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Profit Realisation and the Necessity of Credit Expansion: A Stock–Flow Consistent Analysis of the Monetary Gap

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Abstract:

Why can aggregate profit be persistently realised in monetary production economies despite the apparent insufficiency of the initial money supply? Existing explanations have attributed this problem to a variety of institutional mechanisms, including bank credit, capitalist expenditure, government deficits, financial innovation, and foreign trade. This paper argues that these explanations identify different manifestations of a deeper structural phenomenon without explaining its underlying cause. Building on the Credit-Debt Reproduction Mechanism (CDRM), the paper demonstrates that, in debt-based monetary production economies, the endogenous reproduction of financial claims systematically exceeds the reproduction of the means of monetary settlement. Consequently, the realisation of aggregate profit in monetary form requires a recurrent source of exogenous purchasing power. The argument is developed in three stages. First, a historical analysis shows that debt remissions, usury restrictions, monetary debasement, and successive monetary innovations can be interpreted as institutional responses to a recurring shortage of circulating liquidity. Second, a formal mathematical model establishes the necessary relationship between profit realisation, debt accumulation, and monetary scarcity. Third, a stock-flow consistent model and dynamic simulations demonstrate that, in the absence of sufficient public-sector monetary injections, the monetary gap expands cumulatively, whereas appropriate fiscal recycling of purchasing power stabilises the system. The paper concludes that the recurrent need for monetary accommodation is not a market imperfection but an endogenous property of debt-based monetary production, offering a unified theoretical explanation for a wide range of historical and contemporary monetary phenomena.

The central argument advanced in this paper is that, in debt-based monetary production economies, the endogenous reproduction of financial claims systematically outpaces the reproduction of the means of monetary settlement. Consequently, recurrent exogenous monetary accommodation is not an optional policy instrument but a structural requirement for the monetary realisation of aggregate profit. The remainder of the paper develops this argument through historical analysis, formal mathematical derivation, and stock-flow consistent simulation.

JEL: N1, E4, E5, B5, C6

Introduction:

The fascination with large numbers goes beyond discipline, manifesting with equal gravity in the biological doubling process of cell division and the mathematical compounding of interest. A seminal illustration of exponential growth was recorded by the 13th-century scholar Ibn Khallikan. In his biographical dictionary (Translated by de Slane in 1843), Ibn Khallikan describes the legend of Sissah, the inventor of chess, who presented the game to his sovereign.

Captivated by the invention, the King offered Sissah any reward he desired. Sissah made a seemingly modest request:

"…that a grain of wheat be placed in the first square of the chessboard, two in the second, and that the number of grains be progressively doubled until the last square is attained; whatever this quantity may be" (Ibn Khallikan, trans. 1843).

While the King initially underestimated the request, he soon discovered the mathematical reality: the cumulative sum exceeded the global wheat supply. This narrative serves as a profound metaphor for the "concept of shortage" that arises when a synthetic reproduction process, such as debt or compound interest, is applied to a lifeless system. It highlights the inevitable friction between infinite mathematical expansion and the finite constraints of the physical world.

In this illustration, exponential growth inevitably culminates in a grain deficit; however, the severity of this shortage is a function of the growth rate. If the requested increment were reduced from a 100% increase (doubling) to a 50% compound rate, the physical requirements would shift dramatically.

Assuming an average grain weight of 0.02 g, the doubling sequence produces approximately 369 billion metric tons of rice, vastly exceeding current global agricultural production capacity. In contrast, a 50% growth sequence produces approximately 7,360 metric tons, a quantity that lies well within the capacity of modern agricultural systems. The analytical significance of this comparison lies in its implicit assumption: that a shortage can be mitigated only if technological advancement or productivity gains can outpace the rate of systemic expansion.

The Sissah story serves only as an investigative device. Its purpose is not to equate biological growth with monetary dynamics, but to illustrate a more general principle: cumulative claims may expand according to a logic that eventually exceeds the capacity of the system responsible for their realisation. Whether this principle also characterises monetary production economies cannot be inferred from the story itself. It requires both a historical examination of how societies have addressed recurrent shortages of the means of payment and a theoretical explanation of why such shortages emerge within modern credit economies. The following section develops this argument by first distinguishing biological reproduction from monetary circulation and then tracing the historical evolution of institutional responses to the problem of monetary scarcity.

The paper addresses three closely related questions. First, why has monetary scarcity repeatedly emerged throughout the historical evolution of debt-based economies despite continual monetary innovation? Second, can this phenomenon be derived as a necessary consequence of the accounting structure of a profit-oriented monetary production economy? Third, what institutional conditions are required to reconcile the endogenous expansion of financial claims with the monetary realisation of aggregate profit?

1. From Biological Reproduction to Monetary Realisation

The distinction between biological reproduction and monetary circulation is central to the theoretical framework developed in this paper. In Sissah's grain story, scarcity reflects the finite capacity of biological reproduction: exponential claims eventually exceed the physical harvest. In a monetary production economy, by contrast, monetary scarcity is endogenous to the credit-debt reproduction mechanism (CDRM), the recursive process through which credit creation simultaneously generates debt obligations whose servicing and repayment require further rounds of credit expansion, unless additional non-interest-bearing purchasing power is introduced exogenously into the economy, typically through public-sector monetary injections. Monetary shortages therefore emerge from the institutional rules governing the creation, circulation, and settlement of credit-money and the associated balance-sheet constraints, rather than from the physical limits of production itself. Because money is not a self-reproducing commodity, the aggregate profit of the economy is not a matter of economic efficiency, and it cannot be realised in monetary form using only the initial stock of money in circulation.

The primary contribution of this paper is not to identify yet another institutional mechanism through which aggregate profit may be realised, but to explain why all such mechanisms ultimately become necessary. Through formal mathematical derivation and stock-flow consistent simulation, the paper demonstrates that the recurrent need for additional purchasing power is an endogenous consequence of the credit-debt reproduction mechanism itself. The various historical and theoretical solutions proposed in the literature, including debt jubilees, monetary expansion through debasement, government deficits, bank credit, and financial innovation such as paper or fiat money, can therefore be understood as institutionally distinct responses to the same underlying structural constraint.

The structural distinction mentioned above also provides a useful perspective, allowing us to evaluate previous attempts in explaining the realisation of aggregate profit in monetary economies. Although Circuitist scholars have proposed a variety of institutional mechanisms, including bank credit, capitalist expenditure, government deficits, etc, their explanations ultimately converge on a common conclusion: aggregate profit cannot be realised solely through the initial stock of money generated within the production process. Regardless of the institutional form, each solution requires an additional injection of purchasing power from outside the existing circuit of commodity exchange. As discussed in greater detail below, the present paper interprets this common conclusion not as a coincidence, but as evidence of a more fundamental property of the credit-debt reproduction mechanism[1].

This theoretical impasse invites a critical and historical inquiry: Does the documented evolution of political economy provide evidence that profit-oriented systems, whether

[1] For a complete review of the Circuitists' theories on profit realisation, see Seccareccia (1996), Rochon (2005), and Javidanrad (2020)

denominated in commodities, bullion, or credit, systemically culminate in a shortage of the circulating media?

To answer this question, the analysis proceeds in two stages. It first establishes the conceptual distinction between biological reproduction and monetary reproduction, demonstrating why the logic governing the expansion of financial claims differs fundamentally from that governing physical production. It then examines the historical evolution of monetary institutions before formalising the argument within a stock-flow consistent framework.

1.1. The Issue of the Shortage Through History: A Structural Analysis

The tension between expanding debt obligations and the finite availability of the circulating means of payment has shaped the institutional evolution of monetary economies for millennia. Across different historical periods and civilisations, excessive indebtedness repeatedly generated social instability, concentration of property ownership, debt bondage, and periodic economic crises. In response, states, legal systems, and religious traditions developed institutional mechanisms intended to restrain the accumulation of debt or periodically restore the economy's capacity to reproduce itself. Although these measures differed in form and justification, they reveal a recurring historical concern with the sustainability of monetary obligations relative to the available means of settlement.

Some of the earliest documented responses emerged in ancient Mesopotamia. As Hudson (2018) documents, rulers of Sumer, Akkad, Babylon, and Assyria periodically proclaimed debt remissions ("clean slates") to cancel personal agricultural debts, release debt servants, and restore land to its original cultivators. These proclamations were not exceptional acts of generosity, but rather recurring administrative policies designed to preserve productive capacity, maintain military service, and prevent the concentration of wealth through the accumulation of irreversible debt. Likewise, the Code of Hammurabi imposed legal restrictions on lending contracts and collateral, reflecting an early recognition that unrestricted debt expansion could undermine economic stability.

Comparable concerns are evident throughout the Abrahamic religious traditions. The Hebrew Bible prohibits charging interest on loans to fellow Israelites (The Bible, Exodus 22:25; Leviticus 25:35–37; Deuteronomy 23:19–20) and establishes the Jubilee institution as a periodic cancellation of debts and restoration of property rights (Leviticus 25). Early Christianity largely inherited these principles, and for centuries the Church regarded usury as incompatible with justice, a position reinforced through medieval canon law and ecumenical councils. Similarly, Islamic jurisprudence prohibits riba (The Qur'an, 2:275–279; 3:130), while encouraging commercial partnerships and profit-sharing arrangements in place of predetermined interest on loans. Although grounded in distinct theological traditions, these legal and religious frameworks converge in recognising that unrestricted debt accumulation poses systemic risks extending beyond individual borrowers and lenders.

Within this historical context, Hudson (2018) argues that even the prohibition against coveting a neighbour's household in the Ten Commandments may be understood not simply as a moral injunction but as part of a broader institutional framework aimed at limiting the social consequences of debt. According to this interpretation, households, including family members, servants, livestock, and land, were frequently pledged as collateral against unpayable debts, making the commandment an indirect protection against the progressive transfer of productive assets through creditor claims.

The remarkable recurrence of debt remissions, usury restrictions, and periodic debt forgiveness across geographically and culturally distinct societies suggests that these institutions addressed more than ethical concerns alone. Rather, they point towards a persistent structural tension within monetary economies: the tendency for financial obligations to expand more rapidly than the circulating means available for their settlement. The following sections argue that this historical regularity is best understood through the credit-debt reproduction mechanism, which explains why monetary shortages emerge endogenously even in economies with continuously expanding credit.

1.2. The Structural Wedge: Interest Rates in Antiquity

The exceptionally high nominal interest rates observed in ancient economies did not create the monetary shortage; rather, they amplified an underlying structural imbalance already inherent in debt-based exchange. Once credit contracts stipulated repayment exceeding the principal originally advanced, the aggregate claims outstanding necessarily exceeded the quantity of money initially introduced into circulation. High interest rates accelerated this divergence by increasing the pace at which debt obligations accumulated, thereby exposing more rapidly the insufficiency of the available means of payment.

