

Liquidity and Inefficient Investment

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Abstract

We study the role of fiscal policy in a complete market model where the only friction is the non-pledgeability of human capital. We show that the competitive equilibrium is constrained inefficient, leading to too little risky investment. We also show that fiscal policy following a large negative shock can increase ex ante welfare. Finally, we show that if the government cannot commit to the promised level of fiscal intervention, the ex post optimal fiscal policy will be too small from an ex ante perspective.

Key Words: liquidity, money, banking.

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The Great Recession and the ensuing policy debate on the fiscal stimulus have spurred a renewed interest among economists in the role of government intervention over the business cycle. What is the fundamental inefficiency in the economy that justifies an intervention? Is this intervention justifiable only *ex post*, in the face of an unexpected shock, or are there some *ex ante* benefits if it is anticipated? If the intervention is correctly anticipated, will the government have the incentive to stick to the *ex ante* optimal fiscal policy or is there a time inconsistency?

We build a very simple general equilibrium model to answer these questions. We consider an economy subject to an aggregate shock. Suppose that this shock is verifiable and so can be insured against—indeed contingent security markets exist. Assume that if the shock hits, firm profits are low and so is consumer wealth. Then the shock can lead to a fall in demand for consumption goods, and a fall in prices and employment. If there are no imperfections (e.g., sticky prices) there will be no inefficiency: the negative shock lowers welfare but there is nothing that a benevolent government can do to improve matters.

We argue that this conclusion changes if we add just one reasonable friction: consumers cannot pledge their future human capital. Under these conditions (non-human) assets play two roles: they serve as liquidity (because they allow consumers to credibly transfer future income), and they represent part of the consumer's wealth. If the value of pledgeable wealth falls then consumers will cut back on purchases not just because they want to consume less but also because they *have* to consume less.

Anticipating this possibility even risk neutral consumers will want to insure against the aggregate shock by holding an asset whose value is relatively stable, e.g., this insurance need generates a demand for a riskless asset. We show that this asset will be held even if it is dominated in expected present value terms by riskier assets. However, we also show that the economy does not get the balance between risky and riskless assets right. Because of liquidity constraints there is an externality: liquidity-constrained consumers over-herd the riskless asset, driving up prices of goods and hurting other liquidity-constrained consumers who are trying to buy the same goods. We show that the market equilibrium is not even second-best efficient: the government can intervene to obtain a Pareto improvement. Indeed our analysis provides a foundation for certain kinds of fiscal policy that are observed in the real world. We show that a government "gift" of bonds to consumers will increase output and welfare, even if all consumers are *ex ante* identical and consumers fully anticipate that this gift will be paid for in the future through higher taxes.

Our model, which is a finite horizon one, contains two groups of agents, whom we call doctors and builders. Doctors buy building services from builders and then builders buy doctors' services from doctors, or the other way round. Each builder requires a doctor at a different date and typically one with

different skills from the doctor she is building for, and vice versa for doctors. In other words there is no simultaneous double coincidence of wants (see Jevons (1876) and, for a modern treatment, Kiyotaki and Wright (1989)). Since we assume that future labor income is not pledgeable, this generates a need for means of payment.

Agents are endowed with wheat. Wheat can be invested in various projects. We assume that all asset returns can be pledged but that the returns from some activities are risky. Moreover, risky returns are positively correlated, i.e., there are aggregate shocks. We also suppose that uncertainty about returns is resolved before trading in doctors' and builders' services takes place. Then, a high return realization of risky assets provides a large amount of liquidity for the economy, while a low return realization provides a low amount of liquidity. Since there are diminishing returns to liquidity—the marginal value of liquidity falls to zero when the gains from trade have been exhausted—this induces the equivalent of risk aversion in agents: the yield on the high return assets is discounted in the good state, and safe assets are favored for liquidity purposes.

We study how competitive firms will invest and issue Arrow securities. In the first-best, the economy operates at full capacity and all wheat is invested in risky projects. In the second-best, however, if the total amount of pledgeable wealth in the bad state is low, the competitive economy will overinvest in the riskless technology and overproduce “money”. Investing in liquidity, a doctor who buys building services before he sells doctor services imposes a negative externality on other doctors since his actions increase the price of building services. Moreover, this externality has first order welfare consequences given that doctors are liquidity constrained. Because of this externality too much wheat is invested in the safe asset to create liquidity instead of being invested in socially productive projects.

As a result of this inefficiency, the government can improve on the competitive equilibrium by restricting the amount of investment in the riskless asset. Alternatively, the government can increase the efficiency of the economy by introducing a riskless asset. In our finite horizon economy fiat government money can exist only if the government can tax individuals. We assume that the government can impose sales taxes and that agents can pay these sales taxes with government notes. In our model government notes or government money are equivalent, because both of them must be backed up by future taxes. Therefore, since the intervention we consider does not affect the wealth of each consumer, but only the temporal distribution of this wealth, we label it *fiscal policy*.

We find that government fiscal policy in bad states not only can increase output more than one-to-one (fiscal multiplier), but also can improve ex-ante welfare. We also find that if the government cannot commit to the optimal level of fiscal policy, it will do *too little* of it ex post, i.e. the ex post optimal fiscal policy is different from the ex ante one. This is reminiscent of the famous Kydland and

Prescott (1977) result, with two differences: it applies to fiscal policy and it goes in the opposite direction (too little rather than too much).

Our work is closely related to Holmstrom and Tirole's recent (2011) book (henceforth HT) and Lorenzoni (2008). Both HT and Lorenzoni focus on firms' liquidity needs in the face of an aggregate shock when firms' cash flow is not fully pledgeable and consumers cannot pledge future endowments. By contrast, we focus on consumers' liquidity needs in a world where securities income is fully pledgeable, but consumers' future labor income is not. Our model requires only one friction: the inability of consumers to pledge their future human capital rather than two: the inability of firms to pledge future returns and the inability of consumers to pledge future endowments.

Yet, several of the same forces are at play in all these works. As in HT the government's ability to tax gives it a role in improving matters by injecting liquidity, e.g., by creating a riskless asset and taxing consumers later to finance repayments. As in Lorenzoni (but unlike in HT's main model (Chapter 3) and in Holmstrom and Tirole (1998)), the equilibrium without government money creation is second-best efficient. This is because in HT firms do not use their extra liquidity to drive up prices (e.g., of assets) faced by other firms in the same way that doctors drive up the price of building services. HT do consider a variation of their model (Chapter 7), in which some firms fail and others succeed, and show that firms may invest excessively in liquidity to buy up assets at fire-sale prices, but we think it fair to say that this variation is something of a sideline to the main model used in their book.

The two perspectives lead to different policy implications. HT's preferred solution is an injection of liquidity to firms, while our preferred one is a fiscal policy to improve consumers' liquidity position. In the context of the last recession, HT's model would push toward a more aggressive monetary policy, while ours might suggest a renegotiation of underwater mortgages to alleviate consumers who were liquidity constrained.

Our model also bears a resemblance to Stein (2012) and Gennaioli et al. (2012). Stein (2012) derives an inefficiency in the provision of money based on the assumption that agents have a discontinuous demand for a riskless claim (money) and that there is a friction in financial markets (patient investors cannot raise additional money). Gennaioli et al. (2012) develop a model in which infinitely risk-averse investors have an appetite for riskless assets and, since these investors ignore unlikely risks, financial intermediaries over-supply these assets. In contrast to these papers we endogenize the demand for riskless assets and any financial frictions and we do not assume any investor irrationality.

