**Developmental Dictatorship and Middle Class-driven Democratisation**

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Developmental Dictatorship and Middle Class-driven Democratisation*

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**Abstract**

I investigate the motives behind economic growth under a dictatorship, exploring the trade-off between pursuing higher future gains, which come with growing threats from the demand for democracy from the emerging middle class, and accepting lower gains for a relatively more stable regime. I propose a model where a dictator invests and acquires a rent, citizens educate their children for skilled jobs, and these children adopt democratic values through education. I find that a dictator invests in an underdeveloped economy for future gains. As the economy matures, investment decreases because more citizens get democratic values from higher education. Democracy follows an opposite investment trend: little investment is made when the economy is underdeveloped, but more investment is made as it develops. The analysis is generalised to cases where the dictator is legitimised by higher economic growth than in democracies, and where the dictator oppresses the middle class through high taxation.

**Keywords**: Dictatorship, Growth, Democratisation, Middle Class, Democratic Values
**JEL Classification**: D02, D72, O12, O43

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## 1 Introduction

Social scientists have long debated whether democracies or dictatorships are more prone to economic growth. While no consensus has yet been reached, recent studies point to positive results for democracy (Acemoglu et al., 2019; Colagrossi et al., 2020; Madsen et al., 2015). Notwithstanding, the evidence is clear that some dictatorships pursue economic development. The four Asian tigers, for example, demonstrated remarkable economic growth, with an annual growth rate of over 6 percent for three decades. Several impoverished nations have escaped poverty under the rule of pro-growth dictators (Glaeser et al., 2004) and, more interestingly, some autocracies have achieved faster economic growth than democracies over many periods (Luo and Przeworski, 2019). Why these developmental dictatorships pursue economic growth is not immediately apparent, especially if they have in mind the *modernisation hypothesis*: As economic development industrialises the society and provides higher incomes, people will undergo a series of social and cultural changes that lead to the adoption of democratic institutions (Lipset, 1959).

Central to the modernisation process is the emergence of the middle class, a key protagonist in democratic transitions since the advent of modern democracy in the nineteenth century. As underscored by Moore (1966) with his assertion that "no bourgeoisie, no democracy", the role of the middle class persists in contemporary contexts. The expansion of the urban middle classes consistently contributed to mass mobilisation during the "third wave" and, more recently, during the Arab Spring in Egypt and Tunisia (Huntington, 1993; Haggard and Kaufman, 2016). This newly emerged class receives higher education in a more stable environment, fostering a generation with heightened demands for democratic rights and institutions, thereby cultivating a democratic political culture.[1] Historical evidence demonstrates that a well-educated citizenry is associated with the emergence of broad-based opposition groups and popular uprisings against monarchies, leading to the downfall of numerous European monarchies. Similar dynamics have been observed in other regions, including East Asia, the former Soviet Union, and Eastern Europe, culminating in the overthrow of dictatorial regimes (Glaeser et al., 2007). To understand developmental dictatorships, therefore, it is important to explore the relationship between economic growth and the emergence of a middle class.

In this paper, I analyse growth under a dictatorship with an emergence of middle class as a catalyst for democratic transitions. Dictators in poor countries face limitations in extracting rents due to a scarcity of economic resources. Consequently, some dictators may opt to forego immediate rent-seeking and instead invest in the anticipation of larger future rents. While pursuing economic development can potentially bring greater affluence to a dictator, it also exposes him to growing demands for democratisation from a burgeoning middle class. As a result, a natural trade-off emerges between extracting tiny rents from a politically stable regime and obtaining

[1] Banerjee and Duflo (2008) finds that the middle class tends to have fewer children and invest more in their education and health, possibly due to their job security and the sense of control over their future. According to Inglehart and Welzel (2005), individuals who grew up with less education, economic insecurity, and physical insecurity tend to internalise materialistic values, which are associated with xenophobia and authoritarianism. Conversely, those who have grown up with higher levels of education and stable financial and physical circumstances are more likely to embrace post-materialistic values, which are aligned with egalitarian norms and democratic political cultures.

substantial rents from a resource-abundant, yet politically insecure, regime.

I construct an overlapping generations model with a dictator, where the dictator considers current and expected rents in the next period, decides how much to invest and extract rents, and remains in power until the transition to democracy occurs. In overlapping generations, parents provide education to bequeath a skilled job to their children, and democratic values emerge naturally from education.[2] Young citizens participate in collective action for democratisation based on a global game framework. Among them, those who adopt democratic values from education have a stronger demand for democracy.

I find that the dictator's optimal strategy is to invest more when the economy is underdeveloped, and to invest little or nothing when it is developed. In underdeveloped economies, fewer people adopt democratic values due to lower levels of education. As a result, the dictator is more likely to stay in power and has the opportunity to increase future profits through investment. However, as the economy grows and more skilled workers are employed, the average level of education rises and more people adopt democratic values and actively participate in collective action. Faced with a higher probability of losing power, the dictator prioritises immediate gains over long-term investment. The dictator therefore chooses to invest little or nothing. This investment strategy contrasts with that of a democracy. Based on Persson and Tabellini (2021)'s probabilistic voting model, investment is low when the economy is underdeveloped and high when the economy is developed. This is because economic growth, coupled with higher levels of education, fosters a society with a greater number of policy-oriented democratic citizens who act as a deterrent to rent-seeking behaviour by politicians.

This prediction offers valuable insights into the incentives and desired trajectories of economic growth that growth-oriented dictators aim to pursue: the optimal strategy involves a delicate balance between material gains and maintaining political stability. It extends the theoretical framework of long-lasting dictators with vested interests in economic performance and aligns with empirical evidence indicating that political instability can hinder economic growth (Olson, 1993; Alesina and Perotti, 1996; Przeworski et al., 2000; Aisen and Veiga, 2013). And, more importantly, it illuminates the empirical puzzle surrounding the regime and economic growth: Democratic economies exhibit faster growth when certain conditions are met (for example, participatory culture (Rodrik, 2000), human capital (Doucouliagos and Ulubaşoğlu, 2008), and secondary education (Acemoglu et al., 2019)), but some autocracies grow faster than all democracies in low-developed stages (Luo and Przeworski, 2019).

I offer two extensions to the model. First, I incorporate the legitimacy concerns of the dictator. Establishing legitimacy is of paramount importance for dictatorships, as one of the main triggers for the collapse of such regimes is the erosion of their legitimacy (Przeworski, 1986). Dictatorships often seek economic growth as a means of justifying their rule. Therefore, I model legitimacy as follows: if the dictatorship delivers higher economic growth than democracy, the public's demand for democratisation is not rooted in economic reasons; rather, they demand democratisation solely for political ones. As a result, investment is seen as crucial to maintaining legit-

[2] I assume that individuals possess either materialistic or democratic values, and that democratic values are correlated with higher levels of education.

imacy and increasing future revenues. This extension captures contradicting features inherent in the modernisation hypothesis: economic development stabilises the regime while increasing the probability of democratisation due to increased political mobilisation for political liberties (Kennedy, 2010).

The results confirm the pattern previously observed for underdeveloped economies: investment initially increases and then declines as the demand for regime change increases. When investment becomes pivotal in maintaining legitimacy, the dictatorship allocates the necessary resources to meet these legitimacy demands, resulting in increased investment. However, the dictatorship chooses not to invest further after a certain level of development because the costs associated with maintaining legitimacy are too high for the regime.

Second, I extend the analysis by examining the dictator's strategy of wage suppression through the introduction of a high tax rate. It is a common phenomenon in autocratic regimes that wage levels are significantly lower than in democracies (Przeworski et al., 2000). Intuitively, dictators may choose to impose higher tax rates in response to the potential threat posed by a burgeoning middle class. By doing so, they can curb the growing demand for democracy by limiting the resources available for education and social mobility, despite the cost of economic inefficiency. I therefore endogenise the tax rate, allowing the dictator to choose the implemented tax rate in the next period with a quadratic deadweight loss. The results indicate that the implemented tax rate exceeds the revenue-maximising tax rate and increases with the level of economic development, which results in a decrease in the revenue for the dictator. This provides a rational explanation of policy making under dictatorships that induces economic inefficiency.

This paper proceeds as follows: In Section 2, I briefly review the related literature on the idea of middle class-driven democratisation and dictatorial decision-making. In Section 3, I describe democratic transitions through collective action by the public. In Section 4, I describe an overlapping generations economy with a dictator, which includes education choice by parents, collective action by young citizens, and the dictator's investment decision. In Section 5, I describe the economy after democratisation and compare the investment between the two regimes. And I extend the model by incorporating legitimacy from economic growth in Section 6.1 and by endogenising the tax rate, which describes wage suppression, in Section 6.2.

## 2 Contributions to the Related Literature

This study contributes to the literature on formal models of dictatorships by elucidating a potential mechanism through which dictatorships can foster economic growth. Prior research discusses dictatorships in enhancing regime stability by repression (Tyson, 2018; Dragu and Przeworski, 2019; Gitmez and Sonin, 2023), by power-sharing (Svolik, 2009; Boix and Svolik, 2013), and by control of information (Edmond, 2013; Shadmehr and Bernhardt, 2015; Guriev and Treisman, 2019, 2020). It also explores how dictatorships weigh competence and regime stability: appointment of subordinates (Egorov and Sonin, 2011; Zakharov, 2016), acceptance of free media (Egorov et al., 2009). However, to the best of my knowledge, few theoretical models have delved into the study of dictatorships with a focus on the interplay between economic growth and regime stability, particularly examining why certain dictatorships accommodate rapid economic growth. I analyse how the dictatorship may balance growth and stability in its own interest, suggesting a rapid development made in underdeveloped economies.

The rise of the middle class is crucial to the fall of dictatorship in this model. The middle class supports democratic ideals and works against authoritarian regimes (Luebbert, 1991; Huber et al., 1993; Huntington, 1993; Glassman, 1995, 1997). Haggard and Kaufman (2012) demonstrates that the demands of the new social classes, the emerging bourgeoisie and the urban working class, played an important role in the gradual extension of the franchise. The impact of the emergence of a middle class on democratic institutions depends on education: Education can change political culture and create a conducive environment for civil society to flourish, leading to the establishment of democracy. This idea can be traced back to de Tocqueville (1835), who argued that widespread education in America was key to a flourishing of democracy; several studies examine the empirical plausibility of this idea (Kam and Palmer, 2008; Berinsky and Lenz, 2011; Mayer, 2011). On transitions to democracy, Bourguignon and Verdier (2000) predict that a more equal distribution of education leads to earlier democratisation in oligarchic societies. Glaeser et al. (2007) explains that education increases political participation, leading to transitions. And Murtin and Wacziarg (2014) find that primary education and per capita income lead to democratisation.

Higher education from the emergence of the middle class makes citizens more likely to embrace democratic values, which is the main difference between this study and others in education and democratisation. This assumption is related to the formation of post-materialistic values (Inglehart and Baker, 2000; Inglehart and Welzel, 2005; Inglehart, 2017, 2018) and the emphasis of rich voters on values rather than material interest (Enke et al., 2023). The evolution of democratic values is relevant to the cultural transmission literature (Bisin and Verdier, 2001, 2011) in that parents bequest values through education.[3] This study also contributes to the recently emerging literature on culture and institutions (Tabellini, 2008b; Bisin and Verdier, 2023b; Besley and Persson, 2019) by examining the emergence of democratic values in democratisation. In particular, this study is closely related to Besley and Persson (2019) who highlight how democratic values beget democratic institutions. The main difference is that this study shows how democratic values arise from economic change rather than the values evolved by itself.[4] Previous studies discuss that the democratic values are a pivotal component of economic growth under democratic institutions (Putnam et al., 1992; Rodrik, 2000; Persson and Tabellini, 2009). This study suggests that democratic values have contrasting effects in dictatorships compared to democracies.

Finally, this study contributes to the literature on formal theoretical models of democratic transitions. Boix (2003) and Acemoglu and Robinson (2000, 2001, 2006) focus on class conflict and argue that elites must democratise by expanding the franchise to counter the threat of revolt from the poor for redistribution. Leventoğlu (2014) extends the framework of Acemoglu and

[3] The literature on cultural transmission assumes that parents educate their offspring to bequeath their values, whereas this study assumes that parents are motivated to bequeath skilled jobs.

[4] Lowes (2022) points to the need for future discussion of the interaction between culture and economic policy. This study contributes by illustrating the dynamic interplay between culture, institutions and economic factors.

Robinson (2006) to discuss social mobility in democratisation. Lizzeri and Persico (2004) find that the cause of enfranchisement is in the demand from provision of public goods. The existing literature mainly looks at democratisation from the perspective of economic interests. However, as Tabellini (2008a) points out, there are limitations to explaining institutional change solely in terms of economic incentives. In this respect, this study contributes to the literature by focusing on an increase in the demand for democracy itself as a cause of democratisation, and explaining that this demand for democracy is driven by an increase in education as a result of economic change.