Historical evidence illustrates the magnitude of this pressure. As shown in Table 1, grain loans in Mesopotamia frequently carried annual interest rates between 20 and 50 per cent, while silver loans commonly ranged from 10 to 40 per cent (Homer and Sylla, 2011). These rates undoubtedly intensified the debt burden borne by households and cultivators. Nevertheless, the structural difficulty cannot be reduced to the level of interest itself. Even substantially lower interest rates would not eliminate the underlying accounting requirement that aggregate monetary obligations eventually exceed the purchasing power originally injected into circulation unless additional money enters the system. High interest, therefore, represents an accelerator of an existing structural contradiction rather than its fundamental cause.

Table 1: Interest Rates on Grain & Silver in the Ancient World

Era / Empire	Period	Interest on Grain	Interest on Silver
Sumer	3000 – 1900 B.C.	33.3%	20 – 25%
Babylonia	1900 – 539 B.C.	20 – 33.3%	10 – 20%
Assyria	9th – 7th Cent. B.C.	30 – 50%	20 – 40%
Persian Empire	6th Cent. B.C.	40%	40%

Source: Homer and Sylla (2011)

The persistent inability of the ordinary people to fulfil these obligations, compounded by environmental shocks and restricted access to productive resources, necessitated the codification of rigorous legal frameworks. From the Code of Ur-Nammu (2050 B.C.) to the

Code of Hammurabi (1754 B.C.), these laws were designed to manage the fallout of widespread insolvency (Hudson, 2018; Graeber, 2014).

1.3. Systemic Stabilisers: Debt Jubilees and Monetary Debasement

Historical responses to recurrent debt crises can be interpreted as alternative methods of restoring consistency between expanding monetary obligations and the available means of settlement. Some interventions reduced outstanding liabilities directly through debt remissions and jubilees, while others increased effective liquidity by expanding the money supply. Although these policies differed institutionally, both addressed the same structural imbalance: the inability of the existing stock of money to support the realisation of accumulated financial claims.

In Ancient Mesopotamia and Egypt, rulers frequently utilised the "Clean Slate" or *andurarum* (debt jubilee). These were not acts of mere charity but functional necessities to prevent the total dissolution of the labour force into debt slavery. By periodically cancelling agrarian debts, the state ensured that the "shortage" of means of repayment did not lead to a permanent transfer of land and labour to a small creditor class, which would have undermined the tax base and military readiness.

In the West, this evolution continued with the Laws of Solon in Greece (c. 600 B.C.), which abolished debt bondage and restored the agricultural base (Harris, 2002)[2]. While ordinary people sought relief through jubilees, rulers addressed the shortage of means of payment (here, money) through monetary debasement. Either by replacing silver with gold or by reducing the precious metal content in coinage and replacing it with cheap metals, or in some cases in Europe, through clipping the coins (Chown, 1994). The Roman Empire, the Chinese Empire, and various Medieval European monarchies effectively increased the nominal volume of currency in circulation. (Verhulst, 2002; Aliber et al., 2015; Butcher, 2020). This "forced expansion" of the money supply allowed the state to meet its obligations and provided a temporary reprieve from the scarcity of bullion, a precursor to modern inflationary credit expansion.

In the Chinese Empire, a persistent "currency famine" (*qianhuang*) occurred because the supply of bronze coins could not meet the needs of a growing economy. This scarcity eventually imposed the world’s first transition from metallic coins to a paper-based system (Von Glahn, 2023).

[2] In contrast, while the Law of the Twelve Tables in Ancient Rome (450–449 B.C.) attempted to cap interest rates, the Roman state increasingly prioritised the “sanctity of contract” over social reproduction. As Rome transitioned from a republic to an empire, the systemic refusal to implement debt jubilees created a terminal liquidity crisis. This resulted in the “Colonate” system, where debt-stricken freeholders were bound to aristocratic estates, facilitating the eventual feudalisation of the European economy as a response to a collapsed monetary circuit (Banaji, 2007; Kindleberger and Aliber, 2005).

1.4. From Feudalism to the Capitalist Era: The Scholastic Compromise and the Transition to Modern Credit

In the Feudal era, the expansion of trade and the increasing complexity of commercial transactions intensified the demand for circulating money. This period witnessed a significant intellectual struggle led by the Scholastic scholars, or "Schoolmen", who functioned as the era's primary economic theorists under the umbrella of Church doctrine. As noted by Poitras (2000), a central preoccupation of these thinkers was the reconciliation of the theological prohibition against usury with the practical necessity of credit for a developing trade. The Schoolmen sought to identify a "fair" interest rate, or "just price" for money, through the development of concepts like *lucrum cessans* (profit forgone) and *damnum emergens* (loss incurred), effectively creating a moral and legal framework for debt contracts that could accommodate the rising need for liquidity.

By the Mercantilist era, this struggle for liquidity shifted toward the aggressive accumulation of "specie" (gold and silver). Mercantilist doctrine was fundamentally a response to the perceived shortage of money in a profit-led monetary economy; states pursued trade surpluses to mitigate the systemic stagnation caused by a deficit in the physical means of payment. This reliance on metallic standards, however, created a rigid ceiling on growth, as the money supply remained joined to the exogenous pace of bullion extraction rather than the endogenous requirements of production. Keynes (1936) explicitly explains the constant mercantilist's monetary dilemma in seeking new sources of gold & silver to reduce the economy's interest rate.

In General Theory, Keynes (1936, 214) refers to John Locke's explanation in a letter to a friend concerning usury (1621): “High interest decays Trade. The advantage from interest is greater than the Profit from Trade, which makes the rich Merchants give over, and put out their Stock to Interest, and the lesser Merchants Break”.

The transition to fiat money in modern economies marked a logical and necessary shift in the resolution of this shortage. By decoupling currency from physical commodities, the modern state and the central banking system gained the capacity to expand the money supply according to systemic needs. In the contemporary capitalist era, the "remedy" for the shortage has been institutionalised through the continuous expansion of bank credit. Within this framework, money originates as interest-bearing debt, ostensibly providing the liquidity necessary for the realisation of profit.

Yet, as the theoretical models in this paper will demonstrate, this transition from the "Clean Slate" of antiquity and the "Just Price" of the Scholastics to the "Fiat Credit Expansion" of modernity represents a change in form rather than a resolution of the underlying paradox. While fiat systems remove the physical constraints of bullion, they replace them with a mathematical imperative: because today's circulating money is yesterday's debt, the system requires a perpetual injection of new credit to prevent a recursive liquidity crisis. The "shortage" is no longer a failure of the harvest or the mine, but a structural feature of the monetary production economy itself.

Taken together, these historical responses suggest that monetary scarcity is not an accidental feature of institutional arrangements but a recurrent structural property of debt-based monetary economies. The following sections formalise this proposition through the Credit-Debt Reproduction Mechanism.

1.5. The Shortage of Money in Models:

The subsequent analysis constructs a series of theoretical models to demonstrate that the "shortage of money in circulation" persists as a fundamental, unresolved theoretical impasse, notwithstanding contemporary post-Keynesian assertions to the contrary. Critically, these models reveal that a theoretical solution is absent in all scenarios except those characterised by active and direct government intervention in the intersectoral distribution of profit. This specific case is analysed in the concluding section of this paper.

While the remaining models lack a formal theoretical solution, this does not preclude the existence of practical, short-term remedies within empirical economies. In a real-world context, both financial and non-financial institutions realise monetary profits. However, due to inherent liquidity constraints within a closed system, the source of these profits cannot be found within the existing stock of money in circulation. This is what we know as the "paradox of profit" (Marx, Vol. 2, 1867).

To allow profit-seeking sectors (firms and banks) to realise monetary gains, an exogenous injection of credit, supplied by the banking sector, is required as a "practical remedy" (Javidanrad et al., 2024). Given the zero-sum nature of monetary balances across sectors, this profit realisation is functionally chained to the accumulation of debt in the non-profit-seeking sectors (households and government). Consequently, the system remains viable only as long as these sectors can sustain increasing debt-to-income ratios. This trajectory is fundamentally unsustainable; it is a temporal delay rather than a resolution, inevitably leading to a point where a sector, most notably the household sector, can no longer service its obligations and is forced to withdraw from circulation, and this is the starting point of the crisis.

The search for these compensatory funding sources introduces broader dynamics of contemporary capitalism, specifically financialisation, which is defined as the process of accumulation in which “financial motives, financial markets, financial actors and financial institutions in the operation of the domestic and international economies” receive greater weight and attention in all decision-making procedures. (Epstein, 2005; Krippner, 2005).

The term "practical remedy" that we used before is employed here deliberately to distinguish these temporary palliatives from a "final decisive cure". Since the advent of the monetary production economy, the primary remedy has been the continuous expansion of private/bank credit. This liquidity injection temporarily obscures the paradox of profit and alleviates short-term scarcity, yet it simultaneously initiates a self-reinforcing process of financialisation. Due to its progressive nature, the underlying debt burden eventually resurfaces as a systemic crisis.

Before moving to the next section, we need to clarify the concept of profit from the accounting and economics points of view. Throughout this paper, profit refers to realised monetary profit rather than merely accounting profit. Although mark-up pricing allows firms to record a positive accounting surplus, the realisation of this surplus requires an equivalent monetary flow to return to the firm sector. The analysis, therefore, focuses on the monetary conditions under which accounting profits can be transformed into realised profits.

2. Model Methodology and Scope

The theoretical models developed herein serve a specific experimental purpose: to demonstrate the inherent shortage of circulating money generated by interest-bearing debt and mark-up pricing mechanisms in the absence of exogenous monetary injections. To maintain analytical clarity, these models abstract from the sequential complexities mentioned above, focusing instead on tracing aggregate monetary flows between sectors.

The primary objective is to determine whether monetary profit can be realised within each profit-seeking sector using the initial money stock within a single period, without recourse to intertemporal analysis. The focus remains on the *possibility* of monetary realisation rather than the specific ownership of profit. These models are static and built upon simplifying assumptions and varying levels of abstraction. While the mathematical proofs increase in complexity as the models move toward greater empirical realism, the foundational logic remains consistent.

2.1. The Formal Modelling Procedure

The construction and analysis of these models follow a five-step protocol:

Sectoral Identification: Defining the primary economic sectors and the aggregate monetary transactions between them.

Simplification: Establishing assumptions to isolate variables and reduce computational complexity.

Formalisation: Applying mathematical notation to derive expressions for sectoral monetary flows through the balance sheet matrix and the transaction flow matrix, which are the main instruments of the Stock–Flow Consistent (SFC) Representation.

Defining the Behavioural Equations for the SFC model: This is the vital part of the SFC model for simulation. We need these equations to understand the dynamics of each sector and its connections.

Equilibrium Analysis: Identifying monetary surpluses or deficits within each sector to evaluate the total monetary balance of the macroeconomy.

In the beginning, we start with a very simple model with three main sectors, mainly households (h), firms (f), and banks (b). The banking sector is the creator of money in the system, and it provides the total supply of money and credit for production.

2.2. The Models:

All models are conducted within a closed-economy framework. This assumption is not adopted merely for analytical simplicity but is central to the theoretical objective of the paper. The purpose of the model is to examine whether the existing stock of money generated within a self-contained monetary economy is sufficient to meet the monetary requirements of production, debt servicing, and profit realisation. By excluding external sectors, the model isolates the internal monetary dynamics of the system and allows any shortage of money to be identified without recourse to external sources of demand or liquidity.

