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Abstract

We study the impact of foreign exchange interventions during periods of tight credit constraints. Expanding on the Gabaix and Maggiori (2015) model, we predict that long-lived spot interventions have larger effects on exchange rates than short-lived swaps, unanticipated interventions are more impactful, and tighter credit constraints amplify effects. Using high-frequency data on Brazilian Central Bank interventions from 1999 to 2023, we find that unanticipated spot sales of USD reserves lead to significant domestic currency appreciation and reduced covered interest parity deviations. Spot interventions outperform swaps, especially when global intermediaries are constrained, and enhance market efficiency by lowering USD borrowing costs.

JEL classification: E44; E58; F31; G14
Keywords: Exchange Rate; Central Bank; Interventions; Yield Curve; Asset Pricing

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1 Introduction

Researchers in international finance have long studied how central banks influence currency markets through foreign exchange interventions (FXI). While it is widely accepted that non-sterilized interventions affect exchange rates by altering the money supply, the effectiveness of sterilized interventions and the mechanisms through which they operate remain subjects of ongoing debate (see Sarno and Taylor, 2001, for a review that remains relevant today).

This debate persists largely because identifying the causal effects of FXI on financial and macroeconomic variables poses two significant empirical challenges. Specifically, clean identification requires interventions that are unanticipated by market participants and not driven by the central bank's endogenous responses to current market conditions (see Fratzscher et al., 2019, for a discussion).

While we do not fully resolve both of these empirical challenges, we make significant progress on the first by employing a high-frequency dataset released in 2023 that details every intervention by the Brazilian Central Bank (BCB) since 1999—over 8,000 interventions in total. This comprehensive dataset enables us to distinguish between interventions that were announced ahead of time and those that were not, thereby surprising market participants. By focusing on these unanticipated events, which can be viewed as news relative to the information set of market participants, we can trace the effects of interventions on market variables, following a methodology similar to that used in event studies on monetary policy surprises (see Gürkaynak et al., 2021, for a discussion).[1]

Our main contribution is to show that the impact of FXI depends on the degree to which global FX dealers are constrained in supplying U.S. Dollar (USD) liquidity. When intermediaries are constrained, FXI has a greater capacity to affect spot and forward rates because the supply of cross-border capital is less elastic. We identify this mechanism as the *dollar intermediation* channel, which can be seen as a version of the standard

[1]From an econometric point of view, the use of market surprises in event studies is widely accepted, as these surprises are considered news relative to the information sets of market participants. However, identifying structural policy shocks and their causal effects requires that the instrument capture innovations that are both news to market participants and orthogonal to the economic state. The orthogonality assumption may be violated if the information sets of policymakers and market participants diverge. We acknowledge this caveat in our methodology and leave further exploration of this issue to future studies.

portfolio balance channel discussed in the literature. The channel arises from the imperfect sustainability of domestic currency and USD, and from the existence of financial frictions limiting access to USD liquidity.[2]

To shed light on the dollar intermediation channel, we study the interaction between currency and interest rate markets through the covered interest rate parity (CIP) condition, an important no-arbitrage condition in international finance. Violations of CIP can indicate a relative scarcity of USD liquidity in cross-border financial markets. Central banks can alleviate this scarcity, using FXI to supply USD liquidity and reduce friction in FX intermediation. We exploit the history of BCB interventions to test whether unanticipated sales of USD reserves by the BCB lead to systematic domestic currency appreciation and a decline in CIP violations.

To motivate our empirical framework, we extend the Basic Gamma Model of Gabaix and Maggiori (2015), where financial frictions play a central role. In our extension, we distinguish between spot and swap interventions and allow for partially or fully anticipated interventions. The model involves two countries, Home and Foreign, each with a representative household, domestic tradable and non-tradable goods, and a domestic bond. Households consume domestic and international goods but only hold domestic bonds due to the lack of access to international bond markets.

A foreign exchange Financier intermediates bond trade, constructing a zero-cost portfolio of Home and Foreign bonds to maximize expected profit subject to a credit constraint. The Central Bank conducts short-lived swap interventions that are unwound after one period and long-lived spot interventions that are maintained for two periods, intervening when trade shocks exceed set thresholds. These thresholds are communicated as distributions to manage anticipation of interventions.

The model yields three key empirical predictions: long-lived spot interventions have larger exchange rate effects than short-lived swaps because they alter portfolio compositions for longer; unanticipated interventions have larger effects than anticipated ones because agents cannot adjust consumption and investment plans in advance to offset them; and

[2]The BCB does not have any active foreign-currency liquidity swap line with the Fed and hence has to commit its reserves to supply USD to domestic financial firms.

tighter credit constraints amplify the effects of all interventions by rendering the Financier's supply of intermediation less elastic.

To test model predictions, we exploit a large historical database on BCB interventions. The database includes a high-frequency timestamp of each intervention, the intervention type (spot or swap), and the sign (buy or sell of USD) and size of the intervention. We classify interventions as either anticipated or unanticipated depending on whether the announcement of the intervention occurs on the same day as the auction that implements it.

We combine our database of interventions with spot and futures prices at the 5-minute intervals from Thomson Reuters Tick History. Using this data, we measure the effects of FX buy and sell interventions on prices, interest rates, and CIP violations. Our analysis uses the local projections method in Jordà (2005), a procedure that controls for the intervention amount and for feedback effects in prices.

First, we estimate the impact of unanticipated interventions conditional on intervention type. In response to BCB spot sell interventions, the Brazilian Real (BRL) appreciates by approximately 150 basis points over a seven-hour horizon per 1 USD billion sold. The appreciation is statistically significant at a 95% confidence level. In response to BCB spot buy interventions, the BRL depreciates by approximately 10 basis points over the same horizon, but the depreciation is not statistically significant. The asymmetry in effect sizes suggests an important role for the dollar intermediation channel, whereby spot sell interventions improve USD liquidity and loosen intermediary constraints, as predicted by our model.[3] In response to BCB swap interventions, both traditional and reverse, the BRL shows no significant response, again consistent with the predictions of our theoretical model.

Second, we examine the effects on cross-border funding by measuring the difference in borrowing costs between BRL and USD. In practice, borrowing USD synthetically

[3]Historically, BCB spot sell interventions are clustered around economic stress periods for Brazil: 2001–02, 2009, and 2020–21. These periods saw sharp drops in intermediary capital ratios, as we show in Figure B.3 in the Online Appendix. In contrast, BCB spot purchase interventions tend to occur in periods with healthy intermediary capital ratios. Our theoretical model predicts a stronger exchange rate response to FXI when intermediaries are constrained. With capital ratios proxying for intermediary constraints, our empirical findings support this prediction.

by converting BRL to USD in the forward market is more expensive. This inefficiency is reflected in CIP violations, which measure the difference in BRL and USD interest rates after hedging exchange rate risk with a forward contract. CIP violations reflect several factors, including the scarcity of USD liquidity, as USD is the reserve currency and global supply is constrained. We test whether FXI impacts CIP violations and find that unanticipated sell interventions lead to a decline in the magnitude of these violations. The decline improves cross-border market efficiency by reducing the relative cost of borrowing USD through forward markets, aligning with our model's predictions that FXI alleviates dollar intermediation constraints.

Third, we test whether the impact of FXI on spot prices and CIP violations is more pronounced during periods of tighter intermediary constraints. When USD liquidity is limited, the central bank conducts USD sale operations, anticipating stronger FXI price effects due to the inelastic USD supply. We use the dealer capital ratio from He et al. (2017) as a proxy for intermediary constraints, which measures the capital-to-assets ratio for key global FX dealers. Our findings show that spot sales have significant effects primarily during periods of tighter intermediary constraints. Specifically, while swap FXI generally does not significantly impact the spot rate, it does have significant effects when constraints are tighter.

Finally, we explore an alternative FXI channel: signaling future interest rates. Using high-frequency and daily interest rate data, we analyze the immediate and long-term effects of unanticipated FXI on interest rates. Our findings suggest that unanticipated spot sales of USD result in an economically insignificant short-term increase in interest rates based on intra-day data, consistent with FXI sterilization. Longer-term analysis provides weak evidence of a systematic effect on interest rates following both spot and swap FXI. Overall, the evidence lends less support to the signalling channel than to the dollar intermediation channel as the relevant theory explaining our findings.

Related Literature. Our paper contributes to a literature on the channels through which FXI affects exchange rates (e.g. Sarno and Taylor, 2001; Neely, 2005; Vitale, 2006; Menkhoff, 2010). These studies discuss primarily two channels through which sterilized FXI may operate: the portfolio balance and signaling channels. In a standard sterilized intervention, the Central Bank sells foreign currency reserves while simultaneously purchasing domestic bonds. In the portfolio balance channel, if foreign currency is perceived as riskier than domestic bonds, private agents will absorb more foreign currency in their portfolios only if its relative price decreases. In the signalling channel, selling foreign currency reserves signals an anticipated increase in domestic interest rates. In our paper, the *dollar intermediation* channel, a variation of the portfolio balance channel, emphasizes how financial constraints affect intermediaries' ability to supply USD liquidity. We find strong empirical support for the dollar intermediation channel of FXI in Brazil, but weak support for the signaling channel.

Our paper relates to empirical studies on the impact of FXI on exchange rates and other markets (e.g. Kearns and Rigobon, 2005; Menkhoff et al., 2021; Fratzscher et al., 2019, 2020; Naef, 2024; Dominguez et al., 2013; Arango-Lozano et al., 2020), as well as studies on BCB interventions (Nakashima, 2012; Kohlscheen and Andrade, 2013; Janot and Macedo, 2016; Santos, 2021). For example, Kohlscheen and Andrade (2013) found an impact of 29 basis points per billion USD using intra-day spot and futures prices, while Nedeljkovic and Saborowski (2019) reported up to 100 basis points per billion USD with an IV approach. Barroso (2019) estimated impacts between 51 to 118 basis points using IV analysis, and Santos (2021) found a peak effect of 30 basis points using intra-day data on futures. Turning to swap interventions, da Costa Filho (2021) identified a smaller effect of 3-5 basis points per billion USD. We find larger effect sizes when considering BCB spot sell interventions separately, but similar effect sizes when considering buy and sell interventions jointly.

We contribute to the literature on FXI as a central banking policy tool (e.g. Ferreira et al., 2019; Fratzscher et al., 2019, 2020; Levy-Yeyati and Gómez, 2022; Naef, 2024; Rodnyansky et al., 2023). Fratzscher et al. (2019) show that central banks use FXI to stabilize exchange rates by countering private market trades, while Rodnyansky et al. (2023) show that central banks use FXI to mitigate spillovers from foreign monetary policy, protecting firms with high foreign-currency debt. While our focus is on the effects of FXI and its operating channels, not the central bank's motivations or policy goals, our theoretical model does assume an FXI reaction function aimed at exchange rate stabilization, consistent with Fratzscher et al. (2019). The reaction function allows the central bank to remain inactive until intervention is warranted, and allows the central bank to surprise market participants.

We also engage with theoretical work on FXI in models with financial frictions (see Gabaix and Maggiori, 2015; Agénor et al., 2020; Fanelli and Straub, 2021; Agénor and da Silva, 2023; Beltran and He, 2024; Candian et al., 2024; Pelin, 2024; Yago, 2024). Agénor et al. (2020) and Agénor and da Silva (2023) highlight real macroeconomic effects of sterilized interventions through a bank portfolio channel. Beltran and He (2024) model the FXI needed in EMEs during currency demand shocks. Candian et al. (2024) develop a framework in which central banks use opaque FXI to prevent overreactions to information about economic fundamentals. Yago (2024) models FXI and monetary policy combinations that mitigate the inflation-output trade-off. Relative to these models, we introduce distinctions between spot versus swap and anticipated versus unanticipated interventions that interact with financial frictions and yield testable predictions for our empirical analysis.

Finally, we contribute to the literature on emerging market CIP violations (Du and Schreger, 2016; Hartley, 2020; Bahaj and Reis, 2020; Cerutti and Zhou, 2024; Hertrich and Nathan, 2023), which explores CIP violations, sovereign risk, and central bank swap lines. We find that BCB spot sell interventions reduce USD scarcity and CIP violations for BRL/USD, complimenting the work of Hertrich and Nathan (2023), who find that Bank of Isreal spot sell interventions reduce CIP violations for ILS/USD. Our theoretical model offers a formal explanation for this finding, and our empirical results for Brazil—an emerging market economy with greater external vulnerabilities—strengthen the robustness of the finding.

In summary, our contributions to the existing literature are fourfold. First, we emphasize the dollar intermediation channel of FXI and provide empirical support for FXI models that incorporate financial frictions. Second, we analyze a large number of interventions over a long sample period at high frequency, using the precise timing of interventions for identification. Third, we differentiate between and separately estimate the effects of spot versus swap interventions, as well as anticipated versus unanticipated interventions. Fourth, we provide evidence that central banks can reduce CIP violations to improve FX market efficiency by supplying USD liquidity. Together, these contributions enhance our understanding of FXI.

The remainder of the paper is structured as follows. In Section 2, we introduce a theoretical model of FXI with testable predictions for exchange rates and CIP violations. In Section 3, we introduce the BCB's institutional setting and describe our data. In Section 4, we present our findings for the effects of FXI on the exchange rate and CIP violations, and test for the signaling channel. In Section 5, we conclude.

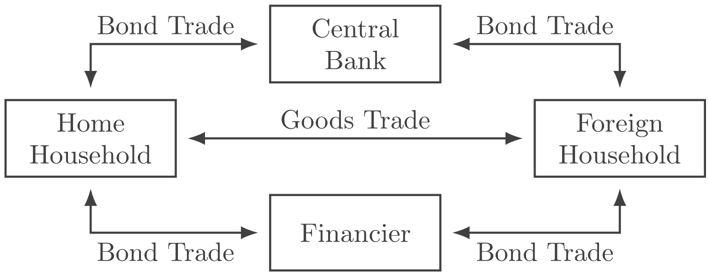
2 Theoretical Model

We extend the Basic Gamma Model of Gabaix and Maggiori (2015) and derive predictions that motivate our empirical investigation of Central Bank interventions in foreign exchange markets. In our extension, we introduce a rudimentary distinction between spot and swap interventions and we allow interventions to be partially or fully anticipated.

There are two countries, Home and Foreign, each with a representative Household, a domestic tradable and non-tradable good, and a domestic bond. Households receive endowments of domestic tradable and non-tradable goods and consume baskets of domestic and international goods. Households have access to international goods markets but no access to international bond markets, and therefore hold only domestic bonds.

A representative FX dealer intermediates bond trade, known as a Financier in Gabaix and Maggiori (2015). The Financier begins each period with zero assets and constructs a zero-cost portfolio of Home and Foreign bonds to maximize expected single-period profit. By trading in both bond markets, the Financier absorbs imbalances in demand for Home and Foreign bonds. However, creditors impose constraints on the Financier that limit its ability to intermediate optimally.

Figure 1: Structure of the Gamma-Eta Model

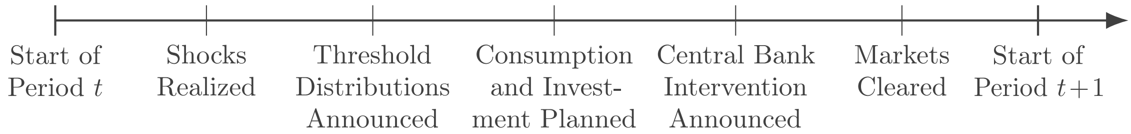


Notes. The figure depicts the structure of the Gamma-Eta Model, with trade flows in goods and financial markets and four decision makers: the Home Household, the Foreign Household, the Financier, and the Central Bank. Figure A.1 in the Online Appendix provides a more complete picture of the model, showing individual markets and distinguishing between real flows and currency flows.

A Central Bank conducts two types of sterilized FXI: short-lived swap interventions that are unwound after one period and long-lived spot interventions that are maintained for two periods. The Central Bank intervenes when trade shocks exceed exogenous thresholds that it sets. The Central Bank withholds its exact threshold values from the Household and Financier and instead communicates distributions of possible values. The Central Bank sets the parameters of the distributions (through a unmodeled communication policy) to control the anticipation of its interventions. Households and the Financier form plans based on the probability of an intervention, before the Central Bank announces its intervention decision each period.

The model makes three central empirical predictions: first, that long-lived spot interventions have larger exchange rate effects than short-lived swap interventions because they alter portfolio compositions for longer; unanticipated interventions have larger effects than anticipated ones because agents cannot adjust consumption and investment plans to offset them; and tighter credit constraints amplify the effects of all interventions by rendering the Financier's supply of intermediation less elastic. Figure 1 provides an overview of the model structure, and Figure 2 illustrates the timing assumptions we make in the model. We present the model and state empirical predictions below and provide additional derivations and proofs in Appendix B.

Figure 2: Phases of Periods Zero and One in the Gamma-Eta Model



Notes. Periods zero and one are divided into five phases. In the first phase, preference shocks are realized and observed by all. In the second phase, the Central Bank announces distributions for its intervention thresholds. If trade shocks exceed the thresholds, the Central Bank will carry out a foreign exchange intervention. In the third phase, the Home and Foreign Households and the Financier form consumption and investment plans. In the fourth phase, the Central Bank reveals its foreign exchange intervention. In the fifth phase, prices are determined and markets clear.

The Home Household. The Home Household solves a three-period intertemporal utility maximization problem,

$$\max_{\{C_{NTs},C_{Hs},C_{Fs}\}_{s=t}^{2}} \sum_{s=t}^{2} \beta^s \, \mathrm{E}_t^{(-)}[\varphi_s \ln C_s] \,,$$

where $t \in \{0,1,2\}$, $\mathrm{E}_t^{(-)}[\cdot]$ denotes a conditional expectation operator that includes period-t information on realized preference shocks but excludes information on the exact intervention thresholds the Central Bank sets in period t, β is the subjective discount factor, $\varphi_t \equiv \chi_t + \alpha_t + \iota_t$ is a sum of stochastic preference shocks, and where

$$C_t \equiv \left[(C_{NTt})^{\chi_t}(C_{Ht})^{\alpha_t}(C_{Ft})^{\iota_t}\right]^{\frac{1}{\varphi_t}} \tag{1}$$

denotes the Home Household consumption basket, a sub-utility function composed of Home non-tradable goods C_{NTt}, Home tradable goods C_{Ht}, and Foreign tradable goods C_{Ft}. The intertemporal maximization is subject to period budget constraints of the form

$$Q_{Ht} + p_{Ht}C_{Ht} + p_{Ft}C_{Ft} + C_{NTt} = Y_{NTt} + p_{Ht}Y_{Ht} + R_{Ht}Q_{Ht-1} \,, \tag{2}$$

where Q_{Ht} denotes the real value of Home holdings of the Home bond, with $Q_{H2} = 0$, p_{Ht} and p_{Ft} are relative prices of the Home and Foreign tradable goods, and Y_{NTt} and Y_{Ht} are stochastic endowments of the Home non-tradable and tradable goods, respectively. The Home non-tradable good is the numéraire for the Home economy.

The Home Household's intertemporal utility maximization problem yields the following optimality conditions,

$$\mathrm{E}_t^{(-)}[\lambda_s] = \mathrm{E}_t^{(-)}\left[\chi_s / C_{NTs}\right] \quad \text{and} \quad \mathrm{E}_t^{(-)}[\lambda_s] = \beta\, \mathrm{E}_t^{(-)}[\lambda_{s+1} R_{Hs+1}]\,, \tag{3}$$

where λ_s denotes the Lagrange multiplier on the Home Household's period-t budget constraint and R_{Hs+1} is the gross real return on the Home bond, with $t \leq s < 2$.

The Home Household also solves an intratemporal utility maximization problem, allocating expenditure across domestic and international goods. Specifically, the Home Household maximizes the logarithm of its sub-utility function in (1),

$$\max_{C_{NTt}, C_{Ht}, C_{Ft}} \chi_t \ln C_{NTt} + \alpha_t \ln C_{Ht} + \iota_t \ln C_{Ft}\,,$$

subject to a consumption expenditure constraint,

$$p_t C_t = C_{NTt} + p_{Ht} C_{Ht} + p_{Ft} C_{Ft}\,, \tag{4}$$

where p_t denotes the Home price index in terms of the Home non-tradable numéraire, and where the Home Household takes total consumption expenditure $p_t C_t$ as fixed in the intratemporal problem. The Home Household's intratemporal problem yields the following optimality conditions for consumption expenditure on Home and Foreign tradable goods,

$$p_{Ft} C_{Ft} = (\chi_t / C_{NTt}) \iota_t \quad \text{and} \quad p_{Ht} C_{Ht} = (\chi_t / C_{NTt}) \alpha_t\,. \tag{5}$$

We maintain several simplifying assumptions that Gabaix and Maggiori (2015) make in their Basic Gamma Model: namely, that the Home non-tradable endowment adjust proportionally to fluctuations in the Home preference for non-tradable goods, $Y_{NTt} = \chi_t$, and that the Home Household does not discount future utility, $\beta = 1$. These assumptions simplify the model and focus our investigation on understanding the impact of Central Bank interventions on the real exchange rate.

The maximization problems of the Foreign Household are analogous to those of the

Home Household, and we maintain a similar set of simplifying assumptions for the Foreign Household. We present details of the Home Household's maximization problems and our specification of the Foreign Household in Appendices A.1 and A.2.

The Financier. The Financier intermediates bond trade and absorbs cross-country imbalances in Home and Foreign Household demand for bonds. While Households borrow and lend only in their respective domestic bonds, they trade in international goods markets where cross-country imbalances can arise. The Financier maximizes single-period firm value,

$$V_t = \beta \, \mathrm{E}_t^{(\cdot)} \left[R_{Ht+1} Q^f_{Ht} + \frac{e_{t+1}}{e_t} R^*_{Ft+1} Q^f_{Ft} \right], \tag{6}$$

where V_t denotes the Financier's value. The Financier's holdings of Home and Foreign bonds are denoted Q^f_{Ht} and Q^f_{Ft}, respectively, and the real return on the Foreign bond is R^*_{Ft+1}. The real exchange rate e_t is defined in units of the Home numéraire per unit of the Foreign numéraire. The Financier transfers any gains or losses to the Foreign Household each period.

The Financier has limited capacity to bear risk. This limitation derives from a constraint that creditors impose on the Financier to prevent it from diverting funds. The constraint aligns incentives such that the value to the Financier of ceasing to intermediate and diverting funds never exceeds the value of continuing to intermediate bond trade. To motivate the functional form of the constraint, Gabaix and Maggiori (2015) suggest that the Financier can more easily divert funds when its portfolio is larger. The constraint is given by

$$\frac{V_t}{\mathrm{E}_t^{(\cdot)}[e_t]} \geq \underbrace{\left| \frac{Q^f_{Ht}}{\mathrm{E}_t^{(\cdot)}[e_t]} \right|}_{\substack{\text{Creditor} \\ \text{Claims}}} \times \underbrace{\Gamma \left| \frac{Q^f_{Ht}}{\mathrm{E}_t^{(\cdot)}[e_t]} \right|}_{\substack{\text{Divertible} \\ \text{Fraction}}}, \tag{7}$$

where the parameter $\Gamma > 0$ governs the Financier's ability to divert funds, and therefore governs the risk-bearing capacity of the Financier. When Γ approaches zero, the Financier's risk-bearing capacity approaches infinity. When Γ approaches infinity, the Financier's risk-bearing capacity approaches zero and the economy approaches financial autarky.[4]

We assume the Financier begins each period with zero assets, so its balance sheet constraint is given by

$$Q_{Ht}^{f} + Q_{Ft}^{f} = 0\,, \tag{8}$$

and any bond position the Financier takes must cost nothing to construct.[5]

The Financier's profit maximization problem yields the following optimality condition, which pins down the amount of bond trade the Financier chooses to intermediate,

$$\frac{Q_{Ht}^{f}}{\mathrm{E}_0^{(-)}[e_t]} = \frac{1}{\Gamma}\,\mathrm{E}_0^{(-)}\left[R_{Ht+1} - \frac{e_{t+1}}{e_t}R_{Ft+1}^{*}\right]. \tag{9}$$

The expression shows how higher Γ corresponds to lower risk-bearing capacity and less intermediation.

Central Bank. The Central Bank conducts sterilized foreign exchange interventions by constructing zero-cost portfolios of Home and Foreign bonds. The Central Bank has no currency liabilities and transfers any gains or losses from its interventions to the Foreign Household each period. Accordingly, the Central Bank's balance sheet satisfies the following condition,

$$Q_{Ht}^{cb} + Q_{Ft}^{cb} = 0\,,$$

where Q_{Ht}^{cb} and Q_{Ft}^{cb} denote the real values of the Central Bank's holdings of Home and Foreign bonds.

The Central Bank intervenes when trade shocks rise above or fall below intervention thresholds that the Central Bank sets exogenously. In particular, the Central Bank's

[4] Gabaix and Maggiori (2015) allow Γ to depend positively on the variance of the real exchange rate. We find limited empirical support for a positive relationship between financial constraints on intermediaries and the variance of the real exchange rate outside of crisis periods. See the Online Appendix B.2 for more details. For this reason, we treat Γ as an exogenous parameter in our model.