## 3 Democratic Values and Transitions to Democracy

In this section, I develop a model of democratisation where transitions arise from the distribution of citizens' value type. Suppose that there is a continuum of mass 1 of citizens. Each young citizen $i \in [0, 1]$ in period $t \in \mathbb{N}$ has the type equal to value $v_{it} \in \{d, m\}$, where $d$ and $m$ represent "democratic" and "materialistic," respectively. Let $\bar{d}_t = \int_0^1 \mathbf{1}[v_{it} = d]di$ denote the proportion of democratic citizens in period $t$. Assume for now that the proportion of democratic citizens $\bar{d}_t$ is exogenously given, which will be endogenised in section 4.

A young citizen $i$ decides whether to participate $a_{it} = 1$, or not $a_{it} = 0$, in collective action. Participation in general is costly because citizens are concerned about suffering from dictatorial repressions as a result of their participation. I assume that democratic citizens have a lower participation cost than materialistic citizens. That is, $0 < b_d < b_m < 1$ where $b_d$ and $b_m$ are the participation cost for democratic and materialistic citizens, respectively. Denote the average participation cost as $\bar{b}_t = \bar{d}_t b_d + (1 - \bar{d}_t)b_m$.

Regime change is desirable for all citizens: Participants earn positive payoffs when the collective action succeeds.[5] When it fails, they receive negative payoffs due to participation cost. And those who do not participate in the collective action get zero. The preferences of a young citizen $i$ are represented by $\{\mathbf{1}[M_t > 1 - \theta_t] - b_{it}\}\, a_{it}$ where $\mathbf{1}[\cdot]$ is an indicator function. Collective action is successful and the regime changes if the mass of participants $M_t = \int_0^1 a_{it}di$ exceeds a threshold $1 - \theta_t$ and fails otherwise. Here, $\theta_t$ represents the regime vulnerability, which is independent and identically distributed for all $t$ and follows a uniform distribution with its domain $[\underline{\theta}, \bar{\theta}]$ where $\underline{\theta} < -\sigma$ and $\bar{\theta} > 1+\sigma$ and $\sigma \in (0, 1/2]$. When $\theta_t \geq 1$, the regime naturally collapses on its own, while $\theta_t \leq 0$ means that there is no hope of removing the dictator from power through collective action. The mean $\mathbb{E}[\theta_t]$ is assumed to be between 0 and 1.

Citizens do not have prior information about $\theta_t$. Instead, they receive a private signal $s_{it} = \theta_t + \sigma\varepsilon_{it}$ where the random error $\varepsilon_{it}$ follows a uniform distribution on $[-1, 1]$ and is independent and identically distributed for all $i \in [0, 1]$ and $t$. Based on the signal received, they construct a belief about the realisation of $\theta_t$, make an inference about the beliefs of others, and decide whether to participate ($a_{it} = 1$) or not ($a_{it} = 0$).

I describe the unique equilibrium and explain how the likelihood of transition depends on

[5] In section 6.1, I extend the discussion that some citizens may not want the transitions to democracy if the dictatorship provide higher economic growth than democracies.

the population of democratic citizens. It is well known from the global games literature that the cutoff-type strategy is the unique Bayes-Nash equilibrium in the participation choice, and the heterogeneity in the participation cost between democratic and materialistic citizens results in different cutoffs in the equilibrium (Sakovics and Steiner, 2012).

**Proposition 1.** *There is a unique Bayes-Nash equilibrium such that each young citizen with signal $s$ and value $v \in \{d, m\}$ follows a cutoff strategy:*

$$a_{it}(s, v) = \begin{cases} 1 & \textit{if} \quad s \geq s_t^*(v) \\ 0 & \textit{if} \quad s < s_t^*(v) \end{cases} \tag{1}$$

*where $s_t^*(d) = \sigma(2b_d - 1) + \bar{b}_t$ and $s_t^*(m) = \sigma(2b_m - 1) + \bar{b}_t$.*

Proposition 1 says that each citizen participates in the collective action only when their signal is above a certain type-specific cutoff. This cutoff is lower for materialistic than for democratic type, which is due to the difference in participation cost. Intuitively, materialistic citizens need more confidence than democratic citizens to join the collective action, as participation is more costly to them. Also, both cutoffs are decreasing in $\bar{d}_t$, which implies that both types are more likely to participate if there is a higher proportion of democratic citizens. Because of these positive externality, the proportion of democratic individuals is crucial for the regime's survival. When the precision of the citizen's signal improves, that is, $\sigma \to 0$, the equilibrium cutoffs for both democratic and materialistic citizens converge to the same cutoff, which is the average participation cost $\bar{b}_t$.

What is the relationship between the population of democratic citizens and the *ex-ante* likelihood of collective action success, and how is the regime vulnerability $\theta_t$ related to the regime stability and regime change? Proposition 2 answers these questions by proposing a threshold for regime vulnerability, above which the regime is to be overthrown.

**Proposition 2.** *Collective action succeeds if $\theta_t \geq \bar{b}_t$ and the regime remains a dictatorship if $\theta_t < \bar{b}_t$. The ex-ante probability of collective action success $\Pr[M_t \geq 1 - \theta_t | \bar{d}_t]$ is*

$$\Pr[M_t \geq 1 - \theta_t | \bar{d}_t] = \frac{\bar{\theta} - \bar{b}_t}{\bar{\theta} - \underline{\theta}}. \tag{2}$$

According to proposition 2, the average participation cost serves as the threshold for regime change. And the precision of the signals $\sigma$ does not matter for the probability. The result indicates that an increase in the proportion of democratic citizens lowers the average participation cost, thereby reducing the likelihood of the regime's survival.

## 4 Economic Growth and Evolution of Democratic Values

Having modelled the regime change, I build an overlapping generation model with a dictator to describe how the proportion of democratic citizens evolves from the economic growth. A continuum of citizens with unit mass is born in each period and live for only two periods. I call citizens in their first and second periods "young citizens" and "parents" respectively. Each young citizen $i \in [0,1]$ acquires education from parent $i$, becomes either a democratic or materialistic type, and decides whether to participate in the collective action. In the subsequent period, she earns wages, consumes for herself, and educates her offspring. The dictator compares extracting the immediate rent to obtaining greater stakes in the next period by promoting economic growth, and remains in power until the collective action succeeds.

The economy begins with an initial level of infrastructure $A_1 > 0$ and infrastructure is accumulated according to $A_{t+1} = (1-\delta)A_t + I_t$ where $\delta \in (0,1]$ is the depreciation rate and $I_t$ is the investment of the dictator in period $t$. The production of industrial economy in period $t$ is

$$Y_t = 2\pi_h \sqrt{A_t} q_t - \frac{q_t^2}{\varphi} \tag{3}$$

where $q_t \in [0, \bar{q}]$, for $\bar{q}$ less than 1, is the proportion of skilled labour occupation, $\pi_h$ is a production parameter for skilled labour, and $\varphi$ is a social cost parameter that is an inverse measure of the cost of providing high-skilled labour jobs.[6] Here, $q_t^2/\varphi$ reflects social costs incurred by structural change in the economy, such as issues related to urbanisation with housing, congestion and crime. Skilled jobs are provided by competitive firms, so skilled occupations $q_t = \varphi\pi_h\sqrt{A_t}$ are provided with the pre-tax wage $\pi_h\sqrt{A_t}$ for $q_t \leq \bar{q}$. This means that an increase in infrastructure increases the share of skilled workers. Those who do not get a skilled occupation work as unskilled labour in the rural economy, with wages $w_{lt} = \pi_l\sqrt{A_t}$ and $\pi_h > \pi_l > 0$. [7]

The dictator imposes an exogenous proportional income tax $\tau \in (0,1)$ only on the industrial economy. And the production in the rural economy remains untaxed.[8] The government revenue $G_t$ is $\tau Y_t$. So, it is indispensable for the dictator to modernise the economy to enjoy more resources at his disposal. Post-tax income for skilled workers $w_{ht}$ is $(1-\tau)\pi_h\sqrt{A_t}$. I assume that the tax rate satisfies $0 < \tau < 1 - \pi_l/\pi_h$, i.e., the post-tax income for skilled workers is greater than the income for unskilled workers.

From the government revenue, the dictator makes investment decision that maximises his preferences which constitute immediate rent gain and expected rent gain in the subsequent period. He does not know the realisation of the regime vulnerability $\theta_t$, but knows the distribution. The preferences of the dictator in period $t$ are represented by

$$\{G_t - \kappa I_t\} + \beta \Pr[M_t < 1 - \theta_t | \bar{d}_t] \{G_{t+1} - \kappa I_{t+1}\} \tag{4}$$

where $\beta \in (0,1]$ is a time discount rate and $\kappa > 0$ is the cost of generating 1 unit of infrastructure.[9] And I say that an investment $I_t$ is *feasible* if $\kappa I_t \leq G_t$. Note that if $\kappa$ is greater than the

[6] I focus on $q_t$ that is smaller than 1 for both technical and realistic reasons: Even if the society becomes highly industrialised, it is impossible for everyone to work in an industrial sector. And technically, if every citizen can get a skilled job, there will be no incentives to provide education. Thus, I denote by $\bar{q} \in (0,1)$ the maximum proportion of skilled jobs.

[7] Note that unskilled wages in the rural economy is also affected by the level of infrastructure, which can be interpreted as a diffusion of technology and wealth.

[8] This exogenous tax rate is endogenised in section 6.2.

[9] The dictator's preferences can be interpreted in several ways: First, the limited lifespan and the turnover of

marginal gain from investment when $\bar{d}_t$ is the lowest, the dictator never invest; in contrast, if $\kappa$ is lower than the marginal gain when $\bar{d}_t$ is the highest, the dictator will invest the entire budget regardless of the circumstances. I assume that $\kappa$ is between these two extremes:

**Assumption 1.** Unit infrastructure cost $\kappa$ satisfies

$$\beta \left\{ \frac{\bar{b}_t(\eta) - \theta}{\bar{\theta} - \underline{\theta}} \right\} \tau \varphi \pi_h^2 < \kappa < \beta \left\{ \frac{b_m - \theta}{\bar{\theta} - \underline{\theta}} \right\} \tau \varphi \pi_h^2. \tag{5}$$

Here, $\bar{b}_t(\bar{d}_t)$ is the average participation cost when the proportion of democratic citizen is $\bar{d}_t$ and $\eta \in (0, 1]$ is the upper bound of $\bar{d}_t$. From the given economic circumstances, each parent gets either skilled or unskilled wages and educates their offspring. Let $e_{it}$ denote the provision of education by parent $i$ to young citizen $i$ and $\bar{e}_t := \int_0^1 e_{it} di$ the average education in period $t$. The preferences of a parent $i$ are represented by

$$\left\{ w_{it} - \frac{e_{it}^2}{2} \right\} + \mu \mathbb{E}[w_{it+1} | e_{it}, \bar{e}_t, q_{t+1}] \tag{6}$$

where $\mu > 0$ is an empathy parameter. The first term indicates consumption and the second term parents' empathy to their offspring: parents are more satisfied when their children's expected wage is higher. Each parent is constrained by $e_{it}^2/2 \leq w_{it}$. The probability of getting a skilled job depends on the number of jobs created in the next period, citizen's education achievement, and the average education. That is, the number of available skilled labour jobs $q_{t+1}$ is rationed by the relative level of education. Define the probability that young citizen $i$ gets skilled job in the next period, $h(e_{it}, \bar{e}_t, q_{t+1})$, as

$$h(e_{it}, \bar{e}_t, q_{t+1}) = h_1(e_{it}, \bar{e}_t, q_{t+1}) + \{1 - h_1(e_{it}, \bar{e}_t, q_{t+1})\} h_2(e_{it}, \bar{e}_t, q_{t+1}) \tag{7}$$

where

$$h_1(e_{it}, \bar{e}_t, q_{t+1}) = \begin{cases} 1 & \text{if } e_{it} > 0 \text{ and } \int_0^1 \mathbf{1}[e_{it}] di < q_{t+1}, \\ \min\{q_{t+1}(e_{it}/\bar{e}_t), 1\} & \text{if } e_{it} > 0 \text{ and } \int_0^1 \mathbf{1}[e_{it}] di \geq q_{t+1}, \\ 0 & \text{if } e_{it} = 0 \end{cases} \tag{8}$$

and $h_2(e_{it}, \bar{e}_t, q_{t+1}) = \{q_{t+1} - \int_0^1 h_1 di\}/\{1 - \int_0^1 h_1 di\}$. In words, the function $h_1$ means that (a) if the number of educated citizens is fewer than the available high-skilled wage jobs, citizens with education get high-skilled wage jobs; (b) if the number of educated citizens exceeds the supply of high-skilled wage jobs, citizen $i$ with double the education level of citizen $j$ is twice as likely to become a high-skilled wage worker compared to citizen $j$; and (c) if a citizen is not educated, she does not receive a high-skilled wage job. The function $h_2$ means if any high-skilled wage jobs remains after hiring from $h_1$, they are allocated to those who did not get the high-skilled wage

dictators within the inner circle give rise to two-period concerns in the decision-making process. Second, there is a common trait among dictatorships to prioritise short-term outcomes over long-term planning. For example, authoritarian regimes often implement short-term economic plans.

