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Bargaining, bargaining power and the composition of investment with an outside option*

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Abstract

We modify a canonical two-agent bargaining game with investments in a joint project, by allowing agents to also invest in outside options that improve their bargaining positions. Absent outside options, it is well known that equal bargaining power maximizes output. However, this is no longer true when investment in outside options is possible and the joint-project technology exhibits stronger substitutability than Cobb-Douglas. When this is so, equal bargaining power *minimizes* project output while *maximizing* total investment in unused outside options. Paradoxically, when inputs are sufficiently strong substitutes, starting at equal bargaining power, each agent would gain from reductions in their own bargaining power.

Keywords: Bargaining, bargaining power, outside option, hold-up problem
JEL Code: C78, D25, L24

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1 Introduction

In this paper, we consider a canonical two-agent bargaining game in which both agents make private costly investments in a joint project and then bargain over how to divide the project's proceeds. Ex ante, the sole source of heterogeneity is possibly unequal bargaining power. With complete contracts, the agents can obviously obtain efficient investments in the joint project before bargaining. However, this is the real world, and such contracting is infeasible, leading to the well-known hold-up problem. We add to this canonical model the real world feature that in addition to investing in the joint project, agents can invest in their outside options thereby improving their bargaining positions. We show that this one change can completely reverse our understanding of the basic economics of the hold-up problem.

In the canonical model with otherwise symmetric agents where the joint project features a CES technology and no investment in outside options, an equal division of bargaining power is always optimal, leading to investment choices that maximize joint project output and (constrained second-best) total agent utility. This result reflects that unequal bargaining abilities magnify the under-investment in the joint project by the agent with the weaker bargaining power, which reduces joint output due to complementarities in inputs, and the strict convexity of the investment cost function, which makes averaging investment efforts cost-reducing. As the hold-up problem remains present when investment in an (unused) outside option is possible, one might expect that equal bargaining powers would still maximize joint project output. In fact, as the joint production technology is concave and the effort investment cost function is convex, it is almost immediate that an equal division of bargaining power (locally) maximizes total combined investment in the joint project and outside options, and (locally) minimizes the total costs of investment that agents incur.

Indeed, equal bargaining powers still maximize joint output given a linear investment technology for the outside option *if* the CES joint project technology exhibits stronger complementarities in agent investments than Cobb-Douglas. However, it turns out that a Cobb-Douglas technology represents a dividing line. With a Cobb-Douglas technology, for all levels of bargaining power that give rise to interior solutions (investment in both the joint project and outside options), the balance in bargaining power has *no* impact on the level of output from the joint project. That is, starting at equal levels of bargaining power, *reducing one agent's bargaining power reduces his investment in the joint project by an amount that is exactly offset by an even greater investment by the other agent that is just enough to leave joint output unchanged.*

Standard predictions are overturned when the joint technology exhibits stronger substitutability between inputs than Cobb-Douglas. Two stark results obtain. First, equal bargaining powers are both a local (and sometimes global) *minimizer* of joint project output and a local *maximizer* of total (wasted) investments in the outside options of agents. Second, starting with equal bargaining power, standard economic logic suggests that a marginal increase in an agent's bargaining power should increase that agent's payoff. This reasoning fails when agents can invest in outside options: when inputs to the joint project are sufficiently strong substitutes, starting at equal bargaining power, each agent would be strictly better off if their own bargaining power were *reduced*. In short, when the other agent can invest in an outside option, having greater ex-ante bargaining power can *harm* an agent.

To understand the economics, first recognize that the degree of complementarity between agents' investments in the joint production function also determines the extent to which an agent's investment in the joint project is substitutable with investment in his outside option. Specifically, if investments in the joint project are highly complementary—such that one agent's investment significantly increases the marginal productivity of the other's—an agent's investment in his outside option becomes a poor substitutes for investment in the joint project, This is because investing in the outside option carries a high opportunity cost in terms of foregone joint output. As a result, with strong production complementarities, agents substitute minimally between the two investment types, making the optimal investment levels largely independent. Consequently, the standard hold-up problem logic from the no-outside option scenario extends, and equal divisions of bargaining power maximize both joint output and total agent utility.

Conversely, when the agents' investments in the joint project are highly substitutable, an agent's investment in the joint project also becomes more substitutable with investment in his outside option. In this case, the composition of an agent's investment becomes highly sensitive to his bargaining power. Increasing substitutability beyond the Cobb-Douglas case amplifies the consequences of bargaining power asymmetry. Starting from equal bargaining power, reducing one agent's bargaining power leads that agent to reduce his investment in the joint project and to increase his investment in the outside option by an even greater amount. Meanwhile, the other agent responds by investing more in the joint project and cutting back even further on his outside investment. At the Cobb-Douglas benchmark, although investments levels in the joint project are asymmetric, total investment rises just enough to keep output unchanged. When inputs are more substitutable than Cobb-Douglas, asymmetries in agents' investments reduce output by less, and the increase in total investment becomes more consequential. As a result, asymmetries in bargaining power *increase* total output, as long as bargaining power remains sufficiently balanced that all equilibrium investments are interior.

In this case, equal division of bargaining power not only locally minimizes the output—and the total investment in the joint project—but it also locally maximizes total investment in outside alternatives, i.e., the extent of agents' free-riding. Paradoxically, *greater bargaining power can harm an agent even though it also increases total output.* This reflects that the agent with less bargaining power free-rides more, increasing investment in his outside alternative by even more than he reduces investment in the joint project.

There is a vast literature on the hold-up problem and possible ways to mitigate it (see Tirole, 1986; Hart and Moore, 1988; Baker et al., 2002; Che and Sakovics, 2008; the last provides a comprehensive survey). In many papers, the problem is posed as a single agent making a non-contractible investment in a joint project or relationship over which they will subsequently bargain. In this one-sided investment setting, the hold-up problem exists as long as the agent making the investment does not have all of the bargaining power. In symmetric two-sided investment settings, both sides under-invest because each investing agent incurs all of the cost from an investment, but only receives a share of the output from the investment. However, in these two-sided investment settings, equal bargaining powers still minimize the under-investment consequences. These models do not consider the possibility that the agents can invest in outside options to improve their bargaining positions. In practice, there are many settings where the agents have that option. For example, a worker can search for an alternative employer and the firm can search for replacement workers or technologies that could replace workers. So, too, intermediate good suppliers can search for other firms that might value their product while a final good producer can search for potential matches with other intermediate good suppliers. In such settings the agents may devote efforts both to building their relationships as well as improving their fall-back options. We show that the ability to invest in outside options can radically alter the qualitative impact of bargaining power allocations on outcomes, showing that equal bargaining powers can minimize joint output and total agent utility; and that having greater ex ante bargaining power can harm an agent.

