpin (2006) in particular (and to a lesser extent Johnson and Keane (2013)) lead to interesting but unwieldy equilibrium structures: labor markets have to clear jointly in all periods because of rational forward looking behavior of workers. Equilibrium is difficult to define. Numerical
solutions to equilibrium structures are potentially hard to verify. By relying on a static labor supply framework, we arrive at a transparent and tractable equilibrium structure (summarized in Section 5.3) which allows us to perform estimation with stable market clearing solutions as the estimator moves across the estimation parameter space. Our framework allows for the explicit characterization of labor market equilibrium in terms of gender-specific equilibrium wages.

A digression on discrimination. Consider now a comparison with Hsieh et al. (2019), which provides a useful point of departure to discuss discrimination in the setting of an equilibrium model. Hsieh et al. (2019) account for labor market discrimination as a “tax” on individual earnings for discriminated groups. Similar to Hsieh et al. (2019), we allow for differential demand for male and female labor by skill (and we additionally allow for differences by occupation). To the extent that changes in the relative labor demand for women are due to changes in discrimination, this is captured by our CES labor share parameters (the effects of which can be magnified by the elasticity parameter). However, our framework differs from Hsieh et al. (2019) in that changes in our share parameter can reflect productivity (or changes in the demand for gender-specific skills) as well as tastes. Both Hsieh et al. (2019) and our paper rely upon observed wages and labor market occupation shares for estimation. It is our understanding that these types of aggregated data (which do not include firm-specific outputs) cannot distinguish between tastes and productivity. In fact the distinction is blurred given that discrimination in the shape of, for instance, workplace sexual harassment causes gender productivity differences (Folke and Rickne 2020; Cici et al. 2021).

Gender gaps and the evolution of inequality. We now discuss, in turn, contributions of our analysis to debates on inequality, and to analysis of the gender wage gap in other, including OECD, countries. Alongside the stream of work on rising wage inequality in the US (Katz and Autor 1999; Acemoglu and Autor 2011), is a stream of work on declining wage inequality in Latin America (since the late 1990s) (López-Calva and Lustig 2010; Levy and Schady 2013; Lustig et al. 2016; Galiani et al. 2017; Fernández and Messina 2018) and in both the college premium is a key driver. These studies tend to take total labor supply and occupation-skill-gender specific labor supply as fixed. Their failure to account for increases in women’s participation is a significant omission in the Latin American setting where

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15. Similar to us, Lee and Wolpin (2006) assume fertility is exogenous, while (Johnson and Keane 2013) do not consider fertility. Fertility is endogenized in the partial equilibrium literature on female labor force participation using discrete choice dynamic programming problems, see (Keane, Todd, and Wolpin 2011).

16. In the context of gender, implicitly, Hsieh et al. (2019) assume two things (1) perfect substitutability of male and female workers, and (2) equal productivity of male and female workers of the same skill, and they argue that differential labor demand for otherwise homogeneous workers arises due to a Becker (1981) loss of utility loss via discrimination rather than due to productivity. Similar assumptions are made by Morchio and Moser (2021) who assume that workers are perfect substitutes across gender and skill level, and do not allow for occupational choice.
there have been large increases with potentially large distributional effects. Ours is the first study to endogenize FLFP and estimate changes in the skill premia of men and women in a general equilibrium framework. Our results provide new insight into the causes of compression of wage inequality among men, suggesting that sluggish wage growth among high-skilled men in Latin American countries may be partly explained by the incorporation of college educated women into the workforce.\footnote{Acemoglu, Autor, and Lyle (2004) argue that the entry of women into the labor force post-World War II contributed to increasing inequality between secondary and college educated men. Although they do not estimate the elasticity of substitution between men and women by skill or by occupation, they similarly allude to women being closer substitutes to high school men than to men with higher or lower skills. However most of the literature does not discuss the role of participation or skill-upgrading among women.} In contrast to a lively literature on male inequality, female inequality has not attracted much attention. We provide the first evidence showing that there was no similarly large decline in wage inequality among women.

We expect that the patterns we identify in Mexico will replicate not only in other Latin American countries but possibly more widely. This is because there is evidence of similar fundamentals—a higher elasticity of substitution between male and female labor in occupations intensive in analytical skills is likely to be fairly universal, occupations intensive in analytical skills usually place in the right-tail of the pay distribution (Autor, Levy, and Murnane 2003; Acemoglu and Autor 2011; Goos, Manning, and Salomons 2014), and occupational segregation by gender tends to be larger in lower-paying occupations (Black and Juhn 2000; Blau, Brummund, and Liu 2013).

Female labor force participation rates (of married women) increased rapidly between about 1940 or 1950 and 1980 in OECD countries. For instance, FLFP increased by about 50% (from around 25% to 52%) between 1940 and 1980 in the US. The average gender pay gap increased in favor of men in this period\footnote{It started to narrow in the late 1970s and this convergence has slowed since 1990 (Bailey and DiPrete 2016; Blau and Kahn 2017).}, but we are unaware of evidence of how it evolved across the wage distribution, or how it influenced wage inequality within gender-group. Ours is one of the first attempts to study the distributional consequences of women joining the labor force. Our equilibrium modeling approach and estimation procedure are relevant to taking forward research on immigration which similarly creates a shift in the size and composition of the workforce. The immigration literature tends to estimate relative wages as a function of relative supplies using a partial equilibrium framework, and to take no account of occupational sorting. The elasticity of substitution between immigrants and natives is a key parameter in the discussion but, as discussed in Borjas, Grogger, and Hanson (2012), there is considerable disagreement over its size.
2 Framework

We use a simplified version of the full model from Section 5 to fix ideas. In particular, we highlight the role of gender substitutability in demand, the relevance of endogenizing occupational participation in supply, and the importance of considering equilibrium wage responses.

At time $t$, given a CES production function that aggregates labor across males and females ($\text{gen} \in \{k, f\}$) of some skill level within three task-content of occupation sub-nests ($o \in \{o_1, o_2, o_3\}$), demand optimality in a competitive equilibrium requires that:

$$
\log \left( \frac{W_{o,k,t}}{W_{o,f,t}} \right) = \log \left( \frac{\alpha_{o,t}}{1 - \alpha_{o,t}} \right) - \frac{1}{\sigma_{p,o}} \log \left( \frac{L_{o,k,t}}{L_{o,f,t}} \right),
$$

(2.1)

where $L_{o,k,t}$ and $L_{o,f,t}$ are labor inputs from male and female workers respectively, $W_{o,k,t}$ and $W_{o,f,t}$ are the wages of male and female workers respectively, $\alpha_{o,t}$ is the ‘share’ parameter that varies over time due to gender-biased demand changes, and $\rho_o \in (-\infty, 1]$ is a function of the elasticity of substitution ($\sigma_{p,o}$) between male and female labor: $\sigma_{p,o} \equiv \frac{1}{1-\rho_o}$. In a departure from most previous related work, all values are occupation-specific. In the full model, we also allow for skill- and occupation-biased technological changes, as well as heterogeneous substitutability between skilled and unskilled labor by occupation groups.

On the supply-side, male and female workers choose among three market occupations and home production. In a setting with random utility, the labor supply functions are:

$$
L_{o,\text{gen},t}^s = L_{\text{gen},t}^{\text{pop}} \times \psi \left\{ \psi W_{o,\text{gen},t} + \pi' B_{\text{gen},t} \right\}_{o=1}^{3},
$$

(2.2)

where supply $L_{o,\text{gen},t}^s$ is occupation- and gender-specific. $L_{\text{gen},t}^{\text{pop}}$ is the gender-specific number of potential workers at time $t$. $B_{\text{gen},t}$ is a vector of observed variables that shifts the relative utility of occupations for each gender and across occupations. While many studies focus on a single determinant, in the full model, we consider all of the main factors discussed in the literature. Specifically, we incorporate time-series of fertility, marital status, women’s economic rights, and appliance availability as shifters of LFP and occupation-skill specific labor supplies. We additionally analyze gender-skill upgrading and emigration as drivers of demographic change among potential workers.

What we wish to communicate with this brief sketch of the model is that this simple structure is general enough to contain four key channels through which FLFP and the wage structure are related: imperfect substitutability of female for male labor ($\sigma_{p}$), non-neutral (gender-biased) technological changes ($\alpha_t$), shifters of occupational (and overall LFP) participation rates ($\pi$), and demographic compositional changes ($L_{\text{gen},t}^{\text{pop}}$). In contrast to large sections of the related literature, we will
model these channels as operating in a context in which labor supply is allowed to respond to changes in the wage structure via $\psi$.

As discussed in the previous section, much of the literature on women’s labor supply takes demand as given. Among those papers that apply the supply-demand framework, most estimate some variation of Equation 2.1 under the assumption of predetermined inelastic short-run labor supply. This often implies a fixed probability for $F_0(\cdot)$, which shuts down $\pi$ and $\psi$. In our setting, non-zero $\pi$ allows supply shifters to affect participation and occupation decisions, for both genders. Non-zero $\psi$, on the other hand, allows LFP and occupation participation rates to respond to changes in the wage structure, arising from either supply-side ($B_{\text{gen,t}}, L_{\text{pop,gen,t}}$) or demand-side ($\alpha_t$) forces. Finally, the magnitude of equilibrium effects and their propagation across occupations depend on occupation-specific gender substitutability and the substitutability of task-based occupations themselves.

Consider the elasticity of substitution between male and female labor in Equation (2.1). If male and female labor are not very substitutable in occupation $o$, that is, if $\sigma_{p_o}$ is small, a large increase in female labor supply will impose downward pressure on female wages in occupation $o$ and, to a much lesser extent, on male wages, leading to an increase in the gender wage gap. Now, when $\psi$ is positive, the greater downward wage pressure on female wages depresses female labor supply, attenuating the impact of FLFP on the gender wage gap. Furthermore, changes specific to one occupation are amplified, through equilibrium, to all occupations and worker groups, and they can have distributional effects. By endogenizing labor supply, our framework allows us to study these general equilibrium effects.

Our framework for equilibrium labor force participation and wage analysis introduces parameters on the demand- and supply-side that have to be pinned down by time-series variation in observed equilibrium labor quantities ($L$) and wages ($W$) across gender, occupation, and skill groups. The data series needs to have sufficient variation to distinguish the separate effects on $L$ and $W$ of i) supply- vs. demand-side parameters, and ii) demand trends and elasticities of substitution by gender, skill, and occupation. Building on standard assumptions on the demand-side (polynomial restriction on trends) and supply-side (multinomial discrete choice), we characterize the equilibrium solution, discuss identification challenges as the time-series of available quantities and prices expands, and provide a scalable and efficient empirical equilibrium estimation routine, see Section 5.3. The modeling and estimation are potentially of value in formulating and addressing questions beyond the question at hand in this paper.

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20. Related research implicitly assumes this elasticity is invariant across occupations. We relax this assumption, allowing variation across the wage distribution in the degree to which the gender wage gap widens (or narrows).
3 Data and Descriptive Statistics

3.1 Data

Appendix A provides data sources and definitions, and explains variable choices and sample construction. Here we briefly describe the key points. We use 13 rounds of the nationally representative Mexican Household Income and Expenditure Survey (ENIGH) from 1989 to 2014. We restrict attention to individuals age 25-55 (hereafter prime-age workers). Wages are constructed as real hourly labor earnings for full-time workers.21

We link women’s labor force participation decisions to fertility and marriage trends, gender discrimination in work-related legislation as captured by the Women, Business and the Law (WBL) index, and home appliance availability. These variables are specific to gender and skill groups and capture potential changes in preferences and the technology of home production over time. Calculations and data sources for the four supply-side shifters, on the skill and gender composition of Mexican workers, and on the share of emigrants by gender and skill are discussed in Appendix Section A.2.

Using the 18 groups in the Mexican occupation classification, we construct three groups defined by whether the activities performed on the job are predominantly manual, routine, or analytical, following Autor, Levy, and Murnane (2003). This division captures the different aptitudes of men and women for different tasks and it also aligns with the wage distribution, more information is in Appendix A.3 and the final division used is shown in Table 1. The three occupational groups each represents about a third of the workforce. The Table shows substantial occupational sorting by gender.

3.2 Descriptive Statistics

Labor force participation. In 1989, the labor force participation rate of the entire prime-age population in Mexico was about 64.2 percent, the female participation rate (FLFP) was only 36 percent, and women accounted for 29 percent of the workforce. By 2014 this picture had changed drastically: the overall participation rate was 76 percent, the FLFP rate was close to 58 percent, and women represented 41 percent of the workforce (see Panels (a) and (c) Figure 1). This increase of about 50 percent in FLFP during the last 25 years was the largest in the Latin American region (Ñopo 2012), and one of the largest in the world (The World Bank 2012). The preceding statistics are from the ENIGH survey. Using decadal census data corroborates these broad trends. It allows us to depict trends going back to 1960 (Panels (b) and (d) of Figure 1), see Bhalotra and Fernández (2021) for analysis
of the longer time series. Differences in level and trend between the Census and ENIGH data are discussed in the Appendix Section A.1.

Three stylized facts characterize the evolution of labor force participation during this period (Table 2). First, most of the increase was of low skilled women (defined as women with at most secondary education), their LFP rising from 35.7 to 55.4 percent between C.1992 and C.2012, while that of high skilled women (defined as college educated) rose from 71.7 to 77.4 percent. While the volume of the increase in FLFP came from low-skilled women, the proportional change in participation of high skilled women was large because the initial share of the female workforce with a college-education was only 14.5 percent in C.1992, rising to 24.0 percent in C.2012 (Table D.1). Second, there was a substantial increase in participation across all age groups within the 25-55 range. Third, the LFP rate of prime-age men was stable at about 94 percent across the period.

Potential workers. The number of workers of any gender and skill emerging on the labor market depends not only on the labor force participation rate, but also on population growth and changes in the gender and skill composition of the population. Between 1989 and 2014, the Mexican prime age population—individuals who are born in Mexico and remain in Mexico—increased by 90 percent, from 25.2 million to 48.0 million. There were two significant trends that altered the gender-skill composition.

One was gender-biased educational (skill) upgrading. The share of skilled women among potential female workers increased from 6.4% to 19.7%, corresponding to an addition of 4.2 million potential skilled female workers (see Panel (a) of Figure 6). This led to convergence with the share of skilled men among potential male workers, which increased from 15.9% to 21.3%, amounting to an increase of 3.0 million potential skilled male workers. The share of women within prime age population was stable at around 53% between 1989 and 2014.

The other was the trend in emigration of prime-age Mexican-born population. Using information from Brücker, Capuano, and Marfouk (2013), we find that between 1989 and 2014, Mexican-born prime age population increased by 101 percent, from 27.3 million to 54.9 million. The share that emigrated increased from 8% to 13%, with most of the increase representing unskilled males (see Panel (c) of Figure 6). The share of emigrants among Mexican-born population increased for all groups other than skilled women: it decreased from 15% to 13% for skilled females.

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22. For certain summary statistics, we merged surveys from 1989, 1992 and 1994 (C.1992), and from 2010, 2012 and 2014 (C.2012) to increase sample size and smooth over year-specific changes.

23. The share of college-educated workers in the population increased from 15.7% to 21.0% among men and from 7.9% to 18.8% among women. The share of the female population completing college had converged to the corresponding share of men by 2014. Since increases in LFP among women were skill-biased, the corresponding increases in the share of college-educated workers in the labor force were 15.6% to 20.8% for men and 14.5% to 24% for women—see Table D.1 which shows shares conditional on working, in contrast to Figure 6 which shows population shares.
increased from 10% to 12% for skilled males, increased from 6% to 11% for unskilled females, and increased from 8% to 15% for unskilled males. We analyze these trends by implementing counterfactuals that shut down their growth over time.

**Wage structure.** At the same time that women were increasingly joining the workforce, the wage structure changed substantially. Figure 2 shows a striking pattern whereby the wages of men evolved more favorably than the wages of women at the lower end of the wage distribution, while the reverse was the case at the upper end of the distribution. This is a motivating fact for the analysis in this paper.

The following stylized facts underpin this. First, there was an overall tendency for real wages to decline since the early 1990s, associated with the ‘Tequila Crisis’ of 1994 and the Great Recession of the late 2000s. Second, the male wage distribution contracted sharply over the period, driven by male wage growth in the lower-tail of the distribution being higher than in the upper-tail, a pan Latin America phenomenon (López-Calva and Lustig 2010; Levy and Schady 2013; Lustig et al. 2016; Galiani et al. 2017; Fernández and Messina 2018). Third, we do not see a similar compression of the female wage distribution. Wage growth for females was ever so slightly u-shaped across the distribution, with wages at the bottom and the top performing better than in the middle.

Figure 3 (Panel a) shows how the unconditional gender wage gap evolved across the wage distribution. The gender gap increased by 10 to 32 percent among workers with below median wages, and declined by 5 to 20 percent among workers above the 80th percentile. The pattern can be replicated using alternative data from the 1990 and 2010 Mexican Census (Panel b). We analyze the role of the wage structure in determining the evolution of the gender wage gap in Section 4 below.

**Determinants of women’s participation.** We now consider how the four observable non-wage determinants of female labor force participation that we analyze evolved over the sample frame. The trends are in Figure 5. Fertility, defined as the percentage of women with at least one under-5 child, declined sharply, by about 20 percentage points, and more sharply among skilled (relative to unskilled) women. The share of women who were married or partnered fell 3 to 10 percentage points and, again, the decline was sharper among skilled women. The percentage of women who had a home appliance (refrigerator or washing machine) rose by more than 30 percentage points for the unskilled to reach close to 90% availability by the end of the period, whereas availability among skilled was more or less constant at about 95%. The WBL index, which measures women’s economic rights as captured by

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24. The series in Panel (a) of Figure 3 is calculated by subtracting the values of the male and female series in Figure 2.
legislation facilitating women’s work participation, increased by close to 20 pp.\textsuperscript{25} In contrast to the first three variables for which we have data by gender and skill group, WBL naturally only varies in the aggregate time series. Overall, there was substantial variation in predictors of FLFP over the period studied, consistent with the large increase in FLFP.

4 Unconditional Quantile Decomposition of the Gender Wage Gap

Changes in the gender wage gap following increases in women’s labor force participation could reflect changes in the skill composition of women vs men, or changes in returns to the skills of women vs men (i.e., wage structure changes). In this section, we use the unconditional quantile decomposition method of Firpo, Fortin, and Lemieux (2007, 2009) to illuminate this question. Figures 2 and 3 show how much the change in the gender wage gap varied across the wage distribution, underlining the importance of conducting the decomposition at different percentiles of the distribution. We find that the distribution of changes in the gender wage gap is primarily driven by wage structure changes.

We document changes in the age and educational attainment of prime-age workers between C.1992 and C.2012 in the lower Panel of Appendix Table D.1. The share of women among female labor force participants with at least some college education (skilled) increased from 14.5 to 24.0 percent, while that of men increased more slowly from 15.6 to 20.8 percent. The more rapid growth in the share of college-educated women may be an alternative explanation for the convergence in wages at the top of the distribution. The Table also shows that the average Mexican worker is becoming older, and more so among women (because of the increase in married women’s FLFP). If the gender wage gap increases with age (Barth, Kerr, and Olivetti 2017; Adda, Dustmann, and Stevens 2017), this could be a factor behind the widening of the mean gender gap.

Details of the decomposition methodology we adopt (see Firpo, Fortin, and Lemieux (2007, 2009)) are in Appendix B. As a starting point, consider a transformed wage-setting model of the form:

\[
RIF^q_{r,gen,t} = X'_{gen,t} \gamma_{gen,t} + \epsilon_{gen,t},
\]

where subscript \( gen \) indicates if the worker is male (\( gen = k \)) or female (\( gen = f \)); the subscript \( t \) indicates the period, either initial (\( t = \text{C.1992} \)) or final (\( t = \text{C.2012} \)); \( RIF^q_{r,gen,t} \) represents the value of the re-centered influence function or RIF (see Appendix B) corresponding to the \( \tau \)’th quantile of the wage distribution at time

\textsuperscript{25} It rose from 61.3 in 1989 to 80.6 in 2014. As a point of reference, the average score in high-income OECD countries is 94.7 points, while the countries with the lowest WBL are found in the Middle East and North Africa (MENA) region, where the average score is 49.6 points (Hyland, Djankov, and Goldberg 2020).
\( t \) and for gender \( \text{gen} \); \( X \) is a vector of socio-demographic characteristics including dummies for 7 education categories, 6 age categories in five-year intervals, and all possible interactions of education and age levels; and \( \epsilon_{\text{gen},t} \) is the error term assumed to have zero conditional mean. We can estimate Equation (4.1) for each gender and period separately by OLS, and then express the estimated difference over time of the expected value of the wage quantile \( \hat{q}_\tau \) as:

\[
\Delta t \hat{q}_{\tau,\text{gen}} = (\bar{X}'_{\text{gen},\text{C.2012}} - \bar{X}'_{\text{gen},\text{C.1992}}) \hat{\gamma}_{\text{gen},P} \Delta \bar{q}_{X,\tau,\text{gen}} + \bar{X}'_{\text{gen},P} \hat{\gamma}_{\text{gen},\text{C.2012}} - \hat{\gamma}_{\text{gen},\text{C.1992}}) \Delta \bar{q}_{S,\tau,\text{gen}},
\]

(4.2)

where overbars denote averages, and \( \hat{\gamma}_{\text{gen},P} \) and \( \bar{X}_{\text{gen},P} \) correspond to the estimated vectors of parameters and the explanatory variables from a wage-setting model in which observations are pooled across the two periods. Here, \( \Delta \bar{q}_{X,\tau,\text{gen}} \) corresponds to the composition effect, which captures the part of the change in the \( \tau \)'th wage quantile that is accounted for by changes in the average skill-demographic composition of workers, given that we set the returns at their (weighted) average over the two periods. \( \Delta \bar{q}_{S,\tau,\text{gen}} \) is the wage structure effect, capturing how changes in returns are affecting wages at the quantile \( \tau \), given that the observable characteristics are fixed to be equal to their (weighted) average over time.

Since we are interested in the effects of composition and price changes on the gender wage gap, we construct the following measures at 19 different percentiles:

\[
\Delta t \hat{q}_{\tau,k} - \Delta t \hat{q}_{\tau,f} = (\Delta t \hat{q}_{X,\tau,k} - \Delta t \hat{q}_{X,\tau,f}) + (\Delta t \hat{q}_{S,\tau,k} - \Delta t \hat{q}_{S,\tau,f}).
\]

(4.3)

Results of the decomposition for five selected percentiles are shown in Table 3. At all percentiles considered, wage structure effects are quantitatively more important than compositional effects. They contribute 63 percent of the observed rise in the gender wage gap at the 5th percentile, and close to 90 percent at the 25th percentile. They over-predict the fall in the gap at the 95th percentile (-22.5 log points observed vs. -34.7 log points attributed to the wage structure). The relative importance of wage structure effects in the evolution of the gender wage gap is also evident in Figure 4 which plots the estimates at all 19 percentiles. Wage structure effects line up remarkably well with observed relative wages across the distribution.

Figure 4 also shows that if the wage structure had remained constant at its average level over the two periods, compositional effects would have led to a larger gender wage gap. Thus changes in the skill and age of the workforce contributed to

26. This specific counterfactual allows us to analyze composition and wage structure effects relative to a baseline defined by (weighted) mean returns and (weighted) mean characteristics over the two periods, eliminating the interaction term present in other decompositions (Oaxaca and Ranson 1994).
a widening of the gap at the lower tail, and have impeded further convergence at
the top of the distribution.27

The results from this section motivate our equilibrium analysis, which illu-
minates the factors driving wage structure changes.2829

5 Theoretical Model

5.1 Demand Side

Aggregate production in the economy is a function of labor, we abstract from capital. Agents are divided into four types according to their gender and skill. We define skilled workers as those with at least some college education.30 The technology is described by a three-level nested constant elasticity of substitution (CES) function, with nests corresponding to occupation, skill, and gender.31 At the top level, output is produced by a CES combination of labor in the three types of market occupations by task-content:

\[
Y_t = Z_t \left[ \alpha_{1,t} L_{a,t}^{\rho_1} + (1 - \alpha_{1,t}) \left( \alpha_{2,t} L_{r,t}^{\rho_2} + (1 - \alpha_{2,t}) L_{m,t}^{\rho_2} \right) \right]^{1/\rho_1},
\]

where \(Y_t\) is total output at time \(t\); \(Z_t\) is a scale parameter that is allowed to vary over time to capture neutral productivity changes;32 \(L_{a,t}\), \(L_{r,t}\), and \(L_{m,t}\) are the total demand of labor in analytical, routine, and manual task-intensive occupations, respectively; \(\rho_1 \in (-\infty, 1]\) is a function of the elasticity of substitution \((\sigma_{\rho_1})\) between labor in non-analytical (routine and manual) vs analytical task-intensive occupations \((\sigma_{\rho_1} \equiv \frac{1}{1-\rho_1})\); \(\rho_2 \in (-\infty, 1]\) is a function of the elasticity of substitution \((\sigma_{\rho_2})\)

27. As women’s participation increases, selection implies that the average wage of women will fall, other things equal. If unobservables driving selection into the labor force scale with observables then our finding here that observables (gender, education, and ages) do not account for much of the change in the wage structure suggests that unobservables are unlikely to drive the distributional changes that we document.

28. As discussed, our equilibrium model endogenously generates the wage structure. This is influenced by demographic change (including education upgrading) and by shifters of labor supply and relative demand by gender, skill, and occupation groups.

29. We find that education as a worker characteristic does not play an important role in the decomposition exercise, which takes wages as observed, without allowing that they respond to labor supplies. The structural estimation that ensues will show that rapidly rising rates of college completion (skill or education upgrading) played an important role through the wage structure. We demonstrate this by estimating the counterfactual wages in a world in which education stays fixed at its initial level.

