Surplus Division and Efficient Matching

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Abstract

We study a two-sided matching model with a finite number of agents and with transferable utility: agents on both sides of the market are privately informed about multi-dimensional attributes that determine the match value of each pair. Utility functions are assumed to be quasi-linear, and monetary transfers among agents are feasible. We ask the following question: what divisions of surplus among matched pairs are compatible with information revelation leading to an efficient matching? Our main result shows that the only robust rules compatible with efficient matching are those that divide the achieved surplus in a fixed proportion, independently of the attributes of the pair's members. In other words, for efficient matching it is necessary that all agents expect to get the same fixed proportion of surplus in every conceivable match. A more permissive result is obtained for one-dimensional attributes and supermodular value functions.

1 Introduction

We study a two-sided matching (or assignment) market with a finite number of privately informed agents, called "workers" and "employers". The model is a general incomplete information version of the standard assignment game due to Shapley and Shubik (1972). Agents are characterized by multi-dimensional attributes, allowing for a representation of general preference relations. Attributes are private information and determine the match

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value / surplus potentially created by each employer-worker pair. Thus, for each involved pair, match values are informationally interdependent, a very natural feature in the context of matching models. Utility functions are assumed to be quasi-linear, and monetary transfers among agents are feasible.

We ask the following question: what divisions of surplus among matched pairs are compatible with information revelation leading to an efficient matching, i.e., to a matching that achieves the maximal possible surplus for each realization of attributes in the economy? Our main result shows that the only robust rules inducing efficient matching are those that divide the surplus in each match according to a fixed proportion, independently of the attributes of the pair's members. Thus, for efficient matching it is necessary that all workers expect to get the same fixed proportion of surplus in every conceivable match, and the same thing must hold for employers! More flexibility is possible in the special case where attributes are one-dimensional and match values are supermodular. In that case efficient matching is compatible with any division that leaves each partner with a fraction of the surplus that is also supermodular.

The equilibrium notion used here is the *ex-post equilibrium.* This is a generalization of equilibrium in dominant strategies which is appropriate for settings with interdependent values, and it embodies a notion of no regret: chosen actions must be considered optimal even after the private information of others is revealed. In particular, our agents do not regret being matched to their actual partners. Ex-post equilibrium is also a belief-free notion, and our results do not depend in any way on the distribution of attributes in the population.[1]

An interesting illustration for a fixed-proportion rule is offered by the German law governing the sharing of profit among a public sector employer and an employee arising from the employee's invention activity. The law differentiates between universities and all other public institutions.[2] Outside universities - where, presumably, the probability of an employee making a job-related discovery is either nil or very low - the law allows any **ex-ante** negotiated contract governing profit sharing (see §40-1 in *Bundesgesetzblatt*

[1] See also Bergemann and Morris (2005) for the tight connections between ex-post equilibria and "robust design".

[2] Practically all German universities are public.

III, 422-1). In marked contrast, independently of circumstances, any university and any researcher working there must divide the profit from the researcher's invention according to **a fixed 30%-70% rule**, with the employee getting the 30% share (see §42-4). The rigidity of this "no-exception" rule is additionally underlined by an explicit mention that all feasible arrangements under §40-1 are not applicable within universities (see §42-5).

The inflexible nature of sharing rules - that do not vary from contract to contract and are not the object of negotiation - is a recurrent theme in several interesting literatures that try to explain this somewhat puzzling phenomenon. Newbery and Stiglitz (1979) and Allen (1985) among many others have noted that sharecropping contracts in many rural economies involve shares of around one half for landlord and tenant. This percentage division is observed in widely differing circumstances and has persisted in many places for a considerable length of time.[3] Luo (1997) has surveyed joint ventures among foreign and Chinese firms in the non-consumer electronic industry, and he has found a mean value of about 54% for the domestic share, with a relatively low standard deviation of about 4%. Lafontaine (1992) shows that, while the degree of reliance on franchising depends on particular economics attributes, the terms of franchise contracts are surprisingly uniform within one industry.

Let us also note here that advantages/disadvantages of discretionary versus rigid rules are the subject of a large legal literature dealing with the opposite case of marital match dissolution via divorce (see Mnookin and Kornhauser,1978). Some countries have relatively inflexible 50%-50% property division rules, while others opt for more flexible negotiated schemes that take into account the particular situation / attributes of the involved pair.

Our paper is at the intersection of several important strands of the economic literature. We briefly review below these strands, while emphasizing both the existing relations to our work and the present novel aspects.

1. Matching: A overwhelming majority of studies within the large body of work on two-sided markets has assumed either complete information or *private values* models, i.e., models where agents' preferences only

[3] For example , Chao (1983) has noted that a fixed 50-50 ratio was prevalent in China for more than 2000 years. The French and Italian words for "sharecropping" literally mean "50-50 split".

depend on signals available to them at the time of the decision, but not on signals privately available to others. This holds for both the Gale-Shapley (1962) type of moneyless models with ordinal preferences, and for the branch studying variants and applications (to auctions and double auctions, say) of the assignment model with money due to Shapley and Shubik (1972).

In the Gale-Shapley model, one-sided serial dictatorship where women, say, sequentially choose partners according to their preferences leads to a Pareto-optimal matching. Difficulties occur when the stronger *stability* requirement is invoked: a standard result is that no stable matching can be implemented in dominant strategies if both sides of the market are privately informed (see Roth and Sotomayor (1990)). Chakraborty et al. (2010) have shown that stability may fail even in a one-sided private information model if preferences on one side of the market (colleges, say) depend on information available to agents on the same side of the market.