Our paper is also related to the huge literature on money. Much of this literature is concerned with the role money plays in general equilibrium (e.g., Hahn (1965)). To create such a role, one needs to dispense with the traditional Walrasian auctioneer and explicitly introduce an exchange process. Ostroy and Starr (1990) provide an excellent survey of attempts in this direction. As far as we can tell, none of

these attempts analyze the externality we identify in our paper. The role money plays in our model (i.e., to address the lack of double coincidence of wants) is similar to that in Kyotaki and Wright (1989). Their focus, however, is on what goods can become money and how. Our focus is to what extent private traders can provide the efficient quantity of medium of exchange.

There are parallels between our work and the literature on incomplete markets. In that literature a competitive equilibrium is typically inefficient and a planner operating under the same constraints as the market can do better (see, e.g., Hart (1975) and Geanakoplos and Polemarchakis (1986)). One feature of the incomplete markets literature is that the market structure is taken as given. This raises the question: why can't the private sector create new securities to complete the market? Our work differs in that we endogenize the market structure: markets are complete with respect to verifiable events such as aggregate shocks, but the inability to borrow against human capital creates liquidity problems¹. Related to this, we focus on whether a market economy overinvests in safe assets, something that, as far as we know, the incomplete markets literature has not considered.

Finally, our paper is linked to a vast and growing literature on the welfare effects of pecuniary externalities in the presence of financial constraints (see, e.g., Kehoe and Levine, (1993), Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009)). Yet, this literature typically finds that the competitive equilibrium delivers an inefficiently low investment in liquid assets. We find the opposite: a competitive equilibrium delivers an inefficiently high level of liquidity.

2. The Framework

We consider an economy that lasts four periods:

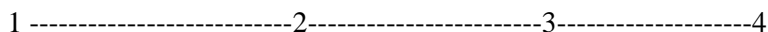


Figure 1

There are two types of agents in equal numbers: doctors and builders. The doctors want to consume building services and the builders want to consume doctor services. Consumption of these services occurs in periods 2 and 3. Doctors and builders can also consume wheat in period 4 and there is no discounting. Each doctor and builder has an endowment of wheat in period 1 equal to e . We will assume that $e > 1$.

We write agents' utilities as:

$$\text{Doctors: } U_d = w_d + b_d - \frac{1}{2}l_d^2$$

¹ In this respect our paper has parallels with Kiyotaki and Moore (1997), which also endogenizes the market structure, although in a different context.

Builders: $U_b = w_b + d_b - \frac{1}{2}l_b^2$

where w_i is the quantity of wheat consumed by individual $i = d, b$; b_d is the quantity of building services consumed by a doctor; l_d is the labor supplied by a doctor; d_b is the quantity of doctor services consumed by a builder; and l_b is the labor supplied by a builder. We assume constant returns to scale: one unit of builder labor yields one unit of building services and one unit of doctor labor yields one unit of doctor services. In words, doctors and builders have a constant marginal utility of wheat, a constant marginal utility of the service provided by the other group of agents, and a quadratic disutility of labor.

In period 1 each agent learns whether he will first buy or sell. Ex ante both events are equally likely. For convenience, we assume that in the east side of town doctors buy builders' services in period 2, while builders buy doctors' services in period 3. In the west side of town, the order is reversed. In periods 2 and 3 the markets for doctor and building services meet and the doctors and builders trade in the order determined in period 1.

We assume there are many doctors and many builders, and so the prices for both services are determined competitively. It is crucial for our analysis that there is *no simultaneous* double coincidence of wants: builders and doctors are in either the market for buying or the market for selling: they cannot do both at the same time. Hence, even if the builder a doctor buys from wants the doctor services from her customer, she cannot buy them at the same time as she is selling building services.

We have deliberately set up the model to be very symmetric; this helps with the welfare comparisons later. Throughout the paper we will analyze the east part of town, where doctors buy in period 2 and builders in period 3; the reverse case is completely symmetric.

Let's turn now to investment. In period 1 wheat can be invested in two technologies. There is a riskless technology (storage) where one unit of wheat is transformed into one unit of wheat in period 4; and there is a risky technology where one unit of wheat is transformed into $R^H > 1$ units of wheat in period 4 with probability π and $R^L < 1$ units with probability $1 - \pi$, where $0 < \pi < 1$ and $\bar{R} = \pi R^H + (1 - \pi)R^L > 1$. *The returns of the various risky projects are perfectly correlated.* Thus there are two aggregate states of the world, $R = R^H$ or $R = R^L$, which we refer to as high (H) or low (L). Agents learn about the aggregate state of the world between periods 1 and 2. All agents are risk neutral.

We suppose that there is free entry of firms possessing the two technologies described above and that these firms face constant returns to scale.

2.1 A benchmark: the Arrow-Debreu equilibrium

In an Arrow-Debreu economy, the state of the world H or L is verifiable. In addition the penalties for default are infinite. Hence doctors can pledge to pay the builders out of income from supplying doctor services that they will earn in period 3.

In such an economy (more precisely, its sequential version), markets for two Arrow securities open in period 1, one paying a unit of wheat in period 4 in state H and the other paying a unit of wheat in period 4 in state L. These securities will be supplied by firms investing in projects. Doctors and builders will purchase these securities. (We assume that doctors know that they will buy before they sell, and builders that they will sell before they buy, before they purchase the Arrow securities; we return to this point below.) By period 2 the state of the world will be known to everyone and thus so will be the payoffs of the two securities. The securities can be re-traded in periods 2 and 3 (since there is no discounting the interest rate will be zero and so the price of the H Arrow security will equal 1 in state H and 0 in state L, and vice versa for the L security). In period 2 the market for building services opens: doctors buy building services using their portfolio wealth and their future labor income. In period 3 the market for doctor services opens: builders buy doctor services using their portfolio wealth and the income they earned in period 2. Finally, in period 4 firms pay out what they owe to the holders of Arrow securities and wheat is consumed.

It is easy to compute the Arrow-Debreu equilibrium. Once we have reached period 2 all the uncertainty is resolved and so we are in a classic competitive equilibrium model without uncertainty, where there are three goods: doctor services, builder services, and wheat. There will be one competitive equilibrium in the high state and another in the low state. Normalize the period 4 price of wheat to be one. Let $p_b^H, p_b^L, p_d^H,$ and p_d^L be the prices of builder and doctor services in the high and low state, respectively, relative to wheat. (We are setting the interest rate to be zero.) Focus on one of the states and for the moment leave off the state superscript. Markets cannot clear if either $p_b > 1$ or $p_d > 1$, since demand will be less than supply for building/doctor services, respectively (consumers will prefer wheat). On the other hand, we cannot have both $p_b < 1$ and $p_d < 1$ because then the demand for wheat would be zero, while the supply is positive. Hence, either $p_b < 1$ and $p_d = 1$, or $p_b = 1, p_d < 1$, or $p_b = p_d = 1$. By symmetry we would expect $p_b = p_d$ (we leave the reader to confirm this). We are left with $p_b = p_d = 1$. Utility maximization then implies $b_d = l_b = d_b = l_d = 1$. Each consumer receives a net surplus of $\frac{1}{2}$ from buying one unit of a service and supplying one unit of labor.

Now go back to period 1. Since all parties are risk neutral it is clear that Arrow security prices will be proportional to probabilities. Also given constant returns to scale neither project can make a profit and at least one must break even. Thus $q^H = \pi / \bar{R}$, $q^L = (1 - \pi) / \bar{R}$, where we normalize the price of period 0 wheat to be one. At these prices only the risky project is employed.