30. To maintain a tractable number of parameters in the model, we do not differentiate worker types by age. This is not unreasonable given that we showed that the increase in FLFP was fairly uniform across the sample age range of 25-55.

31. To test how sensitive the results are to the ordering of the levels in the production technology, we discuss results using alternative model specifications in the robustness checks.

32. \(Z_t\) captures changes in neutral (aggregate) productivity as well as all non-labor inputs residually. In CES demand systems where aggregate labor is combined with non-labor inputs, the relative demand optimality among labor inputs in gender, skill and occupation subgroups is not a function of the prices and productivity parameters of non-labor inputs contained residually in the \(Z_t\) term.
between labor in routine vs manual task-intensive occupations ($\sigma_{\rho_2} \equiv \frac{1}{1-\rho_2}$);\(^{33}\) and the $\alpha$’s are time-varying ‘share’ parameters that we discuss below.

In the second level of the production technology, labor in each occupation is divided into two groups, skilled ($s$) and unskilled ($u$), using a CES combination:

$$L_{\text{occ},t} = \left[ \alpha_{3,\text{occ},t} L_{\rho_3,\text{occ},t}^{\rho_3,\text{occ}} + (1 - \alpha_{3,\text{occ},t}) L_{\rho_3,\text{occ},t}^{\rho_3,\text{occ}} \right]^{1/\rho_3,\text{occ}} \quad \text{for } \text{occ} = a, r, m, \quad (5.2)$$

while at the third level labor is disaggregated in each occupation-skill group between female workers, indexed by $f$, and male workers, indexed by $k$:

$$L_{\text{edu},\text{occ},t} = \left[ \alpha_{4,\text{skl},\text{occ},t} L_{\rho_4,\text{occ}}^{\rho_4,\text{occ}} + (1 - \alpha_{4,\text{skl},\text{occ},t}) L_{\rho_4,\text{occ}}^{\rho_4,\text{occ}} \right]^{1/\rho_4,\text{occ}} \quad \text{for } \text{edu} = s, u, \text{ and } \text{occ} = a, r, m. \quad (5.3)$$

The parameters in the second and third levels have an analogous interpretation to those in Equation (5.1). An innovative feature of our model is that the elasticities of substitution between male and female labor, and between skill groups, are allowed to vary based on the task-content of occupations.

Our set-up is related to Johnson and Keane (2013), who also build gender-, skill-, and occupation-specific relative labor demands based on nested-CES demand aggregation. In contrast to us, they consider occupations directly rather than occupation groups by task-content. Importantly, within occupations, they assume that the elasticities of substitution across gender and education groups are homogeneous—meaning that $\rho_{3,\text{occ}} = \rho_3$ and $\rho_{4,\text{occ}} = \rho_4$ for all $\text{occ}$ groups. Our estimates suggest this is not the case, and that allowing for this heterogeneity can play an important role in explaining the patterns in the data.

The share parameter ($\alpha$) for each CES sub-nest can be interpreted as indexing the share of work activities allocated between different types of labor within each CES combination (Katz and Autor 1999).\(^{34}\) They are allowed to vary over time to capture non-neutral technical change and other factors that shift relative labor demand. As discussed in Section 1.1, we depart from previous studies in allowing technical change to be gender and skill biased and to vary by occupational category. We allow for demand shifts between occupations ($\alpha_{1,t}$ and $\alpha_{2,t}$), capturing forces like

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33. Note that, by assumption, the elasticity of substitution between male and female labor in analytical vs manual task-intensive occupations is the same as the elasticity of substitution between analytical vs routine task-intensive occupations. This seems a natural way of organizing the three occupational groups since we observe them align this way in the wage distribution as low vs. high paying occupations. However, we investigate alternative specifications in the robustness checks section.

34. For example, depending on the elasticity of inputs within a sub-nest, higher $\alpha_{4,\text{skl},\text{occ},t}$ corresponds to higher or lower optimal relative male to female labor demands: If male and female labors are perfect complements ($\rho_{4,\text{occ}} \rightarrow -\infty$), higher $\alpha_{4,\text{skl},\text{occ},t}$ reduces the relative demand for male labor; as male and female labors tend toward perfect substitutes ($\rho_{4,\text{occ}} \rightarrow 1$), higher $\alpha_{4,\text{skl},\text{occ},t}$ increases the relative demand for male labor.
technical change that differentially affect jobs depending on their task-content;\textsuperscript{35} between skilled and unskilled labor within occupations ($\alpha_{3,occ,t}$), capturing skill-biased technical change that can be general or occupation-specific;\textsuperscript{36} and between men and women within occupation-skill groups ($\alpha_{4,skl,occ,t}$), capturing gender-biased technological change;\textsuperscript{37} It may also capture changes in relative labor demand due to changes in Beckerian taste discrimination over time (Hsieh et al. 2019).\textsuperscript{38}

The demand-side of the model has two types of parameters that we need to estimate: \textit{8 parameters that are functions of the elasticities of substitution ($\rho_1$, $\rho_2$, $\rho_{3,a}$, $\rho_{3,r}$, $\rho_{3,m}$, $\rho_{4,a}$, $\rho_{4,r}$, and $\rho_{4,m}$); and a group of time varying demand shift parameters that vary by gender, skill, and occupation ($Z_t$, $\alpha_{1,t}$, $\alpha_{2,t}$, $\alpha_{3,a,t}$, $\alpha_{3,r,t}$, $\alpha_{3,m,t}$, $\alpha_{4,s,a,t}$, $\alpha_{4,s,r,t}$, $\alpha_{4,s,m,t}$, $\alpha_{4,u,a,t}$, $\alpha_{4,u,r,t}$, and $\alpha_{4,u,m,t}$). As argued by Johnson and Keane (2013) and as we discuss in Appendix Section C.2, it is possible to fit the trends in relative wages perfectly if we do not impose any restrictions on the evolution of the relative demand parameters, but this would mean that we would not be able to identify the parameters capturing the elasticities of substitution. We impose a 3rd order polynomial restriction on the trends for the share parameters. For example, the parameter $\alpha_{1,t}$ is allowed to change according to

$$\log \alpha_{1,t} = a_{1,0} + a_{1,1}t + a_{1,2}t^2 + a_{1,3}t^3.$$ (5.4)

Additionally, to flexibly account for neutral technological changes, we allow for $t$-specific $Z_t$ values without parametric restrictions.

As is clear from Equation (2.1), any changes in relative wages that are not explained by movements in relative supplies are absorbed by the relative demand parameters. In total, there are 65 elasticity and share parameters on demand-side that we need to estimate.\textsuperscript{39} Issues of identification and estimation are discussed below.


\textsuperscript{36} As in Katz and Murphy (1992), Machin, Reenen, and Van Reenen (1998), Berman, Bound, and Machin (1998), and Katz and Autor (1999)

\textsuperscript{37} See Acemoglu, Autor, and Lyle (2004), Black and Spitz-Oener (2010), Pitt, Rosenzweig, and Hassan (2012), and Burstein, Morales, and Vogel (2019)

\textsuperscript{38} In a Beckerian taste discrimination framework, the marginal utility of an employer from hiring a female worker can be lower due to utility loss associated with discriminatory tastes (Hsieh et al. 2019; Morchio and Moser 2021). Given labor participation and wage data, we trace out differential gender labor demands. We cannot distinguish demand differences due to discrimination from real productivity differences. Thus the $\alpha_{4,skl,occ,t}$ share parameter we estimate could be capturing changes in discrimination over time or changes in real productivity. Empirically, real productivity and discrimination are also entangled: recent research literature has found that gender-based discrimination (harassment) has real productivity impacts (Folke and Rickne 2020; Cici et al. 2021), see the related discussion in Section 1.1.

\textsuperscript{39} This include $(3 + 3 + 2) = 8$ elasticities of substitution, $(6 + 3 + 2) \times 4 = 44$ coefficients associated with the third order polynomials of $\alpha$ shares, and 13 $t$-specific $Z_t$ parameters.

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5.2 Participation and Occupational Choice on the Supply Side

Male and female workers sort themselves into different market occupations based on preferences over work arrangements and wages\(^{40}\). The model allows gender-specific comparative advantage associated with differences in physical, sensory, motor, and spatial aptitudes\(^{41}\). Comparative advantage will reflect in marginal productivity, and hence influence occupational sorting through wages.

We model occupational choice using a random utility framework where agents of different types choose between the four alternatives according to which provides the highest utility. We model utilities as linear functions that depend on pecuniary and non-pecuniary rewards from each choice. In particular, the utility that a worker of a given type receives from choosing to enter the workforce in one of the three market occupations at time \(t\) is

\[
U(occ | gen, skl, t) = \psi_{gen, skl, occ} + \psi_1 \log (W_{gen, skl, occ, t}) + \epsilon_{gen, skl, occ, t},
\]

where \(\psi_{gen, skl, occ}\) is a time-invariant parameter capturing non-pecuniary rewards (such as occupational job flexibility, or the mission-orientation of a job) from choosing occupation \(occ\); and \(\psi_1\) measures the weight in utility terms that a worker gives to wages \((W_{gen, skl, occ, t})\) in log units.\(^{42}\) \(\epsilon_{gen, skl, occ, t}\) is an idiosyncratic taste shock assumed to be independent and identically distributed extreme value. The assumption about the distribution of the taste shock generates a tractable multinomial logit form for the choice probabilities.\(^{43}\)

The utility from home production is modeled symmetrically for men and women. The literature has linked movements of women into the labor market to changes in contraceptive technology and fertility,\(^{44}\) marriage markets,\(^{45}\) social norms

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\(^{42}\) The log wage assumptions allows the relative odds of choosing to work over leisure to approach zero as wage tends toward zero. Given our data and empirical context, we model only two skill levels, but the framework can be extended to a finer classifications of labor skills. In settings where labor demand is gender- and occupation-, but not skill-specific, wages are sometimes more restrictively assumed to be log-additive given some base efficiency wage unit (Böhm et al. 2019).

\(^{43}\) Given this assumption on the error distribution, occupational selection is driven by individual- and occupation-specific random factors that do not impact productivity. Alternatively, one might assume that the error terms come from individual- and occupation-specific productivity draws that generate heterogeneous earnings given occupation-specific skill-prices (Burstein et al. 2020). Preference and productivity draws might both drive occupational selection, but are difficult to empirically disentangle. We model only preference shocks given the perceived importance of idiosyncratic non-wage factors in determining women’s labor participation decisions.


\(^{45}\) These include Grossbard-Shechtman and Neuman (1988), Fernández and Wong (2014), and Greenwood et al. (2016)
and attitudes towards women’s work, and improvements in technology and capital (e.g., appliances) used for home production. Together, these variables capture changes in preferences and the technology of home production. For instance, fertility will capture preferences for flexibility, and the model allows these preferences to be specific to gender and skill. While the cited studies typically analyze one predictor at a time, we consider them all in a unified framework—this allows us to estimate their relative importance in explaining the patterns in the data. In a departure from partial equilibrium papers in the literature, we allow wages across gender, skill, and occupations to be determined endogenously and jointly, and occupational choices can respond to wage changes when \( \psi_1 \) is nonzero. However we take non-wage shifters of labor force participation as given.

The utility from choosing home production, denoted by \( h \), takes the form:

\[
U(h \mid \text{gen, skl, t}) = \pi_{1,\text{gen}} + \pi_{2,\text{gen, skl}} Pr(\text{child} = 1 \mid \text{gen, skl, t}) \\
+ \pi_{3,\text{gen, skl}} Pr(\text{married} = 1 \mid \text{gen, skl, t}) \\
+ \pi_{4,\text{gen, skl}} Pr(\text{appliance} = 1 \mid \text{gen, skl, t}) \\
+ \pi_{5,\text{gen, skl}} WBL_t + \epsilon_{\text{gen, skl, h, t}}.
\]

(5.6)

\( \pi_{1,\text{gen}} \) are gender-specific intercepts. \( Pr(B = 1 \mid \text{gen, skl, t}) \) are time- and group-specific proportions with young children, a proxy for fertility (child), in stable partnerships (we label this married), and have household appliances (appliance). \( WBL_t \) is the score on a work-related legislation index, and \( \epsilon_{\text{gen, skl, h, t}} \) is a idiosyncratic taste shock assumed to be independent and identically distributed extreme value.

Given the assumed distribution of the taste shocks, the probability that a worker chooses one of the market occupations or home production is

\[
Pr(d_O = 1 \mid \text{gen, skl, t}) = \frac{\exp(\hat{U}(O \mid \text{gen, skl, t}))}{\sum_{\text{occ}=a,r,m,h} \exp(\hat{U}(\text{occ} \mid \text{gen, skl, t}))} \quad \text{for } O = a, r, m, h,
\]

(5.7)

where \( \hat{U} \) is equal to \( U \) without the idiosyncratic shocks. We use these probabilities to find the total labor supply of each type in each occupation. For example, the total supply of female workers with college education in analytical task-intensive occupations is

\[
L^*_f,s,a,t = L^{\text{pop}}_{f,s,a,t} \times Pr(d_a = 1 \mid f, s, t),
\]

(5.8)


where $L_{f,s,t}^{pop}$ is the demographic measure for the total number of in-Mexico prime-age potential female workers with college education at time $t$. As discussed in the data section, $L_{gen,skl,t}^{pop}$ changes over $t$ due to overall increases in population, gender-specific skill upgrading, and gender- and skill-specific patterns of emigration.  

As the example shows, we condition on educational attainment, assuming this is complete before the age of 25, when individuals enter our sample.

We refer to $Pr(d_O = 1 \mid gen, skl, t)$ as the gender- and skill-specific occupation participation rate. Thus the gender- and skill-specific labor force participation rate is $LFP_{gen,skl,t} = \sum_{occ \in \{a,r,m\}} Pr(d_{occ} = 1 \mid gen, skl, t)$. A key feature of our counterfactual analysis is that we consider the potential countervailing effects on FLFP due to shifts in the supply curves. It follows from Equation 5.8 that supply curves may shift with changes in the demographic composition ($L_{gen,skl,t}^{pop}$), or on account of factors that influence $Pr(d_O = 1 \mid gen, skl, t)$, including shifts in fertility, marital patterns, home appliance availability, and more legislation protecting women’s economic rights.

Having laid out our labor supply model, we now explain how it contrasts with the existing literature. There is a long-standing structural literature on female labor supply which focuses on dynamic responses of education, fertility, marriage, and labor market choices to changes in the costs and returns of human capital accumulation and the paths of earnings, taxes, and transfers (often using samples starting at age 16 or 18). These studies generally specify partial equilibrium models that, in contrast to ours, do not allow for endogenous wage adjustments.

In recent papers, Hsieh et al. (2019), Burstein et al. (2020), and Morchio and Moser (2021) have explored equilibrium wage responses in papers that do not focus on FLFP changes over time, but that allow for gender-specific labor supplies. However, in these papers, occupational selection in gender and education cells is based

48. $L_{gen,skl,t}^{pop}$ includes Mexican-born workers who have not emigrated. Given our model assumptions, probabilities in Equation (5.7) are consistent with having an outside option to emigrate that is a function of an exogenously determined foreign wage.

49. Additionally, we refer to $Pr(d_O = 1 \mid gen, t) = \sum_{skl \in \{s,u\}} \left( L_{gen,skl,t}^{pop} / L_{gen,t}^{pop} \right) \times Pr(d_O = 1 \mid gen, skl, t)$ as the gender-specific occupation participation rate, and the gender-specific labor force participation rate is $LFP_{gen,t} = \sum_{skl \in \{s,u\}} \left( L_{gen,skl,t}^{pop} / L_{gen,t}^{pop} \right) \times LFP_{gen,skl,t}$.

50. See reviews in Blundell and Macurdy (1999), Keane (2011), Keane, Todd, and Wolpin (2011), and Blundell (2017). Partial equilibrium labor supply models can handle greater choice complexity and dynamics in effect because only an “inner-loop” for the dynamic life-cycle problem needs to be solved, but there is no need to worry about a potential multi-dimensional “outer-loop” of market clearing conditions.

51. In partial equilibrium and reduced-form models with treated and untreated local labor markets, sometimes general equilibrium effects of policy treatments on local wages can be estimated (Attanasio, Meghir, and Santiago 2012; Breza and Kimball 2021). A general equilibrium model, however, is required to predict equilibrium effects of changes beyond the domain of policy treatment variations. As an exception to this literature, Lee and Wolpin (2006) build a general equilibrium model with occupational selection among blue, white and pink collar jobs, and allow for endogenous education and experience accumulation. However, Lee and Wolpin (2006) take the fertility process as given, do not consider marital status, and, more importantly, they do not consider gender.
on unobserved shocks. In contrast, by linking occupational choices to a rich array of observables, we allow for direct counterfactual comparisons among the different predictors that have been identified in the literature. These non-wage predictors have tended to be analyzed independently, in different settings, while we provide an analysis of them, by gender and skill, in a unified framework. Possibly most closely related to our framework, Johnson and Keane (2013) build an equilibrium model of male and female labor supply with endogenous education and occupation choices. However, since FLFP is not a focus of their paper, they do not consider changes in fertility, marital status, and other observables that we study, instead they account for changes in LFP over time with indirect utility time trends.\(^5\)

### 5.3 Equilibrium and Estimation

The equilibrium model generates a prediction of the wage and labor supply of the four worker types in the three market occupations in every time period. With 13 years of data, there are \((12 + 12) \times 13 = 312\) predictions in total that are a function of the 94 parameters of the model, including 29 on the supply-side and 65 on the demand-side.\(^5\)

We discuss the broad approach to estimation here, elaborating it in an Appendix. In particular, Appendix Section C.1 defines and characterizes the equilibrium as a system of equations for male (or female) wages; Appendix Section C.2 clarifies variations in the data that pin down share and elasticity parameters across nests, and discusses the potential benefits of equilibrium estimation; and Appendix Section C.3 focuses on an estimation strategy that pins down reasonable estimator starting values for the large-dimensional parameter space. The analysis we provide in Appendix Section C.2 could be used to evaluate whether existing papers use the appropriate polynomial order and, accordingly, whether they are appropriately identified.

To provide analytical clarity to the equilibrium problem, we consider the overall nested-CES problem in separate nest-groups. We discuss the de-nesting in Appendix Section C.1.1. Within one period and for one skill-group, the equilibrium wage for women in any given occupation is a function of the equilibrium wages of men across the different occupations (analytical, routine, and manual). The equilibrium wage relationships across genders generate a system of nonlinear equations for female wages that characterizes the equilibrium, which we present in Appendix Sections

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52. In contrast to Johnson and Keane (2013) we focus on prime-age workers between 25 and 55, after most educational decisions have been made, and prior to retirement. We therefore do not model endogenous changes in the share of college educated workers in response to wage changes. However, the educational composition of the labor force is a key demographic characteristic in our setting and, in a counterfactual exercise, we explore equilibrium wage responses to increasing the share of college-educated female workers.

53. The numbers correspond to 12 wages per year for each gender \(\times\) skill \(\times\) occupation triad, and a similar number of labor supplies. Since the gender- and skill-specific occupation participation rates add up to one, labor supplies in the home production occupation are recovered residually.
C.1.2 and C.1.3. In Appendix Section C.1.4, we solve for the equilibrium explicitly via nested root search as well as via a faster but less stable contraction algorithm. While the nested-CES demand system is commonly estimated in the labor economics literature, it is less common to estimate both demand and supply parameters in an equilibrium context. In this paper, we develop an estimation framework that allows us to do this. Given the large number of parameters involved in estimating the model, we discuss the key identification challenges and solutions that arise. Specifically, in Appendix Section C.2.1, we discuss the identification of parameters across nests using relative wages within and across nests. The lowest nest directly faces observed wages and labor quantities, higher nest layers generate aggregate wages and quantities based on lower level parameters and observables. In Appendix Section C.2.2, we discuss the necessary data requirements for jointly identifying $\rho$ and $\alpha$ (the elasticity and share parameters on the demand-side) via equilibrium supply-shifters, and we specify the challenge of this approach in our empirical setting with biennially aggregated data. In Appendix Section C.2.3, we discuss the data requirements for possibly identifying variations in the $\alpha$ parameter over time under polynomial restrictions, a commonly used strategy that we adopt. We show that identification is based on the concept of time-invariance in demand parameters after differencing. In our equilibrium setting where labor supply is elastic to wages, we show in Appendix Sections C.2.4 and C.2.5 that estimation relying only on supply-side participation equations suffers from the potential endogeneity of wages, and estimation relying only on demand-side optimality equations can suffer from bias due to mismeasurement and shocks to relative demand trends. Finally, in Appendix Section C.2.6, we discuss the benefits of the equilibrium solution based estimation that we adopt for this paper.

Estimation proceeds by searching for the set of demand- and supply-side parameters that generates the best fit between equilibrium predictions and the corresponding observed values in the data. However, it is computationally challenging to directly search for minimizing parameters in a 94-dimensional parameter space globally. We estimate the model by first performing a preliminary round of linear and nonlinear least squares estimation of different components of the model to generate reasonable starting values to initialize equilibrium estimation. We generate different starting values as we explore alternative values of the eight $\rho$ (elasticity) parameters. We discuss details of the estimation parameter space in Appendix Section C.3.1. We discuss how parameter values are initialized conditional on $\rho$ in Appendix Section C.3.2. We discuss the error structure and weight matrix from the score of the log-likelihood function in Appendix Section C.3.3.
6 Model Fit and Estimates

Model predictions are based on estimates of the following: the occupation-specific elasticity of substitution between the genders and between the skill groups; occupation-specific gender- and skill-biased technological change; the influence of non-wage determinants of labor supply on participation, and the elasticity of aggregate and occupation-specific labor supply to occupation-specific wage changes. Predictions of gender gaps additionally depend on the size and composition of the labor force (potential labor supply) by gender and skill, which varies with the pace of female relative to male skill-upgrading and unskilled male emigration.

6.1 Model Fit to the Data

In general, the model predictions consistently track long-term trends and short-term variations in the data. Figure 7 shows the skill-weighted relative (i.e male to female) wage and relative supply series by occupation group, and the aggregate labor force participation rates for women and men. Figure 8 Panel (a) shows relative wages by occupation and skill group. It shows the overall downward pattern in relative wages (male relative to female) for skilled workers and the flat or rising trend for relative wages for unskilled workers. Panel (b) shows declining shares of men relative to women in analytical (skilled and unskilled), unskilled manual and unskilled routine occupations. Appendix Table D.2 shows observed and predicted mean wages and occupation shares for all groups at the start and end of the period, showing generally a good fit across all cells.

6.2 Demand Side Elasticity of Substitution

By Occupation and Gender. The elasticities of substitution between male and female labor are estimated to be around 1.1 and 1.3 in manual and routine task-intensive occupations, respectively, and 2.9 in analytical task-intensive occupations (see Table 4). Thus, consistent with our starting premise, male and female labor are closer substitutes in occupations that rely more on analytical skills, and which tend to lie towards the upper end of the wage distribution.

To get a sense of what these values represent, we performed some back-of-the-envelope calculations using the relative demand optimality condition from Equation (2.1). Taking the actual occupation-specific increase in the supply of female relative to male labor, the estimated substitutability of female for male labor implies a widening of the gender wage gap across occupations, and that the gap
widens most in manual and least in analytical task-intensive occupations.\textsuperscript{54} We learn two things. First, that the increase in FLFP exerted substantial downward pressure on the wages of all female workers, but most so in manual occupations that lie towards the bottom of the wage distribution. Second, since the actual widening of the gender wage gap in all occupations (and most of all analytic) was smaller than predicted by the estimated elasticity of substitution, it seems likely that demand trends were favorable to women (especially in analytic occupations).

The only estimates of the elasticity of substitution between male and female labor that we could find are in Acemoglu, Autor, and Lyle (2004) and Johnson and Keane (2013) for the U.S. As discussed in Section 1, their magnitudes are broadly in line with ours. However, their estimates are not occupation-gender specific. We provide the first occupation-task specific elasticities, and demonstrate that the differences across task-based occupation have significant distributional effects. In particular, our finding that female and male labor are more easily substituted in analytic-intensive occupations contributes to explaining the stylized fact that the gender wage gap narrowed at the top of the wage distribution even as it widened lower down in the distribution.

**By Occupation and Skill.** We also contribute to the literature, among the first estimates of elasticities of substitution between skilled and unskilled labor by occupation. Our estimates are 1.4 in analytical, 1.4 in routine and 3.8 in manual task-intensive occupations (see Table 4). Consistent with intuition, the unskilled are closer substitutes for skilled workers in manual occupations.

Since skilled workers are concentrated in analytical occupations, these results on their own suggest that the sharp educational upgrading that occurred in the sample period in Mexico will have exerted downward pressure on the skill premium. However, as we will see, we estimate skill- and gender-biased technological change, which increased the demand for skilled female relative to male labor. As a result, the college premium fell for men and not for women- this is illustrated in the counterfactuals section.

Other studies find estimates of the elasticity of substitution between skilled and unskilled labor that are broadly similar to ours,\textsuperscript{55} but previous studies tend not to provide this elasticity by occupation, which our analysis suggests is a significant

\textsuperscript{54} In manual task-intensive occupations, log (male/female) relative supply fell between about 1992 and 2012 by 56.4 log points (see Appendix Table D.3), so an elasticity of 1.1 implies that the log (male/female) wage ratio should have increased, other things equal, by 51.7 log points. This is considerably larger than the observed 6.4 log point increase. In routine and analytical task-intensive occupations, log (male/female) relative supply fell by 38.8 log points, so the implied elasticities predict an increase in the gender wage gap of 30.4 and 13.2 log points respectively, also significantly larger than the observed 1 and -14 log point changes.

