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Can a Grain of Patience Trigger Cooperation? The Role of an Outside Option*

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Abstract

Cooperation in joint ventures is widespread, despite its vulnerability to defection. It can emerge when the interaction is repeated and agents are patient enough to prefer the benefits of future cooperation over the short-term gains from defection. Thus, if a large fraction of the population consists of impatient exploiters who always defect and agents are randomly paired to play a repeated prisoner dilemma game, patient agents defect as well, and society is in a no-cooperation trap. We show that the existence of an outside option can break this trap even if the fraction of patient agents is arbitrarily small. Impatient agents self-select out of the game, allowing patient agents to cooperate. Patience thus has an evolutionary advantage, leading to widespread cooperation.

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1 Introduction

Cooperation in joint ventures is widespread, despite its vulnerability to exploitation. As the game theory literature shows, cooperation can emerge when the interaction is repeated and agents are patient enough to prefer the future benefits of cooperation over the short-term gains from defection (e.g., Friedman 1971, Fudenberg and Maskin 1986).

It is more challenging to explain cooperation when the population includes many impatient agents, who prefer the short-term gain from exploiting their partners over the future benefits from cooperation. In this scenario, patient agents, fearing exploitation by impatient partners, may also refrain from cooperation. In particular, if agents are randomly matched to play a repeated Prisoner's Dilemma (PD) game and the proportion of patient agents is too low, an equilibrium with cooperation does not exist.

What, then, could explain cooperation among patient agents in an environment with many impatient exploiters? We show that the presence of an outside option (such as home production), that renders participation in the game voluntary, can facilitate cooperation under these conditions.

In our model, agents are randomly paired to play an infinitely repeated PD game, with actions "cooperate" and "defect". There are two types of agents: patient and impatient. Impatient agents are purely present-oriented (discount factor $\delta = 0$) and always defect. Patient agents care about the future (discount factor $\delta > 0$). We introduce an outside option with a payoff that exceeds mutual defection but falls short of mutual cooperation. Agents can settle for the outside option instead of joining the repeated game. If they join the repeated game, they can leave it for the outside option at any future stage (in which case their partner also leaves the game).[1]

Our key result is: in the absence of an outside option, for any discount factor of the patient agents, δ, no matter how close it is to 1, there exists a threshold of the fraction of the patient agents in the population, below which a cooperative equilibrium does not exist. All agents always defect in the game.[2] In contrast, if an outside option is available, there exists a $\delta < 1$, above which a cooperative equilibrium exists for any arbitrary small fraction

[1] To keep the model simple and consistent with the canonical repeated games literature, agents who exit a partnership cannot rematch and form a new partnership. This assumption has no qualitative impact on our main results regarding the role of the outside option in supporting cooperation – see the discussion of the literature on voluntary separation and rematching below.

[2] Note that a nonstationary equilibrium, as in Lehrer and Pauzner (1999) and Lehrer and Yariv (1999), that takes advantage of the different discount factors is impossible here since the impatient agents are completely impatient.

of the patient agents in the population. In that equilibrium, a sufficient number of impatient agents self-select out of the game and all the patient agents join the game and cooperate.

To understand this result, consider an impatient agent choosing to either take the outside option or enter the PD game and defect. The benefit of joining the game is increasing in the probability of being paired with a patient cooperator. If this probability is zero, the outcome of entering the game is mutual defection, and the outside option is preferred. If the probability is one, it is preferable to enter the game and exploit the partner. Thus, there is a threshold probability of being paired with a patient cooperator where the impatient agent is indifferent between the two options.

Patient agents are also indifferent between the outside option and defecting in the game at that probability threshold: the time preference does not matter, as these two alternatives differ only in their first-period payoff (after defection the partnership dissolves and agents receive the outside option payoff in all future periods). However, patient agents have a third viable alternative – to cooperate in the game, and cooperation is strictly preferred at that probability threshold (and above it) if their patience level is above some δ.

Assume now that the discount factor of the patient agents is above that δ and that they all enter the game and cooperate. The number of impatient agents who enter the game (and defect) cannot push the probability of being paired with a patient agent below the impatient agents' indifference threshold, as otherwise, they would strictly prefer to leave the game. As the impatient agents weakly prefer to enter the game, the patient agents indeed strictly prefer to enter and cooperate. Thus, with a sufficiently large discount factor, there is a cooperative equilibrium regardless of how small the fraction of the patient agents is. The existence of the outside option puts a lower bound on the probability that a partner is patient, by allowing the impatient agents to self-select out of the game.

Intuitively, patient agents enter the game because it offers the opportunity for long-term cooperation. Impatient agents enter the game because it offers the opportunity to exploit a patient partner. For both, there is a short-term risk of being exploited upon entering the game. But while for the impatient agent this cost is material, for the (sufficiently) patient agent a one-period loss is negligible. That is, for patient agents, the prospect of long-run cooperation mitigates the cost, but for impatient agents, the short term is all that matters.

From an evolutionary (either cultural or genetic) perspective, with no outside option, if there are only a few patient agents, they do not cooperate and have no fitness advantage. Society is thus trapped in a no-cooperation equilibrium. With an outside option, even an arbitrarily small number of patient agents – a grain – who appear for example by mutation, would cooperate. Their higher payoffs generate an evolutionary advantage and from this grain grows large-scale patience and cooperation. In other words, without an outside option, there are two evolutionary stable steady states – one with few patient agents and no cooperation, and one in which all the agents are patient and cooperate. An outside option eliminates the former.

To formally address the question of the coevolution of patience and cooperation, we include in the model many generations and fertility which is increasing in income, where offspring are of the same patience type as their parents. At the end of each period, agents die with some small, exogenous, probability. We set the discount factor of the patient agents to coincide with the survival rate. In each period, the newborn agents choose between the outside option and entering the game with a random partner. We show that the proportion of patient agents in the population grows over time, even though impatient agents have the fitness advantage in the short run.

We further extend our analysis to the effect of an outside option on cooperation for the entire range of population shares – not just for a grain of patient agents. The preceding results extend: if the proportion of patient agents is not too large, the outside option reduces the discount factor required to sustain cooperation. However, for a sufficiently large proportion, the order is reversed: cooperation requires a higher discount factor with the outside option. This reversal highlights the two opposite effects of the outside option. In addition to enhancing cooperation by attracting impatient agents out of the game – an effect that is crucial when there are many impatient agents – the outside option limits the harshness of punishments of defectors within the game, making cooperation among patient agents harder to support. When there are many patient agents, punishing defectors becomes the binding constraint and the outside option harms cooperation.

It is also worth noting that even if the fraction of patient agents is sufficiently large such that a cooperative equilibrium exists without an outside option, the outside option can increase cooperation. This is the case when the outside option payoff exceeds the payoff that impatient agents obtain in the game without the outside option. Introducing the outside option induces some of the impatient agents to stay out of the game, allowing for more long-run cooperation in society and higher payoffs to all agents.

