European Sovereign Bond and Stock Market Granger Causality Dynamics

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European Sovereign Bond and Stock Market 
Granger Causality Dynamics*

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Abstract

We investigate the lead-lag relationship between weekly sovereign bond yield changes and stock market returns for eight European countries, and how it changed during the period 2008-2018. We use a Markov-Switching Granger Causality method that determines reversals of causality endogenously. In all countries, there were often changes in the direction of the Granger causality between the two markets that coincided with global and idiosyncratic economic events. Stock returns led changes of sovereign bond yields in all countries, particularly during the financial and the Euro Area crisis. Changes of sovereign bond yields occasionally led stock returns in France, Spain, and Portugal.

Keywords: Sovereign Bond Yields, Stock Markets, Vector Autoregression, Markov-Switching, Granger Causality.

JEL classification: C32; C54; C61; G01; G12; G15.

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1 Introduction

The financial crisis of 2008-2009 had adverse consequences for the real economy around the globe and in Europe. Several European countries faced recessions and falling stock indexes and market value of equities, later exacerbated by unsustainable fiscal policies, a consequence of large budget deficits and high government debt levels. Euro-area bond markets faced intense pressure beginning in May 2010, reflecting the sovereign debt crisis. On the one hand, investors demanded higher yields on European sovereign bonds to compensate for their risk. On the other hand, high debt and deficits led investors to lose confidence about the future returns of equities in a higher-bond-yield environment.

Stocks and sovereign bonds, two major components of capital markets, played an essential role in the country risk assessment during the recent crisis. Stocks represent market risk. Sovereign bonds, once generally viewed as safe assets for equity investors to diversify their portfolios, during the crisis they reflected, or were a proxy for, sovereign risk. Both markets are strong indicators of investors’ portfolio choice and were affected by the fragility of the financial sectors and the length and depth of the global recession. In full-information conditions, both markets should assimilate new information simultaneously, and prices should be contemporaneously discovered. Nonetheless, when public and private information are asymmetrically absorbed by one of the markets, a lead-lag relationship can be observed between the prices of sovereign bonds and stocks. In these situations, understading which market leads the other is important, whether for governments and researchers to anticipate specific country risk, or for investors or financial institutions to adapt their financial strategies.

Taking into account the limited transmission of information to the markets, we study the country-specific lead-lag relation between changes in 10-year sovereign (government) bond yields and stock (market) returns. We examine the Granger causality, henceforth causality, between the sovereign bond and stock market, at a weekly frequency, for a set of eight European countries during the period between 2008 and 2018. The usual causality test has a critical limitation: it is susceptible to the sample period, which can reverse the estimates of the causality test statistics and lead to inaccurate conclusions. Our main contribution
is to overcome this limitation by using a methodology that measures the causality and endogenously defines the sample period.

Our methodology based on Markov-Switching Causality, proposed by Psaradakis et al. (2005), consists on a vector autoregressive model with time-varying parameters, and consequently a time-varying pattern of causality. The parameter time-variation is modelled through a hidden Markov chain that reflects changes in causality between the variables of interest, over the sample, endogenously. In the literature, the results of the Vector Autoregressive (VAR) causality tests for a particular country often depend on the selection of the sample period, which generates instability in the causality patterns. To illustrate this problem, we estimate a (VAR) and conduct the Granger Causality tests by splitting the whole sample into three sub-samples to show the instabilities in the causality patterns. Then, we estimate the Markov-switching Causality VAR method that finds endogenously the periods in which the data suggests the presence of causality\(^1\). The method also enables us to calculate the *expected duration* and *actual duration* of the regimes for each country. Knowing the dates of regime switches, we can look at the global or country-specific events that overlap with changes in the direction of causality.

We contribute to the empirical finance literature in three dimensions. First, we find the exact dates when there are shifts in the causality. Second, although the markets are very integrated, we provide some evidence that a global (or regional) crisis affects the countries asymmetrically. Third, in terms of price discovery, we add to the evidence that the direction of the causality is mostly from the stock returns to the first difference in sovereign bond yield.\(^2\) Nevertheless, we find that there are several episodes where causality runs from sovereign bond yields to stock market return.

Our paper is also related to the literature on Markov-Switching VAR, for instance, Taamouti (2012), Droumaguet et al. (2017) and Warne (2000). Taamouti (2012) generalizes the methodology by Timmermann (2000) to find the conditional and unconditional

\(^1\)Nonlinear Granger Causality has been also studied by Song and Taamouti (2018) in a non-parametric setting.

\(^2\)As pointed out by Gyntelberg et al. (2018), this conclusion demands a discretionary interpretation due to the weekly data frequency we are using in the paper.
moments of a Markov-Switching VAR and verifies the relevance of conditional information to asset allocation between a stock index and 10-year government bond. Droumaguet et al. (2017) and Warne (2000) test the Granger causality parameters in the Markov-Switching VAR setting, using a Bayesian and frequentist approach respectively. Our method differs from those because we do not test the Granger Causality parameters; instead, we constraint the regimes in a VAR model to obtain all possible Granger causality patterns and allow the data to select them.

Most of the previous studies that investigate the causality between sovereign risk and stock markets focussed on credit default swap spreads (CDS). Examples include Silva (2014), Coronado et al. (2012) and Corzo-Santamaria et al. (2012). In particular, the later two papers performed VAR-Granger causality tests on the lead-lag relation between CDS and stock market indexes and find that the direction of the Granger Causality depends on the sample period that was defined ad-hoc. However, mostly the stock markets react faster to new information than CDS market. Instead of CDS’s, we use sovereign bond yields as a measure of sovereign risk, for four reasons. First, sovereign bonds yields are issued by governments to investors and their creditworthiness depend on governments perceived ability to repay debts. Second, according to Phillips and Shi (2019) the long-term sovereign bond yields are proxies for the sovereign risk. Thirdly, the sovereign bond markets were not subject to any kind of selling restriction as it happened in 2011 to the CDS markets (Sambalaibat, 2014). The CDS ban led to reducing liquidity in the sovereign CDS market, in particular for Greece, Ireland, Italy, Portugal, and Spain, which rendered this market ineffective for hedging (IMF, 2013). Also, it is difficult to examine the sovereign CDS market in Greece after the International Swaps and Derivatives Association, declared the Greek sovereign default in March 2012. Finally, the literature has offered several papers that measures the lead-lag relationship between sovereign bond yields and CDS, which include Fontana and

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3Other papers have focused on the credit risk and stock market at the corporate level, for instance Longstaff et al. (2005), Norden and Weber (2009), Bystrom (2005), Fung et al. (2008), Forte and Pena (2009), Marsch and Wagner (2012) and Hilscher et al. (2015). Additionally, by focussing on the lead-lag relation between corporate bonds and stock returns Tolikas (2018) finds that the daily stock returns lead the daily bond returns.
Scheicher (2016) and Gyntelberg et al. (2018). In general, they conclude the CDS reacts to new information faster than sovereign bond yields. As we have pointed out, the stock market reacts faster to new information than CDS market and the literature suggests that the CDS Market responds faster than sovereign bond yields. Therefore, the transmission mechanism of price discovering seems to be from stock returns to CDS and then to sovereign bond yields.

