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January 2025 No: 1540

Warwick Economics Research Papers

ISSN 2059-4283 (online)

ISSN 0083-7350 (print)

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Abstract

This paper introduces search frictions in labor and goods markets to explore which condition leads to deviation from LOP, and how asymmetric shocks have an impact on deviation of LOP in an open economy. First, we show that the LOP gap only depends on the ratio of marginal utility of aggregate search across countries. Then, we express the LOP gap in terms of consumption gap across countries and show that asymmetric productivity shocks between countries entail deviations from LOP. This is because asymmetric productivity shocks affect markups via the matching probability, and in turn, induce firms to move across markets. Finally, we also examine responses of macroeconomic variables with respect to country-specific productivity and preference shocks.

Keywords: consumer search, labor market frictions, search and matching, international co-movement

JEL Codes: E24, E31, F41

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1 Introduction

Standard international macroeconomic models have accounted for many features of the international business cycle. However, this class of model ignores to addressing some features of international macroeconomic data. In particularly, one of these features is the fact that deviations from the law of one price (LOP), which implies export price is not equal to the domestic price for same goods when expressed in a common currency. One leading interpretation of the deviation from the LOP is that firms conduct systematic price discrimination across countries, which is called 'pricing to market' by Krugman (1986). In the context of a single good sold in distinct markets with different prices, what cause price discrimination across markets is one of the issues. This paper introduces search and matching frictions in goods markets to explore how a country-specific productivity shock generates deviations of the LOP, and how goods market frictions interact with employment dynamics in an open economy. The main result of the paper is that a country-specific productivity shock leads to deviations from the LOP because it induces consumption gaps, differing search intensives in goods markets across countries.

To account for the role of labor and goods market frictions for the deviation from the LOP, we consider a standard, two-country and two-good model with complete asset markets. Each country specializes in the production of a single good which is traded internationally and produced by using labor as sole input. The model introduces search and matching frictions in both goods and labor markets. Search frictions in labor markets are characterized by specific matching technologies, following Mortensen and Pissarides (1994). Directed search frictions are introduced in goods markets as in Moen (1997), recently used in Bai and Ríos-Rull (2015). Thus, products supplied by firms are consumed only if firms are matched with consumers in goods markets. As firms are assumed to target either the domestic or export market and search efforts are exerted differently in each market, prices firms post could be different based on the search frictions in each market, which leads to the deviations of the LOP.

With the model, we first define the LOP gap to study the conditions which lead to deviations from the LOP. We show the LOP gap depends on the ratio of marginal utility of aggregate search efforts across countries. Thus, if the aggregate search efforts of the home country are different from search efforts of foreign households, the LOP fails to hold. Furthermore, we find that if the utility function does not have a curvature in search efforts, the LOP holds even if search efforts exerted by home and foreign households are different each other.

We begin by exploring the mechanism which causes deviations from the LOP using a simplified static version of the model. We express the LOP gap in terms of aggregate consumption of the home and the foreign countries, and then suggest conditions where deviations from the LOP occur. Namely, through the link between aggregate consumption and aggregate productivity, we find that a country-specific productivity shock generates deviations of LOP. If a country-specific productivity shock in the home country takes places, then households in the home country exert more search efforts to consume more goods in the domestic and the import markets. Higher search efforts of home households in the domestic market lead to the difference of matching probabilities of firms between the domestic and the export markets, which creates a gap between expected profits of a firm in both markets. Thus, firms move across markets due to disparity of profits. At the same time, firms in the domestic market offer lower prices. Since aggregate productivity and marginal costs of posting vacancies are the same across markets, difference in matching probabilities between markets let firms operating in each market offer different prices.

We also examine responses of macroeconomic variables to a country-specific productivity shock with calibrated values of parameters to understand the international propagation of shocks. An increase in home productivity leads to a rise in demand for home goods as well as increased profits of firms. This entails increased vacancies home firms post, and in turn, more employment. Moreover, increasing income leads to a rise in search efforts in the home country. As home households exert more search efforts in domestic and imported goods markets, the matching probability in the domestic market increases. The LOP gap of the home country increases. As a result, output, consumption, and employment in the home economy rises. This leads to an increasing expected demand for foreign goods. Thus, foreign firms also have an incentive to post more vacancies and hire more workers. However, the increase of the LOP gap of foreign goods induce foreign firms to post less vacancies, because movement of firms across markets leads to a fall in the matching probability for firms. Therefore, employment of the foreign country depends on which effect is stronger.

Finally, we study cross-country correlations of output, consumption, and employment, the correlation for the terms of trade and the relative output, the correlation between the real exchange rate and the relative consumption, and the correlation between output and employment within a country. When productivity shocks are the only source of uncertainty, the model show quantitatively lower correlation of output and higher correlation of consumption than data. However, negative correlation between the terms of trade and the relative output as well as negative correlated employment are not produced by the model. When taking into account productivity shocks along with preference shocks, the model generates a negative cross-country correlation for the terms of trade and the relative output, which is consistent with data.

This paper is related to two strands of literature. One set of literature is open economy studies that focus on the international relative prices such as the LOP, taking into account the role of real rigidities. A feature of this kind of literature is that they allow pricing-to-market which means it is possible to impose different prices for the same commodity in the different markets. Alessandria (2009) and Drozd and Nosal (2012) highlight search frictions in the goods market to account for the international prices. Alessandria (2009) addresses the importance of consumer search in generating persistent real exchange rate movements. Drozd and Nosal (2012) show that pricing-to-market is essential to explaining international price dynamics in the aggregate and product level by introducing marketing frictions in goods market. Besides search frictions, there are also other approaches that examine deviations in the LOP such as distribution costs (Corsetti and Pesenti, 2005) market shares (Auer and Schoenle, 2016), and deep habits (Jacob and Uusküla, 2019). Even though we focus on the role of search frictions in goods markets for deviations from the LOP, this paper also suggests the transmission mechanism behind spillover effects of productivity shocks by considering the interaction between search frictions in labor and goods markets.

The other set of literature where this paper can contribute is papers that explore the propagation of shocks over international business cycles considering search frictions either in labor or goods markets. Hairault (2002) account for the observed fluctuations of international business cycles with a model incorporating search frictions in the labor market. He suggests a resolution for the counterfactual correlation of employment by incorporating conventional search and matching frictions to an open economy. Similar to Hairault (2002), Cacciatore (2014) addresses the strong trade linkages causes the greater co-movement of business cycles introducing labor market frictions along with endogenous entry and exit of firms. Meanwhile, Bai and Ríos-Rull (2015) suggest the role of consumer preference shocks instead of the productivity shock to account for the international business cycles, introducing consumer search in goods markets.

The remainder of this paper is organized as follows. Section 2 introduces a two-country, two-good model with labor and goods market frictions. The analytical approach is discussed in section 3. Section 4 reports quantitative results of the model. The final section concludes.

2 Model

The economy is comprised to two countries (home and foreign). Each country is specialized in the production of one good which is traded internationally. Within the home country, there are a measure one of households. Households consume goods and supply labor to domestic firms which sell goods either in the domestic or export markets. There are a measure one of firms in the home country, which consist of $n_{h,t}$ in the domestic market and $n^*_{h,t}$ in the export market. Both the goods and labor markets are subject to search frictions. We assume that each firm posts vacancies, denoted as $v_{h,t}$ by a firm serving in the domestic market and as $v^*_{h,t}$ by a firm serving in the export market, at cost κ in units of domestic goods, to attract unemployed workers. Each household exerts efforts $s_t \in [0, 1]$ to search for goods. In what follows, the home country is focused on in the exposition of the model.

2.1 Matching process

Search frictions in labor and goods markets are characterized by assuming specific matching technologies. In the labor market, vacancies are filled by a Cobb-Douglas matching function as in the conventional Diamond-Mortensen-Pissarides (hereafter

'DMP') model,

$$H_t = \chi u_t^{\phi} \left(v_t\right)^{1-\phi}, \tag{1}$$

where $\chi > 0$. u_t denotes the pool of unemployed workers at the beginning of period t. As there is a single labor market in each country, the total vacancies, v_t, should be equal to the sum of the total vacancies posted by firms serving in the domestic market ($n_{h,t}v_{h,t}$) and the total vacancies posted by firms serving in the export market ($n^*_{h,t}v^*_{h,t}$), where $n_{h,t}$ and $n^*_{h,t}$ is the total mass of firms in the domestic and the export markets, respectively. Thus, $n_{h,t}+n^*_{h,t}$ and $n^*_{f,t}+n_{f,t}$ are the total mass of firms in each economy, measure of one. Defining $\zeta_t \equiv \frac{v_t}{u_t}$ as labor market tightness (vacancies-unemployment ratio), the vacancy filling rate (job finding rate) is $\Phi^v_t \equiv \frac{H_t}{v_t} = \chi\zeta_t^{-\phi}$ ($\Phi^u_t \equiv \frac{H_t}{u_t} = \chi\zeta_t^{1-\phi}$).

Following Blanchard and Galí (2010), we assume workers are immediately productive, such that employment, l_t, evolves according to, $l_t = (1-\rho)\, l_{t-1} + H_t$ where $\rho \in (0,1)$ is the exogenous rate of job separation. The number of searching workers and the number of vacancies, u_t and v_t, are defined as

$$\begin{aligned} u_t &= 1-(1-\delta)l_{t-1} \quad \text{where} \quad l_{t-1} = n_{h,t-1}l_{h,t-1} + n^*_{h,t-1}l^*_{h,t-1}, \\ v_t &= n_{h,t}v_{h,t} + n^*_{h,t}v^*_{h,t}. \end{aligned} \tag{2}$$

As in Bai and Ríos-Rull (2015), we assume a directed search friction in goods market.[1] Households exert efforts to search for goods in either the domestic or import market. Matches are formed by the following Cobb-Douglas functions,

$$M_{i,t} = A\left(s_{i,t}\right)^{\varphi}\left(n_{i,t}\right)^{1-\varphi} \quad \text{for } i = \{h, f\} \tag{3}$$

[1] Under a directed search circumstance, agents select what terms of trade to search for. This implies price is committed ex ante, unlike the undirected search.

where $A > 0$. $n_{h,t}$ ($n_{f,t}$) is the mass of home (foreign) firms serving the home market, whereas $s_{h,t}$ ($s_{f,t}$) is the mass of shoppers search for the home (foreign) goods in the home country. Goods market tightness is source-specific, so $\theta_{h,t} \equiv \frac{n_{h,t}}{s_{h,t}}$ and $\theta_{f,t} \equiv \frac{n_{f,t}}{s_{f,t}}$, are tightness for the domestic and imported goods. In this case, the probability that shoppers are matched with a firm (firms are matched with a shopper) in the domestic market is $\Phi^s_{h,t} \equiv \frac{M_{h,t}}{s_{h,t}} = A\theta^{1-\varphi}_{h,t}$ ($\Phi^n_{h,t} \equiv \frac{M_{h,t}}{n_{h,t}} = A\theta^{-\varphi}_{h,t}$) with similar expression for the import market.

2.2 Households

Households are modelled as an extended family, following Merz (1995). This assumption provides full consumption insurance among members since all members gather their income and consume the same amount. Households have the following inter-temporal utility function, $\sum_{t=0}^{\infty} \beta^t u(c_t, s_t)$, where $\beta \in (0,1)$ is the discount factor. The total consumption of final goods, c_t, is defined over the home ($c_{h,t}$) and foreign ($c_{f,t}$) good and utility from total consumption is increasing and strictly concave. The total mass of shoppers, s_t, consists of $s_{h,t}$ and $s_{f,t}$:

$$c_t = \left[\omega^{1/z} c_{h,t}^{(z-1)/z} + (1-\omega)^{1/z} c_{f,t}^{(z-1)/z}\right]^{z/(z-1)} \text{ and } s_t = s_{h,t} + s_{f,t}, \tag{4}$$

where $\omega \in (0,1)$ denotes measure of openness, and $z > 0$ is the elasticity of substitution between home and foreign goods.

