Pollution, Mortality and Optimal Environmental Policy

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Pollution and the economy

- A large part of the literature on pollution is primarily concerned with climate change and global warming.
- However, there is a growing scientific literature that details a large and significant impact of pollution on health and mortality.
- This has received much less attention in the economics literature and is the basis of this paper.

Pollution and mortality

- According to WHO:
  - China: 656,000 annual premature deaths due to air pollution and an additional 96,000 deaths due to water pollution.
  - India: 537,000 die annually due to air pollution.
  - USA: 46,000 deaths due to air pollution.
- A survey gives a large estimate: up to 40% of all premature mortality is due to pollution (Pimentel, *et al* (2007)).

This paper

- Models the combined dynamics of income, environmental quality and life expectancy.
- Assumes a negative relationship between survival and environmental degradation and a positive income effect on survival
- How optimal policy interacts with pollution and income

Literature

- Growth-pollution-life expectancy.
  - Pautrel [2009],[SJE,2011] and Jouvet *et al.* [JE, 2010])
    - study first/second best environmental policy
    - don't consider possibility of non-convexities and multiple steady states.
  - Mariani *et al.* [JEDC,2010]
    - multiplicity of long-run equilibria via discontinuities in survival probabilities.
  - Varvarigos [WP,2011] and Palivos and Varvarigos [WP,2011]
    - allow for fluctuations in capital along the growth path, also with discontinuties
    - concerned with policies that maximize the probability of survival not welfare.

Our Model

- Discrete time Overlapping Generations model with risk of premature mortality (Chakraborty [2004]).
- Each period a new generation is born, consisting of a continuum of identical agents.
  - Agents
    - born in period t live at most to period $t+1$
    - young at time t survive till old age with probability π_t
    - inelastically supply 1 unit of labour at wage w_t which is used to finance consumption c_t^y and savings for old age s_t
  - young buy annuities from perfectly competitive intermediares who lend out proceeds to firms for investment in capital
  - Production follows constant returns technology $y_t = Ak_t^{\alpha}$
  - set depreciation to 1 so

$$k_{t+1} = s_t,$$

k_{t+1} is capital per worker at time $t+1$

Pollution emission and abatement

- Production causes a proportionate flow of pollution,

$$\zeta_t = \gamma y_t,$$

  $\gamma > 0$.
- The stock z_t of pollutants, evolves as

$$z_t = \zeta_t + \phi z_{t-1}, \text{ where } 1 > \phi > 0 \text{ represent persistence.}$$

- Persistence of pollutants (ozone, $PM_{2.5}$, PM_{10}) up to 3 years in U.K. (Windsor and Toumi (2001)).

Pollution emission and abatement

- Environmental policy consists of an abatement technology that is costly to operate:
  - Funded through a proportional tax τ_t on young agents' income: net wage is $(1-\tau_t)w_t$.
  - The efficiency of abatement is $\chi \geq 0$ and given the technology, the stock of pollution accumulates as

$$z_t = \gamma y_t - \chi \tau_t w_t + \phi z_{t-1}.$$

  - After substituting for w_t and redefining terms, simplifies to

$$z_t = \gamma(1-\psi\tau)Ak_t^{\alpha} + \phi z_{t-1}.$$

  - Where $\psi = \chi(1-\alpha)/\gamma$ is assumed to lie in $[0,1]$.

Survival Probability

$$\begin{aligned}
\pi_t &= \pi(y(k_t), z(k_t)) = \pi(k_t); \\
\pi_y(y,z) &\geq 0,\ \pi_z(y,z) \leq 0, \\
\pi &\in [0,1],\ \ \forall y \geq 0 \ \&\ \forall z \geq 0; \\
\pi(0,z) &= \underline{\pi} \in [0,1]\ \forall z \geq 0; \\
\pi(y,\infty) &= 0\ \forall y \geq 0.
\end{aligned}$$

Preferences

- Agents maximize their utility

$$U = \ln c_t^y + \pi_t \ln c_t^o$$

- subject to life-cycle budget constraints
  - $c_t^y \leq (1-\tau)w_t - s_t$
  - $c_{t+1}^o \leq \frac{r_{t+1}}{\pi_t} s_t$
- The solution to the above problem is

$$s_t = \frac{\pi_t}{1+\pi_t} A(1-\tau)(1-\alpha)k_t^\alpha$$

Equilibrium

- Using the market clearing condition the dynamic path is completely characterised by

$$k_{t+1} = \frac{\pi_t}{1+\pi_t} A(1-\tau)(1-\alpha)k_t^{\alpha}$$

$$k_{t+1} = \mathtt{G}(k_t)$$

- Given k_0 and z_{-1}, the dynamic path of the economy is fully described - given tax policy τ
- Pollution is also a state variable but its path is completely specified once k_t is determined