Our setup also relates to bargaining models in which bargainers' outside options are determined endogenously or evolve stochastically over time.[1] Gul (2001) considers a model where a bargainer can privately choose their investment level prior to negotiation. Fuchs and Skrzypacz (2010) analyze a dynamic bargaining model where an exogenous event arrives randomly during the bargaining process. de Meza and Lockwood (2010) study a setting in which bargainers make ex-ante investments, followed by stochastic matching and ex-post bargaining, and show that over-investment can arise due to endogenously-determined outside options. Hwang and Li (2017) study a setting where a bargainer randomly receives an outside option that she may use, and examine how the transparency of the outside option affects bargaining outcomes. McClellan (2024) analyzes a principal's optimal dynamic negotiation strategy under commitment when an agent's outside option evolves stochastically.

2 Model

There are two agents, $i = A, B$, and two time periods, $t = 0, 1$. At $t = 0$, the agents decide how to allocate non-contractible costly effort to improving a joint project and to improving their outside options that they can pursue in lieu of the joint project. If agent i devotes effort $x_i \geq 0$ to improving his outside option, it would provide i a payoff of

$$w_i(x_i) = \underline{w} + \gamma x_i, \tag{1}$$

[1] In the literature analyzing the impact of (static) outside options on bargaining outcomes, Board and Pycia (2014) show that Coase conjecture breaks down when a privately-informed player has an outside option. Fanning (2023) shows that this result is partially robust when both sides can make offers and a commitment type exists.

if he pursued it, where $\underline{w}$ and γ are strictly positive. The joint project takes a CES production function form. When the agents invest $y_A \geq 0$ and $y_B \geq 0$ in the joint project, the project pays

$$\Theta(y_A, y_B) = \underline{\theta} + \xi \left(\frac{y_A^\rho}{2} + \frac{y_B^\rho}{2} \right)^{\frac{\beta}{\rho}}, \tag{2}$$

where $\xi > 0$, $\rho \leq 1$, $\underline{\theta} \geq 0$, and $0 < \beta < 1$ implies that there are decreasing returns to scale. The elasticity of substitution between y_A and y_B is given by $\sigma = 1/(1-\rho)$. The cost to agent i of efforts x_i and y_i is given by

$$C(x_i, y_i) = c(x_i + y_i)^\alpha, \tag{3}$$

where $\alpha > 2$ and $c > 0$.

At $t = 1$, the agents divide proceeds from the joint project according to the generalized Nash bargaining solution, with each agent's *bargaining power*, $q_i \in (0, 1)$, defined as in Kalai (1977), where $q_A + q_B = 1$. That is, if $\theta > w_A + w_B$, then the output from the joint project allocated to each agent solves

$$\max_{\theta_A \geq w_A, \theta_B \geq w_B} (\theta_A - w_A)^{q_A} (\theta_B - w_B)^{q_B}, \ \text{ s.t. } \ \theta_A + \theta_B \leq \theta. \tag{4}$$

Routine solution yields that agent $i = A, B$ receives

$$\theta_i = w_i + q_i(\theta - w_A - w_B). \tag{5}$$

If $\theta \leq w_A + w_B$ then both agents select their outside options. Thus, agent i's payoff in the game is given by

$$u_i(x_A, x_B, y_A, y_B) = (1 - q_i)w_i(x_i) + q_i\Theta(y_A, y_B) - q_i w_{-i}(x_{-i}) - C(x_i + y_i). \tag{6}$$

Throughout the paper, we assume that the outside option is not sufficiently attractive that the agents want to opt out in equilibrium. This will be the case, for example, when $\underline{\theta}$ is sufficiently large. We focus on pure strategy Nash equilibria.

2.1 Analysis

Note that $\Theta(y_A, y_B) - \underline{\theta}$ and $C(x_i, y_i)$ are homogenous of degree $\beta < 1$ and $\alpha > 2$, respectively. Therefore, the first-order conditions for y_i and x_i are sufficient for deriving the equilibrium efforts, given that their levels are positive. These conditions are given by

$$(1 - q_i)\gamma = c\alpha(x_i + y_i)^{\alpha - 1}; \tag{7}$$

$$0.5^{\frac{\beta}{\rho}} q_i \xi \beta \left(y_A^\rho + y_B^\rho \right)^{\frac{\beta}{\rho} - 1} y_i^{\rho - 1} = c\alpha(x_i + y_i)^{\alpha - 1}. \tag{8}$$

The first-order condition for investment in the outside option, (7), immediately implies the following result:

Lemma 1 *The greater is agent i's bargaining power, q_i, the lower is i's total effort $x_i + y_i$ in the best response function when $x_i > 0$.*

We focus most of our analysis on parameterizations for which both agents invest in both the joint project and their outside options, i.e., $x_i, y_i > 0$, for $i = A, B$. Solving the first-order conditions for $x_i, y_i > 0$ yields

$$y_A^{1-\beta} = \frac{\beta\xi q_A}{2^{\frac{\beta}{\rho}}\gamma q_B}\left(1+\left(\frac{q_A}{q_B}\right)^{\frac{2\rho}{\rho-1}}\right)^{\frac{\beta}{\rho}-1} \quad \text{and} \quad y_B^{1-\beta} = \frac{\beta\xi q_B}{2^{\frac{\beta}{\rho}}\gamma q_A}\left(1+\left(\frac{q_B}{q_A}\right)^{\frac{2\rho}{\rho-1}}\right)^{\frac{\beta}{\rho}-1} \tag{9}$$

$$x_A = \left(\frac{q_B\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}} - y_A \quad \text{and} \quad x_B = \left(\frac{q_A\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}} - y_B. \tag{10}$$

For completeness, in the appendix we provide the first-order conditions for parameterizations for which one or both of the agents do not invest in the outside option or the joint project.

Three important points follow directly from the first-order condition (10) for investment in the outside option. At an interior solution, regardless of the elasticity of substitution between y_A and y_B,

1. Agent i's total effort $x_i + y_i$ is a *decreasing*, concave function of his bargaining power q_i. This follows from re-arranging the solutions in (10) as $x_i + y_i = \left(\frac{(1-q_i)\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}}$.

2. Total investment in the joint project plus outside options is strictly concave in q_A:

$$\sum_{i=A,B}(x_i + y_i) = \left(\frac{\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}}\left(q_A^{\frac{1}{\alpha-1}} + (1-q_A)^{\frac{1}{\alpha-1}}\right). \tag{11}$$

   From symmetry, it follows that balanced bargaining power, $q_A = 0.5$, maximizes total investment.