[5] We assume the microfoundation in Gabaix and Maggiori (2015) to be operating in the background. In their setup, a measure of individual households populates each economy, and a new Household is randomly selected to serve as Financier each period. Households enter this occupation with zero assets, and maximize their single-period profits. Our representative agent model is isomorphic to the micro-founded model.

interventions take the form of bond purchases or sales defined by the following equation,

$$Q_{Ht}^{cb} = q\big[H(\Delta\iota_t - \bar{\iota}_t) - H(\underline{\iota}_t - \Delta\iota_t)\big] = q\Delta H_t\,, \qquad \underline{\iota}_t \leq \bar{\iota}_t\,, \tag{10}$$

where $q > 0$ governs the scale of the intervention, $H(\cdot)$ denotes a Heaviside function defined to equal one when its argument is strictly positive and zero otherwise, $\Delta\iota_t \equiv \iota_t - \iota_t^*$ defines a trade shock as the cross-country difference between Home and Foreign preferences for imports, and $\bar{\iota}_t$ and $\underline{\iota}_t$ denote the upper and lower intervention thresholds that the Central Bank sets exogenously each period. The Heaviside functions govern the sign of the Central Bank's interventions, and to lighten notation we define their difference as $\Delta H_t \equiv H(\Delta\iota_t - \bar{\iota}_t) - H(\underline{\iota}_t - \Delta\iota_t) \in \{-1, 0, 1\}$. Because the intervention thresholds are chosen by the Central Bank, we treat ΔH_t as an exogenous policy variable.

Our model allows for two stylized intervention types: swap and spot interventions. In a swap intervention, the Central Bank takes a bond position in period zero and unwinds the position in period one. The payoff of a swap position is equal to the expected currency return on holding a long position in the home currency bond. In a spot intervention, the Central Bank takes a bond position in period zero and maintains the position in period one. These stylized interventions capture two features of real-world foreign exchange interventions: first, because they operate through bond markets and cost nothing to construct, they are sterilized; second, because the swap intervention is unwound after one period while the spot intervention persists, the spot intervention is longer-lived.

Both interventions can take either sign. That is, the Central Bank responds to sufficiently large positive trade shocks by buying Home bonds and selling Foreign bonds, or to sufficiently large negative trade shocks by selling Home bonds and buying Foreign bonds. In the former case Q_{Ht}^{cb} is positive, in the latter case Q_{Ht}^{cb} is negative. The balance sheet constraint then implies the opposite sign for Q_{Ft}^{cb}. The Central Bank does not intervene when trade shocks remain within its intervention thresholds.

To gain intuition for the cause and effect of a Central Bank intervention, consider the currency flows leading to and resulting from an intervention.[6] A relative rise in the

[6]When we refer to currency flows, we abuse the word currency to mean a claim to the numéraire of

Home preference for Foreign goods (a positive trade shock) induces the Home Household to import more. The Home Household exchanges Home currency for Foreign currency and Home currency weakens. The Central Bank responds by buying Home bonds and selling Foreign bonds. The Central Bank exchanges Foreign currency for Home currency and Home currency strengthens. Thus, the Central Bank buys Home bonds to strengthen Home currency in response to a positive trade shock. The parallel construction also holds for a negative trade shock.

The Household and the Financier have imperfect information about the Central Bank's intervention thresholds. In period zero, the Central Bank communicates distributions with parameters θ_0 for its thresholds, $F(\bar{\iota}_t;\theta_0)$ and $F(\underline{\iota}_t;\theta_0)$ for $t \in \{0,1\}$, but not the exact values of the thresholds. In period one, the Central Bank can update the distributions for its period-one thresholds, $F(\bar{\iota}_1;\theta_1)$ and $F(\underline{\iota}_1;\theta_1)$. Withholding information about its intervention thresholds allows the Central Bank to conduct unanticipated interventions with larger exchange rate effects, as we later show.[7]

The parameters θ_0 and θ_1 govern the ability of Households and the Financier to anticipate interventions when forming consumption and investment plans. In period t, the expected intervention in period $s \geq t$ is

$$\mathrm{E}_t^{(-)}\left[Q_{Hs}^{cb}\right] = q\,\mathrm{E}_t^{(-)}[\Delta H_s] = q\Delta\eta_{s|t}\,, \tag{11}$$

where we define $\Delta\eta_{s|t} \equiv \mathrm{Prob}(\bar{\iota}_s < \Delta\iota_s;\theta_t) - \mathrm{Prob}(\Delta\iota_s < \underline{\iota}_s;\theta_t) \in [-1,1]$, and treat $\Delta\eta_{s|t}$ as an exogenous policy variable controlled by the Central Bank.[8] Figure 3 depicts distributions

the economy, mirroring the language in Gabaix and Maggiori (2015), as described in their footnote 5.

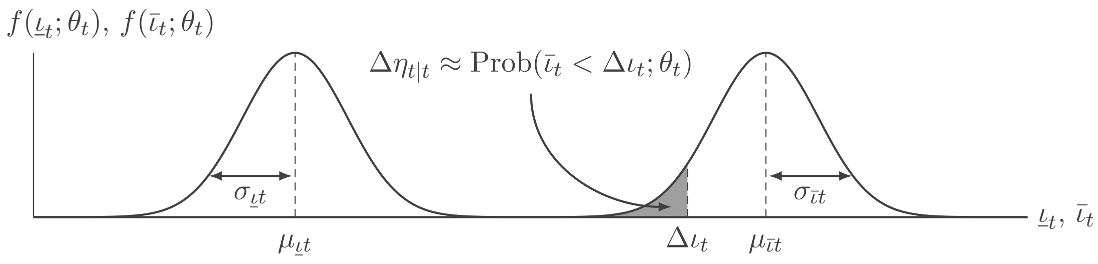
[7]Since the Central Bank can intervene twice at most, we abstract from credibility considerations and do not require the Central Bank's actions to be consistent with its communications. However, the model's empirical predictions are more sensible when the Central Bank's actions and communications are reasonably consistent—for example, when it avoids communicating that it will buy bonds before intervening to sell them. Given the limited number of Central Bank interventions, we do not allow agents to learn from Central Bank actions. We note that learning is not possible even in some infinite-horizon settings with explicit reaction functions, such as the model of Candian et al. (2024).

[8]The expectation of the Heaviside function equals the probability that it's argument is positive. For example, the probability of a positive intervention in $t = s$ is given by

$$\mathrm{E}_t^{(-)}\left[H(\Delta\iota_t - \bar{\iota}_t)\right] = \int_{-\infty}^{\infty} H(\Delta\iota_t - \bar{\iota}_t)\; d\,F(\bar{\iota}_t;\theta_t) = \int_{-\infty}^{\Delta\iota_t} 1\; d\,F(\bar{\iota}_t;\theta_t) = \mathrm{Prob}(\bar{\iota}_t < \Delta\iota_t;\theta_t)\,,$$

where the trade shock $\Delta\iota_t$ is known and the Central Bank controls the parameters in θ_t through

Figure 3: Distributions of Intervention Thresholds.



Notes. The figure shows normal density functions $f(\underline{\iota}_t;\theta_t)$, $f(\bar{\iota}_t;\theta_t)$ for the Central Bank's intervention thresholds $\underline{\iota}_t$ and $\bar{\iota}_t$, as perceived by Households and the Financier in period t. The distributions are characterized by means and standard deviations, $\theta_t = (\mu_{\underline{\iota}t},\ \mu_{\bar{\iota}t},\ \sigma_{\underline{\iota}t},\ \sigma_{\bar{\iota}t},\ \ldots)$, which the Central Bank communicates to the Household and Financier. The shaded region shows the probability, as perceived by the Household and Financier, that the Central Bank will intervene positively by purchasing Home bonds and selling Foreign bonds to strengthen Home currency. The value $\Delta\eta_{t|t}$ approximately equals the shaded probability in the figure, because the probability of a negative intervention is approximately zero. That is, $\Delta\eta_{t|t} \approx \text{Prob}(\bar{\iota}_t < \Delta\iota_t;\theta_t) - 0$.

of the Central Bank's upper and lower intervention thresholds and the probability that the Household and the Financier assign to the event that the Central Bank intervenes positively in response to an observed trade shock. The figure assumes normally distributed thresholds with means and standard deviations such that the probability of a positive intervention is positive and the probability of a negative intervention is approximately zero.

Equilibrium Real Exchange Rate. We find it convenient to work with linearized equations in deviations from the non-stochastic steady state of the economy for the remainder of our analysis. We use hats to denote deviations from steady state.

Bond markets clear in equilibrium. With bonds in zero net supply, market clearing for the Home bond requires that the demands of the Home Household, the Financier, and the Central Bank sum to zero,

$$\hat{Q}_{Ht} + \hat{Q}^{f}_{Ht} + \hat{Q}^{cb}_{Ht} = 0\,. \tag{12}$$

communication policy that we do not model. A similar expression holds for the expectation of a negative intervention in the current period. For the expectation of interventions in future periods ($t < s$), the trade shock $\Delta\iota_s$ is random and the joint distribution of $\Delta\iota_s$ and $\bar{\iota}_s$ must be considered. But the joint distribution will also depend on θ_t and can therefore be controlled by the Central Bank. Hence, we treat $\Delta\eta_{s|t}$ as an exogenous policy variable for $t \leq s < 2$.

Bond holdings are zero for all parties in the non-stochastic steady state, so we linearize bond holdings in levels and divide by the non-stochastic steady state value of the Household preference for imports as a normalization.[9] To obtain an expression for the real exchange rate, we combine linearized demand equations for the Home Household, the Financier, and the Central Bank with the Home bond market clearing condition in (12).

Home Household demand for Home bonds derives from the period budget constraints in (2). Under our simplifying assumptions, after imposing market clearing for the Home good, the budget constraints take the following forms,

$$\begin{aligned} \hat{Q}_{H0} &= \hat{e}_0 - \Delta\hat{\iota}_0 + O\left(\epsilon^2\right) \\ \hat{Q}_{H1} &= \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + O\left(\epsilon^2\right) \\ 0 &= \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\left(\epsilon^2\right), \end{aligned} \tag{13}$$

where the third constraint requires that total variation in the real exchange rate equals total variation in trade shocks and gives rise to the "boomerang" effect of Central Bank interventions that Gabaix and Maggiori (2015) find. For a given sequence of trade shocks, interventions that shifts the real exchange rate in one period must be offset by compensating shifts in future periods. The strength of this effect depends on the model's finite horizon.

The Financier's demand for Home bonds derives from the Financier's optimality condition in (9). Linearizing this condition leads to the following demand for Home bonds,

$$\hat{Q}^f_{Ht} = \frac{1}{\Gamma}\,\mathrm{E}_t^{(-)}\left[\hat{e}_t - \hat{e}_{t+1}\right] + O\left(\epsilon^2\right), \tag{14}$$

which again shows that higher Γ reduces the intermediation undertaken by the Financier, all else equal. The gross real returns on Home and Foreign bonds that appear in (9) are zero to a first-order approximation; this result derives from the simplifying assumption that

[9] For example, for the Home Household, we define $\hat{Q}_{Ht} \equiv (Q_{Ht} - Q_H)/\iota$, where $Q_H = 0$ is the Home Household's steady-state bond demand and $\iota \equiv 1$ is the Home Household's steady-state preference for imports. Because bond holdings are linearized in levels, the clearing condition in (12) is an exact condition rather than a first-order approximation. We linearize most other variables in logs, and the bond demand expressions in (13) are first-order approximate.

non-tradable endowments adjust proportionally to fluctuations in preference parameters for non-tradable goods.[10]

Finally, we rewrite the Central Bank's policy rule in (12) as follows,

$$\hat{Q}^{cb}_{Ht} = q\Delta H_t\,, \tag{15}$$

where we use the fact that the Central Bank's intervention is zero in the non-stochastic steady state.[11] We provide derivations of the linearized demand equations in (12)–(15) in Appendix A.3.

Combining the linearized demand equations with the market clearing condition for the Home bond, we obtain solutions for the path of expected and realized real exchange rates in periods zero, one, and two. Our primary interest lies in the response of the real exchange rate in period zero to Central Bank interventions in the anticipated and unanticipated cases when financial constraints are tight or loose. For this reason, we relegate the full set of solutions and a brief analysis of exchange rate dynamics to Appendices A.4 and A.5. The real exchange rate in period zero is given by

$$\begin{aligned}\hat{e}_0 = \Delta\hat{\iota}_0 - \frac{2\Delta\hat{\iota}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_1] - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_2]}{3+\Gamma} + \Gamma\frac{\Delta\hat{\iota}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\ - q\frac{\Delta H_0 - \Delta\eta_{0|0}}{1+\Gamma} - \Gamma q\frac{\Delta H_0}{1+\Gamma} - \Gamma q\frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\big(\epsilon^2\big)\,.\end{aligned} \tag{16}$$

To build intuition for the terms that appear in (16), consider separately the first and second lines on the right-hand side of the expression, focusing on the extreme cases of zero and infinite credit constraints.

If the Central Bank is passive ($q = 0$), the real exchange rate has three determinants in the first line on the right-hand side of (16): the pure effect of the trade shock, the intertemporal smoothing effect of bond trade, and a credit constraint friction. With severe

[10] As Gabaix and Maggiori (2015) explain, this simplification renders the Household's marginal utility of non-tradable consumption constant; bonds are denominated in the non-tradable good, so precautionary and intertemporal consumption smoothing motives cancel and returns are also constant.

[11] Recall that we linearize bond holdings in levels. For the Central Bank, we have $\hat{Q}^{cb}_{Ht} \equiv (Q^{cb}_{Ht} - Q^{cb}_{H})/\iota = Q^{cb}_{Ht}$, where the last equality follows from $Q^{cb}_{H} = 0$ and $\iota = 1$. Hence, (15) is an exact expression, not an approximation of (10).

credit constraints ($\Gamma \to \infty$), only the first term survives and trade shocks drive the real exchange rate in autarky. Without credit constraints ($\Gamma \to 0$), the first and second terms survive and bond trade smooths the real exchange rate optimally. With limited credit constraints ($0 < \Gamma < \infty$), all three terms survive and the real exchange rate deviates from the frictionless ideal. If trade shocks follow a random walk, zero bond holdings are optimal and the second and third terms again vanish.

If the Central Bank is active ($q > 0$), interventions operate through three channels in the second line on the right-hand side of (16): a surprise channel, a private intermediation replacement channel, and a private intermediation support channel. Without credit constraints ($\Gamma \to 0$), only the first term survives and Central Bank interventions are effective when they surprise the Household and Financier. With severe credit constraints ($\Gamma \to \infty$), only the second terms survives; the Central Bank completely replaces the Financier and intermediates all bond trade. With limited credit constraints ($0 < \Gamma < \infty$), all three terms survive and the Central Bank supports the Financier by reducing the Financier's intermediation burden.

Empirical Predictions. The Gamma-Eta Model yields an intuitive expression for the real exchange rate that captures the interplay between Central Bank interventions, market anticipation, and credit constraints. We now derive qualitative predictions from the model to test empirically in the sections that follow. Specifically, we derive predictions on the direction of intervention effects, on the effects of swap versus spot interventions, on the effects of anticipated versus unanticipated interventions, on the effect of interventions on the quantity of private intermediation, and on the effect of interventions during periods of looser and tighter credit constraints. We provide proofs for the predictions in Appendix A.6.

Prediction 1 (Direction of Intervention Effects)**.** *Consider a Central Bank intervention $\Delta H_0 \neq 0$, whereby the Central Bank constructs a portfolio of Home and Foreign bonds. The real exchange rate responds to the currency flows arising from the portfolio construction.*

The direction of the response depends on the direction of the intervention as follows,

$$\operatorname{sign}\frac{\partial \hat{e}_0}{\partial q} = -\operatorname{sign}\Delta H_0\,.$$

A positive intervention ($\Delta H_0 > 0$), whereby the Central Bank buys Home bonds and sells Foreign, strengthens Home currency relative to Foreign and the real exchange rate falls. A negative intervention ($\Delta H_0 < 0$), whereby the Central Bank sells Home bonds and buys Foreign bonds, weakens Home currency relative to Foreign and the real exchange rate rises.

Prediction 2 (Anticipated versus Unanticipated Interventions)**.** *Consider a Central Bank communication that raises the expectation of a positive intervention in period zero ($\Delta\eta_{0|0} > 0$). The Financier anticipates stronger Home currency in period zero and sells Home bonds to buy Foreign bonds. The anticipation weakens Home currency and the real exchange rate rises,*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{0|0}} > 0\,.$$

If the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by less if the Financier anticipates the intervention in period zero.

If the Financier faces no credit constraint ($\Gamma = 0$) and the period-zero intervention is fully anticipated ($\Delta\eta_{0|0} = \Delta H_0 \neq 0$), then

$$\frac{\partial \hat{e}_0}{\partial q} = 0$$

and interventions have no effect. In this case, the Central Bank can build foreign exchange reserves without disturbing the real exchange rate.

Prediction 3 (Spot versus Swap Interventions)**.** *Consider a fully anticipated Central Bank intervention in period one such that $\Delta\eta_{0|0} = \Delta H_0 \neq 0$. If the Financier faces a credit constraint ($\Gamma > 0$), then*

$$\left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0}=\Delta H_0} > \left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0}=0}.$$

A fully anticipated long-lived spot intervention ($\Delta\eta_{1|0} = \Delta H_0$) has a greater effect on the real exchange rate than a fully anticipated short-lived swap intervention ($\Delta\eta_{1|0} = 0$).

Prediction 4 (Private Intermediation)**.** *Consider a Central Bank communication that raises the expectation of an intervention in period zero, such that the Financier believes the period-zero intervention to be at least half as likely as a period-one intervention of the same sign ($\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$). The Financier responds to the Central Bank's communication by adjusting the amount of its private intermediation as follows,*

$$\operatorname{sign} \frac{\partial \hat{Q}^f_{H0}}{\partial q} = -\operatorname{sign} \Delta\eta_{0|0} \,.$$

Suppose the Financier expects the Central Bank to intervene positively by buying Home bonds and selling Foreign, such that $\Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0} \geq 0$. The Financier will then expect to earn a reduced excess return on the home bond, and will therefore reduce the amount of private intermediation it undertakes.

Prediction 5 (Credit Constraints)**.** *Consider a tightening of credit constraints. If the Central Bank is passive ($q = 0$), tighter credit constraints lower the amount of intermediation the Financier undertakes,*

$$\operatorname{sign} \frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\operatorname{sign} \hat{Q}^f_{H0} \,.$$

The Financier's Home and Foreign bond positions shrink when credit constraints tighten, limiting the ability of Households to smooth consumption intertemporally.

If the Central Bank is active ($q > 0$) and the Financier believes the period-zero intervention to be at least half as likely as a period-one intervention of the same sign ($\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$), then tighter credit constraints amplify the effects of interventions,

$$\operatorname{sign} \frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} \,.$$

This regularity condition admits the anticipation of period-zero swap interventions ($\Delta\eta_{0|0} >$

$\Delta\eta_{1|0} \to 0$) *and the anticipation of spot interventions* ($\Delta\eta_{0|0} = \Delta\eta_{1|0} > 0$).[12]

To map these predictions to the empirical analysis that follows, we treat Brazil as Home, with currency BRL, and the U.S. as Foreign, with currency USD. In this setup, the central bank is the BCB, and a positive intervention refers to an intervention to sell USD, either through a spot or swap transaction. In the following sections, we describe the data and present the results of our empirical analysis testing the predictions of the Gamma-Eta Model.

3 Datasets

In our empirical analysis, we focus on the Brazilian case and employ a public dataset released by the BCB in 2023, along with a number of supporting datasets on foreign exchange, interest rates, intermediary constraints, and macroeconomic aggregates. We briefly describe the BCB's policy framework before introducing the data and defining key variables.

3.1 Policy Framework

Since early 1999, Brazil's monetary system has operated under an inflation-targeting framework established by Presidential Decree No. 3,088. The National Monetary Council, composed of the finance and planning ministers and the BCB governor, sets the inflation target based on the Extended National Consumer Price Index (IPCA).

The BCB uses the Selic rate as its primary tool to achieve this inflation target. The Selic rate is the weighted average interest rate of overnight interbank loans, which are collateralized by federal government securities and settled through the Special System for Settlement and Custody (Selic, in Portuguese). The BCB's Monetary Policy Committee

[12]Define a positive spot intervention as $\Delta H_0 = 1 \cap \Delta H_1 = 1$. If the Financier expects only spot interventions, then $\text{Prob}(\Delta H_1 = 1 \mid \Delta H_0 = 1) = \text{Prob}(\Delta H_0 = 1 \mid \Delta H_1 = 1) = 1$. It follows that

$$\begin{aligned}\text{Prob}(\Delta H_0 = 1 \cap \Delta H_1 = 1) &= \text{Prob}(\Delta H_0 = 1) \times \text{Prob}(\Delta H_1 = 1 \mid \Delta H_0 = 1) = \text{Prob}(\Delta H_0 = 1) \\ &= \text{Prob}(\Delta H_1 = 1) \times \text{Prob}(\Delta H_0 = 1 \mid \Delta H_1 = 1) = \text{Prob}(\Delta H_1 = 1)\,.\end{aligned}$$

The same holds for negative spot interventions. Definitions of $\Delta\eta_{0|0}$ and $\Delta\eta_{1|0}$ then imply $\Delta\eta_{0|0} = \Delta\eta_{1|0}$.

(Copom) meets regularly to set the target for the Selic rate, with the goal of keeping the IPCA inflation rate within a predefined range, specified by upper and lower bands, over the relevant monetary policy horizon.

Regarding exchange rate policy, the BCB operates a floating regime, which the IMF defines as a largely market-determined exchange rate without a predictable path, where intervention is permitted to prevent excessive volatility and manage liquidity but not to maintain a specific rate. The BCB's exchange rate regime does not meet the IMF's *free*-floating definition, which requires that authorities intervene at most three times in any six-month period, with each intervention lasting no more than three business days (Nowzad et al., 2021).

3.2 BCB FX Interventions

The primary goals of the BCB's FXI are to maintain an orderly market, control volatility, and ensure liquidity, while also accumulating reserves during periods of capital inflows (Central Bank of Brazil, 2019). To achieve these goals, the BCB uses spot and swap interventions. Spot interventions are often used when market imbalances are severe. In contrast, swap interventions are often used to hedge currency mismatches on bank balance sheets; for example, currency swap interventions helped mitigate the spillovers of the US Taper Tantrum on bank balance sheets (Chamon, 2019; Gonzalez et al., 2021; da Costa Filho, 2021).

To execute interventions, the BCB's International Reserve Department transacts with 14 accredited financial institutions known as Dealers-14. These institutions are mainly large banks authorized by the BCB that operate in an inter-dealer market. They play a crucial role in supplying USD liquidity to customers in the Brazilian FX market.

In 2023, the BCB released a comprehensive public dataset of its historical foreign exchange interventions since 1999.[13] The dataset details the sign of each intervention (USD purchase or sale), the auction amount, the announcement date and high-frequency announcement timestamp, the operation date, and the intervention type. Four intervention

[13]The database can be found on the BCB's Dados Abertos portal `https://www.bcb.gov.br/conteudo/dadosabertos/BCBDepin/historico-atuacoes-mercado-cambio.csv`.

types are reported: spot purchases, spot sales, traditional swaps, and reverse swaps.

In *spot operations*, the BCB buys or sells USD in exchange for BRL in the interbank foreign exchange market for settlement in two business days.

In *traditional swap operations*, the BCB takes a buying position in swap contract auctions. While the BCB classifies these derivative auctions as swaps, they typically function as non-deliverable futures contracts settled in BRL (Nedeljkovic and Saborowski, 2019). Unlike a standard swap contract, there is no exchange of principals. Instead, the BCB pays the dealer the variation in the BRL/USD exchange rate, plus an onshore USD rate known as the *cupom cambial*, and receives the domestic BRL Selic rate for the same period.[14] Therefore, the traditional swap protects the dealer in the event of a USD appreciation and is useful for hedging net USD liabilities.