The cooperative equilibrium discussed so far (all the patient agents join the venture and cooperate while impatient agents mix between the outside option and defecting in the game) is not the only cooperative equilibrium that exists due to the outside option. Another perfect Bayesian equilibrium is a "screening" one: all the patient agents join the venture, defect in the first round of the game, and cooperate in all following rounds (using standard punishment strategies). All the impatient agents choose the outside option. The defection in the first round deters the impatient agents from entering the game and does not deter the (sufficiently) patient agents.[3] However, this equilibrium is not renegotiation-proof: once two agents meet to play the repeated game, there is common certainty among them that they are both patient. They can thus renegotiate to cooperate in the first round as well, but then the impatient agents are not deterred from entering. We therefore focus on the equilibrium in which all patient agents cooperate in the first round of the game, which (as we show) is renegotiation-proof.

Our predictions have several applications. One is that it can explain how many societies have achieved large-scale repeated cooperation among strangers. Our model shows that with an outside option, initial widespread patience is not a prerequisite for cooperation. Patience and cooperation could have coevolved, starting from a grain of patient agents. In contrast, the message of conventional models of repeated PD games, which abstract from the outside option, is that having a sufficient proportion of patient agents is a prerequisite for cooperation. Otherwise, the unique equilibrium is defection by all agents.[4] In such models, a mutation that introduces a small fraction of patient agents will not allow for cooperation and the patient agents will not gain an evolutionary advantage. Thus, the emergence of large-scale patience must be explained by an independent mechanism (such as benefits to patience in agriculture – see e.g. Galor and Ozak 2016).

Our model also identifies a property of the technology for production with strangers that is crucial for explaining the emergence of patience and cooperation. Consider the stage in which humanity lived in small, economically isolated tribes. The outside option in our model corresponds to production alone or with other tribe members (where immediate social punishments can prevent exploitation). A PD-like technology for repeated joint production with strangers is introduced, with a payoff from mutual cooperation that is higher than the tribal option. Mutual cooperation in our model corresponds to adopting the technology and forming proto-states with large-scale cooperation with non-tribe members. Our model suggests that the new technology will enable the transition only if the payoff from mutual defection is lower than that of the outside option. If the technology's payoff from mutual defection exceeds that of the outside option, cooperation will not emerge – agents would form

[3]This first-round defection resembles a "trust-building" phase in the literature on voluntary separation and rematching games, discussed below.

[4]More precisely, defection by all agents is the unique equilibrium under the reasonable assumption that the types of randomly matched partners are their private information.

joint ventures but defect in them, and evolution will not reward patience.

Our model can also be used to study differences between countries in cooperation among strangers. Some papers explain such differences using multiple steady states (e.g., Tabellini 2008, Anderlini and Terlizzese 2017). Such models provide an explanation for the persistence of cooperation and wealth of nations. However, since the explanation of differences today is based on the existence of these differences in the past, they do not provide any testable predictions that point to an exogenous source of variation. In contrast, our results – a unique long-run evolutionary equilibrium – can provide testable implications. This means that current levels of cooperation in a country could be traced to differences in exogenous factors – the payoffs of the cooperative technology and of the outside option. Alternatively, a country with a low level of cooperation could be in the early stages of transition to a cooperative equilibrium, and this could be traced to different historical developments such as the timing of the transition to agriculture or the emergence of states (which is affected by exogenous geographical factors; see e.g., Mayshar et al., 2022).

Moreover, according to models of multiple equilibria with no outside option, poor countries have low levels of cooperation because most agents defect in joint ventures. With an outside option, cooperation in poor countries is low because most agents avoid joint ventures altogether. This is consistent with the fact that in poor countries the majority of men and women are self-employed, whereas in developed countries the vast majority of workers are employed by non-family members (or are employers).

Another implication of the model relates to the effect of states' capacity to provide law and order. An immediate observation is that if there is no (relevant) outside option, law enforcement can assist cooperation in repeated joint ventures only by affecting the behavior of exploiters within the game – incentivizing them to cooperate. With an outside option, law enforcement can increase cooperation by pushing exploiters out of the venture. Moreover, with an outside option, external punishments that reduce the exploiters' gains from defection increase cooperation gradually in the range where there are only a few patient cooperators and the impatient agents play mixed strategies. Any punishment that reduces the gains from defection would decrease the fraction of defectors in the game, increasing cooperation. Without an outside option, there is no effect on cooperation unless punishment is sufficiently severe to change the PD nature of the game, regardless of the proportion of exploiters in society.

These observations may contribute to the study of how the emergence of states, which provide law and order, has affected current development and cooperation (see e.g. Putterman and Weil 2010). If a relevant outside option was available (i.e. tribe production was better than the mutual defection payoff when producing with strangers), then modest punishments might have been sufficient to trigger an evolutionary process that led to the current large-scale cooperation. Without a sufficiently productive outside option, only a "strong" state, that could change the payoffs of the game so that it is not PD-like, could have triggered cooperation.

There are many attempts to explain cooperation in PD-like situations. The canonical game theory explanation is that cooperation emerges in repeated situations with patient players (the "folk theorems"). This explanation, however, is vulnerable to the introduction of heterogeneity in the patience of agents. That is, if agents are randomly matched to play a repeated PD game with anonymous partners and a large proportion of the population is impatient, cooperation may stop being an equilibrium. Our contribution is to highlight the role of an outside option in restoring cooperation even when there are many impatient agents.

There are several papers worth mentioning that have addressed the role of voluntary participation in supporting cooperation, even though their focus is not on repeated interactions (and thus do not address the issue of whether a small patient population can cooperate). These papers introduce assumptions such as false beliefs, observability of the opponent's type, or situations in which cooperation is sometimes the dominant strategy. In our model, in contrast, an outside option supports cooperation even though cooperation is dominated in the short run, agents are fully rational, and they are randomly matched with anonymous partners, who reveal their type only by their actions.

Frank (1988) assumes that agents cannot hide their emotions that indicate their intentions, and thus defector and cooperator types can be identified, even if not perfectly. This allows agents to settle for the outside option when interacting with agents who are likely to be exploiters. In contrast, in our model, agents are observably identical, and types are revealed solely from their actions, by Bayesian updating.

Orbell and Dawes (1991) and Macy and Skvoretz (1998) assume that individuals tend to project their cooperative attitudes onto others (the "false consensus bias" of Ross et al. (1977)). As a result, cooperators are optimistic about the prospects for cooperation, so they participate in the game, whereas defectors are pessimistic and avoid participation, and thus the outside option supports cooperation. Such beliefs are inconsistent with evolution, as mutants in the form of defector types with correct beliefs would succeed and prosper. Our model, in contrast, allows for cooperation under full rationality, i.e. with beliefs that are consistent with reality. Moreover, in an evolutionary setting, the patient (cooperator) types have higher income and prosper.