3. Total collective investment costs are a strictly convex function of q_A and hence minimized at $q_A = 0.5$:

$$\sum_{i=A,B} C(x_i, y_i) = c\left(\frac{\gamma}{c\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left((1-q_A)^{\frac{\alpha}{\alpha-1}} + q_A^{\frac{\alpha}{\alpha-1}}\right).$$

We next provide a condition for agents to invest in both the joint project and their outside options.

Lemma 2 *Suppose that*

$$\xi < \bar{\xi} = \frac{2}{\beta}\frac{\gamma^{\frac{\alpha-\beta}{\alpha-1}}}{\alpha^{\frac{1-\beta}{\alpha-1}}} \tag{12}$$

and $\underline{\theta}$ is sufficiently large. Then there exists an $\epsilon(\xi) > 0$ such that for $q_A \in (1/2-\epsilon(\xi), 1/2+\epsilon(\xi))$, negotiation occurs and agents invest both in the joint project and the outside option, i.e., $x_i, y_i > 0$ for $i = A, B$.

When agents have the same bargaining power, i.e., $q_i = 0.5$, the equilibrium is symmetric. When $\underline{\theta}$ is sufficiently large, it is not optimal for agents to pursue their outside options in equilibrium, and hence $y_i > 0$.

Substituting for a symmetric solution, noting that y_i is increasing in ξ while x_i is decreasing, we explicitly solve for the value of $\bar{\xi}$ such that the first-order conditions just hold for $x_A = 0$. By continuity, for any $\xi < \bar{\xi}$, there exists an $\epsilon(\xi) > 0$ such that $x_i, y_i > 0$ for $i = A, B$ for all $q_i \in (1/2 - \epsilon(\xi), 1/2 + \epsilon(\xi))$.

As a benchmark, we first consider the canonical setting where investment in the outside option is not possible.

Proposition 1 *With no outside alternative, i.e., with $\gamma = 0$, equal bargaining power, $q_A = q_B = 0.5$, always maximizes joint output, inducing the 'second-best' efficient investments that maximize total agent utility.*

Proof. See the Appendix. ■

The result follows from symmetry and the concavity of the joint production function together with the strict convexity of the investment cost function, which imply that it is most efficient to use equal inputs. Each agent i only internalizes $q_i\Theta(y_A, y_B)$ when investing, failing to internalize the benefit to the other party. The total distortion is minimized by assigning equal bargaining powers. For example, when $q_A > 0.5$, agent A internalizes more of the investment benefits, but B internalizes less, and the consequences of B's greater under-investment outweigh the benefits of A's increased investment. More generally, the optimality of balanced bargaining power extends whenever the productivity of joint investment is sufficiently high relative to the outside option that both agents do not invest in their outside options, i.e. $x_A = x_B = 0$.

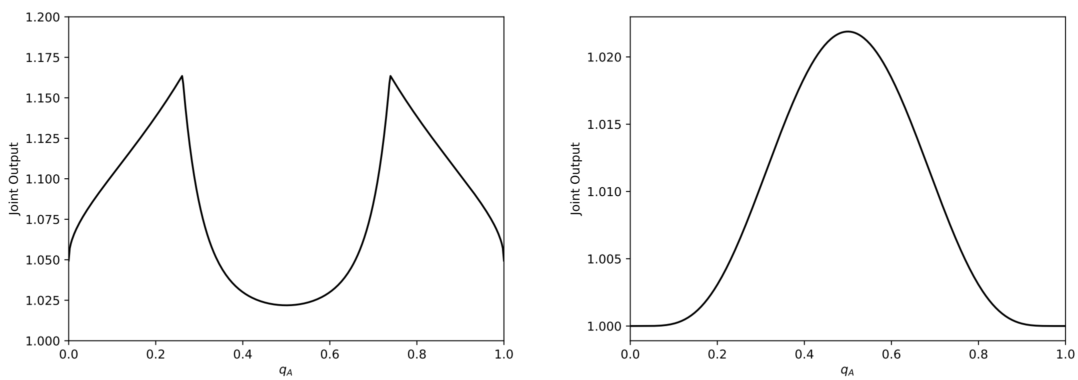

Figure 1: Level of output from the joint project for $\gamma = 1$, $\xi = 1.5$, $\alpha = 3$, $\beta = 0.9$, $\underline{\theta} = 1$, and $c = 1$. In the left panel $\rho = 0.3$; in the right panel $\rho = -0.3$.

With these benchmarks in mind, we now investigate how the possibility of investment in an outside option affects outcomes in settings where investments are interior, $x_i, y_i > 0$, $i = A, B$. Figure 1 illustrates that whether balanced bargaining power maximizes joint output or minimizes joint output hinges on the degree of substitutability σ between inputs in the CES joint production technology. In the left-panel, where $\sigma > 1$, output is globally minimized at $q_A = 0.5$—more generally, $q_A = 0.5$ is a local minimizer of output when $\sigma > 1$. In the right panel, where $\sigma < 1$, we get the familiar result that output is globally maximized at $q_A = 0.5$.

Proposition 2 below provides the formal result.

Proposition 2 *Under the conditions of Lemma 2,*

1. *If $\sigma = 1/(1-\rho) > 1$, then equal bargaining powers, $q_A = q_B = 0.5$, is a local minimizer of joint output.*
2. *If $\sigma < 1$, then equal bargaining powers, $q_A = q_B = 0.5$, is the global maximizer of joint output.*

To understand the intuition first consider the critical case where $\sigma = 1$ ($\rho = 0$), i.e., the joint production function takes a Cobb-Douglas form, $f(y_A, y_B) = \xi(y_A y_B)^{\beta/2}$. In this case, output does not vary with q_A in the neighborhood of $q_A = 0.5$ (i.e., for all q_A such that $x_A, y_A, x_B, y_B > 0$): an increase in q_A induces a reduction in y_B that is exactly offset by an even larger increase in y_A that leaves their product $y_A y_B$ unchanged and hence joint project output unchanged. To see this, substitute (7) into the first-order conditions for y_i to obtain

$$\frac{\beta q_A \xi}{2} y_A^{\frac{\beta}{2}-1} y_B^{\frac{\beta}{2}} = q_B \gamma; \tag{13}$$

$$\frac{\beta q_B \xi}{2} y_A^{\frac{\beta}{2}} y_B^{\frac{\beta}{2}-1} = q_A \gamma. \tag{14}$$