In *reverse swap operations*, the BCB takes a selling position in swap contracts. Similar to a traditional swap, there is no exchange of principals. In this case, the BCB pays the BRL Selic rate, and the dealer pays the *cupom cambial*, plus the variation in the BRL/USD exchange rate. This setup protects the dealer against a depreciation of the USD and can be used to hedge net USD asset positions.[15]

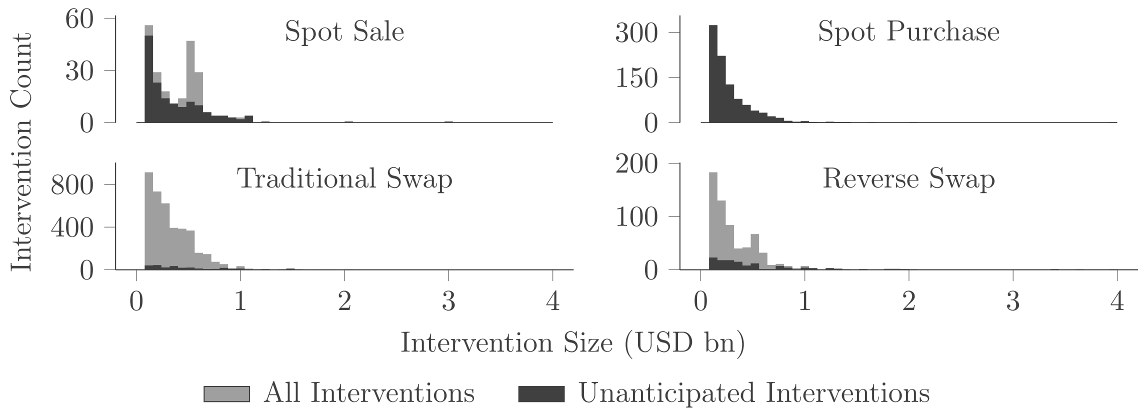
The intervention announcement dates and high-frequency timestamps in the database indicate when the BCB informed the public about a future foreign exchange intervention or, in the case of auctions in interbank spot or forward markets, when an intervention had begun. The intervention operation date specifies when the intervention was conducted, but it does not include a high-frequency timestamp for the operation. The announcement and operation dates allow us to classify interventions as unanticipated or anticipated. We classify interventions as unanticipated if the operation date equals the announcement date and as anticipated if the operation date occurs after the announcement date.

Figure 4 illustrates the size distribution of interventions by type. The mean unanticipated spot buy (sell) intervention in the database is 0.19 (0.17) billion USD with a standard deviation of 0.24 (0.22) billion USD, and the mean unanticipated traditional (reverse)

[14]The *cupom cambial* equals the spread between the overnight interbank deposit rate and the expected exchange rate variation in the BRL/USD. See Nedeljkovic and Saborowski (2019) for more details.

[15]For more information, we refer readers to the institutional details of traditional and reverse swap operations on the BCB website `https://www.bcb.gov.br/estabilidadefinanceira/swapcambial`.

Figure 4: Histograms of BCB FX Market Interventions



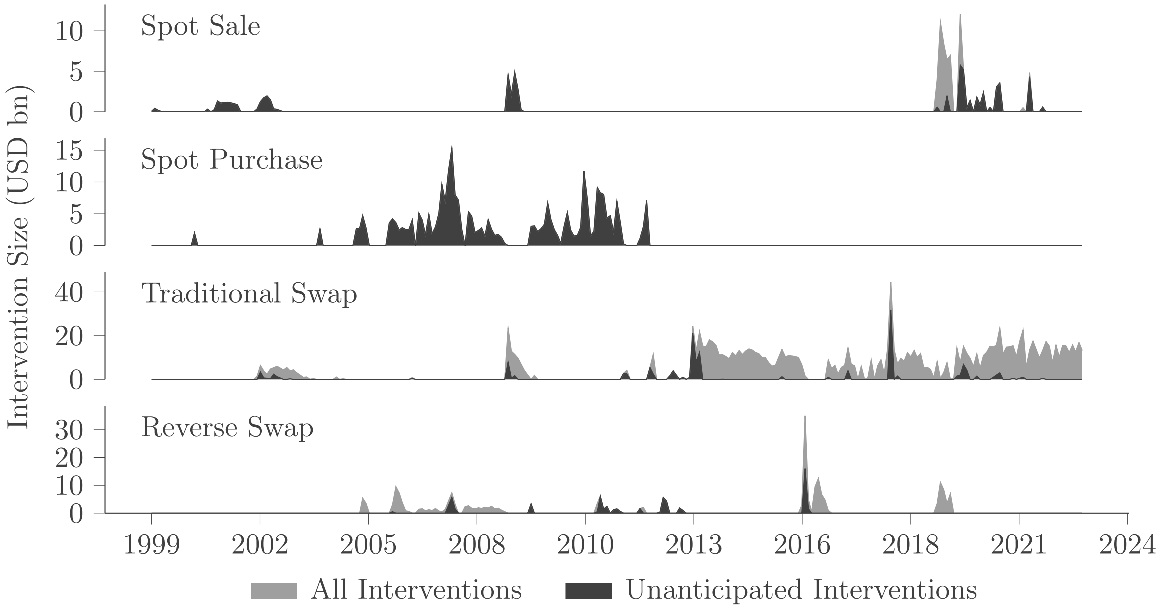
Notes. These figures show 50-bin histograms of BCB's interventions in the FX markets, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at daily frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions. Unanticipated interventions are defined as interventions with an announcement equal to the operation date, while anticipated interventions are defined as interventions with an announcement date that precedes the operation date. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

swap intervention is 0.43 (0.35) billion USD with a standard deviation of 0.41 (0.45) billion USD. Unanticipated interventions are generally larger than anticipated interventions for all intervention types. There are 1483 (385) unanticipated and 0 (87) anticipated spot buy (sell) interventions in the database, while there are 345 (174) unanticipated and 5094 (846) anticipated traditional (reverse) swap interventions. Table B.1 in the Online Appendix provides additional summary statistics on the BCB's foreign exchange interventions.

Figure 5 illustrates patterns in BCB interventions over time, showing a shift towards swaps as the principal policy instrument in recent years and suggesting two distinct periods of FXI use (Central Bank of Brazil, 2019).[16] From 2005 to 2012, the BCB primarily used FXI to lean against capital flows and accumulate foreign reserves through USD spot purchases, except during periods of economic stress, like the Global Financial Crisis in 2008 and 2009, when it conducted spot sales to provide USD liquidity to the FX market. Since 2012, the BCB has used FX swaps as its primary tool to manage price asymmetries, providing hedging options when market supply is limited or abundant (Gonzalez et al.,

[16]Figure B.1 in the Online Appendix shows cumulative intervention amounts by type over time.

Figure 5: Time Series of BCB FX Market Interventions



Notes. These figures show BCB FX market interventions over time, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at monthly frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions, defined as interventions with an announcement equal to the operation date.

2021; Nedeljkovic and Saborowski, 2019), except during the COVID-19 pandemic, when spot sales were again used.

FX swaps are attractive instruments because the derivatives market is larger and more liquid than the spot market, which may explain their frequent use by the BCB. Additionally, the balance sheet effects of swaps are short-lived: with a traditional swap, the BCB effectively extends a collateralized loan in USD that is repaid upon swap maturity, automatically restoring the BCB's USD reserves. In contrast, spot sales of USD have potentially longer-lived effects, reducing the BCB's precautionary savings in USD and weakening its insurance against periods of tight intermediary constraints (Cheng, 2015; Jeanne and Sandri, 2020).

While the BCB does not explicitly differentiate between anticipated and unanticipated swaps in its official framework, empirical patterns suggest that unanticipated swaps typically have short maturities (less than 2 days), whereas anticipated swaps have a broad range of maturities, from overnight to 3 months. These differences in maturity could reflect different policy goals. Although further research is required, one plausible interpretation is that the BCB deploys unanticipated swaps to manage short-term liquidity needs of specific dealers to mitigate roll-over risk and ensure interbank market stability, while deploying anticipated swaps to address longer-term liquidity concerns and alleviate maturity and currency mismatches on bank balance sheets. Figure B.2 in the Online Appendix shows the maturity breakdown of traditional and reverse currency swaps for both anticipated and unanticipated interventions.

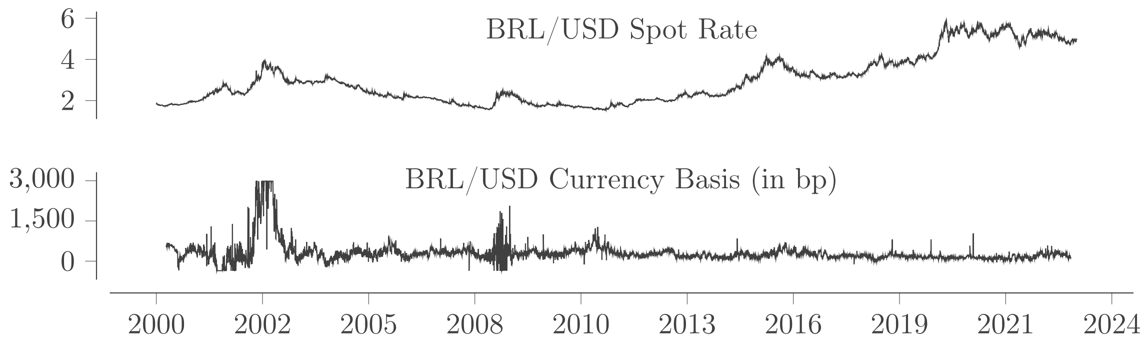
3.3 Additional Data Sources

B3 Exchange Tick Database. We use tick-by-tick data on futures and interest rates, obtained from Brazil's main stock exchange, B3, via an FTP link that was available for a limited time. The dataset covers transactions from 28/02/2019 to 23/01/2020, spanning 224 trading days, and includes trading prices, volumes, aggressor buy-sell signals, market participant trading codes, and bid-ask quotes. For this study, we focus on the BRL/USD WDO contract, a mini USD futures contract traded on B3.[17] Each contract represents an exposure of 10,000 USD and is quoted in BRL per USD. WDO contracts expire monthly and allow investors to take positions on the future BRL/USD exchange rate. An alternative contract, the DOL futures contract, offers similar characteristics but represents a larger 50,000 USD exposure. We analyze the WDO contract instead of the DOL contract for our analysis due to its higher liquidity and trading activity in our sample.

Reuters Benchmark Spot and Forward Data. We supplement the B3 data with high-frequency spot and forward indicative quotes from Thomson Reuters Tick History. The quotes are given at 5-minute intervals, include bid and ask prices, and cover the period 1999 to 2023. While the Reuters quotes are lower frequency than the B3 tick-by-tick data, the Reuters quotes offer a longer time series that better aligns with the historical intervention dataset from the BCB. For this reason, we use the Reuters quotes for our

[17] For more details, see the B3 official website: `https://www.b3.com.br/en_us/products-and-services/trading/exchange-rates/mini-u-s-dollar-futures.htm`

Figure 6: BRL/USD Spot Rate and Currency Basis



Notes. The upper panel plots the spot exchange rate expressed in units of BRL per USD. An increase in the exchange rate indicates a depreciation of BRL and an appreciation of USD. The lower panel plots the daily 1-month maturity BRL/USD currency basis measured in basis points. A positive currency basis is a violation of CIP and indicates a premium to swap BRL into USD in forward and swap markets. To improve legibility in the plot, we winsorize the BRL/USD currency basis at the 1% level. The sample period runs from 1999-01-22 to 2023-04-27.

primary econometric analysis in Section 4. In our analysis, we use the mid-quote of the spot and forward prices.

Figure 6 plots the BRL/USD spot rate and currency basis. We express the spot rate in units of BRL per USD, so an increase indicates a BRL depreciation. The USD has strengthened against BRL since 2010, rising from below 2 BRL per USD in 2010 to above 5 BRL per USD in 2020.

Interest Rates. We obtain daily interest rates from the Brazilian Financial and Capital Markets Association (ANBIMA) through the Institute of Applied Economic Research (IPEA) data repository. For our baseline analysis, we use the 1-month maturity, based on the yield curve of the National Treasury Bill (LTN) with a term of 21 business days.[18]

Cross-Currency Basis. The CIP condition states that equivalent assets denominated in different currencies should earn equal returns after hedging exchange rate risk with a forward contract.[19] Empirically, the CIP condition is frequently violated, and the

[18] ANBIMA uses Svensson (1994) to estimate the term structure of interest rates from Treasury securities. See `https://www.anbima.com.br/data/files/12/D0/11/B0/C7CFD71028DFACD76B2BA2A8/Metodologia_estrutura-termo_out21_ingles.pdf` for more details

[19] To illustrate, an investor can invest one USD at t to receive a risk-free payment in USD at $t+h$, or exchange one USD for BRL and invest in BRL at t to receive a risk-free payment in BRL at $t+h$. In the latter case, the investor can hedge exchange rate risk by entering a forward contract at t to exchange

cross-currency basis, which we denote $x_{t,t+h}$, quantifies the CIP violation. Following Du et al. (2018), we define the cross-currency basis as a wedge between the *synthetic* and direct USD risk-free rate,

$$\left(1 + r^{USD}_{t,t+h}\right)^h = \left(1 + r^{BRL}_{t,t+h} - x_{t,t+h}\right)^h \frac{S_t}{F_{t,t+h}}, \tag{17}$$

where $r^{USD}_{t,t+h}$ denotes the U.S. nominal risk-free rate in USD, $r^{BRL}_{t,t+h}$ denotes the Brazilian nominal risk-free rate in BRL, S_t is the nominal spot exchange rate expressed in BRL per USD, and $F_{t,t+h}$ is the nominal h-period forward exchange rate in BRL per USD. From this expression, we approximate the cross-currency basis as

$$x_{t,t+h} \approx r^{BRL}_{t,t+h} - r^{USD}_{t,t+h} - \rho_{t,t+h}, \tag{18}$$

where $\rho_{t,t+h} \equiv \left(f_{t,t+h} - s_t\right)/h$ is the forward premium, using lower-case variables to denote logs of the spot and forward exchange rates.

A positive BRL/USD currency basis ($x_{t,t+h} > 0$) indicates a CIP violation such that market participants require a premium to swap BRL into USD, suggesting a relative scarcity of USD funding in cross-border markets. The BRL/USD currency basis is measured in basis points and is persistently positive over the period from 1999 to 2023, as shown in Figure 6.

Intermediary Constraints. The Gamma-Eta model predicts that foreign exchange interventions have greater effects when intermediaries are constrained, as measured by the model's parameter Γ. To measure intermediary constraints empirically, we adopt the *intermediary capital ratio* of He et al. (2017) as an empirical proxy for Γ. The ratio, denoted HKM_t, is defined as

$$HKM_t = \frac{\sum_i \text{Market Equity}_{it}}{\sum_i \left(\text{Market Equity}_{it} + \text{Book Debt}_{it}\right)}, \tag{19}$$

BRL for USD at $t + h$. The CIP condition states that both strategies should offer the same return.

where book debt is computed by subtracting total common equity from total assets, and where i indexes individual intermediaries. We follow Cerutti and Zhou (2024) and construct a daily measure of HKM_t using the subset of primary dealers that deal with emerging market currencies based on the Euromoney FX survey.[20]

A decline in HKM_t indicates lower risk-bearing capacity at the largest emerging-market currency dealers. The intuition is that intermediaries with lower capital ratios face tighter borrowing constraints and demand fewer risky assets to comply with regulatory capital requirements. Thus, HKM_t is an important indicator of the ability of primary dealer banks to intermediate USD transactions in cross-border financial markets. If Prediction 5 of the Gamma-Eta model holds, we would expect tighter intermediary constraints to reduce USD intermediation and strengthen the exchange rate response to Central Bank interventions.

Macroeconomic Variables. Finally, we use monetary announcements of the Federal Reserve and BCB, NBER recession dates, and measures of global and sovereign risk as control variables. To measure global risk, we use the VIX index. To measure sovereign risk of emerging markets, we use the JPMorgan EMBI+ (Emerging Markets Bond Index Plus) from the IPEA data repository. The EMBI+ tracks the daily performance of debt securities from emerging markets relative to U.S. Treasury securities.

4 Empirical Analysis

We divide our empirical analysis into four sections. In Section 4.1, we use tick-by-tick data to closely examine two dates on which the BCB conducted both unanticipated and anticipated spot sales of USD. This case study provides preliminary support for the Gamma-Eta Model's Prediction 1 on the direction that FXI affects the exchange rate,

[20]The entities used in our analysis include BNP Paribas, Barclays, Bank of America, Citigroup, Credit Suisse, Deutsche Bank, Goldman Sachs, HSBC, JP Morgan, Morgan Stanley, Societe Generale, Standard Chartered, State Street, and UBS. Several of these institutions——BNP Paribas, Bank of America, Citigroup, Credit Suisse, Goldman Sachs, JP Morgan, and Morgan Stanley——are part of the BCB's Dealers-14, reinforcing the relevance of this HKM measure for our study. See: `https://www4.bcb.gov.br/pec/dealers/principal.asp`.

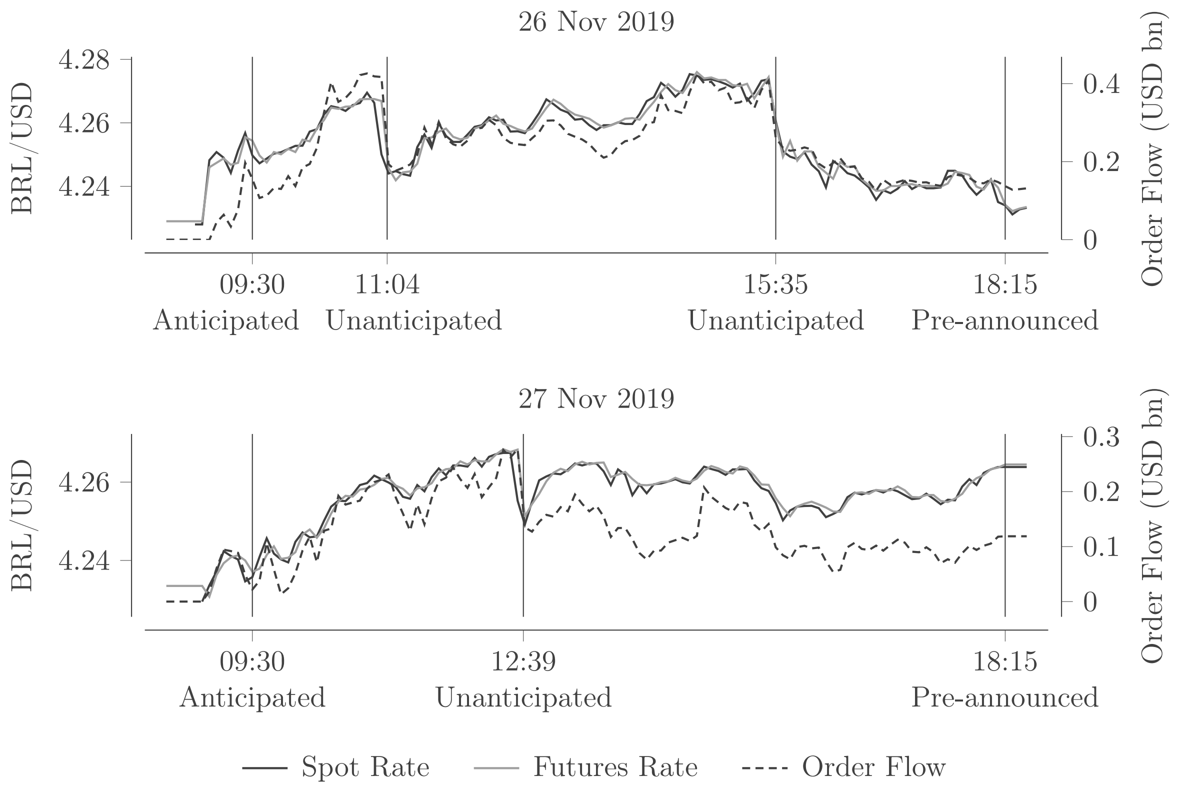
as well as Prediction 2 on the relative impact of anticipated and unanticipated FXI. In Section 4.2, we analyze nearly twenty-five years of BCB interventions using the BCB's historical FXI database, strengthening and extending our case study findings. In addition to Predictions 1 and 2, we test Prediction 3 on spot versus swap FXI and Prediction 4 on private intermediation and FXI, with the cross-currency basis, a measure of CIP violations, serving as a proxy for USD intermediation supplied by Financiers. In Section 4.3, we explore the effects of BCB interventions in periods with tighter and looser intermediary constraints. We find that tighter constraints amplify the exchange rate and cross-currency basis responses to FXI, in line with Prediction 5. Finally, in Section 4.4, we investigate an alternative to the dollar intermediation channel by testing whether FXI signals changes in future interest rates.

4.1 High-Frequency Case Study

To initiate our analysis, we examine the impact of interventions on 26 and 27 November 2019, when the BCB carried out both unanticipated and anticipated spot sales of USD. We focus on these intervention dates due to the availability of tick-by-tick data on transaction prices and order flow for the liquid BRL/USD futures contracts from the B3 exchange.

The upper and lower plots in Figure 7 depict intra-day BRL/USD spot and futures prices, as well as cumulative order flow on these two dates. In the plots, the left vertical axes measure spot and futures prices and the right vertical axes measure cumulative order flow. Vertical lines mark intervention times. Order flow refers to net initiated buy orders over sell orders, a measure of buying pressure derived from aggressor buy-sell indicators in our data. An increase in order flow indicates an increase in net buying pressure for USD. On 26 November, the BCB implemented an anticipated spot sale of USD at 09:30, unanticipated spot sales of USD at 11:04 and 15:35, and pre-announced an intervention at 18:15 that it planned to implement the following morning. On the following day, the BCB implemented an anticipated (pre-announced) spot sale of USD at 09:30, and an unanticipated spot sale of USD at 12:39, before pre-announcing a future intervention at 18:15.

Figure 7: High-Frequency Case Study of Two BCB Intervention Days



Notes. The figures show the BRL/USD exchange rate and cumulative order flow on two dates when the BCB conducted anticipated and unanticipated spot sales of USD. The vertical axes on the left show the BRL/USD exchange rate, and the vertical axes on the right show cumulative order flow in USD billions. Order flow is defined as the net of buyer initiated transaction volume in the 1 month BRL/USD futures market. The upper figure plots the exchange rate and order flow for 26 November 2019, when the BCB conducted one anticipated and two unanticipated interventions and pre-announced an intervention to be conducted on the following day. The lower figure plots the exchange rate and order flow for 27 November 2019, when the BCB conducted one anticipated and one unanticipated intervention and pre-announced an intervention to be conducted on the following day.

The plots show a sharp appreciation of BRL spot and futures prices around unanticipated interventions and an equally sharp decrease in order flow, indicating an accelerated execution of seller-initiated orders in the minutes following unanticipated announcements. In contrast, the plots show muted responses to anticipated interventions and pre-announcements of future interventions. On these two dates, the BCB appears to be reacting to BRL depreciation. In the lead-up to these interventions, BRL volatility had increased and the BRL had weakened against the USD. For instance, on 25 November 2019, the PTAX reference rate for dealers in the B3 currency futures market rose by nearly 12%. This depreciation may be attributable to a reduction in Brazil's benchmark Selic interest rate from 6.40% to 4.90% or to trade tensions between the U.S. and China.

This case study provides initial evidence at tick-by-tick frequency that the BCB can strengthen the BRL by intervening in spot markets to sell USD and purchase BRL, and achieve stronger effects by using unanticipated interventions, consistent with Predictions 1 and 2 of the Gamma-Eta Model. In the following section, we examine the BCB's historical interventions database, where we strengthen and extend the preliminary findings from our case study.

4.2 Historical BCB FXI Analysis

The BCB's database of historical FXI allows us to examine a greater variety of interventions over a longer time series than our B3 data. Specifically, we analyze over 8,000 unanticipated and anticipated spot and swap interventions in buy and sell directions between 1999 and 2023, utilizing indicative BRL/USD spot exchange rate quotes at 5-minute intervals from Thomson Reuters Tick History.

For our baseline empirical specification, we adopt the local projections method of Jordà (2005). This procedure is useful for identifying the impulse response of outcome variables to FXI, while controlling for feedback effects through lags of the outcome variable and appropriate controls. Specifically, we test the effects of FXI on outcome variable y_{t+h} at horizon h,

$$\begin{aligned} y_{t+h} - y_{t-1} = &\beta_h^z INT_t^z \times SAD_{t+h} + \gamma_h^z INT_t^z \times (1 - SAD_{t+h}) \\ &+ \delta_h SAD_{t+h} + \theta_h HKM_t \\ &+ \text{Daily-Freq Controls}_t + \text{High-Freq Controls}_t + u_{t+h} \,, \end{aligned} \tag{20}$$

where the outcome variables are BRL/USD spot prices, forward premia, and the level of CIP violations. Both spot and forward prices are expressed in logarithms.

In this specification, HKM_t denotes the intermediary capital ratio, INT_t^z denotes the BCB intervention amount in USD, with z indicating the intervention type (spot sale, spot purchase, traditional swap, or reverse swap), and SAD_{t+h} indicates whether y_{t+h} falls on the same day as y_t, when the intervention was announced. For example, if an intervention occurs at 10am, SAD_{t+h} takes a value of one for values of h until the close of trading on that day, allowing us to control for gaps in trading between consecutive days. Formally, INT_t^z and SAD_{t+h} are defined as

$$INT_t^z \equiv \begin{cases} \text{USD Amount} & \text{if intervention } z \text{ announced at } t \\ 0 & \text{otherwise}\,, \end{cases}$$

$$SAD_{t+h} \equiv \begin{cases} 1 & \text{if } t+h \text{ on the same day as } t \\ 0 & \text{otherwise}\,. \end{cases}$$

Daily frequency controls include interest rates, term interest rate spreads, spot rate volatility, the total amount of interventions in USD at date t across all instruments, US and Brazilian monetary policy announcement dates, and the EMBI+ emerging-market sovereign risk measure. Up to 10 days of lagged values are included for the total amount of interventions. High-frequency control include up to 10 lags of the outcome variable.