The budget constraint of the households is,

$$\sum_i P_{i,t} c_{i,t} + \mathbb{E}_t Q_{t,t+1} B_{t+1} = W_t l_t + B_t + \Pi_t, \tag{5}$$

where Π_t are profits, B_t are domestic currency state-contingent assets ($Q_{t,t} \equiv 1$), and

$W_t l_t$ is labor income.

In the goods market, the realized output is different from the amount of goods supplied by firms because of the goods market friction. Furthermore, aggregate realized output is consumed by households and also used by firms to post vacancies due to a labor market friction. Thus, under goods and labor market frictions, the aggregate realized output in each market should be equal to aggregate expenditure which consists of consumption and costs of posting vacancies, at per unit cost $\kappa P_{h,t}$, for home firms, and at per unit cost $\kappa^* P^*_{f,t}$, for foreign firms.

We express the home shopping constraint in domestic and imported goods market as:

$$\begin{aligned} s_{h,t}\Phi^s_{h,t}y_{h,t} &= c_{h,t} + \kappa n_{h,t}v_{h,t} \qquad (6) \\ \frac{1}{e_t}P_{f,t}s_{f,t}\Phi^s_{f,t}y_{f,t} &= \frac{1}{e_t}P_{f,t}c_{f,t} + \kappa^* P^*_{f,t}n_{f,t}v_{f,t}, \end{aligned}$$

where e_t denotes the nominal exchange rate that means the price of home currency in terms of unit of foreign currency. The shopping constraints imply that the consumption of the home good, $c_{h,t}$, is equal to the mass of shoppers in that market, $s_{h,t}$, multiplied by the probability of a match, $\Phi^s_{h,t}$, and the goods supplied by firms, $y_{h,t}$, net of the cost of posting vacancies, $\kappa n_{h,t}v_{h,t}$, where $\kappa > 0$ is a parameter, and $n_{h,t}v_{h,t}$ is the total mass of vacancies in the domestic market - the number of vacancies multiplied the mass of firms serving the market. Since each export firm of the foreign economy sells foreign products at price $P_{f,t}$ and posts vacancies, $v^*_{h,t}$, at per unit cost $\kappa^* P^*_{f,t}$ to employ workers, they should consider the difference between the domestic and export price of foreign goods.

Households choose consumption, search effort, and state-contingent assets; $\{c_{i,t}$, $s_{i,t}$, $B_{t+1}\}$, to maximize expected discounted utility taking price, quantity, and market tightness $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$, as given:

$$\begin{aligned} & \max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, s_t\right) \\ & s.t. \\ & \sum_i P_{i,t} c_{i,t} + \mathbb{E}_t Q_{t,t+1} B_{t+1} \leq W_t l_t + B_t + \Pi_t \\ & c_{h,t} \leq s_{h,t} \Phi^s_{h,t} y_{h,t} - \kappa n_{h,t} v_{h,t} \\ & c_{f,t} \leq s_{f,t} \Phi^s_{f,t} y_{f,t} - \kappa n_{f,t} v_{f,t} \left(\frac{e_t P^*_{f,t}}{P_{f,t}} \right) \end{aligned} \tag{7}$$

This leads to the following first-order conditions,

$$u_{c_i}\left(t\right) + \frac{u_{s_i}\left(t\right)}{\Phi^s_{i,t} y_{i,t}} = \lambda_t P_{i,t} \quad \text{for } i = \{h, f\} \tag{8}$$

$$\mathbb{E}_t Q_{t,t+1} = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t}, \tag{9}$$

where λ_t is the Lagrangian multiplier on the household budget constraint. Note that the optimal condition in Equation (8) is different from one in a standard Walrasian model. There is an additional term, $u_{s_i}\left(t\right) / (\Phi^s_{i,t} y_{i,t})$, because of the relation of consumption and search efforts.

Given search effort, shoppers choose how to conduct their shopping, i.e., which market to go to either the domestic or imported goods market. With directed search, this means choosing price, quantity, and market tightness; $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$. When defining the value function, $J\left(a_t\right) = \max\left[u\left(c_t, s_t\right) + \beta \mathbb{E}_t J\left(a_{t+1}\right)\right]$, we have following conditions that characterize the shopper's choices:

$$
\begin{aligned}
J_{P_i}(a_t) &= -\lambda_t c_{i,t} && (10)\\
J_{\theta_i}(a_t)\,\theta_{i,t} &= (1-\varphi)\,J_{y_i}(a_t)\,y_{i,t} && (11)\\
J_{y_i}(a_t) &= \left[u_{c_i}(t) - \lambda_t P_{i,t}\right]\Phi^s_{i,t} s_{i,t} && (12)
\end{aligned}
$$

for $i = \{h, f\}$ as the remaining equations for the household problem. These conditions account for how households' values change with respect to price, quantity, and market tightness suggested by firms. Thus, firms also consider shoppers' choices to optimize their profits. How these conditions affects on a firm's choice is in the next section.

2.3 Firms

A representative firm makes two choices, i.e. how much labor to hire for production and what bundle $\{P_{i,t}, y_{i,t}, \theta_{i,t}\}$ to offer for a match with shoppers. In the domestic (export) market, a firm j, posts $v_{h,t}(j)$ ($v^*_{h,t}(j)$) vacancies, employs $l_{h,t}(j)$ ($l^*_{h,t}(j)$) workers, and produce a final good, $y_{h,t}(j) = a_t l_{h,t}(j)$ ($y^*_{h,t}(j) = a_t l^*_{h,t}(j)$), where a_t is a productivity parameter. Following Bai and Ríos-Rull (2015), firms target either the domestic or export market. Profits of a firm in domestic or export markets are:

$$
\begin{aligned}
\pi_{h,t}(j) &= P_{h,t}(j)\,\Phi^n_{h,t}(j)\,y_{h,t}(j) - W_t l_{h,t}(j) - \kappa P_{h,t} v_{h,t}(j) && (13)\\
\pi^*_{h,t}(j) &= e_t P^*_{h,t}(j)\,\Phi^{*n}_{h,t}(j)\,y^*_{h,t}(j) - W_t l^*_{h,t}(j) - \kappa P_{h,t} v^*_{h,t}(j).
\end{aligned}
$$

Due to search frictions in the goods market, a firm can sell its goods to households only if the firm is matched with a shopper. Therefore, profits of firms in the domestic (export) market depend on the probability that a firm is matched, $\Phi^n_{h,t}(j)$ ($\Phi^{*n}_{h,t}(j)$).

Moreover, labor market frictions cause additional costs to post vacancies.[2]

If a firm targets the domestic market, it chooses $\{v_{h,t}(j), l_{h,t}(j), P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\}$ to maximize its profits, $\pi_{h,t}(j)$, subject to technology, evolution of employment, and a household participation constraint,

$$J(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)) \geq \mathcal{J}_h(a_t) \tag{14}$$

where $J(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j))$ is the value of the representative household and $\mathcal{J}_h(a_t)$ is the value of the household if it shops optimally in the domestic market. The participation constraint implies that firms must suggest bundles no worse than the most attractive one available in the market to attract shoppers.

Accordingly, the representative firm's maximization problem in the domestic market is given by:

$$\begin{aligned}
&\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left(\frac{\lambda_{t+s}}{\lambda_t}\right) \{\pi_{h,t+s}(j)\} \qquad (15)\\
&s.t.\\
&y_{h,t+s}(j) \leq a_{t+s} l_{h,t+s}(j)\\
&l_{h,t+s}(j) \leq (1-\rho) l_{h,t+s-1}(j) + \Phi^v_{t+s} v_{h,t+s}(j)\\
&\mathcal{J}_h(a_{t+s}) \leq J[a_{t+s}; P_{h,t+s}(j), y_{h,t+s}(j), \theta_{h,t+s}(j)]
\end{aligned}$$

As all firms in the domestic market are identical, we can eliminate the $'j'$ in the optimization condition. Using the first order conditions of firms serving the domestic

[2]Note that the cost per hire for an individual firm is expressed in terms of the bundle of domestic final goods, $\kappa P_{h,t}$, in both markets as in Campolmi and Faia (2015). This is because firms evaluate their profits in terms of domestic price index.

market, the job creation condition in the domestic market is

$$\left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t - W_t + (1-\rho)\,\mathbb{E}_t\beta_{t,t+1}\left(\frac{\kappa}{\Phi_{t+1}^v}\right) P_{h,t+1}, \tag{16}$$

where $\beta_{t,t+1}$ denotes the stochastic discount factor between period t and $t+1$, which is given by $\beta_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. With an analogous process, we also derive the job creation condition in the export market.[3]

There are two differences between this model and a standard Walrasian model. The domestic price depends on both the vacancy filling rate, Φ_t^v (labor market), and the probability that a firm is matched with a shopper, $\Phi_{h,t}^n$ (goods market). Absent goods market frictions, vacancy costs, $\kappa > 0$, drive a wedge between the price of labor, W_t, and the marginal productivity, a_t. With goods market friction, this wedge also depends on $\Phi_{h,t}^n$.

Since there is a single labor market in each country, an individual firm serving to either domestic or export markets faces the same wage and in turn the same marginal cost. Thus, the only difference of the job creation condition in the export market is the marginal benefit, $\left(\frac{1}{1-\varphi}\right) e_t P_{h,t}^* \Phi_{h,t}^{*n} a_t$, in Equation (16). Comparing job creation conditions of domestic and export markets, we have the following equation:

$$e_t P_{h,t}^* = \left(\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}}\right) P_{h,t}, \tag{17}$$

which states the export price denoted in terms of home currency depends on the ratio of matching probabilities as well as the relative marginal productivity in the domestic and export markets. According to Equation (17), the relative price of home goods in the domestic and the export markets is linked to the market tightness, $\theta_{h,t}$ and $\theta_{h,t}^*$, in

[3] The details of the derivation are in Appendix.

both goods markets.

Using the first order conditions of firms and optimal shopper's choices, the optimal search efforts in the domestic and the export markets are:

$$\begin{aligned} s_{h,t} &= \varphi c_{h,t} \left(\frac{u_{c_h}(t)}{-u_{s_h}(t)} \right) \tau_{h,t} \\ s^*_{h,t} &= \varphi c^*_{h,t} \left(\frac{u_{c^*_h}(t)}{-u_{s^*_h}(t)} \right) \tau^*_{h,t}, \end{aligned} \tag{18}$$

where

$$\frac{1}{\tau_{h,t}} \equiv (1-\varphi) + \varphi \frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t} y_{h,t}} \text{ and } \frac{1}{\tau^*_{h,t}} \equiv (1-\varphi) + \varphi \frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t} y^*_{h,t}},$$

and φ is a matching technology parameter in the goods market.

Note that the optimal search effort in the domestic market is affected by the relative amount of consumption for the home good ($c_{h,t}$) and aggregate realized output ($s_{h,t}\Phi^s_{h,t}y_{h,t}$) in the domestic market. As consumption for the home good ($c_{h,t}$) is equal to aggregate realized output ($s_{h,t}\Phi^s_{h,t}y_{h,t}$) net of aggregate vacancy posting costs ($\kappa n_{h,t} v_{h,t}$) in the domestic market, vacancy costs create a wedge between the consumption and the realized output. Without labor market friction, i.e. $\kappa = 0$, $\tau_{h,t}$ and $\tau^*_{h,t}$ disappear, going back to the goods market friction model as in Bai and Ríos-Rull (2015). However, even though there are search frictions in labor market, $\kappa \neq 0$, we find the following Lemma 1.

[Lemma 1]

The ratio of the realized output and the consumption in each market is equal to each other:

$$\frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t}y_{h,t}} = \frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} = \frac{c^*_{f,t}}{s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}} = \frac{c_{f,t}}{s_{f,t}\Phi^s_{f,t}y_{f,t}}.$$

□ Proof. See the appendix.

According to Lemma 1, the ratio of output provided for job posting to the output is same across markets. Lemma 1 also implies that $\tau_{i,t} = \tau^*_{i,t} = \tau_t$ for $i = \{h, f\}$, which also allows equations such as the international risk sharing condition to be simplified.