Hauert et al. (2002) introduce an outside option in an evolutionary public good game, showing that it supports some cooperation. However, their game is not PD-like: defection is not always dominant. Rather, when the number of participants in the public good game is small, cooperation is by assumption the dominant action. The role of the outside option is then to reduce the number of effective participants. Thus, unlike our model, their theory does not explain how cooperation can prevail and prosper when the short-term incentive is always to defect.[5]

In our model, rematching is not allowed. When a partnership is dissolved, agents are forced to settle for the outside option for the rest of their lives. We make this assumption to keep the model simple and to make it comparable with the canonical repeated games literature by making minimal modifications. Adding the option to rematch would not affect the qualitative nature of our main result, that the outside option is critical for cooperation when there are many impatient agents. It does, however, lead to restrictions on the degree of cooperation that are inherent in such models.

The literature on infinitely repeated games with voluntary separation and rematching highlights the fact that if cooperation at the outset of the interaction were full, defection would bear no punishment: a defector would immediately be matched to a new agent who cooperates for sure. The literature thus points to alternative discipline devices, in the form of reduced or delayed cooperation when rematching. This could be unemployment before a match with a new employer (Shapiro and Stiglitz 1984), gradual cooperation at the beginning of a partnership (Datta 1996, Kranton 1996), a testing phase when some agents are myopic (Ghosh and Ray 1996), gift exchange (Carmichael and MacLeod 1997), or an asymmetric equilibrium in which some players cooperate and others defect (Fujiwara-Greve and Okuno-Fujiwara 2009, and Fujiwara-Greve, Okuno-Fujiwara and Suzuki 2015).[6]

[5]Specifically, their model has three types: cooperators and defectors, who participate in a public good game, and 'loners' who take the outside option. In each round, a random sample with a fixed number of individuals is picked to play the game. Since the loners do not play, the number of agents who effectively participate in the public good game is random. This leads to cycles: when there are many loners, few agents play the game and cooperation is dominant. The number of cooperators in the population thus grows, increasing the number of effective players. But with a large number of participants in the public good game, defection becomes dominant. As a result, the number of defectors grows, making the loner strategy best, so loners prosper, and so forth.

[6]Ghosh and Ray (1996) have patient and myopic types as in our model (but no outside option). Their focus, however, is on whether cooperation in a game with rematching can be sustained in a renegotiation-proof way – as the required delay in cooperation is not renegotiation-proof without myopic types.

Even though we do not allow for rematching, the outside option does allow for voluntary separation, and thus also limits the punishment of defections within a partnership. More patience is therefore required to maintain cooperation than in the absence of the outside option. Adding rematching in our model would set a limit to cooperation. If the probability of being matched with a patient cooperator were one, patient agents in the game would have insufficient incentive to cooperate. Thus, if the proportion of patient agents is large, in equilibrium they mix and sometimes defect in the first round – this reduces the payoff in a rematch and restores the incentive to cooperate in the game. As a result, even a long evolutionary process cannot lead to full cooperation.

2 Model

There are infinitely many discrete periods. In the first period, $t = 0$, a population of agents with measure 1 is born. Each agent survives from period t to $t+1$ with probability $\delta \in (0, 1)$; the $1 - \delta$ agents who die are replaced by $1 - \delta$ newborn agents, so the population remains 1. There are two types of agents, "patient" (P) and "impatient" (I). The patient agents' discount factor equals the survival rate δ. Thus, their period-t_0 discounted utility from payoff stream $c_{t_0}, c_{t_0+1}, \ldots$ is

$$u_{t_0}\left(c_{t_0}, c_{t_0+1}, \ldots\right) = (1-\delta)\sum_{t=t_0}^{\infty}\delta^{t-t_0}c_t.$$

Impatient agents care only about the current period payoff (discount factor 0).[7] All agents are risk-neutral. We denote the proportion of patient agents among the period-t newborn by λ_t. The proportion of patient agents in the whole period-t population is thus $\mu_t = \delta\mu_{t-1} + (1-\delta)\lambda_t$ for all $t \geq 1$.

2.1 The joint venture

Agents in the first period of their life ("newborn") could be randomly matched with a partner for a joint venture, which is modeled as an infinitely repeated prisoner's dilemma (PD) game with stage payoffs:

[7] For convenience, we normalize the patient agents' discounted utility functions such that the sum of weights over all periods is 1. Thus, the discounted utility from a fixed payoff stream $c_t = c$ is c.

	C	D
C	x, x	$0, x+y$
D	$x+y, 0$	$1, 1$

Assumption 1 (Game is PD): Mutual cooperation (CC) is better than mutual defection (DD), and defection is strictly dominant: $x > 1$ and $y > 0$. Moreover, mutual cooperation is socially beneficial: $x > y$.

Remark 1: For simplicity of exposition and computations, we assume that the events of death for partners are fully correlated, i.e., with probability δ both survive and with probability $1-\delta$ both die. This assumption has no qualitative effect on the results.

We study the effect of introducing an outside option, in which an agent can obtain a payoff z per period. When there is an outside option, agents can opt for it in each period. That is, they are not obliged to join the venture, and once in the venture they can always leave it. We also assume, for simplicity, that only newborn agents have the option to join the venture.[8]

Assumption 2 (Non-trivial outside option): The outside option payoff is lower than that of mutual cooperation and exceeds that of mutual defection: $z \in (1, x)$.

2.2 Evolution

Agents have offspring of their own type. Their reproductive success increases in income: an agent's expected number of offspring in period $t+1$ is proportional to his period-t income plus some fixed $\omega > 0$ (ω reduces the effect of income on fitness and thus slows the evolutionary process). Thus, in period $t+1$, the proportion of patient agents among the $1-\delta$ newborn is $\lambda_{t+1} = \frac{\omega + W_t^P}{\omega + W_t}\mu_t$ (or equivalently $\frac{\lambda_{t+1}}{1-\lambda_{t+1}} = \frac{\omega + W_t^P}{\omega + W_t^I}\frac{\mu_t}{1-\mu_t}$), where W_t^I, W_t^P and W_t are the average incomes among the impatient, patient, and full population. We assume that the initial population composition has agents of both types: $\lambda_0 \in (0, 1)$.

[8] That agents cannot be rematched after they leave a venture affects the degree of possible cooperation but does not change our qualitative results – see the discussion in the introduction of the literature on voluntary separation. That agents cannot dwell in the outside option and then join a venture is only a technical assumption with no qualitative implications.