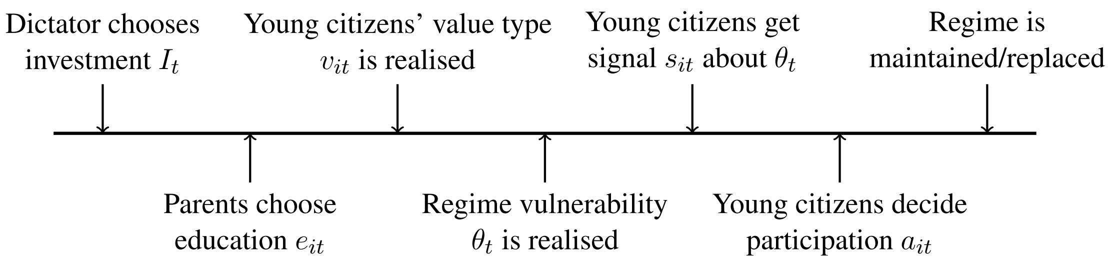



Figure 1: Timeline of events in period $t$

jobs with equal probability.[10]

In addition to better job opportunities, a higher level of education increases the likelihood of young citizens embracing democratic values. The probability of embracing democratic values given education is defined as $\Pr[v_{it} = d|e_{it}] = \min\{e_{it}^2, \eta\}$. The upper bound $\eta$ captures the possibility that a citizen with higher education may not embrace democratic values. The literature on cultural transmissions (Bisin and Verdier, 2001, 2023a; Tabellini, 2008b) also explored how values evolve through education. The main difference is that the literature finds educational motivations from instilling specific values in their children, while this study assumes that education decisions are primarily driven by economic factors.

The evolution of values from parents to children through education has been discussed in the literature on the cultural transmissions. The main distinction from the literature lies in their assumption that parents intentionally instill specific values in their children, whereas I posit that parental decisions are primarily driven by economic factors.

The timing of events is described as follows (see Figure 1):

(i) Given the government budget $G_t$, dictator chooses investment $I_t$.

(ii) After observing $I_t$, each parent $i \in [0, 1]$ receives wages $w_{it}$ and educates $e_{it}$ the offspring. Young citizens become either democratic or materialistic from the education.

(iii) Nature chooses the regime vulnerability $\theta_t$, each young citizen receives a private signal $s_{it}$ about $\theta_t$ and decides whether to participate in collective action.

(iv) If the collective action is successful, democracy begins from period $t + 1$; otherwise the dictator maintains power in period $t + 1$.

In equilibrium, the dictator's investment decision is the optimal action given the best responding action choices of parents' education provision and young citizens' participation in the collective action given their realised value type and signal about the regime vulnerability.

[10] To check whether the distribution rule $h$ works, $0 \leq \int_0^1 h_1 di \leq q_{t+1}$ because $\int_0^1 \min\{q_{t+1}\{e_{it}/\bar{e}_t\}, 1\} di \leq \int_0^1 q_{t+1}\{e_{it}/\bar{e}_t\} di = q_{t+1}$. And $0 \leq h_2 \leq 1$ and $h_2 = 0$ only when $\int_0^1 h_1 di = q_{t+1}$. Therefore, $\int_0^1 h di = q_{t+1}$, which means that all high-skilled labour jobs are distributed according to the education.

**Parental Education**

In this model, education has no real effect on production, but is a means of allocating scarce skilled jobs. Even if education does not accumulate human capital, parents' preference to inherit a skilled occupation leads them to invest in education, which is the only way to bequeath:

**Lemma 1.** *There is no equilibrium such that $e_{it} = 0$ for any $i \in [0, 1]$.*

Then how much do they spend for the education in equilibrium? I analyse each parent's choice of education and how it depends on the current and future economic conditions. For convenience of analysis, I introduce the following assumption:

**Assumption 2.** Empathy parameter $\mu$ satisfies

$$\frac{\eta\varphi\pi_h}{\bar{q}^2\left\{(1-\tau)\pi_h - \pi_l\right\}} < \mu < \frac{2\kappa\pi_l}{(1-\tau)\pi_h - \pi_l}\left\{\kappa^2(1-\delta) + \tau\kappa\varphi\pi_h^2\right\}^{-1/2}. \tag{9}$$

The second inequality implies that parents' empathy for their offspring is not extreme; with high $\mu$, parents prioritise their expenditure on education provision due to their strong interest in their child's future career. Without this assumption, the unskilled household will expend all their wages for social mobility motives for a sufficiently high $A_{t+1}$.[11]

**Proposition 3.** *Under Assumption 2, the equilibrium education satisfies $e_{it} = \bar{e}_t$ for all $i \in [0, 1]$. Furthermore, $\bar{e}_t$ strictly increases in $A_t$ and $I_t$.*

The result suggests that households are strongly encouraged to provide higher education by the increase in the level of infrastructure, as the value of education for a high-skilled job increases both through an increase in wages and an increase in the number of jobs. As a result of increased education, the demand for democracy also increases, as more young citizens adopt democratic values. This describes the emergence of the middle class as correlated with the increasing demand for democracy.

**Dictator's Investment Decision**

Economic growth allows for higher expected returns by increasing government revenues. On the other hand, it raises the average level of education, leading to strong pressure for regime change. Therefore, the dictator faces a trade-off between a more secure regime with fewer resources for rent seeking and a less secure regime with more resources to manage. The main objective is to analyse the dictator's optimal investment decision under these trade-offs.

The dictator maximises his preferences, which depend on the current rent and the expected revenue in the next period. The optimal investment of the dictator, $I_t^{\text{dict}}$, is the solution to the

[11] The first inequality is introduced for the relationships between $\eta$ (upper bound of democratic citizens) and $\bar{q}$ (upper bound of skilled labour): In equilibrium, the highest possible percentage of democratic citizens is achieved before the society has completed its maximum allowed transition from a rural to an industrial economy.

following:

$$
\begin{aligned}
\max_{I_t \geq 0} \quad & \{G_t - \kappa I_t\} + \beta \Pr[M_t < 1 - \theta_t | \bar{d}_t]\{G_{t+1} - \kappa I_{t+1}\} \\
\text{s.t.} \quad & A_{t+1} = (1-\delta)A_t + I_t. \\
& e_{it} = \arg\max_{\tilde{e}_{it} \geq 0} \left\{ w_{it} - \frac{e_{it}^2}{2} \right\} + \mu \mathbb{E}[w_{it+1} | e_{it}, \bar{e}_t, q_{t+1}], \quad \forall i \in [0,1], \\
& \bar{d}_t = \int_0^1 \min\{e_{it}^2, \eta\} di, \\
& I_t \text{ is feasible.}
\end{aligned}
\tag{10}
$$

The constraints correspond to the infrastructure accumulation, parents' optimal education decision, and the proportion of democratic citizens, respectively. This problem says that the dictator may give up some of his current rent and make an investment to increase the rent in the next period. This investment makes each parent $i$ choose the higher $e_{it}$ by improving the prospect for social mobility, which lowers the continuation probability $\Pr[M_t < 1 - \theta_t | \bar{d}_t]$. Therefore, optimal investment finds a balance between increasing revenues without losing too much likelihood of maintaining the regime.

When $A_t$ is low, each household provides a lower level of education, resulting in a smaller proportion of democratic citizens. So, the regime is very likely to remain in power in the next period. Because investments are profitable, the dictator is strongly motivated to invest in the economy. In contrast, when $A_t$ is high, each household can afford a high level of education, leading to the emergence of a larger population of democratic citizens. Due to the threat posed by these democratic citizens, the regime is less likely to continue in the next period. Despite the profitability of investments, the dictator has a weaker incentive to invest in the economy.

**Proposition 4.** *Under Assumptions 1 and 2, there are thresholds of infrastructure $\underline{A}$ and $\overline{A}$ such that optimal investment $I_t^{\text{dict}}$ strictly increases in $A_t$ for all $A_t \leq \underline{A}$, strictly decreases in $A_t$ for all $A_t \in [\underline{A}, \overline{A}]$, and $I_t^{\text{dict}} = 0$ for all $A_t \geq \overline{A}$. There is a unique steady state $A_{ss}^{\text{dict}}$ in $(\underline{A}, \overline{A})$.*

Figure 2 graphically shows the result of Proposition 4. The economy grows when investment exceeds depreciation and declines when it falls below, which is consistent with the main argument of Przeworski et al. (2000): dictatorships stall economic growth when the regime is expected to lose power. The steady state is the level of $A_t$ that equalises investment and depreciation. Also, it suggests that the developmental dictatorship would not promote perpetual growth; instead, the dictator stimulates growth for the mediocre economy. This provides a possible mechanism to explain why it is hard to find highly developed autocracies and, with the analysis of the democratic economy in section 5, why growth is sometimes faster under dictatorships among poorer countries, but slower above a certain level of development (Luo and Przeworski, 2019).

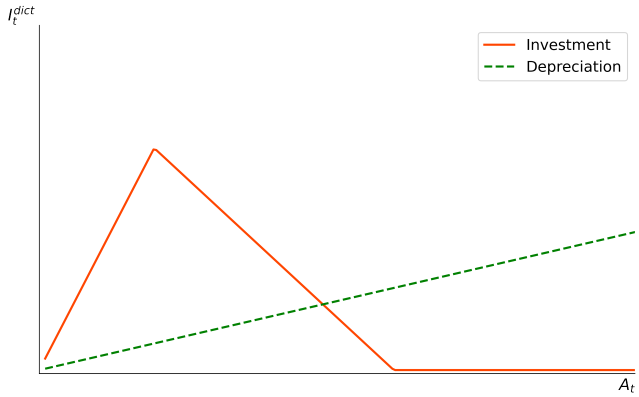



Figure 2: Optimal Investment under Dictatorship

### 4.1 Comparative Statics

#### Degree of Urbanisation

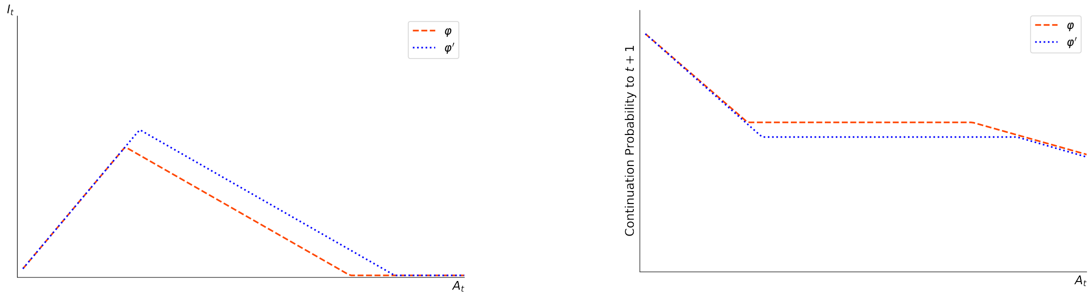
The formation of the industrial and rural sectors and the degree of urbanisation are considered important factors for democratic movement. Urbanised societies facilitate organised mass mobilisation from the middle class and make democratisation more likely (Dahlum et al., 2019). Industrialisation is also a crucial part of this study: it provides a tax base for the dictator and is the source of incentives for education. Therefore, as a first comparative static, I discuss the different sensitivities of industrial transitions caused by economic growth.

In equation (3), the social cost parameter $\varphi$ represents how the change in infrastructure reflects the change in the proportion of skilled labour. When the parameter increases from $\varphi$ to $\varphi'$, economic growth generates much higher government revenues than before, as the number of taxable workers increases more rapidly. On the other hand, a higher $\varphi$ means that economic growth provides more skilled job opportunities than before, resulting in a higher value of education. As a result, an increase in average education leads to more democratic citizens and a greater likelihood that collective action will succeed. This means that it is not obvious whether the dictator increases or decreases the investment. The analysis shows that the dictator increases investment: It is more profitable for the dictator to invest more and bear the burden of increasing democratic pressure. The probability that the regime will continue in the next period also decreases in the equilibrium (see Figure 3).

#### Productivity of Rural Population

In the model, the output of the rural economy has no direct effect on the dictator's decision making, due to the inability to levy taxes. For citizens, however, wages in the rural economy work as reservation wages. It follows that low unskilled wages in the rural economy imply a high income inequality between skilled and unskilled workers.

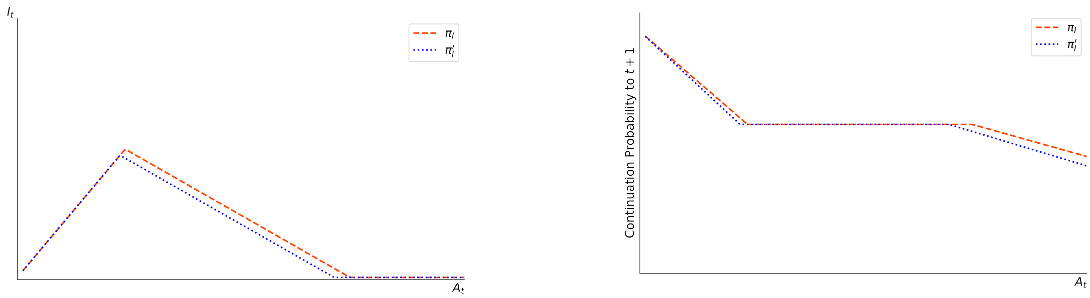
Formal models of democratic transitions consider distributive conflicts and emphasise inequality as a cause of democratisation. Boix (2003) predicts that unequal societies are more




(a) Investment

(b) Likelihood of Regime Survival at $t + 1$

Figure 3: Comparative Static: Urbanisation




(a) Investment

(b) Likelihood of Regime Survival at $t + 1$

Figure 4: Comparative Static: Inequality between industrial and rural workers

difficult for democratic transitions and consolidation because the rich want to avoid the redistributive consequences. In contrast, Acemoglu and Robinson (2000, 2001, 2006) focus on the increasing incentives for the poor to revolt for redistribution as inequality grows. However, more than 40 percent of democratisations are not explained by these distributive conflicts, and many protests that cause transitions were dominated by middle or upper middle class groups (Haggard and Kaufman, 2012). Thus, this comparative static shows the equilibrium effect of inequality on democratisation without distributive conflict.