Dynamics

- A steady state consists of a π, k, y, z that satisfy the following equations
    - $\pi = \pi(y(k), z(k)) = \pi(k)$
    - $k = \mathtt{G}(k) = \frac{\pi(k)}{1+\pi(k)} A(1-\tau)(1-\alpha)k^{\alpha}$
    - $z = \frac{\gamma(1-\psi\tau)Ak^{\alpha}}{1-\phi}$
    - $y = (1-\tau)Ak^{\alpha}$

The steady state mapping

$\mathtt{G}(k)$: $R^+ \to R^+$ describes the steady state mapping.

$\mathtt{G}(0) = 0$ there is always a trivial steady state because

$$\mathtt{G}(0) = \frac{\pi}{1+\underline{\pi}}\Gamma(0)^\alpha = 0$$

Lemma 1 For any $\alpha \in (0,1)$ and $\tau \in (0,1)$ there exists an $\widehat{A}$ and a $\widehat{k}$ and associated $\widehat{\Gamma}$: $\widehat{\Gamma} = \frac{1+\pi(\widehat{k})}{\pi(\widehat{k})}\widehat{k}^{1-\alpha}$ such that $\Gamma > \widehat{\Gamma}$, $\mathtt{G}(\Gamma, \widehat{k}) > \widehat{k}$.

Proof: $G(k)$ can be rearranged so that $\Gamma = \frac{1+\pi(k)}{\pi(k)}k^{1-\alpha}$. Pick $\widehat{k}$ which defines $\widehat{\Gamma}$. With this choice for $\widehat{\Gamma}$, it follows that

$$\widehat{k} = \frac{\pi(\widehat{k})}{1+\pi(\widehat{k})}\Gamma\widehat{k}^\alpha = \frac{1+\pi(\widehat{k})}{\pi(\widehat{k})}\widehat{k}^{1-\alpha}\widehat{k}^\alpha = \widehat{k}.$$

For any $\Gamma > \widehat{\Gamma}$, it follows that $G(k) > k$.

Proposition 1 If the disembodied productivity, A, is large enough, and $\lim_{k\to 0}\pi'(k) < \infty$ then there are two interior steady states, $k_1^\star$ and $k_2^\star$ such that $k_1^\star < k_2^\star$

Proof: $\lim_{k\to 0} \mathtt{G}'(k) = \lim_{k\to 0}\left[\frac{\Gamma k^\alpha}{1+\pi(k)}\right]\lim_{k\to 0}\left\{\alpha\frac{\pi}{k} + \frac{\pi'(k)}{1+\pi(k)}\right\} = 0$ only if $\lim_{k\to 0}\pi'(k) < \infty$. This ensures that at low levels of k that $G(k) < k$.

Lemma 1 ensures that $G(k) > k$ for A sufficiently large. For sufficiently large k it is easy to show that $k > G(k)$. The existence of k_1 such that $k_1 < k_2$ follows from the intermediate value theorem.

The transformation mapping

- For any $k_0 \in (0, k_1^\star)$, economy converges to trivial steady state
- For any $k_0 \in (k_1^\star, k_2^\star)$, economy converges to $k_2^\star$
- $k_1^\star$ represents a poverty trap for two reasons: it has lower output but also is a threshold for which any $k < k_1^\star$ will diverge to the trivial steady state.

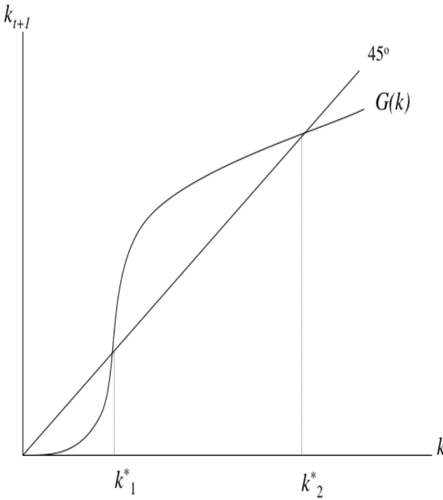

Figure 1

Survival rate along steady state

- The survival rate is

$$\pi(k) = \frac{k^{1-\alpha}}{\Gamma - k^{1-\alpha}}.$$

- Note $\Gamma = A \cdot (1-\tau)(1-\alpha)$ is a constant.
- This is increasing in k.
- Note that $\lim_{k \to 0} \pi = \underline{\pi}$.