We begin by studying the direction and strength of the exchange rate response to interventions. Our empirical findings relate to Predictions 1 through 2 of the Gamma-Eta Model. We then examine the role of intermediaries in the BRL/USD foreign exchange market and how interventions affect the quantity of intermediation supplied in this market. Our findings with respect to intermediation relate to Prediction 4 of the Gamma-Eta Model.

We focus our analysis on unanticipated interventions, which are news with respect to the information set of market participants and not yet reflected in market prices, providing a clear identification of market responses. Importantly, high-frequency announcement timestamps are available for unanticipated interventions, but not for anticipated interventions. Anticipated interventions are pre-announced before markets close on the preceding day, allowing exchange rates to adjust before the auctions take place the following day. Furthermore, anticipated interventions are expected to have weaker effects, due to the forward looking behaviour of rational agents, and in line with Prediction 3 of the Gamma-Eta Model. We document empirical support for this prediction in Online Appendix B.5, using Brazilian market opening times as event dates for anticipated interventions.

Exchange Rate Response to Unanticipated FXI. To understand how different types of unanticipated interventions impact the exchange rate, we categorize the BCB's historical interventions based on type (spot or swap) and direction (buy or sell). We then estimate our baseline local projections specification in (20) for each category separately.

The upper group of plots in Figure 8 displays our exchange rate results. Unanticipated spot sale interventions have the most significant effect on the BRL/USD exchange rate, leading to an approximate 1.5 percentage point BRL appreciation within 7 hours of the announcement. In contrast, unanticipated spot purchase interventions have much weaker effects. After a spot purchase, we observe a 0.1 percentage point BRL depreciation 5 hours post-intervention, with little statistical significance. For unanticipated swap interventions, we find weaker and more transitory effects. Unanticipated traditional swap interventions result in a 10 basis point BRL appreciation over a 1-hour period, while unanticipated reverse swap interventions lead to a 15 basis point BRL depreciation over the same horizon, with little statistical significance.

To shed light on the speed of adjustment, we report coefficient estimates for select horizons of 15 minutes, 1, 3 and 7 hours in Online Appendix B.3. Focusing on spot sell interventions, where our results are the strongest, we find the BRL appreciates by 33 basis points within 15 minutes of the intervention, and continues to appreciate over a 7-hour horizon after the intervention. The speed of adjustment that we estimate over the full sample period is influenced by a subset of intervention dates, such as the 26 November 2019 date from our case study in Section 4.1, where multiple interventions are clustered within a span of several hours. Roughly one third of the 385 unanticipated spot sale interventions in the BCB's historical interventions database are clustered in this way. As one would expect, we find that clustering strengthens and prolongs the effects of interventions, and that second or third interventions produce larger effects than the first intervention in a cluster. Online Appendix B.7 discusses these results in more detail.

Taken together, our results suggest that USD sale interventions appreciate BRL, while purchase interventions depreciate BRL. Spot interventions have stronger effects than swap interventions, and unanticipated interventions have stronger effects than anticipated interventions. These empirical findings are broadly consistent with Predictions 1 through 2 of the Gamma-Eta Model.

Our disaggregation of interventions by type and direction reveals clear heterogeneity in effects. Permanent effects on the spot rate appear to arise solely from spot sale interventions, while effects from swap interventions are transitory. The Gamma-Eta Model, by distinguishing between long-lived spot interventions and short-lived swaps, provides a coherent, though stylized, theoretical explanation for this empirical finding.[21]

Interestingly, we find that spot purchase interventions are less effective than sale interventions. One reason for this asymmetry may be that USD purchase interventions are typically conducted during periods of relaxed intermediary constraints. During these periods, the Gamma-Eta Model predicts weaker effects on spot exchange rates, as intermediaries with loose constraints are well-positioned to absorb the excess supply of BRL without assistance from the Central Bank. We examine the role of intermediary constraints in more detail in the following section.

We also observe that unanticipated interventions tend to follow intra-day periods of depreciation. This is consistent with the case study of unanticipated interventions on 26-27 November 2019 in Section 4.1. The BCB reacts to developments that lead to a weakening of the domestic currency. Comparing the different types of instruments, we find the strongest pre-trends for spot sales, which is intuitive because spot sale interventions historically occur in periods with scarce USD liquidity in Brazilian FX markets. For spot sales, we find an approximate 1 percentage point BRL depreciation in the hours leading up to the intervention. Therefore, a clear reversal follows the intervention, suggesting that FXI is a useful stabilization tool.

We briefly compare our findings to prior studies on the exchange rate effects of BCB interventions (Kohlscheen and Andrade, 2013; Nedeljkovic and Saborowski, 2019; Barroso, 2019; Santos, 2021), which typically estimate a 30 to 100 basis point impact from a 1

[21]Figure A.2 in the Online Appendix illustrates the dynamic response of the exchange rate to Central Bank interventions as implied by the Gamma-Eta Model for a variety of calibrations. The figure shows, among other things, that the exchange rate response to an unanticipated swap intervention is shorter-lived than the response to an unanticipated spot intervention.

billion USD sale of reserves. The differences in our estimates are partly due to differences in the time periods studied (previous work often focuses on 2011-2013 and shorter subsamples) and in the identification and estimation strategies employed. More importantly, we estimate separate effects for interventions in each direction—buy versus sell in the case of spot interventions and traditional versus reverse in the case of swap interventions. Our impact estimates are attenuated when interventions in both directions are pooled together, as shown in Appendix B.4. In the pooled analysis, we estimate the exchange rate response to BCB spot interventions at approximately 50 basis points, in line with previous literature.

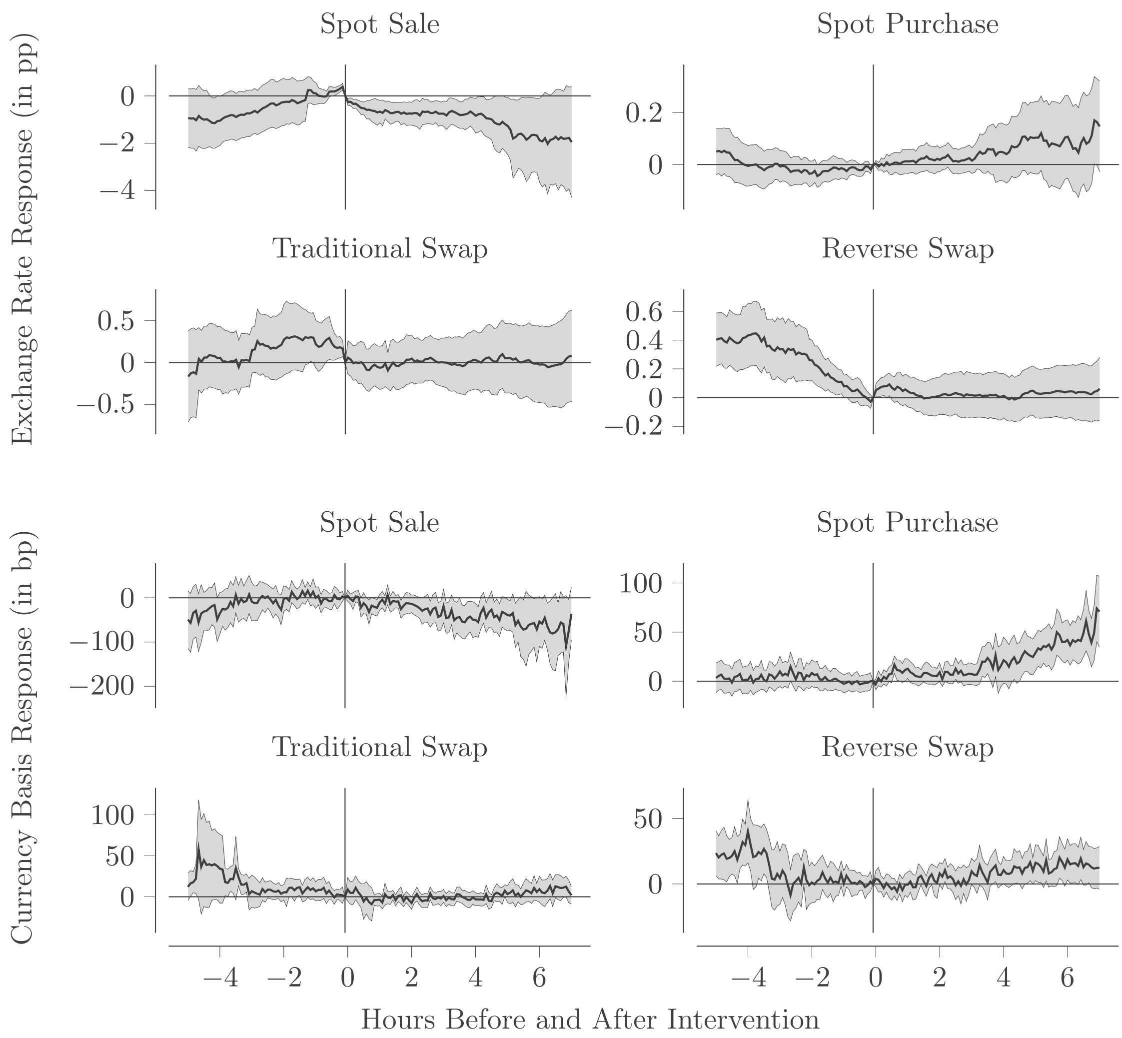
Intermediation Response to Unanticipated FXI. Next, we examine the effects of BCB interventions on the quantity of private intermediation supplied in the BRL/USD foreign exchange market. Prediction 4 of the Gamma-Eta Model states that the amount of private USD intermediation provided decreases following a USD spot sale by the BCB. To test this prediction, we require an empirical proxy for the supply of intermediation.

We use the cross-currency basis defined in (18) for this purpose. The cross-currency basis measures the premium that market participants pay to swap currencies in forward markets. We reason that the Financier's supply of intermediation in (9) is a function of the return made by going short in the USD bond and long in the BRL bond. If forward markets are efficient, the future spot rate is predicted by the forward rate and this return is equivalent to the cross-currency basis. By this logic, the intermediation that Financiers supply should be proportional to the cross-currency basis.[22]

To test Prediction 4, we estimate the baseline local projection specification in (20) with the cross-currency basis as the outcome variable. We show the results in the lower group of plots in Figure 8. Spot sales have the most largest effect on the cross-currency basis among all intervention types, narrowing CIP violations by approximately 100 basis points over a 7-hour horizon, lowering the relative cost of swapping BRL into USD through forward and swap contracts, thereby reducing USD liquidity costs for Financiers. Our

[22]Formally, we refer to model equation (9) which states the amount of bond trade the Financier chooses to intermediate. Under the expectations hypothesis $\mathrm{E}[e_{t+h}] = F_{t,t+h}$, the amount of intermediation undertaken by the financier is proportional to the cross-currency basis.

Figure 8: Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions



Notes. The upper group of figures show BRL/USD exchange rate responses to unanticipated BCB interventions measured in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions measured in basis points (bp). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

finding supports Prediction 4 from the Gamma-Eta Model.

We find weak and insignificant effects for swap FXI (both traditional and reverse) and a widening of CIP violations by approximately 50 basis points over a 7-hour horizon following spot purchase FXI. While our theory predicts that traditional swaps should affect CIP violations, we note that these swaps generally occur in our sample during periods when intermediaries are unconstrained, which may reduce their effectiveness, as per Prediction 5 of the Gamma-Eta Model.[23]

4.3 FXI and Intermediary Constraints

Prediction 5 of the Gamma-Eta Model states that tighter intermediary constraints strengthen the exchange rate response to FXI and lower the quantity of intermediation that Financiers supply. Applied to the Brazilian case, the Gamma-Eta Model predicts that constrained intermediaries will limit USD liquidity to the Brazilian economy, and BCB operations to sell USD and counteract the constraints will have larger exchange rate effects.

To test Prediction 5, we augment the baseline specification in (20) by including dummy variables that indicate the state of intermediary constraints as either tight or loose. The intermediary capital ratio HKM_t defined in (19) serves as our measure of intermediary constraints, with low HKM_t values indicating a tight constraint.

We adopt the following augmented version of our baseline specification in (20),

$$\begin{aligned} y_{t+h} - y_{t-1} = {} & \beta_h^z INT_t^z \times SAD_t \times D_{HKM,t} \\ & + \gamma_h^z INT_t^z \times SAD_{t+h} \times (1 - D_{HKM,t}) + \delta_h SAD_{t+h} + \theta_h HKM_t \\ & + \text{Interactions}_t + \text{Daily-Freq Controls}_t + \text{High-Freq Controls}_t + u_{t+h}\,, \end{aligned}$$

where the outcome variable y_{t+h} is either the BRL/USD exchange rate or the cross-currency basis, and $D_{HKM,t}$ denotes an indicator for intermediary constraints, taking a value of one when the intermediary capital ratio measure HKM_t lies below its 50th percentile and zero otherwise. We interact $D_{HKM,t}$ with intervention size INT_t^z for intervention type z.[24]

The upper group of plots in Figure 9 illustrates how FXI and intermediary constraints interact to influence the exchange rate. We plot the conditional exchange rate response to unanticipated BCB spot and swap interventions. For spot USD sales under tight

[23]Additional results on the forward premium can be found in Section B.6 of the Online Appendix. The forward premium behaves in line with the observed response of CIP violations, which links CIP behavior to the relative demand for currency forwards. If FXI increases USD liquidity in FX swap markets, it raises the forward premium (in BRL per USD). This effect is most pronounced for spot FXI sales of USD reserves.

[24]Additional interactions in the specification are between $1 - SAD_t$ with periods of tight ($D_{HKM,t} = 1$) and loose ($D_{HKM,t} = 0$) constraints.

intermediary constraints ($D_{HKM,t} = 1$), we find that the BCB can statistically significantly appreciate the BRL against the USD by 2 percentage points over a 7-hour horizon per USD 1 billion sold. In comparison, the unconditional effect reported in Section 4.2 was smaller, with a 1.5 percentage point appreciation over the same horizon.

For traditional swaps under tight constraints ($D_{HKM,t} = 1$), the BCB can appreciate the BRL by approximately 20 basis points over a 7-hour window per USD 1 billion. In contrast, no statistically significant effect is observed for traditional swaps under loose constraints, although we do detect a noisy 40 basis point depreciation of the BRL against the USD. For spot purchases of USD and reverse swaps, we find no statistically significant effects.

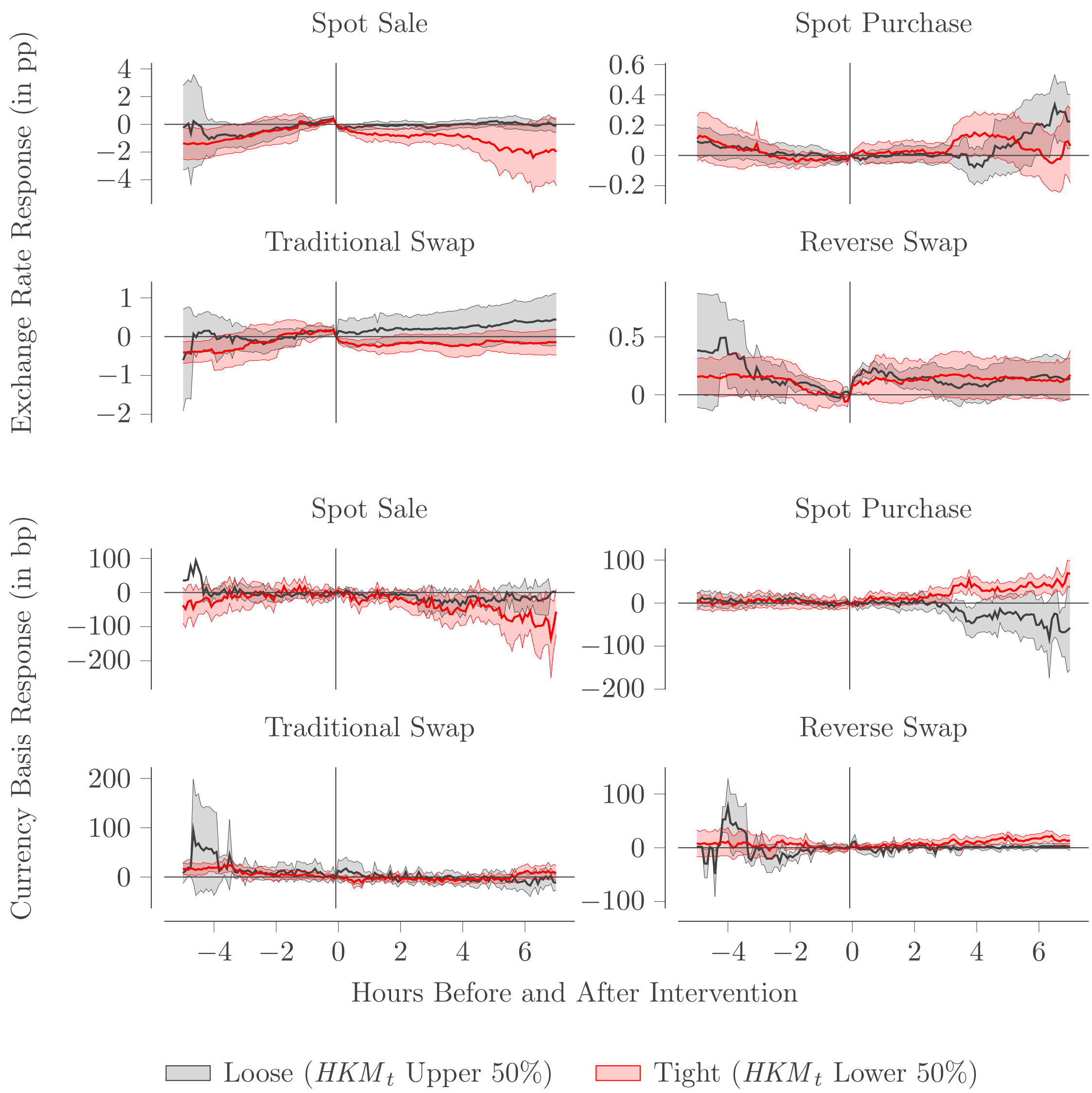
Next, we examine how intermediary constraints affect the quantity of intermediation, again using the BRL/USD cross-currency basis as a proxy. According to Prediction 5, FXI should relax intermediation constraints and reduce the cost of providing USD liquidity, which would be reflected in a decline in the BRL/USD cross-currency basis. While we found no unconditional effects of swap FXI on CIP violations, significant effects emerge when accounting for intermediary constraints. The lower group of plots in Figure 9 shows the conditional response of the cross-currency basis to unanticipated BCB interventions.[25]

Similar to the exchange rate results, we find that the impact of spot USD sales on the cross-currency basis is amplified during periods of tight intermediary constraints ($D_{HKM,t} = 1$), leading to an approximate 50 basis point reduction in CIP violations. In contrast, spot USD purchases under tight constraints widen CIP violations by 70 basis points, with no significant effects for traditional or reverse swaps.

In summary, our empirical findings support Prediction 5 of the Gamma-Eta Model. The data show that unanticipated interventions are more effective when intermediaries are constrained, whereas their effectiveness diminishes when constraints are looser. The economic intuition behind this result is clear: the BCB's spot USD sales and traditional

[25]A potential concern with our measure of intermediary constraints is that it might reflect a structural break in our sample, as dealer capital levels tend to be lower after 2008. To address this, in Section B.8 of the Online Appendix, we use the magnitude of CIP violations as an alternative proxy for intermediary constraints. Using this proxy, we find similar results: spot sales and traditional swap interventions during periods of above-median CIP violations lead to BRL appreciation and a narrowing of the cross-currency basis.

Figure 9: Conditional Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions



Notes. The upper group of figures show BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp), while the lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bp). Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the intermediary capital ratio HKM_t in equation (19). Red indicates tight constraints, defined as periods with HKM_t in the lower 50% of values in our sample. Gray indicates loose constraints, defined as periods with HKM_t in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

swaps increase USD liquidity. When intermediaries face tighter constraints, this added liquidity enhances FX market intermediation and causes larger exchange rate movements. However, when intermediaries are unconstrained, or during spot USD purchases or reverse swaps, the existing USD liquidity is sufficient for efficient intermediation, leading to no significant exchange rate effects.

4.4 Signaling Channel

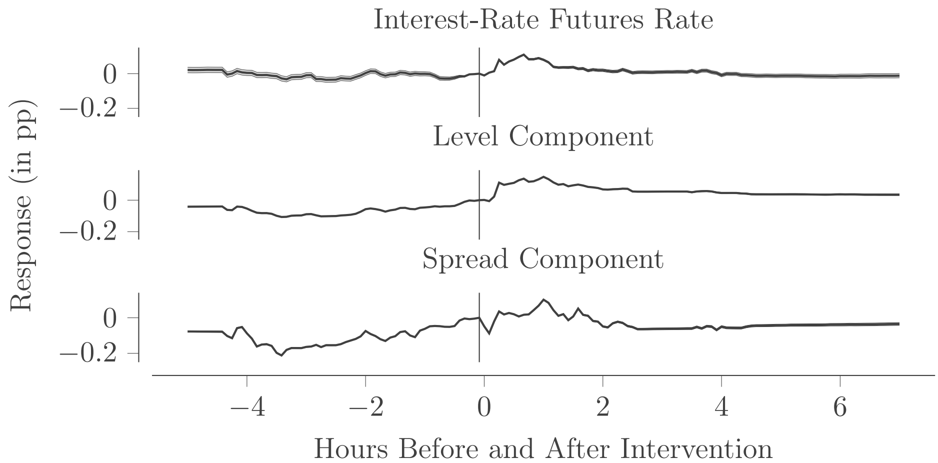
So far, we have found evidence for the dollar intermediation channel, which is a specific case of the portfolio balance channel with constrained financial intermediaries. We now test the effects of FXI on the future path of interest rates, contributing to the debate on the relative importance of the signaling and portfolio balance channels of FXI (Sarno and Taylor, 2001). The signaling channel posits that FXI provides information to market participants on the future path of interest rates. For example, a spot sale of USD reserves can signal that the central bank intends to strengthen the domestic currency in the future, which, all else being equal, requires higher future domestic interest rates.

Intra-day Interest Rate Futures. We first test the signaling channel using intra-day interest rate futures data from B3 as the outcome variable in our baseline specification in (20). Figure 10 plots the response of interest rate futures to the unanticipated spot sale interventions that occur during the period of our sample of B3 data coverage.[26] The 1-month interest rate futures increase by up to 15 basis points 2 hours after the announcement, and then drop to approximately 2.5 basis points 6 hours after the announcement.

Additionally, we plot the level component (average across maturities) of interest rate futures contracts and the spread between 5-year and 1-month interest rates. The average interest rate increase reaches a peak of 20 basis points and declines to 10 basis points after 7 hours, while the spread between the 1-month and 5-year yields increases in the immediate hours but is close to zero over the day. The effects are quantitatively and economically insignificant: a 1 USD billion change in USD increases interest rates by between 2.5 basis points for the 1-month and up to 10 basis points when averaging across maturities. In comparison, a 1 USD billion spot sale leads to a 1.5 percentage point (150 basis points) change in spot appreciation and a 50-100 basis point narrowing of the

[26] Similar to our B3 case study in Section 4.1, this includes unanticipated interventions on 27 August, 26 November, and 27 November 2019, for which we have intra-day transaction data from the B3 exchange.

Figure 10: High-Frequency Interest Rate Response to Unanticipated BCB Spot Sell Interventions



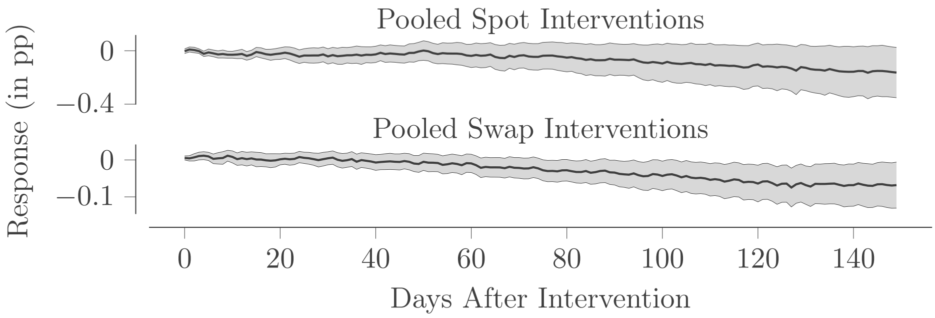
Notes. The figures show high-frequency responses of Brazilian interest rates to BCB spot sale interventions in the BRL/USD FX market measured in percentage points (pp). The upper figure shows the overall response of B3 interest-rate futures for the most liquid contract. The middle figure shows the response of the level component of interest-rate futures rates. The bottom figure shows the response of the spread component of interest-rate futures rates. The sample period of B3 interest rate futures runs from 2019-02-08 to 2020-01-23. Four unanticipated spot sale interventions occurred during the sample period, in total 2.481 billion USD. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors.

cross-currency basis.