3 Benchmark: No outside option

With no outside option, all the agents born in period t are randomly matched to play the repeated game, and each pair stays together until death. Impatient agents, who care only about current payoffs, always defect. We now show that the patient agents also defect if their fraction in the population is small. In that case, since patient and impatient agents play the same actions and have the same payoffs, evolutionary forces do not change the population composition and the society is in a no-cooperation trap. In particular, a mutation that introduces a very small fraction of patient agents into an impatient society will not start an evolutionary process that will increase the patient agent's share over time. Formally:

Proposition 1 *(No-cooperation trap) In any equilibrium of the model with no outside option:*

1. *For any $\delta < 1$ there exists $\lambda(x, y, \delta) > 0$ such that if $\lambda_t < \lambda(x, y, \delta)$ then all agents born in period t always defect.*
2. *Consequently, if $\lambda_0 < \lambda(x, y, \delta)$ then $\lambda_t = \lambda_0$ for all $t \geq 0$.*

The proof is in the appendix.

4 Outside option

We augment the model with the outside option. If an agent opts for the outside option, their payoff will be $z \in (1, x)$ from that period on (and if they are already matched, their partner is forced into the outside option too).

Our main result is that if δ is sufficiently large, there exists an equilibrium in which patient agents cooperate, no matter how small their number. That is, there exists a threshold $\overline{\delta}$, that depends only on the game's payoffs x, y and on the outside option's payoff z, such that if δ is above $\overline{\delta}$ a cooperative equilibrium exists for any $\lambda_t > 0$. This equilibrium is also renegotiation proof, in the sense that for two agents in a partnership the continuation payoffs at each stage – on and off the equilibrium path – are always Pareto dominant (i.e. there is no profitable joint deviation, regardless of whether the deviation is an equilibrium).[9] Formally:

[9]Note that the two partners' renegotiation proofness concept used here does not require robustness against joint deviations by the entire population. Indeed, the equilibrium described below may be Pareto-dominated by other equilibria. However, none of them is partnership-renegotiation-proof.

Proposition 2 *If $\delta > \overline{\delta}(x,y,z)$, where $\overline{\delta}(x,y,z) \equiv \max\left(\frac{y+\frac{x-z}{z}}{y+(x-z)}, \frac{y}{x}, \frac{z}{x}\right) < 1$, then there exists a Perfect Bayesian Equilibrium, which is also Renegotiation Proof, in which all of the patient agents opt for the joint venture and always play C.*

The **equilibrium strategies** are as follows.

- An impatient agent joins the PD game (i.e., joins the pool of agents who are randomly paired to play the game, rather than opting for the outside option) with probability $\frac{\lambda_t(1-q^I)}{q^I(1-\lambda_t)} \in (0,1)$ if $\lambda_t < q^I$ and probability 1 if $\lambda_t \geq q^I$, where $q^I = \frac{z-1}{x+y-1} \in (0,1)$; in the game the agent always plays D.[10]

- A patient agent joins the PD game with probability 1 and plays C in the first round. Then, if one of the partners played D in the first round the agent leaves the game. If both played C the agent remains in the game.

- If the game continues to the second round (implying, by Bayesian updating, that both partners are patient), the two partners begin a cooperative phase, where both play C every period. If a single partner i deviates to D, they switch to a "punishment phase" for i. In this phase partner i plays C and the other partner plays D; play remains in this phase until the first time partner i indeed plays C, at which point play returns to the cooperative phase.[11]

The proof that these strategies constitute a Renegotiation-Proof PBE is in the appendix.

The proposition requires that the discount factor δ be larger than $\delta_1 = \frac{y+\frac{x-z}{z}}{y+(x-z)}$, $\delta_2 = \frac{y}{x}$ and $\delta_3 = \frac{z}{x}$. $\delta > \delta_1$ guarantees that patient agents strictly prefer to join the PD game when the impatient agents are indifferent. $\delta > \delta_2$ guarantees that one round of punishment is sufficient to maintain cooperation between patient agents in the cooperative phase of the game . $\delta > \delta_3$ guarantees that a one-round punishment is not too severe so that a defector agrees to bear the punishment phase rather than quit for the outside option.

The condition $\delta > \delta_2$ is not crucial and is introduced to simplify the equilibrium strategies above. It can be removed at the cost of a more complex punishment phase, that further reduces the defector's continuation payoff. Specifically, we can set the length of the punishment

[10] q^I is the probability that the partner plays C in the first period, which makes an impatient agent indifferent about whether to join the game.

[11] This part follows the "penance" strategies as described by Fudenberg and Tirole (1991)

phase to $n > 1$ rounds where $\delta^n x = z$. This reduces the continuation payoff to z so that defection is deterred as long as $(1-\delta)\,y \leq \delta\,(x-z)$ and this can be shown to hold for any $\delta \geq \delta_1 \equiv \frac{y+\frac{x-z}{z}}{y+(x-z)}$.[12]

As n is generically non-integer, what we mean by n rounds of punishment is that the players first go through $int(n)$ rounds of punishment. After that, they take a lottery and go through one more round of punishment with probability $p = \frac{1-\delta^{frac(n)}}{1-\delta}$ (which is the solution to $p\delta^{int(n)+1} + (1-p)\,\delta^{int(n)} = \delta^n$). Note that the lottery's outcome must be observable to the defector – otherwise the punisher will profit by punishing (playing D) with probability 1.

The constraint that $\delta > \delta_3$ cannot be removed without making stronger assumptions. Recall that if $\delta < \delta_3$, a one round punishment is too severe as the defector prefers to quit for the outside option. In that case, the punishment phase must have $n < 1$ rounds, i.e. one round of punishment with probability $p < 1$. But then we must assume that the defector, having observed that the outcome of the randomization is "punishment", cannot quit for the outside option at the current stage. (Note that allowing the defector to act upon observing the randomization is not a problem in the case of $n > 1$ above. This is because, when the randomization occurs, the $int\,(n) \geq 1$ stages of punishment are irreversible.)

Note that the constraint that $\delta > \delta_3$ could be removed at the cost of losing the RP property, by changing the punishment for defection to quitting the game. In this case, the punished player is forced to accept the continuation payoff z. The new equilibrium is not renegotiation-proof: because both players obtain z after defection, they will renegotiate to forgo the punishment and resume cooperation instead.

Remark 2: The equilibrium described above is not the only cooperative equilibrium. Another PBE has patient agents deterring the impatient agents from entering the PD game, by defecting in the first round. They cooperate in the following rounds, under the usual threat that defection will be punished. Defection in the first round makes the game less attractive than the outside option for impatient agents, who care only about present payoffs, but still attractive for (sufficiently) patient agents who value the guaranteed long-run cooperation. This equilibrium, however, is not renegotiation-proof: In the first round of the PD game it is common knowledge among the two partners that they are both patient, and thus they can negotiate to start cooperation without delay.

[12] An alternative, simpler PBE has players punish defection by quitting for the outside option – which directly reduces the continuation payoff to z. However, this equilibrium is not renegotiation-proof: since the two players know that they are patient, they can renegotiate to remain in the game and to continue to cooperate.