Dividing the two equations yields $y_A = (q_A/q_B)^2 y_B$. Substituting into the first-order conditions yields

$$y_A = \left(\frac{\xi\beta}{2\gamma}\right)^{\frac{1}{1-\beta}} \frac{q_A}{q_B} \quad \text{and} \quad y_B = \left(\frac{\xi\beta}{2\gamma}\right)^{\frac{1}{1-\beta}} \frac{q_B}{q_A}. \tag{15}$$

To see that output is independent of q_A, note that q_A and q_B cancel when substituting y_A and y_B into the Cobb-Douglas production function. As we vary q_A, the set of optimal production inputs satisfies $\sqrt{y_A y_B} = \left(\frac{\xi\beta}{2}\right)^{\frac{1}{1-\beta}}$, i.e., we exactly trace out the isoquant of the production function. Note further that the collective inputs $y_A + y_B = \left(\frac{\xi\beta}{2}\right)^{\frac{1}{1-\beta}} \left(\frac{q_A}{1-q_A} + \frac{1-q_A}{q_A}\right)$ has a derivative proportional to $\frac{1}{(1-q_A)^2} - \frac{1}{q_A^2}$, implying that total inputs to the joint project are increasing in $q_A > 1/2$, i.e., the increase in y_A exceeds the decrease in y_B.

This effect gets magnified as ρ is increased, because the marginal product of y_A is less sensitive to changes of y_B. Because total inputs increase and the mix of inputs matters less as ρ is increased, it follows that total output increases when q_A is increased. Most transparently, consider ρ close to 1. The production function is close to $f(y_A, y_B) = \xi(0.5y_A + 0.5y_B)^{\beta}$. The marginal product at $q_A = 0.5$ is close to $\mathrm{MP_A} = \beta\xi\bar{y}^{\beta-1}$, where $\bar{y} = y_A = y_B$. The first-order condition implies that $q_A \mathrm{MP}_A(y_A, y_B) = (1 - q_A)\gamma$. As we raise q_A marginally above 0.5, MP_A must fall. At the same time agent B's investment drops close to zero. Thus, agent A's investment $\tilde{y}_A$ must satisfy $\mathrm{MP}_A(\tilde{y}_A, 0) < \mathrm{MP}_A(y_A, y_B)$, which implies that $\tilde{y}_A > y_A + y_B$. Thus, the additional investment by agent A must more than make up for the decrease in B's investment. Because ρ is close to 1, the increase in total investment implies that output must increase.

Conversely if we reduce ρ so that inputs are more complementary than Cobb-Douglas, the mix of inputs matters more and output is maximized by balanced bargaining power, as in the canonical setting. To see this most clearly, suppose that ρ is very negative so that y_A and y_B are close to being perfect complements in production. As (11) shows, total effort is maximized by equal bargaining power. Thus, raising q_A past one-half lowers total effort. If the inputs y_A and y_B to the production function are sufficiently complementary, then x_i and y_i are also not very substitutable from the perspective of agent i. This lack of substitutability implies that both $y_A + y_B$ and $x_A + x_B$ decrease as q_A increases, and hence output decreases. The logic underlying why output decreases with enough complementarity mirrors that for the standard hold-up problem: As bargaining power favors agent A, agent B's incentives to invest in the joint project are reduced, which in turn lowers agent A's investment below the socially desirable level. The outside option does not affect this classical result due to the lack of substitutability between investments in the joint project and the outside options.

We just showed that when inputs are more substitutable than Cobb-Douglas, output from the joint project is minimized by equal bargaining powers. We now complement this finding by showing that, in addition, equal bargaining powers *maximize* the (wasted) investments by agents in their outside options:

Corollary 1 *Suppose that the elasticity of substitution* $\sigma > 1$. *Then under the conditions of Lemma 2, total investment in agents' outside options is locally maximized by equal bargaining powers,* $q_A = q_B = 0.5$.

Recall that (11) implies that total investment $\sum_{i=A,B} x_i + y_i$ is maximized at $q_A = 0.5$. Proposition 2 shows that $y_A + y_B$ assumes a local a minimum at $q_A = 0.5$, which, in turn, implies that $x_A + x_B$ assumes a local maximum. In equilibrium, the outside alternatives are never taken, implying that all investments in the outside option are wasted from a social perspective. The fact that this wasteful spending is maximized by balanced bargaining power, further underscores how the presence of the outside option can reverse the standard hold-up result in symmetric settings that equal bargaining power is optimal.

Note that $\sigma > 1$ is a sufficient, but far from necessary, condition for wasteful spending to be maximized by equal bargaining powers. To see this most clearly, consider the Cobb-Douglas setting, where (i) output of the joint project does not vary with q_A when bargaining power is sufficiently balanced, and (ii) wasted investment, $x_A + x_B$, is maximized at $q_A = q_B = 0.5$. From (10), we have

$$x_A + x_B = \left(\frac{\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}} \left((1-q_A)^{\frac{1}{\alpha-1}} + q_A^{\frac{1}{\alpha-1}}\right) - \left(\frac{\xi\beta}{2}\right)^{\frac{1}{1-\beta}} \left(\frac{q_A}{1-q_A} + \frac{1-q_A}{q_A}\right). \tag{16}$$

Because $\alpha > 2$ it is immediate that $(1-q_A)^{\frac{1}{\alpha-1}}$ and $q_A^{\frac{1}{\alpha-1}}$ are strictly concave, while $q_A/(1-q_A)$ and $(1-q_A)/q_A$ are both strictly convex. Thus, $x_A + x_B$ is a concave function of q_A and by symmetry the maximum is obtained at $q_A = 0.5$. Thus, there exists a $\tilde{\sigma} < 1$ such that the insight of Corollary 1 also holds for all $\sigma > \tilde{\sigma}$.

We have shown that total output is minimized and the socially wasteful investments in the outside options are maximized at $q_A = 0.5$ when the elasticity of substitution exceeds 1. Figure 2 illustrates that, as a

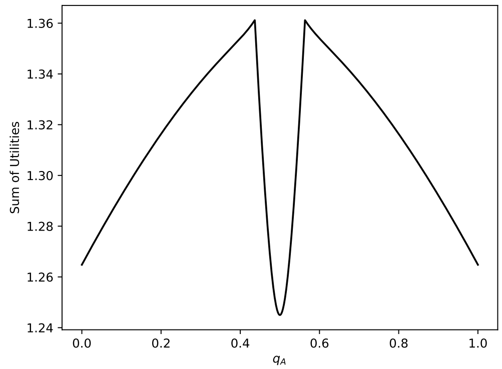

Figure 2: The sum of agents' utilities for $\xi = 1.4$, $\alpha = 3$, $\beta = 0.6$, $\underline{\theta} = 1$, $c = 1$, $\gamma = 1$, and $\rho = 0.9$.

consequence, equal bargaining power, $q_A = q_B = 0.5$, can be a global minimizer of social welfare. We state the formal result below.