This assumption is also consistent with the long-standing debate on capitalist accumulation and profit realisation. As argued by Rosa Luxemburg (1913), capitalist economies may partially overcome limitations arising from insufficient internal demand through expansion into non-capitalist markets and external trade relations. In an open economy, therefore, foreign markets, external borrowing, capital inflows, or trade surpluses may provide additional monetary resources that alleviate or postpone the pressures generated by the domestic monetary constraint. Such mechanisms can serve as alternatives to domestic credit expansion for sustaining accumulation and profit realisation.

By deliberately abstracting from these external channels, the present model seeks to determine whether a monetary shortage emerges even under the most favourable conditions for internal circulation. Consequently, if a monetary gap is shown to exist in a closed economy, its emergence cannot be attributed to external leakages but must instead be understood as a structural feature of the domestic monetary system itself.

The starting point is the simplest model in Section 2.3, and then we move to the fundamental model without a government in Section 2.4, which is a more complicated model with some relaxed assumptions in the simplest model, and finally, in Section 2.5, we talk about the extended model with a government.

2.3. The Simplest Model (Key Assumptions):

The model is constructed under the following simplifying assumptions:

1. The economy is closed, and there is no government sector. The economy consists of only three sectors: households (h), firms (f), and banks (b).

2. There is a single representative bank, a single representative firm, and n households. The entire population is assumed to participate in the labour force; hence, labour is not a scarce factor of production.

3. The banking sector operates without labour costs or operational expenses (like money-lending operations in history). The bank is assumed to supply credit elastically whenever demanded and to receive repayments without additional cost.

4. Individual wages are homogeneous and fixed over the period under consideration. Managerial salaries may be interpreted as multiples of the representative wage and are

therefore incorporated into $\boldsymbol{n}$. The size of the labour force remains constant throughout the period.

5. Households spend all wage income on consumption $W = C$, where $W$and C represent total wage and total consumption, respectively. Therefore, there are no household savings.
6. The nominal interest rate on bank loans is fixed at $\boldsymbol{r}$ in the model.
7. The time horizon is one production period (one year). Consequently, monetary transactions occur only between households and firms in the goods and labour markets, and between firms and banks in the credit market.
8. At the beginning of the period, households and firms hold no initial monetary balances. The total and initial money stock (M) in the economy consists exclusively of bank-created credit of size L_0, extended to firms at the start of the period, i.e., $M = L$, where L represents the initial loan to the production firm. All financial obligations are settled at the end of the period.
9. Production follows a simplified labour-based production function:

$$Q = f(n)$$

Capital is assumed either to be costless or unnecessary for production (like picking apples from a tree). Accordingly, all borrowed funds are used to finance labour costs, implying:

$$L = W$$

So, the total cost of the production is the cost of labour and the cost of borrowing at a rate r, i.e.:

$$TC = W + rL = L(1 + r)$$

10. The only production costs are labour costs and interest payments on bank borrowing. No additional input or operational costs are assumed.
11. The firm sets the price with mark-up μ over the average cost, so: $P = (1 + \mu)\frac{TC}{Q}$.
12. The firm can sell all its products in the market to realise its maximum potential profit.

2.3.1. Balance Sheet Matrix and Transaction Flow Matrix (The Simplest Model)

Following the stock–flow consistent (SFC) framework introduced by Marc Lavoie and Wynne Godley (See Lavoie, 2022), this paper employs both a balance sheet matrix and a transaction flow matrix to ensure full accounting consistency between stocks and flows within the economy. The balance sheet matrix records the stock positions of each sector at a given point in time and identifies the corresponding financial assets and liabilities held by households, firms, and banks. This guarantees that every financial asset simultaneously appears as a liability elsewhere in the system. The transaction flow matrix, by contrast, records the monetary flows taking place between sectors during a given period, including wages, consumption expenditures, interest payments, and new credit creation.

Each monetary transaction is recorded simultaneously as an outflow for one sector and an inflow for another, thereby preserving the consistency of aggregate accounting relations. The combined use of these matrices is essential to the present analysis, as the model examines the relationship between the existing stock of money and the monetary flows required for debt servicing, wage payments, and profit realisation. The SFC framework, therefore, provides the formal accounting structure necessary to determine whether the available money stock is sufficient to meet the monetary obligations generated within the system. In doing so, it allows the model to identify the emergence of a structural monetary gap and the endogenous reproduction of credit and debt dynamics.

The balance sheet matrix for the simplest model (Table 2) shows the stock of assets and liabilities held by each sector at a point in time. For example, the household sector holds deposits M_h (which is the total amount of the initial loan $L = L_f = L_b$ or, equivalently, the total amount of the wage bill) as a positive value, and the firm owes the loan, and as it is a liability, it comes with a negative value.

The Key accounting rule for every row suggests that

$$\sum Assets + \sum Liabilities = 0$$

Table 2: The Balance Sheet Matrix of the Simplest Model

	Households (h)	***Firms (f)***	***Bank (b)***	Σ
Deposits M	$+M_h$	0	$-M_b$	0
Loans L	0	$-L_f$	$+L_b$	0
Net Worth	NW_h	NW_f	NW_b	0

The transaction flow matrix (Table 3) shows the flow of payments during a period. For example, the firm's column shows the amount of money that was received by (positive values) and paid out (negative values) by this sector[3].

Table 3: The Transfer Flow Matrix of the Simplest Model

Flow	***h***	***f***	***b***	Σ
Wages W	$+W_h$	$-W_f$	0	0

[3] In this table, the indices h, f, and b, do not change the value of the variables. So, $W_h = W_f = W, C_h = C_f = C, M_h = M_b = M,$ and $L_f = L_b = L$

Consumption C	$-C_h$	$+C_f$	0	0
Interest on loans rL	0	$-rL_f$	$+rL_b$	0
New loans ΔL	0	$+\Delta L_f$	$-\Delta L_b$	0
Change in deposits ΔM	$+\Delta M_h$	0	$-\Delta M_b$	0

2.3.2. Core Equations (The Simplest Model):

Based on the assumptions in Section 2.3, the derivation of the total revenue and profit of the firm would be:

$$TR = P \cdot Q = (1+\mu)TC$$

and

$$\Pi_F = TR - TC = \mu \cdot TC$$

respectively. So, the total monetary claims required would be

$$W + rL + \Pi_F = L + rL + \mu L(1+r) = L\big(1 + r + \mu(1+r)\big) = L(1+r)(1+\mu)$$

Which is apparently bigger than the total money available in the system:

$$M = L$$

The inequality

$$\text{Required money} > \text{Available money}$$

$$L(1+r)(1+\mu) > L \qquad (\boldsymbol{A})$$

always true for:

$$r > 0, \mu > 0$$

The interpretation is clear: Even if all wages are spent, and all goods are sold, there is still not enough money in the system to pay wages, pay interest, and realise profit. What we are going to show in the rest of this section is that profit realisation is not merely facilitated by credit expansion; it is structurally dependent on it

2.3.3. From Static to Dynamic (The Simplest Model):

The mathematical logic in Section 2.3 shows that the monetary circuit (loop) cannot be closed without a new credit injection. This means the circuit breaks unless a new loan (a new supply of credit) is granted. This means:

$$\Delta L > 0$$

and the bank injects new credit

$$\Delta M = \Delta L$$

or

$$M_t = M_{t-1} + \Delta L_t$$

The required amount of credit at the time $\boldsymbol{t}$ to close the circuit, is

$$\begin{aligned} Req_t &= W_t + rL_{t-1} + \Pi_t \\ &= M_{t-1} + rL_{t-1} + \Pi_t \end{aligned}$$

And the gap at the time $\boldsymbol{t}$ would be:

$$Gap_t = Req_t - M_{t-1} = rL_{t-1} + \Pi_t$$

So, the rule of credit will be:

$$\Delta L_t = \varphi \cdot Gap_t$$

Where, $0 < \varphi \leq 1$, but in this case, we assume for all models that the new loan covers the whole shortage (gap), i.e. $\varphi = 1$.

2.3.4. Dynamic Closure of the Simplest Model:

Tables 1 and 2 (Section 2.2.2) show that the existing (initial) stock of money M_t is insufficient for the required flows $W_t + rL_{t-1} + \Pi_t$ and for that reason, an additional flow of credit ΔL is needed to close the monetary circuit and sustain profit realisation. So, to make the system run, the dynamic closure of the simplest model requires the following equations:

$$\Delta L_t = Gap_t$$

$$L_t = L_{t-1} + \Delta L_t \quad \text{(Debt Accumulation)}$$

$$M_t = M_{t-1} + \Delta L_t \quad \text{(Deposit Accumulation)}$$

In a dynamic model, using a simulation method and assuming the interest rate is $r = 0.05$ and the markup profit is $\mu = 20\%$ with the initial loan (or initial amount of money) of $L_0 = M_0 = 100$ units for $T = 30$ periods, the divergence between the initial amount of credit and what is required to realise the profit is growing exponentially (see Picture 1) and locks the economic system into a path of explosive, co-dependent exponential growth in both total debt (L) and money supply (M), highlighting a fundamental instability within the credit-debt reproduction cycle, in which credit creates debt above its initial level and more credit is needed to redeem the debt (Javidanrad, 2020).

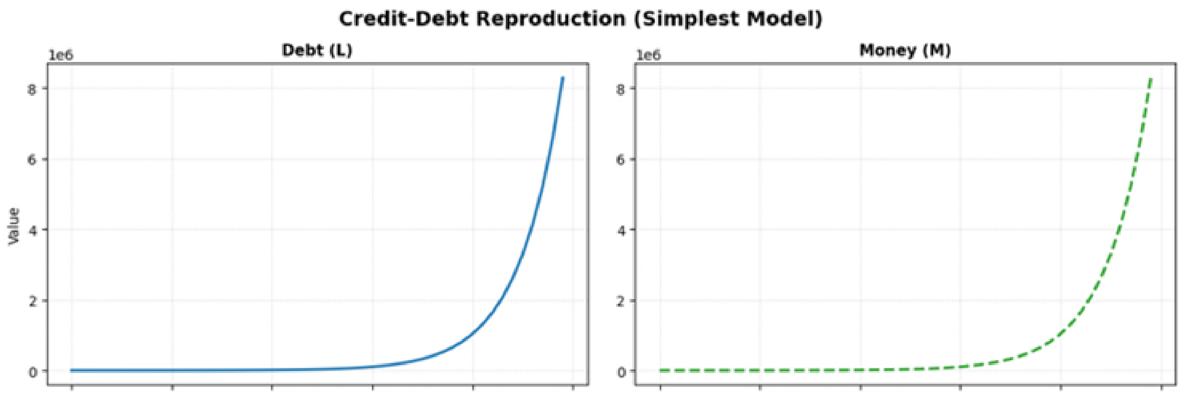
The simulation results for the first 10 years are provided in Table 4. It shows that the monetary gap remains positive in every period and increases over time as interest obligations accumulate. Since the existing stock of money is insufficient to satisfy these obligations, additional borrowing becomes necessary, leading to a continuous expansion of debt. The simulation therefore supports the central proposition of the model: the interaction between interest-bearing debt and a fixed stock of money generates an endogenous credit–debt reproduction mechanism.

