Endogenous Growth and Wave-Like Business Fluctuations

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Motivation

Shumpeter: *"Why is it that economic development does not proceed evenly..., but as it were jerkily; why does it display those characteristics ups and downs?"*

when searching for an answer he drew attention on the critical fact that

"(innovations) appear en masse at intervals", "discontinuously in groups or swarms", which "signifies a very substantial increase in purchasing power all over the business sphere."

Shumpeter was indeed more interested in medium frequency movements lasting around 10 years (Juglar cycles).

Are these medium-term movements observable in the data (e.g. in the U.S. output)?

Average periodicity of 11 years; average amplitude 8%.

3 COMIN AND GERTLER: MEDIUM-TERM BUSINESS CYCLES

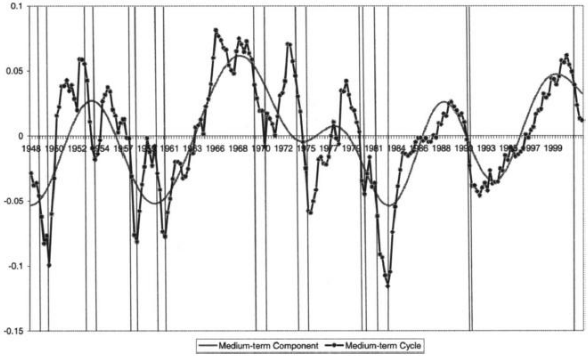

FIGURE 1. NONFARM BUSINESS OUTPUT PER PERSON 16–65

A natural candidate for the study of Schumpeterian wave-like business fluctuations is the observed long delay elapsed between the realization of R&D activities and the implementation and adoption of the associated innovations. (e.g. Mansfield: mean adoption delay of twelve mayor 20th-century innovations is 8 years.)

This source of fluctuation is suggested by the same author:

"the boom ends and the depression begins after the passage of the time which must elapse before the products of the new enterprise can appear on the market."

Therefore the aim of our contribution is to show under which conditions (if any) **wave-like fluctuations may emerge by adding an implementation-adoption delay to an otherwise textbook endogenous growth model with expanding product variety.**

Related Literature

- Dynamic general equilibrium with time delays:
  - Vintage capital literature (Benhabib and Rustichini 1991; Boucekkine et al. 1997; Caballero and Hammour 1994)
  - Time-to-build (Kydland and Prescott 1982; Bambi 2008; Bambi et al. 2012)
- Endogenous competitive equilibrium cycles:
  - Multi-sectors growth models (Benhabib and Nishimura 1979, 1985)
  - Animal spirits (Francois and Lloyd-Ellis 2003)
  - General Purpose Technology literature.

Description of the Model

Production:

Three sectors of production

- Consumption goods sector

$$c_t = n_{t-d}^{\nu+1-\frac{1}{\alpha}} \left(\int_0^{n_{t-d}} x_t(j)^{\alpha} \mathrm{d}j \right)^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1 \qquad (1)$$

where $x_t(j)$ amount of intermediate j and $[0, n_{t-d}]$ range of intermediates at time t. ν is the elasticity of the externality (but also the return to specialization).

If $\nu = 1$ then c_t and n_t grow at the same rate on a BGP.

The Consumption good sector is competitive.

- `Intermediate goods sector` Each intermediate good is produced by a monopolistically competitive firm j with production function

$$x_t(j) = l_t(j), \tag{2}$$

- `Research sector` - New ideas leading to new intermediates can be discovered

$$\dot{n}_t = A n_{t-d}(1 - L_t), \qquad A > 0 \tag{3}$$

where $1 - L_t$ is labor assigned to R&D production. n_{t-d} is an externality. $\dot{n}_t > 0$ otherwise research sector is inactive. The R&D sector is competitive.

Feasibility Constraint

The first key equation can be already derived. At a “symmetric” market equilibrium the three equations can be combined:

$$\dot{n}_t = A\left(n_{t-d} - c_t\right). \tag{4}$$

The AK structure of the model can be easily seen if the extent of product variety n_{t-d} is interpreted as (intangible) capital.

Market Equilibrium

Symmetric in the various intermediates. We may skip the j index.