Proposition 3 *Under the conditions of Lemma 2, social welfare is locally minimized at $q_A = 0.5$ if the elasticity of substitution is sufficiently large. Conversely, if $\sigma \leq 1$, then social welfare is maximized at $q_A = 0.5$.*

For $\sigma < 1$ we have shown that $q_A = 0.5$ is the global maximizer of total output. Further, total costs (11) are convex and minimized at $q_A = 0.5$. Hence, social welfare is maximized. The strict convexity of costs implies that this argument extends to σ slightly above one.

Now consider larger $\sigma > 1$. Quite generally, the proof of Proposition 2 establishes that the second derivative of total output Θ with respect to q_A at $q_A = 0.5$ equals

$$\left(\frac{\xi\beta}{2\gamma}\right)^{\beta} \frac{16\xi\beta(\sigma - 1)}{(1-\beta)}. \tag{17}$$

Thus, this derivative becomes arbitrarily large as $\sigma \to \infty$. This reflects that, with sufficient substitutability, small changes of q_A away from 0.5 lead to large increases in θ. In contrast, total costs (11) are independent of σ, and hence the positive second derivative of Θ determines the overall sign when σ is large.

We conclude analysis by investigating the impact of bargaining power on agents' utilities. If there is no investment in the outside option, we know from Proposition 1 that equal bargaining power maximizes social welfare. If we raise say q_A slightly above 0.5, then we should expect A's utility to rise, albeit at a distortion cost which results in B's utility dropping by even more. If bargaining power is raised by too much, then the distortion effect starts to dominate and A's utility begins to decrease in q_A. This classical hold-up problem intuition extends to our setting as long as inputs are not too substitutable. For example, the right panel of Figure 3 shows that A's optimal level of bargaining power strictly exceeds the social optimum of 0.5.

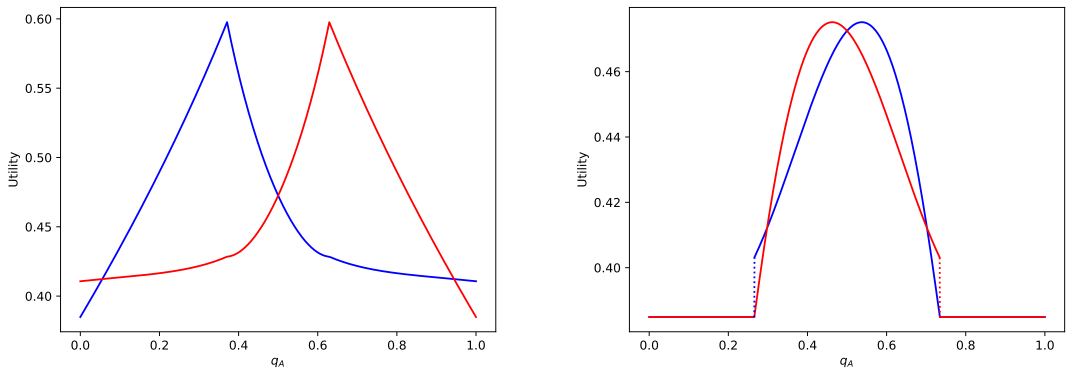

Figure 3: The agents' payoffs (blue for agent A; red for agent B) for $\xi = 1.4$, $\alpha = 3$, $\beta = 0.6$, $\underline{\theta} = 0.7$, $c = 1$, and $\gamma = 1$. Left panel: $\rho = 0.5$. Right panel: $\rho = -2$.

This standard intuition fails when the elasticity of substitution is sufficiently large. The left panel of Figure 3 shows that, from A's perspective, the optimal q_A is less than 0.5. This leads to the seemingly counterintuitive observation that at $q_A = 0.5$, both agents would prefer to *reduce* their own bargaining power. Posed differently, the agent with the lower bargaining power earns a higher payoff than the agent with the higher bargaining power. We now prove that this result is general in nature:

Proposition 4 *Suppose that the conditions of Lemma 2 hold. Then for every β there exists a $\bar{\rho} > 0$ such that for all $\rho \in [\bar{\rho}, 1)$: Increasing agent A's bargaining power from the symmetric case $q_A = 0.5$ strictly reduces A's payoff and strictly increases B's payoff.*

When the elasticity of substitution is sufficiently large, greater bargaining power can harm an agent even though it also results in increased total output. This reflects that the agent with greater bargaining power personally incurs the costs of his increased investment in the joint project; while the agent with reduced bargaining power increases his free riding on the joint project and raises investment in his outside alternative, strengthening his bargaining position.

2.2 Robustness

Our result that output is minimized by equal bargaining power for sufficiently large σ is robust to altering basic assumptions. First, suppose investment in the outside option exhibits decreasing returns to scale, with $w(x) = \underline{w} + \gamma x^{\delta}$, where $\delta < 1$. Trivially, as $\delta \downarrow 0$, the model converges to one where the value of the outside option is fixed at $\underline{w} + \gamma$, so we are back in the standard model, where Proposition 1 shows the socially efficient outcome is obtained when bargaining powers are equal. Conversely, by continuity of the first-order conditions, Proposition 2 implies that if $\sigma > 1$ and δ is sufficiently close to 1, then output is minimized at $q_A = 0.5$. Thus, for every (finite) $\sigma > 1$ there exists a $\bar{\delta}$ such that output is locally minimized at $q_A = 0.5$ when $\delta \geq \bar{\delta}$. Again, what underlies this result is that there must be enough substitutability between investments in the outside options and investments in the joint project. This substitutability is decreased when δ is smaller.

Second, consider the structure of costs. In our model, costs depend on the sum of efforts x_i and y_i. If, instead, the effort costs are separable, i.e., $\frac{\partial^2 C(x,y)}{\partial x \partial y} = 0$, then the first-order conditions for x_i and y_i are separable. As a consequence, there is no substitution between x_i and y_i so output is again maximized by $q_A = 0.5$. Thus, our result requires the cross derivative be sufficiently positive. This highlights that the driver of our result is that when q_A increases, agent B will substitute toward increased x_B, reducing y_B further. In turn, lowering y_B by more reduces the marginal product of y_A by more. That is, the economics are driven by the two avenues of substitution, y_A for y_B in the joint production, and x_i for y_i for each individual agent.