Our sample period captures several global events. It starts at the onset of the financial crisis. It then encompasses the European sovereign debt crisis, when sovereign bonds became central to investors concerned with the ability of some European countries to repay their debts, increasing the bond yields. Later, in 2015, the European Central Bank (ECB) launched the Quantitative Easing (QE) program, known as a bond-buying program, to keep bond yields low. As pointed by Flavin and Lagoa-Varela (2019), the whole context drove stock market investors to use long-term sovereign bonds as a hedge for their financial decisions during the stock market turbulence, depending on the countries and market conditions. As a result of this of the asymmetric flight-to-safety tendencies, domestic sovereign bond became the most important element of heterogeneity across the countries. Finally, the sample also captures many country-specific events, such as the Brexit Referendum, the Spanish Bank Bailout, the Greek International Bailout or the Portuguese Financial crisis.

We find that economic events, whether they are global or country-specific, can trigger reversals in the causality between these two variables. For instance, we find there is a shift in causality in most of the countries, coinciding with the global financial crisis. Additionally, our results contradict the common knowledge that the stock markets always lead the bond markets. The results suggest that the direction of the causality depends on the period, country and nature of the crisis. For instance, we find that sovereign bond yields cause stock returns in some periods in France, Portugal and Spain. The actual duration of the causality regimes also varies across countries.

The remainder of the paper is organized as follows. Section 2 motivates our analysis. First, it provides a description of economics events during the sample period, together with
the description of the data. Second, it reports the estimation of a VAR to highlight how the results on causality depend on the sample choice. Section 3 describes the Markov Switching Causality methodology. Section 4 shows the results and Section 5 concludes.

2 Motivation

2.1 Narrative of the crisis

We start with brief overview of economic events that happened in Europe since 2008. Our methodology identifies endogenously dates in which the direction of causality changed and we will relate these to the events associated with the global and European financial crisis, which peaked between 2009 and 2012.

The European sovereign debt crisis began when the government of Greece reported errors in past budgetary data, which was higher than the country had let on. As a consequence, their 10-year bond spreads increased significantly. Compounded by the global financial crisis, Greek deficit and debt reached high levels soon after which caused distress about its ability to pay its debts and, in late 2008, fears of a deep recession escalated in the Eurozone. Borrowing costs in Greece, Portugal, Ireland and Spain reached prohibitive levels. Unable to roll over their debts, they had to receive bailouts from the European Stability Mechanism (ESM), International Monetary Fund (IMF), or both.

Despite the IMF and the EU’s bailouts, the concerns about the financial crisis led the European Union members, in a meeting on the 22nd of June in 2012, to support a second bailout program for Greece, together with the IMF, to prevent the crisis spreading across Europe. Although Greece and its creditors agreed to a debt restructuring for the bailout funds in 2012, growing risk that Greece will default and the possibility of contagion led to a fall in investors confidence.\textsuperscript{4}

The period from 2009 to 2014 was the hardest period for Portuguese economy, which was affected by both the global financial crises and the sovereign debt crisis. On January

\textsuperscript{4}For instance, Afonso et al. (2012) documented contagion in stock returns and bond yields across different European countries following downgrades in sovereign credit ratings.
2010, fears over the liquidity and stability of Eurozone bonds spread to Portugal, leading bond yields to accelerate to unsustainable levels. This caused the Portugal government to pursue emergency austerity measures. In 2014, the fears of a recession spreading to the Eurozone’s core and breaking up the single currency returned. In macroeconomic terms, inflation was low and even negative in Spain, Portugal and Greece, increasing their debt burdens. These concerns led European stocks to temporarily crash. This last period led the ECB to announce the Quantitative Easing (QE) program in February 2015 in order to stabilise the inflation, stimulate the economy, and maintain low bond yields.

2.2 Data

For the empirical analysis, we use weekly observations on changes of 10-year sovereign bond yields, denoted by $\Delta YLD$ for eight European countries: Germany, France, Spain, Portugal, Italy, Greece, Ireland and the United Kingdom. For the stock market returns, denoted by $\Delta R$, the sample consists of the changes in weekly closing price for DAX (Germany), CAC 40 (France), IBEX 35 (Spain), FTSEMIB (Italy), FTSE 100 (UK) and ISEQ (Ireland), PSI 20 (Portugal) and ASE (Greece). The data is provided by DataStream. Our sample comprises the period from January of 2008 to July of 2018, which gives us the total of 503 observations. A few considerations about our key choices are in order.

One possible alternative was to rely on daily data but, in our case, it would introduce three main difficulties. First, at daily frequency, the data is very noisy, imposing a large computational burden. Secondly, we are not interested in measuring time-varying volatility, where the information content in daily basis could be more important, Instead, we focus on the mean equation and a VAR-type estimator. Finally, although more observations is preferable for econometric precision, using higher frequency data raises the number of outliers that can impose some bias in our estimates, as we are using Markov-Switching methods. An outlier might trigger a switch that has not occurred. Additionally, the studies on inter-market linkages that uses daily data have been criticizing due to the differences at the end of the day markets which could lead an upward-bias of stock prices (Vijh, 1988). For these
reasons, we believe the information contained in weekly data is preferable for our exercise (Goodhart and O’Hara, 1997).

We opted to use the (difference in) yields in levels rather than in spreads because the spreads are usually calculated, taking into account the bonds yield level of Germany. In our set of countries, we are using Germany, which would be left out otherwise. Besides, the findings of Phillips and Shi (2019) suggest that after 2008 there are not many differences between the bond yields and bond yield spreads to detect financial collapses, which is the primary mechanism of our method to trigger the shifts from one regime to another.