Summarizing, we have

Proposition 1: In the unique Arrow-Debreu equilibrium all wheat is invested in the risky project and $p_b^H = p_b^L = p_d^H = p_d^L = 1$, $q^H = \pi / \bar{R}$, $q^L = (1 - \pi) / \bar{R}$. The utilities of the doctors and builders are $U_d = e\bar{R} + \frac{1}{2}$, $U_b = e\bar{R} + \frac{1}{2}$, respectively.

We see from Proposition 1 that the Arrow-Debreu allocation and prices are independent of the initial endowment e and \bar{R} (except for consumption of wheat, which varies one to one with $e\bar{R}$), and each agent's utility is linear in wealth. Note that in the Arrow-Debreu equilibrium there is a separation between trade in goods and investment. Given that agents can pledge their human capital, investment income is not need to finance trade and so investments are chosen entirely on efficiency grounds.

One implication of Proposition 1 is that, since agents' utilities are linear in wealth, there is no demand for insurance before an agent learns his type, i.e., whether he will buy first or sell first. To put it another way the Arrow-Debreu equilibrium maximizes the expected utility of an agent before he knows whether he will buy or sell first (or equivalently whether he is a doctor or a builder): $\frac{1}{2}U_d + \frac{1}{2}U_b = e\bar{R} + \frac{1}{2}$. In what follows we will refer to the Arrow-Debreu equilibrium allocation as the first-best.

2.2 Equilibrium with Non-Pledgeable Labor Income

We now drop the assumption that default penalties are infinite. Instead we assume that doctors who have pledged their period 3 labor income to pay for period 2 goods can just disappear. Knowing this builders will accept only securities as a means of payment in period 2, i.e., doctors will face liquidity constraints. We continue to assume that the state of the world H or L is verifiable, and that markets for the H and L securities exist in period 1. These securities will be supplied by firms investing in projects. Although there are no explicit default penalties, the securities will be collateralized by the project returns in each

state and so there will be no default in equilibrium (we are supposing that asset returns cannot be stolen by firms' managers).

The Arrow securities will be used as a means of exchange in the period 2 and 3 doctor and builder markets. Let x_d^H and x_d^L be the quantities of the two Arrow securities bought by doctors and x_b^H and x_b^L the quantities bought by builders in period 1. As above, write q^H and q^L as the respective prices of the Arrow securities. Markets for builder and doctor services open in periods 2 and 3. Let p_b^H, p_b^L, p_d^H , and p_d^L be the prices of builder and doctor services in the high and low state, respectively; the price of the H (resp., L) Arrow security is 1 in periods 2 and 3 if H (resp., L) occurs, and 0 otherwise.

Consider a doctor's utility maximization problem. In equilibrium the price of building services in period 2 cannot exceed 1 since otherwise doctors would strictly prefer wheat to building services and so the building market would not clear. Thus we can assume for the purpose of calculating utility that doctors use all the proceeds from Arrow securities to buy building services. (By a parallel argument the price of doctor services in period 3 cannot exceed 1 and so for purposes of calculating utility we can assume that builders spend all their income on doctor services—see below.) Next consider a doctor's labor supply decision in period 3. Suppose that we have arrived in one of the states, and ignore the superscript on the state. Then a doctor will choose his labor supply l_d to maximize $p_d l_d - \frac{1}{2} l_d^2$, i.e., set $l_d = p_d$. Note that it is too late for the doctor to buy more building services and so his marginal return from work is p_d (he will use the proceeds to buy wheat). A doctor's labor yields revenue p_d^2 , which he redeems for wheat in period 4; in addition he incurs an effort cost of $\frac{1}{2} p_d^2$, and so his net utility is $\frac{1}{2} p_d^2$.

It follows that in period 1 a doctor chooses x_d^H and x_d^L to solve:

$$(*) \quad \text{Max} \quad \pi \left[\frac{x_d^H}{p_b^H} + \frac{1}{2} (p_d^H)^2 \right] + (1 - \pi) \left[\frac{x_d^L}{p_b^L} + \frac{1}{2} (p_d^L)^2 \right]$$

subject to

$$q^H x_d^H + q^L x_d^L \leq e.$$

Note that firm profits are zero in equilibrium given constant returns to scale, and so we do not need to keep track of any dividends received by consumers.

A similar calculation applies to builders. The difference is that a builder in period 2 chooses her labor supply l_b to maximize $\frac{p_b}{p_d} l_b - \frac{1}{2} l_b^2$. The reason is that a builder's marginal return from work is

$\frac{p_b}{p_d}$, since she will use her income to buy doctor services. Thus a builder's net utility from work is

$\frac{1}{2} \left(\frac{p_b}{p_d} \right)^2$. Hence in period 1 a builder chooses x_b^H and x_b^L to solve:

$$(**) \quad \text{Max} \quad \pi \left[\frac{x_b^H}{p_d^H} + \frac{1}{2} \left(\frac{p_b^H}{p_d^H} \right)^2 \right] + (1 - \pi) \left[\frac{x_b^L}{p_d^L} + \frac{1}{2} \left(\frac{p_b^L}{p_d^L} \right)^2 \right]$$

subject to

$$q^H x_b^H + q^L x_b^L \leq e.$$

Let y^s and y^r be the quantity of period 1 wheat invested respectively in the safe and risky technology. As noted, profit maximization and constant returns to scale imply zero profit: the value of the return stream of each technology cannot exceed the cost of investing in that technology (i.e., 1); and if the inequality is strict the technology will not be used. In other words,

$$(2.1) \quad q^H + q^L \leq 1 \quad \text{where } y^s = 0 \text{ if the inequality is strict;}$$

$$(2.2) \quad q^H R^H + q^L R^L \leq 1 \quad \text{where } y^r = 0 \text{ if the inequality is strict.}$$

The market clearing conditions in the securities and wheat market in period 1 are given respectively by

$$(2.3) \quad x_d^H + x_b^H = y^s + y^r R^H$$

$$(2.4) \quad x_d^L + x_b^L = y^s + y^r R^L$$

$$(2.5) \quad y^s + y^r = 2e$$

Finally, market clearing conditions in the builder and doctor markets in periods 2 and 3 in each state are:

$$(2.6) \quad p_b^H \leq 1. \text{ If } p_b^H < 1, \text{ then } \frac{x_d^H}{p_b^H} = \frac{p_b^H}{p_d^H}. \text{ If } p_b^H = 1, \text{ then } x_d^H \geq \frac{1}{p_d^H}$$

$$(2.7) \quad p_b^L \leq 1. \text{ If } p_b^L < 1, \text{ then } \frac{x_d^L}{p_b^L} = \frac{p_b^L}{p_d^L}. \text{ If } p_b^L = 1, \text{ then } x_d^L \geq \frac{1}{p_d^L}$$

$$(2.8) \quad p_d^H \leq 1. \text{ If } p_d^H < 1, \text{ then } \frac{x_b^H + x_d^H}{p_d^H} = p_d^H. \text{ If } p_d^H = 1 \text{ then } x_b^H + x_d^H \geq 1.$$

$$(2.9) \quad p_d^L \leq 1. \text{ If } p_d^L < 1, \text{ then } \frac{x_b^L + x_d^L}{p_d^L} = p_d^L. \text{ If } p_d^L = 1 \text{ then } x_b^L + x_d^L \geq 1.$$

(2.6) - (2.9) reflect the fact that, if the price of building or doctor services equals 1, consumers are indifferent between buying the service and wheat and so the market clears as long as liquidity is at least equal to supply.

In summary, the above describes a standard Arrow-Debreu equilibrium with one wrinkle: consumers cannot borrow against future labor income. In what follows we refer to this simply as a “competitive equilibrium”.