Suppose that $\pi_h$ is the same but $\pi_l$ changes to $\pi_l'$, where $\pi_l' < \pi_l$. Then $Y_t$ remains unchanged, but the wage differential $\Delta w_t := w_{ht} - w_{lt}$ increases. This means that education is now more important than before, causing citizens to have a stronger desire for upward mobility. From the dictator's point of view, the resulting advanced education would pose a threat to the regime. Given these conditions, how does the dictator's optimal investment and the possibility of moving towards democracy change?

If $A_t$ is low, the regime is less likely to be maintained with $\pi_l'$ than with $\pi_l$, and it is equally likely to be maintained if $A_t$ is above a certain level. This is because the dictator balances the return on investment and the likelihood of maintaining the regime. Then, when $A_t$ is high, the equilibrium with $\pi_l'$ again gives a lower probability because the dictator would not invest anymore and $\pi_l'$ has a higher average education than $\pi_l$ when $A_t$ and $I_t$ are equal. Overall, inequality increases the transition probability, but this is offset by the dictator's reduced investment (see Figure 4).

## 5 Economic Growth under Dictatorship and Democracy

So far, I have discussed the dictator's optimal investment under the threat of a democratic middle class. The natural question arises as to how the pattern differs under democratic regimes. Therefore, the main objective of this section is to analyse how the pattern of economic growth under democracy differs from that under dictatorship.[12]

In my model of dictatorship, the emergence of a democratic culture from economic development was the driving force behind the democratic transition. This democratic culture is seen as a crucial factor for economic growth in democratic regimes (Putnam et al., 1992; Rodrik, 2000; Persson and Tabellini, 2009). To take this into account when comparing regimes in economic growth, I use a probabilistic voting model of electoral competition following Persson and Tabellini (2021).

Right after ruling out the dictator through the period $t-1$ collective action, a democratic election is held with two political parties, $A$ and $B$, competing for the democratic government. At the beginning of each period $s \geq t$, each party $j \in \{A, B\}$ proposes a policy $\alpha_t^j \in [0,1]$ that indicates the proportion of the budget to be obtained as rent. When the policy $\alpha_t^j$ is adopted, the investment made and the rent obtained by party $j$ are $(1-\alpha_t^j)G_t$ and $\alpha_t^j G_t$, respectively. And each party enjoys a non-materialistic gain $\bar{\alpha} \geq 0$ from the incumbency. The preferences of each party $j$ are represented by

$$\mathrm{p}_s^j(\alpha_s^A, \alpha_s^B)\left\{\alpha_s^j + \bar{\alpha}\right\} \tag{11}$$

where $\mathrm{p}_s^j(\alpha_s^A, \alpha_s^B)$ is party $j$'s probability of winning the election.

Each citizen $i \in [0,1]$ in her second period prefers high investment and has equal vote share. Voters vary in their sensitivity to the proposed policy, with those highly sensitive to the policy being more concerned about voting against a party that implicitly proposes high rent-seeking. This policy sensitivity depends on each citizen's value type $v_{is-1}$ obtained in her first period. Let $\lambda_{v_{is-1}}$ represent this policy sensitivity. And let $\bar{\lambda}(\bar{d}_t) := (1-\bar{d}_t)\lambda_m + \bar{d}_t\lambda_d$ be the average policy sensitivity. I restrict these sensitivity parameters and $\bar{\alpha}$ to be within a reasonable range, so that no rent extractions and full rent extractions do not constitute the equilibrium policy outcomes and economic growth may occur:

**Assumption 3.** $\lambda_m$, $\lambda_d$ and $\bar{\alpha}$ satisfy $\{2(1+\bar{\alpha})\}^{-1} < \lambda_m < \lambda_d < \{2\bar{\alpha}\}^{-1}$ and $\bar{\lambda}(\eta) > \bar{\lambda}^{\text{growth}}$ where $\bar{\lambda}^{\text{growth}} = \tau\varphi\pi_h^2\{2\tau\varphi\pi_h^2(1+\bar{\alpha}) - 2(1-\delta)\}^{-1}$.

Each citizen has a different partisan preference. Let $\xi_{is} \sim \mathrm{Unif}[-1/2, 1/2]$ and $\zeta_s \sim \mathrm{Unif}[-1/2, 1/2]$ be the partisan preference of the voter $i$ over the party $A$ and the average popularity of the party $A$, which are distributed independently and identically in all $i \in [0,1]$ and $s \geq t$. After observing $\alpha_s^j$ for $j \in \{A, B\}$, citizen $i$ votes for the party $A$ if

$$\lambda_{v_{is-1}}\left\{\alpha_s^B - \alpha_s^A\right\} + \xi_{is} + \zeta_s > 0 \tag{12}$$

[12] I ignore the possibility of a reversal to dictatorship as this study focuses on the transition to democracy and how autocratic and democratic economies differ in terms of economic growth.

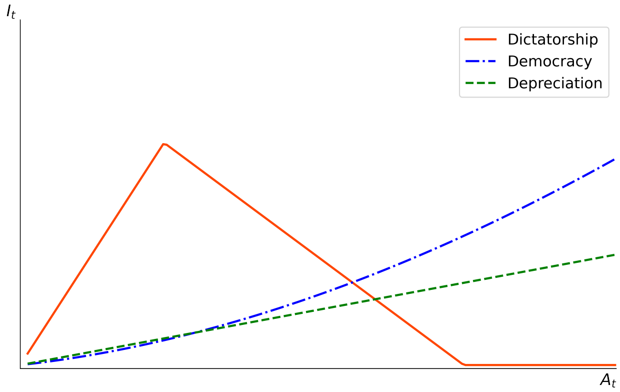


Figure 5: Investment by regime after democratisation

and for party $B$ if the inequality is reversed. Party $A$'s winning probability in period $s$ is derived as

$$\mathrm{p}_s^A(\alpha_s^A, \alpha_s^B) = \begin{cases} 1 & \text{if} \quad \alpha_s^A \leq \alpha_s^B - 1/2\bar{\lambda}, \\ 0 & \text{if} \quad \alpha_s^A \geq \alpha_s^B + 1/2\bar{\lambda}, \\ \frac{1}{2} + \bar{\lambda}\left\{\alpha_s^B - \alpha_s^A\right\} & \text{otherwise} \end{cases} \tag{13}$$

and the equilibrium policies proposed by both parties are

$$\alpha_s^A = \alpha_s^B = \frac{1}{2\bar{\lambda}(\bar{d}_t)} - \bar{\alpha}. \tag{14}$$

Denote this equilibrium policy by $\alpha_s^*$. By Assumption 3, $\alpha_s^*$ is in the interior of $[0, 1]$. This solution demonstrates that the proportion of rent extraction is decreasing in $\bar{d}_t$. In the following proposition, I present a comparison of economic growth between dictatorship and democracy.

**Proposition 5.** *Under Assumptions 1, 2 and 3, suppose that there is an economy under dictatorship and a newly democratised economy with the same infrastructure $A_t$. Then there is a threshold level of infrastructure $\tilde{A}$ such that the equilibrium investment under the dictatorship is higher if $A_t < \tilde{A}$ and lower if $A_t > \tilde{A}$.*

As society democratises, underdeveloped economies lose the vitality of economic growth; with a less institutionalised democratic culture, the policies implemented expend government revenue mostly on rents rather than investment. On the other hand, in a developed economy, rent-seeking by politicians is limited by the democratic checks and balances of citizens, represented by policy sensitivity, thus leading to economic prosperity.

What remains is the long-term effect of the regimes on economic growth. As shown in Proposition 4, the economy under the dictatorship converges to a unique steady state. To analyse the long-term effect of democratic institutions, I find out whether a steady state exists and how it is different from the steady state under the dictatorship.

**Proposition 6.** *Under democracy, there is $A_{ss}^{\text{dem}} > A_{ss}^{\text{dict}}$ such that $A_t$ converges to $0$ for $A_t < \bar{A}_{ss}$ and converges to $A_{ss}^{\text{dem}}$ for $A_t > \bar{A}_{ss}$ for some threshold $\bar{A}_{ss}$ where $\bar{A}_{ss}$ decreases in $\lambda_m$ for $\lambda_m < \bar{\lambda}^{\text{growth}}$ and $0$ for all $\lambda_m \geq \bar{\lambda}^{\text{growth}}$.*

Proposition 6 suggests that democratic economies may lead their paths to downfall or prosperity. When the initial infrastructure after transition is lower than $\bar{A}_{ss}$, depreciation of the existing infrastructure is greater than the new infrastructure created by equilibrium investment. On the contrary, when the initial infrastructure after transition is greater than $\bar{A}_{ss}$, the implemented policy leads to an increase in the net infrastructure. As there is higher expected income with more skilled jobs, parents are incentivised to give their children more educational opportunities than they had themselves, resulting in a more democratic citizens and increased investment. This result is consistent with Persson and Tabellini (2009): higher democratic capital promotes growth, which in turn consolidates democracy through the accumulation of democratic capital.

# 6 Extensions

## 6.1 Legitimising the Dictatorship from Economic Growth

In the main model, the threat of collective action from the emerging middle class leads the dictatorship to halt economic growth. But history shows that the strength of a regime is greater in times of economic growth and weaker in times of economic hardship. For example, the difficult economic situation in France in the late 18th century contributed to the French Revolution. Economic crises with crop failures and high living costs led to widespread poverty and suffering. This, in turn, fuelled the revolutionary fervour that led to the overthrow of the monarchy. Similarly, the Bloody Sunday massacre of 1905 in Russia occurred as the working class suffered from harsh working conditions and low wages leading to widespread poverty. This means that dictators cannot help but look to economic performance to justify their regime.

In particular, some autocracies with rapid economic growth have argued that their system can deliver better economic prosperity than democratic institutions. China's rapid economic growth, for example, gives birth to the so-called "China model", which offers a new vision for many autocracies. To prove its superiority, the Chinese government has been urged to maintain a higher rate of economic growth than democracies. The regime may be able to stave off demands for democracy that are based on economic concerns by providing economic prosperity, leaving only politically oriented demands for democracy.[13]

In this section, I extend the model that takes into account the legitimacy coming from economic growth. Specifically, I analyse the dictator's investment decision with the investment of democracy as a reference point. Introducing legitimacy provides the dictator another layer of horse race: investing in the economy enhances the regime strength in the current period, at the same time it increases the likelihood of regime change in the next period.

[13] According to Haggard and Kaufman (1999), sustaining a good performance may not preclude purely political protest; despite successful reforms and significant economic growth, non-crisis transitions occurred in Chile (1990), Korea (1986), Thailand (1983), and Turkey (1983) due to a variety of international and domestic political pressures.

Suppose that each young citizen $i$ with value type $v_{it} \in \{d, m\}$ receives a positive satisfaction $\gamma_{v_{it}}$ when she participates in the collective action and it succeeds. This satisfaction, which comes from gaining expanded political rights and living under a democratic regime, is greater for democratic citizens than for materialistic ones: $\gamma_m < 1 < \gamma_d$. Furthermore, participation leading to regime change gives 1 when investment under democracy $I_t^{\text{dem}}$ is greater than that under dictatorship $I_t^{\text{dict}}$ and 0 otherwise. This means that democratisation is desirable for democratic citizens regardless of the investment, whereas it is desirable for materialistic citizens only if the dictatorship does not provide sufficient economic growth. This setting contrasts with the one in the previous section in that it assumes that regime change is always desirable for any young citizen. The participation cost is normalised to 1 regardless of the type. The preferences of young citizen $i$ in period $t$ are represented by

$$\left\{\left(\mathbf{1}[I_t^{\text{dem}} > I_t^{\text{dict}}] + \gamma_{v_{it}}\right)\mathbf{1}[M_t > 1 - \theta_t] - 1\right\} a_{it} \tag{15}$$

where $a_{it} \in \{0, 1\}$ indicates whether or not to participate in the collective action. Assume that the parents' problem and democratic investment are the same as in the previous sections. The dictator's optimal decision $I_t^{\text{dict}}$ is the solution to the following problem:

$$\begin{aligned}
\max_{I_t \geq 0} \quad & \{G_t - \kappa I_t\} + \beta \mathbf{1}[I_t^{\text{dem}} > I_t] \Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t \geq I_t^{\text{dem}}] \{G_{t+1} - \kappa I_{t+1}\} \\
& + \beta \left\{1 - \mathbf{1}[I_t^{\text{dem}} > I_t]\right\} \Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\text{dem}} > I_t] \{G_{t+1} - \kappa I_{t+1}\} \\
\text{s.t.} \quad & A_{t+1} = (1 - \delta) A_t + I_t \\
& e_{it} = \arg\max_{\tilde{e}_{it} \geq 0} \left\{ w_{it} - \frac{e_{it}^2}{2} \right\} + \mu \mathbb{E}[w_{it+1} | e_{it}, \bar{e}_t, q_{t+1}], \quad \forall i \in [0, 1], \\
& \bar{d}_t = \int_0^1 \min\{e_{it}^2, \eta\} di, \\
& I_t \text{ is feasible.}
\end{aligned}$$

The main difference from the dictator's problem in (10) is that the continuation probability depends on whether $I_t^{\text{dict}} \geq I_t^{\text{dem}}$ or not. So, I first analyse the collective action problem for each case, and then describe the shape of optimal investment.