We chose these eight countries to represent the variety of cases within the European Union. Germany and France are the most robust economies of the Eurozone. The U.K. did not belong to the Eurozone and started the process of leaving the European Union, which we believe it works as an interesting counterfactual. Italy has robust economy, but with economic and political turmoil. The remaining four countries were rescued due to their financial fragility during the analysed period.

Table 1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean YLD</th>
<th>Max. YLD</th>
<th>Min. YLD</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-0.006</td>
<td>0.36</td>
<td>-0.37</td>
<td>0.10</td>
<td>0.13</td>
<td>3.58</td>
</tr>
<tr>
<td>France</td>
<td>-0.006</td>
<td>0.41</td>
<td>-0.49</td>
<td>0.10</td>
<td>0.16</td>
<td>5.31</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.004</td>
<td>0.72</td>
<td>-1.26</td>
<td>0.18</td>
<td>-1.16</td>
<td>12.61</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.02</td>
<td>0.60</td>
<td>-1.13</td>
<td>0.15</td>
<td>-0.83</td>
<td>10.49</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.006</td>
<td>0.39</td>
<td>-0.56</td>
<td>0.11</td>
<td>0.02</td>
<td>4.16</td>
</tr>
<tr>
<td>Ireland</td>
<td>-0.006</td>
<td>1.46</td>
<td>-2.27</td>
<td>0.24</td>
<td>-0.96</td>
<td>23.13</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.004</td>
<td>2.14</td>
<td>-1.69</td>
<td>0.34</td>
<td>0.12</td>
<td>12.06</td>
</tr>
<tr>
<td>Greece</td>
<td>-0.0009</td>
<td>8.16</td>
<td>-20.69</td>
<td>1.26</td>
<td>-7.73</td>
<td>139.89</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>Mean R</th>
<th>Max. R</th>
<th>Min. R</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.11</td>
<td>14.94</td>
<td>-24.34</td>
<td>3.19</td>
<td>-1.01</td>
<td>11.07</td>
</tr>
<tr>
<td>France</td>
<td>0.02</td>
<td>12.43</td>
<td>-25.05</td>
<td>3.13</td>
<td>-1.21</td>
<td>11.40</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.05</td>
<td>11.10</td>
<td>-23.82</td>
<td>3.48</td>
<td>-0.89</td>
<td>7.58</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.09</td>
<td>10.24</td>
<td>-25.11</td>
<td>3.61</td>
<td>-1.16</td>
<td>8.26</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.05</td>
<td>12.58</td>
<td>-23.63</td>
<td>2.60</td>
<td>-1.40</td>
<td>17.80</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.03</td>
<td>14.47</td>
<td>-32.90</td>
<td>3.51</td>
<td>-1.97</td>
<td>19.57</td>
</tr>
<tr>
<td>Portugal</td>
<td>-0.12</td>
<td>8.50</td>
<td>-20.56</td>
<td>3.04</td>
<td>-1.04</td>
<td>7.52</td>
</tr>
<tr>
<td>Greece</td>
<td>-0.32</td>
<td>17.56</td>
<td>-22.54</td>
<td>4.81</td>
<td>-0.44</td>
<td>4.71</td>
</tr>
</tbody>
</table>

Note: Weekly observations on changes of 10-year sovereign bond yields (ΔYLD) and stock market returns (ΔR). Sample from January of 2008 to July of 2018, with 503 observations.
Table 1 provides the descriptive statistics for each country’s weekly changes of sovereign bond yields and stock returns. The graphs for the two variables are shown in Appendix. The maximum change in sovereign bond yields in Greece, Portugal, Spain, Ireland and Italy are significantly higher than Germany, France and the UK for the entire period. The mean of stock returns is positive in Germany, Ireland, the UK and France while it is negative in the remaining countries. Regarding the kurtosis of the stock returns, Greece has the lowest kurtosis while Ireland and Germany has the highest. In contrast, changes in sovereign bond yields have kurtosis greater than 3, with Greece having the highest and Germany the lowest.

2.3 VAR Model: Causality between Sovereign Bonds and Stock Returns

We motivate our methodology by analysing the dynamic co-movement of weekly changes of sovereign bonds and stock returns using a standard VAR conditional on an exogenous variable. In line with Norden and Weber (2004), we estimate the following two-dimensional VAR model:

\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\mu_1 \\
\mu_2
\end{bmatrix} + \sum_{k=1}^{h} \begin{bmatrix}
\phi_1^{(k)} & \psi_1^{(k)} \\
\phi_2^{(k)} & \psi_2^{(k)}
\end{bmatrix} \times \begin{bmatrix}
\Delta YLD_{t,t-k} \\
\Delta R_{t,t-k}
\end{bmatrix} + \begin{bmatrix}
\varphi_1 \\
\varphi_2
\end{bmatrix} \times \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

where \(\Delta R_t\) is the stock index return at \(t\), \(\Delta YLD_t\) is the first differences of sovereign bond yield at \(t\), \(h\) is the lag order index, \(\varepsilon_t\) is the disturbance term at \(t\). In addition to the endogenous variables, we condition the model to a variable that reflects a global risk. Other articles have used several variables to indicate global risk (factor); for instance, Gomes and Taamouti (2016) uses risk factors based on Google search data. Instead, we conditioned our model to CBOE Volatility Index, \(VIX_t\).\(^5\) One advantage of using this variable is that it reflects the expected volatility based on past values. Therefore, our model does not suffer from endogeneity problems, as \(VIX_t\) is contemporaneous to the endogenous variables. Additionally, \(VIX_t\) is a volatility index, which could represent an unexpected shock in the mean equation.

\(^5\)The original \(VIX_t\) is divided by \(\sqrt{52}\) to reflect the weekly frequency.
We use the Augmented Dickey-Fuller test with optimal lag length selection based on Akaike’s information criterion to check the stationary for all series. Both stock returns and government bond yields changes are stationary for all countries. We found the optimal lag of the VAR to be 2, by computing for information criteria: the likelihood ratio (LR), final prediction error (FPE), Akaike’s information criteria (AIC), Hannan-Quinn information criteria (HQIC) and Schwarz’s Bayesian information criteria (SBIC). The presence of the VIX controls for the volatility that varied substantially in the financial markets during the sample period. We follow other authors, such as Corzo-Santamaria et al. (2012), in performing the Granger causality test for each country. This implies testing the parameters \( \psi_1^{(k)} \) and \( \psi_2^{(k)} \).