3 Conclusion

We modify a canonical two-agent bargaining game with joint investment, by allowing agents to also invest in outside opportunities that improve their bargaining positions. We show that incorporating this one realistic feature can sharply change the familiar predictions from the classical hold-up problem. In particular, when agents cannot invest in outside alternatives, equal bargain powers maximize output and social welfare, as unequal bargaining abilities cause the agent with the weaker bargaining power to under-invest by more, reducing output due to the complementarities in inputs. When agents can invest in outside options, equal bargaining powers remain output-maximizing if the project technology exhibits stronger complementarities in inputs than Cobb-Douglas, reflecting that complementarities magnify the benefits of more balanced investments for outputs.

However, these standard predictions are completely over-turned when the technology exhibits stronger substitutability: equal bargaining power minimizes project output and maximizes total investment in unused outside options. With greater substitutability, asymmetries between agents in their investments reduce output by less, and the greater total investment in the joint project matters more so that asymmetries in bargaining power now *increase* total output as long as bargaining power is sufficiently balanced that all equilibrium investments are interior. Strikingly, when inputs are strong substitutes, starting at equal bargaining power, each agent would gain from reducing their own bargaining power. The weaker bargaining power serves to commit an agent to investing in his outside option, free-riding on the other agent's extensive investments in the joint project.

4 Appendix

To begin, we solve for the relevant first-order conditions, when investments by both agents are not interior.

Case 2: No investment in the the outside option by one of the agents.

Suppose that $x_A = 0$, but $x_B, y_A, y_B > 0$. In this case, (7) only describes agent B's investment in the outside option. Reorganizing the FOCs, it follows that y_A and y_B jointly solve

$$0.5^{\frac{\beta}{\rho}} q_A \xi \beta \left(y_A^\rho + y_B^\rho\right)^{\frac{\beta}{\rho}-1} y_A^{\rho-1} = c\alpha y_A^{\alpha-1}; \tag{18}$$

$$0.5^{\frac{\beta}{\rho}} q_B \xi \beta \left(y_A^\rho + y_B^\rho\right)^{\frac{\beta}{\rho}-1} y_B^{\rho-1} = \gamma q_A. \tag{19}$$

Combining the above two equation yields

$$y_A^{\alpha-\rho} = \frac{\gamma q_A^2 y_B^{1-\rho}}{c\alpha q_B} \text{ and } x_B = \left(\frac{q_A \gamma_B}{c\alpha}\right)^{\frac{1}{\alpha-1}} - y_B. \tag{20}$$

The case of $x_B = 0$ and $x_A, y_A, y_B > 0$ is symmetric. Again, if agent i invests in both the outside option and the joint project, then his total effort decreases in his bargaining power q_i. In contrast, if agent i only invests in the joint project, then, fixing the other agent's investment, agent i's investment increases with q_i. However, agent i's investment also increases in y_{-i}, and increases in q_i induce agent $-i$ to reduce y_{-i}, leaving the net impact of increasing i's bargaining power on y_i ambiguous.

Case 3: No investment in the outside option by both agents.

Suppose that $x_A = x_B = 0$. Then only (8) applies. Solving yields

$$y_A^{\alpha-\beta} = \frac{\beta \xi q_A}{2^{\frac{\beta}{\rho}} c\alpha} \left(1 + \left(\frac{q_A}{q_B}\right)^{\frac{\rho}{\rho-\alpha}}\right)^{\frac{\beta}{\rho}-1} \text{ and } y_B^{\alpha-\beta} = \frac{\beta \xi q_B}{2^{\frac{\beta}{\rho}} c\alpha} \left(1 + \left(\frac{q_B}{q_A}\right)^{\frac{\rho}{\rho-\alpha}}\right)^{\frac{\beta}{\rho}-1}. \tag{21}$$

Note that if agents only invest in the joint project, then the agent with the greater bargaining power invests more than the other agent, but agent i's investment may or may not increase with his bargaining power.

Case 4: One agent only invests in the outside option, while the other agent only invests in the joint project.

Let $x_A = y_B = 0$, and $x_B, y_A > 0$. Solving (7) and (8) in this scenario yields

$$x_B = \left(\frac{q_A \gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}} \text{ and } y_A = \left(\frac{q_A \xi \beta}{2^{\frac{\beta}{\rho}} c\alpha}\right)^{\frac{1}{\alpha-\beta}}. \tag{22}$$

In this case, investments by both agents increase with the bargaining power of the agent who invests in the joint project. This reflects that increasing q_A causes A to internalize more of the payoff from y_A; and since A invests more, project output rises, inducing B to invest more in his outside alternative, as he now gains more from improving his bargaining position.

Finally, note that it can never be the case that one agent only invests in his outside option, while the other agent invests both in the joint project and his outside option. That is, if $y_i = 0$ in equilibrium, then either $x_j = 0$ or $y_j = 0$, where $i, j \in \{A, B\}$ and $j \neq i$. To see this, suppose by way of contradiction that $y_A = 0$ but $x_A, x_B, y_B > 0$. By continuity there exist model parameters such that the first-order conditions of the interior solution apply also for y_A. However, (9) implies that y_A can only be close to zero if q_A is close to 0. However, (9) implies that y_B becomes arbitrarily large, and hence the solution for x_B in (10) becomes negative, a contradiction.

Proof of Lemma 2. First observe that if $\underline{\theta}$ is sufficiently large, some investment in the joint project is optimal. Next, note that (9) and (10) imply that $x_A > 0$ if and only if

$$y_A < \left(\frac{q_B \gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{23}$$

Substituting $q_A = 0.5$ yields

$$\left(\frac{\beta\xi}{2\gamma}\right)^{\frac{1}{1-\beta}} < \left(\frac{\gamma}{2c\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{24}$$

Solving for ξ yields the result. ■

Proof of Proposition 1. Let y_A and y_B be the investment levels that satisfy the first-order conditions, $q_i \text{MP}_i = \alpha y_i^{\alpha-1}$, and suppose that $q_A \neq 0.5$ so that $y_A \neq y_B$. Note that the MRTS $= (y_A/y_B)^{\rho-1}$. Fix the isoquant through (y_A, y_B). Then there exists a constant, k, such that $\text{MP}_\text{i} = k y_i^{\rho-1}$. Thus, the first-order condition is $k q_i y_i^{\rho-1} = \alpha y_i^{\alpha-1}$, which we rewrite as

$$k q_i = \alpha y_i^{\alpha-\rho}. \tag{25}$$

Note that the right-hand side is strictly convex because $\alpha > 2$ and $\rho < 1$. Thus, adding the equations in (25) for A and B and dividing by 2 yields:

$$\frac{k}{2}\left(q_A + (1-q_A)\right) = \frac{1}{2}\alpha y_A^{\alpha-\rho} + \frac{1}{2}\alpha y_B^{\alpha-\rho} > \alpha\left(\frac{y_A}{2} + \frac{y_B}{2}\right)^{\alpha-\rho}. \tag{26}$$