### Collective Action

Suppose that $I_t^{\text{dict}} \geq I_t^{\text{dem}}$, i.e., only democratic citizens prefer to initiate a regime change. As the optimal strategy, every materialistic citizen chooses not to participate ($a_{it} = 0$) and each democratic citizen $i$ uses a cutoff strategy such that $a_{it} = 1$ if $s_{it} \geq s_t^*(\bar{d}_t)$ and $a_{it} = 0$ otherwise, with the cutoff point $s_t^*(\bar{d}_t) = \{\bar{d}_t + 2\sigma\}/\gamma_d + (1 - \bar{d}_t - \sigma)$. The dictator's continuation probability to the next period is

$$\Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\text{dict}} \geq I_t^{\text{dem}}] = \frac{\theta^*(\bar{d}_t) - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \tag{16}$$

where the threshold $\theta^*(\bar{d}_t)$ of regime vulnerability is

$$\theta^*(\bar{d}_t) = 1 - \bar{d}_t\left[1 - \frac{1}{\gamma_d}\right]. \tag{17}$$

In this analysis, I focus on $\eta$ and $\gamma_d$ that are sufficiently large; otherwise, there will be no threat from emerging democratic citizens.

Next, suppose that $I_t^{\text{dem}} > I_t^{\text{dict}}$, i.e., both materialistic and democratic citizens prefer a democratic transition. The collective action problem is equivalent to the problem in section 3 with $b_d = (1+\gamma_d)^{-1}$ and $b_m = (1+\gamma_m)^{-1}$. As in the main model, I assume that Assumption 1 holds. The dictator's continuation probability is

$$\Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\text{dem}} > I_t^{\text{dict}}] = \frac{\bar{b}_t - \underline{\theta}}{\bar{\theta} - \underline{\theta}} \tag{18}$$

where $\bar{b}_t = \bar{d}_t b_d + (1-\bar{d}_t) b_m$ is the average participation cost. It is obvious that $\bar{b}_t$ is greater than $\theta^*$. The difference between $\theta^*$ and $\bar{b}_t$ can be interpreted as the effect of providing higher economic growth than democracies.

### Optimal Investment

I first analyse the investment by regime types when the infrastructure is low. Section 4 shows that it is optimal for the dictator to allocate the entire budget to investment when the level of infrastructure is low. On the other hand, in section 5, low investment is made in the democracy when the infrastructure is low. Thus, $I_t^{\text{dict}} > I_t^{\text{dem}}$ for a sufficiently low $A_t$.

Next, when $A_t$ becomes higher, proposition 4 suggests that the dictatorship reduces investment as the economy grows, mainly due to the increasing threat from democratic citizens. However, the dictator has a limit on how much investment he can reduce. When investment falls below $I_t^{\text{dem}}$, the regime loses its legitimacy as a developmental dictator and faces significant pressure to democratise. Thus, the dictatorship stops reducing investment and maintains it at the same level as under democracy.

In highly developed economies, however, the democratic citizenry expands, leading the dictatorship to face increased demands for democracy from this segment of the population. In addition, maintaining the same level of investment as in a democratic system becomes increasingly costly as it rises in proportion to the number of democratic citizens. As a result, the dictator chooses to reduce investment and focuses on immediate rent gains.

**Proposition 7.** *For a sufficiently high $\gamma_d$, there are thresholds $\underline{A}^{\text{leg}}$ and $\overline{A}^{\text{leg}}$ such that the dictator's optimal investment $I_t^{\text{dict}}$ is greater than $I_t^{\text{dem}}$ for all $A_t < \underline{A}^{\text{leg}}$, $I_t^{\text{dict}} = I_t^{\text{dem}}$ for $A_t \in [\underline{A}^{\text{leg}}, \overline{A}^{\text{leg}}]$ and $I_t^{\text{dem}} > I_t^{\text{dict}}$ for all $A_t > \overline{A}^{\text{leg}}$.*

The result implies that the dictatorship drives economic growth through economic incentives when the economy is underdeveloped. Subsequently, the dictatorship continues to invest in the economy due to the incentive for legitimacy once it surpasses a certain level of economic de-

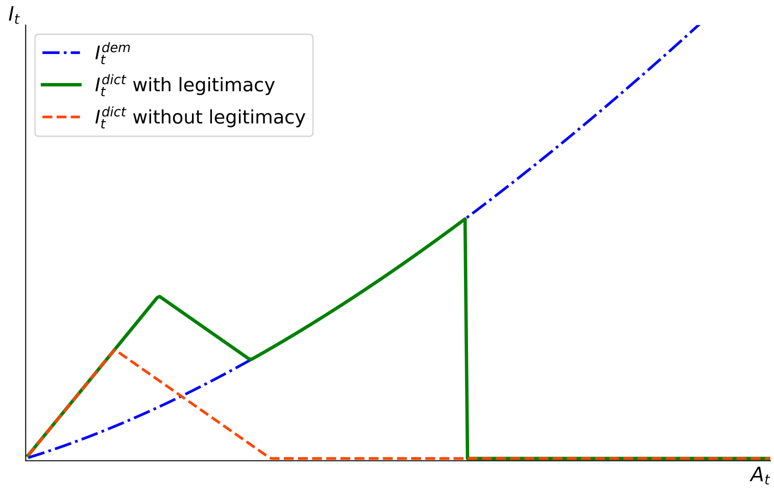



Figure 6: Legitimacy in Investment Decision

velopment. However, when the economic cost of investment exceeds the legitimacy benefit, the dictatorship transitions away from the developmental phase.

Although not fully described in the model, another implication is the institutionalisation of democratic aspects within the regime. Many contemporary authoritarian regimes willingly curtail their authority by introducing elements of democracy. In particular, they adopt economic freedom as a means of stimulating economic activity. Through these institutional constraints, the marginal satisfaction of citizens from the regime transition may be decreased, i.e., lower values of $\gamma_d$ and $\gamma_m$. Therefore, the dictator with institutional constraints will have a greater incentive to promote economic growth at a level equivalent to that of a democratic regime, as this would ensure a more secure regime.

### 6.2 Suppressing the middle class through taxation

So far, I have assumed that a dictator adheres to an exogenously given tax rate $\tau$. In reality, dictators often manipulate the tax system for their own ends. For example, they may implement higher taxation in order to curb the influence of the emerging middle class. In this section, I analyse the dictator's optimal strategies when given the power to set the tax rate and examine how the previous results change.

Suppose that the dictator in period $t$ chooses tax rate in the next period $\tau_{t+1}$. I assume that implementing higher taxation incurs an increased deadweight loss. Define the government revenue in period $t$, $G_t$, as

$$G_t = \tau_t \left(1 - \frac{\tau_t}{2\hat{\tau}}\right) Y_t \tag{19}$$

for $0 < \hat{\tau} < \{\pi_h - \pi_l\}/2\pi_h$. The term $\tau_t/2\hat{\tau}$ in the parenthesis indicates the deadweight loss. Upper bound of $\hat{\tau}$ ensures that post-tax skilled wages are greater than unskilled wages when government revenue is positive. Notice that $G_t$ is maximised when $\tau_t = \hat{\tau}$. And I adjust Assumptions

1 and 2 to

$$\beta\left\{\frac{\bar{b}_t(\eta)-\underline{\theta}}{\bar{\theta}-\underline{\theta}}\right\}\frac{\hat{\tau}}{2}\varphi\pi_h^2<\kappa<\beta\left\{\frac{b_m-\underline{\theta}}{\bar{\theta}-\underline{\theta}}\right\}\frac{\hat{\tau}}{2}\varphi\pi_h^2, \tag{20}$$

$$\frac{\eta\varphi\pi_h}{\bar{q}^2\left\{(1-2\hat{\tau})\pi_h-\pi_l\right\}}<\mu<\frac{2\kappa\pi_l}{(1-\hat{\tau})\pi_h-\pi_l}\left\{\kappa^2(1-\delta)+\tau\kappa\varphi\pi_h^2\right\}^{-1/2}. \tag{21}$$

I first analyse whether the dictator may adopt a tax rate that is lower than the revenue-maximising tax rate $\hat{\tau}$, in the following lemma:

**Lemma 2.** *Any tax rate $\tau_{t+1}<\hat{\tau}$ is not optimal for the dictator in period $t$.*

The rationale behind this result is that reducing the tax rate from $\hat{\tau}$ not only reduces the tax revenue but also increases post-tax skilled wages, thereby boosting the equilibrium education and makes the regime more unstable. Thus, any tax rates below $\hat{\tau}$ can be safely disregarded in the analysis.

Due to deadweight loss, a tax rate strictly higher than $\hat{\tau}$ results in both lower government revenue and lower post-tax skilled wages compared to $\hat{\tau}$. At a glance, this tax rate may seem like a Pareto-inferior tax scheme, and $\hat{\tau}$ may be preferred to the dictator. However, the following proposition demonstrates that this revenue-maximising tax rate does not represent an optimal tax scheme for a dictator.

**Proposition 8.** *Optimal tax rate $\tau^*_{t+1}$ is greater than $\hat{\tau}$ and strictly increases in $A_{t+1}$ for all $A_{t+1}\leq\eta/\left[\varphi\pi_h\{(1-2\hat{\tau})\pi_h-\pi_l\}\right]$.*

Although no one benefits materially, the dictator implements such a destructive tax rate to suppress skilled wages with the aim of preventing the rise of democratic citizens. Furthermore, as the economy develops, the dictator increases the tax rate, thereby extinguishing a higher proportion of output. This finding aligns with Przeworski et al. (2000) that dictatorships, on average, tend to offer lower wages than democracies, and that even wealthier dictatorships suppress workers' wages and utilise labour inefficiently.

Our final question is how the dictator's optimal investment differs when he can choose the tax rate. Given that the increase in the tax rate, as depicted in Proposition 8, diminishes future revenue, it remains uncertain whether the incentives for a dictatorship to make an investment are increasing or decreasing compared to a dictatorship with a fixed tax rate. Hence, I build the baseline model with the tax rate fixed at $\tau_t=\hat{\tau}$ for all $t$. I denote by $\underline{A}^{\text{base}}$ and $\bar{A}^{\text{base}}$ the thresholds $\underline{A}$ and $\bar{A}$ in Proposition 4 for the baseline model.

**Proposition 9.** *When the dictator is allowed to choose the tax rate, there are thresholds $\underline{A}^{\text{tax}}>\underline{A}^{\text{base}}$ and $\bar{A}^{\text{tax}}>\bar{A}^{\text{base}}$ such that the dictator's optimal investment increases in $A_t$ for $A_t\leq\underline{A}^{\text{tax}}$, decreases in $A_t$ for $A_t\in[\underline{A}^{\text{tax}},\bar{A}^{\text{tax}}]$, and no investment for all $A_t\geq\bar{A}^{\text{tax}}$.*

This result suggests that when the authority to adjust tax rates is vested in the dictator's hands, they can sustain incentives for economic growth even at higher levels of infrastructure. By accepting the trade-off of reduced revenue, the dictator can improve the regime stability by mitigating the pressure for democratisation through the implementation of a higher tax scheme. This strategy has a similar motivation to 'power sharing' in dictatorships (Boix and Svolik, 2013), where the dictator relinquishes some of his authority to prevent a complete loss of power.

## 7 Concluding Remarks

In this study, I analyse the dictator's optimal investment decisions under the threat of the emerging middle class and compare them with the economic outcomes in democratic institutions. In an environment where education correlates with embracing democratic values over materialistic ones, economic growth expands job opportunities and skilled job wages. This growth incentivises citizens to pursue further education in pursuit of skilled employment, consequently fostering a prevalence of democratic values in society. From the dictator's standpoint, economic growth promises greater future revenue while heightening the risk of regime instability owing to the encroachment of emerging democratic values. This dilemma exposes the dictator to a trade-off between maintaining a 'stable poor' or venturing into an 'unstable rich' regime. The findings indicate that the dictator allocates a larger portion of the revenue to investment when the economy is underdeveloped, gradually reducing it to zero as economic development advances. This pattern contrasts with the investment behaviour in democratic societies. In a probabilistic voting model by Persson and Tabellini (2021), citizens with democratic values vote based on policy rather than partisanship. The prevalence of these democratic values in developed economies leads to high-investment policies, which establishes a trajectory contrasting with that of dictatorships.