2.4 Wage Bargaining

we determine the wage by assuming Nash wage bargaining between workers and firms.[4] Although bargaining takes place in each market, in equilibrium, the wage is same in the domestic and the export market because workers are free to move across firms in the home economy. The bargaining power of the household is denoted by α. As a result of Nash bargaining in the domestic market, we have the optimal sharing rule as:

$$\alpha S_t^F = (1-\alpha)\, S_t^H, \tag{19}$$

where S_t^F and S_t^H are the surplus of the household and firm from hiring an additional worker, respectively.

$$\begin{aligned} S_t^H &= W_t^E - W_t^U \\ S_t^F &= \gamma_t, \end{aligned} \tag{20}$$

where γ_t is the Lagrangian multiplier for the evolution of employment in the firm's profit maximization problem and thus represents the marginal value of one worker. W_t^E and W_t^U denote the value of employment to a worker and the value of unemployment,

[4] Since the bargaining over either real wage or nominal wage are same with the flexible prices, we assume the bargaining over the nominal wage to simplify the model.

respectively. They are defined as:

$$\begin{aligned} W_t^E &= W_t + \mathbb{E}_t\beta_{t,t+1}\left[(1-\rho)W_{t+1}^E + \rho\Phi_{t+1}^u W_{t+1}^E + \rho(1-\Phi_{t+1}^u)W_{t+1}^U\right] \\ W_t^U &= \mathbb{E}_t\beta_{t,t+1}\left[\Phi_{t+1}^u W_{t+1}^E + (1-\Phi_{t+1}^u)W_{t+1}^U\right] \end{aligned} \tag{21}$$

The value of employment in the future is divided into three circumstances: the matched job continues in the next period ($(1-\rho)W_{t+1}^E$), the worker finds a new job after job separation ($\rho\Phi_{t+1}^u W_{t+1}^E$), and the worker remains unemployed due to job severance ($\rho(1-\Phi_{t+1}^u)W_{t+1}^U$). Similarly, the value of unemployment in the next period is sum of the value of finding a job and the value of remaining unemployed. Using $S_{t+1}^H = W_{t+1}^E - W_{t+1}^U$, the worker's surplus, S_t^H, can be written as:

$$S_t^H = W_t + \mathbb{E}_t\beta_{t,t+1}\left[(1-\rho)S_{t+1}^H - (1-\rho)\Phi_{t+1}^u S_{t+1}^H\right] \tag{22}$$

Meanwhile, the firm's surplus in the domestic market is

$$S_t^F = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t - W_t + (1-\rho)\mathbb{E}_t\beta_{t,t+1}S_{t+1}^F. \tag{23}$$

The firm's surplus is the additional profits from hiring one worker net of the real wage. This leads to the following wage determination equation,

$$W_t = \alpha\left[\left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t + (1-\rho)\mathbb{E}_t\beta_{t,t+1}\kappa\left(\frac{v_{t+1}}{u_{t+1}}\right)P_{h,t+1}\right]. \tag{24}$$

Notice that wage depends on not only the labor market tightness in the future, $\frac{v_{t+1}}{u_{t+1}}$, but also tightness in the goods market, via the term $\Phi_{h,t}^n$, which is different from the conventional DMP model. As a firm saves future costs of posting vacancies by maintaining the match, the bargained wage is affected by the labor market tightness in the future. Moreover, if we consider the export market, as above, we find that the difference across market is reflected in the difference in tightness across the two goods markets.

2.5 International relative prices and International risk sharing

We now introduce some international relative prices. The real exchange rate is defined as:

$$q_t \equiv \frac{e_t P_t^*}{P_t}. \tag{25}$$

The terms of trade of the home country which is a relative price of imports to exports is expressed as

$$TOT_t = \frac{P_{f,t}}{e_t P_{h,t}^*}. \tag{26}$$

We also define the home and foreign good the law of one price (LOP) gaps, respectively, as:

$$\Psi_{i,t} \equiv \frac{e_t P_{i,t}^*}{P_{i,t}} \quad \text{for } i = \{h, f\}, \tag{27}$$

which states that the LOP gap should be equal to one if the LOP holds.

With the complete asset market assumption, home and foreign households have access to state-contingent assets which is traded internationally. This implies the following international risk sharing condition holds:[5]

$$\frac{u_{c_f^*}(t)}{u_{c_h}(t)} = \frac{e_t P_{f,t}^*}{P_{h,t}}. \tag{28}$$

[5]This expression is equivalent to the below:

$$\frac{u_{c^*}(t)}{u_c(t)} = \left(\frac{e_t P_t^*}{P_t}\right)$$

which expressed as the consumption based real exchange rate.

This condition means that the ratio of marginal utilities between the foreign and the home consumption in domestic markets is associated with the relative price of foreign goods to home goods, expressed in terms of the home currency.

Since we assume that firms are free to target either the domestic or the export market, the expected profits in each market are same in equilibrium ($\pi_{h,t} = \pi^*_{h,t}$). Furthermore, all firms face the identical marginal cost when there is no impediment in the international trade in goods markets, because there is a single labor market in the home economy. Thus, in equilibrium, the mass of vacancies posted by each firm is same, in turn, the employment of a firm is also same in both markets:

$$v_{h,t} = v^*_{h,t} \text{ and } l_{h,t} = l^*_{h,t}. \tag{29}$$

In what follows, we denote vacancies (employment) by a firm in both the domestic and the export market by v_t (l_t) instead of $v_{h,t}$ and $v^*_{h,t}$ ($l_{h,t}$ and $l^*_{h,t}$).

Table 1 summarized the equilibrium conditions for the world economy in terms of optimal allocations, prices of labor, and prices of goods.

3 Analytical results

In this section, we study the conditions which lead to deviation from the law of one price (LOP). First, we take into account specific condition where the LOP holds by deriving the LOP gap in terms of aggregate search efforts. Then, we explain how a country-specific productivity shock generates deviations of the LOP in otherwise symmetric economies.[6]

[6]Under the assumption of symmetric economies, the foreign openness parameter, ω^*, is equal to the home one, ω.

Table 1: Model summary

	Home country	Foreign country
Mass of firms	$1=n_{h,t}+n^*_{h,t}$	$1=n^*_{f,t}+n_{f,t}$
Unemployment	$u_t=1-(1-\rho)\,l_{t-1}$	$u^*_t=1-(1-\rho)\,l^*_{t-1}$
Employment	$l_t=(1-\rho)\,l_{t-1}+\Phi^v_t v_t$	$l^*_t=(1-\rho)\,l^*_{t-1}+\Phi^{*v}_t v^*_t$
Production	$y_t=z_t l_t$	$y^*_t=z^*_t l^*_t$
Wage	$W_t=\alpha\left(\frac{1}{1-\varphi}\right)p_{h,t}\Phi^n_{h,t}z_t$	$W^*_t=\alpha\left(\frac{1}{1-\varphi}\right)p^*_{f,t}\Phi^{*n}_{f,t}z^*_t$
	$+\alpha(1-\rho)E_t\beta_{t,t+1}\kappa\left(\frac{v_{t+1}}{u_{t+1}}\right)p_{h,t+1}$	$+\alpha(1-\rho)E_t\beta^*_{t,t+1}\kappa\left(\frac{v^*_{t+1}}{u^*_{t+1}}\right)p^*_{f,t+1}$
Job creation	$\left(\frac{\kappa p_{h,t}}{\Phi^v_t}\right)=\left(\frac{1}{1-\varphi}\right)p_{h,t}\Phi^n_{h,t}z_t$	$\left(\frac{\kappa p^*_{f,t}}{\Phi^{*v}_t}\right)=\left(\frac{1}{1-\varphi}\right)p^*_{f,t}\Phi^{*n}_{f,t}z^*_t$
	$-W_t+(1-\rho)\,E_t\beta_{t,t+1}\left(\frac{\kappa p_{h,t+1}}{\Phi^v_{t+1}}\right)$	$-W^*_t+(1-\rho)\,E_t\beta^*_{t,t+1}\left(\frac{\kappa p^*_{f,t+1}}{\Phi^{*v}_{t+1}}\right)$
Export price	$e_t p^*_{h,t}=\left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right)p_{h,t}$	$\frac{1}{e_t}p_{f,t}=\left(\frac{\Phi^{*n}_{f,t}}{\Phi^n_{f,t}}\right)p^*_{f,t}$
Int'l prices	$\frac{P_{h,t}}{P_{f,t}}=\left[u_{c_h}(t)+\frac{u_{s_h}(t)}{\Phi^s_{h,t}y_t}\right]/\left[u_{c_f}(t)+\frac{u_{s_f}(t)}{\Phi^s_{f,t}y^*_t}\right]$	$\frac{p^*_{f,t}}{p^*_{h,t}}=\left[u_{c^*_f}(t)+\frac{u_{s^*_f}(t)}{\Phi^{*s}_{f,t}y^*_t}\right]/\left[u_{c^*_h}(t)+\frac{u_{s^*_h}(t)}{\Phi^{*s}_{h,t}y_t}\right]$
Search effort	$s_{h,t}=\varphi c_{h,t}\left[\frac{u_{c_h}(t)}{-u_{s_h}(t)}\right]\tau_t$	$s^*_{f,t}=\varphi c^*_{f,t}\left[\frac{u_{c^*_f}(t)}{-u_{s^*_f}(t)}\right]\tau_t$
	$s_{f,t}=\varphi c_{f,t}\left[\frac{u_{c_f}(t)}{-u_{s_f}(t)}\right]\tau_t$	$s^*_{h,t}=\varphi c^*_{h,t}\left[\frac{u_{c^*_h}(t)}{-u_{s^*_h}(t)}\right]\tau_t$
Shopping	$c_{h,t}=n_{h,t}\Phi^n_{h,t}y_t-n_{h,t}\kappa v_t$	$c^*_{f,t}=n^*_{f,t}\Phi^{*n}_{f,t}y^*_t-n^*_{f,t}\kappa v^*_t$
constraint	$c_{f,t}=n_{f,t}\Phi^n_{f,t}y^*_t-n_{f,t}\kappa v^*_t\left(\frac{e_tP^*_{f,t}}{P_{f,t}}\right)$	$c^*_{h,t}=n^*_{h,t}\Phi^{*n}_{h,t}y_t-n^*_{h,t}\kappa v_t\left(\frac{P_{h,t}}{e_tP^*_{h,t}}\right)$
Risk sharing	$\frac{u_{c^*_f}(t)}{u_{c_h}(t)}=\frac{e_tP^*_{f,t}}{P_{h,t}}$	

3.1 LOP gap

We express the LOP gap of the home country as a function of the probability that a firm is matched with a shopper. Using the international risk sharing and optimal search efforts, we derive the following proposition.

[Proposition 1]

The LOP gap depends on the ratio of marginal utility of aggregate search between countries:

$$\Psi_{i,t}=\left[\frac{u_{s^*_i}(t)}{u_{s_i}(t)}\right]^{\varphi}=\left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi}\ \text{for } i=\{h,f\}$$

□ Proof. See the appendix.

Proposition 1 implies that if the marginal utilities of search efforts are equal to each other across countries, then the LOP holds. Furthermore, it is immediate that if the utility function does not have a curvature in search efforts, i.e. the marginal utility of search efforts is constant, the LOP always holds even if search efforts exerted by home and foreign households are different each other.

Note that home and foreign LOP gaps are the same and depend on the marginal utilities of aggregate searches. This is because the aggregate search is the sum of search efforts in the domestic and the import markets, $s_t = s_{h,t} + s_{f,t}$ and $s_t^* = s_{f,t}^* + s_{h,t}^*$, which means that the marginal utility in a sub-market is same as the marginal utility of total search efforts. Moreover, since both countries have the same utility function, equal marginal utilities of searches implies that aggregate search efforts of the home and the foreign countries are equal each other. Thus, if the aggregate search efforts of the home country are different from search efforts of foreign households, i.e. $s_t \neq s_t^*$, the LOP fails to hold.