\textsuperscript{55} Estimates for five Latin American countries during the 1990s range from 1.25 (Fernández and Messina 2018) to 3 (Manacorda, Sánchez-Paramo, and Schady 2010) and for the U.S. the elasticity is close to 1.5 (Katz and Murphy 1992; Ciccone and Peri 2005; Johnson and Keane 2013)
6.3 Demand Trends by Occupation, Gender and Skill

Figure 9 shows the model predictions for the evolution of the log relative share parameters, \( \log \left( \frac{1 - \alpha}{\alpha} \right) \), see Equation (5.4). We show the evolution of demand for male relative to female labor by skill and occupation. Moving up one nest, we then show the evolution of demand for skilled relative to unskilled labor by occupation. Estimates for the uppermost nest show relative demand trends across occupation groups. We also show estimated aggregate output to productivity trends related to the neutral aggregate productivity \( Z_t \) term. Overall, there is evidence of both gender-biased and skill-biased technical change. The most striking pattern is that demand evolved to favor female relative to male labor, especially among skilled workers. This holds across occupations.

**Relative demand for female labor.** There is an increase in the demand for female relative to male labor in every skill-occupation group (Panels a and b), with the largest increase among college-educated (skilled) women (Panel b). For both skill groups, analytical task-intensive work does not exhibit the largest increase in relative female demand due to a high starting level in 1989, but it maintained the highest ratio of female to male demand throughout the years, approaching parity by 2014. The effect size is large: for skilled workers in analytical task-intensive occupations, the model predicts that demand trends alone would have led the gender wage gap to have narrowed by 39 log points.

The coefficients of the relative demand polynomials are estimated, in effect, residually, to explain changes in equilibrium wages and labor quantities that cannot be explained by changes in observable supply-side factors that impact the number of potential workers and LFP. We are not able to pinpoint the drivers of the relative demand trends. However, our results line up with the broader result in the literature that structural changes faced by most economies in recent decades have been favorable to female labor, with jobs in which women have a comparative advantage or at least no disadvantage gaining ground in the economy. As discussed in the Introduction, some studies emphasize labor reallocation from goods to service industries (Lup Tick and Oaxaca 2010; Akbulut 2011; Ngai and Petrongolo 2017), others the changing skill requirements of the economy with the role of brawn declining and cognitive and social skills rising (Galor and Weil 1996; Blau and Kahn 1997; Weinberg 2000; Rendall 2017; Black and Spitz-Oener 2010; Aguayo-Tellez et al. 2013; Rendall 2013; Juhn, Ujhelyi, and Villegas-Sanchez 2014; Deming 2017; Cortes, Jaimovich, and Siu 2018). Our results are also consistent with a decline in

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56. In their analysis of automation, Acemoglu and Restrepo (2018) use an aggregate CES production function with two levels: the first one combining a unit measure of tasks, the second combining capital and labor in a CES aggregate. In the main model, they do not differentiate labor by skill, but in an extension they do.
aspects of gender discrimination. Growth in the WBL index of legislative protection of women’s economic rights will capture a decline in discrimination against women in hiring, manifest as the modeled impacts of WBL on female labor supply. The estimated changes in the demand share parameters in favor of women reflect changes in employer discrimination that are reflected through wages.

**Relative demand by skill and occupation.** Abstracting from gender, relative demand trends evolved to favor skilled relative to unskilled labor, see Panel (c) of Figure 9. Potential drivers of skill-biased demand shifts in Mexico are trade and investment liberalization (Feenstra and Hanson 1997; Hanson 2003; Sánchez-Páramo and Schady 2003; Behrman, Birdsall, and Szekely 2007; Caselli 2014) and the growth of foreign direct investment (Feenstra and Hanson 1997) in this period. In Panel (d) of Figure 9, we see no change in relative demand in analytical task-intensive occupations (which had higher female shares at baseline), but a slight increase in the first decade in routine relative to manual task intensive work.

**Total labor requirement scaled by productivity.** Panels (e) and (f) of Figure 9 show that the aggregate output to productivity ratio \( \frac{Y_t}{Z_t} \) approximately doubles from 1989 to 2014, which matches closely with a doubling of real GDP per capita in Mexico during the same time span. This implies an overall relatively flat, but slightly downward trending pattern in \( Z_t \), which captures changes in neutral aggregate productivity.

Overall, the results confirm that Mexico experienced skill-biased technical change, a phenomenon that has been widely argued to explain rising income inequality in developed economies. Nevertheless, in contrast to many OECD countries, Mexico experienced a compression of wage inequality among men, with educational upgrading (an increase in the share of workers with college education) more than offsetting the increasing demand for skill. No previous work has investigated either how the changing role of women in the labor market contributed to the compression of male inequality, or documented that there was no marked change in wage inequality among women despite even greater educational upgrading. Our estimates of elasticities of substitution and demand trends by skill and occupation contribute on both fronts.

### 6.4 Supply Side Wage Elasticities

While there is one wage parameter \( \psi_1 \) across gender and skill groups, the effects of wages on occupational choices, including the participation margin, can be heterogeneous. We conduct the analysis at three levels. First, we analyze the effects of

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57. In Appendix Section C.1.1, we show that the optimal labor demand in constant-returns CES problems is a function of the \( \frac{Y_t}{Z_t} \) ratio. In Appendix Section C.2.1, we discuss that \( Z_t \) cannot be separately identified from \( \frac{Y_t}{Z_t} \). \( \frac{Y_t}{Z_t} \) captures a total (labor) factor productivity re-scaled output term that captures the total labor/output requirement from aggregating across skill-, occupation- and gender-specific labor units.
increasing wages in all occupations on the aggregate labor supply. Second, we study the effects of increasing occupation-specific wages on aggregate labor supply. Third, we estimate the effects of increasing wages in one occupation on the labor supply to this and other occupations. The second and third parts of the analysis follow naturally in an occupation-choice framework with occupation-specific equilibrium wages. They are, however, generally overlooked in the literature. We make a contribution in providing occupation-specific own- and cross-wage elasticities. These are also important ingredients for the counterfactual analysis. For the four gender and skill groups, the top panel of Table 5 presents the average marginal effects (AME) of wages on aggregate labor supply, in percentage points units, while the bottom panel presents the respective wage elasticities.

Aggregate gender- and skill-specific labor supply responses to an increase in wages across occupations. We first consider a simultaneous increase in wages in all occupations (first column). We find that labor force participation of women is more sensitive to wages than is the case for men, particularly among the unskilled. That female labor supply is more elastic has been documented extensively (Killingsworth and Heckman 1987; Blundell and Macurdy 1999; Keane 2011), but the magnitude tends to vary widely across studies. We estimate elasticities of 0.529 and 0.341 for unskilled and skilled female workers respectively, and of 0.060 and 0.062 for unskilled and skilled male workers. These numbers are close to the average (across studies) reported in meta-analysis of the literature. They are also in line with structural estimates showing larger elasticities of labor supply among women with lower educational attainment (Blundell et al. 2016).

In Figure 10, we show that female (but not male) aggregate wage elasticities have decreased over time. Among skilled women, the elasticity decreased from around 0.52 to 0.24, and among unskilled women from 0.63 to 0.46. These findings corroborate Heim (2007) and Blau and Kahn (2007) who show that the elasticity of labor supply of American women has strongly declined over time. This has been interpreted as evidence of women’s growing labor market attachment. It also follows from an increase in the share of women working. In the counterfactual analysis of Section 7 we discuss potential drivers of this.

Aggregate gender- and skill-specific labor supply responses to an increase in occupation-specific wages. While the aggregate wage elasticity of

58. For AME, we evaluate the total derivative of aggregate labor participation with respect to wages in the direction of equi-distance increases for all wages. For elasticity, we divide the percentage increase in labor supply by a percentage increase in wages that is common across occupation-specific wages.

59. The average elasticity of female vs male labor supply is 0.43 vs 0.07 (Evers, Mooij, and Vuuren 2008), 0.43 (married women) and 0.59 (single mothers) vs 0.12 (Bargain and Peichl 2016), and 0.28 vs 0.06 (Keane 2011).

60. Blundell et al. (2016) estimate, for the UK, an average Marshallian labor supply elasticity for women of 0.475. For women with at most secondary schooling, it is 0.689 and for those with tertiary education it is 0.331.
labor supply is a useful summary measure, it is only empirically relevant if wages across occupations jointly shift up or down. In fact, changes in LFP are better represented by the dot product of the vector of occupation-specific wage changes and the vector of occupation-wage specific elasticities. Columns 2-4 of Table 5 decompose the results in column 1, where all wages are changing, by the separate effects of wages in manual, routine, and analytical occupations. This separation is relevant if, for instance, trade or technological change moves the wage in one occupational class and not in the others. We find that the aggregate labor force participation of unskilled women and men is most responsive to the wage in manual task-intensive occupations, and least responsive to the wage in analytical occupations. Conversely, participation of skilled workers is most responsive to the analytical task wage, consistent with skilled workers being more likely to sort into these occupations once they participate.

*Occupation-, gender- and skill-specific labor supply responses to an increase in occupation-specific wages.* The previous section discussed the net effects of occupation-specific wage changes on gender- and skill-specific labor supply. In this section, we decompose the net effects and consider how changes in occupation-specific wages influence own- and cross-occupation labor supply. See Figure 11 and Appendix Tables D.4 and D.5. All own-wage elasticities are positive, and cross-wage elasticities are negative. Elasticities specific to gender-skill-occupation-year are in the Figures and too numerous to detail, but we summarize the main patterns here. When manual wages change, unskilled men are the most responsive group, and skilled women the least. When routine wages change, the patterns are broadly similar except that now skilled women are more responsive and the own-vs cross-wage elasticity indicate that they will move between analytical and routine task jobs as a function of the relative wage. Changes in analytical task wages produce labor supply responses that are similar between men and women, but differentiated by skill, being larger among skilled workers. While, as known, women’s aggregate participation is more wage-elastic than that of men, less known is that, even though aggregate male labor supply is largely invariant to occupation-specific wages as shown in Table 5, male mobility across occupations is influenced by relative wages.

Overall, our finding of fairly large (and time-varying) occupation-skill-gender specific labor supply responses to the equilibrium wage structure underlines the relevance of accounting for these responses in a general equilibrium model. As explained in the Introduction, this is a contribution this paper makes relative to much of the related literature.

61. For AME, values from columns two to four of Table 5 are partial derivatives that sum up to the total derivative value in the first column.
6.5 Supply Side Non-Wage Determinants of Labor Force Participation

We now discuss estimates of average marginal effects (AME) of the four supply-side shifters on gender-skill-occupation specific labor supply, see Table 6 (Also see Appendix Table D.6). AME is calculated by taking the numerical derivative of the probability of choosing home production with respect to the given variable. The margins are interpreted as the percentage point changes in the probability of leisure conditional on skill and gender given a one percentage point increase in the respective supply-side variable (and, thus, the signs are reversed for occupation-specific labor supply). Our counterfactual results will provide an accounting of the importance of these factors for the evolution of gender gaps in LFP and wages.

**Fertility.** Fertility significantly influences LFP for men and women, and this relationship is strongest among skilled women. We noted earlier that, on average, the percentage of skilled women with young children (fertility) declined by 20 percentage points over the sample period, and more sharply among skilled women. Extrapolating, our estimates indicate that a 10 percentage point (pp) decrease in fertility corresponds to increasing the participation of skilled women by a considerable 6 pp. In contrast, fertility decline decreases the LFP of unskilled women (and unskilled men) albeit these are much weaker effects, at 0.63 pp (and 1.3 pp), consistent with a tendency for unskilled women to take work to meet consumption needs, and for men to work harder when endowed with the responsibility of fatherhood. Skilled male labor supply is almost perfectly inelastic to fertility, consistent with the strong labor market attachment of this group.

**Stable partnerships.** We estimate that the same percentage point decline in the share of stable partnerships would lead to similarly sized increases in the LFP of skilled men and women, and larger increases in the participation of unskilled men. It would reduce participation of unskilled women.

**Household appliance availability.** The availability of appliances is a significant determinant of female participation. Starting again with the level in 1989 and extrapolating from our estimates, if we reduced appliance availability by 10 pp. for unskilled (from 63.0%) and skilled (from 95.6%) women respectively, FLFP would decrease by 5 and 18 percentage points, respectively. However, as the skilled group had close to complete uptake at baseline, the significant growth in uptake of appliances over the period was among unskilled women, see Figure 5. Thus, as seen in the counterfactuals that follow, appliance availability was a key driver of FLFP only for unskilled women.

**Women’s economic rights.** Improvements in gender and work related laws and regulations, as captured by the WBL index, have a small positive effect on FLFP, twice as large for skilled as for unskilled women. In 1989, holding all else constant, if there were a 10 pp increase in the index, there would be a 2.5 pp increase in the
LFP rates for skilled women. The corresponding impact on unskilled women would be 1.3 pp. The marginal effects of an increase in the WBL index on male LFP are negative but close to zero. Improvements in the WBL index will include impacts of legislation designed to limit discrimination against women in the workplace and, to the extent that legislative changes follow changes in cultural and social norms regarding female work (Doepke and Zilibotti 2005; Platteau and Wahhaj 2014), this is consistent with research showing the relevance of shifting norms for the growth in FLFP (Fernández, Fogli, and Olivetti 2004; Fogli and Veldkamp 2011; Fernández 2013).

7 Counterfactuals

Using the estimated parameters of the equilibrium model, in this section we evaluate and compare, in one internally-consistent framework, various factors considered by different strands of the literature as important for changes in gender wage and participation gaps.

We evaluate eight variables. These are four non-wage predictors of labor force participation (in total and by skill and occupation), two contributors to changes in the gender-skill composition of the potential workforce, and two demand-side variables, namely skill and gender-biased technological change (by occupation).

We compute general equilibrium (GE) wage responses given counterfactual changes in demand and supply over time. Where relevant, we compute supply-side partial equilibrium (PE) results given observed wages in each year. Comparisons of GE with PE estimates illuminate the role of endogenous wage responses. We demonstrate that PE tends to over-state the contribution of supply shifters by virtue of ignoring equilibrium wage adjustments that generate countervailing impacts on labor supplies. This is important because research that estimates structural models of female labor supply has typically taken wages (and labor demand) as given and provided PE counterfactuals (Keane, Todd, and Wolpin 2011; Böhm et al. 2019). Accounting for wage adjustments is also important in estimating the effects of changes in demographic composition. We find that accounting for GE effects in this case reverses the sign of the PE effects.62

Our main findings are as follows. The baseline model predicts an overall narrowing of the aggregate gender LFP gap of 19.9 percentage points (similar to the observed increase in female LFP). A narrowing is evident across all occupation and skill groups. Our counterfactual analysis indicates that increasing appliance availability, which increased unskilled FLFP, was the largest contributor, accounting for 28% of the overall narrowing of the LFP gap. The increasing share of skilled

62. Partial equilibrium (PE) results may approximate GE if the economy is a small open economy facing exogenously determined wages. As this is not the case in Mexico, or in general, the GE analysis is more empirically relevant.
women among potential female workers is the main force pushing in the opposite
direction to widen the participation gap, offsetting three quarters of the appliance
effect.63

Alongside a narrowing of the gender participation gap, there was a narrowing
of the overall gender wage gap by 6.3 log points. This is a weighted average over
skilled workers, for whom the wage gap fell by 10.4 log points, and unskilled workers
for whom it increased by 9.6 points. The weight on the skilled group was increasing
over time, reflecting both increasing college completion rates, and the fact that
decreasing fertility encouraged participation conditional on college. The narrowing of
the skilled gender wage gap reflects that demand dominated supply in this group.64
The increase in the unskilled gender wage gap reflects that the relative labor supply
of women, driven by appliance availability and the emigration of unskilled men,
dominated the rising demand for women in this group.

Overall, across skilled and unskilled women, the single largest influence on
the aggregate gender wage gap in this period was the growing demand for female
labor (and the implied outward shift in the demand for female labor explains why
the participation gap and the wage gap move in the same direction). We estimate
that, if there were no increase in the relative demand for women then, rather than
decreasing, the overall gender wage gap would have grown by 17.6 log points.

We now elaborate on each result. In each counterfactual exercise, we fix the
variable or parameter of interest to its value in 1989, and keep it constant across the
years. We then compare the predicted equilibrium wages and labor supplies under
the counterfactual scenario with the baseline model predictions. In Tables 7 and
8, each cell reports changes over time in the gender differences in labor supply and
wages by skill and by occupation-skill groups. The first column shows the prediction
of the baseline model, successive columns present results from counterfactuals that
shut down the mechanism indicated in the column header. Thus if a mechanism in
column 5 (for example) has a large impact on the outcome, this is reflected in a large
difference between the estimate in column 5 and the estimate in the first column.
For ease of exposition, the counterfactual results are also visualized in Figures 12,
13, and 14.

7.1 Non-Wage Determinants of LFP

The gender participation gap. The estimates are in the first block of
Table 7. The headline result is that the rapid rise in household appliances among
unskilled women was the main driver of their LFP, while fertility decline was the

63. The intuition for this is as follows. The rising share of skilled women dampened wage growth
for skilled women which, in turn, cramped further rises in their participation. It also crowded out
some of the potential increase in the share of unskilled women joining the labor force, essentially
because growth in the relative demand for women’s labor was skill-biased.

64. Demand for skilled women rose more rapidly than demand for unskilled women or men.
main driver of the LFP of skilled women. Male labor supply is not substantially impacted by either.

Refer to Figure 13, where the results are visualized. Under PE, in the absence of any increase in appliances, unskilled women would have increased their participation in routine task-intensive occupations from 10% to 12%, whereas in fact this rate increased to about 16%. In the absence of fertility decline, skilled women would have increased their routine work participation rate from about 8.5% to 14.5%, whereas in fact this rate increased to about 18%. In contrast, the counterfactual and the actual participation paths largely overlap for unskilled women under the fertility counterfactual, and for skilled women under the appliance counterfactual. This is despite a large average marginal effect of appliances on skilled women’s work participation (see Table 6), because appliance availability for skilled women was high at the start and did not change substantially. This pattern of results holds by occupation too. Visualizations are provided in Panels (b-d) of Figure 13.

The GE estimates additionally account for the fact that, as more women are driven into work, women’s wages fall, and this inhibits further increases in women’s labor supply. The PE-GE difference is large, for example, it halves the positive impact of appliance availability on the FLFP of unskilled women. Since a larger number of unskilled women participate, equilibrium wage effects are larger in this group.

In aggregate, the model predicts a narrowing of the gender gap in LFP in favor of women by 19.9 pp. Without the observed increase in household appliances, the gap would have narrowed by 14.3 pp (28% of 19.9 pp) under GE, and 7.5 pp (62% of 19.9 pp) under PE. Without fertility decline, it would have narrowed by 17.9 pp (10% of 19.9 pp) under GE and 16.5 pp (17% of 19.9 pp) under PE. The relatively small share of skilled women in the population reduces the aggregate effects of the fertility counterfactual.

Increasing WBL contributed to reducing LFP gaps in all skill and occupation groups, and the decreasing share of stable partnerships slightly widened the LFP gap among the unskilled. The time-series of aggregate LFP gap changes is shown in Panel (a) of Figure 13 and changes between end-points are visualized along the x-axis of Appendix Figure D.2.

The gender wage gap. Under both GE and PE, increasing appliance availability increases the gender wage gap, while decreasing fertility reduces it. Marriage and women’s rights have relatively small impacts on relative wages. These results are visualized in Panel (a) of Figure 12, and the results for marriage and economic

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65. Between 1989 and 2014, the share of unskilled women with a refrigerator or washing machine increased dramatically from 63% to 89%, while the share among skilled women barely increased, from 96% to 98%. On the other hand, all women experienced fertility decline, albeit this was sharper among skilled women. The share of skilled women with children under the age of 5 fell from 54% to 30%. The share of unskilled women with young children fell from 43% to 27%. See Figure 5.
rights are in Panel (a) of Appendix Figure D.2.

The baseline prediction is a narrowing of the aggregate gender wage gap of 6.3 log points. The PE counterfactual shows that, absent the rise in appliance availability, the wage gap would have narrowed by 10.6 (68.2% more than the baseline), which establishes that rising appliance availability has tended to increase the wage gap. The PE effect works by changing the skill composition of the workforce—appliances increase the relative supply of unskilled women who earn lower wages. The GE estimate is a narrowing of the wage gap by 12.2 log points (93.6 percent more than in baseline). GE additionally allows that, as the supply of unskilled female labor increases, their relative wage falls and this leads to further divergence of women’s wages relative to those of men.

The PE counterfactual shows that, absent fertility reduction, the gender wage gap would have narrowed by 3.3 points. The corresponding GE estimate is 5.6 log points. Both numbers are smaller than the baseline of 6.3 points, indicating that, the reduction in fertility decreased the gender wage gap. The PE effect is again compositional but now lower fertility increases the participation of skilled women who earn higher wages, with little effect on unskilled women. The additional channel in the GE effect is that increased participation of skilled women generates downward pressure on skilled female wages, leading to a smaller GE reduction of the wage gap compared to PE.

The extent to which changes in labor supply affect the gender wage gap is determined by $\rho_{4,occ}$, or the elasticity of substitution between male and female labor. Our finding that this elasticity is small in lower-paying manual and routine task-intensive occupations, which employ a disproportionate share of unskilled workers, acts to sharpen the relative wage effects of the appliance counterfactual. The greater ease with which women can substitute men in analytical tasks, which are more likely to be occupied by skilled workers, blunts the relative wage effects of the fertility counterfactual. In particular, downward wage pressure from higher skilled female participation is transmitted to male wages, moderating the downward pressure on the gender wage gap.

Previous studies analyzing the role of, for example, appliances, and fertility, have not studied their important distributional effects.

**The skill premium.** The skill premium in Mexico (and Latin America more generally) has attracted attention, with many studies highlighting falling wage inequality among men. We are unaware of studies showing a more stubborn wage premium for skilled women. Our baseline model predicts a log (skilled/unskilled) wage ratio among women that is fairly stable (-1.4 log points decline). Beneath the surface though, are some large movements that happen to offset one another. In the absence of fertility decline, the female skill premium would have increased by 7.5 pp, while in the absence of increased appliance ownership, it would have decreased by
These counterfactuals demonstrate the responsiveness of the skill premium (or college premium) to factors that shift labor supply differentially for skilled and unskilled workers.

Changes in women’s participation can also impact the skill premium for males, a phenomenon largely ignored in the literature on wage inequality, whether in the US or in Mexico, see Section 1.1. The size of this impact will depend on the elasticity of substitution between male and female labor. The baseline model shows that the male skill premium declined over the period by 21.5 log points. The counterfactuals show that, if not for appliance growth, it would have declined by 20.5 points and, if not for fertility decline, it would have declined 18.3 points. Fertility decline had a larger impact on the male skill premium because of the higher substitutability of men and women in skill-intensive occupations, which transmits more readily the downward pressure on skilled female wages to skilled male wages. However, the magnitude of the fertility effect is limited by the relatively small number of skilled women.

### 7.2 Demographic Changes

**Relative skill upgrading.** As explained in Section 3.2, skill upgrading (college completion rates) proceeded more rapidly among women than men over the analysis period. We construct a counterfactual scenario in which we fix the gender composition of the skilled population at its 1989 level. We do this by allowing the number of skilled males and the gender-specific population to increase as observed, maintaining the female share among the skilled at 31.2%, its initial level. By 2014, under this counterfactual, there are 2.2 million skilled women rather than the actual 5.0 million, and the difference is added to the number of unskilled women.

As the share of skilled female workers increases, since they earn higher wages and have higher LFP rates (see Panel (b) of Figure 6), under PE, the compositional impact is to reduce the gender gap in participation and wages. Thus under the counterfactual that shuts down the actual increase in skilled women among potential workers, the gender wage gap increases by 5.3 log points in contrast to the baseline decline of -6.3 log points, see the second block of Table 8. The gender participation gap declines by 17.8 pp instead of the baseline of 19.9 pp, see Table 8 (first block).

Under GE, there is feedback from equilibrium wages to labor supplies. This counterfactual illustrates nicely the role that the substitutability of male and female labor, the substitutability of skilled and unskilled labor, and the wage elasticity of supply play in the propagation of this feedback. This highlights the relevance of allowing variation in these elasticities by occupation.

We now elaborate the mechanisms at play. Expansion of the share of women in the population with a college degree—an increase in skilled female labor supply—pushes down skilled female wages. Now the high substitutability of male and female...
labor in skilled-worker-dominated analytical task-intensive occupations ($\rho_{4,a}$) plays a role: skilled labor demand moves in favor of women and the demand for skilled men contracts, driving down skilled male wages. At this stage, the high degree of substitutability of skilled vs unskilled labor in manual task-intensive occupation ($\rho_{3,m}$) plays a role: as skilled wages are lower, there is a contraction in demand for male and female unskilled workers. This contraction is stark due to the sharp growth in the relative productivity of skilled female workers (decreasing $\alpha_{4,skl,occ,t}$ and increasing $\alpha_{3,occ,t}$). Overall, there is a reduction in wages across gender and skill groups.\textsuperscript{66}

We next consider implications for gender gaps in participation and wages. The GE effect of skill upgrading on the gender LFP gap is to widen it (-24.2 pp, which exceeds the baseline of -19.9 pp), see Panel (a) of Figure 14. This is the reverse of the PE effect. The reason is essentially that the overall fall in wages has a larger impact on female than on male LFP, given that the aggregate wage elasticity is larger for females (see Panel (b) of Figure 14).\textsuperscript{67}

While accounting for GE effects reverses (widens) the direction of changes in the gender LFP gap implied by PE (a narrowing), GE magnifies the narrowing of the gender wage gap that was established under PE—the counterfactual coefficients are +9.6 (GE) and +5.3 (PE) log points instead of the baseline -6.3 log points—see the third block of Table 8. GE magnifies (about threefold) the narrowing of the gender wage gap for skilled workers, and the widening of the gender gap for unskilled workers. Overall, the relative wage effects on unskilled workers dominate because they are a much larger share of the population.