Becker (1973) has popularized a special case of the complete information Shapley-Shubik model where agents have one-dimensional attributes and where the match value is a supermodular function of these attributes. In particular, agents on one side of the market have the same ordinal preferences over the members of the other side, and the efficient (and stable) matching is assortative. If two-sided incomplete information is introduced in the Becker model one immediately obtains *interdependent values*, i.e., agents' preferences also depend on signals available to others. But, somewhat surprisingly, there are only very few such models, and most of the literature has circumvented the ensuing problems by assuming either complete information, or one-sided private information (which yields private values) or a continuum of types (so that aggregate uncertainty disappears). An exception is Hoppe et al. (2009) who analyzed a two-sided matching model with a finite number of privately informed agents on each side of the market, characterized by complementary one-dimensional attributes. In their model match surplus is divided in a fixed proportion, and they have shown that efficient, assortative matching can arise as one of the equilibria of a bilateral signaling game. This finding is consistent with our present result for the one-dimensional case.

Another strand using Becker's specification and complete information combines matching with ex-ante investment: before matching, agents undertake costly investments that affect their attributes and hence, ultimately, their match values.[4] In a recent study in this vein, Mailath et al. (2012) have focused on the role of what they call "premuneration values", i.e., the surplus accruing to agents from matching, net of monetary transfers. They have noted that in certain circumstances premuneration values are not easily adjusted.[5] Under personalized pricing - that must finely depend on the attributes of the matched pairs - an equilibrium which entails efficient investment and matching always exists, no matter how surplus is shared. This is, mainly, a consequence of the large market hypothesis. In contrast, when personalized pricing is not feasible, surplus division affects the incentives to invest, and under-investment may occur (even if agents are efficiently matched) in situations where the seller's premuneration value depends on the buyer's characteristic.

2. Property Rights: Our study is also related to the large literature analyzing the effects of the ex-ante allocation of property rights on bargaining outcomes, following Coase (1960). Traditionally, this literature has not placed the bargaining agents in a fully described market context. The interplay between private information and ex-ante property rights in private value settings has been emphasized by Myerson-Satterthwaite (1983) and Cramton-Gibbons-Klemperer (1987) in a buyer-seller framework and a partnership dissolution model, respectively. Several papers in this vein allow for interdependent values: for example, Fieseler et al. (2003) have offered a unified treatment that encompasses both the above private values models and Akerlof's (1970) market for lemons, while Brusco et al. (2007) and Gärtner and Schmutzler (2010) have looked at mergers among firms, a setting which is more related to the present study. In all these papers agents have one-dimensional private types and a value maximizing allocation can be implemented via standard Clarke-Groves-Vickrey schemes. Whenever inefficiencies for certain allocations of property rights occur, these stem from the inability to design budget-balanced and individual rational transfers that sustain the value maximizing allocation.[6]

[4] Thus, as in the contracting literature, hold-up situations may occur. The basic question in this and other related papers is to what extent market competition can alleviate hold-up and coordination problems.

[5] In addition, legal restrictions and prevailing social norms (which may enforce an implicit imbalance of power within the partnership) could also restrict the ways in which surplus is divided.

[6] With several buyers and sellers, the Myerson-Satterthwaite model becomes a one-

In marked contrast to all the above papers, our present analysis completely abstracts from budget-balancedness and individual rationality: we put no constraints on feasible monetary transfers. The fixed-proportion sharing rules are dictated here by the mere requirement of value maximization / efficient matching together with incentive compatibility.

3. Multi-dimensional Attributes and Mechanism Design: The prevalent assumption that workers, firms, or potential spouses, say, can be described by a single trait such as skill, technology, wealth, or education is often not tenable. Workers, for example, have very diverse job-relevant characteristics, which are only partially correlated. Thus, it is imperative to better understand the economics of matching markets with multi-dimensional attributes. Tinbergen (1956) pioneered the analysis of labor markets where jobs and workers are described by several characteristics. The study of complete information assignment models with a continuum of traders and multi-dimensional attributes has been pioneered by Gretsky et al. (1992, 1999). Dizdar (2012) has generalized the matching cum ex-ante investment model due to Cole et al. (2001) along this line.[7]

The present combination of multi-dimensional attributes, private information and interdependent values usually generates technical difficulties. Jehiel et al. (2006) have shown that, generically, only trivial social choice functions - where the outcome does not depend on the agents' private information - can be ex-post implemented when values are interdependent and types are multi-dimensional. In a model with linear utilities, Jehiel and Moldovanu (2001) have shown that, generically, the efficient allocation cannot be implemented even if the weaker Bayes-Nash equilibrium concept is used.

Our present insight can be reconciled with those general negative results by noting that the two-sided matching model is not generic. In particular,

dimensional, linear, incomplete information version of the Shapley-Shubik assignment game. Only in the limit, when the market gets very large, one can reconcile, via almost efficient double-auctions, incentives for information revelation with budget-balancedness and individual rationality.

[7]He has shown that the existence of an efficient equilibrium does not hinge on single-crossing conditions and that, in addition to over/under investment, inefficiencies can also arise from false specialization and mismatches Like other recent, related literature (e.g. Chiappori et al. 2010) his analysis uses tools borrowed from optimal transportation theory (see Villani 2003 for an introductory textbook).

we assume here that the match value of all pairs share the same functional form (as a function of the respective attributes), and that the match value of any pair depends neither on how agents outside that pair match, nor on what their attributes are. These features are natural for the matching model but are "non-generic", allowing here a positive - but still tightly determinate - result concerning the necessity of fixed-proportion sharing rules. The sufficiency of these inflexible rules is related to the presence of utilities that admit *cardinal potentials*, as defined by Jehiel et al. (2008).[8] The analysis of the special case of one-dimensional types and supermodular match values is based on an elegant characterization result due to Bergemann and Välimäki (2002) , which generalized previous insights due to Jehiel and Moldovanu (2001) and Dasgupta and Maskin (2000).

Finally, in a recent contribution, Che et al. (2012) have shown that efficiency is not compatible with incentive compatibility in a one-sided assignment model where agents' values over objects are allowed to depend on information of other agents. Their signals are one-dimensional but inefficiency occurs there because of the assumed lack of monetary transfers.

The paper is organized as follows: in Section 2 we present the matching model. In Section 3 we state our results, both for the multi-dimensional case and for the special case where attributes are one-dimensional, and where the match value is supermodular. Section 4 concludes. All proofs are in the Appendix.