We first estimate the VAR for the whole sample and conduct the Granger causality test. We then repeat the estimation and the test in three sub-samples of equal length. The results are reported in Table 2. According to the causality test for the full sample, stock returns cause sovereign bond yields in only three countries: Germany, France and the United Kingdom. Using the full sample, there is no causality from sovereign bond yields to stock markets in any country. However, the conclusion of causality is dependent on the choice of the sample period. In all countries except Portugal and Spain, there is at least one sub-period where stock returns cause sovereign bond yields. Also, in one sub-sample in

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>0.000*</td>
<td>0.012*</td>
<td>0.074</td>
<td>0.163</td>
<td>0.803</td>
<td>0.420</td>
</tr>
<tr>
<td>France</td>
<td>0.000*</td>
<td>0.008*</td>
<td>0.125</td>
<td>0.012*</td>
<td>0.648</td>
<td>0.584</td>
</tr>
<tr>
<td>Spain</td>
<td>0.613</td>
<td>0.277</td>
<td>0.550</td>
<td>0.777</td>
<td>0.529</td>
<td>0.953</td>
</tr>
<tr>
<td>Italy</td>
<td>0.135</td>
<td>0.015*</td>
<td>0.137</td>
<td>0.921</td>
<td>0.362</td>
<td>0.395</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.000*</td>
<td>0.000*</td>
<td>0.107</td>
<td>0.313</td>
<td>0.503</td>
<td>0.831</td>
</tr>
<tr>
<td>Ireland</td>
<td>0.060</td>
<td>0.196</td>
<td>0.293</td>
<td>0.106</td>
<td>0.667</td>
<td>0.514</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.545</td>
<td>0.270</td>
<td>0.933</td>
<td>0.932</td>
<td>0.966</td>
<td>0.360</td>
</tr>
<tr>
<td>Greece</td>
<td>0.320</td>
<td>0.655</td>
<td>0.310</td>
<td>0.009*</td>
<td>0.589</td>
<td>0.784</td>
</tr>
</tbody>
</table>

Note: The null hypothesis of the Granger Causality test is that there is no causality. We report the p-values of the test. The * signals Granger causality. The VAR in equation 1 included the \( VIX_t \) as an exogenous variable was estimated with two-lags with 504 observations (full sample).
the United Kingdom (from 2/15 to 7/18) and in Ireland (8/11 to 2/15) changes in sovereign bond yields cause stock returns.

The choice of sub-period is arbitrary and clearly affects the results. Our main contribution is to employ the Markov-Switching Causality methodology to verify how the causality pattern changes throughout the sample and determine endogenously the timing of the switches.

3 Markov Switching Causality

3.1 Setting

The Markov-Switching Causality was first proposed by Psaradakis et al. (2005), is a vector autoregression where some parameters are constrained to allow different patterns of Granger causality and the switching between each pattern follows a hidden Markov process. The model is as follows:

\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix}
= D_t + \sum_{k=1}^{h} A_t^{(k)} \begin{bmatrix}
\Delta YLD_{t-k} \\
\Delta R_{t-k}
\end{bmatrix} + Z_t \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}, \quad t = 1, 2, \ldots, T, \tag{2}
\]

Where \([\Delta YLD_t, \Delta R_t]\) are the change in sovereign bond yields and stock market returns, \(\varepsilon_{1t}\) and \(\varepsilon_{2t}\) are the reduced form residuals of the two equations, and \(D_t, A_t^{(k)}\) and \(Z_t\) are state-dependent parameter matrices given by

\[
D_t = \begin{bmatrix}
\mu_{10} + \mu_{11}s_{1,t} \\
\mu_{20} + \mu_{21}s_{2,t}
\end{bmatrix}, \quad A_t^{(k)} = \begin{bmatrix}
\psi_{10}^{(k)} + \psi_{11}^{(k)}s_{1,t} + \psi_{12}^{(k)}s_{2,t} \\
\psi_{20}^{(k)} + \psi_{21}^{(k)}s_{2,t}
\end{bmatrix}, \quad Z_t = \begin{bmatrix}
\varphi_{10} + \varphi_{11}s_{1,t} \\
\varphi_{20} + \varphi_{21}s_{2,t}
\end{bmatrix}
\]

Additionally, the model is conditioned to an exogenous variable, \(VIX_t\). The four regimes can be summarized as:

\[
S_t = \begin{cases}
1 & \text{if } s_{1,t} = 1 \text{ and } s_{2,t} = 1 \quad \text{bidirectional causality } (\Delta R_t \leftrightarrow \Delta YLD_t) \\
2 & \text{if } s_{1,t} = 0 \text{ and } s_{2,t} = 1 \quad \text{sovereign bonds cause stock returns } (\Delta YLD_t \rightarrow \Delta R_t) \\
3 & \text{if } s_{1,t} = 1 \text{ and } s_{2,t} = 0 \quad \text{stock returns cause sovereign bonds } (\Delta R_t \rightarrow \Delta YLD_t) \\
4 & \text{if } s_{1,t} = 0 \text{ and } s_{2,t} = 0 \quad \text{no causality } (\Delta R_t \not\leftrightarrow \Delta YLD_t)
\end{cases}
\]

or explicitly:
For $S_t = 1$:
\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\mu_{10} + \mu_{11} \\
\mu_{20} + \mu_{21}
\end{bmatrix} + \sum_{k=1}^{h} \begin{bmatrix}
\phi_{10}^{(k)} + \phi_{11}^{(k)} \\
\phi_{20}^{(k)} + \phi_{21}^{(k)}
\end{bmatrix} \begin{bmatrix}
\Delta YLD_{t-k} \\
\Delta R_{t-k}
\end{bmatrix} + \begin{bmatrix}
\varphi_{10} + \varphi_{11} \\
\varphi_{20} + \varphi_{21}
\end{bmatrix} \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