- Final good sector is competitive, therefore price of intermediates in term of the final good (numeraire) is equal to the marginal productivity:

$$p_t = n_{t-d}. \tag{5}$$

- Intermediate sector is monopolistically competitive. Optimal price rule

$$p_t = \frac{1}{\alpha} w_t, \qquad \Rightarrow \quad w_t = \alpha n_{t-d} \tag{6}$$

$\frac{1}{\alpha}$ is the markup over marginal cost (technology is linear in labor).

Profits:

$$\pi = (p - w)x = (1-\alpha)p\ell = \frac{(1-\alpha)pL}{n_{t-d}} \tag{7}$$

$$= (1-\alpha)L = (1-\alpha)\frac{c_t}{n_{t-d}} \tag{8}$$

- R&D sector is also competitive. Therefore the value of a patent

$$v_t = \frac{w_t(1 - L_t)}{\dot{n}_t} = \frac{w_t}{An_{t-d}} = \frac{\alpha}{A}. \quad (9)$$

This is indeed possible since patent law guarantees that v_t is equal to the present value of the associated flow of monopolistic profits.

$$r_t = \frac{\pi_t}{v_t} + \frac{\dot{v}_t}{v_t} = \frac{\pi_t}{v_t}. \quad (10)$$

- Households problem

$$\max \int_0^\infty \log(c_t)\, \mathrm{e}^{-\rho t} \mathrm{d}t \tag{11}$$

subject to the instantaneous budget constraint

$$c_t + v_t \dot{n}_t = w_t + \pi_t n_{t-d} \tag{12}$$

and the initial condition $n_t = \bar{n}_t$, for $t \in [-d, 0]$, where $\bar{n}_t$ is an exogenously given continuous positive function

Euler-type equation:

$$\frac{\dot{c}_t}{c_t} = \underbrace{\frac{\pi_t}{v_t}}_{R\&D\ returns} \cdot \underbrace{\frac{c_t}{c_{t+d}} e^{-\rho d}}_{discount\ factor} - \rho$$

and using the previous relations we may rewrite it as

$$\frac{\dot{c}_t}{c_t} = \frac{1-\alpha}{\alpha} A\, \mathrm{e}^{-\rho d} \frac{c_t}{n_t} - \rho \tag{13}$$

A **market equilibrium** is a path (c_t, n_t), for $t \geq 0$, verifying the feasibility condition

$$\dot{n}_t = A(n_{t-d} - c_t), \quad t \geq 0, \tag{14}$$

the Euler-type equation

$$\frac{\dot{c}_t}{c_t} = \frac{1-\alpha}{\alpha} A\, \mathrm{e}^{-\rho d} \frac{c_t}{n_t} - \rho, \quad t \geq 0, \tag{15}$$

the initial condition $n_t = \bar{n}_t$, $\bar{n}_t \in C([-d, 0]; \mathbb{R}_{++})$, the transversality condition

$$\lim_{t\to\infty} \frac{n_t}{c_t} \mathrm{e}^{-\rho t} = 0, \tag{16}$$

and the inequality constraints.

Balanced Growth Paths

The solution of the system (14)-(15) is a BGP if and only if the conditions below are satisfied:

i) $A \geq \frac{\alpha\rho e^{\rho d}}{1-\alpha} =: A^e_{\min},$;

ii) the growth rate is given by the unique positive solution g_e of

$$Ae^{-gd} - g = \frac{\alpha(g+\rho)e^{\rho d}}{1-\alpha}, \tag{17}$$

iii) the initial condition $\bar{n}_t$ has the form $\bar{n}_t = n_0 e^{g_e t}$ with $n_0 > 0$ and $t \in [-d, 0]$;

iv) given n_0, the initial consumption c_0 is equal to

$$c_0 = \frac{\alpha(g_e+\rho)e^{\rho d}}{(1-\alpha)A} n_0, \tag{18}$$

Transitional Dynamics

Assume that the initial condition are not specified as in iii).

QUESTION: May persistent cycles around a BGP emerge?

STRATEGY TO ANSWER THE QUESTION:

1. Find the detrended and linearized version of the system (14)-(16) around a steady state.
2. Look at the associated characteristic equation

$$h(\lambda) := \lambda^2 - \rho\lambda - \lambda A e^{-(g_e+\lambda)d} + A(g_e+\rho)e^{-(g_e+\lambda)d} - A(g_e+\rho)e^{-g_e d} = 0$$

   and find conditions on the parameter d, let us say $d = d_H$, under which a couple of purely imaginary roots emerges. This is one of the necessary condition to have periodic orbits through an Hopf bifurcation.