2 The Matching Model

There are I employers and J workers. Each employer e_i ($i \in \mathcal{I} = \{1, ..., I\}$) privately knows his type $x_i \in X$, and each worker w_j ($j \in \mathcal{J} = \{1, ..., J\}$) privately knows his type $y_j \in Y$.

If an employer of type $x \in X$ and a worker of type $y \in Y$ form a match, they subsequently create a *match surplus* of $v(x, y) \in \mathbb{R}_+$ where $v : X \times Y \to \mathbb{R}_+$ is continuously differentiable. Unmatched agents create zero surplus, and all agents have quasi-linear utilities.

The supports of agents' possible types, X and Y, are open connected sub-

[8] They have presented several non-generic cases where ex-post implementation is possible. See also Bikchandani (2006) for other such cases, e.g. certain auction settings.

sets of Euclidean space $\mathbb{R}^n$ for some $n \in \mathbb{N}$. The set of possible alternatives $\mathcal{M}$ is the set of all possible one-to-one matchings of employers and workers. If $I \leq J$, these are the injective maps $m : \mathcal{I} \to \mathcal{J}$. A matching $m \in \mathcal{M}$ will be called *efficient* for a type profile $(x_1, ..., x_I, y_1, ..., y_J)$ if and only if it maximizes aggregate welfare $u_{m'}(x_1, ..., x_I, y_1, ..., y_J) = \sum_{i=1}^{I} v(x_i, y_{m'(i)})$ among all $m' \in \mathcal{M}$ (analogous definitions apply for the case $J \leq I$). Efficient matchings can be obtained as the solutions of a finite linear program (see Shapley and Shubik, 1972).

2.1 Sharing Rules

A sharing rule specifies the division of match values: if e_i and w_j are matched in $m \in \mathcal{M}$, then employer e_i's share of surplus is $v_m^{e_i}(x_1, ..., x_I, y_1, ..., y_J) = \gamma(x_i, y_j)v(x_i, y_j)$, and worker w_j's share is $v_m^{w_j}(x_1, ..., x_I, y_1, ..., y_J) = (1 - \gamma(x_i, y_j))v(x_i, y_j)$ where we assume that $\gamma : X \times Y \to [0, 1]$ is continuously differentiable.[9] In other words, a sharing rule specifies a “complete contract" that determines how surplus is to be divided within the match, at a point in time when the respective attributes are revealed within the match. This division of surplus can be either physically / exogenously given by the “premuneration values" of the agents (see Mailath et al. (2012) where the match surplus is then taken to be the sum of these primitive values), or, in many interesting cases - recall the introductory examples- the sharing rule is determined by some prevailing social norm, or it is designed by some law making agency that standardizes contracts and / or specifies property rights. Finally, if e_i remains unmatched in m we have $v_m^{e_i}(x_1, ..., x_I, y_1, ..., y_J) = 0$ (similarly, $v_m^{w_j}(x_1, ..., x_I, y_1, ..., y_J) = 0$ if w_j stays unmatched).

Together with a sharing rule, the above described matching model gives rise to a natural social choice setting with interdependent values. Every agent attaches a value to each possible alternative, i.e. to matchings of employers and workers. This value depends both on own type and on the type of the partner, but not on the private information of other agents. Moreover, this value does not depend on how other agents match. Thus there are no allocative externalities, and there are no informational externalities across matched pairs.

[9] In principle, we could allow γ to depend on the identities of the partners. As our results below suggest, this feature would not be conducive to efficiency.

2.2 Mechanisms

By the *Revelation Principle*, we may restrict attention to direct revelation mechanisms where truthful reporting by all agents forms an ex-post equilibrium. A direct revelation mechanism (mechanism hereafter) is given by functions $\Psi : X^I \times Y^J \to \mathcal{M}$, $t^{e_i} : X^I \times Y^J \to \mathbb{R}$, $\forall i \in \mathcal{I}$ and $t^{w_j} : X^I \times Y^J \to \mathbb{R}$, $\forall j \in \mathcal{J}$. Ψ selects a feasible matching as a function of reports, t^{e_i} is the monetary transfer to employer e_i, and t^{w_j} is the monetary transfer to worker w_j as functions of reports.

Truth-telling is an ex-post equilibrium if for all employers e_i, $i \in \mathcal{I}$, for all workers w_j, $j \in \mathcal{J}$, and for all type profiles $p = (x_1, ..., x_I, y_1, ..., y_J)$, $p' = (x_1, .., x_i', .., x_I, y_1, ..., y_J)$ and $p'' = (x_1, ..., x_I, y_1, .., y_j'', .., y_J)$ it holds that

$$\begin{aligned} v^{e_i}_{\Psi(p)}(p) + t^{e_i}(p) &\geq v^{e_i}_{\Psi(p')}(p) + t^{e_i}(p') \\ v^{w_j}_{\Psi(p)}(p) + t^{w_j}(p) &\geq v^{w_j}_{\Psi(p'')}(p) + t^{w_j}(p''). \end{aligned}$$

3 The Main Results

For each realization of attributes the value maximizing matching can be computed here without specifying final individual utilities within each match (hence without specifying the agents' utilities in each social alternative). Each particular division of surplus among members of matched pairs determines individual utilities for that matching, and vice versa. We ask the following question: which sharing rules γ, if any, are compatible with information revelation leading to an efficient matching? In other words, using the mechanism design terminology, we ask for which utility functions we can implement the value-maximizing social choice function in ex-post equilibrium.

For our main results we need an assumption known as the *twist condition* in the mathematical literature on optimal transportation (see Villani 2003). This is a multi-dimensional generalization of the well-known Spence-Mirrlees condition. While in optimal transportation - where measures of agents are matched - the condition is invoked in order to ensure that the optimal transport, corresponding here to the efficient matching, is unique and deterministic, we use it for quite different technical reasons (see the Lemmas in the Appendix).