Daily Interest Rates. In addition to using intra-day data, we use daily interest rates as the outcome variable in our baseline specification in (20), to test the signalling channel in our longer historical sample. At a daily frequency, we control for potential endogeneity by identifying a reaction function of FXI. We use macroeconomic variables to estimate the reaction function, including the level and volatility of exchange rates, interest rates, recession dates, the HKM dealer capital ratio, and the VIX index. In our regressions, we use the residual intervention component that is unexplained by macroeconomic fundamentals and is, therefore, plausibly exogenous to interest rates. This approach addresses the potential feedback from interest rates to FXI in our daily-frequency analysis. For supplementary information on the construction of our residualized FXI, we refer readers to Appendix B.9.

For spot and swap FXI at daily frequency, we examine the effect of pooled buy and

Figure 11: Daily-Frequency Interest Rate Response to Pooled Unanticipated BCB Interventions



Notes. The figures show 1-month interest rate responses to pooled unanticipated BCB spot and swap interventions at daily frequency, in percentage points (pp). Unanticipated interventions are measured in USD Billion. In the case of spot interventions, we pool spot sales with spot purchases in a single regression. In the case of swap interventions, we pool traditional swaps with reverse swaps in a single regression. Shaded indicates a 95% confidence interval using White heteroscedasticity-robust standard errors.

sell interventions. We find no significant effects of FXI on daily interest rates, as shown in Figure 11. Interest rates decline weakly by up to 25 basis points in response to (pooled) spot FXI over 6 months, however the effects are statistically insignificant. In summary, we find weak evidence for the central bank signaling changes in interest rates through either spot or swap interventions.

5 Conclusion

In this paper, we investigate the importance of the *dollar intermediation* channel of FXI, hypothesizing that the effects of FXI are more pronounced during periods of tight intermediary constraints.

To motivate our empirical framework, we extend the Basic Gamma Model of Gabaix and Maggiori (2015), distinguishing between spot and swap interventions and considering both anticipated an unanticipated interventions. The model involves two countries with Households that consume domestic and international goods but hold only domestic bonds. A foreign exchange Financier intermediates bond trade subject to credit constraints. The Central Bank uses interventions in the FX market to respond to trade shocks, with thresholds communicated as distributions to manage intervention anticipation. The model predicts that long-lived spot interventions and unanticipated interventions have larger exchange rate effects, and that interventions are more impactful under tighter credit constraints.

Our analysis uses an extensive database of BCB interventions from 1999 to 2023, combined with high-frequency data on spot and forward prices, enabling us to examine the intra-day effects of FXI. We highlight three key findings. First, unanticipated sell interventions of USD reserves lead to an appreciation of the domestic currency by approximately 150 basis points per USD billion, whereas buy interventions have insignificant effects. Second, unanticipated sell interventions reduce CIP violations, improving market efficiency. Third, the impact of FXI on spot prices and CIP violations is more pronounced during periods of tighter intermediary constraints, with significant effects observed primarily during these times.

Finally, we test whether FXI works through an alternative channel by signaling changes in the stance of monetary policy. We find small, transitory interest rate effects following interventions, suggesting that the portfolio balance channel, not central bank signaling, is more important in explaining the observed effects of FXI.

Our findings are salient for central banks using FXI as a tool to improve the functioning of dollar intermediation and FX markets. We contribute to the debate on the relative importance of the portfolio balance and signaling channels of FXI, and the relative efficacy of spot and swap instruments. We show that FXI can alleviate constraints on global financial intermediaries, enhancing market efficiency, with unanticipated spot sales of USD reserves being the most effective instrument.

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Appendix

A Theory

In this section of the Online Appendix, we provide details of the Gamma-Eta model presented in Section 2 of the main paper. We provide a detailed the structure of the model and its key assumptions in Figure A.1, followed by a formal derivation of the key equilibrium conditions in Section A.2. We linearize the model in Section A.3 and solve for the real exchange rate in Section A.4. We provide a brief discussion of exchange rate dynamics in Section A.5. We prove the model's key predictions in Section A.6. Finally, we discuss sterilized interventions in Section A.7.

A.1 Home Household Problem

The Lagrangian formulation of the Home Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned}
\mathcal{L}_0 = \sum_{s=t}^{2} \beta^s \, \mathrm{E}_0^{(-)}[\chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs}] \\
+\beta^0 \lambda_0 \, \mathrm{E}_0^{(-)}[Y_{NT0} + p_{H0} Y_{H0} - C_{NT0} - p_{H0} C_{H0} - p_{F0} C_{F0} - Q_{H0}] \\
+\beta^1 \lambda_1 \, \mathrm{E}_1^{(-)}[Y_{NT1} + p_{H1} Y_{H1} - C_{NT1} - p_{H1} C_{H1} - p_{F1} C_{F1} - Q_{H1} + R_{H1} Q_{H0}] \\
+\beta^2 \lambda_2 \, \mathrm{E}_2^{(-)}[Y_{NT2} + p_{H2} Y_{H2} - C_{NT2} - p_{H2} C_{H2} - p_{F2} C_{F2} + R_{H2} Q_{H1}] \,,
\end{aligned} \tag{21}$$

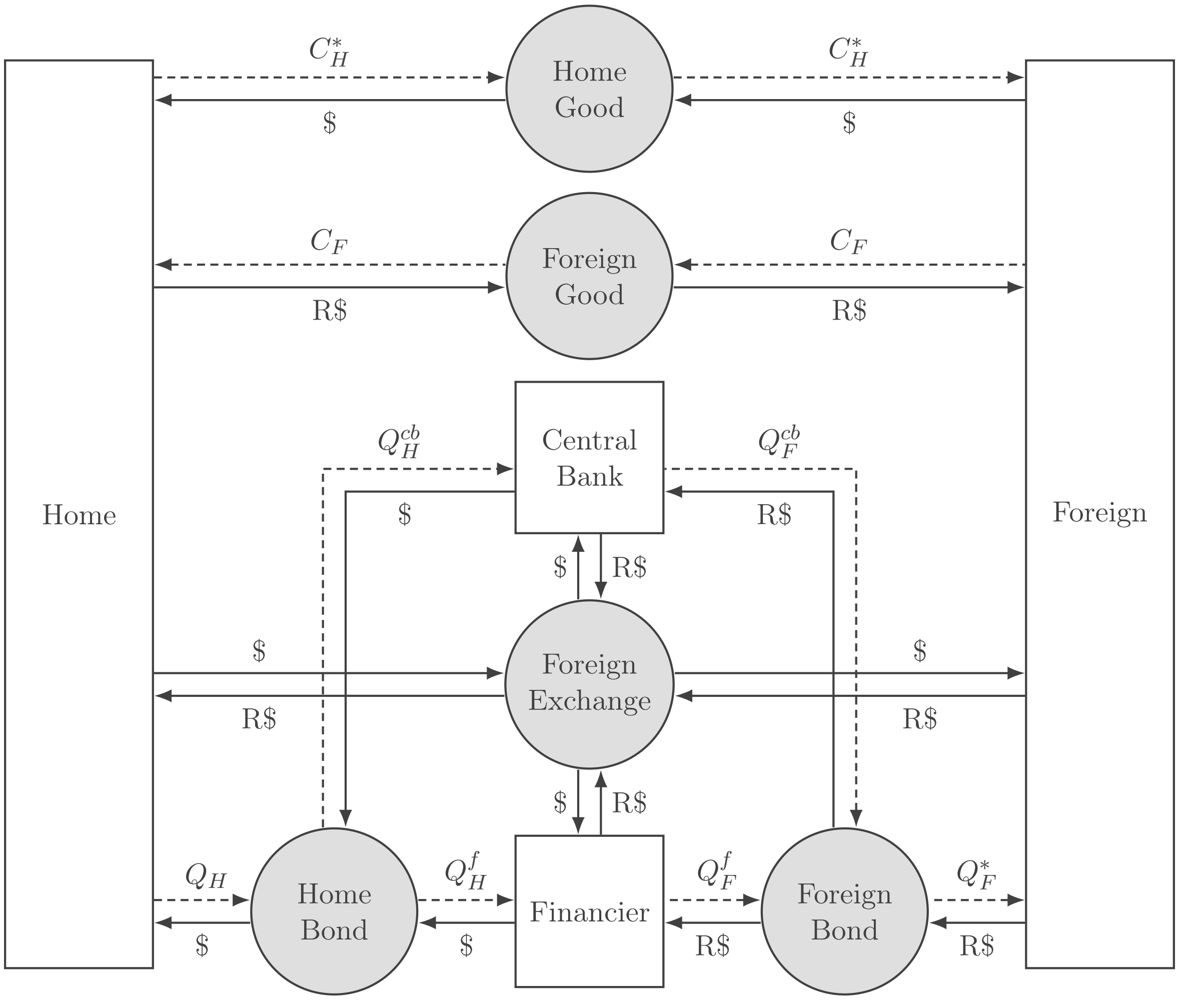
and derivatives $\partial \mathcal{L}_0 / \partial C_{NTs}$ and $\partial \mathcal{L}_0 / \partial Q_{Hs}$ lead to the optimality conditions in (3) for $s < 2$. Similar Lagrangian formulations can be written for the Household's intertemporal problems in $t = 1, 2$.

The Lagrangian formulation of the Home Household's intratemporal objective function is given by

$$\mathcal{L}_t = \chi_s \ln C_{NTt} + \alpha_s \ln C_{Ht} + \iota_s \ln C_{Fs} + \lambda_t (p_t C_t - C_{NTt} - p_{Ht} C_{Ht} - p_{Ft} C_{Ft}) \,, \tag{22}$$

and combining the derivative $\partial \mathcal{L} / \partial C_{NTt}$ with the derivatives $\partial \mathcal{L} / \partial C_{Ht}$ and $\partial \mathcal{L} / \partial C_{Ft}$

Figure A.1: Real and financial flows in the two-country model.



Notes. The figure depicts trade flows resulting from a positive trade shock $\Delta\iota_t$ that induces the Home Household to sell Home bonds and the Financier and Central Bank to buy Home bonds. Decision makers are depicted as rectangles, markets are depicted as circles, and trade flows are depicted as arrows. An arrow into a market depicts a supply flow, and an arrow out of a market depicts a demand flow. Real flows are shown as dashed lines, currency flows are shown as solid lines with labels "$" for USD and "R$" for BRL. Within-country goods trade, as well as profit or loss transfers from the Financier and the Central Bank to the Foreign Household are omitted.

yields the optimality conditions in (5).

A.2 Foreign Household Problem

The Foreign Household solves a three-period intertemporal utility maximization problem,

$$\max_{\{C^*_{NTs}, C^*_{Hs}, C^*_{Fs}\}_{s=t}^{2}} \sum_{s=t}^{2} \beta^{*s} \, \mathrm{E}_t^{(\cdot)}[\varphi_s^* \ln C_s^*] \,, \tag{23}$$

β^* denotes the Foreign Household's subjective discount factor, $\varphi_t \equiv \chi^*_t + \iota^*_t + \alpha^*_t$ denotes a sum of stochastic preference shocks, and where

$$C^*_t \equiv \left[(C^*_{NTt})^{\chi^*_t} (C^*_{Ht})^{\iota^*_t} (C^*_{Ft})^{\alpha^*_t} \right]^{\frac{1}{\varphi^*_t}} \tag{24}$$

denotes the Foreign Household consumption basket, a sub-utility function composed of Foreign non-tradable goods C^*_{NTt}, Home tradable goods C^*_{Ht}, and Foreign tradable goods C^*_{Ft}. The intertemporal maximization is subject to period budget constraints of the form

$$Q^*_{Ft} + p^*_{Ht} C^*_{Ht} + p^*_{Ft} C^*_{Ft} + C^*_{NTt} = Y^*_{NTt} + p^*_{Ft} Y^*_{Ft} + R^*_{Ft} Q^*_{Ft-1} + \pi^*_t + \tau^*_t \,, \tag{25}$$

where Q^*_{Ft} denotes the real value of Foreign holdings of the Foreign bond, with $Q^*_{F-1} = Q^*_{F2} = 0$, where p^*_{Ht} and p^*_{Ft} denote relative prices of the Home and Foreign tradable goods, where Y^*_{NTt} and Y^*_{Ft} denote stochastic endowments of the Foreign non-tradable and tradable goods, respectively, and where π^*_t and τ^*_t denote profits and taxes transferred to the Foreign HOusehold by the Financier and the Central Bank, respectively. The Foreign non-tradable good is the numéraire for the Foreign economy.

The Lagrangian formulation of the Foreign Household's intertemporal objective function for $t = 0$ is given by

$$\begin{aligned} \mathcal{L}^*_0 = \sum_{s=t}^{2} & \beta^{*s} \, \mathrm{E}^{(-)}_0 [\chi^*_s \ln C^*_{NTt} + \iota^*_s \ln C^*_{Ht} + \alpha^*_s \ln C^*_{Fs}] \\ & + \beta^{*0} \lambda^*_0 \, \mathrm{E}^{(-)}_0 [Y^*_{NT0} + p^*_{F0} Y^*_{F0} - C^*_{NT0} - p^*_{H0} C^*_{H0} - p^*_{F0} C^*_{F0} - Q_{H0}] \\ & + \beta^{*1} \lambda^*_1 \, \mathrm{E}^{(-)}_1 [Y^*_{NT1} + p^*_{F1} Y^*_{F1} - C^*_{NT1} - p^*_{H1} C^*_{H1} - p^*_{F1} C^*_{F1} - Q^*_{H1} + R^*_{F1} Q^*_{F0}] \\ & + \beta^{*2} \lambda^*_2 \, \mathrm{E}^{(-)}_2 [Y^*_{NT2} + p^*_{F2} Y^*_{F2} - C^*_{NT2} - p^*_{H2} C^*_{H2} - p^*_{F2} C^*_{F2} + R^*_{F2} Q^*_{F1}] \,, \end{aligned} \tag{26}$$

and derivatives $\partial \mathcal{L}^*_0 / \partial C^*_{NTs}$ and $\partial \mathcal{L}^*_0 / \partial Q^*_{Fs}$ lead to the following optimality conditions,

$$\mathrm{E}^{(-)}_t [\lambda^*_s] = \mathrm{E}^{(-)}_t \left[\chi^*_s / C^*_{NTs} \right] \quad \text{and} \quad \mathrm{E}^{(-)}_t [\lambda^*_s] = \mathrm{E}^{(-)}_t \left[\lambda^*_{s+1} R^*_{Fs+1} \right] , \tag{27}$$

where λ^*_s denotes the Lagrange multiplier on the Foreign Household's period-t budget constraint and where R^*_{Fs+1} denotes the gross real return on the Foreign bond, with $t \leq s < 2$.

The Foreign Household also solves an intratemporal utility maximization problem, allocating expenditure across domestic and international goods. Specifically, the Foreign Household maximizes the logarithm of its sub-utility function in (24),

$$\max_{C^*_{NTt}, C^*_{Ht}, C^*_{Ft}} \chi^*_t \ln C^*_{NTt} + \iota^*_t \ln C^*_{Ht} + \alpha^*_t \ln C^*_{Ft} \,, \tag{28}$$

subject to a consumption expenditure constraint,

$$p^*_t C^*_t = C^*_{NTt} + p^*_{Ht} C^*_{Ht} + p^*_{Ft} C^*_{Ft} \,, \tag{29}$$

where p^*_t is the Foreign price index in terms of the Foreign non-tradable numéraire, and where the Foreign Household takes total consumption expenditure $p^*_t C^*_t$ as fixed in the intratemporal problem.

The Lagrangian formulation of the Foreign Household's intratemporal objective function is given by

$$\mathcal{L}^*_t = \chi^*_s \ln C^*_{NTt} + \iota^*_s \ln C^*_{Ht} + \alpha^*_s \ln C^*_{Fs} + \lambda^*_t (p^*_t C^*_t - C^*_{NTt} - p^*_{Ht} C^*_{Ht} - p^*_{Ft} C^*_{Ft}) \,, \tag{30}$$

and combining the derivative $\partial \mathcal{L}^* / \partial C^*_{NTt}$ with the derivatives $\partial \mathcal{L}^* / \partial C^*_{Ht}$ and $\partial \mathcal{L}^* / \partial C^*_{Ft}$ yields the following optimality conditions for consumption expenditure on Home and Foreign tradable goods,

$$p^*_{Ft} C^*_{Ft} = (\chi^*_t / C^*_{NTt}) \alpha^*_t \quad \text{and} \quad p^*_{Ht} C^*_{Ht} = (\chi^*_t / C^*_{NTt}) \iota^*_t \,. \tag{31}$$

As we did for the Home Household, we make two simplifying assumptions for the Foreign Household: the Foreign non-tradable endowment adjusts proportionally to fluctuations in the Foreign preference for non-tradable goods, $Y^*_{NTt} = \chi^*_t$, and the Foreign Household does not discount future utility, $\beta^* = 1$.

A.3 Deviations from Steady State

To solve the model, we write key equations in terms of deviations from the non-stochastic steady state. We first derive the steady-state equilibrium, and then derive the expressions in deviations from the steady state for bond demands and bond market clearing that stated in (12)–(15) in the main text.

Steady State. To indicate non-stochastic steady-state values, we omit time subscripts from variables. Under our simplifying assumptions, the Household optimality conditions in (3) and (27) imply that steady-state gross real returns on Home and Foreign bonds equal one,

$$R_H = R_F^* = 1\,. \tag{32}$$

The Financier's value function (6) is linear in the Financier's bond holdings, so the Financier's the credit constraint (7) holds with equality. Using the Financier's value function, credit constraint, and balance sheet constraint (8), we obtain the optimality condition in (9). The Financier's optimality condition in (9) then implies zero steady-state bond holdings for the Financier.

The Central Bank's steady-state bond holdings in (10) depend on the steady-state value of the trade shock and the steady-state values of the Central Bank's intervention thresholds. We assume that preferences for imports equal one in the non-stochastic steady state, $\iota = \iota^* = 1$, which implies a steady-state trade shock of zero, $\Delta\iota = 0$. We assume the Central Bank sets its intervention thresholds to zero in the non-stochastic steady-state, $\underline{\iota} = \bar{\iota} = 0$. The Heaviside function is defined to take a value of zero when its argument is zero, so $\Delta H = H(\Delta\iota - \bar{\iota}) - H(\underline{\iota} - \Delta\iota) = 0$. The Central Bank's steady state bond holdings are therefore zero.

Bond market clearing then implies zero bond holdings for the Home Household, so that

$$Q_H = Q_H^f = Q_H^{cb} = 0\,. \tag{33}$$

Market clearing for Home tradable and non-tradable goods requires that

$$Y_{Ht} = C_{Ht} + C^*_{Ht} \quad \text{and} \quad Y_{NTt} = C_{NTt}\,. \tag{34}$$

Combining goods market clearing conditions in (34) with the Home Household period budget constraint in (2) evaluated at the non-stochastic steady state, using goods expenditures in (5) and (31) with our simplifying assumptions, using steady-state bond holdings in (33), and using the Law of One Price for the Home tradable good ($p_{Ht} = e_t p^*_{Ht}$), we obtain the following steady state real exchange rate,

$$e = 1\,, \tag{35}$$

where we have used $\iota = \iota^* = 1$. The steady-state results in (32), (33), and (35) suffice for the derivations of bond demands that we turn to next.

Bond Demands. We begin by deriving the Home Household's Home bond demand in (13). The Home Household's period zero budget constraint can be rewritten using market clearing conditions for the Home tradable and non-tradable in (34), the Law of One Price for the Home tradable good, goods expenditures in (5) and (31), and our simplifying assumptions on preferences to obtain

$$Q_{H0} + \iota_0 = e_0 \iota^*_0\,.$$

Linearizing around the non-stochastic steady state, in logs with respect to e_t, ι_t, and R_{Ht+1} and levels with respect to Q_{Ht}, we obtain

$$(Q_{H0} - Q_H) + \iota\hat{\iota}_0 = e\iota^*\hat{e}_0 + e\iota^*\hat{\iota}^*_0 + O\big(\epsilon^2\big)\,,$$

where we have used $Q_H = 0$, and where we define $\hat{e}_t \equiv \ln e_t - \ln e$, $\hat{\iota}_t \equiv \ln \iota_t - \ln \iota$ and $\hat{\iota}^*_t \equiv \ln \iota^*_t - \ln \iota^*$. Letting $\hat{Q}_{Ht} \equiv (Q_{Ht} - Q_H)/\iota$ and $\Delta\hat{\iota}_t \equiv \hat{\iota}_t - \hat{\iota}^*_t$, and using $e = \iota = \iota^* = 1$, we obtain

$$\hat{Q}_{H0} = \hat{e}_0 - \Delta\hat{\iota}_0 + O\big(\epsilon^2\big)\,. \tag{36}$$

Following a similar procedure for the period one budget constraint, we first obtain

$$Q_{H1} + \iota_1 = e_1\iota_1^* + R_{H1}Q_{H0}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned} (Q_{H1} - Q_H) + \iota\hat{\iota}_1 &= e\iota^*\hat{e}_1 + e\iota^*\hat{\iota}_1^* + R_H Q_H \hat{R}_{H1} + R_H(Q_{H0} - Q_H) + O\big(\epsilon^2\big) \\ \Leftrightarrow \quad \hat{Q}_{H1} &= \hat{e}_1 - \Delta\hat{\iota}_1 + \hat{Q}_{H0} + O\big(\epsilon^2\big)\,, \end{aligned}$$

where we have used $e = \iota = \iota^* = 1$. We combine the linearized expression with (36) to obtain

$$\hat{Q}_{H1} = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + O\big(\epsilon^2\big)\,. \tag{37}$$

For the period two budget constraint, we first obtain

$$\iota_2 = e_2\iota_2^* + R_{H2}Q_{H1}\,,$$

which we linearize around the non-stochastic steady state to obtain

$$\begin{aligned} \iota\hat{\iota}_2 &= e\iota^*\hat{e}_2 + e\iota^*\hat{\iota}_2^* + R_H Q_H \hat{R}_{H2} + R_H(Q_{H1} - Q_H) + O\big(\epsilon^2\big) \\ \Leftrightarrow \quad 0 &= \hat{e}_2 - \Delta\hat{\iota}_2 + \hat{Q}_{H1} + O\big(\epsilon^2\big)\,, \end{aligned}$$

where we have used $e = \iota = \iota^* = 1$. We combine the linearized expression with (37) to obtain

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\big(\epsilon^2\big)\,. \tag{38}$$

The expressions in (36)–(38) appear in (13) in the main text.

The Financier's linearized Home bond demand derives from the Financier's optimality condition in (9). We linearize (9) as follows,

$$\frac{1}{e}\hat{Q}^f_{Ht} - \frac{Q^f_H}{e}\,\mathrm{E}^{(-)}_t[\hat{e}_t] = \frac{1}{\Gamma}\,\mathrm{E}^{(-)}_t\left[R_H\hat{R}_{Ht+1} + R^*_F\left(\hat{e}_{t+1} - \hat{e}_t + \hat{R}^*_{Ft+1}\right)\right] + O\left(\epsilon^2\right), \tag{39}$$

where we define $\hat{Q}^f_{Ht} \equiv (Q^f_{Ht} - Q^f_H)/\iota$. This expression simplifies because $Q^f_H = 0$ and $e = R_H = R^*_F = 1$, and because $\mathrm{E}^{(-)}_t\left[\hat{R}_{Ht+1}\right] = \mathrm{E}^{(-)}_t\left[\hat{R}^*_{Ft+1}\right] = 0$. The latter result derives from linearizations of the Home and Foreign Household intertemporal optimality conditions in (5) and (31). Linearizing (5), assuming $\chi_t = C_{NTt}$ and $\beta = 1$,

$$\mathrm{E}^{(-)}_t\left[\hat{\lambda}_s\right] = \mathrm{E}^{(-)}_t\left[\hat{\chi}_s - \hat{C}_{NTs}\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}^{(-)}_t\left[\hat{R}_{Hs+1}\right] = 0 + O\left(\epsilon^2\right),$$

and linearizing (31), assuming $\chi^*_t = C^*_{NTt}$ and $\beta^* = 1$,

$$\mathrm{E}^{(-)}_t\left[\hat{\lambda}^*_s\right] = \mathrm{E}^{(-)}_t\left[\hat{\chi}^*_s - \hat{C}^*_{NTs}\right] = 0 + O\left(\epsilon^2\right) \quad \Rightarrow \quad \mathrm{E}^{(-)}_t\left[\hat{R}^*_{Fs+1}\right] = 0 + O\left(\epsilon^2\right).$$

These results combine with (39), using $Q^f_H = 0$ and $e = R_H = R^*_F = 1$, to yield the Financier's linearized Home bond demand in (14).

The Central Bank's bond demand written in deviations from steady state derives from the policy rule in (10). We can rewrite the left-hand side of (10) as follows,

$$Q^{cb}_{Ht} = Q^{cb}_{Ht} - Q^{cb}_H = (Q^{cb}_{Ht} - Q^{cb}_H)/\iota = \hat{Q}^{cb}_{Ht}, \tag{40}$$

where we define $\hat{Q}^{cb}_{Ht} = (Q^{cb}_{Ht} - Q^{cb}_H)/\iota$ and use $Q^{cb}_H = 0$ and $\iota = 1$. We leave the right-hand side of the Central Bank's policy rule unchanged, and use (40) to obtain the linearized policy rule in (15).