For $S_t = 2$:
\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\mu_{10} \\
\mu_{20} + \mu_{21}
\end{bmatrix} + \sum_{k=1}^{h} \begin{bmatrix}
\phi_{10}^{(k)} \\
\phi_{20}^{(k)} + \phi_{21}^{(k)}
\end{bmatrix} \begin{bmatrix}
\Delta YLD_{t-k} \\
\Delta R_{t-k}
\end{bmatrix} + \begin{bmatrix}
\varphi_{10} + \varphi_{11} \\
\varphi_{20}
\end{bmatrix} \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

For $S_t = 3$:
\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\mu_{10} + \mu_{11} \\
\mu_{20}
\end{bmatrix} + \sum_{k=1}^{h} \begin{bmatrix}
\phi_{10}^{(k)} + \phi_{11}^{(k)} \\
0
\end{bmatrix} \begin{bmatrix}
\Delta YLD_{t-k} \\
\Delta R_{t-k}
\end{bmatrix} + \begin{bmatrix}
\varphi_{10} + \varphi_{11} \\
\varphi_{20}
\end{bmatrix} \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

For $S_t = 4$:
\[
\begin{bmatrix}
\Delta YLD_t \\
\Delta R_t
\end{bmatrix} = \begin{bmatrix}
\mu_{10} \\
\mu_{20}
\end{bmatrix} + \sum_{k=1}^{h} \begin{bmatrix}
0 \\
\phi_{20}^{(k)}
\end{bmatrix} \begin{bmatrix}
\Delta YLD_{t-k} \\
\Delta R_{t-k}
\end{bmatrix} + \begin{bmatrix}
\varphi_{10} \\
\varphi_{20}
\end{bmatrix} \begin{bmatrix}
VIX_t \\
VIX_t
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix}
\]

Notice that, the parameters $\psi_{1}^{(k)}$ and $\psi_{2}^{(k)}$ are the parameters that give the temporary Granger causality, henceforth temporary causality: $S_t = 2$ is the regime where sovereign bonds temporarily cause stock returns and $S_t = 3$ is the case when the stock returns temporarily cause sovereign bonds. The state $S_t = 1$ is the state where there is a dual temporary causality and $S_t = 4$ is the state where there is no-temporary causality. Beside the imposed differences in the temporary causality patterns in the four regimes, the regimes differ on other parameters, namely $\mu_{11}, \mu_{21}, \varphi_{11}, \varphi_{21}, \phi_{11}, \phi_{21}$ and the regime-dependent variance-covariance matrix of the structural error term, that we define later.

To complete the specification of the model, as defined by Psaradakis et al. (2005), the Markov process that defines the behaviour of the regimes can be described as:

\[
p_{t+1}^{(i)} = \mathcal{P}(s_{t+1} = j | s_{t} = i), \text{ where } i, j = 0, 1 \text{ and } l = 1, 2
\]

Notice that $p_{t+1}^{(i)}$ probability of being at the regime at time $t + 1$ conditioned to the regime at $t$ and the regimes $s_{1,t}$ and $s_{2,t}$ are independent. Therefore, the transition matrix is:

\[
P = \begin{bmatrix}
\begin{array}{cccc}
p_{11}^{(1)} \times p_{11}^{(2)} & (1 - p_{00}^{(1)}) \times p_{11}^{(2)} & p_{11}^{(1)} \times (1 - p_{00}^{(2)}) & (1 - p_{00}^{(1)}) \times (1 - p_{00}^{(2)}) \\
p_{11}^{(1)} \times (1 - p_{11}^{(2)}) & (1 - p_{00}^{(1)}) \times p_{11}^{(2)} & p_{11}^{(1)} \times (1 - p_{00}^{(2)}) & (1 - p_{00}^{(1)}) \times (1 - p_{00}^{(2)}) \\
p_{00}^{(1)} \times p_{11}^{(2)} & (1 - p_{00}^{(1)}) \times (1 - p_{11}^{(2)}) & p_{00}^{(1)} \times p_{00}^{(2)} & (1 - p_{00}^{(1)}) \times p_{00}^{(2)} \\
p_{00}^{(1)} \times (1 - p_{11}^{(2)}) & (1 - p_{00}^{(1)}) \times (1 - p_{11}^{(2)}) & p_{00}^{(1)} \times p_{00}^{(2)} & (1 - p_{00}^{(1)}) \times p_{00}^{(2)}
\end{array}
\end{bmatrix}
\]

\[(3)\]
3.2 Expected Duration

As a by-product of the transition matrix, we can provide a theoretical metric that summarizes in how long each regime is expected to last, in the absence shocks. The expected duration is calculated directly from the estimates of the transition matrix as suggested by Hamilton, 1989:

\[
ED_m = \sum_{i=1}^{\infty} i \times \left[ \pi_{m,m} \right]^{i-1} \times \left[ 1 - \pi_{m,m} \right] = (1 - \pi_{m,m})^{-1}
\]  

(4)

where \( \pi_{m,m} \) are the main diagonal elements of the transition matrix \( P \). In our case, we are interested in how long the states where the sovereign bonds temporarily cause stock returns and stock returns temporarily cause the sovereign bonds last. These are given by:

\[
ED_{Y \rightarrow R} = (1 - \pi_{1,1})^{-1} + (1 - \pi_{2,2})^{-1}
\]

\[
ED_{R \rightarrow Y} = (1 - \pi_{1,1})^{-1} + (1 - \pi_{3,3})^{-1}
\]

\[
\sum ED = ED_{Y \rightarrow R} + ED_{R \rightarrow Y} - (1 - \pi_{1,1})^{-1}
\]

\[
ED_{Y \leftrightarrow R} = (1 - \pi_{4,4})^{-1}
\]

Where \( \sum ED \) is the expected duration of at least one of the variables causes each other.

This metric is important because it shows the degree of persistence. The closer the probability \( \pi_{m,m} \) is to one, the longer it takes to switch to another regime. Also, unlike the probability, the expected duration provides a measurement unit as it is measured in weeks.