Condition 1 *i) For all $x \in X$, the continuous mapping from Y to $\mathbb{R}^n$ given by $y \mapsto (\nabla_X v)(x, y)$ is one-to-one.*

ii) For all $y \in Y$, the continuous mapping from X to $\mathbb{R}^n$ given by $x \mapsto (\nabla_Y v)(x, y)$ is one-to-one.

Match surplus functions that fulfill Condition 1 model many interesting complementarities between multi-dimensional types of workers and employers. In particular, v is not additively separable with respect to x and y, so that the precise allocation of match partners really matters for efficiency. As a simple example consider the bilinear match surplus: $v(x, y) = x \cdot y$, where $\cdot$ denotes the standard inner product on $\mathbb{R}^n$. Then $(\nabla_X v)(x, y) = y$ and $(\nabla_Y v)(x, y) = x$, and Condition 1 is satisfied.

We can now state our main result:

Theorem 1 *Let $n \geq 2$, $I, J \geq 2$, and assume that Condition 1 is satisfied. Then the following are equivalent:*

i) The efficient matching is implementable in ex-post equilibrium.

ii) There is a constant $\lambda_0 \in [0, 1]$ and functions $g : X \to \mathbb{R}$ and $h : Y \to \mathbb{R}$ such that for all $x \in X$, $y \in Y$ it holds that

$$(\gamma v)(x, y) \;=\; \lambda_0 v(x, y) + g(x) + h(y)$$

Moreover, h is constant if $I < J$, and g is constant if $I > J$.

Proof. See Appendix. ∎

Intuitively, for efficient implementation, the incentives of each individual agent must be aligned with the social desire to choose the efficient matching.[10] In particular, this means that, within a pair, the incentives of the pair's members must be aligned with each other. The proof of the Theorem proceeds by translating this insight into precise conditions that connect the functions $v = \gamma v + (1 - \gamma)v$ and γv. The needed alignment is easily established when a fixed proportion rule is used, but impossible otherwise.

[10] It is possible to provide *strict* incentives for truthful revelation of types only if the part of the share that is proportional to (non-separable) match surplus is strictly positive for both sides of the market, i.e. if $\lambda_0 \in (0, 1)$.

Condition 1 ensures that the subset of types for which several distinct matchings are efficient is a well-behaved manifold. The proof of Theorem 1 shows that the property stated in ii) is sufficient for efficient ex-post implementation even if Condition 1 does not hold. The difficult part of the Theorem, where we invoke the twist condition, is showing that i) implies ii).

Corollary 1 *The only sharing rules that can implement the efficient matching irrespective of whether employers or workers are on the short side of the market are of the form* $(\gamma v)(x,y) = \lambda_0 v(x,y) + c$*, where* c *is a constant.*

Remark 1 *Mezzetti (2004) has shown that efficiency is always attainable with two-stage mechanisms where an allocation is chosen at stage one, and where, subsequently, monetary transfers that depend on the realized ex-post utilities of all agents at that particular allocation are executed at stage two. Such two-stage mechanisms with ex-post transfers across matched pairs seem rather unrealistic in the present matching environment.*

Our second main result deals with the special case where agents' attributes are one-dimensional. If $n = 1$, then Condition 1 implies that $y \mapsto (\partial_x v)(x,y)$ is either strictly increasing or strictly decreasing. Consequently, v either has strictly increasing differences or strictly decreasing differences in (x,y).[11] That is, v is either strictly supermodular or strictly submodular. This is the classical one-dimensional assortative/anti-assortative framework à la Becker (1973). We treat here the supermodular case. The submodular one is analogous.

In the one-dimensional supermodular case we find that the class of sharing rules that is compatible with efficient matching is larger, and strictly contains the class of constant rules obtained above.

Theorem 2 *Let* $n = 1$*,* $I, J \geq 2$ *and assume that* v *is strictly supermodular. Then, the efficient matching is implementable in ex-post equilibrium if and only if both* γv *and* $(1-\gamma)v$ *are supermodular.*

Proof. See Appendix. ■

Instead of a direct proof, we derive the above result by applying a characterization result due to Bergemann and Välimäki (2002). These authors

[11] See also Topkis (1998).

have derived a necessary as well as a set of sufficient conditions for efficient ex-post implementation for one-dimensional types. The logic of our proof is as follows. We first verify that monotonicity in the sense of their Definition 4 is satisfied for strictly supermodular match surplus. This is the first part of their set of sufficient conditions (Proposition 3). Then, we show that their necessary condition (Proposition 1) implies that γv and $(1-\gamma)v$ must be supermodular. Finally, we show that the second part of the sufficient conditions is satisfied as well if γv and $(1-\gamma)v$ are supermodular.

4 Conclusion

We have introduced a novel two-sided matching model with a finite number of agents, two-sided incomplete information, interdependent values, and multi-dimensional attributes. We have shown that constant-proportion sharing rules are the only ones conducive for efficiency in this setting. While our present result is agnostic about the preferred proportion, augmenting our model with, say, a particular ex-ante investment game will introduce new, additional forces that can be used to differentiate between various constant sharing rules.

Appendix

For the proof that Condition ii) in the statement of Theorem 1 is necessary for efficient ex-post implementation we first need several Lemmas. The key step is Lemma 4 below.

It will be very useful to introduce a cross-difference (two-cycle) linear operator F, which acts on functions $f: X \times Y \to \mathbb{R}$. The operator F_f has arguments $x^1 \in X^1 = X$, $x^2 \in X^2 = X$, $y^1 \in Y^1 = Y$ and $y^2 \in Y^2 = Y$ and it is defined as follows:[12]

$$F_f(x^1, x^2, y^1, y^2) := f(x^1, y^1) + f(x^2, y^2) - f(x^1, y^2) - f(x^2, y^1).$$

We also define the sets

$$A := \{(x^1, x^2, y^1, y^2) \in X \times X \times Y \times Y | F_v(x^1, x^2, y^1, y^2) = 0\},$$

[12] We choose superscripts here because x_1 is already reserved for the type of employer e_1, and so on.

and

$$A_0 := \{(x^1, x^2, y^1, y^2) \in A | \nabla F_v(x^1, x^2, y^1, y^2) \neq 0\},$$

where

$$\nabla F_v(x^1, x^2, y^1, y^2) = (\nabla_{X^1} F_v, \nabla_{X^2} F_v, \nabla_{Y^1} F_v, \nabla_{Y^2} F_v)(x^1, x^2, y^1, y^2).$$

Whenever $x_1 \neq x_2$ or $y_1 \neq y_2$, Condition 1 implies that $\nabla F_v(x_1, x_2, y_1, y_2) \neq 0$. This is repeatedly used below.