It is useful to compare the competitive equilibrium with what a planner could achieve. In the first-best the planner maximizes the expected utility of an agent who does not know whether he will buy or sell first (equivalently whether he will be a doctor or a builder) subject to the aggregate feasibility constraints. That is, the planner solves:

$$\text{Max} \left\{ \pi [w_d^H + b_d^H - \frac{1}{2}(l_d^H)^2 + w_b^H + d_b^H - \frac{1}{2}(l_b^H)^2] + (1 - \pi) [w_d^L + b_d^L - \frac{1}{2}(l_d^L)^2 + w_b^L + d_b^L - \frac{1}{2}(l_b^L)^2] \right\}$$

subject to:

$$b_d^H = l_b^H$$

$$d_b^H = l_d^H$$

$$b_d^L = l_b^L$$

$$d_b^L = l_d^L$$

$$w_d^H + w_b^H = y^s + y^r R^H$$

$$w_d^L + w_b^L = y^s + y^r R^L$$

$$y^s + y^r = 2e$$

where w_d^i, w_b^i stands for wheat consumption of doctors and builders in state $i=H,L$, l_d^i, l_b^i , stands for labor services of doctors and builders in state i , etc.

The solution is easily seen to be

$$b_d^H = l_b^H = d_b^H = l_d^H = b_d^L = l_b^L = d_b^L = l_d^L = 1$$

$$y^s = 0 \text{ and } y^r = 2e.$$

Of course, this is the same outcome as in the Arrow-Debreu equilibrium.

The next two lemma and proposition characterize the competitive equilibrium (in the absence of labor income pledgeability) and provide a comparison to the first-best.

Lemma 1: In a competitive equilibrium, the prices of doctor and builder services equal one in the high state (i.e., $p_d^H = p_b^H = 1$).

Proof: See Appendix.

The (rough) intuition is that, given $R^H > 1$ there is enough liquidity in the high state to support efficient trade at prices of 1.

Lemma 2: A necessary and sufficient condition for a competitive equilibrium to be first-best optimal is $2eR^L \geq 1$.

Proof: See Appendix.

The (rough) intuition is that if $2eR^L \geq 1$ there is enough liquidity to support efficient trade when all resources are invested in the risky technology, even in the low state.

Proposition 2: If $2eR^L \geq 1$, then a competitive equilibrium delivers the first-best. If

$1 > 2eR^L \geq \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ then a competitive equilibrium is such that investment is efficient (only the risky technology is used), but trading in doctor and building services is inefficiently low. If

$2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ a competitive equilibrium is such that investments and trading in labor services are both inefficient: the riskless technology is operated at a positive scale and trade is inefficiently low.²

Proof: See Appendix.

² In each region the competitive equilibrium is unique. We leave the proof to the reader.

Proposition 2 says that the competitive equilibrium is in one of three regions. If $2eR^L$ exceeds 1 then we achieve the first-best (which we already know from Lemma 2). If $2eR^L$ is smaller than but close to 1 then there is not enough liquidity in the low state and so trade is inefficient. However, all resources are invested in the risky technology –this follows from the fact that the safe technology is strictly unprofitable at the prices supporting the first-best and so will continue to be unprofitable in a neighborhood of the first-best (see the Appendix). Finally, if $2eR^L$ is much below 1 then the demand for liquidity is sufficiently great relative to the supply that some resources are switched to the safe technology, and so both trade and investment are inefficient.

Note that if $R^L = 0$, the economy will never be at the first-best.

Another way to understand Proposition 2 is as follows. Given that $2eR^L < 1$, what determines the type of distortion present is the comparison between the total amount of pledgeable wealth in the bad state and the ratio of the expected capital loss in the bad state ($(1 - \pi)(1 - R^L)$) to the expected capital gain in the good one ($\pi(R^H - 1)$). If the expected capital loss is small relative to the expected capital gain, then it is still optimal to invest all resources in the risky technology (the first-best outcome) and the inefficiency is limited to trading in doctor and builder services. If the expected capital loss is relatively large, then the optimal allocation will require some investment in the storage technology; this reduces the overall losses in the bad state and also the inefficiency in trading.

A natural question to ask is one we raised with respect to an Arrow-Debreu equilibrium. Since individuals know their type before they trade they could in principle obtain insurance against their type. Will they want to do so? As before the answer is no. It is easy to show that, in every region in Proposition 2, in equilibrium both doctors and builders buy some high state Arrow securities. Thus, in period 1 the builders' marginal utility of income will equal $\frac{\pi}{p_b^H q^H}$ and the doctors' marginal utility of income will equal $\frac{\pi}{p_d^H q^H}$. However, we have shown in Lemma 1 that $p_b^H = p_d^H = 1$. Hence the marginal utilities of the two groups are equal, and there is no role for individual insurance.

2.3 Overinvestment in safe assets

We now consider whether a planner operating under the same constraints as the market can do better. We assume that the planner can constrain the production decisions of firms, i.e., choose y^f (or equivalently y^s), but cannot interfere in markets in other ways.

We will focus on the case $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1}\right)^{\frac{4}{3}}$. It is easy to see from the proof of

Proposition 2 that in a neighborhood of the competitive equilibrium both technologies are used and $x_d^L > 0, x_b^L = 0$. Therefore, from (2.4)-(2.5) there is a one-to-one relationship between y^r and x_d^L : decreases in the former correspond to decreases in the latter. In what follows we therefore assume that the planner picks $x_d^L = x^{CP}$ rather than y^r (or equivalently y^s). We will show that the planner can increase surplus by reducing x^{CP} below the competitive level.

Suppose that the planner picks $x_d^L = x^{CP}$ in a neighborhood of the competitive equilibrium.

Then the market clearing conditions (2.7) and (2.9) yield

$$p_d^L = (x^{CP})^{\frac{1}{2}}$$

$$p_b^L = (x^{CP})^{\frac{3}{4}}.$$

Also q_H, q_L, q_H, q_L will satisfy (2.1)-(2.2) with equality.

The doctors' utility, which is given by $\pi \left[x_d^H + \frac{1}{2} \right] + (1-\pi) \left[\frac{x_d^L}{p_b^L} + \frac{1}{2} (p_d^L)^2 \right]$, becomes

$\pi \left[\frac{e - q^L x^{CP}}{q^H} + \frac{1}{2} \right] + (1-\pi) \left[(x^{CP})^{\frac{1}{4}} + \frac{1}{2} x^{CP} \right]$. Similarly, the builders' utility, which is given by

$\pi \left[x_b^H + \frac{1}{2} \right] + (1-\pi) \left[\frac{1}{2} \left(\frac{p_b^L}{p_d^L} \right)^2 \right]$, becomes $\pi \left[\frac{e}{q^H} + \frac{1}{2} \right] + (1-\pi) \left[\frac{1}{2} (x^{CP})^{\frac{1}{2}} \right]$.