This study provides a growth model of dictatorships, enriching the literature on formal models of non-democracies. It links decision-making under dictatorship to the stylised fact that the emerging middle class significantly influenced democratisation, and the finding suggests a novel mechanism for the puzzle of economic growth: economic growth is higher in autocracies when the economy is underdeveloped but higher in democracies when the economy is developed. (Luo and Przeworski, 2019).

This framework leaves several avenues open for future research. First, it would be interesting to investigate institutional reforms within the context of economic growth and the emerging middle class. Although not explored in this research, it is worth noting that several autocratic regimes impose institutional restrictions to safeguard their authority (Treisman, 2020). Second, it would also be interesting to study the manipulation of education by dictatorships in the context of economic growth. Education is a double-edged sword for dictators: While education brings high productivity, it also threatens regimes by increasing the demand for democracy. In response, they may introduce the curriculum and promote education to align the preferences of citizens with the regime (Alesina et al., 2021; Cantoni et al., 2017). However, this improvement in education can be poisonous for dictators in the long run, as the net effect of increased education leads to more people demanding a democratic order, or to stronger popular resistance if they lose the legitimacy of their power.

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## A Proofs

### Proof of Proposition 1

Suppose that all citizens use the cutoff strategies with $s_t^*(d)$ and $s_t^*(m)$ depending on their value types. When a citizen $i$ gets signal $s_{it}$, her posterior belief of $\theta_t$ is uniform on $[s_{it}-\sigma, s_{it}+\sigma]$. If $\theta_t > s_t^*(m)+\sigma$, every materialistic citizen gets signal above $s_t^*(m)$. And if $\theta_t < s_t^*(m)-\sigma$, all of them get signals below $s_t^*(m)$. Accordingly, The mass of participants who are $v_{it}=m$ is $1-\bar{d}_t$ if $\theta_t > s_t^*(m)+\sigma$ and 0 if $\theta_t < s_t^*(m)-\sigma$. If $\theta \in [s_t^*(m)-\sigma, s_t^*(m)+\sigma]$, it is $(1-\bar{d}_t)\left\{\theta_t+\sigma-s_t^*(m)\right\}/2\sigma$. Similarly, the mass of participants who are $v_{it}=d$ are $\bar{d}_t$ if $\theta_t > s_t^*(d)+\sigma$, 0 if $\theta_t < s_t^*(d)-\sigma$, and $\bar{d}_t\left\{\theta_t+\sigma-s_t^*(d)\right\}/2\sigma$ if $\theta_t \in [s_t^*(d)-\sigma, s_t^*(d)+\sigma]$.

Suppose that materialistic citizen $i$ gets signal $s_{it}=s_t^*(m)$. Materialistic citizen's posterior belief of $\theta_t$ is uniform on $[s_t^*(m)-\sigma, s_t^*(m)+\sigma]$. Then choosing $a_{it}=1$ and $a_{it}=0$ is indifferent, which means $\Pr[M_t \geq 1-\theta_t|s_{it}=s_t^*(m), v_{it}=m]=b_m$. Before deriving $M_t$, I introduce a lemma to explore the distance between the two cutoffs.

**Lemma 3.** *The distance between cutoffs $s_t^*(m)$ and $s_t^*(d)$ is less than $2\sigma$.*

*Proof.* Suppose that the distance between $s_t^*(m)$ and $s_t^*(d)$ is greater than or equal to $2\sigma$. If a materialistic citizen $i$ gets a signal $s_{it}=s_t^*(m)$, $\Pr[M_t > 1-\theta_t|s_{it}=s_t^*(m), v_{it}=m]=b_m$. Since the distance between the cutoffs is greater than $2\sigma$, $M_t=\bar{d}_t+(1-\bar{d}_t)\left\{\theta_t+\sigma-s_t^*(m)\right\}/2\sigma$, i.e., all democratic citizens participate. This means

$$
\begin{aligned}
&\Pr[M_t > 1-\theta_t|s_{it}=s_t^*(m), v_{it}=m] \\
&=\Pr\left[\theta_t > \frac{(1-\bar{d}_t)(\sigma+s_t^*(m))}{1-\bar{d}_t+2\sigma}\middle| s_{it}=s_t^*(m), v_{it}=m\right] \\
&=\frac{1}{2\sigma}\left[s_t^*(m)+\sigma-\frac{(1-\bar{d}_t)(\sigma+s_t^*(m))}{1-\bar{d}_t+2\sigma}\right]
\end{aligned}
$$

and it follows that $s_t^*(m)=b_m(1-\bar{d}_t+2\sigma)-\sigma$.

Next, assume that a democratic citizen $j$ gets a signal $s_{jt}=s_t^*(d)$. Then it satisfies that $\Pr[M_t > 1-\theta_t|s_{jt}=s_t^*(d), v_{jt}=d]=b_d$ and $M_t=\bar{d}_t\left\{\theta_t+\sigma-s_t^*(d)\right\}/2\sigma$. Hence

$$
\begin{aligned}
&\Pr[M_t > 1-\theta_t|s_{jt}=s_t^*(d), v_{jt}=d] \\
&=\Pr\left[\theta_t > \frac{\bar{d}_t(s_t^*(d)-\sigma)+2\sigma}{\bar{d}_t+2\sigma}\middle| s_{jt}=s_t^*(d), v_{jt}=d\right] \\
&=\frac{1}{2\sigma}\left[s_t^*(d)+\sigma-\frac{\bar{d}_t(s_t^*(d)-\sigma)+2\sigma}{\bar{d}_t+2\sigma}\right]
\end{aligned}
$$

and I get $s_t^*(d)=b_d(\bar{d}_t+2\sigma)+1-\bar{d}_t-\sigma$.

The cutoffs $s_t^*(d)$ and $s_t^*(m)$ must satisfy $s_t^*(m)-s_t^*(d)\geq 2\sigma$. Substituting the cutoffs and proceeding the calculation,

$$
0 \geq (1-b_m)(1-\bar{d}_t)+2\sigma(1-b_m+b_d)+d_tb_d \tag{22}
$$

which is impossible. Therefore, the distance between the cutoffs is less than $2\sigma$. $\square$

From Lemma 3, the cutoffs $s_t^*(m)$ and $s_t^*(d)$ are closer than $2\sigma$. So, for given $\theta_t$, the density $M_t$ is

$$M_t = \bar{d}_t \left\{ \frac{\theta_t + \sigma - s_t^*(d)}{2\sigma} \right\} + (1 - \bar{d}_t) \left\{ \frac{\theta_t + \sigma - s_t^*(m)}{2\sigma} \right\}. \tag{23}$$

Hence,

$$\begin{aligned}
&\Pr[M_t \geq 1 - \theta_t | s_{it} = s_t^*(m), v_{it} = m] \\
&= \Pr \left[ \theta_t \geq \bar{d}_t \left\{ \frac{s_t^*(d) + \sigma}{2\sigma + 1} \right\} + (1 - \bar{d}_t) \left\{ \frac{s_t^*(m) + \sigma}{2\sigma + 1} \right\} \middle| s_{it} = s_t^*(m), v_{it} = m \right] \\
&= \frac{1}{2\sigma} \left[ s_t^*(m) + \sigma - \bar{d}_t \left\{ \frac{s_t^*(d) + \sigma}{2\sigma + 1} \right\} - (1 - \bar{d}_t) \left\{ \frac{s_t^*(m) + \sigma}{2\sigma + 1} \right\} \right]
\end{aligned}$$

and it follows that $s_t^*(m) = \left\{ 2\sigma(2\sigma + 1)b_m + \bar{d}_t s_t^*(d) - 2\sigma^2 \right\} / (\bar{d}_t + 2\sigma)$. Similarly, $s_t^*(d)$ is derived as $s_t^*(d) = \left\{ 2\sigma(2\sigma + 1)b_d + (1 - \bar{d}_t)s_t^*(m) - 2\sigma^2 \right\} / (1 - \bar{d}_t + 2\sigma)$. Using these two, the equilibrium cutoffs are $s_t^*(m) = \sigma(2b_m - 1) + \bar{b}_t$ and $s_t^*(d) = \sigma(2b_d - 1) + \bar{b}_t$ where $\bar{b}_t = \bar{d}_t b_d + (1 - \bar{d}_t)b_m$ is average participation cost. It is easy to check that $s_t^*(d) < s_t^*(m)$. And it is shown in Morris and Shin (2001) and Sakovics and Steiner (2012) that the cutoff strategy is the unique BNE, which is achieved from iterated elimination of strictly dominated strategies.

**Proof of Proposition 2**

Suppose that $\bar{d}_t$ is given. For all $\theta_t$ weakly smaller than $s_t^*(d) - \sigma$, the probability of collective action success $\Pr[M_t \geq 1 - \theta_t]$ is 0 because all citizens receive signals lower than the cut-off points, so that no one participates. Next, suppose that $\theta_t \in (s_t^*(d) - \sigma, s_t^*(m) - \sigma]$. Then only democratic citizens participate, so the mass of participants $M_t$ is $\bar{d}_t \left\{ \theta_t + \sigma - s_t^*(d) \right\} / 2\sigma$ and

$$\Pr[M_t \geq 1 - \theta_t] = \Pr \left[ \theta_t \geq \frac{2\sigma + \bar{d}_t(s_t^*(d) - \sigma)}{2\sigma + \bar{d}_t} \right]. \tag{24}$$

When $\theta_t = s_t^*(m) - \sigma$, the highest value in the interval,

$$\theta_t - \frac{2\sigma + \bar{d}_t(s_t^*(d) - \sigma)}{2\sigma + \bar{d}_t} = -\frac{2\sigma}{2\sigma + \bar{d}_t} \left\{ (2\sigma + 1)(1 - b_m) \right\} < 0 \tag{25}$$

where the second equality is obtained by substituting the values $s_t^*(m)$ and $s_t^*(d)$. This means that, for any $\theta_t$ on the interval, the collective action is not successful. Finally, suppose that $\theta_t \in (s_t^*(m) - \sigma, s_t^*(d) + \sigma]$. On this interval, the mass of participants $M_t$ is $\bar{d}_t \left\{ \theta_t + \sigma - s_t^*(d) \right\} / 2\sigma + (1 - \bar{d}_t) \left\{ \theta_t + \sigma - s_t^*(m) \right\} / 2\sigma$. And the probability of regime change is

$$\Pr[M_t \geq 1 - \theta_t] = \Pr \left[ \theta_t \geq \bar{d}_t \left\{ \frac{s_t^*(d) + \sigma}{2\sigma + 1} \right\} + (1 - \bar{d}_t) \left\{ \frac{s_t^*(m) + \sigma}{2\sigma + 1} \right\} \right]. \tag{26}$$

It is trivial to see that $M_t < 1 - \theta_t$ when $\theta_t = s_t^*(m) - \sigma$. If $\theta_t = s_t^*(d) + \sigma$, substituting $s_t^*(d)$ and $s_t^*(m)$,

$$\theta_t - \bar{d}_t \left\{ \frac{s_t^*(d) + \sigma}{2\sigma + 1} \right\} - (1 - \bar{d}_t) \left\{ \frac{s_t^*(m) + \sigma}{2\sigma + 1} \right\} = \frac{2\sigma}{2\sigma + 1} b_d (1 + 2\sigma) > 0. \tag{27}$$

The left-hand side of (27) is continuous and strictly increasing in $\theta_t$. By the intermediate value theorem, there is a unique $\bar{\theta}_t$ such that $M_t = 1 - \bar{\theta}_t$. From the algebra, $\bar{\theta}_t = \bar{b}_t$, which means that the regime changes if the regime vulnerability $\theta_t$ is greater than the average participation cost $\bar{b}_t$ and continues otherwise.

**Proof of Lemma 1**

Suppose that $e_{jt} = 0$ for all $j \neq i$. Then $e_{it} = \varepsilon$ for sufficiently small $\varepsilon > 0$ makes $h(\varepsilon, 0, q_{t+1}) = 1$, which gives greater payoff than choosing $e_{it} = 0$. Therefore, $e_{it} = 0$ for all $i \in [0, 1]$ cannot constitute an equilibrium. Next, suppose that $e_{jt} > 0$ for some $j \neq i$, so that $\bar{e}_t > 0$. Because the marginal utility of the parent $i$ when $e_{it} = 0$ is positive, $e_{it} = 0$ cannot be the best response. Therefore, $e_{it} > 0$ for all $i \in [0, 1]$ in equilibrium.

**Proof of Proposition 3**

It is shown in Lemma 1 that $e_{it} > 0$ for all $i \in [0, 1]$. Using the first-order condition, the best response for parent $i$ is derived as

$$e_{it} = \min \left\{ \mu(w_{ht+1} - w_{lt+1}) \frac{q_{t+1}}{\bar{e}_t}, \sqrt{2 w_{it}} \right\} \tag{28}$$

By Assumption 2, choosing $e_{it} = \sqrt{\mu(w_{ht+1} - w_{lt+1}) q_{t+1}}$ for all $i \in [0, 1]$ is interior of the budget constraint, so that it constitute an equilibrium. Let this level of education be $\bar{e}_t$. I now explore whether there is another equilibrium such that the budget constraint binds for some parents. Suppose that there is such an equilibrium. Then the average education in equilibrium $\bar{e}'_t$ must be strictly greater than $\bar{e}_t$. The marginal utility of parent $i$ indicates

$$-e_{it} + \mu(w_{ht+1} - w_{lt+1}) \frac{q_{t+1}}{\bar{e}'_t} < -e_{it} + \mu(w_{ht+1} - w_{lt+1}) \frac{q_{t+1}}{\bar{e}_t} = 0 \tag{29}$$

so that the best response $e'_{it}$ must be strictly less than $e_{it}$, the best response in the previous equilibrium. Therefore, $\bar{e}'_t < \bar{e}_t$, which is a contradiction.