Lemma 1 *Let $n \in \mathbb{N}$, $I = J = 2$ and let Condition 1 be satisfied. If the efficient matching is ex-post implementable, then the following implications hold for all (x_1, x_2, y_1, y_2):*

$$F_v(x_1, x_2, y_1, y_2) \geq (\leq)\, 0 \Rightarrow F_{\gamma v}(x_1, x_2, y_1, y_2) \geq (\leq)\, 0, \tag{1}$$

$$F_v(x_1, x_2, y_1, y_2) \geq (\leq)\, 0 \Rightarrow F_{(1-\gamma)v}(x_1, x_2, y_1, y_2) \geq (\leq)\, 0. \tag{2}$$

Proof. There are only two alternative matchings, $m_1 = ((e_1, w_1), (e_2, w_2))$ and $m_2 = ((e_1, w_2), (e_2, w_1))$. Since the efficient matching is ex-post implementable, the taxation principle for ex-post implementation implies that there must be “transfer" functions $t^{e_1}_{m_1}(x_2, y_1, y_2)$ and $t^{e_1}_{m_2}(x_2, y_1, y_2)$ for employer e_1 such that

$$\begin{aligned} F_v(x_1, x_2, y_1, y_2) \quad &> \quad (<)\, 0 \Rightarrow \\ (\gamma v)(x_1, y_1) + t^{e_1}_{m_1}(x_2, y_1, y_2) \quad &\geq \quad (\leq)\, (\gamma v)(x_1, y_2) + t^{e_1}_{m_2}(x_2, y_1, y_2). \end{aligned} \tag{3}$$

For $y_1 \neq y_2$, we have $(\nabla_{X^1} F_v)(x_2, x_2, y_1, y_2) = (\nabla_X v)(x_2, y_1) - (\nabla_X v)(x_2, y_2) \neq 0$ by Condition 1. Hence, in every neighborhood of $x_1 = x_2$, there are x_1' and x_1'' such that $F_v(x_1', x_2, y_1, y_2) > 0$ and $F_v(x_1'', x_2, y_1, y_2) < 0$. Since γv is continuous, relation 3 pins down the difference of transfers as:

$$t^{e_1}_{m_1}(x_2, y_1, y_2) - t^{e_1}_{m_2}(x_2, y_1, y_2) = (\gamma v)(x_2, y_2) - (\gamma v)(x_2, y_1).$$

Plugging this back into 3 yields for all (x_1, x_2, y_1, y_2) with $y_1 \neq y_2$:

$$F_v(x_1, x_2, y_1, y_2) > (<)\, 0 \Rightarrow F_{\gamma v}(x_1, x_2, y_1, y_2) \geq (\leq)\, 0. \tag{4}$$

As $F_v(x_1, x_2, y, y) = F_{\gamma v}(x_1, x_2, y, y) = 0$, relation 4 holds for all (x_1, x_2, y_1, y_2).

However, every neighborhood of any $(x_1, x_2, y_1, y_2) \in A$ contains both points at which F_v is strictly positive and points at which F_v is strictly negative. Whenever $x_1 \neq x_2$ or $y_1 \neq y_2$, this follows immediately from $\nabla F_v(x_1, x_2, y_1, y_2) \neq 0$. Otherwise, if $x_1 = x_2$ and $y_1 = y_2$, one may perturb x_2 by an arbitrarily small amount to some x_2' (staying in A since $y_1 = y_2$) and apply the argument to (x_1, x_2', y_1, y_2).

Using continuity of γv, 4 may thus be strengthened to 1. A completely analogous argument applies for worker w_1 and yields 2. ■

To prove Theorem 1, we only need local versions of 1 and 2 at profiles where the efficient matching changes. These are available for general $I, J \geq 2$:

Lemma 2 *Let $n \in \mathbb{N}$, $I, J \geq 2$ and let Condition 1 be satisfied. If the efficient matching is ex-post implementable, then for all $(x_1, x_2, y_1, y_2) \in A$, there is an open neighborhood $U_{(x_1,x_2,y_1,y_2)} \subset X \times X \times Y \times Y$ of (x_1, x_2, y_1, y_2) such that for all $(x_1', x_2', y_1', y_2') \in U_{(x_1,x_2,y_1,y_2)}$:*

$$F_v(x_1', x_2', y_1', y_2') \geq (\leq)\, 0 \Rightarrow F_{\gamma v}(x_1', x_2', y_1', y_2') \geq (\leq)\, 0, \tag{5}$$

$$F_v(x_1', x_2', y_1', y_2') \geq (\leq)\, 0 \Rightarrow F_{(1-\gamma)v}(x_1', x_2', y_1', y_2') \geq (\leq)\, 0. \tag{6}$$

Proof. Given $(x_1, x_2, y_1, y_2) \in A$, fix the types of all other employers and workers (x_i for $i \neq 1, 2$, y_j for $j \neq 1, 2$) such that there is an open neighborhood $U_{(x_1,x_2,y_1,y_2)}$ of (x_1, x_2, y_1, y_2) with the following property:

for all $(x_1', x_2', y_1', y_2') \in U_{(x_1,x_2,y_1,y_2)}$, the efficient matching for the profile $(x_1', x_2', x_3, ..., x_I, y_1', y_2', y_3, ..., y_J)$ either matches e_1 to w_1 and e_2 to w_2, or e_1 to w_2 and e_2 to w_1 (depending on the sign of $F_v(x_1', x_2', y_1', y_2')$). From here on, the proof parallels the one of Lemma 1. ■

As noted in the text, for efficient implementation, the incentives of each individual agent must be aligned with the social desire to choose the efficient matching. The next Lemma translates this intuitive requirement into a precise geometric statement about the gradients of $F_{\gamma v}$ and F_v.