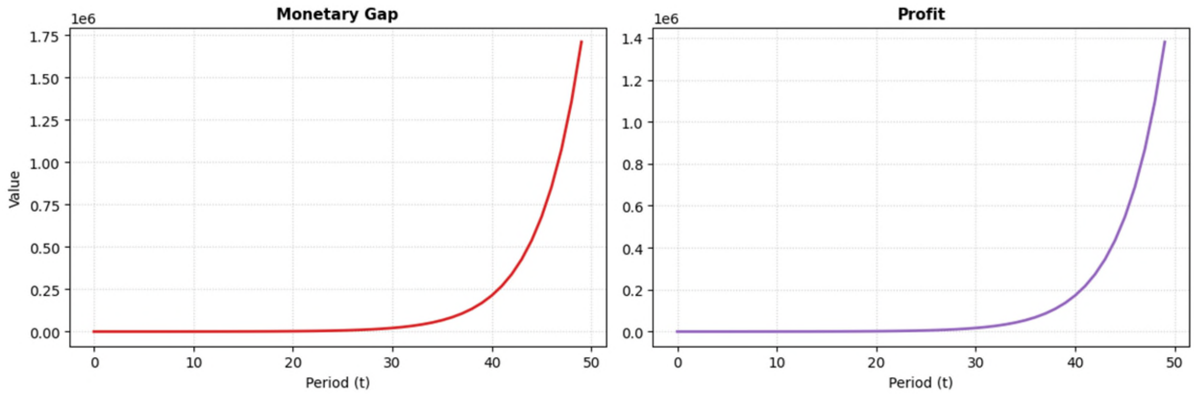
Table 4: Simulation results for the first 10 years for the simplest model

Year	Loans (L_t)	Interest (rL_t)	Monetary Gap
0	100.00	5.00	5.00
1	105.00	5.25	5.25
2	110.25	5.51	5.51
3	115.76	5.79	5.79
4	121.55	6.08	6.08
5	127.63	6.38	6.38
6	134.01	6.70	6.70
7	140.71	7.04	7.04
8	147.75	7.39	7.39
9	155.13	7.76	7.76
10	162.89	8.14	8.14

This compounding expansion occurs because the systemic monetary gap, driven by the joint requirements of interest obligations and firm markup expectations, outpaces the circulating liquidity available for debt servicing. Consequently, to avert widespread default, the banking sector must endogenously inject new credit ($\Delta L_t = Gap_t = Interest + Profit$). This means deposits explode massively because money is being created endogenously to fund its own systemic costs (interest and profit). The model thus illustrates how integrating positive interest rates and capital accumulation targets imposes a structural growth imperative, making long-term macroeconomic stability mathematically impossible without continuous credit expansion (see Picture 1).

Picture 1: The simulation graph of the Simplest Model without Government





We can consider many scenarios in which the interest rate and the markup profit are different, but as the inequality (A) Section 2.2.3 shows for all values of r and μ. There is a mathematical proof of a shortage of money in circulation.

2.4. The Fundamental Model Without Government

The simplifying assumption of a costless lender adopted in the previous model is now relaxed. In the simplest model, the banking sector was represented as a passive intermediary that extended credit and received interest payments without incurring any operating expenses. Under such conditions, interest income received by the bank was not returned to the circular flow of income and expenditure. As a consequence, a portion of the monetary stock was permanently withdrawn from circulation, generating what appears as the paradox of monetary profit. However, as Godley (1996) argues, a closed economy cannot contain a permanent monetary “black hole”. Any income received by one sector must ultimately reappear elsewhere in the system through expenditure on goods, services, labour, or financial assets. Accordingly, the present model introduces a more realistic banking sector that employs labour, incurs operating expenses, and reinjects all monetary income back into the economy.

Since the economy remains closed by assumption, the banking sector is assumed to have no incentive to hoard money or maintain idle cash balances. Any profit earned by the bank is recycled into the economic circuit through wage payments to its employees and through the acquisition of goods or financial assets issued by the productive sector. The objective of this modification is to examine whether the monetary shortage identified in the simplest model persists once the lender's income is fully reintegrated into the circular flow of funds.

The economy consists of three sectors: households, firms, and banks. There is a single representative bank, a single representative firm, and n households. The labour force is divided between the productive and financial sectors, such that

$$n = n_1 + n_2$$

where n_1 denotes the number of employees in the firm and n_2, the number of employees in the banking sector. For simplicity, all workers receive a uniform wage w, while managerial compensation is incorporated as a multiple of the representative wage.

Unlike the previous model, households are allowed to save a proportion α of their wage income in the form of bank deposits. Deposits earn a fixed interest rate i, while banks lend to firms at a higher interest rate r, where $r > i$. Both interest rates are assumed to be constant throughout the period. Households and firms begin the period with no initial monetary balances, implying that all monetary assets are generated within the model itself. Household deposits are formed only after wages have been paid and therefore do not represent inherited wealth or accumulated savings from previous periods.

The production technology remains deliberately simple. Output is produced solely through labour, and the wage bill constitutes the entire production cost before financing expenses. However, unlike the simplest model, firms finance production through a combination of debt and equity. A proportion λ of the wage bill is financed through bank borrowing, so the firm needs to borrow λB from the bank, while the remaining portion $(1 - \lambda)$ is financed through the issuance of equity claims. At this stage, for simplicity, we assume that the equity market is closed to individuals, so the bank is the only investor in the equity market, using customers' deposits and acting as an intermediary between savers and firms without creating additional credit in the system. Equity holders receive a fixed return (dividend) β and no capital gains or asset-price inflation are assumed. The financing structure, therefore, reflects the firm's capital composition rather than introducing speculative dynamics.
The introduction of bank employees requires an additional source of liquidity. Since banks must pay wages before receiving interest payments from firms or deposits from households, the wage bill of bank employees is financed through a separate line of credit created by the banking sector itself. Consequently, the total quantity of money circulating in the economy becomes

$$M = (n_1 + n_2)w = nw$$

This additional injection of purchasing power ensures that aggregate expenditure exceeds the wage bill of production workers alone. Nevertheless, as will be demonstrated below, the presence of additional liquidity does not eliminate the underlying monetary constraint. Although firms may generate accounting profits and all sectors may receive positive gross incomes, the simultaneous achievement of non-negative monetary balances across all sectors remains impossible under the assumed conditions. The paradox of monetary profit, therefore, persists despite the reinjection of banking-sector income into the circular flow.

Because households save a proportion of their income, part of the monetary stock is withdrawn from immediate circulation. For analytical simplicity, households are assumed not to participate directly in the equity market. Instead, the banking sector acts as the sole purchaser of equities issued by firms and finances these purchases using household deposits. This assumption isolates the effects of financial intermediation while avoiding additional complications associated with portfolio choice and asset-market dynamics.

Given the firm's financing structure, total production costs consist of labour costs, interest payments on debt, and payments to equity holders. The total cost of production is therefore

$$TC = n_1 w + \lambda n_1 w r + (1-\lambda) n_1 w (1+\beta)$$

$$= n_1 w (1 + \lambda r) + (1-\lambda) n_1 w (1+\beta)$$

Since the wage bill of production workers is equal to $n_1 w = B$, the total cost can be expressed as

$$TC = B[(1 + \lambda r) + (1-\lambda)(1+\beta)]$$

Assuming a one-period horizon in which all liabilities are settled at the end of the year, firms set prices according to a mark-up rule. The price of output is therefore given by

$$p = (1+\theta)\left[\frac{n_1 w (1+\lambda r) + (1-\lambda) n_1 w (1+\beta)}{Q}\right]$$

where θ denotes the mark-up rate and Q total output.

Under these assumptions, the monetary surplus or deficit associated with each sector can be derived from the corresponding transaction flows and balance-sheet positions. As in the simplest model, the aggregate sum of sectoral net financial balances remains identically equal to zero, reflecting the accounting consistency of the closed economy. The crucial question, however, is whether the existing stock of money is sufficient to satisfy all monetary obligations generated by production, financing, saving, and profit realisation. To address this question, the monetary form of the net accumulation of financial assets (NAFA) is calculated separately for each sector.[4] As in the previous section, the analysis is static and does not seek to identify an equilibrium or steady-state solution. Rather, it examines the monetary feasibility of profit realisation and debt servicing within a single production period.

2.4.1. Balance Sheet Matrix and Transaction Flow Matrix (The Fundamental Model)

The balance sheet of the fundamental model (Table 5) is very similar to that of the simplest model. In both, the fundamental financial instruments are bank deposits and bank loans, which are stock variables. There is only one extra row for equity, but in the transaction flow matrix (Table 6) , there are new features that are primarily flow variables, such as household savings and bank wage payments.

[4] See Javidanrad (2020, Ch. 4) for the calculation of the NAFA of each sector.

Table 5: The Balance Sheet of the Fundamental Model without Government

Assets/Liabilities	***Households*** h	***Firms*** f	***Banks*** b	Σ
Deposits M	$+M_h$	0	$-M_b$	0
Loans L	0	$-L_f$	$+L_b$	0
Equity E	0	$-E_f$	$+E_b$	0
Net Worth	NW_h	NW_f	NW_b	0

Unlike the previous TFM, which reports the monetary value of receivables and payables, the new form of the transaction flow matrix records only the accounting direction of each transaction. Positive and negative signs indicate the receiving and paying sectors, respectively, while zero entries denote the absence of a direct transaction. This presentation follows standard stock–flow consistent modelling practice and highlights the double-entry accounting relationships that underpin the model. The corresponding monetary values are determined subsequently by the behavioural and accounting equations.