The GE feedback effects from wages also have implications for the skill premium. Consider the fourth Panel of Table 8, which shows the impact of holding fixed the gender composition of skilled workers on the gender-specific skill premium. We find that, on its own, the increased share of skilled women significantly decreased the female skill premium by 42.2 log points (from +40.8 under the counterfactual to -1.4 at baseline).

\textbf{Emigration.} There was an increase in emigration rates over the analysis period, led by unskilled men. We conduct a counterfactual in which we fix the share of emigrants in the Mexican-born population by gender and skill at its 1989 level. Rising emigration created upward pressure on wages, especially among the unskilled, increasing their participation rates. Unskilled women reacted more than unskilled men given their larger aggregate wage-elasticity, and this led to a narrowing of the

\textsuperscript{66} The demand-driven fall in unskilled male wages is unambiguous. However, the demand contraction for unskilled female workers is counteracted by the concurrent contraction in female unskilled labor supply. It turns out that, on balance, unskilled female wages fall as well.

\textsuperscript{67} The rising share of skilled women in the labor force was dampened by a fall in their wage. It also crowded out some of the potential increase in the share of unskilled women joining the labor force (as demand favored skilled over unskilled women).
overall gender participation gap.\textsuperscript{68} Since the labor supply of men adjusts less than that of women, male wages absorb most of the impact, and there is an increase in the unskilled and the overall gender wage gap. The magnitude of this effect is similar to that of the appliances counterfactual.

\section*{7.3 Demand Side Share Parameters}

To quantify the impact of relative demand trends, we run two counterfactuals in which we fix either the gender-skill-occupation shares ($\alpha_{4,skl,occ,t}$) or the skill-occupation shares ($\alpha_{3,occ,t}$) to their 1989 values. We find that increasing female-demand-share ($1 - \alpha_{4,skl,occ,t}$) had the largest impact on the narrowing of the gender wage gap of all the mechanisms that we analyze counterfactuals for (see the second block of Table 8). As shown in Section 6.3, relative demand trends strongly favored women relative to men in all occupations and skills groups. Absent this more favorable demand for women, the gender wage gap would have increased by 17.6 log points instead of declining by 6.3 points as predicted in the baseline. In fact, the growing demand for women’s labor counteracted the downward pressure on female wages arising from rapid increases in FLFP.

It is noteworthy that even when we shut down $\alpha_{4,skl,occ,t}$, the labor supply of women increases substantially, closing the participation gap with men, albeit at a slower pace relative to the baseline (see the overall row in the first block of Table 8 and Panels (c) and (d) of Figure 14). But with the growth in relative female demand shut down, the relative female wage adjusts downward (see the overall row in the second block of Table 8).

Fixing the gender-skill-occupation shares ($\alpha_{4,skl,occ,t}$) also has significant distributional effects. The gender wage gap increases in both skill groups, but more among the skilled, reversing the observed pattern. This demonstrates that the actual contraction of the gender wage gap at the top of the wage distribution was primarily driven by stronger labor demand for skilled women (see Figure 9). The latter also translates into an increase in the skill premium for women, and a decrease for men.

In our final counterfactual, we fix the skill-occupation shares ($\alpha_{3,occ,t}$) at their initial value. As these shares do not vary by gender, their effects on the gender wage and participation gaps are indirect and small. What is strongly affected is the skill premium. The counterfactual shows a decline relative to baseline predictions (see the last block of Table 8). This confirms that skill-biased technical change raised the skill premium.

\textsuperscript{68} The counterfactual, phrased in the inverse, is -16.7 pp compared with the baseline -19.9 pp.
8 Robustness Checks

The wage series used for the baseline estimates included only the incomes of full-time workers, see Section 3.1. We now report estimates including income from part-time workers. We also produce estimates replacing the head count measure of labor supply with the total number of hours worked by each group. For the latter exercise, since we do not have a measure of hours worked in home production, we impute those values. We assign each person in home production the average number of hours worked by workers in market occupation with the same age, gender, and level of schooling. The rank-size of the values in each of the levels (nests) is maintained using these alternative measures. Specifically, once we include income from part-time workers, the elasticity of substitution between male and female labor in manual and routine task-intensive occupations is lower, down from 1.09 and 1.28 in the baseline model to 0.80 and 0.97, respectively. This reinforces our conclusions from the previous section that increasing female labor supply led to downward pressure that was particularly large among female workers in these lower-paying occupations. Additionally, using the intensive margin measure of labor supply does not change the estimates in any meaningful way, the results are essentially unchanged.

As discussed, demand is estimated residually and we restrict the $\alpha$ share parameters to follow a cubic trend in their natural logarithm. The cubic trends provided the best fit of the model to the data. Quadratic polynomials did not allow sufficient flexibility, while the coefficients associated with the quartic polynomials were not statistically significant in most cases. Importantly, the estimates are not sensitive to functional form. Results are available upon request.

When modeling the structure of the production technology, two decisions were made that could influence the results but do not have a solid theoretical basis: One, in the three nests, labor is first divided by education and then by gender. This division changes the number of relative demands that are estimated in each dimension, but it should not alter the main results. Second, the model assumes that the elasticity of substitution between analytical and routine task-intensive occupations is the same as that between analytical and manual-task intensive occupations. We check robustness under three revised model specifications: (i) switch the order of the second and third nests of the production technology; (ii) impose that the occupational group that has the common elasticity with the other two is routine task-intensive; and (iii) impose, instead, that the occupational group that has the common elasticity with the other two is manual task-intensive. We find that the rank order of the values of the elasticities of substitution between male and female

69. This suggests that if part-time work were rising disproportionately more for women than for men, this would tend to widen the gender wage gap in favor of men in the occupations that absorb part-time workers. However, this was not the case in Mexico: the share of women in part-time work relative to the share of men in part-time work was fairly stable between about 1990 and 2002, after which it declined (see Figure D.1).
labor is maintained in all cases: the manual and routine task-intensive occupations elasticities lie between 0.7 and 1.2, while the analytical task elasticity lies between 1.9 and 2.6. The corresponding estimates of the parameters from the supply-side of the model under the alternative model specifications remain essentially unchanged compared to the baseline. Results are available upon request.

9 Conclusions

We develop a model that allows that demand and supply forces interact in equilibrium and jointly explain the observed paths of gender-, skill-, and occupation-specific wages as well as changes in gender wage and participation gaps. Our approach marks a departure from most previous work on women’s labor supply, which takes labor demand as fixed. It also marks a departure from the standard labor supply-demand model used, for instance, to analyze immigration, by virtue of endogenizing labor supply. Our estimates suggest that, even in settings where total labor supply is fixed, it is relevant to consider the sorting of labor across occupations, in response to equilibrium wages. We quantify the relative importance of demand, supply, and demographic factors and their distributional consequences, within and between genders and across the wage distribution.

While our empirical results speak specifically to the experience of Mexico as it entered the twenty-first century, similar demand, supply, and demographic mechanisms are likely to be at play in other Latin American countries and beyond (López-Calva and Lustig 2010; Levy and Schady 2013; Lustig et al. 2016; Galiani et al. 2017; Fernández and Messina 2018). Our equilibrium framework and associated solution, identification, and estimation results are potentially applicable in other gender settings and in non-gender settings where a nested-CES production function is appropriate for describing input aggregation, and multinomial discrete choice assumptions can sufficiently capture the relative tradeoffs for participating in alternative occupations.
Tables and Figures

Figure 1: Labor Force Participation by Gender

Absolute Numbers

(a) Survey Data

(b) Census Data

Gender-specific Participation Rates

(c) Survey Data

(d) Census Data

Notes: Participation refers to prime-age population either working or actively searching for a job. The differences between the census and survey data (ENIGH) arise because the census only includes as economically active those individuals whose primary activity was either working or looking for a job. For example, part-time workers whose primary activity was studying are categorized as outside the labor force, leading to an underestimation. Sample weights used in all calculations. See discussions in Section 3.2 and Appendix Section A.1.
Figure 2: Distribution of Changes in Log Hourly Wages by Gender between C.1992 and C.2012

Notes: The series are constructed by computing the change in real log hourly wages between C.1992 and C.2012 at each percentile of the male and female distribution respectively. Sample restricted to prime-age population working more than 35 hours a week. To increase sample size we joined together surveys from 1989, 1992, and 1994 (C.1992), and from 2010, 2012, and 2014 (C.2012). The plot shows that wages in Mexico have tended to decrease over the period. Wage changes for men and women diverge at the two ends of the distribution. See discussions in Section 3.2.
Figure 3: Unconditional Distribution of Changes in the Gender Wage Gap

(a) Survey Data: between C.1992 and C.2012  (b) Census: between 1990 and 2010

Notes: Panel (a) shows the change in log (male/female) hourly wages by percentile between C.1992 and C.2012 calculated using the survey data from ENIGH. Sample is restricted to prime-age population that reported working for more than 35 hours a week. Sample weights used in all calculations. Panel (b) replicates the exercise using information on monthly labor earnings and hours worked from the 1990 and 2010 Mexican CENSUS. See discussions in Section 3.2.
Figure 4: Decomposition of the Gender Wage Gap by Percentile of the Distribution

Notes: The Figure shows results of the Oaxaca-Blinder decomposition of the unconditional change in the log (male/female) wage ratio between C.1992 and C.2012 by percentile. The estimation is done separately for 19 percentiles. Confidence intervals are estimated via bootstrap with 500 replications. Sample weights used in all calculations. The wage structure effect dominates, tracking the data. The composition effect is fairly constant across the distribution and close to zero. See discussions in Section 4.
Figure 5: Trends in Fertility, Marriage, Appliances and Norms Regarding Women’s Work

(a) Fertility

(b) Marital Status

(c) Women, Business and the Law (WBL) index

(d) Household Appliances

Notes: Panel (a) depicts the share of each group with children under the age of 5. Panel (b) depicts the share of each group that is married or has a permanent partner. Panel (c) shows the value of the Women, Business and the Law (WBL) index. Panel (d) shows the share of each group that has both a refrigerator or a washer in the household. The measures of fertility and marriage can only be calculated for a sample restricted to the household head and their spouse or partner; trends for the larger sample used in the estimation are not available. The ENIGH survey started asking the question on marital status to all members of the household in 1996, and the question about the number and age of children since 2004. The sample is restricted to the prime-age population. See discussions in Section 3.2, Section 7.1, and Appendix Section A.2.
Figure 6: Share of Each Gender-Skill Group in the Prime-Age Population, Gender-Skill Specific Participation Rates, and Share of Emigrants in the Mexican Born Prime-Age Population

(a) Share of the Gender-Skill Group in Prime-Age Population

(b) Gender-Skill Specific LFP Rate

(c) Share of Emigrants in Mexican Born Population

(d) Potential Workers (Excluding Emigrants)

Notes: Panel (a) depicts the share of each gender-skill group in the prime-age population. Panel (b) depicts the gender-skill specific participation rates. Panel (c) depicts the share of emigrants in the total Mexican born population, conditional on gender, skill group, and being prime-age. Panel (d) depicts the total number of potential prime-age workers by gender and skill group (excluding emigrants). The number of emigrants by skilled group are taken from Brücker, Capuano, and Marfouk (2013). See discussions in Section 3.2, Section 7.2, and Appendix Section A.2.
Figure 7: Model Fit
Data and Model Predictions for Relative Wages, Relative Supplies and Participation Rates

Analytical Occupations

(a) Log (Male/Female) Wage Ratio

(b) Log (Male/Female) Relative Supply

Routine Occupations

(c) Log (Male/Female) Wage Ratio

(d) Log (Male/Female) Relative Supply

Manual Occupations

(e) Log (Male/Female) Wage Ratio

(f) Log (Male/Female) Relative Supply

Participation Rates

(g) Female

(h) Male

Notes: The different panels depict the series of log (male/female) relative wages, log (male/female) relative labor supplies (= demands), and LFP rates from both the raw data and as predicted from the model, showing a close fit. See discussions in Section 6.1.
Figure 8: Model Fit
Data and Model Predictions for Male to Female Relative Wages and Male and Female Occupation Participation Rates Differences

(a) Log (Male/Female) Wage Ratio

(b) (Male - Female) Occupation Participation Rates

Notes: The panels depict log (male/female) relative wages and (male - female) occupation participation rates differences (See Footnote 49 for definition). The skill- and occupation-specific results from the first block of the model columns of Tables 7 and 8 show the differences between the averages of the first three years and the final three years based on the Model Prediction lines in the present Figure. See discussions in Section 6.1.
Figure 9: Estimates of the Relative Demand Indexes and Total Factor Productivity

Production Technology: $\alpha$ Share Parameter

(a) Level III, Unskilled
Male Share $\alpha_{4,u,occ,t}$ (vs. Female)

(b) Level III, Skilled
Male Share $\alpha_{4,s,occ,t}$ (vs. Female)

(c) Level II
Skilled Share $\alpha_{3,occ,t}$ (vs. Unskilled)

(d) Level I, Routine Share $\alpha_{2,t}$ (vs. Manual),
Analytical Share $\alpha_{1,t}$ (vs. Rou. + Man.)

Production Technology: $\frac{Y}{Z}$ Output Productivity Ratio

(e) $\frac{Y}{Z}$ and Real GDP Per Capita

(f) Total Factor Productivity ($Z$)

Notes: Panels (a)-(d) show the estimated relative demand indexes captured by the natural logarithm of the ratio of $\alpha$ and $1 - \alpha$. Panel (e) shows the estimated output to productivity ratio $\frac{Y}{Z}$, plotted along real GDP per capita relative to 1989 using FRED data. Panel (f) is the natural logarithm of the ratio of real GDP per capita by the $\frac{Y}{Z}$ ratio. See discussions in Sections 6.3 and 7.3.
Figure 10: Estimates of Aggregate Wage Elasticity
The Elasticity of Gender- and Skill-specific Aggregate Labor Supply with Respect to Wages

Notes: The panel depicts elasticities. It shows the ratio of a percentage change in the aggregate labor supply—for each gender and skill group—over a percentage increase in wages. The same percentage increase in wages is applied to all occupation-specific wages concurrently. Year-specific elasticities are computed. The first column in the bottom panel of Table 5 shows the averages of the elasticities over time. See Figure 11 for the own- and cross-elasticities of occupation-specific labor supplies with respect to occupation-specific wages. See discussions in Section 6.4.
Figure 11: Estimates of Own and Cross Wage Elasticity
The Elasticity of Occupation-specific Labor Supply to Occupation-specific Wage

(a) Own and Cross-Elasticities of Manual Wages

(b) Own and Cross-Elasticities of Routine Wages

(c) Own and Cross-Elasticities of Analytical Wages

Notes: The panels depict elasticities. It shows the ratio of a percentage change in occupation-specific labor supply—for each gender and skill group—over a percentage increase in an occupation-specific wage. Triangle, circle, and diamond lines represent the elasticity of manual, routine, and analytical labor supplies with respect to different wages. See Figure 10 for aggregate elasticities. See Appendix Table D.5 for average own-and cross-elasticities across the years. See discussions in Section 6.4.
Figure 12: Counterfactual Exercises
Effects of Non-wage Determinants of LFP, Demographics, and Demand Side Parameters on Changes in the Gender LFP and Wages Gaps between C.1992 and C.2012.

(a) Changes in Gender Participation and Wage Gaps: (C.2012 - C.1992)


Notes: The Table reports the difference between C.1992 and C.2012 of i) the log (male/female) wage ratio and ii) the change in the (male - female) LFP and occupation rates under different counterfactual scenarios. Figure (a) visualizes results from the “Overall” row in the first two blocks of Tables 7 and 8. Figure (b) visualizes results from the skill/occupation-specific rows in the first two blocks of Tables 7 and 8 (Skilled-manual and unskilled-analytical results are not shown for conciseness). Black-dashed lines mark model predictions, and points indicate predictions under key counterfactual scenarios. Points to the right of the vertical dashed-line reduce gender LFP and occupation participation gaps; points to the top of the horizontal dashed-line reduce the gender wage gaps. Under the counterfactuals, we set the share with under-5 children (Fertility), the share with refrigerator or a washing machine (Appliance), the skilled population female share (Skilled Female), the gender/skill-specific emigrant shares (Emigrant), the skill/occupation-specific demand gender share $\alpha_4$, and occupation-specific demand skill share $\alpha_3$ at their 1989 values, respectively. See discussions in Section 7.
Figure 13: Counterfactual Exercises
Non-wage Determinants of LFP and Occupation Participation Rates

(a) (Male - Female) LFP Gap, Change from 1989

Notes: We set observables for non-wage determinants of LFP at their 1989 values, one at a time. In Panel (a), appliance had the largest effect on the aggregate gender participation gap. In Panels (c)-(d), more appliance and less fertility increased LFP for unskilled and skilled women, respectively. In partial equilibrium, we hold wages as observed. In general equilibrium, we resolve for equilibrium wages given supply curve shifts. See discussions in Section 7.1.
Notes: Panels (a) and (c) show differences between the gender LFP gap in 1989 and each subsequent year under various counterfactuals. Panels (b) and (d) show variations in gender-specific LFP rates over time. In the counterfactuals here, we set the skilled population female share and the gender/skill-specific emigrant shares, and the skill/occupation-specific demand gender share ($\alpha_4$) and occupation-specific demand skill share ($\alpha_3$) parameters at their 1989 values. Figures 6 and 9 present changes in these variables and parameters over time. See discussions in Sections 7.2 and 7.3.
<table>
<thead>
<tr>
<th>ENIGH Principal Group</th>
<th>Median Percentile of the Task Measure</th>
<th>Av. Share (x100)</th>
<th>Av. Male Share (x100)</th>
<th>Av. Wage Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Analytical</td>
<td>Routine</td>
<td>Manual</td>
<td></td>
</tr>
<tr>
<td>Managers</td>
<td>90.0</td>
<td>17.0</td>
<td>27.5</td>
<td>Analytical</td>
</tr>
<tr>
<td>Crafts and Trades (Supervisors)</td>
<td>84.0</td>
<td>42.0</td>
<td>62.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Education</td>
<td>83.0</td>
<td>11.0</td>
<td>65.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Professional</td>
<td>83.0</td>
<td>42.0</td>
<td>46.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Technical</td>
<td>71.0</td>
<td>69.0</td>
<td>43.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Arts/Entertainment</td>
<td>66.0</td>
<td>35.0</td>
<td>48.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Sales</td>
<td>61.0</td>
<td>22.5</td>
<td>15.0</td>
<td>Analytical</td>
</tr>
<tr>
<td>Crafts and Trades (Laborers)</td>
<td>40.0</td>
<td>82.0</td>
<td>73.0</td>
<td>Routine</td>
</tr>
<tr>
<td>Clerical (Supervisors)</td>
<td>61.0</td>
<td>63.0</td>
<td>51.5</td>
<td>Routine</td>
</tr>
<tr>
<td>Crafts and Trades ( Helpers)</td>
<td>10.5</td>
<td>62.0</td>
<td>60.5</td>
<td>Routine</td>
</tr>
<tr>
<td>Machine Operators</td>
<td>16.0</td>
<td>62.0</td>
<td>51.0</td>
<td>Routine</td>
</tr>
<tr>
<td>Clerical (Laborers)</td>
<td>41.5</td>
<td>53.0</td>
<td>12.0</td>
<td>Routine</td>
</tr>
<tr>
<td>Transport</td>
<td>19.5</td>
<td>21.0</td>
<td>96.0</td>
<td>Manual</td>
</tr>
<tr>
<td>Agriculture</td>
<td>32.0</td>
<td>27.0</td>
<td>82.0</td>
<td>Manual</td>
</tr>
<tr>
<td>Protective Services</td>
<td>24.5</td>
<td>5.5</td>
<td>76.5</td>
<td>Manual</td>
</tr>
<tr>
<td>Domestic Service</td>
<td>9.0</td>
<td>8.0</td>
<td>76.0</td>
<td>Manual</td>
</tr>
<tr>
<td>Street Sales</td>
<td>38.0</td>
<td>13.0</td>
<td>64.0</td>
<td>Manual</td>
</tr>
<tr>
<td>Service</td>
<td>28.0</td>
<td>25.0</td>
<td>63.0</td>
<td>Manual</td>
</tr>
</tbody>
</table>

Notes: The three task measures were originally constructed for three-digit occupational codes of the U.S. CENSUS by Autor, Levy, and Murnane (2003). For each measure, we first organize the three-digit occupations by percentiles, and then calculate the median percentile within the broader 18 occupational groups of the ENIGH. Each of the 18 occupations is assigned to the group in which the median percentile was highest. See discussions in Section 3.1 and Appendix Section A.3.
<table>
<thead>
<tr>
<th></th>
<th>C.1992</th>
<th></th>
<th>C.2012</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female (x100)</td>
<td>Male (x100)</td>
<td>Female (x100)</td>
<td>Male (x100)</td>
</tr>
<tr>
<td>Overall</td>
<td>38.59</td>
<td>96.49</td>
<td>59.59</td>
<td>95.82</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary</td>
<td>35.75</td>
<td>96.58</td>
<td>55.47</td>
<td>95.89</td>
</tr>
<tr>
<td>College</td>
<td>71.73</td>
<td>96.00</td>
<td>77.42</td>
<td>95.59</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-34</td>
<td>40.54</td>
<td>96.82</td>
<td>59.18</td>
<td>95.73</td>
</tr>
<tr>
<td>35-44</td>
<td>39.52</td>
<td>97.53</td>
<td>62.63</td>
<td>97.29</td>
</tr>
<tr>
<td>45-55</td>
<td>33.52</td>
<td>94.42</td>
<td>56.61</td>
<td>94.26</td>
</tr>
</tbody>
</table>

Notes: The cells report the (conditional) share of the respective column group. For instance, 35.75 percent of the prime-age female population with secondary education (unskilled) participated in the labor force in C.1992. We joined together surveys from 1989, 1992, and 1994 (C.1992), and from 2010, 2012, and 2014 (C.2012) to increase the sample size of the ENIGH data survey. Sample weights used in all calculations. See discussions in Section 3.1.
Table 3: Decomposition of the Gender Wage Gap for Selected Percentiles

<table>
<thead>
<tr>
<th></th>
<th>P5</th>
<th>P25</th>
<th>P50</th>
<th>P75</th>
<th>P95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Change</td>
<td>0.333 [0.054]</td>
<td>0.154 [0.021]</td>
<td>0.133 [0.021]</td>
<td>0.031 [0.027]</td>
<td>-0.225 [0.045]</td>
</tr>
<tr>
<td>Overall Wage Structure</td>
<td>0.211 [0.056]</td>
<td>0.140 [0.021]</td>
<td>0.131 [0.020]</td>
<td>0.052 [0.026]</td>
<td>-0.347 [0.045]</td>
</tr>
<tr>
<td>Overall Composition</td>
<td>0.122 [0.028]</td>
<td>0.014 [0.013]</td>
<td>0.002 [0.015]</td>
<td>-0.021 [0.018]</td>
<td>0.121 [0.025]</td>
</tr>
</tbody>
</table>

Notes: The table shows results of the Oaxaca-Blinder decomposition of the unconditional change in the log (male/female) wage ratio by percentiles. The standard errors in brackets are calculated via bootstrap with 500 replications. Sample weights used in all calculations. See discussions in Section 4.
<table>
<thead>
<tr>
<th>Elastocities of Substitution</th>
<th>Estimate</th>
<th>SE</th>
<th>Implied Elasticity</th>
<th>95% Conf. Int.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gender</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{4,m}$: male, female (manual)</td>
<td>0.084</td>
<td>0.066</td>
<td>1.091</td>
<td>[0.955, 1.273]</td>
</tr>
<tr>
<td>$\rho_{4,r}$: male, female (routine)</td>
<td>0.218</td>
<td>0.067</td>
<td>1.278</td>
<td>[1.093, 1.540]</td>
</tr>
<tr>
<td>$\rho_{4,a}$: male, female (analytical)</td>
<td>0.660</td>
<td>0.078</td>
<td>2.941</td>
<td>[2.022, 5.389]</td>
</tr>
<tr>
<td><strong>Education</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{3,m}$: skilled, unskilled (manual)</td>
<td>0.739</td>
<td>0.036</td>
<td>3.831</td>
<td>[3.010, 5.271]</td>
</tr>
<tr>
<td>$\rho_{3,r}$: skilled, unskilled (routine)</td>
<td>0.301</td>
<td>0.110</td>
<td>1.431</td>
<td>[1.091, 2.078]</td>
</tr>
<tr>
<td>$\rho_{3,a}$: skilled, unskilled (analytical)</td>
<td>0.302</td>
<td>0.125</td>
<td>1.433</td>
<td>[1.058, 2.220]</td>
</tr>
<tr>
<td><strong>Occupation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{1}$: analytical, routine and manual</td>
<td>0.031</td>
<td>0.092</td>
<td>1.032</td>
<td>[0.869, 1.271]</td>
</tr>
<tr>
<td>$\rho_{2}$: routine, manual</td>
<td>-0.154</td>
<td>0.159</td>
<td>0.867</td>
<td>[0.681, 1.192]</td>
</tr>
</tbody>
</table>

Notes: The table reports the point estimates and standard errors of the elasticities of substitution from the production technology. See estimates discussions in Section 6.2, and estimator discussions in Appendix Section C.3.3.
Table 5: Labor Supply Responses to Wage Changes, Marginal Effects and Elasticities