Finally, we write the Home bond market clearing condition in deviations from steady state using the definitions of $\hat{Q}_{Ht}$, $\hat{Q}^f_{Ht}$, and $\hat{Q}^{cb}_{Ht}$ stated above. The expression (12) is exact, given that bond holdings are linearized in levels.

A.4 Real Exchange Rate Solutions

To solve for the real exchange rate, we combine the linearized bond demands of the Home Household in (13), the Financier in (14), and the Central Bank in (15) with the market clearing condition in (12) to obtain a system of three linear equations,

$$0 = \hat{e}_0 - \Delta\hat{\iota}_0 + q\Delta H_0 + \frac{1}{\Gamma}\,\mathrm{E}^{(-)}_0[\hat{e}_0 - \hat{e}_1] + O\big(\epsilon^2\big) \tag{41}$$

$$0 = \hat{e}_0 + \hat{e}_1 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 + q\Delta H_1 + \frac{1}{\Gamma}\,\mathrm{E}^{(-)}_1[\hat{e}_1 - \hat{e}_2] + O\big(\epsilon^2\big) \tag{42}$$

$$0 = \hat{e}_0 + \hat{e}_1 + \hat{e}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1 - \Delta\hat{\iota}_2 + O\big(\epsilon^2\big)\,, \tag{43}$$

in the endogenous real exchange rates $\hat{e}_0$, $\hat{e}_1$, and $\hat{e}_2$ and their conditional expected values. Because conditional expected values appear in the system of three equations, we need additional conditions to pin down equilibrium real exchange rates.

Taking the expectations of (41)–(43) conditional on period-zero information and the expectations of (42) and (43) conditional on period-one information, we obtain five additional conditions, which gives us a system of eight linear equations in total and allows us to solve for $\hat{e}_0$, $\hat{e}_1$, $\hat{e}_2$, $\mathrm{E}^{(-)}_0[\hat{e}_0]$, $\mathrm{E}^{(-)}_0[\hat{e}_1]$, $\mathrm{E}^{(-)}_0[\hat{e}_2]$, $\mathrm{E}^{(-)}_1[\hat{e}_1]$, and $\mathrm{E}^{(-)}_1[\hat{e}_2]$. Standard methods can be used to solve the system. We omit the intermediate algebra and provide final solutions for realized real exchange rates,

$$\begin{aligned}\hat{e}_0 = \Delta\hat{\iota}_0 &- \frac{2\Delta\hat{\iota}_0 - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_1] - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_2]}{3+\Gamma} - \frac{\Gamma\big(\mathrm{E}^{(-)}_0[\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0]\big)}{(1+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} - \Gamma q\frac{\Delta H_0}{1+\Gamma} - \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,,\end{aligned} \tag{44}$$

$$\begin{aligned}\hat{e}_1 = \Delta\hat{\iota}_1 &- \frac{2\Delta\hat{\iota}_1 - \Delta\hat{\iota}_0 - \mathrm{E}^{(-)}_1[\Delta\hat{\iota}_2]}{3+\Gamma} + \frac{(1+\Gamma)\big(\Delta\hat{\iota}_1 - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_1]\big)}{(2+\Gamma)(3+\Gamma)} + \frac{\mathrm{E}^{(-)}_1[\Delta\hat{\iota}_2] - \mathrm{E}^{(-)}_0[\Delta\hat{\iota}_2]}{(2+\Gamma)(3+\Gamma)} \\ &- q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} - q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma}\,, \\ &- \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} - \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\Big(\epsilon^2\Big)\,, \quad \text{and}\end{aligned} \tag{45}$$

$$
\begin{aligned}
\hat{e}_2 = \Delta\hat{\iota}_2 &- \frac{2\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0 - \Delta\hat{\iota}_1}{3+\Gamma} + \frac{\Gamma(\Delta\hat{\iota}_2 - \Delta\hat{\iota}_0)}{(1+\Gamma)(3+\Gamma)} + \frac{2\Delta\hat{\iota}_2 - \mathrm{E}_0^{(-)}[\Delta\iota_2] - \mathrm{E}_1^{(-)}[\Delta\hat{\iota}_2]}{(1+\Gamma)(3+\Gamma)} \\
&+ \frac{\Gamma\left(\Delta\hat{\iota}_2 - \mathrm{E}_1^{(-)}[\Delta\iota_2]\right)}{(1+\Gamma)(3+\Gamma)} + \frac{\Delta\hat{\iota}_1 - \mathrm{E}_0^{(-)}[\Delta\hat{\iota}_1]}{(2+\Gamma)(3+\Gamma)} - \frac{\mathrm{E}_1^{(-)}[\Delta\iota_2] - \mathrm{E}_0^{(-)}[\Delta\iota_2]}{(2+\Gamma)(3+\Gamma)} \\
&+ q\frac{\Delta H_0 - \eta_{0|0}}{1+\Gamma} + \Gamma q\frac{\Delta H_0}{1+\Gamma} + \Gamma q\frac{\eta_{1|0} - \eta_{0|0}}{(1+\Gamma)(3+\Gamma)} \\
&+ q\frac{\Delta H_1 - \eta_{1|1}}{2+\Gamma} + q\frac{(\Delta H_1 - \eta_{1|1}) - (\Delta H_0 - \eta_{0|0})}{2+\Gamma} \\
&+ \Gamma q\frac{\Delta H_1 - \Delta H_0}{3+\Gamma} + \Gamma q\frac{(\Delta H_1 - \eta_{1|0}) - (\Delta H_0 - \eta_{0|0})}{(2+\Gamma)(3+\Gamma)} + O\left(\epsilon^2\right).
\end{aligned}
\tag{46}
$$

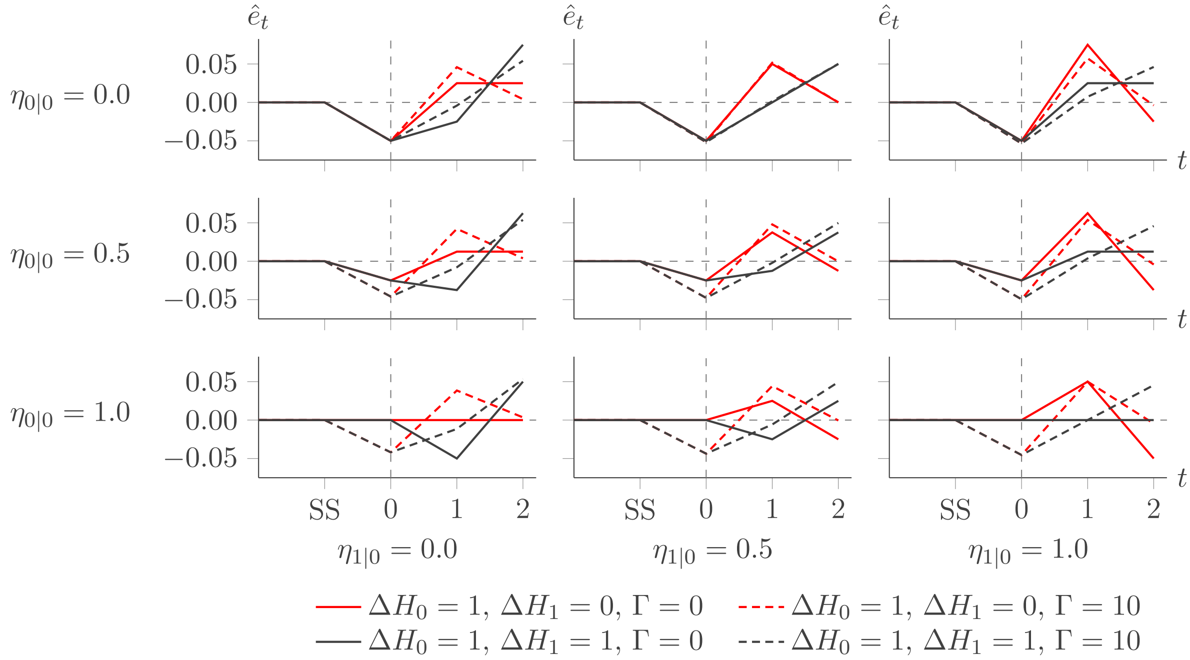
A.5 Real Exchange Rate Dynamics

We limit our theoretical model to three periods because we seek interpretable closed-form expressions that deliver intuitive qualitative predictions on the real exchange rate effects of Central Bank interventions. In our model, interventions are only possible in the first two periods, when bond trading occurs. This timeline gives us enough flexibility to differentiate between short-lived swap and long-lived spot interventions, but not more. The constrained timeline of the model precludes a nuanced study of economic dynamics; however, limited insights can still be gained from analyzing the three-period path of the real exchange rate in response to trade shocks and interventions.

Figure A.2 plots the path of the real exchange rate in response to positive Central Bank interventions at scale $q = 0.05$. To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\iota}_t = 0$. In the figure, the Central Bank intervenes positively to strengthen Home currency and lower the real exchange rate $\hat{e}_0$. In a positive intervention, the Central Bank sells Foreign bonds and buys Home bonds, exchanging Foreign currency for Home currency in the process. The Home currency demand caused by the Central Bank's bond trade strengthens Home currency and put downward pressure on $\hat{e}_0$. The model's equation (13) produces what Gabaix and Maggiori (2015) describe as a "boomerang" effect.

Figure A.2 shows how the path of the real exchange rate depends on the type of intervention (swap versus spot), the degree of anticipation (unanticipated versus partially or fully anticipated), and the credit constraint (tight or loose). The initial real exchange rate response is generally larger when the intervention is unanticipated, as shown in the

Figure A.2: Real Exchange Rates Dynamics



Notes. The figure plots the path of the real exchange rate over the three periods of the model in response to Central Bank interventions at scale $q = 0.05$, beginning from a steady-state position (SS). To isolate the effects of interventions, we set trade shocks to zero, $\Delta\hat{\imath}_t = 0$. Red lines represent short-lived swap interventions. Black lines represents long-lived spot interventions. Solid lines represent loose credit constraints. Dashed lines represent tight credit constraints. Each row corresponds to a value for the probability of a positive period-zero intervention as perceived by Households and the Financier. Each column corresponds to a value for the probability of a positive period-one intervention as perceived by Households and the Financier. Probability values capture three cases: unanticipated ($\eta_{s|t} = 0$), partially anticipated ($\eta_{s|t} = 0.5$), and fully anticipated ($\eta_{s|t} = 1$).

top row of the figure. The effect is largest when the intervention is fully unanticipated and the credit constraint is loose, as shown in the upper left figure. When fully anticipated with loose credit constraints, interventions have no effect, as shown in the bottom left and right figures. In between these extremes, the Central Bank has can shape the path of the real exchange rate in many ways by varying the type of intervention and manipulating Household and Financier expectations.

A.6 Prediction Proofs

Proof 1 (Direction of Interventions)**.** *We now prove Prediction 1. Consider a Central Bank intervention whereby the Central Bank constructs a portfolio of Home and Foreign bonds. The real exchange rate responds to the currency flows arising from the portfolio* *construction as follows,*

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma \Delta H_0}{1+\Gamma} - \frac{\Delta H_0 - \mathrm{E}_0^{(-)}[\Delta H_0]}{1+\Gamma} - \frac{\Gamma\, \mathrm{E}_0^{(-)}[\Delta H_1 - \Delta H_0]}{(1+\Gamma)(3+\Gamma)}\,. \tag{47}$$

The direction of the response depends on the direction of the intervention. Recall that $\Delta H_0 \in \{-1, 0, 1\}$ *and consider two cases: 1) positive intervention, and 2) negative intervention.*

- ***Case 1:*** *Consider a* ***positive intervention****,* $\Delta H_0 = 1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = -1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{<} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftarrow \quad \frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} + \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &< 1 \\
\Leftrightarrow \quad \frac{3}{3+\Gamma} &< 1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

- ***Case 2:*** *Consider a* ***negative intervention****,* $\Delta H_0 = -1$*. In this case,*

$$\begin{aligned}
\frac{\partial \hat{e}_0}{\partial q} = 1 + \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &\overset{!}{>} 0 \\
\Leftrightarrow \quad \frac{(3+2\Gamma)\,\mathrm{E}_0^{(-)}[\Delta H_0]}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma\,\mathrm{E}_0^{(-)}[\Delta H_1]}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftarrow \quad -\frac{3+2\Gamma}{(1+\Gamma)(3+\Gamma)} - \frac{\Gamma}{(1+\Gamma)(3+\Gamma)} &> -1 \\
\Leftrightarrow \quad -\frac{3}{3+\Gamma} &< -1 \\
\Leftrightarrow \quad 0 &< \Gamma\,.
\end{aligned}$$

Hence, for $\Delta H_0 \neq 0$ *and* $\Gamma > 0$*,*

$$\operatorname{sign} \frac{\partial \hat{e}_0}{\partial q} = -\operatorname{sign} \Delta H_0\,.$$

In words, a positive intervention strengthens Home currency and lowers the real exchange *rate, while a negative intervention weakens Home currency and raises the real exchange rate.*

Proof 2 (Spot versus Swap Interventions)**.** *We now prove Prediction 3. Consider four cases of fully anticipated Central Bank interventions: 1) a positive spot intervention, 2) a positive swap intervention, 3) a negative spot intervention, and 4) a negative swap intervention.*

- ***Case 1:*** *Consider a fully anticipated* ***positive spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1$*. In this case, from* (47)*,*

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma}\,.$$

- ***Case 2:*** *Consider a fully anticipated* ***positive swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = 1,\, \eta_{1|0} = 0$*. In this case, from* (47)*,*

$$\frac{\partial \hat{e}_0}{\partial q} = -\frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}\,.$$

For $\Gamma > 0$*, cases one and two imply that*

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = \eta_{1|0} = 1} < \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0 = \eta_{0|0} = 1,\, \eta_{1|0} = 0},$$

where the subscripts indicate the values at which the derivative is evaluated.

- ***Case 3:*** *Consider a fully anticipated* ***negative spot*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = \eta_{1|0} = -1$*. In this case, from* (47)*,*

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma}\,.$$

- ***Case 4:*** *Consider a fully anticipated* ***negative swap*** *intervention in period one such that* $\Delta H_0 = \eta_{0|0} = -1,\, \eta_{1|0} = 0$*. In this case, from* (47)*,*

$$\frac{\partial \hat{e}_0}{\partial q} = \frac{\Gamma}{1+\Gamma} \times \frac{2+\Gamma}{3+\Gamma}\,.$$

For $\Gamma > 0$, cases three and four imply that

$$\left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0=\eta_{0|0}=\eta_{1|0}=-1} > \left.\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta H_0=\eta_{0|0}=-1,\, \eta_{1|0}=0},$$

where the subscripts indicate the values at which the derivative is evaluated. Hence, for $\Delta H_0 = \Delta\eta_{0|0} \neq 0$ and $\Gamma > 0$, cases one through four imply that

$$\left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0}=\Delta H_0} > \left|\frac{\partial \hat{e}_0}{\partial q}\right|_{\Delta\eta_{1|0}=0}.$$

In words, a fully anticipated long-lived spot intervention ($\Delta\eta_{1|0} = \Delta H_0$) has a greater effect on the real exchange rate than a fully anticipated short-lived swap intervention ($\Delta\eta_{1|0} = 0$), for both positive and negative interventions.

Proof 3 (Anticipated versus Unanticipated Interventions)**.** *We now prove Prediction 2. Consider a Central Bank communication that raises the expectation of a positive intervention in period zero ($\Delta\eta_{0|0} > 0$). The second partial derivative of (47) with respect to the parameter that $\eta_{0|0}$ that governs the expectation of a period zero intervention is given by*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{0|0}} = \frac{3 + 2\Gamma}{(1+\Gamma)(3+\Gamma)} > 0\,.$$

In words, if the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by less if Households and the Financier anticipate the intervention in period zero. If $\Gamma = 0$ and $\Delta H_0 = \eta_{0|0}$, the right-hand size of (47) equals zero as can be seen by inspection.

We obtain a second prediction in this context, which we omitted from Prediction 2 in the body of the paper for brevity. Consider a Central Bank communication that raises the period-zero expectation of a positive intervention in period one — e.g. to maintain an on-going spot intervention. The second partial derivative of (47) with respect to the parameter $\eta_{1|0}$ that governs the expectation of a period-one intervention is given by

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Delta\eta_{1|0}} = -\frac{\Gamma}{(1+\Gamma)(3+\Gamma)} < 0\,.$$

Thus, if the Central Bank carries out a positive intervention in period zero, the real exchange rate will fall by more than it otherwise would, if Households and the Financier anticipate that the intervention will be maintained in period one. This results suggests that forward guidance may have a role to play Central Bank FXI policy.

Proof 4 (Private Intermediation)**.** *We now prove Prediction 4. From the Financier's linearized bond demand in* (14)*, using the real exchange rate solutions for periods zero and one in* (44) *and* (44)*, we obtain*

$$\hat{Q}^f_{H0} = \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]}{3+\Gamma} + \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(1+\Gamma)(3+\Gamma)} - q\left[\frac{\Delta\eta_{0|0}}{1+\Gamma} - \frac{\Gamma(\Delta\eta_{1|0} - \Delta\eta_{0|0})}{(1+\Gamma)(3+\Gamma)}\right] + O(\epsilon^2)$$

Taking the derivative of $\hat{Q}^f_{H0}$ with respect to q, we obtain

$$\frac{\partial\hat{Q}^f_{H0}}{\partial q} = \frac{\Gamma\Delta\eta_{1|0}}{(1+\Gamma)(3+\Gamma)} - \frac{(3+2\Gamma)\Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)}\,.$$

This derivative takes the opposite sign of the expected intervention if

$$\frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} < 2 + \frac{3}{\Gamma}\,, \tag{48}$$

which holds for any $\Gamma > 0$ as long as $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$.

In words, the condition $\Delta\eta_{0|0}/\Delta\eta_{1|0} > 1/2$ requires the probability of a period-zero intervention to exceed half the probability of a period-one intervention of the same sign, conditional on information available to the Financier in period zero. In the previous sentence, probability refers to the "net" probability of a positive intervention, because $\Delta\eta_{t|0}$ gives the probability of a positive intervention minus the probability of a negative intervention in period t, conditional on information available in period zero.

Proof 5 (Credit Constraints)**.** *We now prove Prediction 5. From the Financier's bond* *demand in* (14) *and the real exchange rate solutions in* (44) *and* (45)*,*

$$\begin{aligned}\hat{Q}^f_{H0} = \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]}{3+\Gamma} + \frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(1+\Gamma)(3+\Gamma)} \\ - q\frac{\Delta\eta_{0|0}}{1+\Gamma} + \Gamma q \frac{\Delta\eta_{1|0} - \Delta\eta_{0|0}}{(1+\Gamma)(3+\Gamma)} + O\big(\epsilon^2\big)\,.\end{aligned} \tag{49}$$

Suppose the Central Bank is passive $(q = 0)$ *and consider a tightening of credit constraints,*

$$\frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_1]}{(3+\Gamma)^2} - 2(2+\Gamma)\frac{\Delta\hat{\imath}_0 - \mathrm{E}_0^{(-)}[\Delta\hat{\imath}_2]}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{50}$$

By inspection of (49) *and* (50)*, tighter credit constraints lower the amount of intermediation the Financier undertakes if the Central Bank is passive* $(q = 0)$*,*

$$\operatorname{sign}\frac{\partial \hat{Q}^f_{H0}}{\partial \Gamma} = -\operatorname{sign}\hat{Q}^f_{H0}\,.$$

The Financier's portfolio positions in Home and Foreign bonds shrink when credit constraints tighten, limiting the ability of Households to smooth consumption intertemporally.

Suppose the Central Bank is active $(q > 0)$ *and consider the impact that tighter credit constraints have on the real exchange rate effect of an intervention. From* (47)*,*

$$\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \frac{\big[(3-\Gamma^2) - (3+\Gamma)^2\big]\Delta\eta_{0|0}}{(1+\Gamma)^2(3+\Gamma)^2} - \frac{(3-\Gamma^2)\Delta\eta_{1|0}}{(1+\Gamma)^2(3+\Gamma)^2}\,. \tag{51}$$

To establish the sign of the partial derivative in (51)*, we evaluate four cases.*

- ***Case 1:*** *Let* $\Delta\eta_{0|0} > 0$*,* $\Delta\eta_{1|0} > 0$*, and* $3 - \Gamma^2 > 0$*. From* (51)*,*

$$\begin{aligned}\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2) - (3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\ &\Leftrightarrow \quad -\frac{6 + 6\Gamma + 2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}}\,,\end{aligned}$$

which is satisfied under the assumptions in Case 1.

- ***Case 2:*** *Let* $\Delta\eta_{0|0} > 0$*,* $\Delta\eta_{1|0} > 0$*, and* $3 - \Gamma^2 < 0$*. From* (51)*,*

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{<} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2)-(3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
&\Leftrightarrow \quad \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0}-\Delta\eta_{0|0}}{\Delta\eta_{0|0}} \\
&\Leftarrow \quad 1 > \frac{\Delta\eta_{1|0}-\Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0} \,.
\end{aligned}$$

Hence, the inequality is satisfied in Case 2 when $\Delta\eta_{0|0} > \frac{1}{2}\Delta\eta_{1|0}$*.*

- ***Case 3:*** *Let* $\Delta\eta_{0|0} < 0$*,* $\Delta\eta_{1|0} < 0$*, and* $3 - \Gamma^2 > 0$*. From* (51)*,*

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2)-(3+\Gamma)^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
&\Leftrightarrow \quad -\frac{6+6\Gamma+2\Gamma^2}{(3-\Gamma)^2} < \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \,,
\end{aligned}$$

which is satisfied under the assumptions in Case 3.

- ***Case 4:*** *Let* $\Delta\eta_{0|0} < 0$*,* $\Delta\eta_{1|0} < 0$*, and* $3 - \Gamma^2 < 0$*. From* (51)*,*

$$\begin{aligned}
\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} \overset{!}{>} 0 \quad &\Leftrightarrow \quad \frac{(3-\Gamma^2)-(3+\Gamma)^2}{(3-\Gamma)^2} > \frac{\Delta\eta_{1|0}}{\Delta\eta_{0|0}} \\
&\Leftrightarrow \quad \frac{(3+\Gamma)^2}{3-\Gamma^2} > \frac{\Delta\eta_{1|0}-\Delta\eta_{0|0}}{\Delta\eta_{0|0}} \\
&\Leftarrow \quad 1 > \frac{\Delta\eta_{1|0}-\Delta\eta_{0|0}}{\Delta\eta_{0|0}} \quad \Leftrightarrow \quad \Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0} \,.
\end{aligned}$$

Hence, the inequality is satisfied in Case 4 when $\Delta\eta_{0|0} < \frac{1}{2}\Delta\eta_{1|0}$*.*

From (47) *and* (51) *and Cases 1–4, under the regularity condition that* $|\Delta\eta_{0|0}| > \frac{1}{2}|\Delta\eta_{1|0}|$*, and assuming the Central Bank's communications are consistent with its actions* $(\operatorname{sign}\Delta\eta_{0|0} = \operatorname{sign}\Delta\eta_{1|0} = \operatorname{sign}\Delta H_0 \neq 0)$*, we have*

$$\operatorname{sign}\frac{\partial^2 \hat{e}_0}{\partial q \partial \Gamma} = \operatorname{sign}\frac{\partial \hat{e}_0}{\partial q} \,,$$

and tighter credit constraints amplify interventions.

Table A.1: Central Bank Balance Sheet View of Sterilized Intervention.

Step	Assets	Liabilities
1. Purchase Foreign Currency	Assets in Foreign Currency (+1)	Circulating Home Currency (+1)
2. Sell Home Bonds	Assets in Home Currency (−1)	Circulating Home Currency (−1)
3. Net Effect	Assets in Foreign Currency (+1) Assets in Home Currency (−1)	Circulating Home Currency (±0)

Notes. The tables illustrate the process of sterilizing a Central Bank intervention with respect to Home currency. Step 1 shows the Central Bank purchasing Foreign currency with Home currency, increasing the Home currency in circulation. Step 2 shows the Central Bank selling Home-currency bonds to offset the rise in Home currency from Step 1. Step 3 shows the net effect of Steps 1 and 2: the Home money supply remains unchanged, while the Central Bank's Foreign currency assets increase and its Home currency assets decrease. This sterilized intervention keeps the supply of Home currency constant but alters the composition of the central bank's assets.

A.7 Sterilized Intervention

Central banks can manage exchange rates without altering the money supply through sterilized intervention. Consider a spot purchase of Foreign currency as an example intervention. In a non-sterilized intervention, the Central Bank purchases Foreign currency with Home currency, increasing the Home money supply. To "sterilize" the impact, the Central Bank sells Home-currency bonds to absorb the excess Home currency. If the sterilization is perfect, the money supply remains constant, while the ratio of Home-currency and Foreign-currency bonds held by the public and the central bank changes.