Table 6: Transaction Flow Matrix of the Fundamental Model without Government

Current Transactions

Flows (Income & Expenditure Circuit)	***h***	***f***	***b***	Σ
Firm wages W_f	+	-	0	0
Bank wages W_b	+	0	-	0
Consumption C_h	-	+	0	0
Deposit interest iM_h	+	0	-	0
Loan interest rL_f	0	-	+	0
Dividend payment βE_f	0	-	+	0

Financial transactions

Financial Flows (Change in the Balance Sheet)	***h***	***f***	***b***	Σ
New deposits ΔM_h	+	0	-	0
New loans ΔL_f	0	+	-	0
Equity issuance ΔE_f	0	+	-	0

2.4.2. Core Equations (The Fundamental Model):

Based on the assumption of the fundamental model, some of the core behavioural equations and the profit of the firm and the bank will be as follows:

Household Income: $Y_h = W_f + W_b + iM_h$ (where, $W_f = B = n_1 w,$ and $W_b = n_2 w$)

Consumption: $C_h = (1 - \alpha)Y_h$

Saving: $S_h = \alpha Y_h$

Deposit Accumulation: $\Delta M_h = S_h$ (or equivalently $M_h(t) = M_h(t-1) + S_h(t)$)

Production Function: $Q = f(n_1)$

Firm Borrowing: $L_f = \lambda W_f = \lambda B$ (where $B = n_1 w$)

Equity Financing: $E_f = (1-\lambda)W_f = (1-\lambda)B$

Firm Profit: $\Pi_f = C_h - W_f - rL_f - \beta E_f$

Bank Profit: $\Pi_b = rL_f + \beta E_f - iM_h - W_b$

It can be proved mathematically that there will be a shortage of money (monetary gap) and, hypothetically, not all three sectors can have a surplus value in their accounts in a static model (see Javidanrad, 2020). So, if the monetary obligations generated through the period can be shown by:

$$W_f + W_b + rL_f + \beta E_f + \Pi_f + \Pi_b$$

Therefore, the monetary gap will be:

$$Gap_t = \left(W_f + W_b + rL_f + \beta E_f + \Pi_f + \Pi_b\right) - M_h$$

2.4.3. Dynamic Closure of the Fundamental Model

The dynamic closure of the fundamental model is similar to that of the simplest model. So, again we have:

$$\Delta L_t = Gap_t$$

$$L_t = L_{t-1} + \Delta L_t \quad \text{(Debt Accumulation)}$$

$$M_t = M_{t-1} + \Delta L_t \quad \text{(Deposit Accumulation)}$$

In a dynamic model, we consider two scenarios:

Repayable Equity Claims (Scenario A): In this scenario, the principal value of equity is repaid within one period (e.g. one year)

Standard Equity Finance (Scenario B): In this scenario, only dividends are paid, and equity ownership continues over different periods, possibly with different ownership.

The emergence of a monetary gap does not depend on the assumption that equity principal is repaid. Even under a conventional equity-financing structure, where only dividends are paid, the cash on hand remains insufficient to meet the monetary obligations generated by production and financing. Consequently, additional credit creation is required to sustain the circulation of money and the realisation of profits, resulting in the endogenous reproduction of credit and debt over time.

The simulation is conducted using a set of baseline parameter values chosen for illustrative purposes rather than empirical calibration. The household saving rate is set at ($s = 0.10$), implying that households save 10% of their disposable income and spend the remaining 90% on consumption. Firms finance 80% of their production costs through bank borrowing ($\lambda = 0.80$) and the remaining 20% through equity issuance. The lending interest rate is fixed at ($r = 0.05$), while deposits earn ($i = 0.02$), generating a positive interest margin for the banking sector. Firms apply a mark-up rate of ($\mu = 0.20$) over unit costs when determining prices. The labour force consists of 90 production workers and 10 bank employees, all of whom receive a normalised wage of 1 monetary unit. These values are selected to represent a stylised capitalist economy with moderate saving behaviour, partial debt financing, and positive interest-bearing liabilities. The objective is not to reproduce the characteristics of a particular economy but to examine whether the monetary gap and the associated credit–debt reproduction mechanism emerge under plausible parameter values.

To assess the robustness of the results, future work may extend the analysis by conducting sensitivity tests on the saving rate, financing structure, interest rates, and mark-up rate. However, the purpose of the present simulation is to demonstrate the existence of the mechanism rather than to estimate its quantitative magnitude.

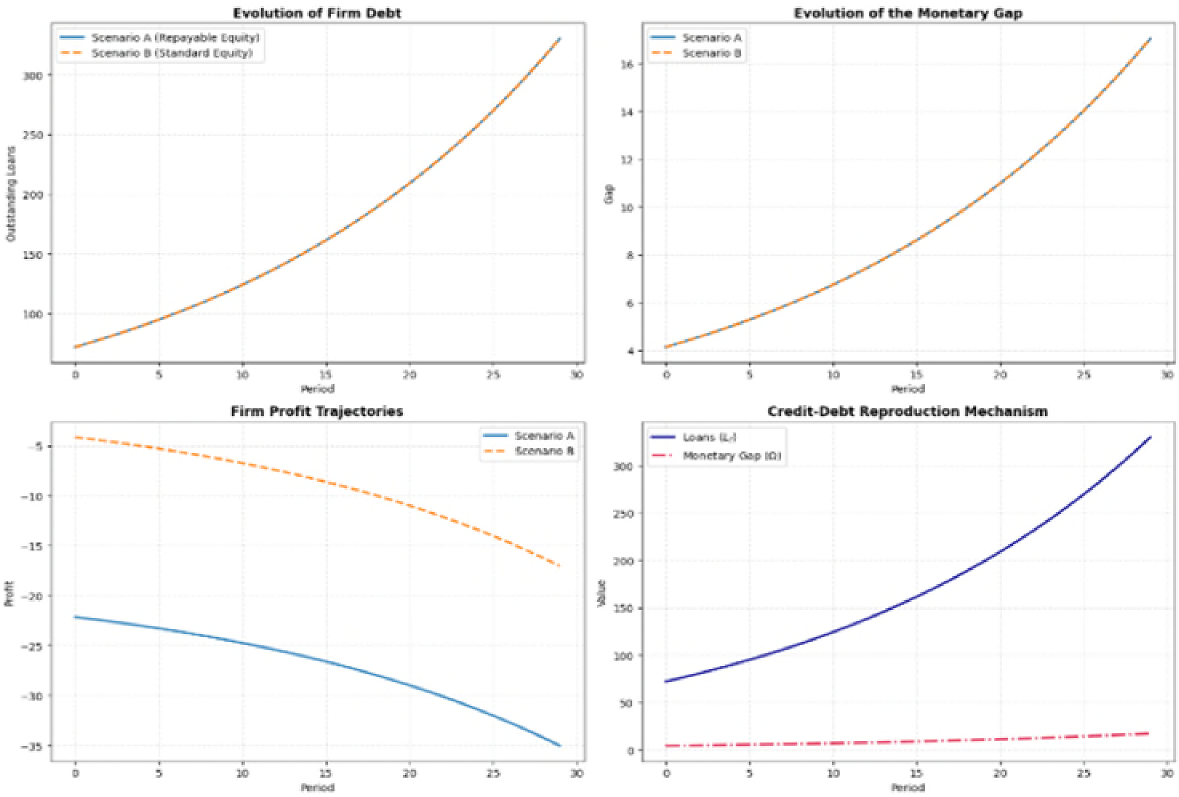
Table 7 reports the simulated evolution of the fundamental model over the first ten years. The results indicate that the initial stock of money is insufficient to meet the monetary obligations arising from production, interest payments, and financial commitments. As a consequence, a positive monetary gap emerges in every period and must be covered through additional credit creation. The accumulation of new debt increases future interest obligations, which in turn enlarges the monetary gap and generates a further demand for borrowing. The simulation, therefore, reveals a self-reinforcing credit–debt reproduction mechanism in which debt expansion becomes a necessary condition for maintaining the circulation of money and the settlement of contractual obligations.

Table 7: The result of the simulation for the Fundamental Model under scenarios A and B

Year	$Loans_A$	Gap_A	$FirmProfit_A$	$Loans_B$	Gap_B	$FirmProfit_B$
0	72.00	4.14	-22.14	72.00	4.14	-4.14
1	76.14	4.35	-22.35	76.14	4.35	-4.35
2	80.49	4.56	-22.56	80.49	4.56	-4.56
3	85.05	4.79	-22.79	85.05	4.79	-4.79
4	89.84	5.03	-23.03	89.84	5.03	-5.03
5	94.87	5.28	-23.28	94.87	5.28	-5.28
6	100.15	5.55	-23.55	100.15	5.55	-5.55
7	105.70	5.82	-23.82	105.70	5.82	-5.82
8	111.52	6.11	-24.11	111.52	6.11	-6.11
9	117.63	6.42	-24.42	117.63	6.42	-6.42

The comparison between Scenarios A and B further demonstrates that the existence of the monetary gap does not depend on the assumption that equity principal must be repaid. Although the magnitude of firm profits differs across the two scenarios, the shortage of circulating money persists in both cases (see Picture 2). Consequently, the need for additional credit arises regardless of whether firms rely on repayable financial claims or conventional equity finance. The results suggest that the monetary shortage is a structural characteristic of the model rather than a consequence of a particular financing arrangement.

Picture 2: The Simulation Graph of the Fundamental Model without Government



More broadly, the simulation supports the central theoretical proposition of this paper: in a closed monetary production economy, the stock of money created in previous periods is insufficient to realise all monetary claims simultaneously. Under such conditions, the continuous expansion of credit becomes necessary to sustain production, profit realisation, and debt servicing. The observed growth of debt is therefore not merely a behavioural outcome but an accounting requirement arising from the interaction among money creation, financial obligations, and the circular flow of income.

From the perspective of financialisation, the simulation highlights how the expansion of financial claims can become increasingly detached from the underlying process of real production. As the stock of debt grows through time, an increasing share of economic activity is devoted to meeting financial obligations rather than supporting productive accumulation. The credit–debt reproduction mechanism, therefore, provides a theoretical explanation for the persistent growth of indebtedness and the expanding role of finance in modern economies.

2.5. The Extended Model with Government

The next step is to introduce the government sector and examine whether its presence alters these conclusions.

Adding the government sector brings the model closer to the institutional structure of a modern monetary economy. At the same time, this extension requires additional assumptions and a more complex analytical framework. Before constructing the model, it is necessary to clarify the economic role of government and the channels through which it interacts with the other sectors.

Following Friedman (1955), the government in a free private enterprise economy performs several essential functions. These include preserving the rules of exchange by enforcing contracts and protecting property rights, preventing the emergence of monopolistic structures that undermine effective competition, addressing negative externalities generated by market activity, and protecting individuals whose rights and freedoms may otherwise be compromised. Beyond these functions, governments also provide and invest in public goods and services that generate substantial positive externalities and contribute to the reproduction of economic activity. Through taxation, public expenditure, borrowing, and redistribution, the government becomes an integral component of the monetary circuit and the flow of funds between sectors.

The inclusion of government, therefore, raises an important question: can fiscal intervention alleviate the monetary constraints identified in the previous models, or does it merely redistribute the existing money shortage within the system? To address this question, several fiscal policy scenarios will be examined. Before doing so, however, many additional assumptions must be introduced.

First, the government sector is assumed to have no authority to create money. Consequently, it operates using the same stock of money and credit available to the rest of the economy. Government revenue, therefore, originates from two sources: taxation and borrowing through the issuance of Treasury Bills (TB). Treasury Bills are assumed to be one-period financial assets that can be purchased by all other sectors. They yield a uniform rate of return ($\boldsymbol{r_{TB}}$) and are traded without transaction costs.

Second, the government is assumed to follow a balanced-budget rule. Total government expenditure is therefore equal to total government revenue, implying neither a surplus nor a deficit in the government account. The government sector employs ($\boldsymbol{n_3}$) employees, each receiving the same wage rate ($\boldsymbol{w}$), as workers employed in the other sectors. For simplicity of the simulation process, we assume that the government impose the same tax rate $\boldsymbol{\tau}$ for all sectors.