3.3 Distribution

To estimate the model using maximum likelihood we need to assume a particular distribution for the residuals. In the original article, Psaradakis et al. (2005), the residuals are assumed to be normally distributed. However, as illustrated in Table 2, high-frequency financial time-series data are more leptokurtic than macroeconomic quarterly data. Taking this feature into consideration, we assume that the residuals follow a Generalized Error Distributed (GED).
The bivariate cumulative density function of the GED is described by Giller (2005) as:

\[
F(x|\Phi_m, \Sigma_m, \kappa_m) = \frac{x}{\sqrt{\pi^2 \Sigma_m}} \frac{\Gamma(2)}{\Gamma(1 + 2\kappa_m)} \exp - \frac{\Gamma(3\kappa_m)}{\Gamma(\kappa_m)} \left( \frac{(\varepsilon_t)' \Sigma_{m}^{-1} (\varepsilon_t)}{2\kappa_m} \right) \left( \frac{1}{\Sigma_{m}} \right)^{1/2}
\]

where \( \Sigma_m \) is the covariance matrix, \( \Phi_m \) is the parameter vector and \( \kappa_m \) is the distributional parameter reflecting the kurtosis. We allow both the covariance matrix and the distributional parameter to vary across the four regimes \( m = 1, 2, 3, 4 \).

### 3.4 Estimation Method

The parameters of this Markov-Switching Granger Causality model are estimated by maximum likelihood (MLE), assuming that the conditional distribution of \([\Delta YLD_t, \Delta R_t]\) with respect of all past values of variables and states is GED\(^6\). There is large evidence that Markov Switching models are strongly dependent on the initial values, and sometimes the results depend on their choice. Taking this into account, we construct a grid search of the initial values for some crucial parameters, namely the ones related to the distribution (\( \kappa \)) and the transition probability matrix (\( P \)). To obtain initial values of the parameters, we estimate a set of unconstrained and constrained linear regression of the variables and combine these estimates. Two grid methods were used, one that varies the values of the transition probabilities (from 0.500 to 0.999 in steps of 0.001) and the distributional parameter (from 0.25 to 2 in steps of 0.20) and another one that just varies the values of the transition probabilities (from 0.500 to 0.999 in steps of 0.001) with the distributional parameter fix to the value of 0.5 which corresponds to the assumption of the error term being normally distributed. In total, about four thousands initial values points are evaluated, and the point that returns the highest likelihood is picked to calculate the final estimates of the parameters. The standard errors of the estimates are obtained by the outer product of the scores as an estimator for the information matrix (see Davidson and MacKinnon (1993)).

\(^6\)The optimization algorithm for MLE is the secant update of the Hessian matrix, also known by Broyden-Fletcher-Goldfarb-Shano.
4 Markov Switching Granger Causality - Results

4.1 Estimation Results

We set the lags of the Markov-Switching VAR to \( h = 2 \), the same as the unrestricted VAR in Section 2.3. This was further supported by the Box-Pierce Q-test and by the literature, in particular Kapetanios (2001). The key estimated parameters are presented in Table 3 and the remaining estimates are reported in Appendix.

The parameters that dictate the temporary causality from stock market to sovereign bonds are \((\psi_1^{(k)})\) and from sovereign bonds to stock markets are \((\psi_2^{(k)})\). We find that \((\psi_1^{(k)})\) are significant for all countries, whereas \((\psi_2^{(k)})\) is significant only for France, Portugal and Spain. In all countries stock returns cause the sovereign bonds at some point in the sample and sovereign bonds cause stock returns at some point in France, Portugal and Spain. In Germany and Italy, the regime without a lead-lag relationship between stock returns and bond yields is expected to be very persistent. The expected duration of state with no-temporary causality, is more than 60 weeks for Germany and Italy, more than 30 weeks for Greece and Ireland, and between 8 and 20 weeks for the remaining countries.\(^7\) In all countries, either \(\varphi_{20}\) or \(\varphi_{10}\) are significant, meaning that \(\text{VIX}_t\) affected contemporaneously the two variables during the no-temporary causality periods. For Portugal, the expected duration of temporary causality was about 40 weeks, which is the longest. For the remaining countries it varied between from 5 to 30 weeks.

The table 4 shows the actual duration and its calculated by considering all smoothed regime probabilities, \(\mathcal{P}(S_t = \ell|X_{1-h}, ..., X_T; \hat{\Phi})\), that exceeds 0.85 (higher than the 0.5 threshold used by Hamilton (1989)). Then:

\[
AD_n = \sum_{i=1}^{n} I(\mathcal{P}(S_t = \ell|X_{1-h}, ..., X_T; \hat{\Phi}) > 0.85)
\]  

(5)

Where \(S_t = \ell, \ell = 1, \ldots, 4\) are the associated regimes, \(X_t = [\Delta R_t, \Delta YDL_t]\) and the function