To sum up, with an outside option, if the patient agents are sufficiently patient they cooperate no matter how small their number. The idea is that due to the outside option, the impatient agents do not flood the game. Thus, for any $\lambda_t > 0$ the proportion q_t of patient agents in first round of the game satisfies $q_t \geq q^I$, where q^I is the q_t under which an impatient agent is indifferent whether to join the game. Under such q_t, patient agents strictly prefer to cooperate. Without an outside option, $q_t \equiv \lambda_t$, and thus no matter how patient the patient agents are, there always exists a sufficiently small λ_t (given δ) such that cooperation is not an equilibrium.

4.1 Evolution

Recall that patient agents' decisions are based on discounting future income using the same discount factor as the survival rate δ. Since they strictly prefer their own equilibrium strategy (entering the game and playing C) over the impatient agents' equilibrium strategy (staying home or entering the game and playing D), one could expect that they will have more offspring and eventually dominate the population.

While in the sequel we show that patient agents indeed have more offspring, the argument is more complex. As the population composition may change over time, the fact that impatient agents (who have higher immediate income) have offspring earlier, who may themselves bring more offspring earlier, might counteract the long-run advantage of patient agents. From a mathematical perspective, the evolutionary success depends on the average payoff in a *given period* over all the *currently living* patient vs. impatient agents. These sets of agents comprise agents born in different periods in the past who survived from period to period with probability δ. Unless the population composition is in a steady state (λ_t unchanging) then this average "snapshot" payoff may differ from the δ-discounted lifetime income of agents. Thus, to show that patient agents have the evolutionary advantage, we need to employ finer arguments.

Proposition 3 *There exists $\delta(\omega, x, y, z) < 1$ such that if $\delta > \delta(\omega, x, y, z)$ then for any $\lambda_0 > 0$, $\{\lambda_t : t \geq 1\}$ is increasing in t and converges to* 1.

The proof is in the appendix.

5 How much patience is needed for cooperation?

In Section 4 we show that with an outside option there exists a patience level δ above which cooperation, even among a grain of the patient agents, is possible, here we investigate the minimum patience that is required for cooperation.

5.1 The effect of the outside option payoff z

We start by analyzing the minimum δ for cooperation by even a grain of patient agents, as a function of the attractiveness of the outside option payoff z, i.e. as z varies in the range between 1 (payoff from mutual defection) and x (payoff from mutual cooperation). Outside this range, cooperation among a grain of patient agents is impossible for any $\delta < 1$.

As shown in Section 4, patient agents enter the game and cooperate if $\delta \geq \min(\delta_1, \delta_3)$ where $\delta_1 = \frac{y+\frac{x-z}{z}}{y+(x-z)}$ (the threshold above which patient agents enter the game at the q for which impatient ones are indifferent), and $\delta_3 = \frac{z}{x}$ (the threshold above which patient agents agree to bear a one-period punishment had they deviated).[13] Figure 1 plots the two thresholds δ_1 and δ_3 as a function of $z \in (1, x)$ (with $x = 2$ and $y = 0.1$). The entry decision threshold δ_1 is captured by the convex curve (blue) which is 1 at the extremes (recall that cooperation among a grain of patient agents requires $1 < z < x$) and has a unique minimum at an intermediate z. The agreeing-to-a-one-period-punishment constraint δ_3 is captured by the increasing linear line (red).[14]

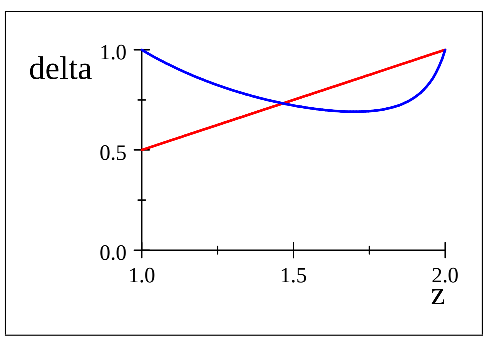

Figure 1: Required patience for cooperation as function of the outside option payoff

[13]Note that the condition $\delta > \delta_3$ is required only for renegotiation proofness of the equilibrium. A simple PBE which holds for any $\delta > \delta_1$ has defection by one player followed by both players quitting the game for the outside option – fixing both payoffs to z. This however is not RP, as they would renegotiate to continue cooperating instead of quitting. Our more complicated punishment strategies are RP but require the additional constraint $\delta > \delta_3$.

[14]The figure depicts a case in which δ_3 is binding for large z. With a sufficiently large y (e.g. $y = 1$) the blue curve would be entirely above the red line, so δ_3 would not be binding for any z. The reason why δ_1 is increasing in y, is that a larger y attracts more impatient defectors into the game thereby reducing the expected payoff to patient agents. Therefore more patience is required for them to enter and cooperate.

The δ_1 graph captures the fact that, for any game parameters x and y, the range of z for which cooperation is an equilibrium grows (in the inclusion sense) with δ, but never reaches 1 or x for $\delta < 1$. To understand why it never reaches 1 or x, consider the attractiveness of entering the game over the outside option for both types of agents. An impatient agent weakly prefers to enter (and defect) if the proportion of patient cooperators q in the game is a least $q_I(z)$, while a patient agent weakly prefers to enter (and cooperate) if it is at least $q_P(z, \delta)$. Cooperation among an arbitrarily small grain λ of patient agents is thus possible only if $q_I(z) \geq q_P(z, \delta)$.

The benefit for patient agents from entering the game increases with the discount factor δ and thus $q_P(z, \delta)$ is decreasing in δ. As $q_I(z)$ is independent of δ, it immediately follows that the range of z that enables cooperation grows with δ. At the extremity $z = 1$, $q_I(1) = 0$ as an impatient defector gets at least 1 in the game, while $q_P(1, \delta)$ is strictly positive, since a patient cooperator gets 0 with probability $1 - q$ in the game. By continuity, $q_P(z, \delta) > q_I(z)$ also in a neighborhood of $z = 1$ for any $\delta < 1$. At the other extremity, $z = x$, $q_I(x) < 1$ as at $q = 1$ an impatient defector gets $x + y > z$ in the game, while $q_P(x, \delta) = 1$, since a patient cooperator gets no more than x in the game. By continuity, $q_P(z, \delta) > q_I(z)$ also in a neighborhood of $z = x$ for any $\delta < 1$.

5.2 Is the outside option beneficial when there are more than a few patient agents?

Our analysis thus far focused on whether the patient agents can cooperate even if their proportion λ_t is arbitrarily small. We now study the required patience δ for cooperation among the patient agents for any λ_t, and compare the required patience with and without an outside option. We show that while for small λ_t our result extends, i.e. cooperation requires less patience with an outside option, the reverse holds for λ_t above some threshold.