<table>
<thead>
<tr>
<th>Increase Wages in</th>
<th>Increase Occupation-specific Wages:</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Occupations</td>
<td>Manual Wage</td>
<td>Routine Wage</td>
<td>Analytical Wage</td>
<td></td>
</tr>
</tbody>
</table>
| Average Marginal Effects on LFP Rates: | \(\psi_1 = 0.966\) | | | | \\
| \(\text{values are in percentage points}\) | | | | | \\
| female, unskilled | 0.107 | 0.060 | 0.026 | 0.020 |
| female, skilled   | 0.036 | 0.003 | 0.009 | 0.025 |
| male, unskilled   | 0.023 | 0.013 | 0.008 | 0.002 |
| male, skilled     | 0.008 | 0.001 | 0.002 | 0.005 |

Elasticity of Labor Supply with Respect to Wages: \(\text{values are elasticities}\)

| female, unskilled | 0.529 | 0.099 | 0.071 | 0.067 |
| female, skilled   | 0.341 | 0.009 | 0.044 | 0.160 |
| male, unskilled   | 0.060 | 0.025 | 0.022 | 0.010 |
| male, skilled     | 0.062 | 0.005 | 0.012 | 0.041 |

Notes: Given log wage coefficient \(\psi_1 = 0.966\), we show in the top panel the Average Marginal Effects of wages on the gender- and skill-specific LFP rates. In the bottom panel, we show the average elasticities of gender- and skill-specific labor supply with respect to wages. We average over year-specific values. Column 1 shows the the effects of changing all three occupation-specific wages jointly. We evaluate the marginal effects given equi-distance increases in all wages; We evaluate the elasticity given equal-percentage increases in all wages. Columns 2–4 present results when only the wage for one of the three occupations increases. See discussions in Section 6.4.
Table 6: Occupational Choice Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>SE</th>
<th>Average Marginal Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fertility</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{2,f,u}$: female, unskilled</td>
<td>-0.257</td>
<td>0.135</td>
<td>-0.063</td>
</tr>
<tr>
<td>$\pi_{2,f,s}$: female, skilled</td>
<td>2.735</td>
<td>0.810</td>
<td>0.602</td>
</tr>
<tr>
<td>$\pi_{2,k,u}$: male, unskilled</td>
<td>-2.281</td>
<td>0.097</td>
<td>-0.132</td>
</tr>
<tr>
<td>$\pi_{2,k,s}$: male, skilled</td>
<td>-0.044</td>
<td>0.016</td>
<td>-0.003</td>
</tr>
<tr>
<td><strong>Marriage</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{3,f,u}$: female, unskilled</td>
<td>-0.265</td>
<td>0.247</td>
<td>-0.065</td>
</tr>
<tr>
<td>$\pi_{3,f,s}$: female, skilled</td>
<td>0.267</td>
<td>0.355</td>
<td>0.059</td>
</tr>
<tr>
<td>$\pi_{3,k,u}$: male, unskilled</td>
<td>3.017</td>
<td>0.115</td>
<td>0.178</td>
</tr>
<tr>
<td>$\pi_{3,k,s}$: male, skilled</td>
<td>0.916</td>
<td>0.050</td>
<td>0.055</td>
</tr>
<tr>
<td><strong>Appliance</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{4,f,u}$: female, unskilled</td>
<td>-2.075</td>
<td>0.144</td>
<td>-0.508</td>
</tr>
<tr>
<td>$\pi_{4,f,s}$: female, skilled</td>
<td>-8.348</td>
<td>0.218</td>
<td>-1.808</td>
</tr>
<tr>
<td>$\pi_{4,k,u}$: male, unskilled</td>
<td>0.845</td>
<td>0.440</td>
<td>0.049</td>
</tr>
<tr>
<td>$\pi_{4,k,s}$: male, skilled</td>
<td>-3.031</td>
<td>0.025</td>
<td>-0.178</td>
</tr>
<tr>
<td><strong>WBL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi_{5,f,u}$: female, unskilled</td>
<td>-0.514</td>
<td>0.296</td>
<td>-0.126</td>
</tr>
<tr>
<td>$\pi_{5,f,s}$: female, skilled</td>
<td>-1.151</td>
<td>0.211</td>
<td>-0.252</td>
</tr>
<tr>
<td>$\pi_{5,k,u}$: male, unskilled</td>
<td>0.712</td>
<td>0.522</td>
<td>0.042</td>
</tr>
<tr>
<td>$\pi_{5,k,s}$: male, skilled</td>
<td>1.102</td>
<td>0.105</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Notes: For the fertility (share having under-5 children), marriage (share married or having a permanent partner), appliance (share having a refrigerator or a washing machine), and WBL (an index measuring laws and regulations that restrict women’s economic opportunities) variables, the Average Marginal Effect is the percentage points increase in leisure probability—averaged across years—given 1 percentage point increase in the respective supply variables, holding all else the same. See estimates discussions in Section 6.5 and estimator discussions in Appendix Section C.3.3.
Table 7: Counterfactual Exercises with Non-wage Determinants of Labor Supply

<table>
<thead>
<tr>
<th></th>
<th>Change in Gender Participation and Wage Gaps: C.2012 - C.1992</th>
<th>Partial Equilibrium</th>
<th>General Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Path of Wages as Observed</td>
<td>Wages Adjust as Supply Curves Shift</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1) Model</td>
<td>(2) Fertility</td>
</tr>
<tr>
<td>100 × Δ (Male - Female) LFP and Occupation Participation Rates</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>-19.9</td>
<td>-16.5</td>
</tr>
<tr>
<td>Skilled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td></td>
<td>-3.9</td>
<td>-2.2</td>
</tr>
<tr>
<td>Routine</td>
<td></td>
<td>-0.9</td>
<td>-0.3</td>
</tr>
<tr>
<td>Manual</td>
<td></td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Unskilled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td></td>
<td>-4.5</td>
<td>-4.4</td>
</tr>
<tr>
<td>Routine</td>
<td></td>
<td>-2.9</td>
<td>-2.5</td>
</tr>
<tr>
<td>Manual</td>
<td></td>
<td>-7.6</td>
<td>-7.1</td>
</tr>
<tr>
<td>100 × Δ Log (Male/Female) Wage Ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td>-6.3</td>
<td>-3.3</td>
</tr>
<tr>
<td>Skilled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td></td>
<td>-9.0</td>
<td>—</td>
</tr>
<tr>
<td>Routine</td>
<td></td>
<td>-10.3</td>
<td>—</td>
</tr>
<tr>
<td>Manual</td>
<td></td>
<td>-41.5</td>
<td>—</td>
</tr>
<tr>
<td>Unskilled</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td></td>
<td>3.0</td>
<td>—</td>
</tr>
<tr>
<td>Routine</td>
<td></td>
<td>15.9</td>
<td>—</td>
</tr>
<tr>
<td>Manual</td>
<td></td>
<td>7.1</td>
<td>—</td>
</tr>
<tr>
<td>100 × Δ Log (Male/Female) Wage Ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skilled</td>
<td></td>
<td>-10.4</td>
<td>—</td>
</tr>
<tr>
<td>Unskilled</td>
<td></td>
<td>9.6</td>
<td>—</td>
</tr>
<tr>
<td>100 × Δ Log (Skilled/Unskilled) Wage Ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td>-21.5</td>
<td>—</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td>-1.4</td>
<td>—</td>
</tr>
</tbody>
</table>

Notes: The Table reports the difference between C.1992 and C.2012 of i) the log (male/female) wage ratio and ii) the change in the (male - female) gender-specific LFP and occupation participation rates (See Footnote 49 for definition; See Figure 12 and Appendix Figure D.2 for visualizations). The paths of wages are held as observed under PE. Wages adjust given supply curve shifts under GE. Occupation- and skill-specific relative wages are invariant under PE; the overall relative wage shifts under PE due to compositional changes. The first column corresponds to model predictions. The Fertility columns correspond to the counterfactual predictions once we set the gender- and skill-specific shares of individuals having a child under the age of 5 to the values of 1989, and constant across the years. This variable is decreasing over time (Panel (a) of Figure 5). The Marriage columns fix the gender- and skill-specific shares of individuals married or having a permanent partner at 1989 levels, and constant across the years. This variable decreases slightly over time (Panel (b) of Figure 5). The WBL columns fix the Women, Business and the Law index at 1989 levels. This variable increases over time (Panel (d) of Figure 5). The Appliance columns fix the gender- and skill-specific shares of individuals with a refrigerator or a washing machine at 1989 levels. This variable increases substantially over-time (Panel (c) of Figure 5). See discussions in Section 7.1.
Table 8: Counterfactual Exercises with Demographic Variables and Demand Side Parameters

<table>
<thead>
<tr>
<th>Path of Wages as Observed</th>
<th>Wages Adjust as Supply/Demand Curves Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Partial Equilibrium</strong></td>
<td><strong>General Equilibrium</strong></td>
</tr>
<tr>
<td>Model</td>
<td>Demographics</td>
</tr>
<tr>
<td>(1)</td>
<td>(2) Skilled</td>
</tr>
<tr>
<td></td>
<td>Gender $\alpha_4$</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>(6) Skilled</td>
</tr>
<tr>
<td></td>
<td>Gender $\alpha_4$</td>
</tr>
</tbody>
</table>

### 100 × ∆ (Male - Female) LFP and Occupation Participation Rates

<table>
<thead>
<tr>
<th>Overall</th>
<th>-19.9</th>
<th>-17.8</th>
<th>-19.6</th>
<th>-24.2</th>
<th>-16.7</th>
<th>-16.5</th>
<th>-16.9</th>
</tr>
</thead>
</table>

**Skilled**

| Analytical | -3.9 | 0.9 | -4.0 | 1.1 | -3.7 | -3.7 | -2.7 |
| Routine    | -0.9 | 0.7 | -0.9 | 0.3 | -0.9 | -0.1 | -0.7 |
| Manual     | -0.1 | 0.2 | -0.1 | 0.0 | -0.2 | 0.4 | -0.1 |

**Unskilled**

| Analytical | -4.5 | -5.8 | -4.5 | -8.6 | -3.2 | -4.1 | -3.6 |
| Routine    | -2.9 | -4.3 | -2.7 | -5.7 | -2.2 | -3.2 | -2.5 |
| Manual     | -7.6 | -9.5 | -7.4 | -11.3 | -6.6 | -5.8 | -7.3 |

### 100 × ∆ Log (Male/Female) Wage Ratio

<table>
<thead>
<tr>
<th>Overall</th>
<th>-6.3</th>
<th>5.3</th>
<th>-6.3</th>
<th>9.6</th>
<th>-12.6</th>
<th>17.6</th>
<th>-5.1</th>
</tr>
</thead>
</table>

**Skilled**

| Analytical | -9.0 |     |     | -28.6 | -11.5 | 32.3 | -9.4 |
| Routine    | -10.3 |     |     | -42.6 | -14.7 | 56.2 | -11.0 |
| Manual     | -41.5 |     |     | -76.8 | -46.3 | 61.3 | -42.3 |

**Unskilled**

| Analytical | 3.0 |     |     | 12.6 | -1.7 | 12.1 | 1.4 |
| Routine    | 15.9 |     |     | 32.6 | 7.8 | 21.0 | 13.0 |
| Manual     | 7.1 |     |     | 25.2 | -1.8 | 22.9 | 3.9 |

### 100 × ∆ Log (Skilled/Unskilled) Wage Ratio

| Skilled | -10.4 |     |     | -31.9 | -13.3 | 35.0 | -9.8 |
| Unskilled | 9.6 |     |     | 23.5 | 2.7 | 17.8 | 6.1 |

### 100 × ∆ Log (Skilled/Unskilled) Wage Ratio

| Male | -21.5 |     |     | -14.6 | -16.9 | -6.8 | -43.6 |
| Female | -1.4 |     |     | 40.8 | -1.0 | -24.0 | -27.7 |

Notes: The Table reports the difference between C.1992 and C.2012 of i) the log (male/female) wage ratio and ii) the change in the (male - female) gender-specific LFP and occupation participation rates (See Footnote 49 for definition; See Figure 12 for visualization). The paths of wages are held as observed under PE. Wages adjust given supply curve shifts under GE. Occupation- and skill-specific relative wages are invariant under PE; the overall relative wage ratio shifts under PE for demographics counterfactuals due to compositional changes. Participation is invariant under PE for demand counterfactuals. The first column corresponds to model predictions. The Demographics-Skilled Females columns keep the female share among skilled population at 1989 levels, while skilled males and gender-specific populations levels increase as observed. The share of skilled workers increased over time, especially for women (Panel (b) of Figure 6). The Demographics-Emigrant columns keep the gender- and skill-specific shares of emigrants in Mexican born population at 1989 levels. The emigrant share in unskilled Mexican born population increased over time, especially for men (Panel (a) of Figure 6). The Demand-Gender $\alpha_4$ and Demand-Skill $\alpha_3$ columns set the skill- and occupation-specific demand gender share and occupation-specific demand skill share parameters at 1989 values, respectively. Demand trends favored skilled labor and women (Figure 9). See discussions in Sections 7.2 and 7.3.
References


Firpo, Sergio, Nicole Fortin, and Thomas Lemieux. 2007. *Decomposing Wage Distributions Using Recentered Influence Functions.* Unpublished manuscript, PUC-Rio and UBC.


A Data Appendix (online)

A.1 ENIGH Data and Variable Definitions

We compute from the ENIGH survey year-, gender-, skill-, and occupation-specific wages and labor supplies. We have made these data series available for view and download at this link https://github.com/FanWangEcon/PrjLabEquiBFW/blob/main/PrjLabEquiBFW/data/Dataset1.csv with associated key file.

ENIGH and Census comparison. We use 13 waves of the nationally representative Mexican Household Income and Expenditure Survey (ENIGH), covering 1989-2014. For certain summary statistics, we merged surveys from 1989, 1992, and 1994 (C.1992), and from 2010, 2012, and 2014 (C.2012) to increase sample size and smooth over year-specific changes. Figure 1 displays trends in FLFP from ENIGH and the Mexican census. The trends are broadly similar. Differences in level and trend arise from the census only including as economically active individuals whose primary activity was either working or looking for a job. For example, part-time workers whose primary activity was studying are categorized as outside the labor force, leading to census estimates of LFP being lower. For analysis of the longer run trends in FLFP in Mexico seen in the decadal census plots, see Bhalotra and Fernández (2021). Between 1960 and 1990, the FLFP rate increased 11 percentage points, rising from 12 to 23 percent. It accelerated after 1990: between 1990 and 2010 the rate increased by 22 percentage points, reaching 45 percent in 2010.

Wage definition. Labor earnings data refer to the monthly monetary remuneration from labor, including wages, salaries, piecework, and any overtime pay, commissions, or tips usually received, but excluding income received from government transfers. We omit earnings of self-employed workers when calculating labor earnings because, for this group, it is not possible to disentangle remuneration from labor from returns to capital or profits, a common problem in the literature; however, our estimates include labor remuneration for formal and informal workers since self-employment and labor informality are distinct categories. We add up earnings from different occupations if the individual has a secondary job. Monthly earnings are converted into hourly wages by dividing monthly earnings by the worker’s total hours of work per week in all jobs multiplied by the usual number of weeks in a month. Wage rates are transformed into real 2012 U.S. Dollars using the Mexican Consumer Price Index and the purchasing power parity adjusted exchange rate estimated by the IMF. We removed outliers (less than 1 percent in each year), restricting to hourly rates above $0.1 and below $150. The estimates are not sensitive to this.

Full and part time work. We use the sample of workers aged 25 to 55 (prime-age workers). This is done to ameliorate selection problems arising from changes in the educational and retirement choices of younger and older cohorts. Since part-time work is more common among women, to ensure comparability, the
wage series in the main analysis is calculated using full time workers only (35 hours or more in the previous week). The share of workers working part-time is 33 to 38 percent for female workers and 10 to 13 percent for male workers. Importantly, the increase in FLFP over the sample period was clearly not driven by part-time work. In fact, the ratio of female to male part-time workers was stable between 1990 and 2004, after which it declined. Nevertheless, we include results for part-time workers and also results accounting for changes in hours in robustness checks.

A.2 Supply Side Variables

We link women’s labor force participation decision to fertility trends, marriage patterns, gender discrimination in work-related legislation as captured by the Women, Business and the Law (WBL) index, and home appliance availability. These variables capture potential changes in preferences and the technology of home production over time. Additionally, using the ENIGH survey sample and survey weights, we compute the potential prime-age worker population by gender, skill and year. These potential worker counts are impacted by the emigration of Mexican-born workers, something we discuss below. We have made these potential worker and supply-variable data series available for view and download at this link https://github.com/FanWangEcon/PrjLabEquiBFW/blob/main/PrjLabEquiBFW/data/Dataset2.csv with associated key file. Trends in supply side variables are visualized in Figures 5 and 6.

Fertility and marital status. Fertility is defined as the average number of children under the age of five across women. Marital status refers to being married or having a stable partner. We compute these statistics from the ENIGH dataset directly. We generate aggregate proportions by gender and skill group in each year. The measures of fertility and marriage can only be calculated for a sample restricted to the household head and their spouse or partner; trends for the larger sample used in the estimation are not available. The ENIGH survey started asking the question on marital status to all members of the household in 1996, and the question about the number and age of children since 2004. The sample is restricted to the prime-age population.

WBL. As a measure of women’s economic rights, which may also serve as a reverse proxy for discrimination, we use the Women, Business and the Law (WBL) index. The index attempts to capture inequality in legislation against women throughout their working life. Thirty-five legislative issues that correlate with women’s economic empowerment were identified and aggregated to construct the index, with higher values indicating a lessening of restrictions on women’s economic opportunities. The index can range from 0 to 100 and is increasing in the relative equality of rights between men and women. For a detailed description see Hyland, Djankov, and Goldberg (2020).
Appliance. We compute the share of individuals having home appliances. We consider that an individual has access to home appliance if the individual has either a refrigerator or a washing machine.

Emigration. For purposes of the demographic counterfactual that we construct to analyze changes in the gender-skill composition of potential workers, we use the information on emigrant stocks constructed by Brücker, Capuano, and Marfouk (2013). The authors collected data from 20 OECD member states on the immigrant population aged 25 years and older by gender, educational level, and country of birth between 1980 and 2010. Migration is defined according to country of birth rather than foreign citizenship. The final dataset includes estimated stocks of immigrants coming from 195 countries, including Mexico. Although the information is restricted to migrants going to OECD countries, the Pew Research Center estimates that close to 97.3% of Mexican emigrants go to the United States alone. When necessary, we interpolate for emigrants counts in ENIGH survey years.

A.3 Division of Occupations into Manual, Routine, and Analytical Task-Intensive Groups

The ENIGH survey uses the Mexican occupation classification system to categorize workers according to the type of tasks they perform in the main job. The system went through two changes since 1989: first there was an update of the original Clasificación Mexicana de Ocupaciones (CMO) in 1992, and then a full change to the newly introduced Sistema Nacional de Clasificación de Ocupaciones (SINCO) in 2010. These changes make the series incompatible at high levels of disaggregation of the occupational groups, but it is possible to homogenize the SINCO classification to the principal group level of the CMO using the comparability tables produced by INEGI. A.1 The principal group division has 18 distinct occupational groups that can be consistently followed throughout the period of analysis.

The 18 principal level occupations from the ENIGH are classified into three groups defined by whether the activities done in the jobs are predominantly manual, routine (repetitive and easily codifiable tasks), or analytical intensive. The division is based on the measures constructed by Autor, Levy, and Murnane (2003) from different sets of variables of the 1977 Dictionary of Occupational Titles (DOT) of the U.S., and then linked to the three-digit occupation codes of the CENSUS. The DOT evaluated highly detailed occupations along 44 objective and subjective dimensions that include physical demands and required worker aptitudes, temperaments, and interests. Autor, Katz, and Kearney (2006) used a subset of those dimensions to generate a simple typology consisting of three aggregates for analytical, routine, and manual tasks. The analytical task measure corresponds to the average from two

---

variables of the DOT: DCP, which measures direction, control, and planning of activities; and GED-MATH, which measures quantitative reasoning requirements. The routine task measure corresponds to an average from two variables of the DOT: STS, which measures adaptability to work requiring set limits, tolerances, or standards; and FINGDEX, measuring finger dexterity. Finally, the manual task measure uses a single variable, EYEHAND, which measures eye, hand, and foot coordination.\textsuperscript{A.2}

In practice, we first create a cross-walk between three-digit CENSUS codes in the U.S. and the 18 categories of the principal group occupational division of the ENIGH. This task is facilitated by the fact that both the ENIGH and the U.S. CENSUS follow similar international standards when constructing their own occupation classifications. Since the three task measures are ordinal, there is no direct way to use the actual magnitude of the variables to compare occupations across the three dimensions. For each task measure, we first organize the three-digit occupations by percentiles and then calculate the median percentile of the measure within the broader 18 occupational groups of the ENIGH. Each of the 18 occupations is assigned to the group in which the median percentile was highest (see Table 1).

This procedure generated a balanced division with respect to the overall employment share of each group, and it is also consistent with the broad classification of aggregate occupations used in the literature that follows the task-based framework. Two important caveats should be stressed: First, any attempt to homogenize occupation classification systems from different countries involves some subjective choices. In the cases where we found occupations that do not have an immediate correspondence between the two systems, we had to use our judgement, based on documentation about the description of the occupation, to select a corresponding match. Second, the task measures were created specifically for U.S. economy, and it is likely that there are differences in the intensity in which certain skills are used in given occupations between the U.S. and Mexico. Results should be interpreted with these two caveats in mind.

B RIF and Decomposing Changes in Wage Distributions (online)

Firpo, Fortin, and Lemieux (2007, 2009) allow extending the traditional Oaxaca-Blinder decomposition to distributional statistics beyond the mean. This is achieved through the use of influence functions (IF). Influence functions measure the effect that an infinitesimal amount of “errors” have on a given estimator (Cowell and Victoria-Feser 1996), but they also have properties that allow us to model the sensitivity of a given unconditional wage quantile to a change in a set of covariates. To see this, let \( q_\tau(F_W) \) be \( \tau \)th quantile of the distribution of wages, expressed in terms

\textsuperscript{A.2} See the online Appendix in Dorn (2009) for further details. Other papers that have used this measures include Autor, Katz, and Kearney (2006), Goos and Manning (2007), Dorn (2009), Rendall (2013), Autor and Dorn (2013), and Adda, Dustmann, and Stevens (2017).
of the cumulative distribution $F_W(w)$. Define the following mixture distribution:

$$G_{W,\epsilon} = (1-\epsilon)F_W + \epsilon H_W \quad for \quad 0 \leq \epsilon \leq 1$$  \hspace{1cm} (B.1)

where $H_W$ is some perturbation distribution that only puts mass at the value $w$. In that case, $G_{W,\epsilon}$ is a distribution where, with probability $(1-\epsilon)$, the observation is generated by $F_W$, and with probability $\epsilon$, the observation takes the arbitrary value of the perturbation distribution. By definition, the influence function corresponds to:

$$IF(w; q_\tau, F_W) = \lim_{\epsilon \to 0} \frac{q_\tau(G_{W,\epsilon}) - q_\tau(F_W)}{\epsilon}$$  \hspace{1cm} (B.2)

where the expression is analogous to the directional derivative of $q_\tau$ in the direction of $H_W$. Analytical expressions for influence functions have been derived for many distributional statistics.\textsuperscript{B.1} The influence function in the case of the $\tau$th quantile takes the form:

$$IF(w; q_\tau, F_W) = \frac{\tau - 1[w \leq q_\tau]}{f_W(q_\tau)}$$  \hspace{1cm} (B.3)

where $1[\cdot]$ is an indicator function and $f_W$ is the PDF.\textsuperscript{B.2} Using some of the properties of influence functions, a direct link with the traditional Oaxaca-Blinder approach can be established. In particular, a property that is shared by influence functions is that, by definition, the expectation is equal to zero.

$$\int_{-\infty}^{+\infty} IF(w; q_\tau, F_W)dF(w) = 0$$  \hspace{1cm} (B.4)

Firpo, Fortin, and Lemieux (2009) propose a simple modification in which the quantile is added back to the influence function, resulting in what the authors call the Recentered Influence Function (RIF).

$$RIF(w; q_\tau, F_W) = q_\tau + IF(w; q_\tau, F_W)$$  \hspace{1cm} (B.5)

The importance of this transformation lies in the fact that the expectation of the RIF is precisely the quantile $q_\tau$. With this result, Firpo, Fortin, and Lemieux (2009) show that we can model the conditional expectation of the RIF as a linear function of the explanatory variables.

$$E[RIF(w_t; q_{\tau_t}, F_{W,t}|X_t)] = X_t'\gamma_t$$  \hspace{1cm} (B.6)

Moreover, if we apply the law of iterated expectations to Equation (B.6), the

\textsuperscript{B.1} Essama-Nssah and Lambert (2011) provides a comprehensive list of influence functions for different distributional statistics.

\textsuperscript{B.2} Note that the influence function in this case depends on the density. In order to obtain the empirical density the authors propose non-parametric kernel density estimation.
end result is an expression that directly relates the impact of changes in the expected values of the covariates on the unconditional quantile $q_{\tau}$. Note that this result is all that is required to extend the Oaxaca-Blinder decomposition to quantiles, since the basic components of the method are all present in Equation (B.6).

Estimation of Equation (B.6) can be done by OLS, and only requires replacing the dependent variable, $\log w_t$ in the original wage setting model with the RIF of the quantile $q_{\tau}$. The interpretation of the estimates $\hat{\gamma}_t$ can be thought of as the effect of a small change in the distribution of $X$ on $q_{\tau}$, or as linear approximation of the effect of large changes of $X$ on $q_{\tau}$ (Firpo, Fortin, and Lemieux 2007).