The planner maximizes $U^d + U^b$. Differentiating the welfare function with respect to x^{CP} yields

$$(2.10) \quad -\pi \frac{q^L}{q^H} + (1-\pi) \left[\frac{1}{4} (x^{CP})^{-\frac{3}{4}} + \frac{1}{2} \right] + (1-\pi) \frac{1}{4} (x^{CP})^{-\frac{1}{2}}$$

We want to prove that (2.10) is negative when we evaluate it at the market equilibrium,

$x^{CP} = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$. From (2.1) and (2.2) we know that $\frac{1-R^L}{R^H-1} = \frac{q^H}{q^L}$. Hence, we can rewrite (2.10)

calculated at $x^{CP} = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ as

$$-\pi \frac{q^L}{q^H} + (1-\pi) \left[\frac{1}{4} \left(\frac{1-\pi}{\pi} \frac{q^H}{q^L} \right)^{-1} + \frac{1}{2} \right] + (1-\pi) \frac{1}{4} \left(\frac{1-\pi}{\pi} \frac{q^H}{q^L} \right)^{\frac{2}{3}}.$$

Simplifying, this becomes

$$-\frac{3}{4} \pi \frac{q^L}{q^H} + \frac{1}{2} (1-\pi) + \frac{1}{4} (1-\pi) \left(\frac{\pi}{1-\pi} \frac{q^L}{q^H} \right)^{\frac{2}{3}}.$$

Since $\frac{\pi}{1-\pi} \frac{q^L}{q^H} > 1$, we can simplify further to obtain

$$-\frac{3}{4} \pi \frac{q^L}{q^H} + \frac{1}{2} (1-\pi) + \frac{1}{4} (1-\pi) \frac{\pi}{1-\pi} \frac{q^L}{q^H} = -\frac{1}{2} \pi \frac{q^L}{q^H} + \frac{1}{2} (1-\pi) < 0.$$

It follows that the planner can increase surplus by reducing x^{CP} below the competitive level, or equivalently by reducing y^s .

Proposition 3: When $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$, the economy overinvests in safe assets.

In other words, the competitive equilibrium will be inefficient as long as there is sufficiently high aggregate uncertainty before trading takes place ($2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$). The intuition is that the non-pledgeability of future labor income creates an additional demand for relatively safe assets. The reason is that transactional needs generate a form of risk aversion even in risk neutral people. When an agent has the opportunity/desire to buy, having a great deal of pledgeable wealth in some states does not compensate her for the risk of having very little pledgeable wealth in other states, because there are diminishing returns to liquidity: in the former states the gains from trade have been exhausted and the marginal value of liquidity is zero, whereas in the latter states the agents are wealth-constrained and the marginal value of liquidity is high. As a result, agents are willing to hold relatively safe assets even if they have a lower yield. However, in doing so they ignore the negative externality they impose on other agents (doctors).

We have shown that too many resources are invested in manufacturing these relatively safe assets. An example of this overproduction is the huge expansion of the finance sector in the first decade of the new millennium, an expansion that cannot be explained by any of the traditional roles performed by

finance (Philippon (2008)). While the production of AAA mortgage-backed securities might have been privately optimal, our model suggests that it was not necessarily socially optimal.

3. Fiscal Policy

So far we have ignored the role of the government in providing liquidity. We will now relax this assumption. Following Holmstrom and Tirole (1998, 2011) and Woodford (1990), we assume the government can exploit a power it has, which the private sector does not have: the power to tax. In particular, the government can issue notes to consumers, and these notes will be valuable because they are backed by future tax receipts. In our finite horizon model notes or money are equivalent, because both of them must be backed by future taxes. Therefore, since the intervention we consider does not affect the wealth of each consumer, but only the temporal distribution of this wealth, we label it *fiscal policy*.

Holmstrom and Tirole (1998) justify the assumption that it is easier for the government to collect taxes, than for creditors to collect debts, from consumers on the grounds that the government can audit incomes or impose jail penalties. We adopt a different rationale. We suppose that the government can impose sales taxes on certain productive facilities that consumers use and which can be easily monitored. Private lenders cannot duplicate such an arrangement since they do not have the power to require (all) facilities to participate.

To allow for sales taxes we assume that in period 4 our agents consume flour as well as wheat: one unit of flour yields one unit of utility. There is a milling technology for turning wheat into flour: each agent can obtain λ units of flour at the cost of $\frac{1}{2}c\lambda^2$ units of wheat, where $\lambda \geq 0$ is the agent's choice variable. This activity occurs at facilities (mills) that can easily be monitored by the government, and so the government can impose a per unit flour tax t that cannot be avoided.

An agent's utility is now:

$$\text{Doctors: } U_d = w_d + b_d - \frac{1}{2}l_d^2 + (1-t)\lambda_d - \frac{1}{2}c\lambda_d^2$$

$$\text{Builders: } U_b = w_b + d_b - \frac{1}{2}l_b^2 + (1-t)\lambda_b - \frac{1}{2}c\lambda_b^2$$

where t is the tax rate on flour.

We assume that in period 4 each agent has a large endowment of wheat (in addition to any labor income and dividends from investment), so that she is not at a corner solution, and hence λ_d, λ_b satisfy the first order condition

$$(3.1) \quad \lambda_d = \lambda_b = \frac{1-t}{c}.$$

This yields

$$(3.2) \quad U_d = w_d + b_d - \frac{1}{2}l_d^2 + \frac{1}{2c}(1-t)^2$$

$$(3.3) \quad U_b = w_b + d_b - \frac{1}{2}l_b^2 + \frac{1}{2c}(1-t)^2$$

The total amount raised by taxes on doctors and builders is

$$(3.4) \quad T = \frac{2t(1-t)}{c}.$$

Since in the high state there is ample liquidity, it is natural to focus on the case where the government issues notes only in the low state. We will suppose that the government can target those who need the liquidity most: the doctors. (Our analysis so far is consistent with the assumption that it is verifiable in period 2 whether an agent has to buy or sell first.)³

In summary, the government's fiscal policy consists of issuing m notes to each doctor in period 2 if and only if the state L occurs. Each note promises one unit of wheat in period 4.

Given that government notes must be backed by taxes, we have

$$(3.5) \quad m = T = \frac{2t(1-t)}{c}.$$

Note that (3.5) implies that $T=0$ when $t=0$ and T reaches a maximum at $t = \frac{1}{2}$. Thus, it is never optimal

to set $t > \frac{1}{2}$ since the deadweight loss increases in t

From a period 2 perspective this very simple model exhibits some Keynesian features when

$2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$. If, when the state is low, the government intervenes with an (unexpected)

hand-out m in period 2, it will have the effect of boosting the level of output by more than m (fiscal multiplier). To see this, assume that x_d^L and x_b^L are fixed at their competitive equilibrium levels, which are less than 1. Then, after the government hand-out of m to doctors the new equilibrium becomes:

$$\frac{x_d^L + m}{p_b^L} = \frac{p_b^L}{p_d^L}$$

$$\frac{x_d^L + m}{p_d^L} = p_d^L,$$

³ The case where the government cannot target the constrained agents is not qualitatively different, but a bit more cumbersome.

which implies

$p_b^L = (x_d^L + m)^{\frac{3}{4}}$ and $p_d^L = (x_d^L + m)^{\frac{1}{2}}$. Since $l_d^L = p_d^L$ and $l_b^L = \frac{p_b^L}{p_d^L}$, the fiscal policy increases output

(which we measure as $p_d^L l_d^L + p_b^L l_b^L = (p_d^L)^2 + \frac{(p_b^L)^2}{p_d^L}$) from $2x_d^L$ to $2(x_d^L + m)$. Thus, the fiscal multiplier

is 2. Not only does a fiscal policy following a big negative shock increase output more than one-to-one, but it also increases ex ante welfare. To see this it is sufficient to notice that

$$\frac{\partial W^L}{\partial m} = \left[\frac{1}{4} (x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4} (x_d^L + m)^{-\frac{1}{2}} - \frac{(1-t)}{(1-2t)} \right] > 0 \text{ for } m \text{ close to } 0.$$

A more interesting question is what happens when the intervention is fully anticipated. We will consider two cases: one where the government can commit to m ex ante and the other where the government cannot commit. Also in contrast to Section 2.3 we will suppose that the government cannot constrain the production decision of firms: its only policy tool is m .