[Proposition 2]

The ratio of search efforts depends on the ratio of consumption and the ratio of the marginal rate of substitution between consumption and search.

$$\frac{s_t^*}{s_t} = \left[\frac{u_{c^*}(t)/u_{s^*}(t)}{u_c(t)/u_s(t)}\right]\left(\frac{c_t^*}{c_t}\right)$$

□ Proof. This is evident when we take the ratio of the aggregate search efforts. This is because Lemma 1 implies that the aggregate search efforts are expressed by the aggregate consumption:

$$s_t = \varphi\left[\frac{u_c(t)}{-u_s(t)}\right]c_t\tau_t \text{ and } s_t^* = \varphi\left[\frac{u_{c^*}(t)}{-u_{s^*}(t)}\right]c_t^*\tau_t^* \text{ where } \tau_t = \tau_t^* \tag{30}$$

Proposition 2 suggests that if the marginal rate of substitution between consumption and search is a function of consumption and search, the ratio of aggregate search efforts only depends on the ratio of aggregate consumption. This implies that the aggregate search efforts of the home and the foreign households are equal to each other if home and foreign consumption is same.

Therefore, Proposition 2 along with Proposition 1 states that the LOP gap depends on the relative aggregate consumption if the marginal rate of substitution between consumption and search is expressed in terms of consumption and search.

3.2 Deviations from the LOP

In the previous section, we find when the LOP holds in general terms. In this section, we concentrate on the deviation from the LOP when there is a country-specific productivity shock by considering a functional form.

we focus on Greenwood-Hercowitz-Huffman (GHH) preferences over consumption and search efforts. The period utility function is

$$u\left(c_t, s_t\right) = \frac{1}{1-\sigma}\left(c_t - \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma}, \tag{31}$$

where $1/\sigma$ denotes the intertemporal elasticity of substitution. While ψ captures disutility from exerting search efforts, η determines the elasticity of search effort with respect to the return. As the wealth effects in search efforts are eliminated with GHH preferences, shopping efforts are procyclical. This is consistent with empirical research such as Petrosky-Nadeau et al. (2016).

To be able to explore the mechanism which causes deviations from the LOP, we further consider a simplified static version of the model by setting the Armington elasticity, z, to 1 and the separation rate of a job, ρ, to 1. This means a job continues only one period, and in turn, all workers search jobs in every period, i.e. $u_t = 1$. Thus, the job creation condition and the wage equation in the labor market for a firm serving to the domestic market are replaced by the following static conditions:

$$\left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t - W_t \tag{32}$$

$$W_t = \alpha \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^n a_t. \tag{33}$$

Apart from the equilibrium conditions in the labor market, other equations are as reported in Table 1.[7]

With the period utility function, we derive the relation between aggregate search and consumption within a country by Equation (30):

$$s_t^{1+\frac{1}{\eta}} = \tau_t \left(\frac{\varphi}{\psi}\right) c_t \text{ and } s_t^{*1+\frac{1}{\eta}} = \tau_t \left(\frac{\varphi}{\psi}\right) c_t^*. \tag{34}$$

According to Proposition 2, the ratio of search efforts is expressed in terms of ratio of consumption. This implies that the LOP gap is in terms of the ratio of aggregate consumption by Proposition 1:

$$\Psi_{i,t} = \left(\frac{c_t}{c_t^*}\right)^{\frac{\varphi[\sigma(1+\eta)-1]}{1+\eta}} \quad \text{for } i = \{h, f\}. \tag{35}$$

This equation states that if (1) there is no search frictions in goods market, i.e. $\varphi = 0$ or (2) σ is equal to $1/(1+\eta)$, the LOP always holds regardless of the fluctuation of consumption. Otherwise, the LOP does not hold when home and foreign consumption is different which implies that the real exchange rate fluctuates. To see why the LOP

[7] The full equations of the static model are in Appendix

gap is linked with the consumption gap across countries, we need to use the following Proposition 3.

[Proposition 3]

The different productivity shocks across countries, $a \neq a^*$, lead to consumption gaps $(c \neq c^*)$ *across countries, in turn deviations from the law of one price.*

$$\widehat{c_t} - \widehat{c_t^*} = \frac{(2\omega - 1)(1 + \eta)}{\zeta} \left(\widehat{a_t} - \widehat{a_t^*} \right),$$

with

$$\begin{aligned} \zeta &= 2(1-\sigma)(1-\omega)(1+\eta)\left[\varphi(1-\phi) + 2\omega(\phi - \varphi)\right] \\ &\quad + \phi(1+\eta) - \varphi\eta\left[2\omega(1-\phi) + 2\phi - 1\right], \end{aligned}$$

where ˆ denotes the log deviation from the steady state.

□ Proof. See the appendix.

Proposition 3 suggests that a country-specific productivity shock generates consumption gaps, and then deviations from the LOP. If the coefficient of consumption gaps is not equal to 0, different productivity shocks cause a disparity of consumption across countries. Note that if consumption is not home-biased ($\omega = 0.5$), difference of productivity between countries does not link to the consumption gap. This implies that LOP holds even if there is a country-specific productivity shock, in the absence of the consumption home-biasedness. ζ determines whether relative consumption of the home country to the foreign country rises in response to an increase in productivity of home country. Taking into account meaning of each parameter, ζ is positive if consumption is home-biased ($\omega > 0.5$), which states a positive consumption gap ($\widehat{c_t} > \widehat{c_t^*}$) and the positive LOP gap according to Equation (35).

To understand the mechanism behind Proposition 3, suppose there is a country-specific productivity shock in the home country. Households in the home country exert more search efforts to consume more in the domestic and the import markets. Given the number of firms operating in each market unchanged, more search efforts of home households in the domestic market, $s_{h,t}$, cause higher matching probability in the domestic market, $\Phi^n_{h,t}$, than in the export market, $\Phi^{*n}_{h,t}$. The difference of matching probabilities of firms between the domestic and the export markets entails a gap between expected profits of a firm in both markets:

$$\underbrace{P_{h,t}\Phi^n_{h,t}y_t - W_t l_t - \kappa P_{h,t}v_t}_{\pi_{h,t}} \neq \underbrace{e_t P^*_{h,t}\Phi^{*n}_{h,t}y_t - W_t l_t - \kappa P_{h,t}v_t}_{\pi^*_{h,t}}. \tag{36}$$

Thus, firms in the export market move to the domestic market. At the same time, firms in the domestic market offer lower price, $P_{h,t} < e_t P^*_{h,t}$. Firms' movement across markets occurs until the expected profits of a firm in both markets are equal to each other.

Intuitively, the mechanism can be explained by the role of matching probability as well. Taking into account the job creation condition (Equation (32)) with wage condition (Equation (33)) in the domestic and export markets are given by the following expressions:

$$\begin{aligned} P_{h,t} &= \left(\frac{1-\varphi}{1-\alpha}\right)\frac{1}{\Phi^n_{h,t}a_t}\left(\frac{\kappa P_{h,t}}{\Phi^v_t}\right) \\ e_t P^*_{h,t} &= \left(\frac{1-\varphi}{1-\alpha}\right)\frac{1}{\Phi^{*n}_{h,t}a_t}\left(\frac{\kappa P_{h,t}}{\Phi^v_t}\right). \end{aligned} \tag{37}$$

Recall that φ and α denote the matching technology parameter in the goods market and the bargaining power of the household, respectively. $\kappa P_{h,t}/\Phi^v_t$ is the marginal cost of posting an additional job vacancy. According to the above equation, the price which firms offer in goods markets depends on the marginal cost of posting vacancies, matching probability of firms, and aggregate productivity. Since aggregate productivity and marginal costs are the same across markets, difference in matching probabilities between markets let firms operating in each market offer different prices.

Furthermore, Bai and Ríos-Rull (2015) and Bai et al. (2017) highlight the role of matching probability as a productivity shock, measuring aggregate productivity of $\Phi^n_{h,t} a_t$. If there is a preference shock affecting the matching probability only, not a_t, firms adjust their price offers in response to the preference shock, because it plays as a productivity shock via the changing of the matching probability.

4 Quantitative analysis

In this section, we present a quantitative analysis of the model. We study the responses of aggregate variables to productivity shocks. Then, international correlations of business cycles are also reported.

4.1 Calibration

In this section, the calibration of the parameters presented in the model is discussed. We assume the home country is the U.S. and the foreign country is the EU for the calibration of parameters. A period in this paper is set to a quarter. For the parameters related to preferences, we choose standard values used in the literature. The discount factor, β, is assumed 0.99 to adjust the quarterly real interest rate to 1 percent. The CRRA parameter, σ, is set to 2, which implies the inter-temporal elasticity of substitution is 0.5. We assume that the consumption elasticity between home and foreign goods (z) is

1.2, following Ruhl (2008).

The elasticity of match (ϕ) in the labor market is set to 0.5 as in Pissarides (2009). The worker's bargaining power (α) is assumed to be 0.5 so that the Hosios (1990) condition holds. The matching elasticity in goods market, φ, and the parameter η are set to 0.23 and 0.11, respectively, which are calibrated in Bai et al. (2017). Moreover, we set the matching efficiency in goods market, A to 1 as in Bai et al. (2017).

The remaining parameters are calibrated using the steady-state targets. The matching efficiency in labor market, χ and χ^*, is calibrated by setting the steady-state vacancy filling probability to 0.71 suggested by Den Haan et al. (2000). The job separation rate of the home country, ρ, is set to 0.105 as in Gertler et al. (2008), to match the estimates of the U.S. monthly rates suggested by Shimer (2005). We calibrate 0.036 for the job separation rate of the foreign country, ρ^*, using monthly estimates of the EU-15 in Hobijn and Şahin (2009). For parameters of vacancy posting costs, κ and κ^*, we set the targets of unemployment rates at 6% for home and 10% for foreign economy, which is consistent with OECD data.[8] The openness parameter in each country is chosen such that imports are 13% and 18% of aggregate output, respectively, as in Bayoumi et al. (2004). We calibrate the value of disutility parameter for search efforts, ψ, to match the capacity utilization of 81%, based on the series published by the Federal Reserve Board, following Bai and Ríos-Rull (2015). The chosen parameters can be shown in Table 2.

[8] Data are taken from OECD (2022) between 1991 and 2005.

Table 2: Parameters

Targets	Value	Parameter	Value	Source
Parameters set exogenously				
Risk aversion		σ	2	-
Discount factor		β	0.99	$(\beta^{-4}-1)\times 100 \doteqdot 4\%$
Armington elasticity		z	1.2	Ruhl (2008)
Bargaining power		α	0.5	Hosios (1990)
Matching elas. (labor)		ϕ	0.5	Pissarides (2009)
Matching elas. (goods)		φ	0.23	Bai et al. (2017)
Frisch elas. for search		η	0.11	Bai et al. (2017)
Matching efficiency		A	1	Bai et al. (2017)
Job separation rate		$[\rho, \rho^*]$	$[0.105, 0.036]$	Shimer (2005)
Calibrated Parameters				
Vacancy filling prob.	0.71	$[\chi, \chi^*]$	$[0.66, 0.41]$	Den Haan et al. (2000)
SS employment (L)	$[94\%, 90\%]$	$[\kappa, \kappa^*]$	$[0.94, 2.28]$	OECD (2022)
Imports-to-output	$[13\%, 18\%]$	$[\omega, \omega^*]$	$[0.84, 0.79]$	Bayoumi et al. (2004)
Capacity utilization	81%	$[\psi, \psi^*]$	$[4667, 1175]$	Bai and Ríos-Rull (2015)

Note: The calibrated parameters are derived from U.S. (home) and EU (foreign) steady-state targets. The parameters with * refer to the foreign country.

4.2 Responses to shocks

We assume that the aggregate productivity follows bivariate autoregressive process, following Backus et al. (1992).

$$A_{t+1} = \Omega A_t + \varepsilon_{t+1}, \tag{38}$$

where $A_t = [\ln a_t, \ln a_t^*]^T$ and $\varepsilon_{t+1} \sim N(0, V)$. ε_t are considered as serially independent random variables. Thus, the diagonal elements of Ω imply the persistence of country-specific productivity shock, while the off-diagonal elements denote the spillover effects of a productivity shock across countries. We set the values of parameters associated with spillover and persistence of productivity shocks are 0.088 and 0.906, while the variance of shock and correlation of shocks are set to 0.00852^2 and 0.258, as in Backus et al. (1992).

