MECHANISM DESIGN WITH COMMUNICATION CONSTRAINTS[1]

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Abstract

We study mechanism design in a context where communicational constraints prevent the use of revelation mechanisms, and agents behave strategically. We examine a setting with multiple agents, each producing (or purchasing) a single dimensional output with single-dimensional cost (or valuation) parameter satisfying a standard single-crossing property. Necessary and sufficient conditions for Bayesian implementation in arbitrary dynamic communication protocols are obtained. Optimal mechanisms are shown to maximize the Principals objective (with 'virtual' types replacing true types) subject to communication feasibility alone. This implies delegating production (or purchase) decisions to agents strictly dominates centralized decisions. Optimal communication protocols involve multiple rounds of communication in which agents simultaneously send binary messages, if communication costs depend on total time delay. They involve sequential reports when communication costs depend instead on the total number of information bits sent.

KEYWORDS: communication, mechanism design, auctions, decentralization, incentives, organizations

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1 Introduction

Most practical contexts of economic mechanisms such as auctions, regulation or internal organization of firms involve dynamic, interactive communication. Participants exchange bids, cost reports, or budgets through dynamic and time-consuming processes which feed into ultimate decisions concerning production, exchange and financial transfers. These are in sharp contrast with revelation mechanisms commonly studied in the theoretical literature, in which agents make reports of everything they know in a simultaneous, static process.

Part of the reason is that the dimensionality of private information is considerably richer than what can be feasibly be communicated to others in real time. This observation was made by Hayek (1945) in his famous critique of Lange-Lerner socialist resource allocation mechanisms, in which he argued that communication constraints provided much of the justification for a decentralized market economy coordinating decisions through price signals. Hayek's observation motivated a large literature on resource allocation mechanisms that economize on communication costs. Examples are the message space literature (Hurwicz (1960, 1972), Mount and Reiter (1974)) and the theory of teams (Marschak and Radner (1972)).[3] This early literature on mechanism design ignored incentive problems.[4] The more recent literature on mechanism design on the other hand focuses only on incentive problems, ignoring communication costs. As a result the space of relevant mechanisms studied are revelation mechanisms, where agents communicate reports of their entire private information in a single, instantaneous step.

A few recent papers have explored the implications of co-existence of communication costs and incentive problems. Van Zandt (2007) and Fadel and Segal (2009) pose the question of the extent to which incentive problems increase communicational complexity of mechanisms that implement a desired allocation rule, in a general setting. Other authors have sought to characterize optimal incentive mechanisms in settings with restricted message spaces and the standard assumptions of single-dimensional outputs and single-crossing preferences (Green and Laffont (1986, 1987), Melumad, Mookherjee and Reichelstein (1992, 1997), Laffont and Martimort (1998), Blumrosen, Nisan and Segal (2007), Blumrosen and Feldman (2006), Deneckere and Severinov (2004), Kos (20011a, 2011b)).[5] For reasons of tractability, these authors have restricted the communication protocols and range of mechanisms considered. With the exception of Kos (2011b), most authors focus on mechanisms with a single round of communication (with restricted

[3]Segal (2006) surveys recent studies of informationally efficient allocation mechanisms.

[4]Reichelstein and Reiter (1988) examined implications of strategic behavior for communicational requirements of mechanisms implementing efficient allocations.

[5]Battigali and Maggi (2002) study a model of symmetric but nonverifiable information where there are costs of writing contingencies into contracts. This is in contrast to the models cited above which involve asymmetric information with constraints on message spaces.

message spaces).[6] However it is well-known that informational efficiency necessitates dynamic, interactive communication when agents' opportunities to express themselves are intrinsically limited.

The key analytical problem in incorporating dynamic communication protocols into models with strategic agents is in characterizing incentive constraints. From the standpoint of informational efficiency, it is valuable to allow agents to learn about the messages sent by other agents before they send some additional messages. This can create complications in the presence of incentive problems. Additional communication rounds where agents send messages after hearing reports by other agents in previous rounds generates additional incentive constraints over and above those arising in the first round. Moreover, dynamic mechanisms enlarge the range of possible deviations available to participants. Apart from the possibility of mimicking equilibrium strategies of other types of the same agent, additional deviations are possible in a dynamic communication process. Van Zandt (2007) observes that this is not a problem when the solution concept is *ex post incentive compatibility (EPIC)*, where agents do not regret their strategies even after observing all messages sent by other agents. When we use the less demanding concept of a (perfect) Bayesian equilibrium, dynamic communication protocols may impose additional incentive constraints. If so, a trade-off between informational efficiency and incentive problems can arise.

This trade-off has been difficult to study, owing to absence of a precise characterization of incentive constraints for dynamic protocols in existing literature. Fadel and Segal (2009) consider a very general setting, and provide different sets of sufficient conditions which are substantially stronger than necessary conditions. In this paper we restrict attention to contexts with single dimensional outputs and single-crossing preferences for each agent, which is nevertheless general enough to capture a wide range of applications to auctions, regulation and internal organization contexts. In these contexts, we show that the necessary conditions are sufficient for Bayesian implementation for arbitrary dynamic communication protocols (Proposition 1).

This enables us to show (Proposition 2) that under standard regularity conditions (on the hazard rate of distributions of types), the mechanism design problem reduces to selecting an output allocation rule which maximizes a payoff function of the Principal (modified to include the cost of incentive rents paid to agents in a standard way with 'virtual' types replacing actual types) subject to communication feasibility restrictions alone. This extends the standard approach to solving for optimal mechanisms with unlimited communication (following Myerson (1981)). It provides a convenient representation of the respective costs imposed by incentive problems and communicational constraints.