3.1. The case of commitment

We will assume that agents have rational expectations about government actions. Suppose that agents anticipate that m will be injected in the low state. Then, the equilibrium of Section 2.2 changes as follows:

In period 1 a doctor chooses x_d^H and x_d^L to solve:

$$(*) \quad \text{Max} \quad \pi \left[\frac{x_d^H}{p_b^H} + \frac{1}{2} (p_d^H)^2 + \frac{1}{2c} \right] + (1-\pi) \left[\frac{x_d^L + m}{p_b^L} + \frac{1}{2} (p_d^L)^2 + \frac{1}{2c} (1-t)^2 \right]$$

subject to

$$q^H x_d^H + q^L x_d^L \leq e.$$

where (*) reflects the fact that the flour tax is zero in state H and t in state L and the doctors receive m in state L.

Similarly, a builder chooses x_b^H and x_b^L to solve:

$$(**) \quad \text{Max} \quad \pi \left[\frac{x_b^H}{p_d^H} + \frac{1}{2} \left(\frac{p_b^H}{p_d^H} \right)^2 + \frac{1}{2c} \right] + (1-\pi) \left[\frac{x_b^L}{p_d^L} + \frac{1}{2} \left(\frac{p_b^L}{p_d^L} \right)^2 + \frac{1}{2c} (1-t)^2 \right]$$

subject to

$$q^H x_b^H + q^L x_b^L \leq e.$$

The other equilibrium conditions (2.1)-(2.6) and (2.8) stay the same, while (2.7) and (2.9) become

$$(3.6) \quad p_b^L \leq 1. \text{ If } p_b^L < 1, \text{ then } \frac{x_d^L + m}{p_b^L} = \frac{p_b^L}{p_d^L}. \text{ If } p_b^L = 1, \text{ then } x_d^L + m \geq \frac{1}{p_d^L}$$

$$(3.7) \quad p_d^L \leq 1. \text{ If } p_d^L < 1, \text{ then } \frac{x_b^L + m + x_d^L}{p_d^L} = p_d^L. \text{ If } p_d^L = 1 \text{ then } x_b^L + m + x_d^L \geq 1.$$

The government chooses in period 0 to maximize the expected utility of an agent who does not know whether he will buy or sell first (equivalently, whether he will be a doctor or a builder). That is, the government chooses m to maximize the sum of a doctor and builder utilities:

$$(3.8) \quad W = \pi \left[\frac{x_d^H}{p_b^H} + \frac{1}{2} (p_d^H)^2 + \frac{1}{2c} + \frac{x_b^H}{p_d^H} + \frac{1}{2} \left(\frac{p_b^H}{p_d^H} \right)^2 + \frac{1}{2c} \right] + \\ (1-\pi) \left[\frac{x_d^L + m}{p_b^L} + \frac{1}{2} (p_d^L)^2 + \frac{1}{2c} (1-t)^2 + \frac{x_b^L}{p_d^L} + \frac{1}{2} \left(\frac{p_b^L}{p_d^L} \right)^2 + \frac{1}{2c} (1-t)^2 \right]$$

where for each m the x 's and the p 's are given by the market equilibrium corresponding to that m .

The interesting case is when in the absence of fiscal policy investment and trading in labor services are both inefficient, i.e. $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ (see Proposition 2). In this case we have

Proposition 4: If $2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$, a positive injection of notes ($m > 0$) in the low state is

welfare improving.

Proof:

As the proof of Proposition 2 makes clear, the competitive equilibrium in the case $m=0$ has the following features:

$$(3.9) \text{ Both technologies are used and so } q^H + q^L = 1, \quad q^H x_d^H + q^L x_d^L = 1, \text{ i.e. } q^L = \frac{R^H - 1}{R^H - R^L} \text{ and}$$

$$q^H = \frac{1 - R^L}{R^H - R^L}.$$

(3.10) Doctors hold both Arrow securities, while builders strictly prefer the H security ($x_b^L = 0$). The

doctor first order condition implies $\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L}$ and hence $p_b^L = \frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1}$.

$$(3.11) \quad p_d^H = p_b^H = 1, \quad x_d^H = x_b^H = 1.$$

$$(3.12) \quad \frac{x_d^L}{p_b^L} = \frac{p_b^L}{p_d^L}, \quad \frac{x_d^L}{p_d^L} = p_d^L, \text{ which implies } p_d^L = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{2}{3}} \text{ and}$$

$$x_d^L = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}.$$

Furthermore, it is easy to adapt the proof of Proposition 2 to show that (3.9) – (3.11) will hold in a competitive equilibrium when the government injects m , for m close to zero. By contrast, (3.12) becomes

$$(3.13) \quad \frac{x_d^L + m}{p_b^L} = \frac{p_b^L}{p_d^L}$$

$$(3.14) \quad \frac{x_d^L + m}{p_d^L} = p_d^L,$$

which implies

$$(3.15) \quad p_d^L = (p_b^L)^{\frac{2}{3}}, \quad x_d^L + m = (p_b^L)^{\frac{4}{3}}.$$

According to (3.10), the government injection of m does not affect p_b^L . Hence, from (3.15) p_d^L stays the same and so does $x_d^L + m$. We may conclude that there is a 100% crowding out of x_d^L by m .

Clearing in the period 0 wheat market implies:

$$(3.16) \quad x_d^H + x_b^H = y^s + y^r R^H$$

$$(3.17) \quad x_d^L + x_b^L = y^s + y^r R^L$$

$$(3.18) \quad y^s + y^r = 2e$$

and so (since $x_b^L = 0$),

$$(3.19) \quad y^s = \frac{x_d^L - 2eR^L}{1-R^L}$$

$$(3.20) \quad y^r = \frac{2e - x_d^L}{R^L - 1}$$

Hence, a fall in x_d^L leads to a decrease in y^s , an increase in y^r , and an increase in $x_d^H + x_b^H$.

In summary, we have

$$(3.21) \quad \frac{dx_d^L}{dm} = -1, \quad \frac{dy^s}{dm} = -\frac{1}{1-R^L}, \quad \frac{dy^r}{dm} = \frac{1}{1-R^L}.$$

Also from (3.5)

$$(3.22) \quad \frac{dt}{dm} = \frac{c}{2(1-2t)}.$$

To ascertain the effect of m on welfare we differentiate (3.8) with respect to m to obtain:

$$(3.23) \quad \frac{dW}{dm} = \pi \left[\frac{dy^s}{dm} + \frac{dy^r}{dm} R^H \right] + (1-\pi) \left[-\frac{2}{c} (1-t) \frac{dt}{dm} \right] = \pi \left[\frac{R^H - 1}{1-R^L} \right] - (1-\pi) \frac{(1-t)}{(1-2t)}$$

Hence, the optimal choice of m is characterized by the first order condition:

$$(3.24) \quad \frac{\pi}{(1-\pi)} \left[\frac{R^H - 1}{1-R^L} \right] \leq \frac{(1-t)}{(1-2t)} \text{ with equality if } m > 0.$$

Note that the LHS of (3.24) is strictly bigger than 1 given that the expected return of the risky assets strictly exceeds one. Thus, (3.24) cannot be satisfied at $t=0$. We may conclude that a government handout ($m > 0$) in the low state is welfare improving.