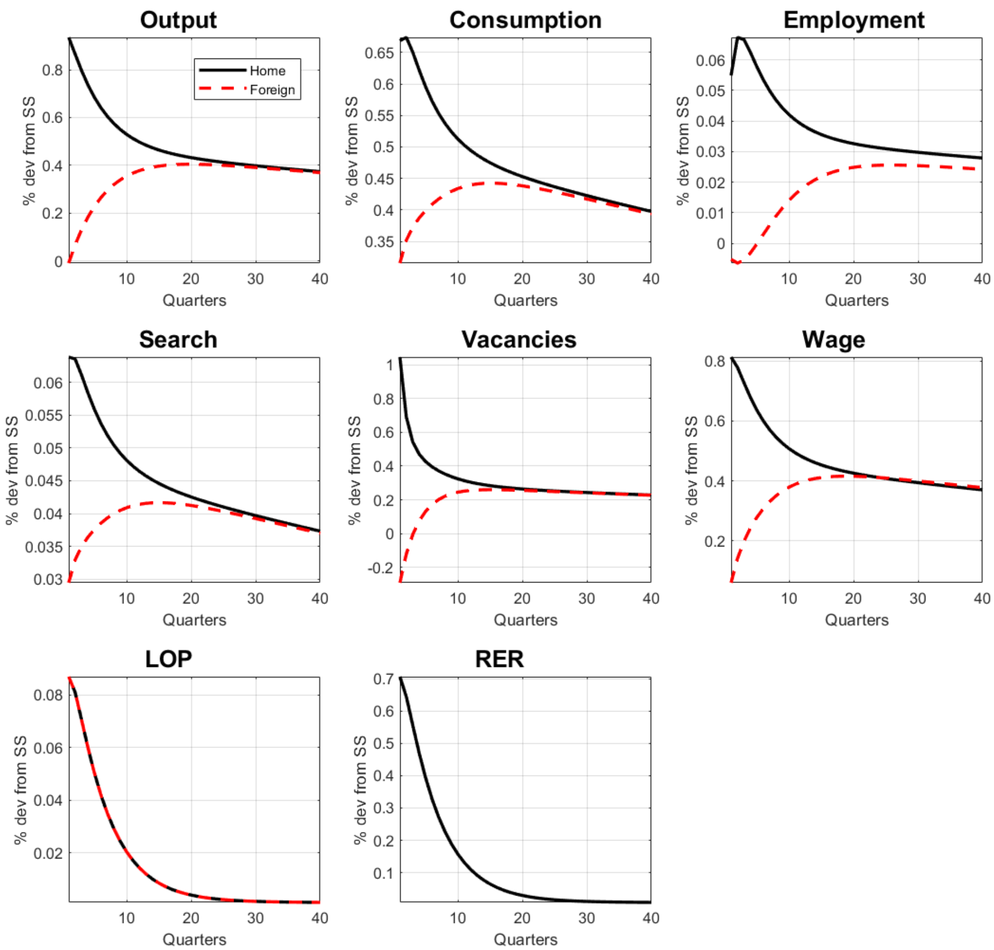

Figure 1: Responses to one s.d. productivity shock in the home country

Note: The home (foreign) country is assumed as the U.S. (EU) economy. One period denotes a quarter on the horizontal axis and the percentage deviation from the steady state is depicted on the vertical axis.

Figure 1.1 shows the impulse responses of chosen variables to one standard deviation of productivity in the home country. An increase in home productivity leads firms in home country post more vacancies, inducing more employment. More employment as well as increasing values of jobs in the home country also make wage increase. Furthermore, increasing income (output) leads home households to exert search efforts more because we assume there is a positive relation between income and search efforts with GHH preferences. An increasing search effort of home households in both domestic and imported goods markets causes higher matching probability in the domestic market. Thus, the LOP gap of the home country increases. As a result, output, consumption and employment in the home country increases.

Due to positive spillover effects of productivity shocks, productivity of the foreign country also increases. Moreover, since a positive home productivity shock causes a rise in home consumption, perfect international risk-sharing leads foreign consumption to increase as a result of complete asset markets. Thus, future consumption in both countries will also increase because the effect of a shock on the home country is persistent. This, in turn, leads expected demand for foreign products to rise. Thus, foreign firms also have an incentive to post more vacancies and hire more workers due to the higher expected returns to jobs. However, the increase of the LOP gap over foreign goods gives another incentive for foreign firms to post less vacancies.

To understand the transmission mechanism behind the responses of foreign employment, it is useful to consider the equilibrium condition of labor market in the foreign country. The equilibrium condition is summarized by the equation:

$$\frac{\kappa}{\Phi_t^{*v}} = \left(\frac{1-\alpha}{1-\varphi}\right)\Phi_{f,t}^{*n} a_t^* - (1-\rho)\,\mathbb{E}_t \beta_{t,t+1}^* \left[\left(\frac{\kappa}{\Phi_{t+1}^{*v}}\right) - \alpha\kappa\left(\frac{v_{t+1}^*}{u_{t+1}^*}\right)\right], \tag{39}$$

where

$$\Phi_t^{*v} = \chi\left(\frac{v_t^*}{u_t^*}\right)^{-\phi}.$$

The left-hand side of the equation indicates the marginal cost to a firm, whereas the right-hand side represents the expected marginal benefit. The first term in the right-hand side is the current earning from hiring an additional worker. The terms in the bracket mean discounted continuation values of a job which is not separated in the next period. Thus, the equilibrium condition implies that employment is determined at the level where the marginal cost and the marginal expected profits of an additional worker are equal.

The marginal product of labor in Equation (39) is different from the standard DMP model due to an additional variable, $\Phi^{*n}_{f,t}$, which comes from goods market frictions. This is because, except for the own marginal productivity, a^*_t, the condition depends on the matching probability in goods market, $\Phi^{*n}_{f,t}$, affected by movement of firms between markets, as well.

Without goods market frictions, if a positive productivity shock happens in the home country, an increase in foreign productivity due to positive spillover effects of productivity shocks and increasing future value of jobs leads firms to post more vacancies. Considering goods market frictions, however, there is an additional effect via the matching probability. Given the mass of firms in each market unchanged, a rise in the LOP gap in foreign countries induces foreign firms to move from the export market to the domestic market, because the profit in the domestic market is temporarily higher. This causes an increase of the market tightness in the domestic market, inducing a fall in the matching probability. Thus, according to the equilibrium condition, foreign firms also have an incentive to post less vacancies with goods market frictions. Therefore, employment of the foreign country depends on which effect has more impacts than the other.

4.3 International correlations

In this section, we calculate cross-country correlations of output, consumption, and employment, the correlation for the terms of trade and the relative output, the correlation between the real exchange rate and the relative consumption, and the correlation between output and employment within a country. Table 3 reports correlations both in the data and in the open-economy models.[9] While the first column reports characteristics found in the data corresponding the U.S. aggregate, the remaining columns are statistics derived from the models: search frictions both in labor and goods markets ('Two-search'), search frictions in goods markets ('Goods search'), and search frictions in labor markets ('Labor search').[10] All entries in the table are Hodrick-Prescott filtered values with a smoothing parameter of 1600.

To analyze correlations of selected variables, we examine the effect of productivity shocks introduced in the previous subsection. We also study the implications of productivity shocks along with preference shocks because Bai and Ríos-Rull (2015) emphasize that the role of preference shocks in the consumer search model to explain the business cycles.[11] To introduce preference shocks, we assume that the disutility parameter of search can vary during the given periods, as in Bai et al. (2017):

$$u\left(c_t, s_t\right) = \frac{1}{1-\sigma}\left(c_t - d_t \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma}, \tag{40}$$

[9]The data column are for the period of 1976:1-2015:4, using data for the US and the aggregate of the EU-15.

[10]For the analysis of the goods search model, the period utility function is assumed as

$$u\left(c_t, s_t, l_t\right) = \frac{1}{1-\sigma}\left(c_t - d_t \psi \frac{s_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)^{1-\sigma} - \iota \frac{l_t^{1+\frac{1}{g}}}{1+\frac{1}{g}},$$

where g set to 0.72.

The period utility function of the labor search model is assumed as

$$u\left(c_t\right) = \frac{c_t^{1-\sigma}-1}{1-\sigma}.$$

[11]Since Bai and Ríos-Rull (2015) and Bai et al. (2017) consider endogenous productivity which consists of aggregate productivity and the matching probability in the goods market, preference shocks play a role as productivity shocks.

Table 3: Business cycle statistics

Correlations	Data	Productivity			Productivity and preference		
		Two-search	Goods search	Labor search	Two-search	Goods search	Labor search
corr(y,y*)	0.55	0.28	0.03	0.30	0.35	0.19	–
corr(c,c*)	0.40	0.87	0.69	0.88	0.54	0.27	–
corr(n,n*)	0.93	−0.01	0.94	0.12	−0.04	0.61	–
corr(TOT,y/y*)	−0.21	0.99	1.00	0.99	−0.27	−0.35	–
corr(q,c/c*)	0.06	1.00	1.00	1.00	1.00	1.00	–
corr(y,n)	0.83	0.94	−0.85	0.95	0.93	−0.94	–

Note: The statistics of the data column are for 1970:1 to 2015:4 using U.S. and the aggregate data of the EU-15. All statistics have been HP-filtered with a smoothing parameter of 1,600.

where d_t follows AR(1) process with respective persistence. We set the persistence and the standard deviation of a shock to 0.99 and 0.61, taking estimates in Bai et al. (2017).[12] We assume that shocks to preferences have no spillover across countries, and there is no correlation between shocks.

When taking into account productivity shocks only, the cross-country correlations of output in all models are less than those of consumption, which is inconsistent with the data. Comparing with the goods search model, the two-search and the labor search models report relatively higher correlations of output and consumption at 0.28 and 0.87 for the former, at 0.30 and 0.88 for the latter.

Regarding employment, only the two-search model shows a negative international correlation of −0.01, which is not consistent with the data. This is because the negative spillover effects caused by frictions in goods markets, as explored with the impulse responses in the previous subsection. Accordingly, without goods market frictions, the positive spillovers are found in the labor search model. The goods search model shows very highly correlated employment despite the negative spillover effects. This

[12]Bai et al. (2017) estimate the process by using U.S. quarterly data from 1967 to 2013 with Bayesian methods.

comes from the property that employment of the home country decreases in response to productivity shocks as well, which is opposite with data. According to the correlation between output and employment within the home country, the goods search model reports negative correlations of -0.85 and -0.94, respectively, in both cases, different from data.

Meanwhile, all models do not explain the correlation between the terms of trade and the relative output, and the correlation for the real exchange rate and the relative consumption in data, because of the assumption of complete financial market.

When there are productivity shocks along with preference shocks, both the two-search model and the goods search model report a negative correlation between the terms of trade and the relative output, which is consistent with data. Moreover, correlations of output and consumption in the two-search model become close to data quantitatively, as the cross-country correlation of output increases to 0.35, whereas the correlation of consumption decreases to 0.54. Note that the labor search model does not take into account preference shocks due to the property of the model.

Different from Bai and Ríos-Rull (2015), the preference shock, i.e. demand shock, is not sufficient to address the international co-movement over business cycles in this paper. Since the utility function in Bai and Ríos-Rull (2015) does not have a curvature in search efforts, LOP always holds in their model. With the curvature of the utility function in search efforts, however, deviations from LOP can happen as discussion in the previous section, which gives a different intuition on the cross-country co-movement of business cycles.

5 Conclusion

In this paper, we analyze deviations from the law of one price and international spillover effects of aggregate productivity and preferences shocks across countries using the two-search model which introduces search and matching frictions in both goods and labor markets. We also examine conditions which lead to deviations from the LOP and the mechanism how productivity shocks make the LOP fail to hold. Finally, we study impulse responses of macroeconomic variables with respect to a positive productivity shock in the home economy, and cross-country correlations. We find the mechanism which causes deviations from the LOP. Since the LOP gap only depends on the ratio of marginal utility of aggregate search across countries and is linked the consumption gap across countries, a country-specific productivity shock entails deviations from the LOP. Moreover, we find the two-search model reports consistent correlations with data, in terms of cross-country correlations of output and consumption, and a negative correlation between the terms of trade and the relative output when productivity and preference shocks considered.