With no outside option (see Section 3), a patient agent playing C rather than D loses y in the first round and gains $x - 1$ in all the following rounds if the partner is patient (probability λ_t). If the partner is impatient (probability $1 - \lambda_t$) the immediate loss is 1 with no change in the following rounds' payoff (1). The gain weakly exceeds the loss if:

$$\delta \lambda_t (x - 1) \geq (1 - \delta)(\lambda_t y + (1 - \lambda_t) 1), \tag{1}$$

or

$$\delta \geq \frac{\lambda_t (y - 1) + 1}{\lambda_t (x + y - 1) + (1 - \lambda_t)}.$$

With an outside option the consideration is similar, with two differences: the future gain from playing C rather than D against a patient partner is $x - z$ rather than $x - 1$, and the probability that the partner is patient is $q_t = \max(\lambda_t, q^I) = \max(\frac{z-1}{x+y-1}, \lambda_t)$ rather than λ_t. The condition for a patient agent to prefer C becomes:

$$\delta q_t (x - z) \geq (1 - \delta)(q_t y + (1 - q_t) 1), \quad (2)$$

or:

$$\delta \geq \frac{q_t (y - 1) + 1}{q_t (x + y - z) + (1 - q_t)}.$$

In the case with an outside option, it is also required that $\delta \geq \delta_3 = \frac{z}{x}$ (agents agreeing to a one-round punishment). Note that this additional constraint does not depend on q_t, as it relates to a stage in the game were both players are known to be patient.

Figure 2 is a plot of the patience threshold required for cooperation as a function of the proportion of patient agents λ_t. We fix $x = 2$, $y = 1$, and $z = 1.5$. The threshold without an outside option is the monotone line starting at 1 (red), while the other (blue) is the threshold with an outside option.

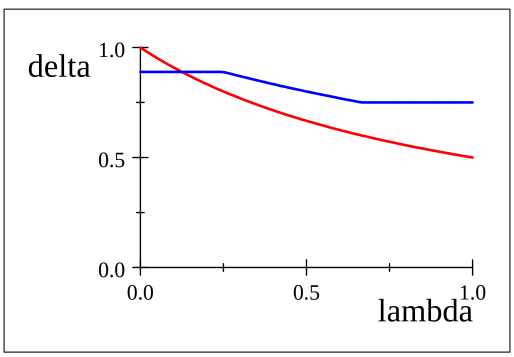

Figure 2: Required patience for cooperation with and without an outside option

As illustrated in the figure, the outside option reduces the patience needed for cooperation for λ_t below some threshold. Above it, the required patience with an outside option is higher, i.e., the existence of an outside option can prevent cooperation.

A closer look at equations 1 and 2 reveals the forces at work. When λ_t is close to 0 the required patience without the outside option (red) goes all the way to 1. This is because λ_t is, in this case, the probability that the random partner is patient. With an outside option, the probability that the random partner is patient, q_t, is bounded below by q^I, and thus for $\lambda_t \leq q^I$ the blue line is flat and below 1. For a sufficiently small λ_t it is thus lower than the red line. When the proportion of patient agents λ_t is larger than q^I, $q_t = \lambda_t$. Thus, the advantage of the outside option no longer exists, and only the cost remains: that the outside option increases the continuation payoff in case of defection from 1 to z, and this reduces the benefit to cooperation in the LHS of the equations. Thus, in this range, the threshold with the outside option is higher. Finally, for λ_t above some threshold, the additional RP condition for the case with an outside option ($\delta \geq \delta_3 = \frac{z}{x}$) becomes binding (blue line flat again).

6 Conclusion

We propose a two-stage model of cooperation in which agents first decide whether to participate in a joint venture and then choose to cooperate or defect within it. By incorporating an outside option that makes participation voluntary, our framework better describes how cooperation unfolds in reality.

Our analysis shows that the outside option can sustain cooperation in repeated joint ventures, even in populations with a substantial proportion of impatient exploiters. In traditional repeated Prisoner's Dilemma models without an outside option, cooperation among patient agents becomes unsustainable when defectors are too numerous. However, when opting out is possible, enough exploiters withdraw, allowing patient agents to cooperate. This enables even a small fraction of patient agents to foster widespread cooperation over time.

Our model helps explain several empirical observations. For instance, it provides a mechanism for the transition from hunter-gatherer societies to modern states, where large-scale cooperation among strangers emerges through the co-evolution of patience and cooperation rather than assuming pre-existing patience. It also accounts for the prevalence of self-employment in poorer countries, where, rather than widespread defection in joint ventures, individuals often choose independent work as an alternative.

Appendix

Proof of Proposition 1:

Since impatient agents care only about current period payoffs, they always (on and off the equilibrium path) play their stage-game dominant action D. As for patient agents, in the first round of the game (payoff weight $1-\delta$), playing D rather than C results in an immediate gain of least $\min(1, y)$. Playing D might be followed by punishment, but its cost is bounded: If the partner is impatient (probability $1 - \lambda_t$) they will in any case play D so there is no cost. And if the partner is patient (probability λ_t), the change in payoff cannot exceed $x + y$ in every period in the future (total payoff weight δ). Thus, the total discounted net gain to playing D rather than C is at least

$$(1 - \delta) \min(1, y) - \delta\lambda_t (x + y),$$

which is positive for $\lambda_t < \lambda(x, y, \delta)$, where $\lambda(x, y, \delta) \equiv \frac{1-\delta}{\delta} \frac{\min(1,y)}{x+y} > 0$.

As in any equilibrium both types play the same action D in the first period. Moving into the second period there is no learning and the posterior probability that the partner is patient remains λ_t. Thus, the same argument holds again and patient agents play D in the second period, and so forth.

Finally, as all agents always play D, their payoffs – patient or impatient – are 1 in each period. Thus the proportion of patient agents always remains constant at λ_0. QED.

Proof that the equilibrium strategies in proposition 2 constitute a Perfect Bayesian Equilibrium:

a. The entry decision

Consider the newborn population of generation t. By the equilibrium strategies, all the patient ones (proportion λ_t) join the game with probability 1 while the impatient ones (proportion $1 - \lambda_t$) join with probability $\frac{\lambda_t(1-q^I)}{q^I(1-\lambda_t)}$ if $\lambda_t < q^I$ and 1 if $\lambda_t \geq q^I$. The posterior probability that an agent in the game is patient is thus:

$$q_t = \begin{cases} q^I & \text{if} \quad \lambda_t \leq q^I \\ \lambda_t & \text{if} \quad \lambda_t \geq q^I \end{cases}.$$

Since in the first round of the game all the patient agents play C and all the impatient ones D, q_t is the probability that one's partner plays C. For an impatient agent (discount factor 0), the expected payoff in the game is thus:

$$V_t^{I,D}(q_t) = q_t (x + y) + (1 - q_t) 1.$$

If $\lambda_t \leq q^I$ then $q_t = q^I = \frac{z-1}{x+y-1}$ implying $V_t^{I,D}(q_t) = z$, so impatient agents playing mixed in the entry decision is indeed optimal. And if $\lambda_t > q^I$ then $q_t = \lambda_t > q^I$ implying $V_t^{I,D}(q_t) > z$ (as $x + y > 1$), so they indeed strictly prefer to enter.