C Solution, Identification and Estimation of the Model (online)

In this section we discuss model solution, identification and estimation. The theoretical model was presented in Section 5 and the labor market participation and wage data for Mexico were described in Section 3.1. First, we characterize the labor market equilibrium and describe algorithms for the equilibrium solution in Section C.1. Second, we discuss the identification of demand and supply side parameters in Section C.2. Third, we provide details of the equilibrium estimation routine in Section C.3. Additionally, we provide a Matlab companion code package and website which provides computational examples for our paper.

C.1 Equilibrium Definition and Solution

In this section, we discuss the equilibrium structure and solutions. In Section C.1.1, we discuss denesting the nested-CES problem and solving each sub-nest as a separate but linked demand problem. In Section C.1.2, we characterize the equilibrium solution with a system of nonlinear equations for female occupation-specific wages. In Section C.1.3, we define the competitive labor market equilibrium. In Section C.1.4, we solve for the equilibrium explicitly via nested root search as well as via a faster but less stable contraction algorithm.

C.1.1 Demand Denesting

Given the demand system presented in Section 5, we consider optimal labor demand in a particular sub-nest of the nested-CES demand system. For notational clarity, we ignore skill subscripts in this section. Without loss of generality, the optimal labor demand equations for routine male and female workers are:

\[
L_{k,r}^{d,*} = L_{r} \cdot \left( \alpha_{k,r} + \alpha_{f,r} \cdot \left( \frac{W_{k,r}}{W_{f,r}} \cdot \frac{\alpha_{f,r}}{\alpha_{k,r}} \right)^{\frac{1}{1-\rho_{4,r}}} \right)^{\frac{1}{\rho_{4,r}}} \\
L_{f,r}^{d,*} = L_{r} \cdot \left( \alpha_{k,r} \cdot \frac{W_{f,r}}{W_{k,r}} \cdot \frac{\alpha_{k,r}}{\alpha_{f,r}} \right)^{\frac{1}{1-\rho_{4,r}}} + \alpha_{f,r}
\]

(C.1)
where $\alpha_{f,r} = 1 - \alpha_{k,r}$ and $L_r$ is the level of aggregate labor demand for this sub-nest. Equation (C.1) contains solutions to the expenditure minimization problem of male and female workers in routine task-intensive occupations for a particular skill group:

$$\min_{L_{k,r}, L_{f,r}} (L_{k,r} \cdot W_{k,r} + L_{f,r} \cdot W_{f,r}),$$

such that

$$L_r = \left(\alpha_{k} L_{k,r}^{\rho_{k,r}} + \alpha_{f} L_{f,r}^{\rho_{f,r}}\right)^{\frac{1}{\rho_{k,r}}}.$$

The full nested-CES problem presented in Section 5.1 can be solved separately as eleven de-nested problems in the form of Equation (C.1). Lower- and higher-level nests are connected via nest-specific aggregate labor demand $L_r$: $L_r$ is the output quantity requirement for lower-level nests and is the input choice for higher-level nests. $L_r$ captures the effects of upper-nest share and elasticity parameters on choices.

For higher-level nests, given constant returns, the cost of acquiring aggregate labor input is a weighted average of the underlying gender-specific wages from the lowest nests. For example, the routine task-intensive occupation-specific aggregate labor price $W_r$ is equal to:

$$W_r = W_{k,r} \left(\alpha_{k,r} + \alpha_{f,r} \left(\frac{W_{k,r} \alpha_{f,r}}{W_{f,r} \alpha_{k,r}}\right)^{\frac{\rho_{k,r}}{\rho_{f,r}}}\right)^{\frac{1}{\rho_{k,r}}}$$

$$+ W_{f,r} \left(\alpha_{k,r} \left(\frac{W_{f,r} \alpha_{k,r}}{W_{k,r} \alpha_{f,r}}\right)^{\frac{\rho_{k,r}}{\rho_{f,r}}} + \alpha_{f,r}\right)^{\frac{1}{\rho_{f,r}}}.$$

(C.2)

C.1.2 System of Equations for Equilibrium Wages

Given the optimal labor supply problem and corresponding aggregate labor supply equations from Section 5.2, at equilibrium, quantity demanded is equal to quantity supplied,

$$L_r \cdot \left(\alpha_{k,r} + \alpha_{f,r} \left(\frac{W_{k,r} \alpha_{f,r}}{W_{f,r} \alpha_{k,r}}\right)^{\frac{\rho_{k,r}}{\rho_{f,r}}}\right)^{\frac{1}{\rho_{k,r}}} = \frac{L_{k,\text{pop}} \cdot \exp\left(\hat{U}_k(r \mid W_{k,r}, B_k)\right)}{\sum_{O \in \{a,r,m,h\}} \exp\left(\hat{U}_k(O \mid W_{k,O}, B_k)\right)},$$

(C.3)

where $B_k$ represents a vector of exogenous gender (and skill) specific attributes, $L_{k,\text{pop}}$ is the gender (and skill) specific population level, and $L_r$ is the aggregate quantity of routine workers demanded of a particular skill level. For notational clarity, we continue to ignore skill subscripts.

Applying some algebra to Equation (C.3) and a symmetric equation for quantity of female workers demanded and supplied, we arrive at two equations where, given aggregate labor demand $L_r$, the female (male) labor wage in routine occupation is a function of male (female) wages in analytical, routine, and manual occupations:
Following Equation (C.4), similar results can be arrived at for manual and analytical occupation wages. In all cases, the equilibrium wage for one gender in one occupation is a function of the equilibrium wages of the other gender across all occupations. Overall, within a year, for either skilled or unskilled workers, six equations for the two genders and three occupational categories characterize the equilibrium solution. The equations can be combined. For example, for female analytical work, we have:

\[
W_{f,a} = W_{f,a} \left( W_{f,a}, W_{f,r}, W_{f,m} \right), W_{k,r} \left( W_{f,a}, W_{f,r}, W_{f,m} \right), W_{k,m} \left( W_{f,a}, W_{f,r}, W_{f,m} \right). \tag{C.4}
\]

Combining all six equations and given aggregate labor demands \( L_m, L_r, L_a \), we arrive at a system of three equations and three unknowns:

\[
\begin{align*}
W_{f,a} &= W_{f,a} \left( W_{f,a}, W_{f,r}, W_{f,m} \right) \\
W_{f,r} &= W_{f,r} \left( W_{f,a}, W_{f,r}, W_{f,m} \right) \\
W_{f,m} &= W_{f,m} \left( W_{f,a}, W_{f,r}, W_{f,m} \right)
\end{align*}
\tag{C.5}
\]

The solution to the system of equations in Equation (C.5) consists of three female wages. Equation (C.4) leads to male wages given female wages. Equation (C.3) leads to labor quantities given wages. During each model period, we solve Equation (C.5) at the third nest level for skilled and unskilled workers separately. C.2

The skilled and unskilled equilibrium solutions are linked via aggregate skilled and unskilled labor demands, \( L_{s,m}, L_{s,r}, L_{s,a} \) and \( L_{u,m}, L_{u,r}, L_{u,a} \).

### C.1.3 Competitive Labor Market Equilibrium

In each period, given the aggregate output and productivity ratio \( \frac{Y}{Z} \), demand parameter vectors \( \alpha \) and \( \rho \), supply parameters vectors \( \psi \) and \( \pi \), the vector of gender- and skill-specific supply characteristics \( B \), and the vector of gender- and skill-specific potential worker levels \( L^{pop} \), the competitive labor market equilibrium consists of wages and aggregate labor quantities, such that,

1. Female wages \( \{W_{f,edu,occ}\}_{edu \in \{s,u\}, occ \in \{a,r,m\}} \) solve Equation (C.5) for all \( edu \) groups.

---

C.2. Unskilled and skilled workers have separate labor supply problems and belong to separate nests under the demand system.
2. Aggregate skill-occupation demands \( \{L_{edu,occ}\}_{edu \in \{s,u\}, occ \in \{a,r,m\}} \) solve Equation (C.1) given aggregate wages and occupation-specific aggregate demands.\(^{C.3}\)

The equilibrium definition distinguishes between two separable components of nested-CES equilibrium problems. On the one hand, only the lowest level of demand nests directly face supply-side equations and wages \( \{W_{f,edu,occ}\}_{edu \in \{s,u\}, occ \in \{a,r,m\}} \). On the other hand, parameters of upper-level nests are linked to the problem at the lowest level of nests via \( \{L_{edu,occ}\}_{edu \in \{s,u\}, occ \in \{a,r,m\}} \).

For generalizability, in terms of demand, the solution to the equilibrium system is scalable to alternative nested-CES demand systems with additional levels of nests and alternative nesting structures. In terms of supply, the structure here assumes that workers make labor supply decisions for the current period given current wages only.\(^{C.4}\)

C.1.4 Solving for Market Clearing Wages

**Explicit Root Search** The system of nonlinear equations in Equations C.5 does not have an analytical solution, but numerical root search routines can be deployed to explicitly solve for equilibrium wages given demand and supply parameters. Specifically, the equilibrium problem can be solved in three nested stages. In stage one, given \( W_{f,r}, W_{f,m} \), we solve for the root \( W_{f,a}^{*} \):

\[
W_{f,a}^{*} (W_{f,r}, W_{f,m}) = \arg \min_{W_{f,a}} \left| W_{f,a} - W_{f,a} (W_{f,a}, W_{f,r}, W_{f,m}) \right|.
\]  
(C.6)

In stage two, we solve for the root \( W_{f,r}^{*} \) given \( W_{f,m} \):

\[
W_{f,r}^{*} (W_{f,m}) = \arg \min_{W_{f,r}} \left| W_{f,r} - W_{f,r} (W_{f,a}^{*} (W_{f,r}, W_{f,m}), W_{f,r}, W_{f,m}) \right|.
\]  
(C.7)

In stage three, we arrive at one equation and one unknown:

\[
W_{f,m}^{*} = \arg \min_{W_{f,m}} \left| W_{f,m} - W_{f,m} (W_{f,a}^{*} (W_{f,r}^{*} (W_{f,m}), W_{f,m}), W_{f,r}^{*} (W_{f,m}), W_{f,m}) \right|.
\]  
(C.8)

\(^{C.3}\) Aggregate occupation-specific demands at higher tier successively solve Equation (C.1) given \( Y \), as well as successively aggregated wages at occupation and skill levels given female skill-occupation specific wages \( \{W_{f,edu,occ}\}_{edu \in \{s,u\}, occ \in \{a,r,m\}} \).

\(^{C.4}\) Under a dynamic labor supply model, households might make labor decisions based on current wages as well as the path of future wages. Under the assumption of rational expectations, one might iterate over parameters until expectations become self-fulfilling and the expected path of wages conforms to the actual path of wages given aggregate labor supply. This solution concept suffers from the curse of dimensionality when additional dimensions of equilibrium wages are added. In our example here, if workers in 1989 consider the path of wages for the next 25 years in making labor market decisions, Equation (C.5) would become a system of equation that requires solving for a 150-dimensional (3 times 2 times 25) market-clearing root.
Equation (C.8) can be solved via triply-nested root-search. Given aggregate demands, \( \{L_{edu, occ}\}_{edu \in \{s, u\}, occ \in \{a, r, m\}} \), Equation (C.8) is solved for skilled and unskilled workers separately and satisfies the first condition for a competitive labor market equilibrium.

Given upper level nest parameters and wage solutions of Equation (C.8), we update \( \{L_{edu, occ}\}_{edu \in \{s, u\}, occ \in \{a, r, m\}} \). The process iterates until the aggregate skill- and occupation- specific demands are consistent with wage solutions of Equation (C.8). This satisfies the second condition for the competitive labor market equilibrium.

**Iterative Wage Contraction** In practice, searching for a three dimensional female wage root vector can be slow. To speed up the estimation procedure, we also solve the problem via iterative wage contraction, based on a modified version of the algorithm used in Johnson and Keane (2013). C.5

Given \( \{L_{edu, occ}\}_{edu \in \{s, u\}, occ \in \{a, r, m\}} \), first, we solve for quantity supplied given wages for skilled and unskilled workers following Equation (5.8). Second, given demand-side first-order conditions from Equation (2.1), we solve for relative wages that would be consistent with the quantity supplied. Third, given relative wages, Equation (C.1) solves for the level of female labor demanded, which is proportional to the quantity supplied from the second step. Fourth, given the log odds ratio formulation of the supply equations from Equation (C.25), we solve for the wage levels that support the level of female labor demanded from the third step. Fifth, given female wage levels, we use the relative wages from step three to find male wage levels. Sixth, we update \( \{L_{edu, occ}\}_{edu \in \{s, u\}, occ \in \{a, r, m\}} \) with new wages, which are the weighted averages of initial wages and new wages computed following Equation (C.2) from step four and five. The process iterates until quantity demanded is equal to quantity supplied. C.6

The iterative wage contraction solution algorithm can be fast. C.7 This algorithm, however, does not guarantee equilibrium convergence. At arbitrary starting points for wages, wage iterations generally converge towards either zero or positive infinity. We start wage iteration at the observed wage levels and solve for converging wages. We check for market clearing in skilled and unskilled nests and across all years separately. When wages do not converge, we reduce the wage updating speed in step six by putting higher weights on wages from prior iterations. In cases where

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C.5. Johnson and Keane (2013) does not explicitly solve for demand quantities, but iterates over marginal products given quantities, and quantity supplied given wages.

C.6. Relative wages matter for quantity demanded, but the level of wages matters for quantity supplied given supply parameters. In the second step, period-specific Lagrange multipliers confound the mapping of period-specific aggregate productivity to wage levels. Given marginal products and corresponding relative wages, steps three to five provide a consistent normalization for wage levels given the \( Y \) ratio.

C.7. The method solves 12 equilibrium wages during 13 periods in less than one second on a home-PC available in 2021.
convergence to a fixed-point still fails, we solve explicitly for equilibrium using the explicit root search routine just described in the prior segment of Section C.1.4. On our companion website, we provide as functions both the iterative wage contraction algorithm and the exact root search routine, along with associated examples and tutorials.

C.2 Identification of Demand and Supply Parameters

While the nested-CES demand system is commonly estimated in the labor literature, it is perhaps less common to estimate both demand and supply parameters in an equilibrium context. In this paper, we develop an estimation framework. We discuss in the following sections key identification challenges and solutions in our estimation framework. Specifically, in Section C.2.1, we discuss the identification of parameters across nests through relative wages within and across nests. In Section C.2.2, we discuss the necessary data requirement for plausibly jointly identifying $\rho$ and $\alpha$ via equilibrium supply-shifters and the challenge of this approach in our empirical context of biennially aggregated data. In Section C.2.3, we discuss the data requirements for possibly identifying variations in $\alpha$ parameter over-time under polynomial restriction, a standard strategy that we adopt. In Section C.2.4, we discuss the challenge to demand-side only estimation posed by potential mismeasurement of equilibrium wages and number of workers as well as shocks to relative demands. In Section C.2.5, we discuss the challenge to supply-side only estimation in the context of our labor market participation model. Finally, in Section C.2.6, we discuss equilibrium solution based estimation.

C.2.1 One Period Data and Relative Wages Within and Across Nests

Given one period of data, conditional on known $\rho$ values, share parameters $\alpha$ are identified given relative wages within and across nests.

Consider a constant-returns two-level nested-CES problem. Level one combines skilled and unskilled workers, and level two combines male and female workers:

$$\begin{align*}
\min_{L_{k,s},L_{f,s},L_{k,u},L_{f,u}} & \quad (L_{k,s} \cdot W_{k,s} + L_{f,s} \cdot W_{f,s} + L_{k,u} \cdot W_{k,u} + L_{f,u} \cdot W_{f,u}) \\
\text{s.t.} & \quad \frac{Y}{Z} = \left(\alpha_s \left(\alpha_{k,s} L_{k,s}^\rho s + (1 - \alpha_{k,s}) L_{f,s}^\rho s\right) + (1 - \alpha_s) \left(\alpha_{k,u} L_{k,u}^\rho u + (1 - \alpha_{k,u}) L_{f,u}^\rho u\right)\right)^{\frac{1}{\rho}}
\end{align*}$$

(C.9)

The problem in Equation (C.9) has eight parameters: $\rho = \{\rho_s, \rho_u\}$, $\alpha = \{\alpha_s, \alpha_{k,s}, \alpha_{k,u}\}$, and $\{Y, Z\}$. From one period of data, we observe four wages $\{W_{k,s}, W_{f,s}, W_{k,u}, W_{f,u}\}$, and four labor quantities $\{L_{k,s}, L_{f,s}, L_{k,u}, L_{f,u}\}$.

First, it is not possible to separately identify output $Y$ from productivity...
where $O_s$ is the productivity-scaled aggregate output from the production function, and it determines the levels of optimal demands. Given the output constraint in Equation (C.9), $Y/Z$ is known when $\rho$ and $\alpha$ are known.

Second, one period of data does not allow for the joint identification of $\alpha$ and $\rho$. However, given $\rho$ values, $\alpha$ values are identified. Specifically, three optimality conditions link respective optimal relative labor demands to relative wages:

$$\log \left( \frac{W_{k,s}}{W_{f,s}} \right) = \log \left( \frac{\alpha_{k,s}}{1 - \alpha_{k,s}} \right) + (\rho_s - 1) \cdot \log \left( \frac{L_{k,s}}{L_{f,s}} \right),$$

Relative wage between skilled males and females

$$\log \left( \frac{W_{k,u}}{W_{f,u}} \right) = \log \left( \frac{\alpha_{k,u}}{1 - \alpha_{k,u}} \right) + (\rho_u - 1) \cdot \log \left( \frac{L_{k,u}}{L_{f,u}} \right),$$

Relative wage between unskilled males and females

$$\log \left( \frac{W_{k,s}}{W_{k,u}} \right) = \log \left( \frac{\alpha_s}{1 - \alpha_s} \right) \left[ \frac{\alpha_{k,s} \cdot L_{k,s}^{\rho_s - 1} \cdot O_u^{\rho_u}}{\alpha_{k,u} \cdot L_{k,u}^{\rho_u - 1} \cdot O_u^{\rho_u}} \right] + \rho \cdot \log \left( \frac{O_s}{O_u} \right),$$

Relative wage between skilled and unskilled males

where $O_s = \left( \alpha_{k,s} L_{k,s}^{\rho_s} + (1 - \alpha_{k,s}) L_{f,s}^{\rho_s} \right)^{1/\rho_s}$ and $O_u = \left( \alpha_{k,u} L_{k,u}^{\rho_u} + (1 - \alpha_{k,u}) L_{f,u}^{\rho_u} \right)^{1/\rho_u}$. The first two equations of Equations (C.10) determine $\alpha_{k,s}$ and $\alpha_{k,u}$, which determine the values inside the square brackets of the third equation and identify $\alpha_s$.

Since $\log \left( \frac{\alpha}{1 - \alpha} \right) : (0, 1) \to \mathbb{R}$, there exists $\alpha$ to fit any positive wages vectors.

Third, using Equation (C.2), $\alpha_s$ is alternatively identifiable by

$$\log \left( \frac{W_k}{w_u} \right) = \log \left( \frac{\alpha_s}{1 - \alpha_s} \right) + \rho \cdot \log \left( \frac{O_s}{O_u} \right),$$

where $W_k$ and $W_u$ are aggregate wages for $O_s$ and $O_u$. In problems with additional layers of nesting, by applying Equation (C.11) iteratively upward, a $\alpha$ vector of up to $2^N - 1 = \sum_{i=0}^{N-1} 2^{N-i-1}$ parameters can be identified given $2^N$ pairs of wage and labor quantity data.

In the context of our empirical problem, the literature does not provide us with occupation-specific gender elasticities nor occupation-specific skill elasticities.

C.8 It is important to note that CES production function parameters are often estimated in a setting with panels or cross-sections of observed input and output data across many individuals, firms or countries. In those settings, there can be individual-specific productivity shocks $Z$, with various layers of subscripts. Shocks that are unobserved by the econometrician and wages that are observed by the econometrician jointly drive individual-specific optimal choices, leading to endogeneity between production function inputs and the error term. The central estimation question is to disentangle the endogeneity between inputs and the productivity shock term, which might capture productivity shocks as well as unobserved inputs. In our setting, however, we have an observed time-series of equilibrium wage and quantity data for each occupation and skill cell. Rather than having individual-specific productivity shocks, at each time period, there is a single aggregate productivity shock $Z$ shared across all occupations and all workers. This $Z$ captures both the aggregate productivity variation across time as well as potential unobserved non-labor inputs. Furthermore, we rely on demand optimality conditions in Equation (C.10) to estimate the model, where the $Y/Z$ term does not appear.
If such values existed, following the above procedure, year-specific demand share parameters might potentially be found that fit the observed data series perfectly.

C.2.2 Two Periods Data and Equilibrium Supply-shifters

Given two periods of data, if equilibrium changes in wages and labor quantities are driven by supply-shifters only, then demand parameters that do not vary over the two periods can be identified. Note that the assumption here is not that there are supply-side shocks (instruments) that are uncorrelated with demand shocks, but that a shift in the supply curve traces out a time-invariant (over two periods) demand curve. We consider estimation issues related to shocks to relative demands in Appendix Section C.2.4.

Given data from \( t = \tau \) and \( t = \tau + 1 \), time-subscripted Equations (C.10) provide six equations for identifying the six \( \rho \) and \( \alpha \) parameters. Each pair of nest-specific and time-invariant \( \alpha \) and \( \rho \) values is pinned down by linearly matching the relative wages and labor quantity in both periods. In this and the following sections, for notational clarity, we ignore the skill and task-intensive occupation subscripts.

It might not, however, be possible to explain observed equilibrium changes with only supply-shifters. For any one particular nest, there exists a continuum of \( \alpha \) and \( \rho \) combinations that can explain observed relative wages and quantities in a period. We can express \( \alpha \) as a function of \( \rho \):

\[
\hat{\alpha}_t(\rho) = \frac{\left(\frac{W_k,t}{W_f,t}\right) \cdot \left(\frac{L_k,t}{L_f,t}\right)^{1-\rho}}{1 + \left(\frac{W_k,t}{W_f,t}\right) \cdot \left(\frac{L_k,t}{L_f,t}\right)^{1-\rho}},
\]

where the hat and the time sub-script indicate that \( \hat{\alpha}_t \) is a function of observables at period \( t \). Between periods \( t = \tau \) and \( t = \tau + 1 \), a condition for the existence of an equilibrium supply-shifter is that there must be a \( \rho^* \in (-\infty, 1] \), where

\[
\hat{\alpha}_t(\rho^*) = \hat{\alpha}_{t+1}(\rho^*).
\]

Equation (C.13) simplifies to

\[
\log \left(\frac{W_{k,\tau}W_{f,\tau+1}}{W_{f,\tau}W_{k,\tau+1}}\right) \cdot \left(\log \left(\frac{L_{k,\tau+1}L_{f,\tau}}{L_{f,\tau+1}L_{k,\tau}}\right)^{-1}\right) > 0.
\]

Equation (C.14) is a necessary condition for the existence of equilibrium supply-shifters, and it simply requires that relative wages must shift in the opposite direction as relative labor quantities.

---

C.9 Because Equation (C.13) is a function of relative wages and quantities, it does not relate to how the aggregate output to productivity ratio \( \frac{Y}{Z} \) might or might not be changing over time. Additionally, as a necessary condition, satisfying Equation (C.14) does not mean that observed changes in wages and quantities are only driven by supply-shifters.
For a nested-CES problem, variations in \( \rho \) and \( \alpha \) in any sub-nest impact demands across all nests. Hence, to use an equilibrium supply-shifters identification strategy, observable changes for all lowest-layer sub-nests must satisfy Equation (C.14). For Mexico, if there are episodes of supply-only policy shifts and corresponding short-interval ex-ante and ex-post observed equilibrium wages and labor quantities, demand parameters can plausibly be identified during each episode and compared over time without parametric assumptions.

Empirically, given our biennially aggregated data, we do not find any data segments during which changes in labor quantities and wages satisfy Equation (C.14) across all level three sub-nests. It is perhaps natural that over the course of 2 to 4 years, there would be sufficient changes in demand-side parameters that can substantially impact equilibrium, precluding the use of supply-shifter only instruments for identification.

### C.2.3 Three and More Periods Data and Demand Share Polynomials

With three or more periods of data, we follow the literature and allow for demand-side share parameters to vary over \( t \) under polynomial restrictions. This means both demand and supply parameters can vary over time. Here, we consider one sub-nest. The logic for identification across nests follows from the discussions in Section C.2.1.

We express the logarithm of \( \alpha_t \) as a \( M \)th degree polynomial:

\[
\log \left( \frac{W_{k,t}}{W_{f,t}} \right) = \log \left( \frac{\exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)}{1 - \exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)} \right) + (\rho - 1) \cdot \log \left( \frac{L_{k,t}}{L_{f,t}} \right). \tag{C.15}
\]

\( T \geq M + 2 \) periods of data are needed to identify the \( M + 1 \) polynomial coefficients, \( \{a_0, a_1, \ldots, a_M, \rho\} \), and \( \rho \). In practice, polynomial coefficients can be found by regressing \( \{\log (\hat{\alpha}_t (\rho))\}_{t=1}^{T} \) on the time matrix \( \{1, t, t^2, \ldots, t^M\}_{t=1}^{T} \). To analyze data requirements for identification, we provide an explicit characterization for data variations that identifies each \( a_i \).