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Appendix

A.1 Data Sources

We collect the data series of US and EU-15 from OECD Quarterly National Accounts (QNA), OECD Economic Outlook (EO), and Federal Reserve Economic Data (FRED) for the period 1976:1–2015:4. The EU-15 comprises 15 European countries, including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, and United Kingdom. While data for GDP and consumption are Gross Domestic Product and Private plus Government Final Consumption Expenditure from QNA, respectively, data for employment come from EO. As employment series are not available for all European countries, we compute the series for EU as the aggregate of 12 European countries weighted with populations in 2015, except Greece, Ireland, and Luxembourg. We use the series real effective exchange rate from FRED for the US real exchange rate. Data for the terms of trade are computed by the ratio of import prices over export prices from QNA.

A.2 Firm's optimal choice

A.2.1 Domestic market

Following Blanchard and Galí (2010), we assume workers are immediately productive, such that employment, $l_{h,t}$, evolves according to, $l_{h,t} = (1-\rho)\, l_{t-1} + \Phi_t^v v_{h,t}$, and $\rho \in (0,1)$ is the exogenous rate of job destruction.

To derive FOCs of a firm j in the domestic market, we need to solve the following optimization problem taking $P_{h,t}$ as given:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\lambda_t}{\lambda_0}\right) \{P_{h,t}(j)\, \Phi_{h,t}^n(j)\, y_{h,t}(j) - W_t l_{h,t}(j) - \kappa P_{h,t} v_{h,t}(j)$$
$$+\xi_t \left[J\left(a_t; P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\right) - \mathcal{J}_h(a_t)\right]$$
$$+\mu_t \left[a_t f\left(l_{h,t}(j)\right) - y_{h,t}(j)\right]$$
$$+\gamma_t \left[(1-\rho)\, l_{h,t-1}(j) + \Phi_t^v v_{h,t}(j) - l_{h,t}(j)\right]\} \tag{A.1}$$

First, consider the choice of $\{v_{h,t}(j), l_{h,t}(j)\}$. We find,

$$\gamma_t = \left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} \text{ and } \gamma_t = \mu_t a_t f'\left(l_{h,t}(j)\right) - W_t + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \gamma_{t+1}$$

$$\rightarrow \left(\frac{\kappa}{\Phi_t^v}\right) P_{h,t} = \mu_t a_t f'\left(l_{h,t}(j)\right) - W_t + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi_{t+1}^v}\right) P_{h,t+1} \tag{A.2}$$

where $\beta_{t,t+1}$ denotes the stochastic discount factor between period t and $t+1$, which is given by $\beta_{t,t+1} = \beta \lambda_{t+1}/\lambda_t$.

Now consider the choice of $\{P_{h,t}(j), y_{h,t}(j), \theta_{h,t}(j)\}$. We find,

$$P_{h,t}(j) \;:\; \Phi_{h,t}^n(j)\, y_{h,t}(j) = -\xi_t J_{P_h} \tag{A.3}$$

$$y_{h,t}(j) \;:\; \mu_t = P_{h,t}(j)\, \Phi_{h,t}^n(j) + \xi_t J_{y_h} \tag{A.4}$$

$$\theta_{h,t}(j) \;:\; \varphi \Phi_{h,t}^n(j)\, P_{h,t}(j)\, y_{h,t}(j) = \xi_t \theta_{h,t}(j)\, J_{\theta_h} \tag{A.5}$$

respectively. As all firms in the domestic market are identical, we can eliminate the 'j' in the FOC equations.

$$y_{h,t}\Phi^n_{h,t} = -\xi_t J_{P_h} \tag{A.6}$$

$$\mu_t = P_{h,t}\Phi^n_{h,t} + \xi_t J_{y_h} \tag{A.7}$$

$$\varphi\Phi^n_{h,t}P_{h,t}y_{h,t} = \xi_t\theta_{h,t}J_{\theta_h} \tag{A.8}$$

Recall the participation constraint in the domestic market from the households' problem.

$$J_{P_h} = -\lambda_t c_{h,t} \tag{A.9}$$

$$J_{\theta_h}\theta_{h,t} = (1-\varphi)\, J_{y_h} y_{h,t} \tag{A.10}$$

$$J_{y_h} = \left[u_{c_h}(t) - \lambda_t P_{h,t}\right]\Phi^s_{h,t} s_{h,t} \tag{A.11}$$

For the price equation in the domestic market, use (A.7), (A.8) and (A.10) to eliminate ξ_t from the firms optimal conditions,

$$\mu_t = P_{h,t}\Phi^n_{h,t} + \xi_t J_{y_h} \quad \text{and} \quad \varphi\Phi^n_{h,t}P_{h,t}y_{h,t} = \xi_t(1-\varphi)\, J_{y_h} y_{h,t}$$
$$\rightarrow \quad \mu_t = P_{h,t}\Phi^n_{h,t}\left(\frac{1}{1-\varphi}\right) \tag{A.12}$$

Thus, the job creation condition in the domestic market is:

$$\left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} = \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi^n_{h,t}a_t f'(l_{h,t}) - W_t + (1-\rho)\,\mathbb{E}_t\beta_{t,t+1}\left(\frac{\kappa}{\Phi^v_{t+1}}\right) P_{h,t+1} \tag{A.13}$$

For the search effort in the domestic market, use (A.6), (A.8) and (A.10) to eliminate

ξ_t:

$$
\begin{aligned}
y_{h,t}\Phi^n_{h,t} &= -\xi_t J_{P_h} \quad \text{and} \quad \varphi\Phi^n_{h,t}P_{h,t}y_{h,t} = \xi_t\theta_{h,t}J_{\theta_h} \\
&\rightarrow \quad y_{h,t}\Phi^n_{h,t} = -\varphi\frac{\Phi^n_{h,t}P_{h,t}y_{h,t}}{J_{\theta_h}\theta_{h,t}}J_{P_h} \\
&\rightarrow \quad -J_{P_h}P_{h,t} = \frac{1}{\varphi}J_{\theta_h}\theta_{h,t} = \frac{1}{\varphi}\left(1-\varphi\right)J_{y_h}y_{h,t} \qquad \text{(A.14)}
\end{aligned}
$$

Eliminating J_{y_h} by using (A.11), we write:

$$
-J_{P_h}P_{h,t} = \frac{1-\varphi}{\varphi}\left[u_{c_h}\left(t\right) - \lambda_t P_{h,t}\right]\Phi^s_{h,t}s_{h,t}y_{h,t} \qquad \text{(A.15)}
$$

Finally, use (A.9) and $u_{c_h}\left(t\right) + \frac{u_{s_h}(t)}{\Phi^s_{h,t}y_{h,t}} = \lambda_t P_{h,t}$ to eliminate $J_{P_h}P_{h,t}$. We find:

$$
\begin{aligned}
\varphi c_{h,t}\lambda_t P_{h,t} &= \left(1-\varphi\right)\left[u_{c_h}\left(t\right) - \lambda_t P_{h,t}\right]\Phi^s_{h,t}s_{h,t}y_{h,t} \\
\varphi c_{h,t}\left[u_{c_h}\left(t\right) + \frac{u_{s_h}\left(t\right)}{\Phi^s_{h,t}y_{h,t}}\right] &= \left(1-\varphi\right)\left[-\frac{u_{s_h}\left(t\right)}{\Phi^s_{h,t}y_{h,t}}\right]\Phi^s_{h,t}s_{h,t}y_{h,t} \\
-s_{h,t}u_{s_h}\left(t\right) &= \varphi c_{h,t}u_{c_h}\left(t\right)\left[\frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t} + \left(1-\varphi\right)s_{h,t}\Phi^s_{h,t}y_{h,t}}\right] \qquad \text{(A.16)}
\end{aligned}
$$

Using the shopping constraint, we can also write the search effort as:

$$
s_{h,t} = -\varphi c_{h,t}\frac{u_{c_h}\left(t\right)}{u_{s_h}\left(t\right)}\left[\frac{c_{h,t} + n_{h,t}\kappa v_{h,t}}{c_{h,t} + \left(1-\varphi\right)n_{h,t}\kappa v_{h,t}}\right] \qquad \text{(A.17)}
$$

A.2.2 Export market

To derive FOCs of a firm j in the export market, we need to solve the following optimization problem, taken $P_{h,t}$ as given:

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\lambda_t}{\lambda_0}\right) \{e_t P^*_{h,t}(j)\, \Phi^{*n}_{h,t}(j)\, y^*_{h,t}(j) - W_t l^*_{h,t}(j) - \kappa P_{h,t} v^*_{h,t}(j)$$
$$+\xi_t \left[J\left(a_t; P^*_{h,t}(j), y^*_{h,t}(j), \theta^*_{h,t}(j)\right) - \mathcal{J}_{h^*}(a_t)\right]$$
$$+\mu_t \left[a_t f\left(l^*_{h,t}(j)\right) - y^*_{h,t}(j)\right]$$
$$+\gamma_t \left[(1-\rho)\, l^*_{h,t-1}(j) + v^*_{h,t}(j)\, \Phi^v_t - l^*_{h,t}(j)\right]\} \tag{A.18}$$

First, consider the choice of $\left\{v^*_{h,t}(j), l^*_{h,t}(j)\right\}$. We find,

$$\gamma_t = \left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} \quad \text{and} \quad \gamma_t = \mu_t a_t f'\left(l^*_{h,t}(j)\right) - W_t + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \gamma_{t+1}$$

$$\begin{aligned} \rightarrow \left(\frac{\kappa}{\Phi^v_t}\right) P_{h,t} &= \mu_t a_t f'\left(l^*_{h,t}(j)\right) - W_t \\ &\quad + (1-\rho)\, \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi^v_{t+1}}\right) P_{h,t+1} \end{aligned} \tag{A.19}$$

Now, consider the choice of $\left\{P^*_{h,t}(j), y^*_{h,t}(j), \theta^*_{h,t}(j)\right\}$. We find,

$$P^*_{h,t}(j) \;:\; e_t \Phi^{*n}_{h,t}(j)\, y^*_{h,t}(j) = -\xi_t J_{P^*_h} \tag{A.20}$$

$$y^*_{h,t}(j) \;:\; \mu_t = e_t P^*_{h,t}(j)\, \Phi^{*n}_{h,t}(j) + \xi_t J_{y^*_h} \tag{A.21}$$

$$\theta^*_{h,t}(j) \;:\; \varphi e_t P^*_{h,t}(j)\, \Phi^{*n}_{h,t}(j)\, y^*_{h,t}(j) = \xi_t \theta^*_{h,t}(j)\, J_{\theta^*_h} \tag{A.22}$$

respectively. As all firms in the export market are identical, we can eliminate the $'j'$ in the FOC equations.

$$\begin{aligned} e_t \Phi^{*n}_{h,t} y^*_{h,t} &= -\xi_t J_{P^*_h} && \text{(A.23)} \\ \mu_t &= e_t P^*_{h,t} \Phi^{*n}_{h,t} + \xi_t J_{y^*_h} && \text{(A.24)} \\ \varphi e_t P^*_{h,t} \Phi^{*n}_{h,t} y^*_{h,t} &= \xi_t \theta^*_{h,t} J_{\theta^*_h} && \text{(A.25)} \end{aligned}$$

Recall the participation constraint in the export market from the foreign households' problem.

$$\begin{aligned} J_{P^*_h} &= -\lambda^*_t c^*_{h,t} && \text{(A.26)} \\ J_{\theta^*_h} \theta^*_{h,t} &= (1-\varphi) J_{y^*_h} y^*_{h,t} && \text{(A.27)} \\ J_{y^*_h} &= \left[u_{c^*_h}(t) - \lambda^*_t P^*_{h,t} \right] \Phi^{*s}_{h,t} s^*_{h,t} && \text{(A.28)} \end{aligned}$$