Lemma 3 *Let $n \in \mathbb{N}$, $I, J \geq 2$ and let Condition 1 be satisfied. If the efficient matching is ex-post implementable, then there is a unique function $\lambda : A_0 \to [0, 1]$ satisfying*

$$\nabla F_{\gamma v}(x_1, x_2, y_1, y_2) = \lambda(x_1, x_2, y_1, y_2) \nabla F_v(x_1, x_2, y_1, y_2) \tag{7}$$

for all $(x_1, x_2, y_1, y_2) \in A_0$.

Proof. Since $\nabla F_v(x_1, x_2, y_1, y_2) \neq 0$ for all $(x_1, x_2, y_1, y_2) \in A_0$, 5 yields a unique $\lambda(x_1, x_2, y_1, y_2) \geq 0$ with

$$\nabla F_{\gamma v}(x_1, x_2, y_1, y_2) = \lambda(x_1, x_2, y_1, y_2)\nabla F_v(x_1, x_2, y_1, y_2).$$

Moreover, $\nabla F_{(1-\gamma)v}(x_1, x_2, y_1, y_2) = (1 - \lambda(x_1, x_2, y_1, y_2))\nabla F_v(x_1, x_2, y_1, y_2)$ and 6 therefore implies $\lambda(x_1, x_2, y_1, y_2) \in [0, 1]$. ∎

The crucial step in the proof follows now. It shows that the function λ must be constant. This constant corresponds then to a particular constant-proportion sharing rule.

Lemma 4 *Let $n \geq 2$, $I, J \geq 2$ and let Condition 1 be satisfied. Then the function λ from Lemma 3 must be constant: there is a $\lambda_0 \in [0, 1]$ such that $\lambda \equiv \lambda_0$.*

Proof. Let us spell out the equalities in 7:

$$\begin{aligned}
(\nabla_X \gamma v)(x_1, y_1) - (\nabla_X \gamma v)(x_1, y_2) &= \lambda(x_1, x_2, y_1, y_2)((\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2)) \\
(\nabla_X \gamma v)(x_2, y_2) - (\nabla_X \gamma v)(x_2, y_1) &= \lambda(x_1, x_2, y_1, y_2)((\nabla_X v)(x_2, y_2) - (\nabla_X v)(x_2, y_1)) \\
(\nabla_Y \gamma v)(x_1, y_1) - (\nabla_Y \gamma v)(x_2, y_1) &= \lambda(x_1, x_2, y_1, y_2)((\nabla_Y v)(x_1, y_1) - (\nabla_Y v)(x_2, y_1)) \\
(\nabla_Y \gamma v)(x_2, y_2) - (\nabla_Y \gamma v)(x_1, y_2) &= \lambda(x_1, x_2, y_1, y_2)((\nabla_Y v)(x_2, y_2) - (\nabla_Y v)(x_1, y_2)).
\end{aligned} \tag{8}$$

Given any $(x_1, x_2, y_1, y_2) \in A_0$, one obtains the same system of equations at $(x_2, x_1, y_1, y_2) \in A_0$, albeit for $\lambda(x_2, x_1, y_1, y_2)$. Thus, the function λ is symmetric with respect to x_1 and x_2. Similarly, it is symmetric with respect to y_1 and y_2. Next, for given $x_1 \in X$ and $y_1 \neq y_2$, the vectors in the first equation of 8 (with $(\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2) \neq 0$ on the right hand side) do not depend on how (x_1, y_1, y_2) is completed by x_2 to yield a full profile that lies in A_0. Consequently, $\lambda(x_1, x_2, y_1, y_2) = \lambda(x_1, x_1, y_1, y_2)$ for all these possible choices.

We next show that for a given x_1, λ does in fact not depend on y_1 and y_2 as long as $y_1 \neq y_2$. To this end, start with any $x_1 \in X$ and $y_1 \neq y_2$. We will show that for all $y_2' \neq y_1$ it holds

$$\lambda(x_1, x_1, y_1, y_2) = \lambda(x_1, x_1, y_1, y_2'). \tag{9}$$

Then, by symmetry of λ, $\lambda(x_1, x_1, y_1, y_2) = \lambda(x_1, x_1, y_2', y_1)$, and repeating the argument will yield that λ is indeed independent of y_1 and y_2 as long as $y_1 \neq y_2$.

So, let us prove 9. Using the first equation of 8, we have (for $y_2' \neq y_2$):

$$
\begin{aligned}
&\lambda(x_1, x_1, y_1, y_2)((\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2)) \\
&= ((\nabla_X \gamma v)(x_1, y_1) - (\nabla_X \gamma v)(x_1, y_2')) + ((\nabla_X \gamma v)(x_1, y_2') - (\nabla_X \gamma v)(x_1, y_2)) \\
&= \lambda(x_1, x_1, y_1, y_2')((\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2')) \\
&+ \lambda(x_1, x_1, y_2', y_2)((\nabla_X v)(x_1, y_2') - (\nabla_X v)(x_1, y_2)).
\end{aligned}
$$

It follows that

$$
\begin{aligned}
&(\lambda(x_1, x_1, y_1, y_2') - \lambda(x_1, x_1, y_1, y_2))\,((\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2')) \\
&+ (\lambda(x_1, x_1, y_2', y_2) - \lambda(x_1, x_1, y_1, y_2))\,((\nabla_X v)(x_1, y_2') - (\nabla_X v)(x_1, y_2)) \\
&= 0. \qquad (10)
\end{aligned}
$$

Two cases must now be distinguished.