Polynomial coefficients can be identified via differences of \( \log (\hat{\alpha}_t (\rho)) \). With data vector \( \{\log (\hat{\alpha}_t (\rho))\}_{t=\tau}^{\tau+M} \), starting at any \( \tau \in [1, T - M] \), the coefficient for the highest polynomial term. \( a_M \), is equal to:

\[
a_M = \frac{1}{M!} \sum_{i=0}^{M} \left( (-1)^i \frac{M!}{(M-i)!i!} \right) \times \log (\hat{\alpha}_{(\tau+M)} (\rho)) \tag{C.16}
\]

\( \text{Alternating binomial coeff.} \)

All data: \( \hat{\alpha}_\tau, \ldots, \hat{\alpha}_{(\tau+M)} \)

C.10. In Equation C.15, we assume that patterns of changes in \( \alpha_t \) are smooth and not subject to shocks, an assumption we relax in Section C.2.4.
where the sum is equal to the $M^{th}$ difference over time of $\log(\hat{\alpha}_t(\rho))$. Specifically, when $M = 3$, given $T > 4$ periods of data available, the coefficient for the highest polynomial is equal to:

$$
\alpha_3 \left( \rho, \left\{ \frac{W_{k,t}}{W_{f,t}}, \frac{L_{k,t}}{L_{f,t}} \right\}_{t=\tau}^{\tau+3} \right) = \frac{1}{3 \cdot 2} \cdot \log \left( \frac{\hat{\alpha}_{\tau+3}(\rho) \cdot \hat{\alpha}_{\tau+1}(\rho) \cdot \hat{\alpha}_{\tau+1}(\rho) \cdot \hat{\alpha}_{\tau+1}(\rho)}{\hat{\alpha}_{\tau+2}(\rho) \cdot \hat{\alpha}_{\tau+2}(\rho) \cdot \hat{\alpha}_{\tau+2}(\rho) \cdot \hat{\alpha}_t(\rho)} \right),
$$

(C.17)

for any $\tau \in [1, T - 3]$. The log relative ratios of the $\hat{\alpha}_t$ across time segments, which are a function of relative wages and labor quantities as shown in Equation (C.12), determine the polynomial coefficients.

Given coefficients for higher order polynomials, coefficients for the $m < M$ lower order polynomials are equal to, for all starting dates $\tau \in [1, T - m]$:

$$
a_m = \sum_{i=0}^{m} \left( (-1)^i ((m - i)!))^{-1} \right) \cdot \left( \log(\hat{\alpha}_{\tau+m-i}(\rho)) - \sum_{j=0}^{M-m-1} a_{M-j} \cdot t^{M-j} \right).
$$

(C.18)

Equation (C.18) identifies $a_m$ from the $m^{th}$ difference over time of $\log(\hat{\alpha}_t(\rho))$, after first differencing out the contribution to $\hat{\alpha}_t(\rho)$ from higher than $m^{th}$ order polynomial terms as shown in Equation (C.18).

Despite the flexibility of a $M^{th}$ order polynomial, intuitively, identification is potentially possible because the $M^{th}$ derivative of a $M^{th}$ order polynomial is, by design, time-invariant. This time-invariance restriction allows for iteratively solving for the coefficients for lower order polynomial terms through differencing.

Following the discussion in Section C.2.2 for two periods of data, in the multiperiod context, it is also possible that there exist no combinations of polynomial coefficients, $\{a_0, a_1, \ldots, a_M\}$ and $\rho$ that could plausibly explain observed equilibrium outcomes. The discussion in Section C.2.2 could be viewed as an analysis under $0^{th}$ order polynomial assumption with $M = 0$.

In the absence of mismeasurement and shocks to relative demands, if the polynomial coefficients generated from Equations (C.16) and (C.18) based on different segments of data with different $\tau$ starting points vary, that indicates a violation of the time-invariance assumption of the $M^{th}$ derivative of the $M^{th}$ order polynomial. Empirically, given the possibility of mismeasurement and relative demand shocks, estimates based on different $\tau$ starting points would not be the same. How-

---

C.11. The first difference is $(\hat{\alpha}_t - \hat{\alpha}_{t-1})$, the second difference is $(\hat{\alpha}_{t+1} - \hat{\alpha}_t) - (\hat{\alpha}_t - \hat{\alpha}_{t-1})$. The $M^{th}$ difference is based on differencing data from over $M + 1$ periods. The number of occurrences of each $\hat{\alpha}_t$ term in the $m^{th}$ difference follows the $(m+1)^{th}$ row of Pascal’s Triangle and is expressed in Equation (C.16) as a finite alternating series with binomial coefficients.
ever, large deviations in coefficients computed based on Equations (C.16) and (C.18) for different data segments might be indicative that the time-invariance assumption given the current order of polynomial is not satisfied (or it might also be indicative of the presence of significant mismeasurement or shocks, see the next section). An increase in $M$ might be needed, with requisite increase in $T$ data availability. In this paper, we model changes in relative demand trends in each sub-nest with 3rd degree polynomials.

C.2.4 Demand Estimation, Mismeasurement, and Shocks

Mismeasurement and Shocks Let \( \{ W_{k,t}, W_{f,t}, L_{k,t}, L_{f,t} \} \) and \( \{ \hat{W}_{k,t}, \hat{W}_{f,t}, \hat{L}_{k,t}, \hat{L}_{f,t} \} \) represent data with and without mismeasurement respectively. Assuming that mismeasurement is classical and log normal, we have

\[
\log(W_{\text{gen},t}) = \log(\hat{W}_{\text{gen},t}) + \epsilon_{\text{gen},t},
\]

and

\[
\log(L_{\text{gen},t}) = \log(\hat{L}_{\text{gen},t}) + \eta_{\text{gen},t},
\]

where \( \epsilon_{\text{gen},t} \sim \mathcal{N}\left(-\frac{\sigma^2_\epsilon}{2}, \sigma^2_\epsilon\right) \) and \( \eta_{\text{gen},t} \sim \mathcal{N}\left(-\frac{\sigma^2_\eta}{2}, \sigma^2_\eta\right) \). As in prior sections, for notational clarity, we continue to ignore skill- and task-intensive occupation subscripts.

Additionally, changes in \( \alpha_t \) over time might not be smooth, and there could be relative productivity shocks \( \nu_t \) to skill- and gender-biased technological changes:

\[
\log\left(\frac{\alpha_t}{1 - \alpha_t}\right) = \log\left(\frac{\hat{\alpha}_t}{1 - \hat{\alpha}_t}\right) + \nu_t,
\]

where the log of \( \hat{\alpha}_t \) follows a smooth polynomial over time. Given Equation (C.20), \( \alpha_t \) is a positive fraction for any \( \nu_t \) draws along the real line, meaning that \( \alpha_t(\hat{\alpha}_t, \nu_t) : (0, 1) \times \mathcal{R} \to (0, 1) \).\(^{12}\) For ease of exposition, we assume \( \nu_t \) to be normal: \( \nu_t \sim \mathcal{N}\left(-\frac{\sigma^2_\nu}{2}, \sigma^2_\nu\right) \).

Scenario One We now consider four possible scenarios based on varying assumptions on \( \sigma^2_\epsilon, \sigma^2_\eta, \) and \( \sigma^2_\nu \). In the first scenario, suppose \( \sigma^2_\epsilon > 0 \), but \( \sigma^2_\eta = 0 \)

\(^{12}\) Given Equation (C.20), for any \( \nu_t \in \mathcal{R} \), we have

\[
\alpha_t(\hat{\alpha}_t, \nu_t) = \hat{\alpha}_t \cdot \left(\frac{\exp(\nu_t)}{1 + \hat{\alpha}_t \cdot (\exp(\nu_t) - 1)}\right) \in (0, 1)
\]

This is because

\[
\lim_{\nu_t \to -\infty} \left(\frac{\exp(\nu_t)}{1 + \hat{\alpha}_t \cdot (\exp(\nu_t) - 1)}\right) = 0 \text{ and } \lim_{\nu_t \to \infty} \left(\frac{\exp(\nu_t)}{1 + \hat{\alpha}_t \cdot (\exp(\nu_t) - 1)}\right) = \frac{1}{\hat{\alpha}_t}.
\]
\[ \log \left( \frac{W_{k,t}}{W_{f,t}} \right) = \log \left( \frac{\exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)}{1 - \exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)} \right) + (\rho - 1) \cdot \log \left( \frac{L_{k,t}}{L_{f,t}} \right) + (\epsilon_{k,t} - \epsilon_{f,t}) \cdot (1 - \rho) \cdot (\eta_{k,t} - \eta_{f,t}) . \] (C.21)

Under Equation (C.21), mismeasurement is on the left-hand-side. The identification discussions for \( \rho \) and polynomial coefficients is the same as before, but now there can be gaps between model predictions and the data.

**Scenario Two** In the second scenario, suppose \( \sigma^2_{\epsilon} > 0 \) but \( \sigma^2_{\eta} > 0 \) and \( \sigma^2_{\nu} = 0 \), but \( \rho \) is known from prior literature, we have:

\[ \log \left( \frac{W_{k,t}}{W_{f,t}} \right)^{1-\rho} = \log \left( \frac{\exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)}{1 - \exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)} \right) + (\epsilon_{k,t} - \epsilon_{f,t}) + (1 - \rho) \cdot (\eta_{k,t} - \eta_{f,t}) \cdot (1 - \rho) \cdot (\eta_{k,t} - \eta_{f,t}) . \] (C.22)

Under Equation (C.22), polynomial share coefficients remain identifiable. A challenge is that as the data-generating true \( \rho \) tends away from perfect substitution (\( \rho = 1 \)) and toward complementarity (\( \rho \to -\infty \)), the mismeasurement is magnified. Lower \( \rho \) values reduce the precision of polynomial share estimates given the same span of data.

**Scenario Three** In the third scenario, suppose \( \sigma^2_{\epsilon} > 0 \), \( \sigma^2_{\eta} > 0 \) and \( \sigma^2_{\nu} = 0 \), and \( \rho \) is not known, we have:

\[ \log \left( \frac{W_{k,t}}{W_{f,t}} \right) = \log \left( \frac{\exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)}{1 - \exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)} \right) + (\rho - 1) \cdot \log \left( \frac{L_{k,t}}{L_{f,t}} \right) + (\epsilon_{k,t} - \epsilon_{f,t}) + (1 - \rho) \cdot (\eta_{k,t} - \eta_{f,t}) . \] (C.23)

In Equation (C.23), the log relative labor ratio is correlated with the error term. Hence, there is standard classical errors-in-variable attenuation bias. As in the second scenario, mismeasurement can be magnified by lower values for data-generating true \( \rho \).

In terms of the measurement errors, for developed economies such as the U.S., there might be administrative records of income and wages as well as detailed firm-level employment data by industry and occupation. In our context, mismeasurement is of greater concern. We compute wages and the number of workers based on aggregating the ENIGH survey data from a full sample of 87,826 housing units. Our

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C.13. Without mismeasurement, given \( M^{th} \) order polynomial and \( T \geq M + 2 \) periods of data, any \( M+1 \) segment of data will generate the same exactly identified demand parameters using Equations (C.16) and (C.18). With mismeasurement, the best-fit is in effect obtained from an averaging of the results from each \( M+1 \) data segment.
focus on occupation leads to 16 occupation-skill-gender data cells. In each survey year, for some cells (e.g., routine-unskilled-men) the sample size is substantial, but for other cells (e.g., manual-skilled-women) the sample size is limited and suffers from sampling error. Additionally, there might be mismeasurement in the underlying reported wage/earning and labor market participation decisions.

**Scenario Four** In the fourth scenario, suppose $\sigma_\epsilon^2 > 0$, $\sigma_\eta^2 > 0$ and $\sigma_\nu^2 > 0$, and $\rho$ is not known, we have:

$$
\log \left( \frac{W_{k,t}}{W_{f,t}} \right) = \log \left( \frac{\exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)}{1 - \exp \left( \sum_{i=0}^{M} a_i \cdot t^i \right)} \right) + (\rho - 1) \cdot \log \left( \frac{L_{k,t}}{L_{f,t}} \right) + (\epsilon_{k,t} - \epsilon_{f,t}) + (1 - \rho) \cdot (\eta_{k,t} - \eta_{f,t}) + \nu_t.
$$

(C.24)

In Equation (C.24), $\alpha_t$ is a function of $\nu_t$, and $\alpha_t$ impacts the labor demand curve. In our setting, when labor supply is elastic with respect to wages (i.e. $\psi_1 > 0$), the equilibrium relative labor ratio $L_{k,t}/L_{f,t}$ is endogenous to $\nu_t$. Hence, when directly estimating Equation (C.24), in addition to issues related to mismeasurement, bias can also come from the correlation between $\nu_t$ and $L_{k,t}/L_{f,t}$.

It is important to note that even when labor supply is inelastic with respect to wages (i.e. $\psi_1 = 0$), bias can still arise if demand shocks $\nu_t$ are correlated with supply shocks—technological shocks might impact both demand and supply curves. However, when $\psi_1 = 0$, it might be possible to identify shocks that only shift supply curves as instruments for estimating Equation (C.24). For example, demographic changes might shift the x-intercepts of the inelastic (with respect to wage) labor supply curves without impacting labor demand. Such instruments, however, do not resolve the endogeneity problem when $\psi_1 > 0$: $L_{k,t}/L_{f,t}$ is an equilibrium outcome determined jointly by demand and supply curves, and the effects of supply shocks on $L_{k,t}/L_{f,t}$ depend on $\nu_t$ even if these supply shocks are uncorrelated with $\nu_t$.

In discussing the four scenarios above, we have clarified the conditions under which bias might arise when demand-side parameters are estimated from demand-side relative optimality conditions alone. In Appendix Sections C.2.6 and C.3, we discuss how equilibrium solution based estimation can resolve the challenges posed by scenarios three and four.
C.2.5 Supply Estimation and Wage Endogeneity

Following Equations (2.2), (5.5), and (5.6), the difference in indirect utility from choosing one of the three occupational categories and leisure is:

\[ U(\text{occ} \mid \text{gen, edu, } t) - U(h \mid \text{gen, edu, } t) = (\psi_{\text{gen, edu, occ}} - \pi_{1, \text{gen}} - \pi_{2, \text{gen}}) - \pi'_{\text{gen, edu}} B_{\text{gen, edu}, t} + \psi_1 \log (W_{\text{gen, edu, occ}, t}) + (\epsilon_{\text{gen, edu, occ}, t} - \epsilon_{\text{gen, edu, h}, t}) \]

(C.25)

Given the extreme value aggregate probability formulation shown in Equation (5.7), we could potentially estimate the parameters of Equation (C.25) via OLS by replacing the left-hand-side of Equation (C.25) with \( \log \left( \frac{L_{\text{gen, edu, O}, t}}{L_{\text{pop}}} \right) - \log \left( \frac{L_{\text{gen, edu, h}, t}}{L_{\text{gen, edu}, t}} \right) \), which represents log differences in observed aggregate labor shares.

In partial equilibrium discrete choice supply (or demand) estimation settings, the potential endogeneity of prices with the error term might require the use of instruments. In the context of the equilibrium model here, equilibrium wage solutions capture all time-varying and occupation-specific share and productivity differences from the demand-side.

C.2.6 Mismeasurement, Shocks, Equilibrium Solution, and Estimation

To conduct the counterfactual analysis of interest, we need both demand- and supply-side parameters. For estimating demand parameters, equilibrium estimation avoids potential bias that might arise from demand-only estimation discussed in Appendix Section C.2.4. For estimating supply parameters, equilibrium estimation provides wages endogenously.

At each \( t \), given vectors of demand parameters \( \{\hat{\alpha}_{4,t}, \hat{\alpha}_{3,t}, \hat{\alpha}_{2,t}, \hat{\alpha}_{1,t}, Y_t, \rho\} \), supply parameters \( \{\psi, \pi\} \), gender- and skill-specific supply-side variables \( B_t \), gender- and skill-specific total potential worker count \( L_t^{\text{pop}} \), and relative productivity shocks \( \nu_t \) (see Appendix Section C.2.4), one could solve for vectors of equilibrium wages \( \hat{W} \) and labor quantities \( \hat{L} \) across twelve occupation-gender-skill categories. Given vectors of measurement error draws \( \{\epsilon_t, \eta_t\} \) (see Appendix Section C.2.4), model predictions could be matched to observed wages \( W \) and labor quantities \( L \). In a
specific gender, skill, and occupation cell, we have, for equilibrium labor quantity,

\[
\log (L_{gen, skl, occ, t}) = \log \left( \tilde{L}_{gen, skl, occ, t} \right) = \log \left( \frac{\hat{y}_{t} \phi_{t}}{\nu_{t} \psi_{t}} \right) + \eta_{gen, skl, occ, t}
\]

and a parallel equation for equilibrium wage.

Following the discussions in Appendix Section C.2.4, for demand only estimation under Equation (C.23), observed relative wages are regressed on observed relative labor quantities, leading to potential bias. Under Equation (C.26), observed wages and labor quantities are both on the left-hand-side of Equation (C.26) and are matched against model equilibrium predictions that are solved at given vectors and parameters, observables, and potential shock draws.

In addition to the identification of supply- and demand-side parameters previously discussed, the variances of relative demand shocks and measurement errors are potentially identifiable as well. On the one hand, \(\nu_{t}\) impacts both \(\hat{L}(\nu_{t})\) and \(\hat{W}(\nu_{t})\), which allows \(\nu_{t}\) to help explain the residual covariance between \(L(\nu_{t})\) and \(W(\nu_{t})\) not explained by the smooth demand trends and supply-side observables. On the other hand, measurement errors for wages (\(\epsilon_{t}\)) and labor quantities (\(\eta_{t}\)) are uncorrelated by assumption and help explain uncorrelated residual differences between model predictions and data.

To jointly identify the variances of these unobservables, given the distributional assumptions from Appendix Section C.2.4, we could repeatedly solve for equilibrium outcomes \(\hat{L}(\nu_{t})\) and \(\hat{W}(\nu_{t})\) given vectors of \(\nu_{t}\) draws, and find the vectors of \(\epsilon_{t}(\nu_{t})\) and \(\eta_{t}(\nu_{t})\) draws that explain the residual differences between model predictions and data using Equation (C.26). These residual differences can be inputs for a simulated maximum likelihood estimator. This approach imposes high computational burdens: given a specific set of parameter values, equilibrium solution needs to be resolved a large number of times to construct one simulated likelihood; the simulated likelihood needs to be reconstructed a large number of times as the estimator searches across the large parameter space.

In Appendix Section C.3, we solve the model along the smooth polynomial trend and set \(\nu_{t} = 0\). This means that the model that we solve is deterministic and the differences between model predictions and observables are explained by measurement (sampling) errors. We do this for computational feasibility considerations but also because in comparison to the overall trend changes in relative productivities over the course of two and half decades, period by period relative productivity shocks \(\nu_{t}\) likely have relatively muted effects on equilibrium outcomes. Given our
estimation results, we find that the residual vectors $\hat{\epsilon}$ and $\hat{\eta}$ have a correlation of 0.063. Following prior discussions, the non-zero correlation indicates that true variance for $\nu_t$ is unlikely zero; however, the relatively small correlation indicates that parameter estimates would likely not differ significantly if the model was estimated and solved given vectors of non-zero $\nu_t$ draws.

C.3 Estimation

We discuss our equilibrium estimation strategy in the following sections. We discuss the estimation parameter space in Section C.3.1. We discuss initializing starting values for estimation in Section C.3.2. We discuss the error structure in Section C.3.3.

C.3.1 Estimation Parameter Space

Let $\Theta$ be the $94 \times 1$ vector of all parameters of the model. This includes 11 supply-side $\psi$ parameters, 18 supply-side $\pi$ parameters, 8 demand-side elasticity parameters, 44 demand-side share polynomial coefficients, and 13 year-specific demand-side output-productivity ratios.

Let $\Theta^\rho = \left\{ \{ \rho_4, O \}, \rho_3, O \right\}_{O \in m, r, a}, \rho_1, \rho_2 \}$ be the $8 \times 1$ vector of elasticity parameters. Let $\mathcal{O}$ be some estimation objective function that is a function of the differences in model prediction and observed data. Let $p(\Theta)$ be the $312 \times 1$ vector of equilibrium wage and labor-quantity predictions of the model. Let $q$ be the observed vector of wages and labor-quantity data taken from ENIGH. Finally, let subscripts $i$ in $q_i$ and $p_i$ denote any time, gender, skill, and occupation specific data and predictions.

The equilibrium estimation problem searches for optimal constrained elasticity parameters, given unconstrained non-elasticity parameters that provide best fit conditional on the elasticity parameters:

\[
\min_{\Theta^\rho \in (-\infty, 1]^8} \left\{ \min_{\Theta \setminus \Theta^\rho} \mathcal{O} \left( \{ q_i - p_i (\Theta) \} \right) \right\} .
\]

(C.27)

Given the large parameter space, it is important to initialize estimation at good starting values. Given a particular combination of $\Theta^\rho$ values, we initialize the estimation of demand- and supply-side parameters at parameters that provide best-fit under demand- and supply-side only estimation. Specifically, given $\Theta^\rho$, we minimize:

\[
\min_{\Theta \setminus \Theta^\rho} \mathcal{O} \left( \{ q_{Lj}^j - p_{Lj}^j (\Theta \mid q^W) \} \right) ,
\]

(C.28)

C.14. There are 13 $\psi$ parameters, however, 2 of them can not be separately identified from gender-specific $\pi_{1, gen}$ parameters.

C.15. $\Theta^\rho$ values are constrained between perfect substitutability and perfect complementarity. All other parameters can take on any positive or negative values.
where $p^L$ is the combined vector of labor quantities predicted by demand and supply equations given data wage vector $q^W$, and $q^L$ is the data vector of labor quantities to match. We estimate demand-side parameters via nonlinear least-square, which we provide details for in the next section, and supply-side parameters via linear least-square. We use the resulting estimates as starting parameter values for equilibrium estimation in Equation (C.27).

### C.3.2 Initializing demand-side parameters

We discuss here the estimation routine to generate starting values for all 56 non-elasticity demand-side parameters. The strategies here follow from the identification discussions in Section C.2.

Given our parametric assumptions on share parameter trends from Equation (5.4), Equation (C.1) for optimal male and female labor demand can be rewritten as:

$$L^*_d,k = L \left( 1 + \left( 1 - \exp \left( \sum_{j=0}^{3} a_j t^j \right) \right) \left( \frac{W_k}{W_f} \cdot \frac{1 - \exp \left( \sum_{j=0}^{3} a_j t^j \right)}{\exp \left( \sum_{j=0}^{3} a_j t^j \right)} \right)^{\frac{\rho}{1-\rho}} - 1 \right)^{-\frac{1}{\rho}}$$

$$L^*_d,f = L \left( 1 + \exp \left( \sum_{j=0}^{3} a_j t^j \right) \left( \frac{W_f}{W_k} \cdot \frac{\exp \left( \sum_{j=0}^{3} a_j t^j \right)}{1 - \exp \left( \sum_{j=0}^{3} a_j t^j \right)} \right)^{\frac{\rho}{\rho-1}} - 1 \right)^{-\frac{1}{\rho}}$$

(C.29)

Conditional on the elasticity parameter $\rho$ and given data on relative prices $\{\frac{W_{k,t}}{W_{f,t}}\}_{t=1}^T$ and gender-specific labor demands $\{L_{g,t}\}_{t=1}^T$, the share trend parameters of Equation (C.29) can be estimated via Equation (C.30):

$$\min_{\{a_j\}_{j=0}^{3}} \sum_{t=1}^{T} \sum_{g \in \{k,f\}} \tau_{g,t} \cdot \left( L_{g,t} - L_{g,t}^d \left( L_t, \{a_j\}_{j=0}^{3}, \rho; t, \frac{W_{k,t}}{W_{f,t}} \right) \right)^2,$$

(C.30)

where $\tau_{g,t}$ are potential estimation weights.\(^{C.16}\)

In Equation (C.30), in addition to unknown share trend parameters, time-varying aggregate labor demand $\{L_t\}_{t=1}^T$ for the sub-nest under consideration are also unknown. These $\{L_t\}_{t=1}^T$ values can first be found as best fitting proportional scalars: slopes estimates with the origin as the y-intercept. Let $\Omega_{g,t} \left( \{a_j\}_{j=0}^{3}, \rho, t, \frac{W_{k,t}}{W_{f,t}} \right) = L_{g,t}^d / L_t$, at each $t$, the best fitting $L_t$ value is:

$$\hat{L}_t = \frac{\Omega_{k,t} \cdot L_{k,t} + \Omega_{f,t} \cdot L_{f,t}}{\Omega_{k,t}^2 + \Omega_{f,t}^2} \cdot \frac{Q_{g,t}}{\sum_{t=1}^{T} \sum_{g \in \{m,f\}} Q_{g,t}}.$$

(C.31)

\(^{C.16}\) For example, $\tau_{g,t} = \frac{Q_{g,t}}{\sum_{t=1}^{T} \sum_{g \in \{m,f\}} Q_{g,t}}$. 