Under these assumptions, the government enters the monetary circuit as both a recipient and distributor of monetary funds. The way it collects revenue and allocates expenditure becomes crucial in determining whether the paradox of monetary profit can be resolved theoretically. In the following section, the paper follows a fiscal policy scenario in which the tax is proportional to the net profit of the other sectors[5].

Therefore, four different scenarios could be considered based on the tax rates and their distributions:

a) 30% tax rate, which is immediately collected and redistributed
b) 30% tax rate, which is collected at the end of the period and redistributed in the next period
c) 100% tax rate, which is immediately collected and redistributed
d) 100% tax rate, which is collected at the end of the period and redistributed in the next period

We can add one more assumption to extend the model further: The household sector can enter into the equity and bonds markets. They can buy/sell the firm's equities and government bonds. Under such assumptions, the balance sheet matrix would be as follows:

2.5.1. Balance Sheet Matrix and Transaction Flow Matrix (The Extended Model with Government)

Since this model extends the Fundamental Model by introducing a government sector, both the Balance Sheet Matrix and the Transaction Flow Matrix (TFM) are modified only to the extent necessary to capture the government's fiscal activities. As before, the Balance Sheet Matrix records the stock positions of all sectors at a given point in time and maintains the stock-flow consistent accounting principle that every financial asset held by one sector corresponds to an equal liability of another sector. Compared with the Fundamental Model, the government enters the balance sheet through the issuance of Treasury Bills, which constitute government liabilities and financial assets for the sectors that purchase them. All other assets and liabilities, including deposits, loans, and equities, remain unchanged. (see Table 8)

The Transaction Flow Matrix, in the same manner, records the monetary flows that occur during a single accounting period and shows how income, expenditure, and financial transactions are distributed across sectors (Table 9) . Following the structure of the Fundamental Model, the extended TFM incorporates the government's current transactions, including wage payments, purchases of goods and services, interest payments on Treasury Bills, and the collection of taxes on the net profits of households, firms, and banks. The financial section of the matrix records changes in deposits, loans, equity holdings, and Treasury Bill holdings. As in the previous models, each row sums to zero, ensuring that every payment by one sector is received by another sector and preserving the consistency between the flow

[5] To conserve space, the lump-sum tax scenario is omitted. Its results mirror those of the previous model, as a fixed tax-and-transfer scheme does not resolve the systemic shortage of money in circulation and therefore leaves the credit-debt reproduction mechanism intact. For the mathematical proof of the existing shortage, see Javidanrad (2020, Ch. 4)

and stock accounts. Together, the Balance Sheet Matrix and TFM provide the accounting framework required to analyse whether the introduction of government and profit taxation can resolve the monetary constraints identified in the earlier models.

Table 8: The Balance Sheet Matrix for the Extended Model with Government

Assets/Liabilities	***Households*** h	***Firms*** f	***Banks*** b	***Government g***	Σ
Deposits M	$+M_h$		$-M_h$		0
Loans L		$-L_f$	$+L_f$		0
Equity E	$+E_h$	$-E$	$+E_b$		0
Treasury Bills TB	$+TB_h$	$+TB_f$	$+TB_b$	$-TB$	0
Net Worth NW	NW_h	NW_f	NW_b	0	0

Where $E = E_h + E_b$, and $TB = TB_h + TB_f + TB_b$.

For simplicity in simulation, we assume all private sectors have the same share of the treasury bills:

$$TB_h = TB_f = TB_b = \frac{TB}{3}$$

Table 9: The Transaction Flow Matrix for the Extended Model with Government

Flows (Income & Expenditure Circuit)	***h***	***f***	***b***	***g***	Σ
Firm wages (W_f)	+	−	0	0	0
Bank wages (W_b)	+	0	−	0	0
Government wages (W_g)	+	0	0	−	0
Consumption (C_h)	−	+	0	0	0
Government purchases (G_f)	0	+	0	−	0
Deposit interest ($i.M_h$)	+	0	−	0	0
Loan interest ($r.L_f$)	0	−	+	0	0
Dividend payment (βE_f)	+	−	+	0	0
Treasury bill interest ($r_{TB}.TB$)	+	0	0	−	0
Profit tax (T_h)	−	0	0	+	0
Profit tax (T_f)	0	−	0	+	0
Profit tax (T_b)	0	0	−	+	0

Financial Flows (Changes in The Balance Sheet)	***h***	***f***	***b***	***g***	Σ
Deposit accumulation (ΔM_h)	+	0	−	0	0
New loans (ΔL_f)	0	+	−	0	0
Equity issuance (ΔE_f)	$+\Delta E_h$	−	$+\Delta E_b$	0	0
Treasury bill issuance (ΔTB)	+	+	+	−	0

2.5.2. The Core Equations (The Extended Model with Government)

By adding the government sector and some new assumptions, the core equations must change accordingly.

Household Income: $Y_h = W_f + W_b + W_g + iM_h + \beta E_h$ (where, $W_f = B = n_1 w, W_b = n_2 w$, and $W_{bg} = n_3 w$)

Consumption: $C_h = (1 - \alpha) Y_h$

Saving: $S_h = \alpha Y_h$

Deposit Accumulation: $\Delta M_h = S_h$ (or equivalently $M_h(t) = M_h(t-1) + S_h(t)$)

Production Function: $Q = f(n_1)$

Firm Borrowing: $L_f = \lambda W_f = \lambda B$ (where $B = n_1 w$)

Equity Financing: $E_f = (1 - \lambda) W_f = (1 - \lambda) B$

Firm Profit: $\Pi_f = C_h + G_f - W_f - rL_f - \beta E$ (where, G_f is the government expenditure on firms' production and $E = E_h + E_b$. For simplicity in simulation, we can also assume that the household and banking sectors have equal shares of the equity market)

Bank Profit: $\Pi_b = rL_f + \beta E_b - iM_h - W_b$

Household Profit: $\Pi_h = W_f + W_b + W_g + iM_h + \beta E_h - C_h$

Tax of the Firm: $T_f = \tau . \max(0, \Pi_f)$

Tax of the Bank: $T_b = \tau . \max(0, \Pi_b)$

Tax of the Household: $T_h = \tau . \max(0, \Pi_h)$

Total Tax: $T = T_h + T_f + T_b$

Government Borrowing Through Treasury Bills: $TB = TB_h + TB_f + TB_b$

Total Receivable by Government: $TR_g = T + TB$

Total Expenditure by the Government: $TE_g = W_g + G_f + r_{TB} . TB$

2.5.3. Dynamic Closure of the Fundamental Model

Under the scenario of immediate tax collection and distribution, we will have the following balanced budget constraint equation:

$$G_f(t) = T(t) + TB(t) - W_g(t) - r_{TB} . TB\ (t)$$

And under the scenario of the tax collection at the end of the period and redistributing it in the next period, we will follow the balanced budget equation:

$$G_f(t) = T(t-1) + TB(t) - W_g(t) - r_{TB}.TB\ (t)$$

There should be one extra dynamic equation after adding the government sector, and that relates to the endogeneity of the stock of treasury bills. Therefore,

$$TB(t) = TB(t-1) + \Delta TB_t$$

Where ΔTB_t represents newly issued treasury bills required to satisfy the government's balanced-budget constraint.

By adding the government sector, the monetary obligations generated during the period $\boldsymbol{t}$ would be**:**

For firm: $W_f + rL_f + \beta E_f + T_f$

For bank: $W_b + iM_h + T_b$

For households: $C_h + T_h$

For Government: $W_g + G_f + r_{TB}.TB$

Total monetary obligations:

$$MO = W_f + W_b + W_g + C_h + rL_f + iM_h + \beta E_f + G_f + r_{TB}.TB + T_f + T_b + T_h$$

So, the monetary gap equation would be:

$$Gap_t = MO_t - M_h(t-1)$$

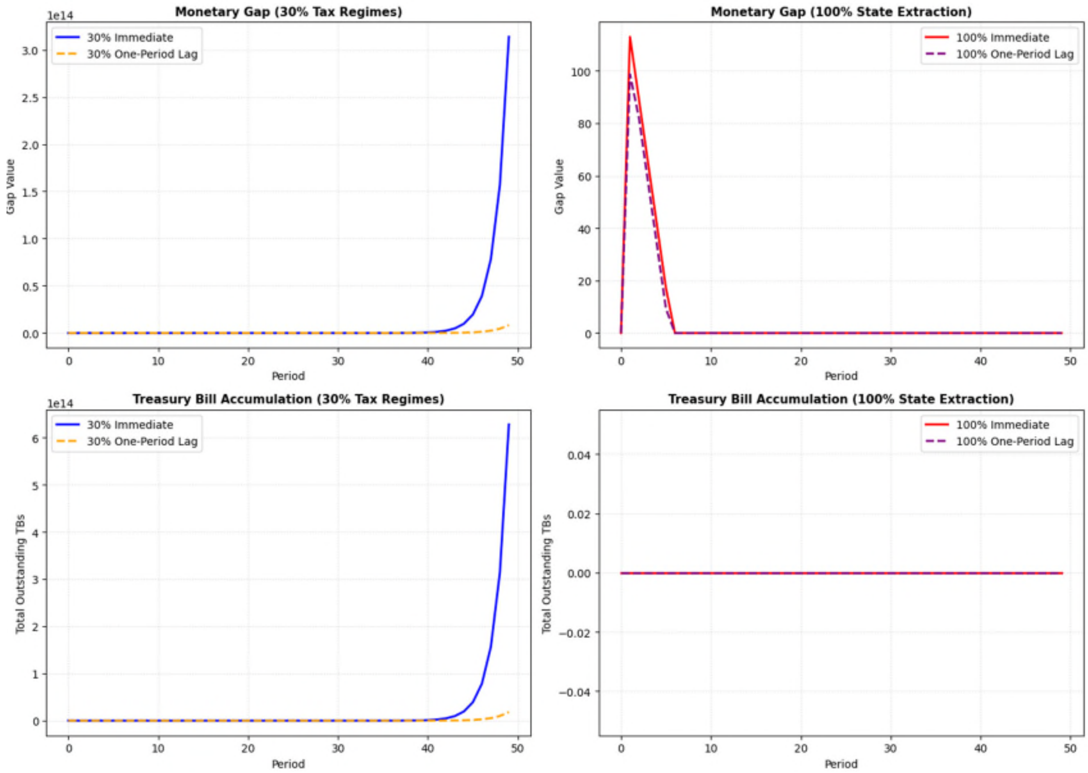
$$Gap_t = (W_f + W_b + W_g + C_h + rL_f + iM_h + \beta E_f + G_f + r_{TB}.TB + T_f + T_b + T_h) - M_h(t-1))$$

To evaluate the dynamics of the monetary circuit under varying fiscal regimes, the baseline stock-flow-consistent model is calibrated with structurally coherent parameters. The labour force is distributed across three primary institutional sectors, comprising $n_1 = 100$ production workers in the goods sector, $n_2 = 10$ financial workers in the banking sector, and $n_3 = 10$ Civil servants in the public sector. The uniform nominal wage rate is normalised to $w = 1$, rendering the baseline institutional wage bill strictly dependent on sectoral employment headcounts. Behavioural and financial parameters are configured to reflect standard Post-Keynesian structural assumptions: the household liquidity preference (saving rate) is set at $\alpha = 0.20$, while the banking sector's advance rate for working capital loans is calibrated to $\lambda = 0.70$, meaning that 70% of firm wages (W_f) are funded via endogenous credit obligations (L_f) and the remaining 30% via equity issuance (E_f). Financial returns and debt-servicing costs are dictated by a fixed interest rate structure where the bank lending rate is set at $r = 0.05$, the deposit rate at $i = 0.01$, the corporate dividend payout rate on equity at $\beta = 0.04$, and the sovereign yield on Treasury Bills at $r_{TB} = 0.03$. The simulation initialises from a standard baseline state where initial circulating bank deposits are set at $M_0 = 100$ and the sovereign debt stock begins at $TB_0 = 0$ This parameter space establishes a compounding liquidity deficit, isolating how the speed and intensity of state extraction stabilise or aggravate the systemic monetary gap over an analytical horizon of $T = 50$ periods. The results of 10 years are provided in Table 10.