Case 1: $(\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2')$ and $(\nabla_X v)(x_1, y_2') - (\nabla_X v)(x_1, y_2)$ are linearly independent. Then, it follows from 10 that $\lambda(x_1, x_1, y_1, y_2') = \lambda(x_1, x_1, y_1, y_2)$.

Case 2: $(\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2')$ and $(\nabla_X v)(x_1, y_2') - (\nabla_X v)(x_1, y_2)$ are linearly dependent. In this case, pick some $y_2'' \in Y$ such that $(\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2'')$ and $(\nabla_X v)(x_1, y_2'') - (\nabla_X v)(x_1, y_2)$ are linearly independent. This is always possible since $(\nabla_X v)(x_1, \cdot)$ maps connected open neighborhoods of y_1 one-to-one into $\mathbb{R}^n$, and since for $n \geq 2$, there is no one-to-one continuous mapping from an open set in $\mathbb{R}^n$ to the real line $\mathbb{R}$.[13]

From Case 1, we obtain $\lambda(x_1, x_1, y_1, y_2'') = \lambda(x_1, x_1, y_1, y_2)$. Since $(\nabla_X v)(x_1, y_1) - (\nabla_X v)(x_1, y_2')$ and $(\nabla_X v)(x_1, y_2') - (\nabla_X v)(x_1, y_2'')$ are also linearly independent, we then get $\lambda(x_1, x_1, y_1, y_2') = \lambda(x_1, x_1, y_1, y_2'')$, and hence 9 follows.

The third equation of 8 may be now used in an analogous way to show that for a given y_1, $\lambda(x_1, x_2, y_1, y_1)$ does not depend on x_1 and x_2, as long as $x_1 \neq x_2$.

The final ingredient is the following observation: for every $(x_1, x_1, y_1, y_2) \in A_0$, there is a $x_2 \neq x_1$ with $(x_1, x_2, y_1, y_2) \in A_0$. Indeed, $(\nabla_{X^2} F_v)(x_1, x_1, y_1, y_2) \neq$

[13] This is a special case of Brouwer's (1911) classical dimension preservation result: For $k < m$, there is no one-to-one, continuous function from a non-empty open set U of $\mathbb{R}^m$ into $\mathbb{R}^k$.

0, so that the set of x_2 for which $(x_1, x_2, y_1, y_2) \in A_0$ is given locally (in a neighborhood of $x_2 = x_1$) by a differentiable manifold of dimension $n-1$. Since $n \geq 2$, this manifold must contain points other than x_1. A similar argument applies to $(x_1, x_2, y_1, y_1) \in A_0$.

To conclude the proof, we show that λ is constant on $\{(x_1, x_2, y_1, y_2) \in A_0 | x_1 \neq x_2 \text{ and } y_1 \neq y_2\}$. This set is non-empty by the previous observation (and we have already seen that $\lambda(x_1, x_2, y_1, y_2) = \lambda(x_1, x_1, y_1, y_2)$ and $\lambda(x_1, x_2, y_1, y_2) = \lambda(x_1, x_2, y_1, y_1)$, so that λ is constant on all of A_0 then). Given any $(x_1, x_2, y_1, y_2), (x_1', x_2', y_1', y_2') \in A_0$ with $x_1 \neq x_2$, $y_1 \neq y_2$, $x_1' \neq x_2'$ and $y_1' \neq y_2'$, we have:

$$\begin{aligned}
\lambda(x_1, x_2, y_1, y_2) \;&=\; \lambda(x_1, x_1, y_1, y_2) \;=\; \lambda(x_1, x_1, y_1', y_2') \\
&= \lambda(x_1, x_2'', y_1', y_2') = \lambda(x_1, x_2'', y_1', y_1') \\
&= \lambda(x_1', x_2', y_1', y_1') \;=\; \lambda(x_1', x_2', y_1', y_2'),
\end{aligned}$$

where $x_2'' \neq x_1$ is any feasible profile completion for (x_1, y_1', y_2'). ■

We are now finally ready to prove Theorem 1.

Proof of Theorem 1. ii)⇒ i): Consider the case $I \leq J$. As in the proof of Lemma 1, we make use of the "taxation principle" for ex-post implementation. For employer e_i, and matching $m \in \mathcal{M}$ define $t_m^{e_i}(x_{-i}, y_1, ..., y_J) := \lambda_0 \sum_{l \neq i} v(x_l, y_{m(l)}) - h(y_{m(i)})$. Then, $(\gamma v)(x_i, y_{m(i)}) + t_m^{e_i}(x_{-i}, y_1, ..., y_J) = \lambda_0 \sum_{l=1}^{I} v(x_l, y_{m(l)}) + g(x_i)$, so that it is optimal for e_i to select a matching that maximizes aggregate welfare. Note that strict incentives for truth-telling can be provided only if $\lambda_0 > 0$. For worker w_j, define

$$\begin{aligned}
t_m^{w_j}(x_1, ..., x_I, y_{-j}) := \quad & (1-\lambda_0) \sum_{k \in m(\mathcal{I}), k \neq j} v(x_{m^{-1}(k)}, y_k) \\
& + g(x_{m^{-1}(j)}) \mathbf{1}_{j \in m(\mathcal{I})} - h(y_j) \mathbf{1}_{j \notin m(\mathcal{I})}.
\end{aligned}$$

Here, $\mathbf{1}_{j \in m(\mathcal{I})} = 1$ if $j \in m(\mathcal{I})$, and $\mathbf{1}_{j \in m(\mathcal{I})} = 0$ otherwise. Note that if $I = J$, then $j \in m(\mathcal{I})$ for all possible matchings m, so that the final (y_j-dependent) term always vanishes. If $I < J$, then h is constant by assumption, and the transfer does not depend on y_j. It follows that if w_j is matched in m, his utility is $((1-\gamma)v)(x_{m^{-1}(j)}, y_j) + t_m^{w_j}(x_1, ..., x_I, y_{-j}) = (1-\lambda_0) \sum_{k \in m(\mathcal{I})} v(x_{m^{-1}(k)}, y_k) - h(y_j)$. Otherwise, his utility is just $t_m^{w_j}(x_1, ..., x_I, y_{-j}) = (1-\lambda_0) \sum_{k \in m(\mathcal{I})} v(x_{m^{-1}(k)}, y_k) - h(y_j)$. Hence, it is optimal for w_j to select a matching that maximizes aggregate welfare, and strict incentives for truth-telling can be provided only if $\lambda_0 < 1$. This proves i) for $I \leq J$. The proof for the case $I \geq J$ is completely analogous.