Next, I show that $\bar{e}_t$ strictly increases in both $A_t$ and $I_t$. Because $A_{t+1} = (1 - \delta) A_t + I_t$ is strictly increasing in $A_t$ and $I_t$, $w_{ht+1}$ and $w_{lt+1}$ are strictly increasing in both $A_t$ and $I_t$, and $q_{t+1}$ is weakly increasing in $A_t$ and $I_t$. Thus, from the first-order condition, the equilibrium education $\bar{e}_t$ is also strictly increasing in both $A_t$ and $I_t$.

**Proof of Proposition 4**

Suppose that every parent $i \in [0,1]$ chooses an equilibrium education $e_{it}$. Let $I_t = G_t$; the dictator invests in the whole budget. When $A_t \to 0$, $G_t \to 0$ so that $A_{t+1} \to 0$. Then $\bar{b}_t \to b_m$ because $\bar{e}_t \to 0$. By Assumption 1, the marginal utility for the dictator at $I_t = G_t$ is positive for a sufficiently small $A_t$ and negative for a sufficiently large $A_t$. Because the marginal utility is continuous and strictly decreasing in $A_t$, by the intermediate value theorem, there is a unique $A_t$, which I denote as $\underline{A}$, that makes the marginal utility equal to zero. Next, fix $I_t = 0$ and let $\overline{A}$ be the infrastructure $A_t$ such that the marginal utility at $I_t = 0$ equals zero. And the marginal utility of the investment at $I_t = 0$ and $\underline{A}$ is strictly positive. It follows that $\overline{A} > \underline{A}$.

Next, suppose that $A, A' \in (\underline{A}, \overline{A})$ and $A' > A$. And let $I$ and $I'$ be the optimal investments for $A$ and $A'$. To obtain a contradiction, assume that $I' \geq I$. As the optimality condition, the marginal utility of investment at $I$ and $A$ is zero. Because the marginal utility decreases in both $I_t$ and $A_t$ in this interval, the marginal utility at $I'$ and $A'$ must be negative, which violates the assumption that $I'$ is optimal for $A'$.

To show the existence of a steady state $A_{ss}^{\text{dict}}$, for $A \in [\underline{A}, \overline{A}]$, the dictator's budget set is $[0, G_t]$, which is compact and continuous in $A_t$. By *Berge's maximum theorem*, optimal investment $I_t$ is continuous in $A_t$; write it as $I(A_t)$. The steady state satisfies $I(A) - \delta A = 0$. Because $I(\underline{A}) - \delta \underline{A} > 0$ and $I(\overline{A}) - \delta \overline{A} < 0$, by the intermediate value theorem, there is $A_{ss}^{\text{dict}} \in (\underline{A}, \overline{A})$ such that $I(A_{ss}^{\text{dict}}) - \delta A_{ss}^{\text{dict}} = 0$. This steady state is unique, as the marginal utility of investment is strictly increasing in $A_t$ on this interval.

**Proof of Proposition 5**

Suppose that every parent $i$ chooses education as their best response. From Proposition 4, optimal investment for the dictator is $I_t^{\text{dict}} = G_t$ for all $A_t \leq \underline{A}$, $I_t^{\text{dict}} = 0$ for all $A_t \geq \overline{A}$. And $I_t^{\text{dict}}$ is continuous and strictly decreasing in $A_t \in [\underline{A}, \overline{A}]$. On the other hand, by Assumption 3, investment under democracy $I_t^{\text{dem}} = (1 - \alpha_t^*) G_t$ is in the interior of $[0, G_t]$ and strictly increases in $A_t$. Then $I_t^{\text{dict}} - I_t^{\text{dem}} > 0$ for all $A_t \leq \underline{A}$ and $I_t^{\text{dict}} - I_t^{\text{dem}} < 0$ for all $A_t \geq \overline{A}$. On $[\underline{A}, \overline{A}]$, because $I_t^{\text{dict}}$ strictly decreases and $I_t^{\text{dem}}$ strictly increases in $A_t$, there is $A_t = \tilde{A}$ such that $I_t^{\text{dict}} - I_t^{\text{dem}} = 0$. Then, $I_t^{\text{dict}} > I_t^{\text{dem}}$ for all $A_t < \tilde{A}$ and $I_t^{\text{dem}} > I_t^{\text{dict}}$ for all $A_t > \tilde{A}$, as desired.

**Proof of Proposition 6**

Let me denote by $\alpha^*(\bar{d}_{t-1})$ the value of $\alpha_t^*$ at $\bar{d}_{t-1}$. The steady state requires that $(1 - \alpha_t^*) G_t = (1 - \delta) A_t$.

Suppose first that $\lambda_m \geq \bar{\lambda}^{\text{growth}}$. Then $(1 - \alpha_t^*) G_t > \delta A_t$ for all $0 < A_t \leq \{\bar{q}/\varphi \pi_h\}^2$. This means that infrastructure increases and no steady state in this range. When $A_t \geq \{\bar{q}/\varphi \pi_h\}^2$, by Assumption 2, government revenue $G_t$ is $\tau \left\{ 2\pi_h \sqrt{A_t} \bar{q} - \bar{q}^2/\varphi \right\}$ and $\alpha_t^* = \alpha^*(\eta)$, the upper bound of $\alpha_t^*$. It is obvious that $(1 - \alpha^*(\eta)) G_t > \delta A_t$ for $A_t = \{\bar{q}/\varphi \pi_h\}^2$. For a sufficiently high

$A_t$,

$$\tau\left\{1-\alpha^*(\eta)\right\}\left\{2\pi_h\sqrt{A_t}\bar{q}-\bar{q}^2/\varphi\right\}<\delta A_t \tag{30}$$

because the left-hand side of the inequality is strictly concave in $A_t$ and the right-hand side is linear in $A_t$. And $(1-\alpha^*(\eta))G_t$ is continuous in $A_t$, so there is a steady state $A_{ss}^{\text{dem}}$ that makes $(1-\alpha^*(\eta))G_t = \delta A_t$. To show that $A_{ss}^{\text{dem}} > A_{ss}^{\text{dict}}$, by Assumption 1, at $A_{ss}^{\text{dict}}$, $\bar{d}_t < \eta$. On the other hand, at $A_{ss}^{\text{dem}}$, $\bar{d}_t = \eta$. Because $\bar{d}_t$ increases in infrastructure, $A_{ss}^{\text{dem}} > A_{ss}^{\text{dict}}$.

Next, suppose that $\lambda_m < \bar{\lambda}^{\text{growth}}$. For $A_t \in \left[0, \{\bar{q}/\varphi\pi_h\}^2\right]$, $(1-\alpha_t^*)G_t$ is convex in $A_t$.[14] When $A_t = \{\bar{q}/\varphi\pi_h\}^2$, $(1-\alpha_t^*)G_t > \delta A_t$. And, for a sufficiently small $A_t$, $(1-\alpha^*(\bar{d}_t))G_t < \delta A_t$. By the intermediate value theorem, there exists $A_t$ such that $(1-\alpha^*(\bar{d}_t))G_t = (1-\delta)A_t$, denoted as $\bar{A}_{ss}$. With this $\bar{A}_{ss}$, $\bar{d}_{t-1}$ must be smaller than $\eta$, which follows from $\{1-\alpha^*(\eta)\}\tau\varphi\pi_h^2 > \delta$. Then $(1-\alpha_t^*)G_t$ for $A_t$ below $\bar{A}_{ss}$ is strictly convex, so $(1-\alpha_t^*)G_t$ is smaller than $\delta A_t$ and converges to 0 as $t\to\infty$. Thus, 0 becomes the low steady state. For $A_t$ greater than $\bar{A}_{ss}$, it is trivial to show that it converges to $A_{ss}^{\text{dem}}$.

Finally, I show that $\bar{A}_{ss}$ decreases in $\lambda_m$. Fix $\lambda_d$. $\bar{A}_{ss}$ satisfies

$$\left\{1-\alpha^*(\bar{d}_{t-1})\right\}G_t = \delta\bar{A}_{ss}. \tag{31}$$

$G_t = \tau\varphi\pi_h^2\bar{A}_{ss}$ as $\bar{A}_{ss} < \left\{\bar{q}/\varphi\pi_h^2\right\}^2$. Hence,

$$\frac{1}{\lambda_d\bar{d}_{t-1}+\lambda_m(1-\bar{d}_{t-1})} = 2\left(1-\frac{\delta}{\tau\varphi\pi_h^2}\right). \tag{32}$$

Let $\lambda_m' = \lambda_m + \varepsilon$ for a small $\varepsilon > 0$. And let $\bar{A}_{ss}'$ be the threshold with $\lambda_m'$. Assume, for contradiction, that $\bar{A}_{ss}' \geq \bar{A}_{ss}$. Because the right-hand side is constant,

$$\lambda_d\bar{d}_{t-1}+\lambda_m(1-\bar{d}_{t-1}) = \lambda_d\bar{d}_{t-1}'+\lambda_m'(1-\bar{d}_{t-1}') \tag{33}$$

where $\bar{d}_{t-1}'$ is the proportion of democratic citizens with $\bar{A}_{ss}'$. Because $\bar{d}_{t-1}' > \bar{d}_{t-1}$, this equality does not hold, which means $\bar{A}_{ss}'$ is not the threshold. Therefore, $\bar{A}_{ss}' < \bar{A}_{ss}$.

**Proof of Proposition 7**

Let $I_t^{\text{dict}}$ be the optimal investment for the dictator. Suppose that the first benchmark investment $I_t^{\alpha}$ is the optimal investment with continuation probability $Pr[M_t < 1-\theta_t|\bar{d}_t] = \left\{\theta^*(\bar{d}_t)-\underline{\theta}\right\}/\left\{\bar{\theta}-\underline{\theta}\right\}$. Similarly, suppose that the second benchmark investment $I_t^{\beta}$ is the optimal investment with continuation probability $Pr[M_t < 1-\theta_t|\bar{d}_t] = \left\{\bar{b}_t-\underline{\theta}\right\}/\left\{\bar{\theta}-\underline{\theta}\right\}$. From the first-order condition, $I_t^{\alpha}$ is analogous to Proposition 4: For some thresholds $\underline{A}$ and $\overline{A}$, $I_t^{\alpha} = G_t$ for $A_t \leq \underline{A}$, $I_t^{\alpha}$ decreases for $A_t \in [\underline{A}, \overline{A}]$ and then $I_t^{\alpha} = 0$ for $A_t \geq \overline{A}$. On the other hand, $I_t^{\text{dem}}$ is low when $A_t$ is low and increases in $A_t$. There is $A_t$ such that $I_t^{\alpha} = I_t^{\text{dem}}$, denoted as $\underline{A}^{\text{leg}}$. It is obvious that $I_t^{\text{dict}} > I_t^{\text{dem}}$ for all $A_t < \underline{A}^{\text{leg}}$. With $A_t = \underline{A}^{\text{leg}} + \varepsilon$ for a sufficiently small $\varepsilon > 0$, $I_t^{\alpha} < I_t^{\text{dem}}$,

[14] Specifically, on the interval, it is strictly convex if $\bar{d}_{t-1} < \eta$ and linear if $\bar{d}_{t-1} = \eta$.

which means that $I_t^{\mathrm{dem}}$ does not satisfy the first-order condition. However, optimal investment is $I_t^{\mathrm{dict}} = I_t^{\mathrm{dem}}$ because the investment below $I_t^{\mathrm{dem}}$ makes the regime unstable from collective action, so that

$$\begin{aligned} &G_t - \kappa I_t^{\mathrm{dem}} + \beta \Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\mathrm{dict}} \geq I_t^{\mathrm{dem}}] G_{t+1}^{\alpha} \\ &> G_t - \kappa I_t^{\beta} + \beta q \Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\mathrm{dem}} > I_t^{\mathrm{dict}}] G_{t+1}^{\beta} \end{aligned} \tag{34}$$

where $G_{t+1}^{\alpha}$ and $G_{t+1}^{\beta}$ are government revenues from investments $I_t^{\mathrm{dem}}$ and $I_t^{\beta}$, respectively. Next, I show that, for a sufficiently large $\gamma_d$, there is $A_t$ such that the inequality is reversed. Because $\Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\mathrm{dict}} \geq I_t^{\mathrm{dem}}] = \{\theta^* - \underline{\theta}\} / \{\overline{\theta} - \underline{\theta}\}$ and $\Pr[M_t < 1 - \theta_t | \bar{d}_t, I_t^{\mathrm{dem}} > I_t^{\mathrm{dict}}] = \{\bar{b}_t - \underline{\theta}\} / \{\overline{\theta} - \underline{\theta}\}$, the following is obtained from deducting the right-hand side from the left-hand side:

$$\kappa(I_t^{\beta} - I_t^{\mathrm{dem}}) + \beta \left\{ \frac{\theta^* - \underline{\theta}}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\alpha} - \frac{\bar{b}_t - \underline{\theta}}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\beta} \right\} \tag{35}$$