Table 10: The result of the simulation for the Extended Model under different scenarios

Year	*30% Immediate Regime: Monetary Gap*	*30% One-Period Lag Regime: Monetary Gap*	*100% Immediate Regime: Monetary Gap*	*100% One-Period Lag Regime: Monetary Gap*
0	0.00	0.00	0.00	0.00
1	1.41×10^2	1.15×10^2	113.20	95.50
2	2.85×10^2	2.10×10^2	85.10	62.40
3	4.41×10^2	3.25×10^2	52.40	31.10
4	6.23×10^2	4.80×10^2	21.80	8.20
5	8.95×10^2	6.95×10^2	3.50	0.00
6	1.32×10^3	9.80×10^2	0.00	0.00
7	2.01×10^3	1.42×10^3	0.00	0.00
8	3.18×10^3	1.95×10^3	0.00	0.00
9	5.22×10^3	2.65×10^3	0.00	0.00
10	8.90×10^3	3.40×10^3	0.00	0.00

Picture 3: The simulation graphs of the Extended Model with Government



The simulation outcomes presented in Picture 3 expose a massive scale divergence and structural asymmetry between a moderate fiscal state (30% taxation) and a total extraction state (100% taxation). Under the 30% tax regime, both the monetary gap and Treasury Bill accumulation explode quadratically or exponentially over time. This rampant growth occurs because a 30% fiscal clawback is fundamentally insufficient to offset private sector savings, compound loan interest commitments, and institutional markup leakages, leading to an ever-widening structural deficit.

Conversely, under the 100% state tax regime, both the monetary gap and the need for Treasury Bill issuance are compressed to virtually zero after an initial, short-lived adjustment spike. By completely reclaiming all corporate net profits and private surpluses, the state permanently breaks the compounding debt-accumulation spiral and establishes long-term macroeconomic steady states. The most critical insight from Picture 3, however, is the astronomically vast difference in scale of the monetary gap between these two tax designs. For the 30% tax regimes, the monetary gap explodes to a staggering peak of over 3×10^{14} (300 trillion units) in the immediate scenario and nearly 1×10^{13} in the one-period lagged scenario. In contrast, the maximum monetary gap under the 100% tax regime peaks at a mere 110 units during the first few periods before permanently dropping to zero. This represents a scale divergence of more than twelve orders of magnitude. While a partial tax rate allows monetary obligations to spiral uncontrollably out of synchronisation with the circulating money supply, a total extraction policy acts as a perfect structural brake. It forces all income flows directly back into a closed loop, completely extinguishing the systemic scarcity of money.

So, one of the important outcomes from the 100% tax scenario is that this practice is not really about taxation; it is something much more fundamental: If every unit of monetary profit is exactly recycled back as non-interest-bearing purchasing power, the endogenous monetary shortage disappears, and there will be no monetary gap. The simulations suggest that the long-run stability of a debt-based monetary economy is determined not by the absolute size of government spending but by whether public-sector monetary injections are sufficient to offset the endogenous monetary leakage arising from private debt accumulation.

Conclusion:

The analysis presented in this paper has demonstrated that the endogenous reproduction of financial claims systematically outpaces the reproduction of the means of monetary settlement, thereby making recurrent monetary accommodation a structural requirement of debt-based production rather than a discretionary policy choice.

This paper has argued that the persistent difficulty of realising aggregate profit is not an episodic consequence of imperfect markets, inadequate demand, or exceptionally high interest rates. Rather, it is a structural characteristic of debt-based monetary production. By distinguishing biological reproduction from monetary reproduction, the analysis demonstrates that the expansion of financial claims follows a fundamentally different logic from the reproduction of physical output. Whereas physical production is constrained by material resources, monetary obligations generated through credit creation accumulate endogenously through balance-sheet relationships and, unless counterbalanced, progressively exceed the available means of settlement.

This proposition was examined from complementary historical, analytical, and computational perspectives. The historical review showed that debt remissions, usury restrictions, monetary debasement, and successive innovations in monetary institutions have repeatedly emerged as responses to recurring shortages of liquidity rather than as isolated legal or political developments. The formal mathematical model established that aggregate monetary obligations systematically grow beyond the purchasing power initially introduced into circulation, while the stock-flow consistent simulations demonstrated how this monetary gap evolves dynamically under alternative institutional arrangements. Together, these results suggest that the recurrent requirement for additional purchasing power is an endogenous property of the credit-debt reproduction mechanism rather than an accidental feature of particular historical episodes or policy regimes.

The principal theoretical contribution of the paper is therefore not the identification of another institutional mechanism through which aggregate profit may be realised, but the explanation of why diverse mechanisms proposed in the literature ultimately converge on the same structural necessity. Whether the additional purchasing power originates through government deficits, central bank operations, financial innovation, foreign-sector surpluses, or other institutional arrangements, each mechanism performs the same macroeconomic function: it relaxes the monetary constraint created by the endogenous expansion of debt obligations relative to the available means of settlement.

The policy implications follow directly from this theoretical result. If monetary scarcity is an endogenous property of debt-based production rather than a temporary market failure, the role of the public sector extends beyond conventional macroeconomic stabilisation. Fiscal and monetary authorities become responsible for maintaining the consistency between the evolving stock of private financial obligations and the quantity of non-interest-bearing net financial assets available to support their settlement. Policies that systematically withdraw purchasing power from circulation without providing an equivalent monetary accommodation risk intensifying financial fragility, whereas appropriately calibrated fiscal injections can sustain monetary circulation without requiring an ever-accelerating expansion of private indebtedness.

The simulation analysis also identifies an important benchmark condition for the long-run sustainability of debt-based monetary production. Under the model's assumptions, when profits are fully taxed and continuously recycled into the economy through public expenditure, whether immediately or with a one-period lag, the endogenous monetary shortage disappears, and the system converges to a stable trajectory without requiring an ever-increasing expansion of private credit. This result should not be interpreted as a literal recommendation for a 100% profit tax. Rather, it demonstrates a more general principle: the sustainability of a monetary production economy depends on the continuous replenishment of non-interest-bearing purchasing power sufficient to offset the monetary leakage associated with the accumulation of private financial claims. In this respect, the source of public expenditure is of secondary importance. What matters is that the public sector injects an adequate quantity of net financial assets into circulation to maintain consistency between the growth of monetary obligations and the means available for their settlement.

More broadly, the Credit-Debt Reproduction Mechanism provides a unified framework for interpreting historical debt crises, the long-run process of financialisation, and the recurrent dependence of capitalist economies on expanding liquidity. Future research may extend the present model by incorporating international trade, endogenous asset-price dynamics, heterogeneous financial institutions, and central bank balance-sheet operations, thereby further exploring the implications of the mechanism across different institutional settings.

Python code for the Simplest Model without the government

```python
import numpy as np
import matplotlib.pyplot as plt

# ======================================================
# PARAMETERS
# ======================================================
T = 50
r = 0.05    # interest rate
mu = 0.2    # markup

# ======================================================
# ARRAYS
# ======================================================
L = np.zeros(T)
M = np.zeros(T)
gap = np.zeros(T)
profit = np.zeros(T)

# ======================================================
# INITIAL CONDITIONS
# ======================================================
L[0] = 100
M[0] = 100

# ======================================================
# LOOP
# ======================================================
for t in range(1, T):

    # wages = previous loan
    W = L[t-1]

    # total cost
    TC = W * (1 + r)

    # profit from markup
    profit[t] = mu * TC

    # required payments
    required = W + r * L[t-1] + profit[t]

    # monetary gap
    gap[t] = required - M[t-1]

    # credit injection (key mechanism)
    dL = max(0, gap[t])

    # update stocks
    L[t] = L[t-1] + dL
    M[t] = M[t-1] + dL

# ======================================================
# PLOTS
# ======================================================
plt.figure(figsize=(10, 6))

# Note: As discussed, M and L perfectly overlap.
# Using a dashed line style '--' allows you to see both.
plt.plot(L, label="Debt (L)", linewidth=2.5)
plt.plot(M, '--', label="Money (M)", linewidth=2)
plt.plot(gap, label="Monetary Gap")
plt.plot(profit, label="Profit")

plt.legend()
plt.title("Credit-Debt Reproduction (Simplest Model)")
plt.xlabel("Period (t)")
plt.ylabel("Value")
plt.grid(True, linestyle=':', alpha=0.6)

plt.show()
```

Python code for the Fundamental Model without Government (1)

Python

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# ==============================================================================
# PARAMETERS & SYSTEM INITIALIZATION
# ==============================================================================
T = 30

# Population / Employment
n1 = 90         # firm workers
n2 = 10         # bank workers
w = 1           # nominal wage rate

# Behavioral & Financial parameters
s = 0.10        # saving rate
lam = 0.80      # debt-financing ratio
r1 = 0.05       # loan interest rate
rd = 0.02       # deposit interest rate
beta = 0.03     # dividend rate
mu = 0.20       # markup

# Baseline Macrodynamic Constants
B = n1 * w            # production wage bill
M = (n1 + n2) * w     # aggregate circulating money stock
Yh = M                # initial household income
C = (1 - s) * Yh      # household consumption flow

# ==============================================================================
# SCENARIO A: REPAYABLE EQUITY CLAIMS
# ==============================================================================
LA = np.zeros(T)
GapA = np.zeros(T)
ProfitFirmA = np.zeros(T)
ProfitBankA = np.zeros(T)

LA[0] = lam * B

for t in range(T):
    E = (1 - lam) * B

    # Total cost structure under Scenario A
    TC = B + r1 * LA[t] + E * (1 + beta)
    R = C  # Effective revenue

    ProfitFirmA[t] = R - TC
    ProfitBankA[t] = r1 * LA[t] + beta * E - n2 * w

    # Structural liquidity deficit
    GapA[t] = (n1 * w) + (n2 * w) + (r1 * LA[t]) + (beta * E) - M

    if t < T - 1:
        LA[t + 1] = LA[t] + GapA[t]
```

Python code for the Fundamental Model without Government (2)