A sterilized intervention can be viewed as a combination of two transactions. First, in the FX market, the Central Bank conducts a non-sterilized intervention by purchasing Foreign currency (or Foreign-currency bonds), first issuing Home currency to fund the purchase. Second, in the money market, the Central Bank "sterilizes" the effect by selling an equivalent amount of Home bonds to absorb the initial increase in the Home money supply. The net effect of a sterilized spot purchase is neutral regarding the Home currency in circulation, but there is a portfolio change in assets, with an increase (decrease) in the share of Foreign-currency assets held by the central bank (public). Table A.1 illustrates these steps in a stylized Central Bank balance sheet.

B Empirics

This section of the Online Appendix provides the empirical details supporting the main findings reported in Section 4 of the paper.

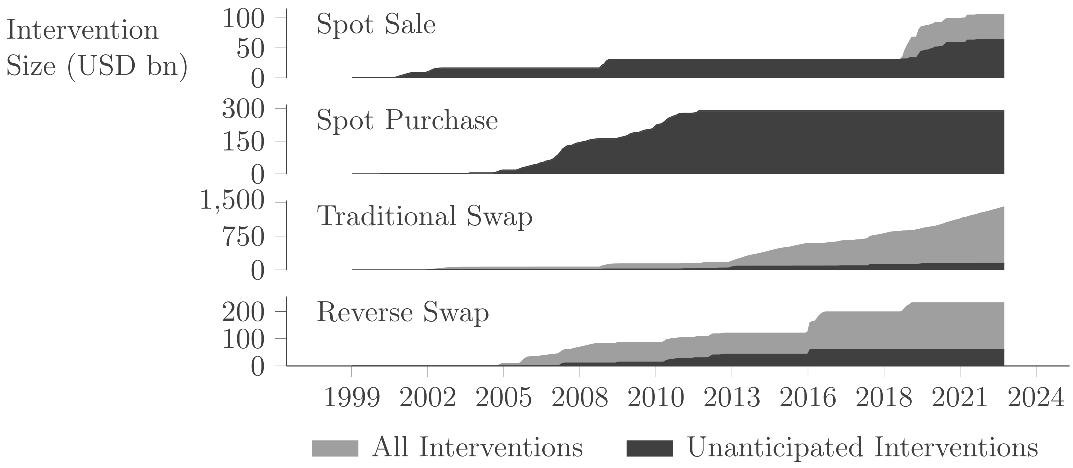
We provide additional descriptive statistics for our datasets in Section B.1. We describe the relationship between Γ and exchange rate volatility in Section B.2, providing empirical support for an assumption in the Gamma-Eta Model. We tabulate the exchange rate response to unanticipated spot sell interventions at different horizons in Section B.3. We estimate dynamic exchange rate responses after pooling buy and sell interventions in Section B.4 for a more direct comparison with prior results reported in the FXI literature. We estimate dynamic exchange rate responses to anticipated interventions in Section B.5 and find generally weaker effects as predicted by the Gamma-Eta Model. We estimate dynamic forward premium responses to unanticipated interventions in Section B.6, providing evidence on linking CIP behavior to the relative demand for currency forwards. We estimate dynamic exchange rate responses to unanticipated interventions on days with single versus multiple interventions in Section B.7. We consider an alternative CPI-based method of conditioning on intermediary constraints when estimating dynamic conditional exchange rate responses to FXI in section B.8. Finally, we describe the procedure we use to compute an FXI residual for our examination of the signalling channel in Section B.9.

B.1 Description of Data and BCB FXI

This section of the Online Appendix presents tables and figures summarizing the BCB's FXI, as well as key statistics on the BRL/USD exchange rate and control variables from the local projections in Section 4.

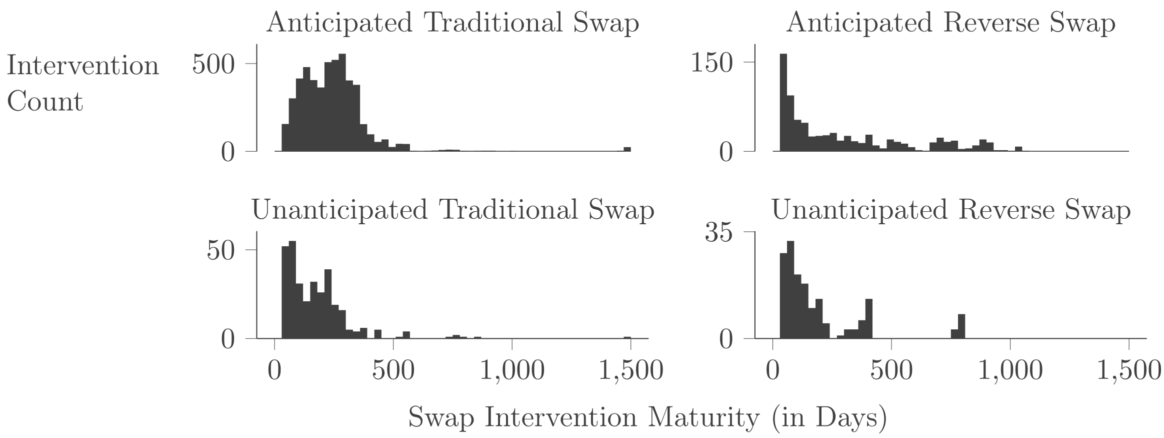
Figure B.1 shows the cumulative size of BCB interventions over time, disaggregated by type (spot sales, purchases, traditional and reverse swaps), and distinguishes between

Figure B.1: Cumulative BCB FXI



Notes. These figures show the cumulative UCB FX market interventions over time, by intervention type. The horizontal axes show the intervention size in USD billions, after aggregating at monthly frequency. The lighter shade shows all interventions, both anticipated and unanticipated, while the darker shade shows the subset of unanticipated interventions, defined as interventions with an announcement equal to the operation date.

Figure B.2: Maturity Breakdown of BCB Swap Interventions



Notes. These figures show 50-bin histograms of the maturities of BCB's interventions in the FX markets, by intervention type. The horizontal axes show the maturity in days. Unanticipated interventions are defined as interventions with an announcement equal to the operation date, while anticipated interventions are defined as interventions with an announcement date that precedes the operation date. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

anticipated and unanticipated interventions. Figure B.2 provides a histogram of swap intervention maturities, further split by traditional and reverse swaps, with the horizontal axis indicating maturity in days.

Table B.1 summarizes the BCB's FX interventions by type (spot sales/purchases and traditional/reverse swaps), showing the mean, standard deviation, maximum, and total

Table B.1: Summary Statistics for BCB FXI

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.17	0.48	0.19	0.00
SD	0.22	0.39	0.24	0.00
Max	1.10	3.00	4.64	0.00
Count	385	87	1483	0.00
	Traditional Swap		Reverse Swap	
	Unanticipated	Anticipated	Unanticipated	Anticipated
Mean	0.43	0.25	0.35	0.20
SD	0.41	0.24	0.45	0.28
Max	1.85	3.50	3.38	4.00
Count	345	5094	174	846

Notes. This table shows the mean, standard deviation (SD), maximum, and total number of counts for BCB's FXI. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27. Mean, standard deviation, and max values are expressed in USD billion.

Table B.2: Summary Statistics for BRL/USD

	BRL/USD Spot Rate	1m BRL/USD	
		Currency Basis	Forward Premium
Mean	2.773	−220.106	186.341
SD	1.105	133.919	111.107
Min	1.207	−639.128	−79.337
25%	1.916	−302.106	101.390
75%	3.268	−124.770	275.808
Max	5.905	147.176	638.495

Notes. This table shows mean, standard deviation (SD), minimum, 25% percentile, 75% percentile, and maximum values for the BRL/USD spot rate, currency basis, and forward premium. The spot rate is expressed in units of BRL per USD. The currency basis and forward premium are expressed in basis points (bp). The sample period for spot rate is from 1999-01-22 to 2023-04-27. The sample period for currency basis and forward premium is from 2003-11-24 to 2023-04-27.

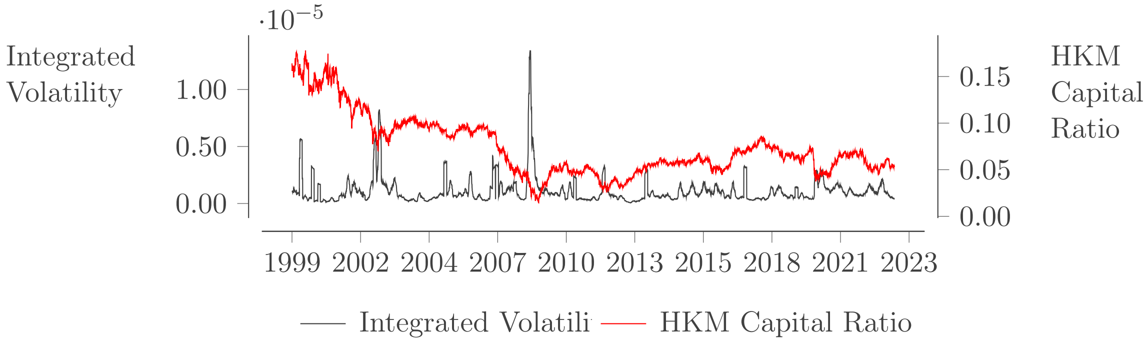
count for both anticipated and unanticipated interventions, expressed in USD billions. Table B.2 provides summary statistics for the BRL/USD exchange rate, currency basis, and forward premium, including the mean, standard deviation, and various percentiles. Table B.3 presents statistics for the control variables used in the local projections regressions, including the HKM intermediary capital ratio, Brazil's EMBI, US and Brazilian interest rates, market volatility, and total intervention size.

Table B.3: **Summary Statistics for Control Variables**

	(1) HKM	(2) $EMBI$	(3) i^{US}	(4) i^{BR}	(5) i_s^{US}	(6) i_s^{BR}	(7) INT^{TOL}	(8) S^{VOL}
Mean	0.074	0.042	1.531	11.712	0.614	0.551	0.644	1.253×10^{-6}
SD	0.032	0.033	1.679	5.103	0.523	2.121	0.785	2.273×10^{-5}
Min	0.014	0.014	0.051	1.888	−3.485	−17.167	0.000	0.000×10^{0}
25%	0.051	0.023	0.149	8.249	0.298	-0.614	0.137	2.238×10^{-7}
75%	0.093	0.046	2.315	14.135	0.881	1.198	0.750	8.197×10^{-7}
Max	0.178	0.244	6.875	37.333	2.820	14.175	8.850	2.051×10^{-3}

Notes. The table shows the mean, standard deviation (SD), minimum, 25% and 75% percentile, and maximum values for daily frequency control variables used in equations (20) and 4.3. The variables include (1) HKM, intermediary capital ratio, (2) $EMBI$, emerging Markets Bond Index Plus for Brazil from JP Morgan, (3)i^{US} one-day US Libor rate, (4) i^{BR} one-day Brazil inter-bank rate, (5) i_s^{US}, one-year minus one-day US Libor rate spread, (6) i_s^{BR} one-year minus one-day Brazil inter-bank rate spread, (7) INT^{TOL}, the total amount of interventions in USD (of all instruments), and (8) S^{VOL}, spot market volatility. Values in (3)–(6) are expressed in percentage points. Values in (7) are expressed in billion USD. The sample period runs from 1999-01-22 to 2023-04-27.

Figure B.3: Integrated BRL/USD Exchange Rate Volatility and HKM Ratio



Notes. This figure plots the 30-day moving averages of integrated volatility for the BRL/USD spot rate (black) and the HKM capital ratio (red). Integrated volatility is calculated using high frequency spot quotes from Thomson Reuters Tick History. The HKM capital ratio is calculated for a subset of primary dealers that deal with emerging market currencies following He, Kelly and Manela (2017) and Cerutti and Zhou (2024). The sample period runs from 1999-01-22 to 2023-04-27.

B.2 Exchange Rate Variance and Intermediary Constraints

The Gamma-Eta Model in Section 2 assumes a fixed value for the parameter Γ that governs the risk-bearing capacity of the Financier. This assumption departs from the Basic Gamma Model of Gabaix and Maggiori (2015), where Γ depends on the variance of the real exchange rate, $\Gamma = \gamma \operatorname{Var}(e_1)^{\alpha}$, with parameters $\gamma \geq 0$ and $\alpha \geq 0$.

Our empirical proxy for Γ is the intermediary capital ratio of He et al. (2017), which we construct using balance sheet data for a subset of primary dealers that handle emerging-market currencies following Cerutti and Zhou (2024). We define this measure in equation (19) and plot it against integrated volatility for the BRL/USD exchange rate in Figure B.3 over a sample period from 1999 to 2023. We compute integrated volatility using high frequency spot quotes from Thomson Reuters Tick History.

We find a weak relationship between the intermediary capital ratio and exchange rate volatility over this period, except perhaps during brief crisis periods, and therefore assume no direct relationship between Γ and $\text{Var}(e)$ in our model.

B.3 Unanticipated Interventions

In Section 4.2 of the main paper, we examine the impact of unanticipated BCB FXI on the BRL/USD exchange rate and currency basis. Figure 8 illustrates our results. In this section of the Online Appendix, we provide a supplemental tabulation of exchange rate effects for each intervention type at discrete post-intervention horizons: 15 minutes, 1 hour, 3 hours, and 7 hours. Table B.4 presents these results.

Table B.4: Exchange Rate Response to Unanticipated BCB Interventions

Panel A: Full Sample

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.332***	−0.663***	−0.752***	−1.951	−0.005	0.012	0.015	0.146	0.002	−0.063	0.001	0.077	0.079**	0.044	0.018	0.062
	(0.096)	(0.241)	(0.276)	(1.193)	(0.013)	(0.024)	(0.025)	(0.089)	(0.104)	(0.131)	(0.147)	(0.276)	(0.032)	(0.056)	(0.078)	(0.111)

Panel B: Tight Intermediary Constraints

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.333***	−0.703***	−0.734**	−1.980	0.025	0.017	0.019	0.063	−0.147***	−0.229***	−0.160	−0.139	0.118**	0.142	0.163*	0.173
	(0.115)	(0.254)	(0.310)	(1.258)	(0.025)	(0.035)	(0.035)	(0.126)	(0.049)	(0.084)	(0.107)	(0.165)	(0.049)	(0.108)	(0.094)	(0.108)

Panel C: Loose Intermediary Constraints

	Spot Sale				Spot Purchase				Traditional Swap				Reverse Swap			
	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours	15 mins	1 hour	3 hours	7 hours
β_h	−0.187**	−0.106	−0.213	−0.115	−0.033**	−0.007	0.009	0.224**	0.094	0.160	0.185	0.437	0.170***	0.193***	0.100	0.144
	(0.081)	(0.138)	(0.155)	(0.224)	(0.016)	(0.024)	(0.030)	(0.091)	(0.155)	(0.172)	(0.198)	(0.344)	(0.032)	(0.033)	(0.067)	(0.089)

Notes. Panel A shows the response of the log spot rate to the BCB's unexpected FXI at intra-day horizons over the full sample period. Panels B and C show the heterogeneous responses of the log spot rate to the BCB's unexpected FXI at intra-day horizons during periods of tight and loose intermediary constraints, respectively. The regressions follow equation 4.3. White heteroscedasticity-robust standard errors are reported in the parenthesis. Results are in percentage points. *, **, and *** denote significance levels at the 10%, 5%, and 1% confidence levels, respectively. The sample period is from 1999-01-22 to 2023-04-27.

As shown in the table, unanticipated spot sale interventions have the largest and most sustained effects on the BRL, while other types of interventions exhibit weaker or transitory impacts. The table helps quantify the speed and magnitude of exchange rate adjustments following each type of intervention.

B.4 Pooled Interventions

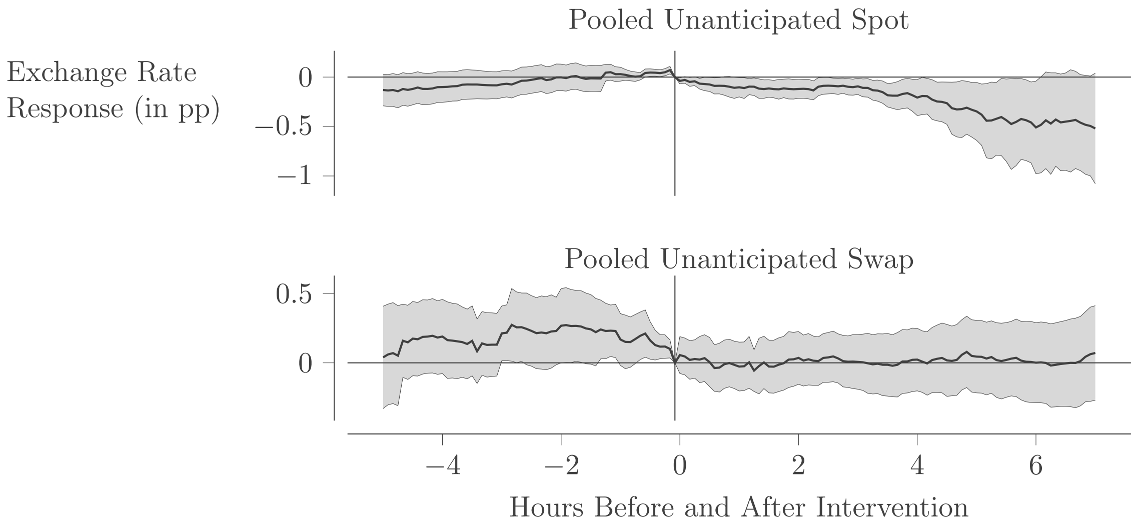
For our main results in Section 4.2 of the paper, we estimate our baseline specification in (20) separately for interventions in each direction—buy versus sell in the case of spot interventions, and traditional versus reverse in the case of swap interventions. In this section of the Online Appendix, we estimate our baseline specification for pooled interventions. In the case of spot interventions, we pool spot sales (with positive sign) and spot purchases (with negative sign) in a single regression. In the case of swap interventions, we pool traditional swaps (with positive sign) and reverse swaps (with negative sign) in a single regression. Figure B.4 shows the dynamic response of the exchange rate to pooled unanticipated BCB interventions.

The top panel of Figure B.4 shows the response to pooled spot interventions, while the bottom panel shows the response to pooled swap interventions. In the pooled analysis, the exchange rate effect of spot FXI is attenuated, falling from our 150 basis point estimate for spot sell interventions separately to 50 basis points for pooled spot buy and sell interventions. The pooled analysis is useful for comparing the effect sizes we estimate with estimate sizes estimated in prior literature. Specifically, our 50 basis point pooled estimate is within the 30–100 basis point range of estimates from the prior literature (Kohlscheen and Andrade, 2013; Nedeljkovic and Saborowski, 2019; Barroso, 2019; Santos, 2021).

B.5 Anticipated Interventions

In the main paper, we focus on unanticipated interventions, where we finder stronger effects. In this section of the Online Appendix, we illustrate the effects of anticipated interventions, using the opening of trading as the event time. Figure B.5 shows the exchange rate response to anticipated BCB spot sell interventions, as well was traditional and reverse

Figure B.4: Exchange Rate Response to Pooled Unanticipated Interventions



Notes. The figure shows the BRL/USD exchange rate response to pooled spot FXI (upper) and pooled swap FXI (bottom), with responses measured in percentage points (pp). In the case of spot interventions, we pool spot sales (with positive sign) and spot purchases (with negative sign) in a single regression. In the case of swap interventions, we pool traditional swaps (with positive sign) and reverse swaps (with negative sign) in a single regression. Unanticipated interventions are measured in USD Billion. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The dotted lines denote a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period is from 1999-01-22 to 2023-04-27.

swap interventions. Recall that all spot purchase interventions are unanticipated.

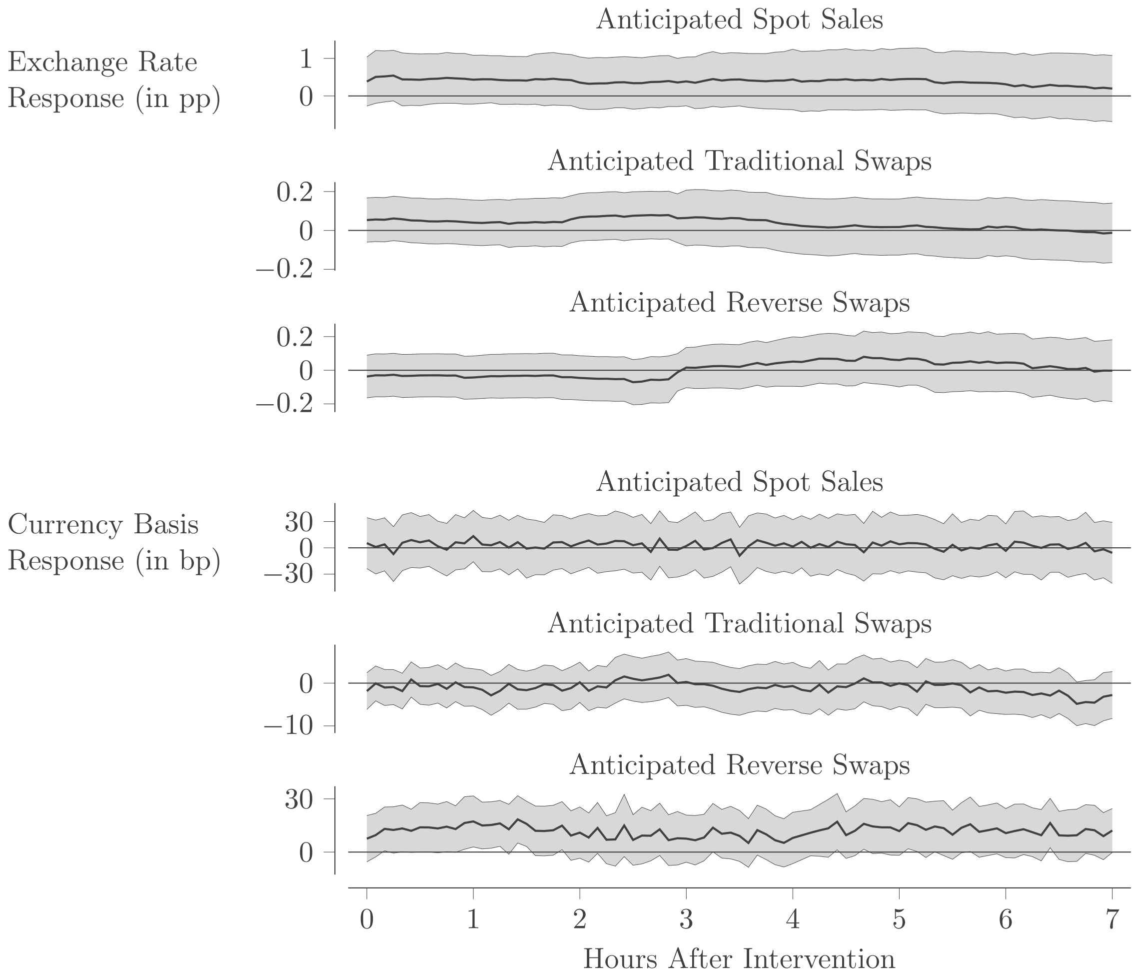
The top panel of Figure B.5 displays the effect of anticipated spot sell interventions, while the bottom panel illustrates the response to traditional and reverse swap interventions. In all cases, the exchange rate response to anticipated interventions is muted, consistent with Prediction 3 of the Gamma-Eta model.

B.6 Forward Premia and Interventions

In this section, we estimate dynamic forward premium responses to unanticipated interventions, providing evidence that links CIP behavior to the relative demand for currency forwards.

Figure B.6 shows the dynamic response of the BRL/USD forward premium to unanticipated FX interventions by the BCB. The figure includes four panels, each representing a different type of intervention: spot sales, spot purchases, traditional swaps, and reverse swaps. The forward premium, calculated as the difference between forward and spot

Figure B.5: Exchange Rate and Currency Basis Responses to Anticipated BCB Interventions

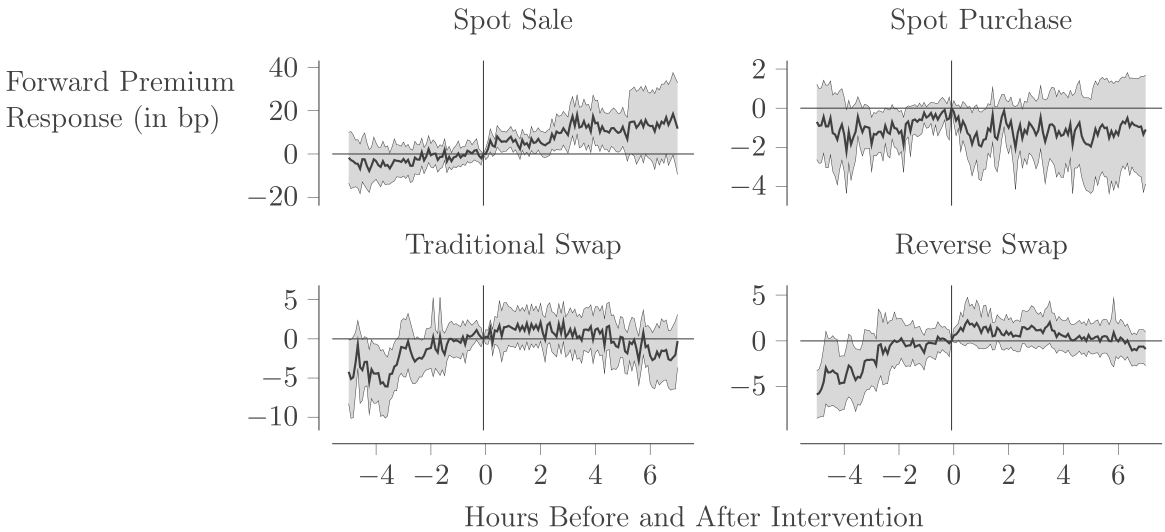


Notes. The upper group of figures show BRL/USD exchange rate responses to anticipated BCB interventions on operational date, measured in percentage points (pp). The lower group shows BRL/USD currency basis responses to anticipated BCB interventions on operational date, measured in basis points (bp). Within each group, the figures show responses to spot sell, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

exchange rates, is measured in basis points (bp). The figure shows a positive and significant response in the forward premium to BCB spot sale interventions.