Note that $((y_A + y_B)/2, (y_A + y_B)/2))$ is strictly above the isoquant because of concavity of the production function. Let $\bar{y} < (y_A + y_B)/2$ be the common value of y such that $(\bar{y}, \bar{y})$ is on the isoquant through (y_A, y_B). Then $0.5k > \alpha\bar{y}^{\alpha-\rho}$, which is equivalent to $0.5k y^{\rho-1} > \alpha\bar{y}^{\alpha-1}$. Using again that $\text{MP}_\text{i} = k y_i^{\rho-1}$, this implies that $0.5\text{MP}_i > \alpha\bar{y}^{\alpha-1}$, for both $i = A, B$. As a consequence, equality in the first-order condition $0.5\text{MP}_i = MC_i$ holds for some $y^* > \bar{y}$. That is, (y^*, y^*) is above the isoquant though (y_A, y_B) and hence output cannot be maximized when $y_A \neq y_B$, i.e., when $q_A \neq 0.5$.

Finally, note that $q_A = 0.5$ also maximizes total welfare. In particular, $2C(\bar{y}) < 2C((y_A + y_B)/2) < C(y_A) + C(y_B)$ where the first inequality follows from $\bar{y} < (y_A + y_B)/2$, and the second follows from the convexity of the cost function. ■

Proof of Proposition 2. The symmetry of agents A and B implies that output must assume a critical value at $q_A = q_B = 0.5$. It is therefore sufficient to sign the second derivative. Let $q_A = q$ and $q_B = 1 - q$. Then in the interior solution, the derivatives of effort levels with respect to q take the form

$$y_A'(q) = y_A(q)f(q)h(q) \quad \text{and} \quad y_B'(q) = -y_B(q)f(q)h(1-q); \tag{27}$$

$$x_A'(q) = -k(1-q) - y_A'(q) \quad \text{and} \quad x_B'(q) = k(q) - y_B'(q), \tag{28}$$

where

$$f(q) = \frac{1}{(1-\beta)q(1-q)}; \tag{29}$$

$$h(q) = 1 + \frac{2(\beta-\rho)}{\rho-1}\frac{\left(\frac{q}{1-q}\right)^{\frac{2\rho}{\rho-1}}}{1+\left(\frac{q}{1-q}\right)^{\frac{2\rho}{\rho-1}}}; \tag{30}$$

$$k(q) = \left(\frac{q\gamma}{c\alpha}\right)^{\frac{1}{\alpha-1}}\frac{1}{q}. \tag{31}$$

Note that

$$f'(q) = f(q)\frac{2q-1}{q(1-q)}, \quad h'(q) = \frac{2\rho}{\rho-1}\frac{h(q)-1}{q(1-q)\left(1+\left(\frac{q}{1-q}\right)^{\frac{2\rho}{\rho-1}}\right)}$$

Next, evaluate the first- and second-order derivatives of $\theta(q)$ at $q = 0.5$. Let $\psi(q) = y_A(q)^\rho + y_B(q)^\rho$, then

$$\theta(q) = \underline{\theta} + \xi\left(\frac{1}{2}\right)^{\frac{\beta}{\rho}}\psi(q)^{\frac{\beta}{\rho}} \tag{32}$$

$$\theta'(q) = \frac{\xi\beta\left(\frac{1}{2}\right)^{\frac{\beta}{\rho}}}{\rho}\psi(q)^{\frac{\beta}{\rho}-1}\psi'(q) \tag{33}$$

$$\theta''(q) = \frac{\xi\beta\left(\frac{1}{2}\right)^{\frac{\beta}{\rho}}}{\rho}\psi(q)^{\frac{\beta}{\rho}-2}\left(\left(\frac{\beta}{\rho}-1\right)\psi'(q)^2 + \psi(q)\psi''(q)\right) \tag{34}$$

with

$$\psi(q) = y_A(q)^\rho + y_B(q)^\rho \tag{35}$$

$$\psi'(q) = \rho f(q)\left(y_A(q)^\rho h(q) - y_B(q)^\rho h(1-q)\right) \tag{36}$$

$$\begin{aligned}\psi''(q) &= \rho f'(q)\left(y_A(q)^\rho h(q) - y_B(q)^\rho h(1-q)\right)\\ &\quad + \rho f(q)\left[\rho y_A(q)^\rho f(q)h(q)^2 + y_A(q)^\rho h'(q) + \rho y_B(q)^\rho f(q)h(1-q)^2 + y_B(q)^\rho h'(1-q)\right]\end{aligned} \tag{37}$$

Let $\bar{y} = y_A(0.5) = y_B(0.5) = (\xi\beta/2\gamma)^{1/(1-\beta)}$, then

$$\begin{aligned}\psi(0.5) &= 2\bar{y}^\rho > 0,\\ \psi'(0.5) &= 0,\\ \psi''(0.5) &= 2\rho f(0.5)\bar{y}^\rho\left[\rho f(0.5)h(0.5)^2 + h'(0.5)\right]\end{aligned}$$

Since $f(0.5) = \frac{4}{1-\beta}$, $h(0.5) = \frac{1-\beta}{1-\rho}$, and $h'(0.5) = -\frac{4\rho(\rho-\beta)}{(1-\rho)^2}$, we have

$$\theta'(0.5) = 0, \tag{38}$$

$$\theta''(0.5) = \left(\frac{\xi\beta}{2\gamma}\right)^{\frac{\beta}{1-\beta}} \frac{16\xi\beta\rho}{(1-\beta)(1-\rho)}. \tag{39}$$

Since the sign of $\theta''(0.5)$ inherits that of ρ, we have shown the desired result.

It is immediate that any investments in an unused outside option are socially inefficient, and, by raising the marginal cost of investment they lower investment in the joint project below the social optimum.