```
# ==============================================================================
# SCENARIO B: STANDARD EQUITY FINANCE
# ==============================================================================
LB = np.zeros(T)
GapB = np.zeros(T)
ProfitFirmB = np.zeros(T)
ProfitBankB = np.zeros(T)

LB[0] = lam * B

for t in range(T):
    E = (1 - lam) * B

    # Total cost structure under Scenario B
    TC = B + r1 * LB[t] + beta * E
    R = C  # Effective revenue

    ProfitFirmB[t] = R - TC
    ProfitBankB[t] = r1 * LB[t] + beta * E - n2 * w

    # Structural liquidity deficit
    GapB[t] = (n1 * w) + (n2 * w) + (r1 * LB[t]) + (beta * E) - M

    if t < T - 1:
        LB[t + 1] = LB[t] + GapB[t]

# ==============================================================================
# RESULTS DATAFRAME GENERATION
# ==============================================================================
results = pd.DataFrame({
    "Year": np.arange(T),
    "Loans_A": np.round(LA, 2),
    "Gap_A": np.round(GapA, 2),
    "FirmProfit_A": np.round(ProfitFirmA, 2),
    "BankProfit_A": np.round(ProfitBankA, 2),
    "Loans_B": np.round(LB, 2),
    "Gap_B": np.round(GapB, 2),
    "FirmProfit_B": np.round(ProfitFirmB, 2),
    "BankProfit_B": np.round(ProfitBankB, 2)
})

print("--- SIMULATION RESULTS TABLE ---")
print(results.to_string(index=False))

# ==============================================================================
# VISUALIZATION SUITE (4-PANEL COMPREHENSIVE PLOT)
# ==============================================================================
fig, axes = plt.subplots(2, 2, figsize=(15, 11))

# Figure 1: Debt Dynamics
axes[0, 0].plot(LA, label="Scenario A (Repayable Equity)", linewidth=2)
axes[0, 0].plot(LB, label="Scenario B (Standard Equity)", linestyle="--", linewidth=
axes[0, 0].set_title("Evolution of Firm Debt", fontsize=12, fontweight='bold')
axes[0, 0].set_xlabel("Period")
axes[0, 0].set_ylabel("Outstanding Loans")
axes[0, 0].legend()
axes[0, 0].grid(True, linestyle=":", alpha=0.6)

# Figure 2: Monetary Gap
axes[0, 1].plot(GapA, label="Scenario A", linewidth=2)
axes[0, 1].plot(GapB, label="Scenario B", linestyle="--", linewidth=2)
axes[0, 1].set_title("Evolution of the Monetary Gap", fontsize=12, fontweight='bold'
axes[0, 1].set_xlabel("Period")
axes[0, 1].set_ylabel("Gap")
axes[0, 1].legend()
axes[0, 1].grid(True, linestyle=":", alpha=0.6)
```

Python code for the Fundamental Model without Government (3)

```
# Figure 3: Firm Profit Profiles
axes[1, 0].plot(ProfitFirmA, label="Scenario A", linewidth=2)
axes[1, 0].plot(ProfitFirmB, label="Scenario B", linestyle="--", linewidth=2)
axes[1, 0].set_title("Firm Profit Trajectories", fontsize=12, fontweight='bold')
axes[1, 0].set_xlabel("Period")
axes[1, 0].set_ylabel("Profit")
axes[1, 0].legend()
axes[1, 0].grid(True, linestyle=":", alpha=0.6)

# Figure 4: Credit-Debt Reproduction Mechanism (Scenario B Baseline)
axes[1, 1].plot(LB, label="Loans ($L_f$)", color="darkblue", linewidth=2)
axes[1, 1].plot(GapB, label="Monetary Gap ($\Omega$)", color="crimson", linestyle="-
axes[1, 1].set_title("Credit-Debt Reproduction Mechanism", fontsize=12, fontweight='
axes[1, 1].set_xlabel("Period")
axes[1, 1].set_ylabel("Value")
axes[1, 1].legend()
axes[1, 1].grid(True, linestyle=":", alpha=0.6)

plt.tight_layout()
plt.show()
```

Python Code for the Extended Model with the Government (1)

```python
# Force Google Colab to render matplotlib outputs directly inside the notebook
import matplotlib
%matplotlib inline

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# ==============================================================================
# GLOBAL PARAMETERS & INITIAL CONDITIONS
# ==============================================================================
T = 50

n1, n2, n3, w = 100, 10, 10, 1.0
alpha, lam = 0.20, 0.70
r_L, i, beta, r_TB = 0.05, 0.01, 0.04, 0.03
M0, TB0 = 100.0, 0.0

# ==============================================================================
# CORE SIMULATION ENGINE
# ==============================================================================
def simulate_extended_model(tax_rate=0.30, lagged=False):
    Mh, TB = np.zeros(T), np.zeros(T)
    Yh, Ch, Sh = np.zeros(T), np.zeros(T), np.zeros(T)
    Pf, Pb, Ph = np.zeros(T), np.zeros(T), np.zeros(T)
    Tf, Tb, Th, Tax, Gf, Gap = np.zeros(T), np.zeros(T), np.zeros(T), np.zeros(T), n

    Mh[0], TB[0] = M0, TB0
    Wf, Wb, Wg = n1 * w, n2 * w, n3 * w
    Lf, Ef = lam * Wf, (1 - lam) * Wf
    Eh, Eb = 0.5 * Ef, 0.5 * Ef

    for t in range(1, T):
        deposit_interest = i * Mh[t-1]
        equity_income_h = beta * Eh
        equity_income_b = beta * Eb
        loan_interest = r_L * Lf
        tb_interest = r_TB * TB[t-1]

        # Household Sector Account
        Yh[t] = Wf + Wb + Wg + deposit_interest + equity_income_h
        Ch[t] = (1 - alpha) * Yh[t]
        Sh[t] = alpha * Yh[t]
        Ph[t] = Yh[t] - Ch[t]
        Th[t] = tax_rate * max(0, Ph[t])

        # Banking Sector Account
        Pb[t] = loan_interest + equity_income_b - deposit_interest - Wb
        Tb[t] = tax_rate * max(0, Pb[t])

        # Government Expenditure Design
        if lagged:
            Gf[t] = max(0, Tax[t-1] + TB[t-1] - Wg - tb_interest)
            Pf[t] = Ch[t] + Gf[t] - Wf - loan_interest - (beta * Ef)
            Tf[t] = tax_rate * max(0, Pf[t])
            Tax[t] = Tf[t] + Tb[t] + Th[t]
        else:
            if tax_rate == 1.0:
                Tf[t] = max(0, Ch[t] - Wf - loan_interest - (beta * Ef))
                Tax[t] = Tf[t] + Tb[t] + Th[t]
                Gf[t] = max(0, Tax[t] + TB[t-1] - Wg - tb_interest)
                Pf[t] = Ch[t] + Gf[t] - Wf - loan_interest - (beta * Ef)
            else:
                Tf_base = tax_rate * (Ch[t] - Wf - loan_interest - (beta * Ef))
                tax_independent = Th[t] + Tb[t] + Tf_base
                Gf[t] = max(0, (tax_independent + TB[t-1] - Wg - tb_interest) / (1 -
                Pf[t] = Ch[t] + Gf[t] - Wf - loan_interest - (beta * Ef)
                Tf[t] = tax_rate * max(0, Pf[t])
                Tax[t] = Tf[t] + Tb[t] + Th[t]
```

Python Code for the Extended Model with the Government (2)

```
        # Stock Variable Accounts
        total_govt_outlays = Wg + Gf[t] + tb_interest
        treasury_gap = max(0, total_govt_outlays - Tax[t])
        TB[t] = TB[t-1] + treasury_gap
        Mh[t] = Mh[t-1] + Sh[t]

        # Unbacked Flow Circuit Evaluation
        total_monetary_demands = (Wf + Wb + Wg + Ch[t] + loan_interest +
                                  deposit_interest + (beta * Ef) + Gf[t] + tb_inter
        Gap[t] = max(0, total_monetary_demands - Tax[t] - Mh[t-1])

    return pd.DataFrame({"Gap": Gap, "TB": TB})

# ==============================================================================
# SCENARIO MATRIX EXECUTION
# ==============================================================================
S1 = simulate_extended_model(tax_rate=0.30, lagged=False)
S2 = simulate_extended_model(tax_rate=0.30, lagged=True)
S3 = simulate_extended_model(tax_rate=1.00, lagged=False)
S4 = simulate_extended_model(tax_rate=1.00, lagged=True)

# ==============================================================================
# COLAB GRAPH RENDER UNIT
# ==============================================================================
# Explicitly initialize the drawing canvas
plt.close('all')
fig, axes = plt.subplots(2, 2, figsize=(14, 10))

# Panel 1: Monetary Gap for 30% Tax Scenarios
axes[0, 0].plot(S1["Gap"], label="30% Immediate", color="blue", linewidth=2)
axes[0, 0].plot(S2["Gap"], label="30% One-Period Lag", color="orange", linestyle="-
axes[0, 0].set_title("Monetary Gap (30% Tax Regimes)", fontsize=11, fontweight='bol
axes[0, 0].set_xlabel("Period")
axes[0, 0].set_ylabel("Gap Value")
axes[0, 0].legend()
axes[0, 0].grid(True, linestyle=":", alpha=0.5)

# Panel 2: Monetary Gap for 100% Tax Scenarios
axes[0, 1].plot(S3["Gap"], label="100% Immediate", color="red", linewidth=2)
axes[0, 1].plot(S4["Gap"], label="100% One-Period Lag", color="purple", linestyle="
axes[0, 1].set_title("Monetary Gap (100% State Extraction)", fontsize=11, fontweigh
axes[0, 1].set_xlabel("Period")
axes[0, 1].set_ylabel("Gap Value")
axes[0, 1].legend()
axes[0, 1].grid(True, linestyle=":", alpha=0.5)

# Panel 3: Treasury Bill Accumulation (30% Regimes)
axes[1, 0].plot(S1["TB"], label="30% Immediate", color="blue", linewidth=2)
axes[1, 0].plot(S2["TB"], label="30% One-Period Lag", color="orange", linestyle="--
axes[1, 0].set_title("Treasury Bill Accumulation (30% Tax Regimes)", fontsize=11, f
axes[1, 0].set_xlabel("Period")
axes[1, 0].set_ylabel("Total Outstanding TBs")
axes[1, 0].legend()
axes[1, 0].grid(True, linestyle=":", alpha=0.5)

# Panel 4: Treasury Bill Accumulation (100% Regimes)
axes[1, 1].plot(S3["TB"], label="100% Immediate", color="red", linewidth=2)
axes[1, 1].plot(S4["TB"], label="100% One-Period Lag", color="purple", linestyle="-
axes[1, 1].set_title("Treasury Bill Accumulation (100% State Extraction)", fontsize
axes[1, 1].set_xlabel("Period")
axes[1, 1].set_ylabel("Total Outstanding TBs")
axes[1, 1].legend()
axes[1, 1].grid(True, linestyle=":", alpha=0.5)

plt.tight_layout()

# Force Colab to process drawing vectors onto the active UI thread
plt.draw()
plt.show()
```

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