Figure B.7 extends the analysis by conditioning responses on the state of intermediary constraints, as measured by the HKM ratio that we define in equation (19). The figure separates the interventions into two categories: periods with loose constraints (HKM ratio in the upper 50%) and periods with tight constraints (HKM ratio in the lower 50%). The responses are again presented for spot sales, spot purchases, traditional swaps, and reverse

Figure B.6: Forward Premium Response to Unanticipated BCB Interventions



Notes. The figures show forward premium responses measured in basis points (bp) to unanticipated BCB spot sales, spot purchase, traditional swap, and reverse swap interventions. The forward premium is defined as the difference between forward and spot BRL/USD rates, measured in basis points. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from from 2003-11-24 to 2023-04-27.

swaps, measured in basis points. This figure shows a slightly more pronounced forward premium response to BCB spot sale interventions.

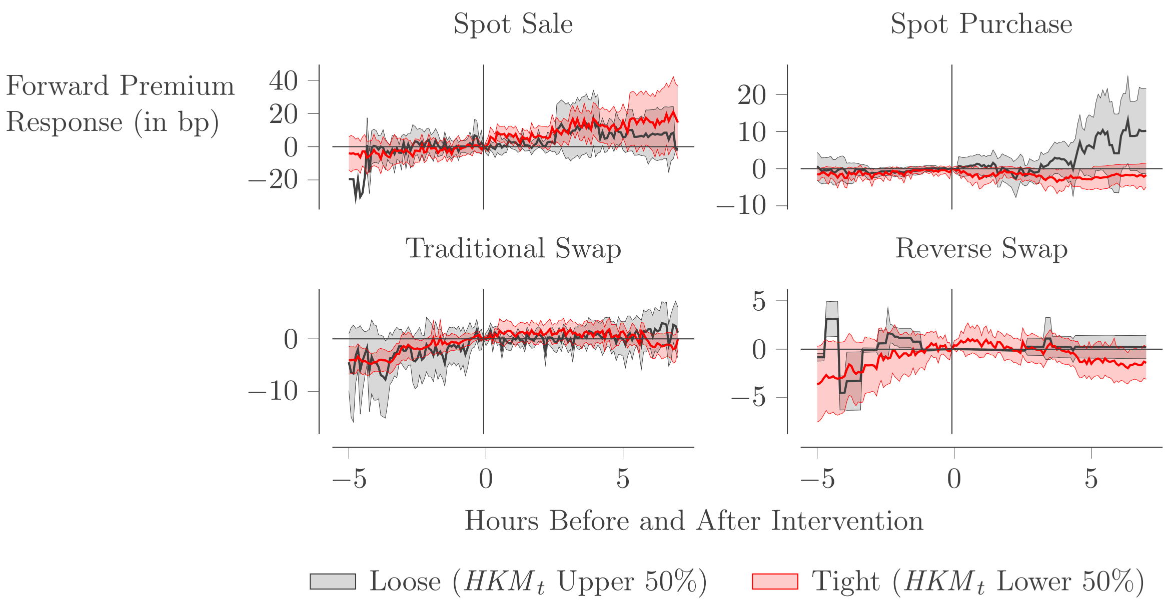
B.7 Clustered Interventions

In this section, we analyze the differential effects of single and multiple same-day unanticipated BCB interventions on the BRL/USD exchange rate. We classify interventions by type—spot sales, spot purchases, traditional swaps, and reverse swaps. The analysis shows how the market responds to isolated interventions versus clusters of interventions occurring within the same day.

Table B.5 shows the distribution of days with single or multiple FX interventions by type. For spot interventions, the upper panel indicates that single interventions are most common, though multiple interventions still occur frequently. In contrast, the lower panel shows that multiple interventions dominate for swaps. This table motivates our analysis of exchange rate responses to FXI, conditional on the number of same-day interventions.

Figure B.8 focuses on days with a single unanticipated intervention, showing muted

Figure B.7: Conditional Forward Premium Response to Unanticipated BCB Interventions



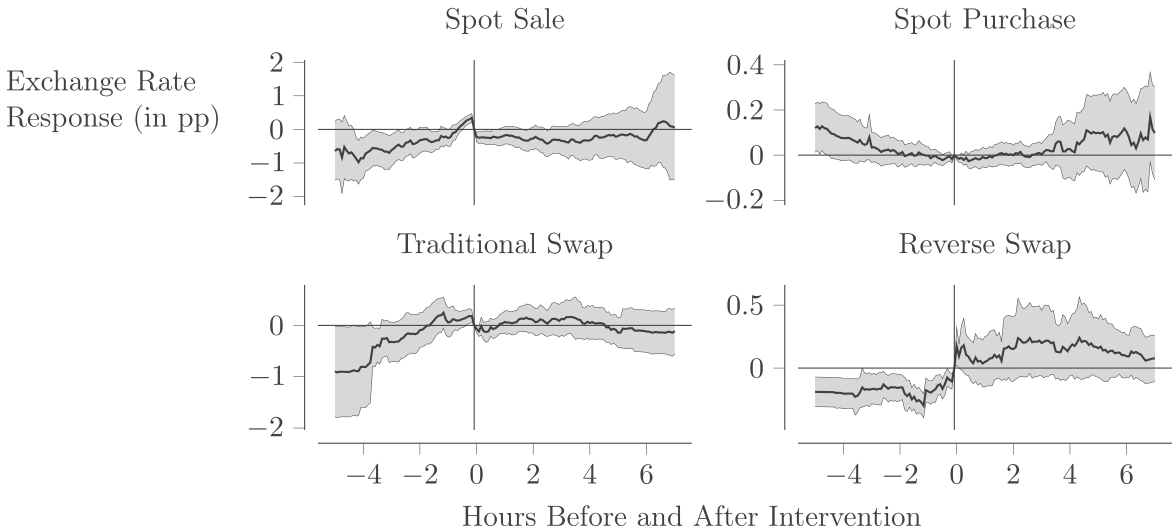
Notes. The figures show BRL/USD 1-month forward premium rate responses in basis points (bp) to unanticipated BCB interventions. The figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Each figure plots responses conditional on the state of FX dealers' balance sheet constraints on the intervention day. We measure balance sheet constraints by constructing the intermediary capital ratio HKM_t of He et al. (2017) for the FX dealer banks listed in Cerutti and Zhou (2024). Red indicates tight constraints, defined as periods with HKM_t in the lower 50% of values in our sample. Gray indicates loose constraints, defined as periods with HKM_t in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

Table B.5: Number of Days with Single or Multiple FXIs

	Spot Sale		Spot Purchase	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	252	85	1157	0
$= 2$	54	1	157	0
≥ 3	8	0	4	0
	Traditional Swap		**Reverse Swap**	
	Unanticipated	Anticipated	Unanticipated	Anticipated
$= 1$	27	67	8	84
$= 2$	48	1055	10	35
≥ 3	48	805	33	175

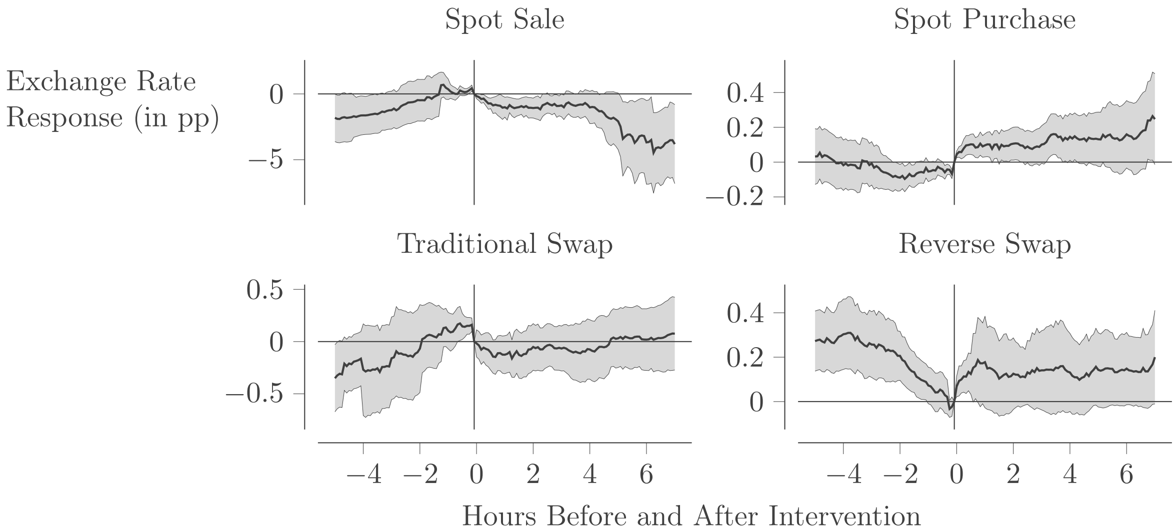
Notes. This table shows the number of days with single or multiple BCB's FXIs for each type of intervention. For unanticipated interventions, the announcement date is equal to the operation date. For anticipated interventions, the announcement date precedes the operation date. Traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The sample period runs from 1999-01-22 to 2023-04-27.

Figure B.8: Exchange Rate Response on Days with One Unanticipated Intervention



Notes. The figure shows the dynamic response of the BRL/USD exchange rate to BCB spot sale (top left), spot purchase (top right), traditional swap (bottom left), and reverse swap (bottom right) FXI on days with exactly one BCB intervention. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

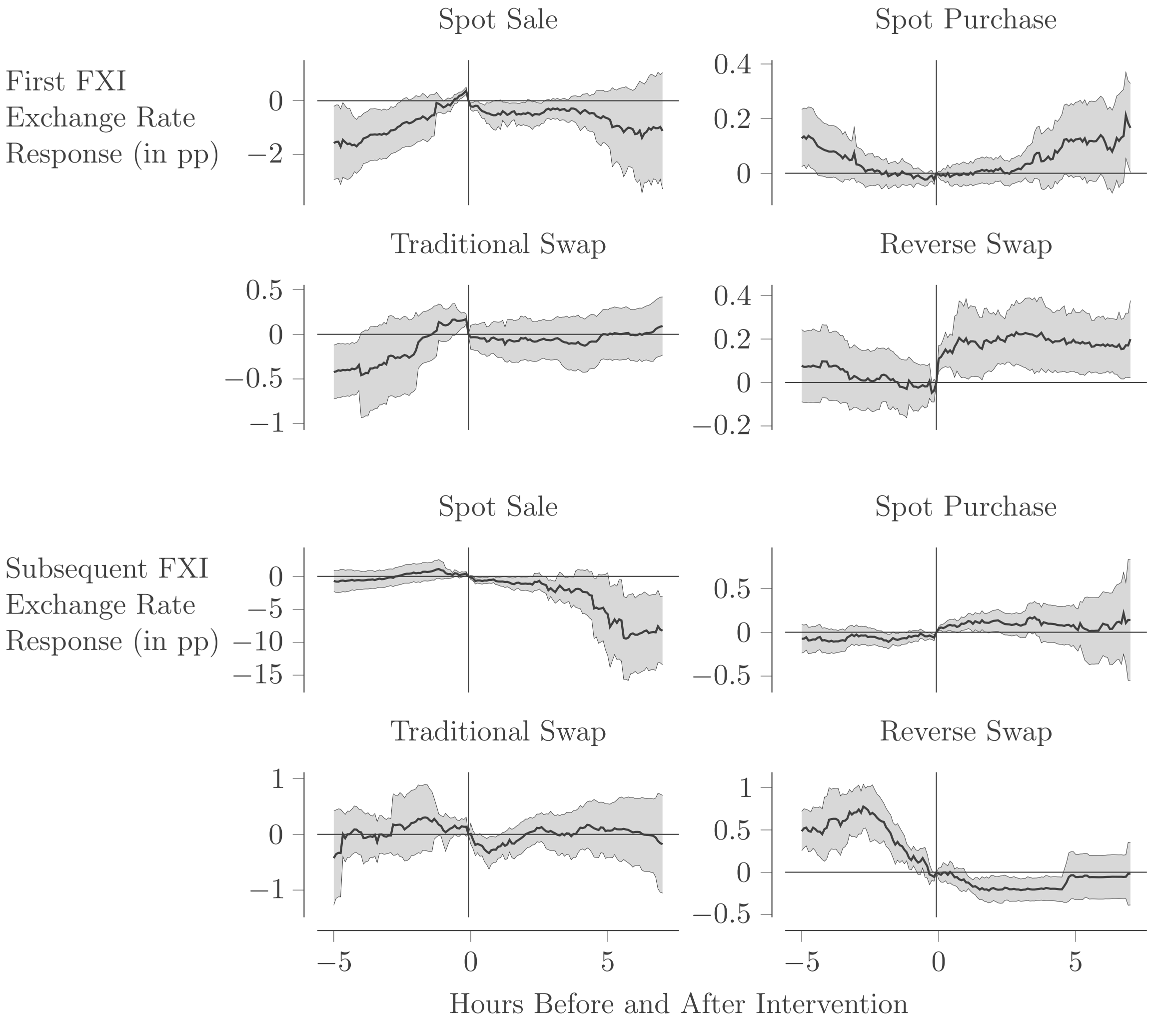
Figure B.9: Exchange Rate Response on Days with Multiple Unanticipated Interventions



Notes. The figure shows the dynamic response of the BRL/USD exchange rate to BCB spot sale (top left), spot purchase (top right), traditional swap (bottom left), and reverse swap (bottom right) FXI on days with more than one BCB intervention. A traditional (reverse) swap is the sale (purchase) of USD at the spot leg of the swap contract. The shaded area denotes a 95% confidence interval using White heteroscedasticity-robust standard errors. The sample is from 2003-11-24 to 2023-04-27.

exchange rate responses for spot sales, spot purchases, traditional swaps, and reverse swaps. Figure B.9 extends focuses on days with multiple unanticipated interventions,

Figure B.10: Exchange Rate Response to First and Subsequent Unanticipated Interventions



Notes. The upper group of figures shows the BRL/USD exchange rate responses to the first unanticipated BCB interventions on days with multiple unanticipated interventions on days with multiple unanticipated interventions, measured in percentage points (pp). The lower group of figures shows the BRL/USD exchange rate responses to subsequent unanticipated same-day BCB interventions. Within each group, the figures show responses to the BCB's spot sale, spot purchase, traditional swap, and reverse swap interventions. Traditional swaps involve the sale of USD while reverse swaps involve the purchase of USD at the spot leg of the swap contract. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 1999-01-22 to 2023-04-27.

showing stronger and more significant exchange rate responses across all intervention types. Finally, Figure B.10 contrasts the effects of the first intervention of the day with subsequent interventions, showing that subsequent interventions generally produce stronger effects.

B.8 Alternative Measures of Intermediary Constraints

In Section 4.3 of the main paper, we use the HKM intermediary capital ratio as our primary measure of intermediary constraints. However, a potential concern with this measure is that it may reflect a structural break in our sample, as dealer capital levels tend to be lower after 2008. This could lead to biased estimates of the impact of foreign exchange interventions, as the HKM measure might capture broader post-crisis trends rather than contemporaneous liquidity constraints.

To address this issue, we perform a robustness check by using CIP violations as measured by the BRL/USD currency basis as an alternative proxy for intermediary constraints. CIP violations offer a market-based measure of USD liquidity pressures, reflecting the premium required to swap BRL into USD. CIP violations are less likely to be influenced by structural shifts in capital ratios, providing a more dynamic measure of short-term liquidity constraints faced by intermediaries.

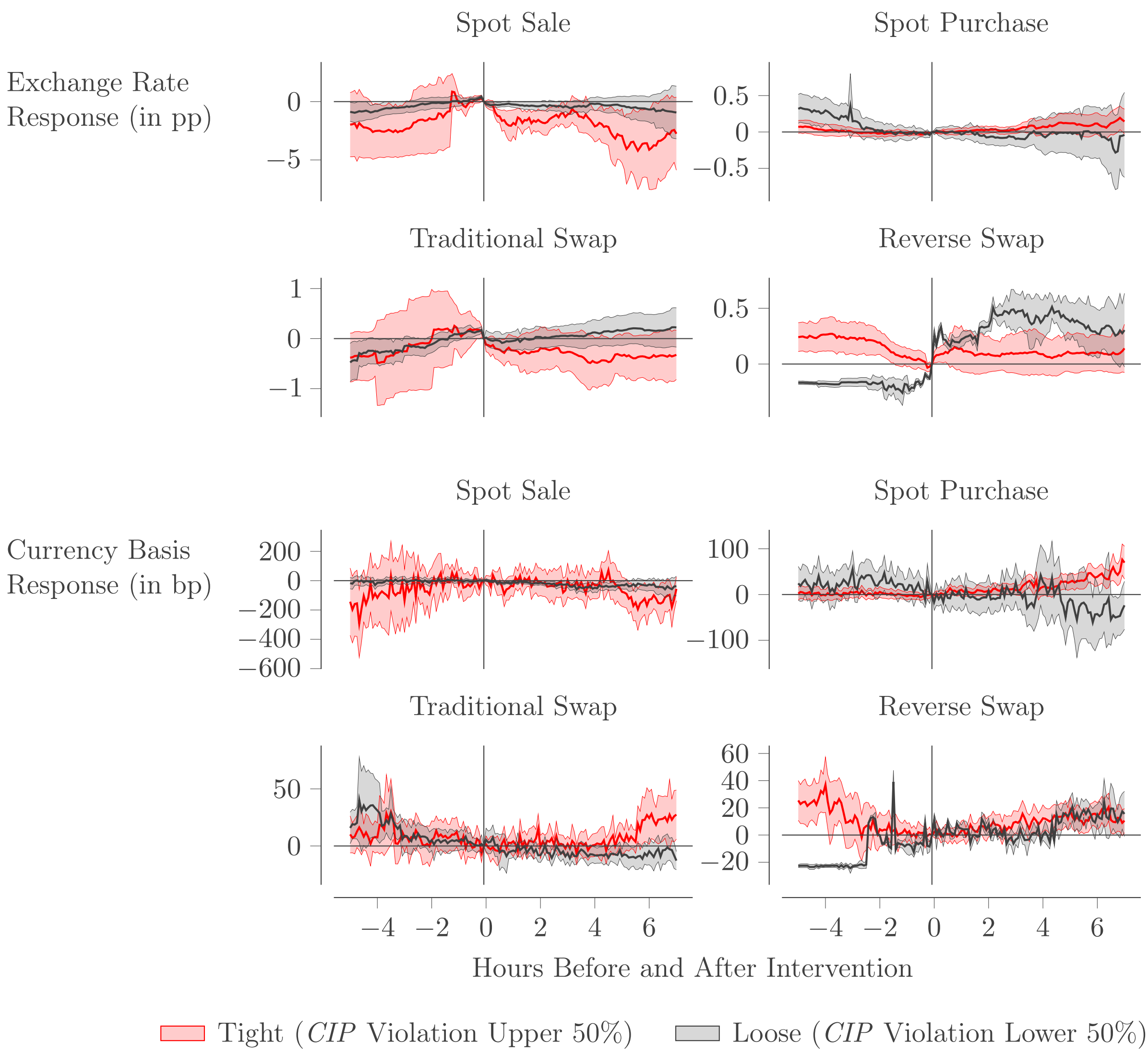
Figure B.11 presents the results of this alternative analysis, showing that spot sales and traditional swap interventions have more pronounced effects on BRL appreciation and the narrowing of the cross-currency basis during periods of above-median CIP violations. These findings are consistent with the results obtained using the HKM measure, reinforcing our conclusion that foreign exchange interventions are more effective when intermediaries face greater USD liquidity constraints.

B.9 Residual Intervention Measure

In Section 4.4, we examine the signalling channel of FXI and in part of our examination we use daily-frequency data to estimate our baseline specification in (20) over a six-month horizon. For this lower-frequency, longer-horizon analysis, we construct a residual measure of FXI in order to mitigate policy endogeneity concerns. In this section of the Online Appendix, we describe our procedure for obtaining the plausibly-exogenous residual component of FXI.

Specifically, we regress the intervention amount FXI_t on a set of control variables, interpreting the residual from this regression as the exogenous component of FXI. In the

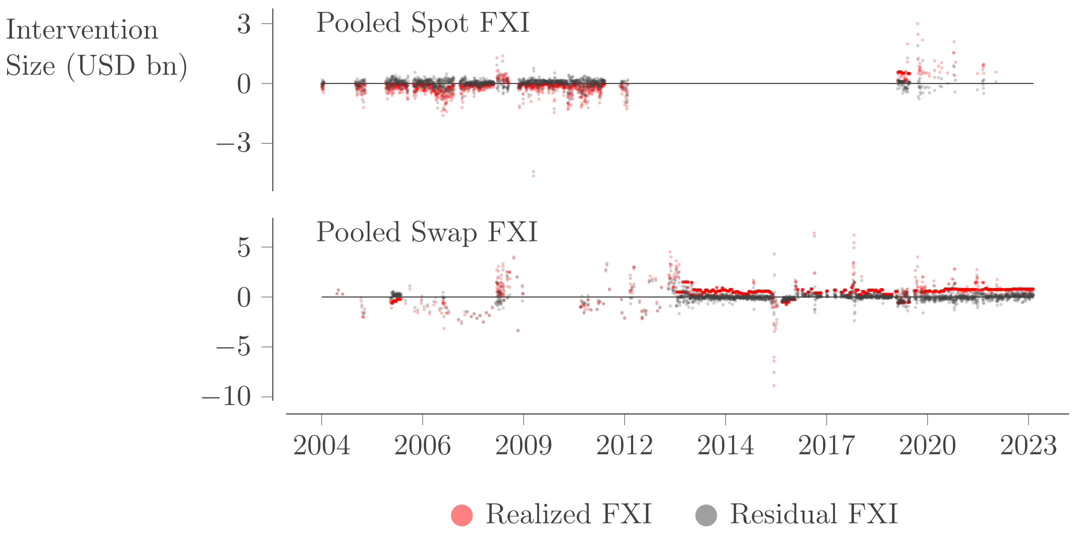
Figure B.11: Conditional Exchange Rate and Currency Basis Responses to Unanticipated BCB Interventions: CIP Violations



Notes. The upper group figures shows BRL/USD exchange rate responses to unanticipated BCB interventions in percentage points (pp). The lower group figures show BRL/USD currency basis responses to unanticipated BCB interventions in basis points (bp). Each group shows responses to spot sale, spot purchase, traditional swap, and reverse swap interventions. The figures plot responses conditional on the level of CIP violation on intervention day. Red indicates tight-constraint periods with CIP violations in the upper 50% of values, while gray indicates loose-constraint periods with CIP violations in the upper 50% of values in our sample. Shading indicates a 95% confidence interval using White's heteroscedasticity-robust standard errors. The sample period runs from 2003-11-24 to 2023-04-27.

case of spot interventions, we pool buy and sell interventions for the regression, where spot sell interventions receive a positive sign and spot buy interventions receive a negative sign. In the case of swap interventions, we pool traditional and reverse swap interventions for the regression, where traditional swap interventions receive a positive sign and reverse swap interventions receive a negative sign.

Figure B.12: Exogenous Residual Component of FXI



Notes. The figure shows realized values of *FXI* in red, along with the plausibly-exogenous residual component of FXI in grey, for BCB FXI aggregated at a daily frequency over the period 1999 to 2023. The upper panel shows realized pooled spot intervention amounts and residuals, and the lower panel shows pooled swap intervention amounts and residuals. The linear FXI prediction model we use to estimate the residual component of FXI is defined in equation (52).

Our specification is given by

$$FXI_t = \alpha + \beta z_{t-1} + \epsilon_t \,, \tag{52}$$

for all dates on which FXI was non-zero, i.e. for all t such that $FXI_t \neq 0$, where FXI_t denotes the dollar amount of FXI in USD billion. The vector of control variables z_{t-1} includes lagged Brazilian interest rate expectations, the BRL/USD spot rate, the HKM intermediary capital ratio, sovereign default risk, exchange rate volatility, U.S. and Brazilian interest rates, monetary policy announcements, US recessions, forward premia, and FX interbank trading volume. We plot the realized FXI values and residuals in Figure B.12.

We interpret the residual from this regression as the plausibly-exogenous component of FXI, and we then the residual in a second stage to estimate our baseline local projections specification in (20). Our procedure is similar to the procedure that Rodnyansky et al. (2023) use to identify FXI.