\(^7\)Notice, this is a theoretical outcome from the transition matrix, and it is different than the actual causality.
Table 3: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>Germany</th>
<th>France</th>
<th>Spain</th>
<th>Italy</th>
<th>U.K.</th>
<th>Ireland</th>
<th>Portugal</th>
<th>Greece</th>
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<tbody>
<tr>
<td>Autoregressive Parameters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_{10}^{(1)}$</td>
<td>-0.1045</td>
<td>-0.0894</td>
<td>-0.1295</td>
<td>-0.0342</td>
<td>-0.0775</td>
<td>0.0262</td>
<td>0.0706</td>
<td>-0.0758</td>
</tr>
<tr>
<td>(0.01)</td>
<td>(0.07)</td>
<td>(0.00)</td>
<td>(0.24)</td>
<td>(0.05)</td>
<td>(0.33)</td>
<td>(0.17)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>$\phi_{10}^{(2)}$</td>
<td>-0.0725</td>
<td>-0.0879</td>
<td>-0.0676</td>
<td>-0.0221</td>
<td>0.0014</td>
<td>-0.1171</td>
<td>-0.1481</td>
<td>0.0110</td>
</tr>
<tr>
<td>(0.05)</td>
<td>(0.04)</td>
<td>(0.08)</td>
<td>(0.32)</td>
<td>(0.49)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.42)</td>
<td></td>
</tr>
<tr>
<td>$\phi_{11}^{(1)}$</td>
<td>0.0793</td>
<td>0.0611</td>
<td>-0.0314</td>
<td>-0.1214</td>
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<td>(0.25)</td>
<td>(0.32)</td>
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<td>(0.16)</td>
<td>(0.06)</td>
<td>(0.26)</td>
<td>(0.18)</td>
<td>(0.31)</td>
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<tr>
<td>$\phi_{11}^{(2)}$</td>
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<td>-0.0422</td>
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<td>0.0749</td>
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<tr>
<td>(0.40)</td>
<td>(0.37)</td>
<td>(0.04)</td>
<td>(0.29)</td>
<td>(0.09)</td>
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<td>(0.10)</td>
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<tr>
<td>$\phi_{20}^{(1)}$</td>
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<td>-0.1875</td>
<td>-0.1103</td>
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<td>(0.01)</td>
<td>(0.09)</td>
<td>(0.02)</td>
<td>(0.16)</td>
<td>(0.28)</td>
<td>(0.12)</td>
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</tr>
<tr>
<td>$\phi_{20}^{(2)}$</td>
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<td>-0.1806</td>
<td>-0.0583</td>
<td>-0.1189</td>
<td>-0.1379</td>
<td>-0.1889</td>
<td>0.0022</td>
</tr>
<tr>
<td>(0.44)</td>
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<td>(0.00)</td>
<td>(0.12)</td>
<td>(0.03)</td>
<td>(0.01)</td>
<td>(0.00)</td>
<td>(0.49)</td>
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</tr>
<tr>
<td>$\phi_{21}^{(1)}$</td>
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<td>-0.2135</td>
<td>-0.1931</td>
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<td>-0.1823</td>
<td>-0.2251</td>
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<td>(0.31)</td>
<td>(0.01)</td>
<td>(0.02)</td>
<td>(0.34)</td>
<td>(0.27)</td>
<td>(0.02)</td>
<td>(0.00)</td>
<td>(0.03)</td>
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<tr>
<td>$\phi_{21}^{(2)}$</td>
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<td>(0.23)</td>
<td>(0.27)</td>
<td>(0.13)</td>
<td>(0.18)</td>
<td>(0.27)</td>
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<tr>
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<td></td>
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<tr>
<td>$\psi_{1}^{(k)}_{R\rightarrow Y}$</td>
<td>-0.0061</td>
<td>-0.0108</td>
<td>-0.0076</td>
<td>-0.0011</td>
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<td>-0.0030</td>
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<tr>
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<tr>
<td>$\psi_{1}^{(2)}_{R\rightarrow Y}$</td>
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<td>(0.02)</td>
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<tr>
<td>$\psi_{1}^{(k)}_{Y\rightarrow R}$</td>
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<td>-0.2781</td>
<td>-0.6421</td>
<td>-0.4779</td>
<td>0.3573</td>
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</tr>
<tr>
<td>(0.18)</td>
<td>(0.44)</td>
<td>(0.44)</td>
<td>(0.34)</td>
<td>(0.37)</td>
<td>(0.21)</td>
<td>(0.03)</td>
<td>(0.23)</td>
<td></td>
</tr>
<tr>
<td>$\psi_{2}^{(2)}_{Y\rightarrow R}$</td>
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<td>(0.06)</td>
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<td>0.0147</td>
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<td>-0.3794</td>
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<td>(0.32)</td>
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<td>(0.00)</td>
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<td>$\varphi_{21}$</td>
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<td>0.9523</td>
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<td>1.0445</td>
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<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
<td>(0.00)</td>
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<tr>
<td>$ED_{R\rightarrow Y}$</td>
<td>16.871</td>
<td>9.732</td>
<td>26.410</td>
<td>20.555</td>
<td>5.434</td>
<td>33.056</td>
<td>29.077</td>
<td>43.887</td>
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<tr>
<td>$ED_{Y\rightarrow R}$</td>
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<td>23.784</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>27.220</td>
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</tr>
<tr>
<td>$\sum ED$</td>
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<td>14.549</td>
<td>39.308</td>
<td>20.555</td>
<td>5.434</td>
<td>33.056</td>
<td>41.289</td>
<td>43.887</td>
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<tr>
<td>$ED_{Y\leftrightarrow R}$</td>
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<td>8.617</td>
<td>32.766</td>
<td>11.591</td>
<td>34.453</td>
</tr>
</tbody>
</table>

The system of equations (2) and (3) with two lags is estimated with maximum likelihood, using 504 observations.

a The terms in the parenthesis are the p-values.
b ED stands for expected duration - How many weeks the state is expected to last.

$I()$ is an indicator function that attributes one when the inputs are greater than 0.85. We divide the sample in three periods: the financial crisis, the European debt crisis and the
Table 4: Actual Duration

<table>
<thead>
<tr>
<th></th>
<th>Financial Crisis</th>
<th></th>
<th>European Debt Crisis</th>
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<td>$\Delta R_t$</td>
<td>$\Delta YDL_t$</td>
<td>$\Delta R_t$</td>
<td>$\Delta YDL_t$</td>
<td>$\Delta R_t$</td>
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Note: The actual duration is measure in weeks. a 11/07/2008 to 27/11/2009. b 04/12/2009 to 26/10/2012. c 06/03/2015 to 10/11/2017

Quantitative Easing.

In all the countries, the stock market caused the bond markets during some periods. With the exception of Ireland, the actual duration of this regime was longer during the financial crisis. The actual duration of the regime in which bond markets caused stock markets in France, Spain and Portugal, was also longer during the financial crisis. This finding is consistent with those from Andersson et al. (2008) who finds a negative correlation between stocks and bond prices during periods of stock market uncertainty, maybe driven by a "flight-to-quality" phenomenon. We also find a negative correlation, shown by the signs of the temporary causality in the table 3, in the "Causal Parameters" rows at lag one of $\psi_1^{(1)}$.

4.2 Smoothed Probabilities

We aggregate the smoothed probabilities according to the temporary causality to provide a clearer interpretation. The probability of being in the regime where stock markets temporarily cause sovereign bond yields is shown in Figure 1 and the probability of being in the regime where sovereign bond yields temporarily cause stock market returns is shown in Figure 2. The criteria we have adopted to defined the direction follows the statistical significance, at 10 percent, of $\psi_1^{(k)}$ and $\psi_2^{(k)}$, which are the estimates that control the temporary causality. Note that if $\psi_1^{(k)}$ is not significant, regime 1 nests the regime 2, and regime 3 nests
the regime 4.

Our results indicate that the global or idiosyncratic shocks coincide with reversals of temporary causality between stocks returns and sovereign bonds. We can observe in Figure 1 that the plots are similar for Germany, France, Spain, Italy and the U.K., in particular during the financial crisis. The main difference, in this case, lies in the actual duration of the regime. As we can also see in Table 4, the actual duration is shorter for the U.K. and Italy. However, the trigger of this temporary causality occurred at the same time for those countries. Another interesting fact is related to the quantitative easing program conducted by the ECB. The QE program triggers the reverse in the temporary causality in both core countries (Germany and France).