We now prove that the local maximum is a global maximum when $\rho < 0$, i.e., when the elasticity of substitution is less than 1. We claim that in the interior solution, if $\rho < 0$, then $\theta'(q) > 0$ for all $q < 0.5$. From (33) and (36), it follows that

$$\operatorname{sgn}(\theta'(q)) = \operatorname{sgn}\left(y_A(q)^\rho h(q) - y_B(q)^\rho h(1-q)\right)$$

Let $z(q) = \left(\frac{q}{1-q}\right)^{\frac{2\rho}{\rho-1}}$. Then

$$\begin{aligned} y_A(q)^\rho &= C_y z(q)^{\frac{\rho-1}{2(1-\beta)}} (1+z(q))^{\frac{\beta-\rho}{1-\beta}} \\ y_B(q)^\rho &= C_y z(1-q)^{\frac{\rho-1}{2(1-\beta)}} (1+z(1-q))^{\frac{\beta-\rho}{1-\beta}} \end{aligned}$$

where $C_y = \left(\frac{\beta\xi}{2^{\frac{\beta}{\rho}}\gamma}\right)^{\frac{\rho}{1-\beta}} > 0$. Therefore,

$$\begin{aligned} \operatorname{sgn}\left(y_A(q)^\rho h(q) - y_B(q)^\rho h(1-q)\right) = \operatorname{sgn}\Big[& z(q)^{\frac{\rho-1}{2(1-\beta)}} (1+z(q))^{\frac{\beta-\rho}{1-\beta}} \left(1 + \frac{2(\beta-\rho)}{\rho-1}\frac{z(q)}{1+z(q)}\right) \\ & - z(1-q)^{\frac{\rho-1}{2(1-\beta)}} (1+z(1-q))^{\frac{\beta-\rho}{1-\beta}} \left(1 + \frac{2(\beta-\rho)}{\rho-1}\frac{z(1-q)}{1+z(1-q)}\right)\Big]. \end{aligned}$$

Since $\rho < 0$, $z(q) < 1$ for $q < 0.5$. Further, $z(1-q) = z(q)^{-1} > 1$. Therefore,

$$\begin{aligned} 1 + \frac{2(\beta-\rho)}{\rho-1}\frac{z(q)}{1+z(q)} &= \frac{(1-\rho) + (1+\rho-2\beta)z(q)}{(1-\rho)(1+z(q))} > 0, \\ 1 + \frac{2(\beta-\rho)}{\rho-1}\frac{z(q)}{1+z(q)} &> 1 + \frac{2(\beta-\rho)}{\rho-1}\frac{z(1-q)}{1+z(1-q)}. \end{aligned}$$

Therefore, it suffices to show that $z(q)^{\frac{\rho-1}{2(1-\beta)}}(1+z(q))^{\frac{\beta-\rho}{1-\beta}} > z(1-q)^{\frac{\rho-1}{2(1-\beta)}}(1+z(1-q))^{\frac{\beta-\rho}{1-\beta}}$. As

$$\begin{aligned} z(q)^{\rho-1}(1+z(q))^{2(\beta-\rho)} - z(1-q)^{\rho-1}(1+z(1-q))^{2(\beta-\rho)} &= z(q)^{\rho-1}(1+z(q))^{2(\beta-\rho)} - z(q)^{1-\rho}\left(\frac{1+z(q)}{z(q)}\right)^{2(\beta-\rho)} \\ &= (1+z(q))^{2(\beta-\rho)}\left(z(q)^{\rho-1} - z(q)^{1+\rho-2\beta}\right) > 0, \end{aligned}$$

where the last inequality is from $\beta < 1$, we have shown the desired result. ■

Proof of Proposition 4. Symmetry implies that $\frac{\partial x_A}{\partial q_A}|_{q_A=0.5} = -\frac{\partial x_B}{\partial q_A}|_{q_A=0.5}$ and $\frac{\partial y_A}{\partial q_A}|_{q_A=0.5} = -\frac{\partial y_B}{\partial q_A}|_{q_A=0.5}$. Further,

$$\frac{\partial y_A}{\partial q_A}\Big|_{q_A=0.5} = \frac{2^{\frac{2\beta-1}{\beta-1}}\left(\frac{\beta\xi}{\gamma}\right)^{\frac{1}{1-\beta}}}{1-\rho} > 0 \text{ and } \frac{\partial x_A}{\partial q_A}\Big|_{q_A=0.5} = -\frac{2^{\frac{\alpha-2}{\alpha-1}}\left(\frac{\gamma}{\alpha c}\right)^{\frac{1}{\alpha-1}}}{\alpha-1} - \frac{\partial y_A}{\partial q_A}\Big|_{q_A=0.5}. \tag{40}$$

Proposition 2 implies that $\frac{\partial\Theta(y_A,y_B)}{\partial q_i}|_{q_A=0.5} = 0$. Thus,

$$\begin{aligned}\frac{\partial u_A(x_A,x_B,y_A,y_B)}{\partial q_A}\Big|_{q_A=0.5} &= q_B\gamma\frac{\partial x_A}{\partial q_A} - q_A\gamma\frac{\partial x_B}{\partial q_A} - \gamma(x_A+x_B) + q_A\frac{\partial\Theta(y_A,y_B)}{q_A} + \Theta(y_A,y_B)\\ &\quad - \alpha c(x_A+y_A)^{\alpha-1}\left(\frac{\partial x_A}{\partial q_A}+\frac{\partial y_A}{\partial q_A}\right) \\ &= \gamma\frac{\partial x_A}{\partial q_A}\Big|_{q_A=0.5} - 2(\underline{w}+\gamma x_A) + \Theta(y_A,y_B) - \alpha c(x_A+y_A)^{\alpha-1}\left(\frac{\partial x_A}{\partial q_A}+\frac{\partial y_A}{\partial q_A}\right).\end{aligned} \tag{41}$$

Note that

$$\Theta(y_A,y_B)|_{q_A=0.5} = \underline{\theta} + \xi\left(\frac{\beta\xi}{2\gamma}\right)^{\frac{\beta}{1-\beta}}. \tag{42}$$

Because $x_A \geq 0$ we get

$$\frac{\partial u_A(x_A,x_B,y_A,y_B)}{\partial q_A}\Big|_{q_A=0.5} = -2\underline{w} + \underline{\theta} - \gamma\left(\frac{\gamma}{2\alpha c}\right)^{\frac{1}{\alpha-1}}\left(\frac{1}{\alpha-1}+2\right) + 2\gamma\left(\frac{\beta\xi}{2\gamma}\right)^{\frac{\beta}{1-\beta}}\left(1+\frac{1}{\beta}-\frac{2}{1-\rho}\right), \tag{43}$$

which is negative if ρ is sufficiently close to 1. Similarly, it follows that $\frac{\partial u_A(x_A,x_B,y_A,y_B)}{\partial q_B}\Big|_{q_A=0.5} > 0$.

The last statement follows immediately from symmetry at $q_A = 0.5$. ∎

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