For the price equation in the export market, use (A.24), (A.25) and (A.27) to eliminate ξ_t from the firms optimal conditions,

$$\begin{aligned} \mu_t &= e_t P^*_{h,t} \Phi^{*n}_{h,t} + \xi_t J_{y^*_h} \quad \text{and} \quad \varphi e_t P^*_{h,t} \Phi^{*n}_{h,t} y^*_{h,t} = \xi_t (1-\varphi) J_{y^*_h} y^*_{h,t} \\ &\rightarrow \mu_t = e_t P^*_{h,t} \Phi^{*n}_{h,t} \left(\frac{1}{1-\varphi} \right) \end{aligned}$$

The job creation condition in the export market is:

$$\left(\frac{\kappa}{\Phi^v_t} \right) P_{h,t} = \left(\frac{1}{1-\varphi} \right) e_t P^*_{h,t} \Phi^{*n}_{h,t} a_t f'\left(l^*_{h,t} \right) - W_t + (1-\rho) \mathbb{E}_t \beta_{t,t+1} \left(\frac{\kappa}{\Phi^v_{t+1}} \right) P_{h,t+1} \tag{A.29}$$

Since wage is common in the domestic and the export markets, we know:

$$
\begin{aligned}
& \left(\frac{1}{1-\varphi}\right) P_{h,t}\Phi_{h,t}^{n}a_t f'\left(l_{h,t}\right) - \left(\frac{\kappa}{\Phi_t^v}\right)\ P_{h,t} + (1-\rho)\,\mathbb{E}_t\beta_{t,t+1}\left(\frac{\kappa}{\Phi_{t+1}^v}\right)\ P_{h,t+1} \\
= & \left(\frac{1}{1-\varphi}\right) e_t P_{h,t}^{*}\Phi_{h,t}^{*n}a_t f'\left(l_{h,t}^{*}\right) - \left(\frac{\kappa}{\Phi_t^v}\right)\ P_{h,t} + (1-\rho)\,\mathbb{E}_t\beta_{t,t+1}\left(\frac{\kappa}{\Phi_{t+1}^v}\right)\ P_{h,t+1} \\
\rightarrow & \quad e_t P_{h,t}^{*} = P_{h,t}\left(\frac{\Phi_{h,t}^{n}}{\Phi_{h,t}^{*n}}\right)\left[\frac{f'\left(l_{h,t}\right)}{f'\left(l_{h,t}^{*}\right)}\right] \qquad \text{(A.30)}
\end{aligned}
$$

If employment in each market is same, i.e. $l_{h,t} = l_{h,t}^{*}$, the price of home goods in the export market is

$$
e_t P_{h,t}^{*} = P_{h,t}\left(\frac{\Phi_{h,t}^{n}}{\Phi_{h,t}^{*n}}\right) \qquad \text{(A.31)}
$$

With an analogous process, we know the search effort in the export market is:

$$
-s_{h,t}^{*}u_{s_h^*}(t) = \varphi c_{h,t}^{*}u_{c_h^*}(t)\left[\frac{s_{h,t}^{*}\Phi_{h,t}^{*s}y_{h,t}^{*}}{\varphi c_{h,t}^{*} + (1-\varphi)\,s_{h,t}^{*}\Phi_{h,t}^{*s}y_{h,t}^{*}}\right] \qquad \text{(A.32)}
$$

A.3 Proof of Lemma 1

First, focus on the derivation of the Lagrangian multiplier λ_t. We can derive λ_t by using not only the FOCs of a home firm in the domestic market, but also the ones of a foreign export firm.

$$
\begin{aligned}
\lambda_t &= (1-\varphi)\left(\frac{u_{c_h}(t)}{P_{h,t}}\right)\left[\frac{s_{h,t}\Phi_{h,t}^{s}y_{h,t}}{\varphi c_{h,t} + (1-\varphi)\,s_{h,t}\Phi_{h,t}^{s}y_{h,t}}\right] \qquad \text{(A.33)} \\
\lambda_t &= (1-\varphi)\left(\frac{u_{c_f}(t)}{P_{f,t}}\right)\left[\frac{s_{f,t}\Phi_{f,t}^{s}y_{f,t}}{\varphi c_{f,t} + (1-\varphi)\,s_{f,t}\Phi_{f,t}^{s}y_{f,t}}\right]
\end{aligned}
$$

Using $\partial c_t / \partial c_{i,t} = P_{i,t}/P_t$ for $i = \{h, f\}$, we have

$$\lambda_t = (1-\varphi)\left(\frac{u_c(t)}{P_t}\right)\left[\frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t} + (1-\varphi)\, s_{h,t}\Phi^s_{h,t}y_{h,t}}\right] \tag{A.34}$$

$$\lambda_t = (1-\varphi)\left(\frac{u_c(t)}{P_t}\right)\left[\frac{s_{f,t}\Phi^s_{f,t}y_{f,t}}{\varphi c_{f,t} + (1-\varphi)\, s_{f,t}\Phi^s_{f,t}y_{f,t}}\right]$$

Accordingly, when defining the bracket as τ_t, we know the following relation holds:

$$\tau_t \equiv \frac{s_{h,t}\Phi^s_{h,t}y_{h,t}}{\varphi c_{h,t} + (1-\varphi)\, s_{h,t}\Phi^s_{h,t}y_{h,t}} = \frac{s_{f,t}\Phi^s_{f,t}y_{f,t}}{\varphi c_{f,t} + (1-\varphi)\, s_{f,t}\Phi^s_{f,t}y_{f,t}} \tag{A.35}$$

which implies that the ratios of the realized output to the consumption in the domestic and imported markets are equal to each other. With an analogous process, we also derive similar conditions for the foreign economy.

$$\tau^*_t \equiv \frac{s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}}{\varphi c^*_{f,t} + (1-\varphi)\, s^*_{f,t}\Phi^{*s}_{f,t}y^*_{f,t}} = \frac{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}}{\varphi c^*_{h,t} + (1-\varphi)\, s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} \tag{A.36}$$

Meanwhile, consider the inverses of τ_t and τ^*_t:

$$\frac{1}{\tau_t} = \varphi\frac{c_{h,t}}{s_{h,t}\Phi^s_{h,t}y_{h,t}} + (1-\varphi) \;\text{ and }\; \frac{1}{\tau^*_t} = \varphi\frac{c^*_{h,t}}{s^*_{h,t}\Phi^{*s}_{h,t}y^*_{h,t}} + (1-\varphi) \tag{A.37}$$

Using the shopping constraints, we can rewrite the above expressions as:

$$\frac{1}{\tau_t} = \varphi\left[1 - \frac{n_{h,t}\kappa v_{h,t}}{n_{h,t}\Phi^n_{h,t}y_{h,t}}\right] + (1-\varphi) \;\text{ and }\; \frac{1}{\tau^*_t} = \varphi\left[1 - \frac{n^*_{h,t}\kappa v^*_{h,t}}{n^*_{h,t}\Phi^{*n}_{h,t}y^*_{h,t}}\left(\frac{P_{h,t}}{e_t P^*_{h,t}}\right)\right] + (1-\varphi) \tag{A.38}$$

Since there is no impediment in the international trade, we know that $v_t = v_{h,t} = v^*_{h,t}$ and in turn $y_t = y_{h,t} = y^*_{h,t}$. Furthermore, with a single labor market in each country, the home LOP gap $(e_t P^*_{h,t}/P_{h,t})$ is equal to the ratio of probabilities that a firm match with a shopper in the domestic and export markets ($\Phi^n_{h,t}/\Phi^{*n}_{h,t}$). Therefore, we have the following relation:

$$\begin{aligned} \frac{1}{\tau_t^*} &= \varphi\left[1-\frac{\kappa v_t}{\Phi^n_{h,t} y_t}\right]+(1-\varphi)=\frac{1}{\tau_t} \\ \Rightarrow &\ \ \tau_t=\tau_t^* \end{aligned} \tag{A.39}$$

This represents that the ratio of the output and the consumption in each market is same.

A.4 Proof of Proposition 1

With a single labor market assumption, we express the LOP gap of the home country as a function of the probability that a firm is matched with a shopper.

$$\begin{aligned} \frac{e_t P^*_{h,t}}{P_{h,t}} &= \frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}} \\ &= \left[\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n^*_{h,t}}{n_{h,t}}\right)\right]^{\varphi} \end{aligned} \tag{A.40}$$

With Lemma 1, we can simplify the condition of the international risk sharing as:

$$\begin{aligned} \frac{u_{s_h^*}(t)}{u_{s_h}(t)} &= \left(\frac{e_t P^*_{h,t}}{P_{h,t}}\right)\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{c^*_{h,t}}{c_{h,t}}\right) \\ &= \left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right)\left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n_{h,t}\Phi^n_{h,t} y_t}{n^*_{h,t}\Phi^{*n}_{h,t} y_t}\right) \\ &= \left(\frac{s_{h,t}}{s^*_{h,t}}\right)\left(\frac{n_{h,t}}{n^*_{h,t}}\right) \end{aligned} \tag{A.41}$$

Plugging the condition of the international risk sharing into the LOP gap, we have the following relation.

$$\frac{e_t P^*_{h,t}}{P_{h,t}} = \left[\frac{u_{s^*_h}(t)}{u_{s_h}(t)}\right]^{\varphi} = \left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi} \tag{A.42}$$

With an analogous process, we also know the LOP gap for the foreign country as:

$$\frac{e_t P^*_{f,t}}{P_{f,t}} = \left[\frac{u_{s^*_f}(t)}{u_{s_f}(t)}\right]^{\varphi} = \left[\frac{u_{s^*}(t)}{u_s(t)}\right]^{\varphi}. \tag{A.43}$$

A.5 Proof of Proposition 3

With the static version of the model setting $\rho = 1$, we can derive the equilibrium condition in the labor market.

$$\begin{aligned} \frac{\kappa}{\Phi^v_t} &= \left(\frac{1-\alpha}{1-\varphi}\right)\Phi^n_{h,t}a_t \text{ where } \Phi^v_t = \chi v_t^{-\phi} \\ \rightarrow \quad v_t &= \left[\left(\frac{\chi}{\kappa}\right)\left(\frac{1-\alpha}{1-\varphi}\right)\Phi^n_{h,t}a_t\right]^{1/\phi} \end{aligned} \tag{A.44}$$

By Lemma 1, we can express the consumption for home goods in the domestic market as:

$$c_{h,t} = k n_{h,t}\Phi^n_{h,t}y_t, \tag{A.45}$$

where k is a constant.

Plugging Equation (A.44) into Equation (A.45), we have

$$
\begin{aligned}
c_{h,t} &= k n_{h,t} \Phi_{h,t}^{n} y_t \\
&= k n_{h,t} \Phi_{h,t}^{n} a_t l_t \\
&= k n_{h,t} \Phi_{h,t}^{n} a_t \left(\chi v_t^{1-\phi} \right) \\
&= k \chi n_{h,t} \Phi_{h,t}^{n} a_t \left(v_t^{1-\phi} \right) \\
&\to \; c_{h,t} = k\chi \left[\left(\frac{\chi}{\kappa} \right) \left(\frac{1-\alpha}{1-\varphi} \right) \right]^{\frac{1-\phi}{\phi}} n_{h,t} \left(\Phi_{h,t}^{n} a_t \right)^{\frac{1}{\phi}}
\end{aligned} \tag{A.46}
$$

Log-linearizing Equation (A.46), we derive

$$
\widehat{c_{h,t}} = \widehat{n_{h,t}} + \frac{1}{\phi} \widehat{\Phi_{h,t}^{n}} + \frac{1}{\phi} \widehat{a}_t, \tag{A.47}
$$

where ˆ denotes the log deviation from the steady state.