i) $\Rightarrow$ ii): By Lemma 4, there is a $\lambda_0 \in [0,1]$ such that for all $x \in X$, $y_1, y_2 \in Y$ with $y_1 \neq y_2$ it holds (the profile may be completed to lie in A_0, e.g. by $x' = x$):

$$(\nabla_X \gamma v)(x, y_1) - (\nabla_X \gamma v)(x, y_2) \;=\; \lambda_0((\nabla_X v)(x, y_1) - (\nabla_X v)(x, y_2)).$$

Integrating along any path from x_2 to x_1 (X is open and connected in $\mathbb{R}^n$, hence path-connected) yields $F_{\gamma v}(x_1, x_2, y_1, y_2) = \lambda_0 F_v(x_1, x_2, y_1, y_2)$. Hence, by linearity of the operator F, we obtain that $F_{(\gamma-\lambda_0)v} \equiv 0$. A function of two variables has vanishing cross differences if and only if it is additively separable, so that we can write $(\gamma v)(x, y) = \lambda_0 v(x, y) + g(x) + h(y)$. This concludes the proof for the case where $I = J$.

It remains to prove that h must be constant if $I < J$ (the proof that g must be constant when $I > J$ is analogous). Given $y_1 \in Y$, Condition 1 implies that $(\nabla_Y v)(\cdot, y_1)$ vanishes at most in one point. Pick then any $x_1 \in X$ with $(\nabla_Y v)(x_1, y_1) \neq 0$. Set $y_2 = y_1$ and complete the type profile for ($i \neq 1$, $j \neq 1, 2$) such that, for an open neighborhood U of (y_1, y_1), the efficient matching changes only with respect to the partner of e_1 : either w_1 is matched to e_1 and w_2 remains unmatched, or w_2 is matched to e_1 and w_1 remains unmatched. For $(y_1', y_2') \in U$, it follows that $v(x_1, y_1') - v(x_1, y_2') \geq (\leq)\ 0$ implies $((1-\gamma)v)(x_1, y_1') - ((1-\gamma)v)(x_1, y_2') \geq (\leq)\ 0$. Hence, there is a $\mu(x_1, y_1) \geq 0$ such that

$$(1 - \lambda_0)(\nabla_Y v)(x_1, y_1) - (\nabla_Y h)(y_1) = \mu(x_1, y_1)(\nabla_Y v)(x_1, y_1).$$

In other words, $(\nabla_Y h)(y_1)$ and $(\nabla_Y v)(x_1, y_1)$ are linearly dependent. Finally, let x_1 vary and note that, by Condition 1, the image of $(\nabla_Y v)(\cdot, y_1)$ cannot be concentrated on a line (recall footnote 13). Thus, we obtain that $(\nabla_Y h)(y_1) = 0$. Since y_1 was arbitrary and Y is connected, it follows that the function h must constant. ■

Proof of Theorem 2. Let $I \leq J$ (the proof for $I \geq J$ is analogous). Consider some $i \in \mathcal{I}$ and a given. fixed type profile for all other agents $(x_{-i}, y_1, ..., y_J)$. Given any such type profile, we re-order the workers and employers other than i such that $x^{(1)} \geq ... \geq x^{(I-1)}$ and $y^{(1)} \geq ... \geq y^{(J)}$.

We now verify the monotonicity condition identified by Bergemann and Välimäki.[14] This requires that the set of types of agent i for which a particular social alternative is efficient forms an interval. Let then m_k, $k = 1, ..., I$ denote the matching that matches $x^{(l)}$ to $y^{(l)}$ for $l = 1, ..., k-1$, x_i to $y^{(k)}$ and $x^{(l)}$ to $y^{(l+1)}$ for $l = k, ..., I-1$. Then, for $k = 2, ..., I-1$ it holds the set

$$\{x_i \in X | u_{m_k}(x_1, ..., x_I, y_1, .., y_J) \geq u_m(x_1, ..., x_I, y_1, .., y_J) \ , \forall m \in \mathcal{M}\}$$

is simply $[x^{(k)}, x^{(k-1)}]$. For $k = I$ the set is $(\inf X, x^{(I-1)}]$, and for $k = 1$ it is $[x^{(1)}, \sup X)$. Monotonicity for workers j is verified in the same way.

Next, the necessary condition of Bergemann and Välimäki, spelled out for our matching model, requires that at all "switching points" $x_i = x^{(k-1)}$ where the efficient allocation changes, it also holds that

$$\frac{\partial}{\partial x_i}((\gamma v)(x_i, y^{(k-1)}) - (\gamma v)(x_i, y^{(k)})) \geq 0.$$

Given x_i and $y' > y$ we can always complete these to a full type profile such that x_i is a change point at which the efficient match for x_i switches from y to y'. Hence $\frac{\partial}{\partial x}((\gamma v)(x, y') - (\gamma v)(x, y)) \geq 0$ for all x and $y' > y$. So, γv must have increasing differences, i.e. it is supermodular. Since $\frac{\partial}{\partial x_i}((\gamma v)(x_i, y^{(k-1)}) - (\gamma v)(x_i, y^{(k)})) \geq 0$ is satisfied for all $x_i \in X$ (not just at switching points !), the second part of the sufficient conditions of Bergemann and Välimäki is satisfied. The argument for workers (yielding supermodularity of $(1-\gamma)v$) is analogous. This completes the proof. ■

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[14] We only verify it for type profiles for which all these inequalities are strict. When some types coincide, it is still straightforward to verify monotonicity but we do not spell out the more cumbersome case distinctions here.

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