The European debt crisis also seems to have triggered the temporary causality in the direction towards the sovereign bonds, except for the U.K. This indicates the absence of contagious to the UK from the stock market uncertainty period in the Eurozone. Some of the remaining shifts appear to be related to idiosyncratic turmoil, for instance the Brexit referendum, that triggered a short lived change of causality in the U.K., but not in the other European countries.

In Portugal, Greece and Ireland, our results link changes in regimes to idiosyncratic events, namely the financial crisis in these countries. These patterns are consistent with other studies that have found that those countries have detached from the "core" countries during this period. Fontana and Scheicher (2016) shows that the CDS-Bond spread in these countries turned negative between 2009 and 2010 and from 2011 onwards. In the Irish case, one reasonable explanation is that the CDS ban has played an essential role in this temporary causality pattern. After the ban, the investor shifted their hedge position to yields. Sambalaibat (2014) documents that a European Commission CDS ban has decreased bond market liquidity, consequently it has reduced the bond prices. The ban might have led the sovereign bonds yields to take over the role of assessing the probability of Ireland debt default.

Turning now to the inverse temporary causality, where sovereign bonds cause stock re-
Figure 1: Smooth Probabilities for Temporary Granger Causality ($\Delta R_t \rightarrow \Delta YLD_t$)
Figure 2: Smooth Probabilities for Temporary Granger Causality ($\Delta YLD_t \rightarrow \Delta R_t$)
turns, it only occurs in three countries: Portugal, Spain and France. The _temporary_ causality pattern is similar during the financial crisis. It is also similar during the European sovereign debt crisis and the period of Quantitative Easing, with countries diverging in the actual duration. This finding shows the importance of our methodology, as the usual methods applied to the whole sample or different ad-hoc sub-samples do not capture these patterns, as shown in Table 2. More importantly, this finding contradicts the previous literature that measures the causality between the country’s credit and market risks, that mainly found the causality from market risk to sovereign risk. One exception is Coronado et al. (2012) that found that in 2010, the CDS took the lead over the stock returns. Nonetheless, with sovereign bond yields, our results indicate that only had happened in Portugal, Spain and France. For the remaining countries other than these three, the probability of having this _temporary_ causal relationship is zero because $\psi_2^{(k)}$ is not significant all lags.

### 4.3 Contagion

Our results also reveal some insights on contagion. Forbes and Rigobon (2002) define contagion as the increase in the economic co-movements during crisis periods. We adopt a similar approach to Ge (2020), interpreting the rise of the co-movements in the smoothed probability during an economic crisis as contagion. From the plots in Figures 1 and 2, we can identify when the co-movements started and which countries have first risen its smoothed probability into a specific regime.

Take the regime where stock markets _temporarily_ cause sovereign bond yields, during the Financial Crisis, Portugal and Greece have entered this regime first, followed by the UK. Germany and Italy entered in the same week, and in two weeks later France and Spain. The co-movements of all countries have happened within seven months of the beginning of our sample. During the European Debt Crisis, Germany, France, Spain and Italy have entered into this regime simultaneously. However, the regime lasted longer for Italy and Spain, as we observe in table (4).

In the regime where sovereign bonds _temporary_ cause stock returns, at the onset of the
Financial Crisis, Spain has entered in this regime first and then was followed by France and Portugal. For most the second half of the Financial Crisis, these countries were in the same regime. During the European Debt Crisis, the co-movement stated simultaneously after the second week of May of 2010 and lasted for five months, and recurred from the third week of August of 2011 to the first week of November of 2011.

5 Conclusions

We have studied the causality between stock returns and changes of the sovereign bond, using the weekly data from 2008 and 2018 from eight European countries. We employ a standard methodology based on a VAR model to analyse the country-specific lead-lag relationship for the whole sample, and an approach based on Markov-switching causality to determine the dates of reversals of the causality endogenously. We consider the stock returns as reflecting the economic environment of a country (or market risk indicator) and the sovereign bonds a market current perception of country default risk (sovereign risk indicator). To this end, the temporary causality can be interpreted as a propagation mechanism from one market to another.

We draw three conclusions from our analysis. First, we find the exact dates when there are shifts in the temporary causality direction. Alongside the shifts in temporary causality, we find actual duration of the regimes are country specific, a sign of asymmetry of how shocks are absorbed by the two markets. Second, an idiosyncratic crisis from a peripheral country has limited strength to define the temporary causality elsewhere. Idiosyncratic crisis drives changes in the peripheral country’s (Ireland, Greece, and Portugal) actual temporary causality patterns, but not seem to affect core countries, Germany, France and the U.K., as well as Italy. These are affected by a global (or regional)-Systemic crisis, which reflects their economic stability. By focusing on the smoothed probabilities, we find that the main difference of temporary causality patterns of the core countries are the actual durations. Still the starting points are often the same and coincide with a global (or regional) crisis.
The evidence of causality from sovereign bond yields to stock market returns is weaker and limited to France, Portugal and Spain. From the perspective of price discovery, we infer that during a systemic crisis, such as the Financial and European debt crisis, stock returns appear to be more informative, but the importance of the sovereign bond yields can not be neglected. In some periods of more idiosyncratic crisis, sovereign risk might lead.

References


6 **Appendix**

6.1 **Data**

1. Sovereign (government) bond yields: The 10-year sovereign bond yields are obtained from the Datastream. The weekly data are generally for the last trading day of the week. Changes are defined as the first differences of the sovereign bond yields. ($\Delta YLD$)

2. Stock Index Returns: The stock market returns are the weekly weighted return obtained from the Datastream. Stock returns are defined as the logarithmic changes of the stock index and the numbers have been multiplied by 100 to express the index’s return as a percentage. ($R_t = \log \text{of Stock Price}$) ($\Delta R_t = \text{Stock Returns}$)
Figure 3: Changes in Sovereign Bond Yields, All Countries
Figure 4: Stock Returns, All Countries
### 6.2 Temporary Granger Causality - Additional Results

Table 5: Estimation Results - Mean and Probabilities Parameters

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Note: The terms in parenthesis are the p-values. $\kappa$ is the distributional parameter, and $\sigma_{11}, \sigma_{22}$ and $\sigma_{12}$ defines $\Sigma$ and both are regime dependent.