With the GHH preferences and conditions for optimal search efforts, we write the matching probability in terms of consumption and mass of firms:

$$
\begin{aligned}
\widehat{\Phi_{h,t}^{n}} &= \varphi \left(\widehat{s_{h,t}} - \widehat{n_{h,t}} \right) \\
&\to \; \widehat{\Phi_{h,t}^{n}} = \varphi \left(\frac{1}{1+\eta} \widehat{c}_t - \widehat{n_{h,t}} \right).
\end{aligned} \tag{A.48}
$$

Plugging Equation (A.48) into Equation (A.47), we calculate

$$
\begin{aligned}
\widehat{c_{h,t}} &= \widehat{n_{h,t}} + \frac{1}{\phi} \widehat{\Phi_{h,t}^{n}} + \frac{1}{\phi} \widehat{a}_t \\
&= \widehat{n_{h,t}} + \frac{\varphi}{\phi} \left(\frac{\eta}{1+\eta} \widehat{c}_t - \widehat{n_{h,t}} \right) + \frac{1}{\phi} \widehat{a}_t \\
&\to \; \widehat{a}_t = (\varphi - \phi) \widehat{n_{h,t}} + \phi \widehat{c_{h,t}} - \frac{\varphi\eta}{1+\eta} \widehat{c}_t.
\end{aligned} \tag{A.49}
$$

With an analogous process, we obtain

$$\widehat{a_t^*} = (\varphi - \phi)\,\widehat{n_{f,t}^*} + \phi\widehat{c_{f,t}^*} - \frac{\varphi\eta}{1+\eta}\widehat{c_t^*}, \tag{A.50}$$

for the foreign country.

Subtracting Equation (A.49) into Equation (A.50), we have

$$\widehat{a_t} - \widehat{a_t^*} = (\varphi - \phi)\left(\widehat{n_{h,t}} - \widehat{n_{f,t}^*}\right) + \phi\left(\widehat{c_{h,t}} - \widehat{c_{f,t}^*}\right) - \frac{\varphi\eta}{1+\eta}\left(\widehat{c_t} - \widehat{c_t^*}\right). \tag{A.51}$$

To understand the relationship between productivity shocks and aggregate consumption gaps, we need to express $\left(\widehat{n_{h,t}} - \widehat{n_{f,t}^*}\right)$ and $\left(\widehat{c_{h,t}} - \widehat{c_{f,t}^*}\right)$ in terms of $\left(\widehat{c_t} - \widehat{c_t^*}\right)$.

(1) $\widehat{n_{h,t}} - \widehat{n_{f,t}^*}$

Using the international risk sharing and the property of CES aggregator, we express the LOP gap of home goods as:

$$\begin{aligned}
\frac{u_{c_f^*}(t)}{u_{c_h}(t)} &= \frac{e_t P_{f,t}^*}{P_{h,t}} \\
&= \frac{e_t P_{h,t}^*}{P_{h,t}}\frac{P_{f,t}^*}{P_{h,t}^*} \\
&= \frac{e_t P_{h,t}^*}{P_{h,t}}\left(\frac{\partial c_t^*/\partial c_{f,t}^*}{\partial c_t^*/\partial c_{h,t}^*}\right) \\
&\rightarrow \frac{e_t P_{h,t}^*}{P_{h,t}} = \frac{u_{c_h^*}(t)}{u_{c_h}(t)}.
\end{aligned} \tag{A.52}$$

With the functional form, we have

$$\begin{aligned}
\frac{\Phi_{h,t}^n}{\Phi_{h,t}^{*n}} &= \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_{h,t}}{c_{h,t}^*}\right)\left(\frac{c_t}{c_t^*}\right)^{\sigma-1} \\
&\rightarrow \left(\frac{c_t}{c_t^*}\right)^{1-\sigma} = \left(\frac{1-\omega}{\omega}\right)\frac{n_{h,t}}{n_{h,t}^*}.
\end{aligned} \tag{A.53}$$

Log-linearizing Equation (A.53), we express the aggregate consumption in terms of the mass of firms:

$$\begin{aligned}
\widehat{c_t} - \widehat{c_t^*} &= \frac{1}{1-\sigma}\left(\widehat{n_{h,t}} - \widehat{n_{h,t}^*}\right) \\
&= \frac{1}{(1-\sigma)(1-n_h)}\widehat{n_{h,t}} \\
&= \frac{1}{(1-\sigma)(1-\omega)}\widehat{n_{h,t}}. \qquad \text{(A.54)}
\end{aligned}$$

where a variable without subscript 't' denotes the steady state value. Note that n_h is equal to ω due to the assumption of symmetric economies.

For the foreign country, we also have

$$\begin{aligned}
\widehat{c_t} - \widehat{c_t^*} &= \frac{1}{1-\sigma}\left(\widehat{n_{f,t}} - \widehat{n_{f,t}^*}\right) \\
&= -\frac{1}{(1-\sigma)\left(1-n_f^*\right)}\widehat{n_{f,t}^*} \\
&= -\frac{1}{(1-\sigma)(1-\omega)}\widehat{n_{f,t}^*}. \qquad \text{(A.55)}
\end{aligned}$$

Thus, we derive

$$\widehat{n_{h,t}} - \widehat{n_{f,t}^*} = 2(1-\sigma)(1-\omega)\left(\widehat{c_t} - \widehat{c_t^*}\right). \qquad \text{(A.56)}$$

(2) $\widehat{c_{h,t}} - \widehat{c_{f,t}^*}$

Log-linearizing the CD aggregator (z=1) for consumption and using the relation of consumptions at the steady state, we calculate

$$\widehat{c_t} = \omega\widehat{c_{h,t}} + (1-\omega)\widehat{c_{f,t}} \qquad \text{(A.57)}$$

$$\widehat{c_t^*} = \omega\widehat{c_{f,t}^*} + (1-\omega)\widehat{c_{h,t}^*}. \qquad \text{(A.58)}$$

Meanwhile, by Lemma 1, we express the the relative consumption for home goods in the domestic and the export markets as:

$$\frac{c_{h,t}}{c_{h,t}^{*}}=\left(\frac{n_{h,t}\Phi_{h,t}^{n}}{n_{h,t}^{*}\Phi_{h,t}^{*n}}\right)=\left(\frac{s_{h,t}}{s_{h,t}^{*}}\right)^{\varphi}\left(\frac{n_{h,t}}{n_{h,t}^{*}}\right)^{1-\varphi}. \tag{A.59}$$

Log-linearizing Equation (A.59), we derive

$$\widehat{c_{h,t}}-\widehat{c_{h,t}^{*}}=(1-\varphi)\left(\widehat{n_{h,t}}-\widehat{n_{h,t}^{*}}\right)+\frac{\varphi\eta}{1+\eta}\left(\widehat{c_t}-\widehat{c_t^{*}}\right). \tag{A.60}$$

With an analogous process, we obtain

$$\widehat{c_{f,t}^{*}}-\widehat{c_{f,t}}=(1-\varphi)\left(\widehat{n_{f,t}^{*}}-\widehat{n_{f,t}}\right)-\frac{\varphi\eta}{1+\eta}\left(\widehat{c_t}-\widehat{c_t^{*}}\right). \tag{A.61}$$

With Equation (A.57), (A.58), (A.60), and (A.61), we have

$$\begin{aligned}\widehat{c_{h,t}}-\widehat{c_{f,t}^{*}} \quad = \quad & \frac{(1+\eta)-2\varphi\eta(1-\omega)}{(2\omega-1)(1+\eta)}\left(\widehat{c_t}-\widehat{c_t^{*}}\right) \\ & -\frac{(1-\varphi)(1-\omega)}{2\omega-1}\left(\widehat{n_{h,t}}-\widehat{n_{h,t}^{*}}-\widehat{n_{f,t}^{*}}+\widehat{n_{f,t}}\right).\end{aligned} \tag{A.62}$$

Using Equation (A.56),

$$\begin{aligned}& \widehat{c_{h,t}}-\widehat{c_{f,t}^{*}} \\ = \quad & \frac{(1+\eta)-2\varphi\eta(1-\omega)-2(1-\varphi)(1-\sigma)(1-\omega)(1+\eta)}{(2\omega-1)(1+\eta)}\left(\widehat{c_t}-\widehat{c_t^{*}}\right).\end{aligned} \tag{A.63}$$

(3) $\widehat{a_t}-\widehat{a_t^{*}}$

Plug Equation (A.56) and (A.63) into Equation (A.51),

$$\widehat{a}_t - \widehat{a}_t^* = \frac{\zeta}{(2\omega - 1)(1+\eta)} \left(\widehat{c}_t - \widehat{c}_t^*\right), \tag{A.64}$$

where

$$\begin{aligned} \zeta &= 2(\sigma - 1)(1-\omega)(1+\eta)\left[\varphi(1-\phi) + 2\omega(\phi - \varphi)\right] \\ &\quad + \phi(1+\eta) - \varphi\eta\left[2\omega(1-\phi) + 2\phi - 1\right]. \end{aligned}$$

A.6 Static model

Table A.1 reports the main equations of static model, assuming $\rho = 1$.

Table A.1: Static Model

	Home country	Foreign country
Mass of firms	$1 = n_{h,t} + n^*_{h,t}$	$1 = n^*_{f,t} + n_{f,t}$
Unemployment	$u_t = 1$	$u^*_t = 1$
Employment	$l_t = \Phi^v_t v_t$	$l^*_t = \Phi^{*v}_t v^*_t$
Production	$y_t = z_t l_t$	$y^*_t = z^*_t l^*_t$
Wage	$W_t = \alpha \left(\frac{1}{1-\varphi}\right) p_{h,t} \Phi^n_{h,t} z_t$	$W^*_t = \alpha \left(\frac{1}{1-\varphi}\right) p^*_{f,t} \Phi^{*n}_{f,t} z^*_t$
Job creation	$\left(\frac{\kappa p_{h,t}}{\Phi^v_t}\right) = \left(\frac{1}{1-\varphi}\right) p_{h,t} \Phi^n_{h,t} z_t - W_t$	$\left(\frac{\kappa p^*_{f,t}}{\Phi^{*v}_t}\right) = \left(\frac{1}{1-\varphi}\right) p^*_{f,t} \Phi^{*n}_{f,t} z^*_t - W^*_t$
Export price	$e_t p^*_{h,t} = \left(\frac{\Phi^n_{h,t}}{\Phi^{*n}_{h,t}}\right) p_{h,t}$	$\frac{1}{e_t} p_{f,t} = \left(\frac{\Phi^{*n}_{f,t}}{\Phi^n_{f,t}}\right) p^*_{f,t}$
Int'l relative prices	$\frac{P_{h,t}}{P_{f,t}} = \left(\frac{\omega}{1-\omega}\right) \frac{c_{f,t}}{c_{h,t}}$	$\frac{p^*_{f,t}}{p^*_{h,t}} = \left(\frac{\omega}{1-\omega}\right) \frac{c^*_{h,t}}{c^*_{f,t}}$
Search effort	$s_{h,t} = \varphi c_{h,t} \left[\frac{u_{c_h}(t)}{-u_{s_h}(t)}\right] \tau_t$	$s^*_{f,t} = \varphi c^*_{f,t} \left[\frac{u_{c^*_f}(t)}{-u_{s^*_f}(t)}\right] \tau_t$
	$s_{f,t} = \varphi c_{f,t} \left[\frac{u_{c_f}(t)}{-u_{s_f}(t)}\right] \tau_t$	$s^*_{h,t} = \varphi c^*_{h,t} \left[\frac{u_{c^*_h}(t)}{-u_{s^*_h}(t)}\right] \tau_t$
Shopping	$c_{h,t} = n_{h,t} \Phi^n_{h,t} y_t - n_{h,t} \kappa v_t$	$c^*_{f,t} = n^*_{f,t} \Phi^{*n}_{f,t} y^*_t - n^*_{f,t} \kappa v^*_t$
	$c_{f,t} = n_{f,t} \Phi^n_{f,t} y^*_t - n_{f,t} \kappa v^*_t \left(\frac{e_t P^*_{f,t}}{P_{f,t}}\right)$	$c^*_{h,t} = n^*_{h,t} \Phi^{*n}_{h,t} y_t - n^*_{h,t} \kappa v_t \left(\frac{P_{h,t}}{e_t P^*_{h,t}}\right)$
Risk sharing	$\frac{u_{c^*_f}(t)}{u_{c_h}(t)} = \frac{e_t P^*_{f,t}}{P_{h,t}}$	