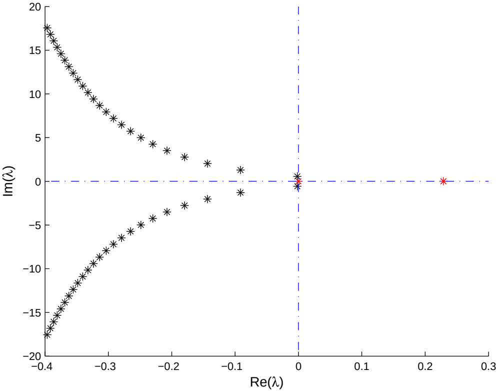

Figure: Spectrum of Roots.

3. Dimensional-reduction of the dynamic system to “throw away” the spurious zero eigenvalue.
4. Apply the Hopf bifurcation theorem to the resulting projected system and prove that there exists a family of periodic orbits $p_d(t)$ for d in a right or left neighborhood of d_H.
5. On such orbits the ratios

$$\frac{\tilde{c}_t}{\tilde{n}_t}, \qquad \frac{\tilde{n}_{t+s}}{\tilde{c}_t}, \qquad \text{and } \frac{\tilde{n}_{t+s}}{\tilde{n}_t}, \quad \forall s \in [-d, 0]$$

   are all periodic functions.
6. The family of periodic orbits $p_d(t)$ satisfies the TVC.

Quantitative Analysis and Medium-Term Movements

Our objective is to show that the conditions required for our economy to be on a permanent cycle equilibrium are quantitatively sensible.

For this purpose, we set the model parameters to

$$d = 8.2, \quad \rho = 0.03, \quad \alpha = 0.9, \quad \nu = 1 \;\; \text{and} \;\; A = 0.786, \tag{19}$$

which allows us to replicate some key features of the US economy.

The adopted value of d is consistent with Mansfield's estimations, and $\alpha = .9$ is in line with estimated markups in Basu and Fernald, implying a markup rate of 11%.

By setting $\rho = .03$, A was chosen for the growth rate g_e to be equal to 2.4% as in Comin and Gertler.

A slight increase in the adoption delay d and two roots transversally cross the imaginary axis and **a periodic orbit emerges with periodicity $T = 11.21$ years.**

To simulate the path of c_t and n_t we have to set the initial conditions.

We assume that during the years 1948 to 1959 the US economy faced medium term movements similar to those estimated by Comin and Gertler for the same period.

To match this movement, we assume that the initial (detrended) condition is represented by the trigonometric function

$$\bar{n}_t e^{-g_e t} = 1 + a \cos(bt/\pi)$$

where parameter a and b are set respectively to 0.0375 and 20/11 to reproduce an amplitude close to 8%.

We simulate the nonlinear system to find the solution of detrended output, measured as $A\tilde{n}_{t-d}$ and normalized to turn around zero.

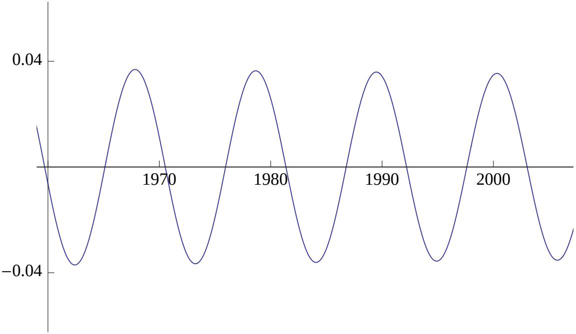

Figure: Simulated path for US output normalized around zero.

We observe that

1. the simulated output converges to a deterministic (Juglar) cycle with periodicity 11.21 years;
2. the periodicity but not the amplitude is independent on the initial conditions;
3. the simulated path implies recessions around 1973, 1984, 1995, and 2006.

Also in the paper:

- We explain why the qualitative and quantitative results are similar for value of ν different but still close to zero.
- We solve the Social Planer Problem of this economy and show that a procyclical R&D subsidy rate designed to half consumption fluctuations increases the growth rate from 2.4% to 3.4% with a 9.6% (compensation equivalent) increase in welfare.