For a patient agent, joining the game will result in cooperation forever if the partner is patient (discounted lifetime payoff x). If the partner is impatient (and thus defects) the payoff is 0 is in the first period and z in every period thereafter. The patient agent's expected discounted payoff in the game is thus:

$$V_t^{P,C}(q_t) = q_t x + (1 - q_t)((1 - \delta) 0 + \delta z) = q_t x + (1 - q_t) \delta z.$$

As $x > z > \delta z$ then $V_t^{P,C}(q_t)$ is increasing and indifference ($V_t^{P,C}(q_t) = z$) holds at:

$$q^P \equiv \frac{z(1-\delta)}{x - \delta z}.$$

The entry threshold of patient agents is lower then that of impatient agents, i.e. $q^P < q^I$, if:

$$\frac{z(1-\delta)}{x-\delta z} < \frac{z-1}{x+y-1}$$

$$\delta > \delta_1 \equiv \frac{y + \frac{x-z}{z}}{y + (x-z)}$$

$$\delta > \delta_1 \equiv \frac{y + \frac{x-z}{z}}{y + (x-z)}.$$

where $\delta_1 < 1$ as $x > z > 1$.

Thus, for $\delta > \delta_1$ (so that $q^P < q^I$), we have:

If $0 < \lambda_t \leq q^I$ then by the equilibrium strategies (impatient agents joining the game with probability $0 < \frac{\lambda_t(1-q^I)}{q^I(1-\lambda_t)} \leq 1$ and patient agents with probability 1), $q_t = q^I$. Thus impatient agents are indifferent and patient agents strictly prefer the game, so the strategies indeed constitute an equilibrium.

If $\lambda_t > q^I$ then by the equilibrium strategies (both types join the game with probability 1), $q_t = \lambda_t > q^I$, so both types indeed strictly prefer the game.

Note that 1 and 2 above imply that $q_t \geq q^I$.

b. The first round of the game

We show that the equilibrium strategies (impatient agents play D in the first round and patient agents C) are indeed optimal. For the impatient agents, D is always the dominant strategy. For a patient agent, the discounted payoff playing C is (see above):

$$V_t^{P,C}(q_t) = q_t x + (1 - q_t) \delta z,$$

while the discounted payoff to playing D is:

$$V_t^{P,D}(q_t) = (1-\delta)(q_t(x+y) + (1-q_t)1) + \delta z$$

(the first round payoff is $x+y$ if the partner is patient and 1 if impatient; the payoff in all the subsequent periods is z.). Note that:

$$V_t^{P,D}(q_t) = (1-\delta) V_t^{I,D}(q_t) + \delta z.$$

To see that $V_t^{P,C}(q_t) > V_t^{P,D}(q_t)$ for $\delta > \delta_1$ (implying $q_t \geq q^I$ as shown above), first note that $V_t^{P,D}(0) = 1 - \delta + \delta z > \delta z = V_t^{P,C}(0)$. Moreover, since $V_t^{I,D}\left(q^I\right) = z$ then $V_t^{P,D}\left(q^I\right) = (1-\delta) V_t^{I,D}\left(q^I\right) + \delta z = (1-\delta) z + \delta z = z$. But in the entry decision part we saw that for $\delta > \delta_1$, $V_t^{P,C}\left(q^I\right) > z$. The claim thus follows from the linearity of both $V_t^{P,C}(q_t)$ and $V_t^{P,D}(q_t)$.

c. Subsequent rounds

After the first round of the game, each player observes their partner's action. If the partner played D in the first round, Bayesian updating implies that they are impatient for sure and will thus play D as long the game continues. Consequently, the best the player can get in each future round of the game is 1 – less then the outside option payoff z. The player thus quits the game and the game ends.

The game continues beyond the first round only if both players played C. In this case, each of them assigns (by Bayesian updating) probability 1 to their partner being a patient agent who will (absent a deviation) play C from period 2 on. Thus, by following the equilibrium strategy (remaining in the game and playing C each period) the player obtains payoff x in each round. Any deviation at any stage is not profitable: quitting the game will result in payoff $z < x$ from that stage on; playing D (thus getting $x+y$ rather then x for one round) will be followed by a punishment in the next round (thus getting 0 rather than x). Defining $\delta_2 = \frac{y}{x}$ we have $y - \delta x < 0$ for any $\delta > \delta_2$, where $\delta_2 < 1$ as $x > y$. This consideration will also hold in any round beyond round 2.

d. Optimality of the strategies also off-path

Finally, we check that also after a deviation, patient agents agree to follow their prescribed strategies (for impatient agents there is nothing to check as they always play D both on and off the equilibrium path).

Note that in the first round, both C and D are played in equilibrium with strictly positive probabilities, so a deviation cannot be detected: the continuation play after both C and D is "on path". Also, after a deviation by which a player quits the game, both players have no additional decisions to make (they are at their outside option irreversibly) so there is nothing to check.

Consider then a deviation at the second or later round by which a (patient) player deviates and plays D rather than C. In this case, the partner gains by punishing, as this yields the highest possible payoff $x + y$ for one round, followed by a resumption of mutual cooperation with a payoff x forever.[15]

Also the defector prefers to follow the punishment scheme, as this minimizes his loss. The defector gets 0 in the next round (plays C against D), while in future rounds cooperation resumes with payoff x in each period, resulting in discounted payoff (in terms of next period) of $(1 - \delta)\, 0 + \delta x = \delta x$. If instead the defector chooses to quit the game for the outside option the payoff is z, which is less then δx if $\delta > \delta_3 \equiv \frac{z}{x}$. (Note that playing D for a few rounds and only then quitting is even worse than quitting immediately.) Another alternative could to delay the return to cooperation. By doing this for one round, the immediate gain is $1 - 0$ and next round loss of $x - 0$ (in further rounds payoffs are the same). This is not profitable if $1 - \delta x < 0$, which holds for $\delta \geq \delta_3 = \frac{z}{x} > \frac{1}{x}$. The same argument shows that further delaying the return to cooperation is even more costly.

e. The equilibrium is Renegotiation-Proof[16]

Finally, the equilibrium is weak-renegotiation proof. Once both players played C in the first round, it is common knowledge among them that they are patient. At each node on the equilibrium path they expect payoffs (x, x) forever. Since $y < x$ there is no continuation payoff that Pareto dominates this. Off the equilibrium path, renegotiation is also impossible: after a defection, the payoffs are $(0, x+y)$ once and (x, x) thereafter, so there is no alternative that dominates it.[17]

Note that once two agents are matched, the above arguments show that the equilibrium they play is strong renegotiation proof. However, our equilibrium is only weak renegotiation proof: because before the matching, the whole population could coordinate on an alternative PBE, the "screening" one described in Remark 2, which may be a Pareto improvement. The "screening" equilibrium, however, is not renegotiation proof by itself: once two agents meet

[15] To be precise, we need to specify the punisher's belief: is the defector a patient agent who deviated just now, or an impatient agent who deviated (played C) in all the previous rounds and can thus be expected to play D also in the next round (in which case the punisher is better off leaving the game)? Since both scenarios have 0 probability, the PBE solution concept allows us to pick either, and we assume the first (which is also more reasonable in this context).