Change in tax

- An increase in tax rate on emissions has following effect on $\mathtt{G}(k)$

$$\frac{\partial \mathtt{G}(k)}{\partial \tau} = \left[-\frac{\pi}{1+\pi} - \frac{\frac{[\pi_z \gamma \psi](1-\tau)Ak^{\alpha}}{1-\phi}}{(1+\pi)^2} \right] (1-\alpha)Ak^{\alpha}$$

- We know $s_t = G(k_t)$ and an increase in τ lowers net wage incomes which at constant π and lowers $\mathtt{G}(k)$.

Change in tax on steady state capital

- As shown, both steady state capital stocks increase,
  - but this leads to a widening of the poverty trap;
  - while the neoclassical state moves rightward.

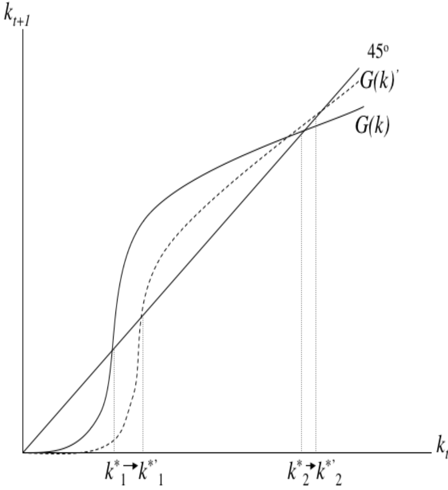

Figure 2

Example

- Assuming the specific functional form:

$$\pi = \frac{\underline{\pi} + y^{\beta}}{1 + y^{\beta}} \frac{1}{1 + z^{\delta}}.$$

- Sufficient conditions for Lemma 1:

$$\min\{\beta, \delta\} > \frac{1}{\alpha} > 1$$

- Following set of parameter values,

$$\alpha = 1/3, A = 2, \gamma = 1, \underline{\pi} = 0.0, \beta = \delta = 5, \psi = 0.8, \phi = 0.1;$$

MATLAB was used to solve for steady states at different values of τ.

τ	k_ℓ^*	k_h^*
0.00	0.0339	0.0965
0.15	0.0404	0.1136
0.35	0.0686	0.1026

Second best policy

- The only consequence of pollution is that it creates a negative external effect on expected lifetimes.
- Given the OLG framework, externality only affects expected lifetime utility of the young
- Hence there is the potential for welfare improvement via a tax on the young with the proceeds going to pollution abatement.
- Sequential optimal abatement policies that, John and Pecchenino [1994], maximize expected lifetime utility of young.

The planner problem

In each period t, a government chooses an optimal pollution tax to maximise lifetime welfare of the generation born in that period:

$$\max_{\tau_t} U^t = \ln c_t^y + \pi_t \ln c_{t+1}^o$$

subject to

- agents' budget constraints
- competitive equilibrium savings behaviour;
- size restrictions on the tax rate: $1 \geq \tau \geq 0$

The planner problem

- After substitution this becomes

$$\max_{\tau_t} V(k_t, \tau_t) = \ln\left(\frac{(1-\tau_t)(1-\alpha)Ak_t^\alpha}{1+\pi(k_t)}\right) + \quad (1)$$

$$\pi(k_t)\ln\left(\frac{\alpha(1-\alpha)^\alpha A^{1+\alpha}(1-\tau)^\alpha k^{2\alpha}}{\pi(k_t)^{1-\alpha}(1+\pi(k_t))^\alpha}\right). \quad (2)$$

- The f.o.c. is:

$$\frac{dV_t}{d\tau_t} = \left[\ln c_{t+1}^o - \frac{2-\alpha+\pi_t}{1+\pi_t}\right] \cdot \frac{\partial \pi_t}{\partial \tau_t} - \frac{1+\alpha\pi_t}{1-\tau_t} \leq 0; \quad (3)$$

where < 0 implies $\tau_t = 0$.