For a sufficiently high $A_t$, it becomes

$$-\kappa I_t^{\mathrm{dem}} + \beta \frac{\theta^*(\eta) - \bar{b}_t(\eta)}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\alpha} + \beta \frac{\bar{b}_t(\eta) - \underline{\theta}}{\overline{\theta} - \underline{\theta}} \left\{ G_{t+1}^{\alpha} - G_{t+1}^{\beta} \right\} \tag{36}$$

$$< -\kappa I_t^{\mathrm{dem}} + \beta \frac{\theta^*(\eta) - \bar{b}_t(\eta)}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\alpha} + \frac{\kappa}{\tau \varphi \pi_h^2} \left\{ G_{t+1}^{\alpha} - G_{t+1}^{\beta} \right\} \tag{37}$$

$$< -\kappa I_t^{\mathrm{dem}} + \beta \frac{\theta^*(\eta) - \bar{b}_t(\eta)}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\alpha} + \frac{\kappa \bar{q}}{\varphi \pi_h} \sqrt{I_t^{\mathrm{dem}}} \tag{38}$$

where the first inequality is obtained from Assumption 1. Because $I_t^{\mathrm{dem}} = (1 - \alpha^*(\eta)) G_t$, it is equal to

$$\kappa \sqrt{(1 - \alpha^*(\eta)) G_t} \left[ -\sqrt{(1 - \alpha^*(\eta)) G_t} + \frac{\bar{q}}{\varphi \pi_h} \right] + \beta \frac{\theta^*(\eta) - \bar{b}_t(\eta)}{\overline{\theta} - \underline{\theta}} G_{t+1}^{\alpha} < 0 \tag{39}$$

when $\theta^*(\eta) - \bar{b}_t(\eta)$ becomes negligible for $\gamma_d$ sufficiently high and the terms within the bracket for the first term is negative for a high $A_t$. Thus, there is $A_t$, denoted by $\overline{A}^{\mathrm{leg}}$, such that (35) becomes 0. Then for all $A_t > \overline{A}^{\mathrm{leg}}$, investing less than $I_t^{\mathrm{dem}}$ is optimal for the dictator.

**Proof of Lemma 2**

Let $A_{t+1}$ be given. To obtain the contradiction, suppose that a tax rate $\tau < \hat{\tau}$ is optimal for the dictator. Let $\tau' = \tau + \varepsilon$ for a small $\varepsilon > 0$ such that $\tau' < \hat{\tau}$. Because $(1-\tau) w_{ht+1} > (1-\tau') w_{ht+1}$ with the same $w_{lt+1}$, equilibrium average education under $\tau$, $\bar{e}_t$, is strictly greater than that under $\tau'$, $\bar{e}_t'$. Also, government revenue under $\tau$, $G_{t+1}$, is greater than that under $\tau'$, $G_{t+1}'$, which means $\tau'$ gives greater payoff than $\tau$, hence $\tau$ is not optimal.

### Proof of Proposition 8

Let $A_{t+1}$ be given. From the result of Lemma 2, any tax rate below $\hat{\tau}$ cannot be the optimal tax rate. So, I focus on $\tau_{t+1} \geq \hat{\tau}$.

As previously shown, $\Pr[M_t < 1 - \theta_t | \bar{d}_t(\tau_{t+1})] = \left\{\bar{b}_t(\tau_{t+1}) - \underline{\theta}\right\} / \left\{\bar{\theta} - \underline{\theta}\right\}$ where $\bar{b}_t(\tau_{t+1})$ is the value of $\bar{b}_t$ with $\bar{d}_t = \{(1 - \tau_{t+1})\pi_h - \pi_l\}\,\varphi\pi_h A_{t+1}$. And $G_t = \tau_t(1 - \tau_t/2\hat{\tau})\varphi\pi_h^2 A_t$. The next period expected utility for the dictator in period $t$ is

$$\left(\frac{\bar{b}_t(\tau_{t+1}) - \underline{\theta}}{\bar{\theta} - \underline{\theta}}\right)\left\{\tau_{t+1}\left(1 - \frac{\tau_{t+1}}{2\hat{\tau}}\right)\varphi\pi_h^2 A_{t+1} - \kappa I_{t+1}\right\}. \tag{40}$$

This is maximised when $I_{t+1} = 0$. Differentiating with respect to $\tau_{t+1}$,

$$\left(\frac{\varphi\pi_h^2 A_{t+1}}{\bar{\theta} - \underline{\theta}}\right)\left\{\left(1 - \frac{\tau_{t+1}}{\hat{\tau}}\right)(\bar{b}_t(\tau_{t+1}) - \underline{\theta}) + \tau_{t+1}\left(1 - \frac{\tau_{t+1}}{2\hat{\tau}}\right)(b_m - b_d)\mu\varphi\pi_h^2 A_{t+1}\right\} \tag{41}$$

As the first-order condition, the choice of $\tau_{t+1}$ needs to make this derivative equal to 0. To show that $\tau_{t+1} = \hat{\tau}$ is not optimal, by substituting $\tau_{t+1} = \hat{\tau}$,

$$\left(\frac{\varphi\pi_h^2 A_{t+1}}{\bar{\theta} - \underline{\theta}}\right)\left\{\frac{\hat{\tau}}{2}(b_m - b_d)\mu\varphi\pi_h^2 A_{t+1}\right\} > 0 \tag{42}$$

so that $\hat{\tau}$ is not optimal for a given $A_{t+1} > 0$. And twice differentiating with respect to $\tau_{t+1}$,

$$\left(\frac{\varphi\pi_h^2 A_{t+1}}{\bar{\theta} - \underline{\theta}}\right)\left\{-\frac{\bar{b}_t(\tau_{t+1}) - \underline{\theta}}{\hat{\tau}} + \left(1 - \frac{\tau_{t+1}}{\hat{\tau}}\right)(b_m - b_d)\mu\varphi\pi_h^2 A_{t+1}\right\} \tag{43}$$

which is negative for $\tau_{t+1} \geq \hat{\tau}$. Therefore, $\tau_{t+1}^*$ must be greater than $\hat{\tau}$.

Next, to show that $\tau_{t+1}^*$ increases in $A_{t+1}$, let $A_{t+1}$ and $A'_{t+1}$ with $A'_{t+1} > A_{t+1}$. The derivative is strictly greater with $A'_{t+1}$ than $A_{t+1}$. Therefore, $\tau_{t+1}^*$ with $A'_{t+1}$ is greater than $\tau_{t+1}^*$ with $A_{t+1}$.

### Proof of Proposition 9

From Lemma 2, $\tau_{t+1}$ below $\hat{\tau}$, and $\tau_{t+1}$ above $2\hat{\tau}$ are not optimal due to the deadweight loss. So I restrict the focus to $\tau_{t+1} \in [\hat{\tau}, 2\hat{\tau}]$. Denote by $I_t^{\text{dict}}$ and $I_t^{\text{tax}}$ the optimal investment when $\tau_t$ is fixed to $\hat{\tau}$ and when $\tau_t$ is optimally chosen by the dictator, respectively. Define $V(A_{t+1}, \tau_{t+1})$ by

$$V(A_{t+1}, \tau_{t+1}) = \Pr[M_t < 1 - \theta_t | \bar{d}_t(\tau_{t+1})] G_{t+1}. \tag{44}$$

Fix $\tau_{t+1}$ and twice differentiate $V(A_{t+1}, \tau_{t+1})$ with respect to $A_{t+1}$,

$$V''_{AA}(A_{t+1}, \tau_{t+1}) = 2\left[\frac{b_d - b_m}{\bar{\theta} - \underline{\theta}}\right]\mu\left\{(1 - \tau_{t+1})\pi_h - \pi_l\right\}\varphi^2\pi_h^3 < 0 \tag{45}$$

so that $V(A_{t+1}, \tau_{t+1})$ is strictly concave in $A_{t+1}$. Let $\tau^*(A_{t+1})$ be the optimal tax rate for a given $A_{t+1}$. Because $[\hat{\tau}, 2\hat{\tau}]$ is compact, by the maximum theorem (see Sundaram, 1996, pp. 237–238),

$V^*(A_{t+1}) := V(A_{t+1}, \tau^*(A_{t+1}))$ is also strictly concave.

I describe optimal investment with the tax manipulation, $I_t^{\text{tax}}$. It is obvious that $I_t^{\text{tax}} = 0$ for $V_A^{*\prime}((1-\delta)A_t) \leq \kappa$, because the marginal cost of the investment, $\kappa$, is greater than the marginal expected revenue for the dictator. And, for $V_A^{*\prime}((1-\delta)A_t) > \kappa$, $I_t^{\text{tax}} = \min\{I_t, G_t\}$ with $I_t$ that satisfies $V_A^*((1-\delta)A_t + I_t) = \kappa$. Because $V^*$ is strictly concave, there is a unique $A^*$ such that $V_A^*(A^*)' = \kappa$.

The next task is to show that $\underline{A}^{\text{tax}} > \underline{A}^{\text{base}}$. Suppose not: $\underline{A}^{\text{base}} \geq \underline{A}^{\text{tax}}$. Let $A_t = \underline{A}^{\text{base}}$ and let $A_{t+1} = (1-\delta)\underline{A}^{\text{base}} + G(\underline{A}^{\text{base}}, \hat{\tau})$ where $G(A,\tau)$ is government revenue with infrastructure $A$ and tax rate $\tau$. By construction, $V_A'(A_{t+1}, \hat{\tau}) = \kappa$. By Proposition 8, $\tau^*(A) > \hat{\tau}$ for $A > 0$. Because $\underline{A}^{\text{base}} \geq \underline{A}^{\text{tax}}$ by assumption, $G(A_t, \hat{\tau}) > G(A_t, \tau^*)$ due to higher deadweight loss from the taxation, and $V^*$ is strictly concave, I have $V_A^{*\prime}(A'_{t+1}) < \kappa$ where $A'_{t+1} = (1-\delta)A_t + G(A_t, \tau^*)$.

The derivative of $V(A,\tau)$ with respect to $A$ is

$$V_A'(A,\tau) = 2\left[\frac{\bar{b}_t(\tau) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right]\tau\left(1 - \frac{\tau}{2\hat{\tau}}\right)\varphi\pi_h^2 - \left[\frac{b_m - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right]\tau\left(1 - \frac{\tau}{2\hat{\tau}}\right)\varphi\pi_h^2 \tag{46}$$

so that

$$\begin{aligned}
V_A'(A_{t+1},\tau^*) - V_A'(A_{t+1},\hat{\tau}) =& \varphi\pi_h^2\left[\tau^*\left(1 - \frac{\tau^*}{2\hat{\tau}}\right)\left\{2\left(\frac{\bar{b}_t(\tau^*) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right) - \left(\frac{b_m - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)\right\}\right] \\
&- \varphi\pi_h^2\left[\frac{\hat{\tau}}{2}\left\{2\left(\frac{\bar{b}_t(\hat{\tau}) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right) + \left(\frac{b_m - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)\right\}\right]
\end{aligned}$$

$$\begin{aligned}
V_A'(A_{t+1},\tau^*) - V_A'(A_{t+1},\hat{\tau}) =& 2\varphi\pi_h^2\left[\tau^*\left(1 - \frac{\tau^*}{2\hat{\tau}}\right)\left\{\frac{\bar{b}_t(\tau^*) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right\} - \frac{\hat{\tau}}{2}\left\{\frac{\bar{b}_t(\hat{\tau}) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right\}\right] \\
&- \varphi\pi_h^2\left(\frac{b_m - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right)\left[\tau^*\left(1 - \frac{\tau^*}{2\hat{\tau}}\right) - \frac{\hat{\tau}}{2}\right]
\end{aligned}$$

From the optimality of $\tau^*$, $V(A_{t+1},\tau^*) > V(A_{t+1},\hat{\tau})$ which leads to

$$\tau^*\left(1 - \frac{\tau^*}{2\hat{\tau}}\right)\left(\frac{\bar{b}_t(\tau^*) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right) > \frac{\hat{\tau}}{2}\left(\frac{\bar{b}_t(\hat{\tau}) - \underline{\theta}}{\overline{\theta} - \underline{\theta}}\right). \tag{47}$$

And $\hat{\tau}/2 > \tau^*(1 - \tau^*/2\hat{\tau})$. Then $V_A'(A_{t+1},\tau^*) > V_A'(A_{t+1},\hat{\tau})$. Because $A_{t+1} > A'_{t+1}$, $V_A'(A'_{t+1},\tau^*) > \kappa$ due to the strict concavity of $V$, which is a contradiction. Therefore, $\underline{A}^{\text{tax}} > \underline{A}^{\text{base}}$. Following the same procedure, I conclude that $\overline{A}^{\text{tax}} > \overline{A}^{\text{base}}$.