[16] The concept we use is weak RP.

[17] Renegotiating in round 1 is also clearly impossible.

to play the repeated game, there is a common certainty among them that they are both patient, and they will renegotiate to cooperate also in the first round. QED.

Proof of Proposition 3:

Recall that the population of patient agents in the period t consists of the patient survivors from period $t-1$ plus the patient newborns:

$$\mu_t = \delta\mu_{t-1} + (1-\delta)\lambda_t.$$

And similarly for the impatient agents:

$$1-\mu_t = \delta(1-\mu_{t-1}) + (1-\delta)(1-\lambda_t).$$

We refer to the right summand as "newborns" and to the left summand as "adults".

Intuitively, the income of patient agents is higher than that of impatient agents among adults, and lower among newborns. For large δ there are many more adults than newborns, so on average we would expect patient agents to have higher income. This is indeed true since, as we show, the ratio of the proportion of patient agents among newborns to their proportion among adults is bounded, i.e. does not grow to infinity as δ approaches 1, and similarly for impatient agents.

To compute the period-t average incomes of patient and impatient agents, W_t^P and W_t^I, we first note that the expected income of:

The $(1-\delta)(1-\lambda_t)$ *newborn impatient agents* is not more than $x+y < z+x+y$.

The $\delta(1-\mu_{t-1})$ *adult impatient agents* is z (as they are all at the outside option).

The $(1-\delta)\lambda_t$ *newborn patient agents* is at least $0 = z-z$.

The $\delta\mu_{t-1}$ *adult patient agents* is at least $q^I x + \left(1-q^I\right) z = z + q^I(x-z)$. (To see why, note that each adult patient agent has encountered, in their first period of life t', a patient partner with probability $\max\left(\lambda_{t'}, q^I\right) \geq q^I$. In this case they are still matched and their current payoff is $x > z$. If the partner was impatient the game has terminated and their current payoff is z).

A lower bound on the expected income of a patient agent is thus:

$$\begin{aligned} W_t^P &> \frac{(1-\delta)\lambda_t(z-z)) + \delta\mu_{t-1}\left(z+q^I(x-z)\right)}{(1-\delta)\lambda_t + \delta\mu_{t-1}} \\ &= z - \frac{(1-\delta)\lambda_t z}{(1-\delta)\lambda_t + \delta\mu_{t-1}} + \frac{\delta\mu_{t-1}q^I(x-z)}{(1-\delta)\lambda_t + \delta\mu_{t-1}} \\ &> z - \frac{1-\delta}{\delta}\frac{\lambda_t}{\mu_{t-1}}z + \frac{1}{\frac{1-\delta}{\delta}\frac{\lambda_t}{\mu_{t-1}}+1}q^I(x-z). \end{aligned}$$

By the evolution equation $\lambda_t = \frac{\omega+W^P_{t-1}}{\omega+W_{t-1}}\mu_{t-1}$, we have $\frac{\lambda_t}{\mu_{t-1}} = \frac{\omega+W^P_{t-1}}{\omega+W_{t-1}} < \frac{\omega+x}{\omega+0}$. Thus:

$$W^P_t > z - \frac{1-\delta}{\delta}\frac{\omega+x}{\omega}z + \frac{1}{\frac{1-\delta}{\delta}\frac{\lambda_t}{\mu_{t-1}}+1}q^I\left(x-z\right).$$

An upper bound on the expected income of an impatient agent is:

$$\begin{aligned}W^I_t &< \frac{\left(1-\delta\right)\left(1-\lambda_t\right)\left(z+x+y\right)+\delta\left(1-\mu_{t-1}\right)z}{\left(1-\delta\right)\left(1-\lambda_t\right)+\delta\left(1-\mu_{t-1}\right)}\\ &= z + \frac{\left(1-\delta\right)\left(1-\lambda_t\right)\left(x+y\right)}{\left(1-\delta\right)\left(1-\lambda_t\right)+\delta\left(1-\mu_{t-1}\right)}\\ &< z + \frac{\left(1-\delta\right)\left(1-\lambda_t\right)\left(x+y\right)}{\delta\left(1-\mu_{t-1}\right)}\\ &= z + \frac{1-\delta}{\delta}\frac{1-\lambda_t}{1-\mu_{t-1}}\left(x+y\right).\end{aligned}$$

By the evolution equation, $1-\lambda_t = \frac{\omega+W^I_{t-1}}{\omega+W_{t-1}}\left(1-\mu_{t-1}\right)$, we have $\frac{1-\lambda_t}{1-\mu_{t-1}} = \frac{\omega+W^I_{t-1}}{\omega+W_{t-1}} < \frac{\omega+x+y}{\omega}$. Thus:

$$W^I_t < z + \frac{1-\delta}{\delta}\frac{\omega+x+y}{\omega}\left(x+y\right).$$

Consequently:

$$\frac{\omega+W^P_{t-1}}{\omega+W^I_{t-1}} > \frac{\omega+z-\frac{1-\delta}{\delta}\frac{\lambda_t}{\mu_{t-1}}z+\frac{1}{\frac{1-\delta}{\delta}\frac{\lambda_t}{\mu_{t-1}}+1}q^I\left(x-z\right)}{\omega+z+\frac{1-\delta}{\delta}\frac{\omega+x+y}{\omega}\left(x+y\right)} \overset{\delta\to 1}{\to} \frac{\omega+z+q^I\left(x-z\right)}{\omega+z} > 1.$$

By the evolution equation $\frac{\lambda_{t+1}}{1-\lambda_{t+1}} = \frac{\omega+W^P_t}{\omega+W^I_t}\frac{\mu_t}{1-\mu_t}$, this implies that there exists $\delta\left(\omega,x,y,z\right) < 1$ such that for any $\delta > \delta\left(\omega,x,y,z\right)$, $\frac{\lambda_{t+1}}{1-\lambda_{t+1}} > \rho\frac{\mu_t}{1-\mu_t}$ for some $\rho > 1$, for all $t \geq 1$, which means that λ_t and μ_t are increasing and converging to 1. QED.

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