- The effects are:
  - Direct effect reduces consumption and savings (last term).
  - Indirect effect: raises π which increases savings but reduces return from savings.

Proposition 2: If k_t is below some threshold level $\underline{k}$, then $\tau_t = 0$.

Proof: From the following f.o.c:

$$\left[\ln c^o_{t+1} - \frac{2-\alpha+\pi_t}{1+\pi_t}\right] \cdot \frac{\partial \pi_t}{\partial \tau_t} - \frac{1+\alpha\pi_t}{1-\tau_t} \leq 0,$$

a necessary condition for $\tau_t > 0$ is that $\left[\ln c^o_{t+1} - \frac{2-\alpha+\pi_t}{1+\pi_t}\right] > 0$ because the effect of an increase in tax on survival is positive, i.e. $\frac{\partial \pi}{\partial \tau} > 0$. For initial capital sufficiently small k_o this will not hold because consumption of the old is so small that the log of this small value approaches minus infinity.

The tax function

Because the general parameterisation so far it is turns out to be difficult ensure second-order conditions for the optimal tax to hold. To proceed, we assume the following parametric form:

$$\pi = \left[\frac{\underline{\pi} + y^{\beta}}{1 + y^{\beta}}\right]\left[\frac{1}{1 + z^{\delta}}\right] \ \min\{\beta, \delta\} \geq \frac{1}{\alpha} > 1$$

Proposition 3: Provided that the second-order condition for the optimal tax is satisfied, and the level of capital is above a threshold $\tilde{k}$

- (i) there exists a function $h : [\tilde{k}, \infty) \longrightarrow [0, 1)$ such that optimal $\tau = h(k)$;
- (ii) $h(k)$ is (weakly) increasing in k.

Proof: (i) follows from the parameterisation of the survival probability and the s.o.c.. (ii) follows from the implicit function theorem

$$\frac{\partial h(k)}{\partial k} = -\frac{\mathtt{H}_k}{\mathtt{H}_r} \geq 0$$

Steady state with optimal taxes

$$\begin{aligned}\tau_t &= h(k_t) \\ k_{t+1} &= \frac{\pi(\tau_t, k_t)}{1+\pi(\tau_t, k_t)} A(1-\tau_t)(1-\alpha)k_t^{\alpha} = \mathtt{G}(\tau_t, k_t)\end{aligned}$$

In a steady state:

$$\begin{aligned}\tau &= h(k) \\ k &= \frac{\pi(k, \tau)}{1+\pi(k, \tau)} A \cdot (1-\tau)(1-\alpha)k\alpha \Rightarrow k = g(\tau)\end{aligned}$$

Local dynamics

The equation of motion is a first-order difference equation in k

$$k_{t+1} = \frac{\pi(h(k_t), k_t)}{1 + \pi(h(k_t), k_t)} A(1 - h(k_t))(1 - \alpha) k_t^{\alpha} = \mathtt{G}(h(k_t), k_t)$$

Linearising around a steady state

$$\left. \frac{dk_{t+1}}{dk_t} \right|_{k^*} = \mathtt{G}'(k^*) + \frac{\partial \mathtt{G}(k^*)}{\partial \tau} h'(k^*)$$

Simplifying:

$$\left. \frac{dk_{t+1}}{dk_t} \right|_{k^*} = \mathtt{G}'(k^*) + g'(\tau^*)(1 - \mathtt{G}'(k^*)) h'(k^*). \quad (4)$$

where

$$g'(\tau^*) = \frac{\frac{\partial \mathtt{G}(k^*)}{\partial \tau}}{1 - \mathtt{G}'(k^*)}.$$

Local dynamics

Type	$\mathtt{G}'(k)$	$g'(\tau)$	$h'(k)g'(\tau)$	**Dynamics**
Neoclassical	< 1	< 0	$> -\mathtt{G}'/(1-\mathtt{G}')$	Stable
Neoclassical	< 1	< 0	$< -\mathtt{G}'/(1-\mathtt{G}')$	Oscillations
Neoclassical	< 1	> 0	> 1	Unstable
Neoclassical	< 1	> 0	< 1	Stable
Poverty Trap	> 1	> 0	< 1	Unstable
Poverty Trap	> 1	> 0	> 1	Stable
Poverty Trap	> 1	> 0	$< -\mathtt{G}'/(1-\mathtt{G}')$	Oscillations