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Given Equation (C.31) and ignoring weights, the optimization problem from Equation (C.30) can be rewritten as:

\[
\min_{\{a_j\}_{j=0}^3} \sum_{t=1}^{T} \left( \frac{L_{k,t} - L_{f,t} \left( \frac{\Omega_{k,t}}{\Omega_{f,t}} \right)}{1 + \left( \frac{\Omega_{k,t}}{\Omega_{f,t}} \right)^2} \right)^2 + \left( \frac{L_{f,t} - L_{k,t} \left( \frac{\Omega_{f,t}}{\Omega_{k,t}} \right)}{1 + \left( \frac{\Omega_{f,t}}{\Omega_{k,t}} \right)^2} \right)^2,
\]

(C.32)

where \(\{\Omega_{g,t}\}_{g \in \{k,f\}}\) are only functions of the share trend parameters \(\{a_j\}_{j=0}^3\). Equation (C.32) assumes a parametric functional form for share parameters, nonparametrically fits \(L_t\), and assumes that \(\rho\) is fixed over time. Equation (C.32) is estimated via non-linear least square.\(^{C.17}\)

The \(\{L_t\}_{t=1}^{T}\) generated initially from Equation (C.32) are best-fitting for the current sub-nest, however, they are not consistent with parameters from higher layer nests. To generate consistent aggregate outputs requirements at lower nests, we repeat the just described estimation procedure but now perform it at higher layers of the nested-CES demand system: we use aggregate wages generated following Equation (C.2) and fit higher layer nest aggregate labor choice predictions against the \(\{L_t\}_{t=1}^{T}\) just generated from lower layer nests.

In this fashion, we estimate Equation (C.32) repeatedly as we move iteratively upwards along each branch of the nested-CES problem. This generates polynomial share parameters along each branch of each nest layer. At the highest nest layer, estimating Equation (C.32) generates best-fitting predictions for the aggregate \(\{Y_t\}_{t=1}^{T}\) ratios. Equipped with all demand-side parameters, we generate \(\{L_t\}_{t=1}^{T}\) aggregate output requirements for the lowest nest layer. This overall procedure can be repeated several times until the \(\{Y_t\}_{t=1}^{T}\) ratio across iterations converge. The demand-side estimation routine discussed in this section is linearly-scalable to nested-CES problems with additional layers and branches. Conditional on \(\Theta\), we use the estimates from this section as the starting parameter values for equilibrium estimation under Equation (C.27).

### C.3.3 Error Structure, Weight Matrix, and Standard Errors

In this section, we discuss the estimation objective function \(O\). We assume a simplified error structure to facilitate estimation. The presence of the error term follows from our discussions in Section C.2.4 on potential mismeasurement due to misreporting or sampling errors. For any given prediction \(i\), we assume that the error

\[
\log \left( \frac{W_{k,t} \cdot \left( \frac{Z_t}{Y_t} \right)^{1-\rho}}{1 + W_{f,t} \cdot \left( \frac{Z_t}{Y_t} \right)^{1-\rho}} \right) = a_0 + a_1 t + a_2 t^2 + a_3 t^3.
\]

(C.33)

This follows from the discussions in Section C.2.3.

---

\(^{C.17}\) Given \(\rho\), for the nest-specific nonlinear data-fitting procedure, starting values for polynomial share coefficients is obtained by estimating the following linear equation:

\[
\log \left( \frac{W_{k,t} \cdot \left( \frac{Z_t}{Y_t} \right)^{1-\rho}}{1 + W_{f,t} \cdot \left( \frac{Z_t}{Y_t} \right)^{1-\rho}} \right) = a_0 + a_1 t + a_2 t^2 + a_3 t^3.
\]
term, $e_i$, at the true parameter vector, $\Theta^*$, follows a normal distribution centred at zero that is independent across $i$. C.18 That is,

$$e_i = q_i - p_i(\Theta^*) \quad ,$$

(C.34)

where $f(e_i) = \frac{1}{\sqrt{2\pi}\sigma_i^2} \exp\left(\frac{e_i^2}{2\sigma_i^2}\right)$. The log-likelihood function takes the form

$$\log L(\Theta) = \sum_i \log f(e_i) = \sum_i \log f(q_i - p_i(\Theta)) \quad ,$$

(C.35)

and the respective score function, $s(\Theta)$, is:

$$s(\Theta) = \frac{\partial \log L(\Theta)}{\partial \Theta} = \sum_i \frac{\partial \log f(q_i - p_i(\Theta))}{\partial \Theta} = \sum_i \frac{1}{\sigma_i^2} \frac{\partial p_i(\Theta)}{\partial \Theta} (q_i - p_i(\Theta)) \quad ,$$

(C.36)

which we can write more compactly in vector form as

$$s(\Theta) = W'(\Theta)(q - p(\Theta)) \quad .$$

(C.37)

Here, $W'(\Theta)$ is $312 \times 94$ weight matrix that depends on the derivatives of the vector of predictions with respect to each of the parameters, and the variance of each prediction error $\sigma_i^2$. At the maximum likelihood estimate, $\hat{\Theta}_{ml}$, the score vector of the log likelihood is set to zero:

$$s(\hat{\Theta}_{ml}) = W'(\hat{\Theta}_{ml})(q - p(\hat{\Theta}_{ml})) = 0 \quad .$$

(C.38)

We use $m = q - p(\Theta)$ as a vector of population moments such that $E(q - p(\Theta)) = 0$, and obtain a a consistent estimator of $\Theta^*$ by GMM:

$$g(\hat{\Theta}_{gmm}) = W'(\hat{\Theta}_{gmm})(q - p(\hat{\Theta}_{gmm})) = 0 \quad ,$$

(C.39)

where $W'$ is a fixed positive definite matrix of instruments. Efficient GMM estimator can be obtained by choosing instruments that are asymptotically equivalent to the weights $W'(\hat{\Theta}_{ml})$ in Equation (C.37). The problem is that we would need to have a consistent initial estimate of $\Theta^*$. Given that we do not have those consistent initial estimates, we follow an iterative process. We start from a plausible set of initial values of the parameters ($\Theta_0$) and use them to estimate the vector of partial derivatives $\frac{\partial \hat{p}_i(\Theta_0)}{\partial \Theta_0}$. The estimates of the variance of each error, $\hat{\sigma}_i^2$, are calculated as the square of the estimated error from this initial set of parameter values. Both of these estimates are then used to construct an initial weight matrix, which allows

C.18. The normality assumption for the error terms follows from the Central Limit Theorem. We compute sample averages from the micro-data. All but 5 of the gender-skill-occupation-year subnest have at least 35 observations based on which sample means are computed.
us to solve the minimization problem.\(^{C.19}\) The estimates obtained after this first iteration\(^{C.20}\) are used to update the weight matrix, and the process continues until the parameter vector converges to a stable point.

Since it is usually not possible to satisfy Equation (C.39), we estimate the parameters of the model using the quadratic form:

\[
\hat{\Theta}_{gmm} = \text{argmin} [q - p(\Theta)]'W(\Theta)W'(\Theta)[q - p(\Theta)]. \tag{C.40}
\]

Finally, the standard errors of the parameter estimates are calculated applying the standard method of moments formula. We presented standard errors of demand- and supply-side parameter estimates in Tables 4 and 6 as well as Appendix Table D.6.\(^{C.21}\) Let \(\Gamma\) be the matrix of partial derivatives of the sample moments \(\bar{m}(\hat{\Theta}_{gmm})\) with respect to the parameters. The ith row corresponds to:

\[
\Gamma_i(\hat{\Theta}_{gmm}) = \frac{\partial \bar{m}_i(\hat{\Theta}_{gmm})}{\partial \hat{\Theta}_{gmm}}, \tag{C.41}
\]

so the variance-covariance matrix can be calculated using:

\[
\text{Var}(\hat{\Theta}_{GMM}) = \left[\Gamma(\hat{\Theta}_{gmm})'\text{Var}[\bar{m}(\Theta_{gmm})]^{-1}\Gamma(\hat{\Theta}_{gmm})\right]^{-1}. \tag{C.42}
\]

---

\(^{C.19}\) The parameter search is done using the interior-point algorithm in Matlab.

\(^{C.20}\) Note that even though the weight matrix is a function of the parameters, it remains fixed during the parameter search.

\(^{C.21}\) Given that the moment structure is overidentified—312 pairs of observables and models predictions in terms of wage and labor quantities (shares) and 94 demand- and supply-side parameters—we can perform a J-overidentification test. Given high critical values associated with larger number of excess moments as well as a close fit of model predictions and observables as shown in Tables 7 and 8 we fail to reject the null that the moments conditions hold under the true model parameters at 10 percent significance level.
D Additional Figures and Tables (online)

Figure D.1: Share of Part-Time Workers by Sex

Notes: An individual is defined as working part-time if he/she reported working less than 35 hours a week. See discussions in Section 8.
Figure D.2: Counterfactual Exercises

(a) Changes in Gender Participation and Wage Gaps: C.2012 - C.1992


Notes: The Table reports the difference between C.1992 and C.2012 of i) the log (male/female) wage ratio and ii) the change in the (male - female) LFP and occupation rates under different supply variable counterfactual scenarios. Figure (a) visualizes results from the “Overall” row in the first two blocks of Table 7. Figure (b) visualizes results from the skill- and occupation-specific rows in the first two blocks of Table 7 (skilled-manual and unskilled-analytical results are not shown for conciseness). Black-dashed lines mark model predictions, and points indicate predictions under key counterfactual scenarios. Points to the right of the vertical dashed-line reduce gender LFP and occupation participation gaps; points to the top of the horizontal dashed-line reduce gender wage gaps. Under the counterfactuals, we set the share with under-5 children (Fertility), the share married or having a permanent partner (Marriage), the WBL index (capturing laws and regulations that restrict women’s economic opportunities), and the share with a refrigerator or a washing machine (Appliance) at their 1989 values, respectively. Figure 5 presents changes in these variables over time. See discussions in Section 7.1.
Panels (a), (b), and (c) show variations in the log (male/female) wage by skill groups under non-wage determinants of LFP, demographic, and demand counterfactuals, respectively. In the counterfactuals, we set the share with under-5 children (Fertility) and with a refrigerator or a washing machine (Appliance), the gender-specific skilled worker share (Skilled Female) and the gender/skill-specific emigrant (Emigrant) shares, and the skill/occupation-specific demand gender share ($\alpha_4$) and occupation-specific demand skill share ($\alpha_3$) parameters at their 1989 values. Figures 5, 6, and 9 present changes in these variables and parameters over time.
<table>
<thead>
<tr>
<th></th>
<th>C.1992</th>
<th>C.2012</th>
<th>Dif. in Dif.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female Share (x100)</td>
<td>Male Share (x100)</td>
<td>∆&lt;sub&gt;c.1992&lt;/sub&gt; (Male - Female)</td>
</tr>
<tr>
<td>Prime-Age Population</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Participation Rate</td>
<td>38.59</td>
<td>96.49</td>
<td>57.89</td>
</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary (unskilled)</td>
<td>92.09</td>
<td>84.27</td>
<td>-7.82</td>
</tr>
<tr>
<td>College (skilled)</td>
<td>7.91</td>
<td>15.73</td>
<td>7.82</td>
</tr>
<tr>
<td>Age</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25-34</td>
<td>44.63</td>
<td>43.61</td>
<td>-1.02</td>
</tr>
<tr>
<td>35-44</td>
<td>32.34</td>
<td>32.84</td>
<td>0.50</td>
</tr>
<tr>
<td>45-55</td>
<td>23.02</td>
<td>23.55</td>
<td>0.52</td>
</tr>
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<td>Prime-Age Workforce</td>
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</tr>
<tr>
<td>Education</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary (unskilled)</td>
<td>85.48</td>
<td>84.37</td>
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<td>45-55</td>
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<td>23.04</td>
<td>2.83</td>
</tr>
</tbody>
</table>

Notes: The table reports participation rates of the prime-age population in the first row. The following rows show shares of the prime-age population (first panel) and shares of the prime-age work force (second panel) in each gender-education and gender-age group. For example, in C.1992, 92.09 percent of the female population had at most a secondary schooling, and 7.91 percent had a college degree. As fractions of the work force these shares were 85.45 and 14.52 percent. Sample weights used in all calculations. See discussions in Section 4.
Table D.2: Model Fit
Data and Model Predictions for Occupation Participation Rates and Wages

<table>
<thead>
<tr>
<th></th>
<th>C.1992 Female</th>
<th>C.1992 Male</th>
<th>C.2012 Female</th>
<th>C.2012 Male</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Wages</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td><strong>College</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td>7.17</td>
<td>6.16</td>
<td>10.25</td>
<td>8.77</td>
</tr>
<tr>
<td>Routine</td>
<td>6.11</td>
<td>4.33</td>
<td>8.29</td>
<td>5.47</td>
</tr>
<tr>
<td>Manual</td>
<td>3.17</td>
<td>2.55</td>
<td>5.23</td>
<td>4.30</td>
</tr>
<tr>
<td><strong>Secondary</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>3.21</td>
<td>4.66</td>
<td>3.77</td>
</tr>
<tr>
<td>Routine</td>
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<td>2.58</td>
<td>3.11</td>
<td>2.34</td>
</tr>
<tr>
<td>Manual</td>
<td>1.92</td>
<td>1.59</td>
<td>2.16</td>
<td>1.85</td>
</tr>
<tr>
<td><strong>Occupation Shares (x100)</strong></td>
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<td></td>
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</tr>
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<td><strong>College</strong></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td>1.61</td>
<td>2.24</td>
<td>5.00</td>
<td>5.31</td>
</tr>
<tr>
<td>Routine</td>
<td>0.41</td>
<td>0.62</td>
<td>1.23</td>
<td>1.26</td>
</tr>
<tr>
<td>Manual</td>
<td>0.08</td>
<td>0.05</td>
<td>0.65</td>
<td>0.39</td>
</tr>
<tr>
<td>Home Production</td>
<td>2.05</td>
<td>1.24</td>
<td>0.53</td>
<td>0.45</td>
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<tr>
<td><strong>Secondary</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analytical</td>
<td>5.19</td>
<td>5.47</td>
<td>6.75</td>
<td>6.45</td>
</tr>
<tr>
<td>Routine</td>
<td>5.29</td>
<td>4.87</td>
<td>13.76</td>
<td>13.55</td>
</tr>
<tr>
<td>Manual</td>
<td>7.59</td>
<td>6.82</td>
<td>17.21</td>
<td>17.58</td>
</tr>
<tr>
<td>Home Production</td>
<td>30.69</td>
<td>31.60</td>
<td>1.96</td>
<td>2.11</td>
</tr>
</tbody>
</table>

Notes: The table reports average wages and occupation participation rates (among all potential workers) in C.1992 and C.2012 both from the raw data and predicted by the model. For this table, the occupation participation rates are not conditional on gender and skill groups, i.e. the female and male columns sum up to 100. See discussions in Section 6.1.
Table D.3: Levels and Changes of Real Hourly Wages by Sex, Education, and Occupation. C.1992 and C.2012

<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
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<td></td>
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<td>Wages</td>
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<td>Wages</td>
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</tr>
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<td>86.54</td>
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<td>20.08</td>
<td>19.62</td>
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<td>[0.13]</td>
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<td>[0.00]</td>
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<td>Wages</td>
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<tr>
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<td>2.95</td>
<td>-5.34</td>
<td>77.88</td>
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<td>2.48</td>
<td>9.85</td>
<td>37.96</td>
<td>15.19</td>
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<td>[0.02]</td>
<td>[1.44]</td>
<td>[0.00]</td>
<td>[0.02]</td>
<td>[0.02]</td>
<td>[1.39]</td>
<td>[0.00]</td>
<td>[2.09]</td>
</tr>
</tbody>
</table>

<table>
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<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical</td>
<td>Skilled</td>
<td>5.34</td>
<td>7.59</td>
<td>35.12</td>
<td>41.75</td>
<td>4.70</td>
<td>5.84</td>
<td>21.58</td>
<td>2.87</td>
<td>-13.55</td>
</tr>
<tr>
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<td></td>
<td>[0.11]</td>
<td>[0.12]</td>
<td>[2.59]</td>
<td>[0.00]</td>
<td>[0.09]</td>
<td>[0.11]</td>
<td>[2.47]</td>
<td>[0.00]</td>
<td>[3.68]</td>
</tr>
<tr>
<td>Routine</td>
<td>Skilled</td>
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<td>3.64</td>
<td>-4.61</td>
<td>98.93</td>
<td>3.00</td>
<td>2.89</td>
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<td>60.21</td>
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<td>[0.04]</td>
<td>[2.10]</td>
<td>[0.00]</td>
<td>[0.05]</td>
<td>[0.03]</td>
<td>[2.16]</td>
<td>[0.00]</td>
<td>[2.94]</td>
</tr>
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<td>Skilled</td>
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<td>2.23</td>
<td>14.41</td>
<td>95.12</td>
<td>1.82</td>
<td>2.25</td>
<td>20.78</td>
<td>38.75</td>
<td>6.36</td>
</tr>
<tr>
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<td></td>
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<td>[0.03]</td>
<td>[2.37]</td>
<td>[0.00]</td>
<td>[0.03]</td>
<td>[0.02]</td>
<td>[1.82]</td>
<td>[0.00]</td>
<td>[3.01]</td>
</tr>
</tbody>
</table>

| Educ.-Occ. | Skilled | Analytical | 7.35 | 10.37 | 34.40 | 85.46 | 6.30 | 8.15 | 25.86 | 16.15 | -8.53 | -69.31 |
|            | Routine  | 6.27 | 8.88 | 34.79 | 70.18 | 4.88 | 5.25 | 7.39 | 13.86 | -27.40 | -56.32 |
| Manual     | 3.75 | 5.27 | 24.06 | [0.11] | [1.86] | [0.00] | [4.50] | [0.00] | [20.50] | [0.01] |

| Unskilled | Analytical | 3.95 | 4.66 | 16.64 | 16.00 | 2.67 | 3.12 | 15.72 | -9.71 | -0.92 | -25.72 |
| Routine    | 3.42 | 3.10 | 102.09 | [0.07] | [0.08] | [3.62] | [0.00] | [4.85] | [0.00] |
| Manual     | 1.92 | 2.16 | 93.58 | [0.03] | [1.72] | [0.00] | [2.96] | [0.00] |

Notes: The table reports the average real hourly wages, the average log (male/female) wages gap, and the log (male/female) relative supply by skill, occupation, and year. Sample is restricted to prime-age workers. The sample for the construction of the wages series is restricted to include only full-time workers. Standard errors are in brackets. Sample weights used in all calculations. See discussions in Section 6.2.
Table D.4: Aggregate Average Marginal Effects of Wages Decomposition

\[
\sum_{t=1}^{T} \frac{1}{T} \frac{d Pr(work| gen, t)}{d w} \quad \text{Increase Wages in All Occupations}
\]

\[
\sum_{t=1}^{T} \frac{1}{T} \frac{\partial Pr(dO=1| gen, edu, t)}{\partial w} \quad \text{Increase Occupation-specific Wages: Manual Wage Routine Wage Analytical Wage}
\]

### Average Marginal Effects with Respect to Gender- and Skill-specific:

#### LFP Rates

<table>
<thead>
<tr>
<th>Gender/Skill</th>
<th>LFP Rates</th>
<th>Manual Occupation Participation Rates</th>
<th>Routine Occupation Participation Rates</th>
<th>Analytical Occupation Participation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>female, secondary</td>
<td>0.107</td>
<td>0.060</td>
<td>-0.015</td>
<td>-0.015</td>
</tr>
<tr>
<td>female, college</td>
<td>0.036</td>
<td>0.033</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>male, secondary</td>
<td>0.023</td>
<td>0.013</td>
<td>-0.075</td>
<td>-0.033</td>
</tr>
<tr>
<td>male, college</td>
<td>0.008</td>
<td>0.001</td>
<td>0.004</td>
<td>0.003</td>
</tr>
</tbody>
</table>

#### Manual Occupation Participation Rates

<table>
<thead>
<tr>
<th>Gender/Skill</th>
<th>LFP Rates</th>
<th>Manual Occupation Participation Rates</th>
<th>Routine Occupation Participation Rates</th>
<th>Analytical Occupation Participation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>0.090</td>
<td>-0.015</td>
<td>-0.015</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>0.008</td>
<td>0.001</td>
<td>0.004</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>0.120</td>
<td>-0.075</td>
<td>-0.033</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>0.018</td>
<td>0.004</td>
<td>0.003</td>
</tr>
</tbody>
</table>

#### Routine Occupation Participation Rates

<table>
<thead>
<tr>
<th>Gender/Skill</th>
<th>LFP Rates</th>
<th>Manual Occupation Participation Rates</th>
<th>Routine Occupation Participation Rates</th>
<th>Analytical Occupation Participation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>-0.015</td>
<td>0.041</td>
<td>0.033</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>-0.001</td>
<td>0.022</td>
<td>0.037</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>-0.075</td>
<td>0.083</td>
<td>0.033</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>-0.004</td>
<td>0.024</td>
<td>0.025</td>
</tr>
</tbody>
</table>

#### Analytical Occupation Participation Rates

<table>
<thead>
<tr>
<th>Gender/Skill</th>
<th>LFP Rates</th>
<th>Manual Occupation Participation Rates</th>
<th>Routine Occupation Participation Rates</th>
<th>Analytical Occupation Participation Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>-0.015</td>
<td>-0.006</td>
<td>0.033</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>-0.004</td>
<td>-0.013</td>
<td>0.037</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>-0.033</td>
<td>-0.021</td>
<td>0.033</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>-0.013</td>
<td>-0.020</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Notes: Values are in percentage points. Given log wage coefficient \( \psi_1 = 0.966 \), we compute the Average Marginal Effects of wages over time for gender and skill groups. Average Marginal Effect in columns 2–4 are the partial derivatives of occupation participation rates—averaged across the years—with respect to wage. Column 1 shows the total derivative of overall LFP rates with respect to all three wages, evaluated towards the direction of equi-distance increases in all wage levels. The table presents a decomposition: in the first block, the values from each row in the first column is the sum of the values from columns 2 to 4; each value from columns 2 to 4 in the first block is the sum of the values in the same cell in subsequent panels. See discussions in Section 6.4.
Table D.5: Elasticity of Aggregate and Occupation-specific Labor Supply to Wage

<table>
<thead>
<tr>
<th></th>
<th>Increase Wages in</th>
<th>Increase Occupation-specific Wages:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Occupations</td>
<td>Manual Wage</td>
</tr>
<tr>
<td>Elasticity of Gender- and Skill-specific Labor Supply to Wages:</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Aggregate Labor Supply</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>female, secondary</td>
<td>0.529</td>
<td>0.099</td>
</tr>
<tr>
<td>female, college</td>
<td>0.341</td>
<td>0.009</td>
</tr>
<tr>
<td>male, secondary</td>
<td>0.060</td>
<td>0.025</td>
</tr>
<tr>
<td>male, college</td>
<td>0.062</td>
<td>0.005</td>
</tr>
<tr>
<td><strong>Manual Labor Supply</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>0.148</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>0.025</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>0.235</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>0.076</td>
</tr>
<tr>
<td><strong>Routine Labor Supply</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>-0.042</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>-0.006</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>-0.201</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>-0.024</td>
</tr>
<tr>
<td><strong>Analytical Labor Supply</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>female, secondary</td>
<td>—</td>
<td>-0.048</td>
</tr>
<tr>
<td>female, college</td>
<td>—</td>
<td>-0.026</td>
</tr>
<tr>
<td>male, secondary</td>
<td>—</td>
<td>-0.128</td>
</tr>
<tr>
<td>male, college</td>
<td>—</td>
<td>-0.111</td>
</tr>
</tbody>
</table>

Notes: Values are elasticities. Given log wages coefficient $\psi_1 = 0.966$, we compute the elasticities of wages for gender and skill groups. Column 1 presents the ratio of a percentage change in aggregate labor supply over a concurrent and equal-percentage increase in wages for all three occupation-specific wages. Averages across the years are shown in the table; Figure 10 visualizes these aggregate elasticities year by year. Columns 2–4 present occupation-specific elasticities—averaged across the years—of aggregate and occupation-specific labor supplies with respect to wages. Appendix Figure 11 visualizes these occupation-specific elasticities year by year. See discussions in Section 6.4.
Table D.6: Additional Supply Side Parameter Estimates

<table>
<thead>
<tr>
<th>Non Pecuniary Rewards/Tastes</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi_{f,u,m}$: female, unskilled, manual</td>
<td>18.514</td>
<td>0.159</td>
</tr>
<tr>
<td>$\psi_{f,u,r}$: female, unskilled, routine</td>
<td>17.687</td>
<td>0.181</td>
</tr>
<tr>
<td>$\psi_{f,u,a}$: female, unskilled, analytical</td>
<td>17.453</td>
<td>0.170</td>
</tr>
<tr>
<td>$\psi_{f,s,r}$: female, skilled, manual</td>
<td>11.145</td>
<td>0.143</td>
</tr>
<tr>
<td>$\psi_{f,s,r}$: female, skilled, routine</td>
<td>12.304</td>
<td>0.185</td>
</tr>
<tr>
<td>$\psi_{f,s,a}$: female, skilled, analytical</td>
<td>13.338</td>
<td>0.168</td>
</tr>
<tr>
<td>$\psi_{k,u,m}$: male, unskilled, manual</td>
<td>9.139</td>
<td>0.126</td>
</tr>
<tr>
<td>$\psi_{k,u,r}$: male, unskilled, routine</td>
<td>8.690</td>
<td>0.118</td>
</tr>
<tr>
<td>$\psi_{k,u,a}$: male, unskilled, analytical</td>
<td>7.511</td>
<td>0.125</td>
</tr>
<tr>
<td>$\psi_{k,s,r}$: male, skilled, manual</td>
<td>2.446</td>
<td>0.078</td>
</tr>
<tr>
<td>$\psi_{k,s,r}$: male, skilled, routine</td>
<td>2.875</td>
<td>0.100</td>
</tr>
<tr>
<td>$\psi_{k,s,a}$: male, skilled, analytical</td>
<td>3.795</td>
<td>0.107</td>
</tr>
</tbody>
</table>

Notes: The table shows the point estimates and standard errors of additional supply side parameters. These parameters are “intercepts” that are specific to each gender and skill group for each one of the three work occupations. See estimates discussions in Section 6.5 and estimator discussions in Appendix Section C.3.3.