QED

We see that, when the government intervention is expected, the inefficient overinvestment in safe assets is reduced. Yet, the level of output in periods 2 and 3 is still inefficient. As (3.21) makes clear, government liquidity completely crowds out private liquidity. As a result, the level of trade remains the same as in the original equilibrium without government intervention. Nevertheless, when the government does intervene in period 2, the multiplier is bigger than 1 as per the analysis above.

Given this tension, what is the optimal level of m ? Assume that c is small (the deadweight cost of taxation is large) and thus it is never desirable for the government to move the economy too far from the non-intervention equilibrium. We can then be confident that we will remain in a neighborhood of $m = 0$ and so (3.9)-(3.11) continue to hold. The first order condition for the optimal m then follows from (3.24):

$$(3.25) \quad \frac{\pi}{(1-\pi)} \left[\frac{R^H - 1}{1-R^L} \right] = \frac{(1-t)}{(1-2t)}$$

Since the RHS is strictly increasing in t and converges to ∞ as $t \rightarrow \frac{1}{2}$ from below, (3.25) has a unique solution and the optimal m can be deduced from (3.25).

In the above we have considered optimal fiscal policy when fiscal policy is the only instrument at the government's disposal. A natural question to ask is what happens if the government can choose fiscal policy *and* control the investment allocation. It is not difficult to show that fiscal policy is still useful : $m > 0$ at the optimum. We leave the details to the reader.

3.2. The case of non-commitment

Suppose that m is characterized by (3.25) but now the government can change m ex post if state L occurs. Will it choose to do so? We assume that the government continues to be benevolent: it maximizes the sum of doctor and builder utilities in the low state. The problem in term of commitment is that x_d^L, x_b^L and production decisions are sunk.

Given that x_d^L and $x_b^L = 0$ are fixed, market prices for doctor and building services will be given by (3.13)–(3.14), where m now varies. Total welfare in the low state (see (3.8)) can be written as

$$(3.26) \quad W^L = \left[\frac{x_d^L + m}{p_b^L} + \frac{1}{2} (p_d^L)^2 + \frac{x_b^L}{p_d^L} + \frac{1}{2} \left(\frac{p_b^L}{p_d^L} \right)^2 + \frac{1}{c} (1-t)^2 \right]$$

$$= \left[(x_d^L + m)^{\frac{1}{4}} + \frac{1}{2} (x_d^L + m) + \frac{1}{2} (x_d^L + m)^{\frac{1}{2}} + \frac{1}{c} (1-t)^2 \right].$$

Since x_d^L is fixed,

$$(3.27) \quad \frac{\partial W^L}{\partial m} = \left[\frac{1}{4} (x_d^L + m)^{-\frac{3}{4}} + \frac{1}{2} + \frac{1}{4} (x_d^L + m)^{\frac{1}{2}} - \frac{(1-t)}{(1-2t)} \right].$$

Now apply (3.25) and use (3.10) and (3.13)–(3.14) to write $x_d^L + m = \left(\frac{\pi}{(1-\pi)} \frac{R^H - 1}{1 - R^L} \right)^{\frac{3}{4}}$. Then,

$$(3.28) \quad \frac{dW^L}{dm} = \left[\frac{1}{4} \frac{\pi}{(1-\pi)} \frac{R^H - 1}{1 - R^L} + \frac{1}{2} + \frac{1}{2} \left(\frac{\pi}{(1-\pi)} \frac{R^H - 1}{1 - R^L} \right)^{\frac{2}{3}} - \frac{(1-\pi)}{\pi} \frac{1 - R^L}{R^H - 1} \right] < 0$$

since $\frac{\pi}{(1-\pi)} \frac{R^H - 1}{1 - R^L} > 1$.

We see that, when the government considers its decision as of period 2, it has an incentive to renege on the previously announced level of fiscal intervention. In other words,

Proposition 5: The government fiscal policy is time inconsistent.

Ex post the government will want to give fewer hand-outs than it said it would. The reason is that the promise to give hand-outs in the low state helps address two problems: the inefficient investment in period 0 and the inefficiently low level of trade in periods 2 and 3. If the government can renege on its promise in period 2, however, it will find that at that time its actions affect only one inefficiency: the low level of trade in periods 2 and 3. Since the government finds it less beneficial to tax people to deal with one inefficiency rather than two, it will deviate in the direction of intervening less than promised.

4. An Extension

So far, we have considered borrowers who can breach any promise to pay out of future labor income by “disappearing”. In practice some specialist agents may be able to keep track of borrowers and force them to repay their debts. This is especially plausible when we assume that storage (or investment) takes place through banks and so all payments are endorsements of some storage/investments held at a bank. Specifically, suppose that all payments for building and doctor services take place through check transfers and that the bank is able to seize these before they are cashed for consumption. In this way labor income becomes contractible. However, a bank cannot force anyone to work. That is, all a bank can do is to ensure that someone who defaults has zero consumption (apart from any non-pledgeable investment income). As a result, uncollateralized lending against future labor income is possible, but there is a repayment constraint. Each worker can borrow up to the point at which ex post he is indifferent between exerting effort and repaying the loan and doing nothing and defaulting.

In a simplified framework (Hart and Zingales (2011)) we show that lending does not resolve the tension between private and social objectives. Nevertheless, lending does improve welfare since it increases the volume of trade without sacrificing the higher return of the alternative investment. Interestingly, lending is not a perfect substitute for the riskless asset. The reason is that with no private liquidity in the system, the amount borrowed by doctors equals the purchasing power in the hands of builders, which in turn equals the revenue received by doctors for their services. But if the revenue equals the debt, it is not in the interest of the doctors to work, given that they have to exert costly effort. Hence, the doctors will default. To have a functioning lending market, we need a minimum amount of private liquidity.

5. Conclusions

We built a simple framework to analyze the role of fiscal policy in attenuating the impact of aggregate shocks on private investment choices and aggregate output. We show that the mere lack of pledgeability of human capital, even in the presence of a complete market for securities, makes the competitive equilibrium constrained inefficient. The market will invest too much in producing safe securities and will dedicate too few resources towards risky investments.

Our work can be distinguished from that of Holmstrom and Tirole (2011) and Lorenzoni (2008). Their analyses are based on the idea that the market underprovides liquidity to firms, whereas in our treatment consumer liquidity needs are the driver. To compare the importance of the two, we looked at the Survey of Small Businesses Finances and Survey of Consumer Finances. The 2004 Survey of Consumer Finances finds that 37% of families are financially constrained, where constrained is defined as a family that applied for credit and has been rejected or has been discouraged from applying by the fear of being rejected. By contrast, the 2003 Survey of Small Businesses Finances finds that only 15% of small firms were constrained, using the same definition. Since small firms are more likely to be constrained than big firms, this evidence seems to suggest that financial constraints are more likely to be a problem for consumers than for firms.

In our simple model a fiscal policy following a big negative shock can increase not only ex post output more than one-to-one (fiscal multiplier), but also ex ante welfare. We have supposed that the government is able to target directly consumers who are in need of liquidity. If we were to drop this assumption, an interesting set of problems would arise. Would it be cheaper for the government to bail out financial intermediaries rather than to hand out money to consumers randomly? If so, how would this benefit trade off against the potential moral hazard problem financial intermediaries face when they expect to be bailed out in major downturns? We analyze these and other issues in a separate paper (Hart and Zingales, 2013).

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Appendix

Lemma 1: In a competitive equilibrium, the prices of doctor and builder services equal one in the high state (i.e., $p_d^H = p_b^H = 1$).

Proof: Suppose $p_d^H < 1$. Then, by (2.8), $x_b^H + x_d^H = (p_d^H)^2 < 1$, which contradicts (2.3), given that $R^H > 1$ and $y^s + y^r > 1$ by (2.5). Hence $p_d^H = 1$.

To prove that $p_b^H = 1$, assume the contrary: $p_b^H < 1$. We first show that $x_d^H \geq x_d^L$. Suppose not: $x_d^H < x_d^L$. Then $x_d^L > 0$. From the first order conditions for (*),

$$(A1) \quad \frac{\pi}{p_b^H q^H} \leq \frac{1-\pi}{p_b^L q^L}$$

That is, the utility rate of return on the low state Arrow security for doctors must be at least as high as that on the high state Arrow security. We also know that there is more output in the high state, so, if $x_d^H < x_d^L$, builders must be buying the high state security, which means that it must give them an attractive return, or, from their first order condition,

$$(A2) \quad \frac{\pi}{q^H} \geq \frac{1-\pi}{p_d^L q^L},$$

where we are using the fact that $p_d^H = 1$.

Putting (A1)-(A2) together yields

$$(A3) \quad \frac{p_b^L}{p_b^H} \leq p_d^L.$$

If $p_b^L = 1$, (A3) implies $p_b^H = 1$, which we have supposed not to be the case. Hence $p_b^L < 1$. Then we have, from (2.6) and (2.7), $(p_b^H)^2 = x_d^H$ and $(p_b^L)^2 = x_d^L p_d^L$. Therefore (A3) becomes

$$\frac{(x_d^L)^{\frac{1}{2}}}{(x_d^H)^{\frac{1}{2}}} \leq (p_d^L)^{\frac{1}{2}} \leq 1$$

or $x_d^H \geq x_d^L$, which is a contradiction.

Hence, $x_d^H \geq x_d^L$. Since a doctor's utility is increasing in x_d^L and x_d^H , a doctor's budget constraint will hold with equality. Thus $q^H x_d^H + q^L x_d^L = e$, which implies $(q^H + q^L)x_d^H \geq e$. Hence, by (2.1), $x_d^H \geq e > 1$, implying $p_b^H = 1$ by (2.6).

Q.E.D.

Lemma 2: A necessary and sufficient condition for a competitive equilibrium to be first-best optimal is $2eR^L \geq 1$.

Proof:

To achieve the first-best the supply of building and doctor services must be 1 in each state. Since the supply of doctor services is given by p_d^H, p_d^L in (2.8), (2.9), it follows that $p_d^H = p_d^L = 1$. The supply of building services is given by $\frac{p_b^H}{p_d^H}, \frac{p_b^L}{p_d^L}$ in (2.6)-(2.7), and, substituting $p_d^H = p_d^L = 1$, we obtain $p_b^H = p_b^L = 1$. Hence, again from (2.6)-(2.7), $x_d^H \geq 1$ and $x_d^L \geq 1$.

In the first-best all wheat is invested in the high yield project: $y^s = 0$ and $y^r = 2e$. Therefore, from (2.4), $2eR^L = x_d^L + x_b^L \geq x_d^L \geq 1$. Hence, $2eR^L \geq 1$ is a necessary condition.

To prove sufficiency consider a candidate equilibrium where the prices of doctor and builder services equal 1 in both states, $q^H = \frac{\pi}{R}, q^L = \frac{(1-\pi)}{R}$, all wheat is invested in the high yield project, and the doctors buy at least one unit of each Arrow security. Since $q^H + q^L < 1 < e$, they can afford to do so. Doctors and builders satisfy their first order conditions and firms maximize profit. Hence this is indeed a competitive equilibrium.

Q.E.D.

Proposition 2: If $2eR^L \geq 1$, then a competitive equilibrium delivers the first-best. If

$1 > 2eR^L \geq \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ *then a competitive equilibrium is such that investment is efficient (only the risky technology is used), but trading in doctor and building services is inefficiently low. If*

$2eR^L < \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}}$ a competitive equilibrium is such that investments and trading in labor services are both inefficient: the riskless technology is operated at a positive scale and trade is inefficiently low.

Proof:

Lemma 2 shows that we achieve the first-best if $2eR^L \geq 1$. Consider the case $2eR^L < 1$. We know from Lemma 1 that $p_d^H = p_b^H = 1$. We show first that doctors will hold both securities. Given $p_b^H = 1$, (2.6) implies $x_d^H > 0$. Suppose $x_d^L = 0$. This is inconsistent with $p_b^L = 1$ in (2.7). But if $p_b^L < 1$, then, from (2.7), $x_d^L = 0$ implies $p_b^L = 0$. This in turn implies that the marginal return on the low state Arrow security for a doctor is infinite, which means that the first order condition in (*) cannot hold. Therefore, $x_d^L > 0$.

Since doctors hold both securities, we have

$$(A4) \quad \frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L}$$

Let's assume first that both technologies are used. Then (2.1) and (2.2) hold as an equality and

$$q^L = \frac{R^H - 1}{R^H - R^L} \text{ and } q^H = \frac{1 - R^L}{R^H - R^L}. \text{ Therefore}$$

$$(A5) \quad p_b^L = \frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} < 1$$

since $\bar{R} > 1$.

It is easy to see that $p_b^L < p_d^L$. This is clear from (A5) if $p_d^L = 1$. Suppose $p_d^L < 1$. Then (2.9) implies $(p_d^L)^2 = x_b^L + x_d^L \geq x_d^L = \frac{(p_b^L)^2}{p_d^L}$ by (2.7). Hence, $p_d^L \geq (p_b^L)^{\frac{2}{3}} > p_b^L$ since $p_b^L < 1$. This proves $p_b^L < p_d^L$. It follows that the rate of return on the low security is strictly less than that on the high security for builders. So builders will not hold the low security (from the first order condition for (**)): $x_b^L = 0$.

From (2.7),

$$(A6) \quad p_d^L x_d^L = (p_b^L)^2,$$

from which it follows, since $p_b^L < 1$ and $p_b^L < p_d^L$, that $x_d^L < 1$. But then (2.9) in combination with $x_b^L = 0$ implies $p_d^L < 1$. Hence, again from (2.9),

$$(A7) \quad x_d^L = (p_d^L)^2.$$

Combining (A6) and (A7) we have

$$(A8) \quad x_d^L = (p_b^L)^{\frac{4}{3}}.$$

If the solution $x_d^L = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}} > 2eR^L$, then this candidate equilibrium where both

technologies are used is feasible. Also trade of doctor and builder services is inefficient since p_b^L, p_d^L are both less than 1.

If $x_d^L = \left(\frac{1-\pi}{\pi} \frac{1-R^L}{R^H-1} \right)^{\frac{4}{3}} \leq 2eR^L$, then we solve instead for an equilibrium with $x_d^L = 2eR^L$,

$x_b^L = 0$. In this case, there will be no investment in the storage technology (thus $y^s = 0$) and the market clearing condition for securities, (2.3)-(2.4), simplifies to

$$x_d^H + x_b^H = 2eR^H,$$

$$x_d^L + x_b^L = 2eR^L.$$

(2.7) and (2.9) become

$$p_d^L = (2eR^L)^{\frac{1}{2}}$$

$$p_b^L = (2eR^L)^{\frac{3}{4}},$$

which are below one since $2eR^L < 1$. Doctors hold both Arrow securities since $p_b^H, p_b^L > 0$ and so we must have

$$\frac{\pi}{q^H} = \frac{1-\pi}{p_b^L q^L} = \frac{1-\pi}{(2eR^L)^{\frac{3}{4}} q^L}.$$

This, together with (2.2) with equality, determines q^H and q^L . Thus, in this equilibrium investment is efficient but the level of trading is not. QED