[6] Kos (2011b) examines a dynamic process in which a seller of an indivisible object asks a sequence of successive binary questions to two buyers concerning their valuations.

A number of implications of this result are then derived. The first concerns the value of delegating production decisions to agents.[7] In contexts of unconstrained communication where the Revelation Principle applies, it is well known that centralized decision-making can always (trivially) replicate the outcome of any delegation mechanism.[8] This is no longer so when communication constraints prevent agents from reporting everything they know to the Principal (as argued by Hayek). In the presence of incentive problems, however, delegation can generate costs owing to opportunistic behavior, as well as benefits from enhanced informational efficiency. In our context, it turns out the benefits of improved information always outweigh attendant incentive costs. Proposition 3 shows that any centralized mechanism is strictly dominated by some mechanism with decentralized production choices made by agents. This vindicates Hayek's arguments in favor of decentralized mechanisms in general. Practical implications include the superiority of taxes over quantitative controls, and of firm organizations which delegate production decisions to workers (Aoki (1990)).

A second set of implications concern the design of dynamic communication protocols. Proposition 2 has strong implications for how communication processes ought to be structured: to maximize the amount of information exchanged by agents. If the underlying cost of communication is time delay (where messages take time to write or send), an optimal protocol involves multiple rounds of communication in which agents simultaneously send messages in each round, and send as little information possible (i.e., binary messages) in each round. But if the limitation is on the total number of binary messages sent, optimal protocols necessitate sequential reporting, where every communication round involves a single agent sending messages.

The paper is organized as follows. Section 2 provides some examples to help explain the informational benefit of dynamic communication protocols, and then the incentive problems they give rise to. Section 3 introduces the general model. Section 4 is devoted to characterizing feasible allocations. Section 5 uses this to represent the design problem as maximizing the Principal's incentive-rent-modified welfare function subject to communicational constraints alone. Section 6 uses this to compare centralized and decentralized mechanisms, while Section 7 describes implications for design of optimal communication protocols. Section 8 concludes.

[7]Earlier literature such as Melumad, Mookherjee and Reichelstein (1992, 1997) and Laffont and Martimort (1998) have focused on a related but different question: the value of decentralized contracting (or subcontracting) relative to centralized contracting. Here we assume that contracting is centralized, and examine the value of decentralizing production decisions instead. We comment further on implications of our analysis for decentralization of contracting in Section 8.

[8]For a formal statement and proof, see Myerson (1982).

2 Examples and Related Literature

Example 1

We start by providing an example illustrating the informational value of dynamic, interactive communication protocols when there are limitations on the amount of information that can be communicated by any agent in any given round, and incentive issues are ignored.

Suppose two agents jointly produce a common output q for the Principal, which takes three possible levels $0, 1, 2$. The corresponding gross revenues $V(q)$ earned by the Principal are $0, 38$ and 50 respectively. Agent i incurs a unit cost θ_i of production. For Agent 1, this cost takes two possible values $0, 10$ which are equally likely *ex ante*. On the other hand θ_2 can take three possible values $0, 30, 100$ where the prior probability of costs 0 and 100 are $\frac{1}{4}$ each and the probability of cost 30 is a half.

Suppose the Principal is concerned about the efficiency measured by the expected value of $V(q) - (\theta_1 + \theta_2)q$. The first best allocation (without any consideration of communication or incentive constraints) is that $q = 2$ if $\theta_1 + \theta_2 < V(2) - V(1) = 12$, $q = 1$ if $V(2) - V(1) = 12 \leq \theta_1 + \theta_2 \leq 38 = V(1)$, $q = 0$ if $V(1) = 38 < \theta_1 + \theta_2$. The corresponding first-best outputs $q^{FB}(\theta_1, \theta_2)$ are $q^{FB}(0, 100) = q^{FB}(10, 100) = q^{FB}(10, 30) = 0$, $q^{FB}(0, 30) = 1$ and $q^{FB}(0, 0) = q^{FB}(0, 10) = 2$. This first best allocation is shown in case (a) of Figure 1.

Now introduce a constraint on communication: each agent can send only a binary message $m_i \in \{0, 1\}$ only once. Agent 2 who has three types then cannot report his true type. Ignoring incentive issues, suppose agents follow instructions provided by the Principal regarding what message to send in different contingencies. In what follows we focus on *threshold reporting strategies*, in which the type space of each agent is partitioned into a number of subintervals (which equals the number of feasible messages), and they report the subinterval in which their type belongs. It is easy to check that this is without any loss of generality, as long as there is a single round of communication for each agent.

Consider first a protocol where the two agents send a binary report simultaneously to the Principal. There are two possible threshold reporting strategies possible, shown shown in (b) and (c) of Figure 1. The Principal now does not have the information available to implement the first-best efficient allocation, lacking information of Agent 2's true type. Conditional on the information available, the constrained efficient allocations are also shown in (b) and (c) of this figure. In the case of simultaneous reporting, the Principal's information is represented by a 'rectangular' partition of the type space, where she knows whether Agent 1's cost is high (10) or low (0), and whether Agent 2's cost is high or low (the precise definition of which depends on the particular threshold strategy used by Agent 2). The threshold for high cost of Agent 2 is 100 in case (b) and is 30 in case (c).

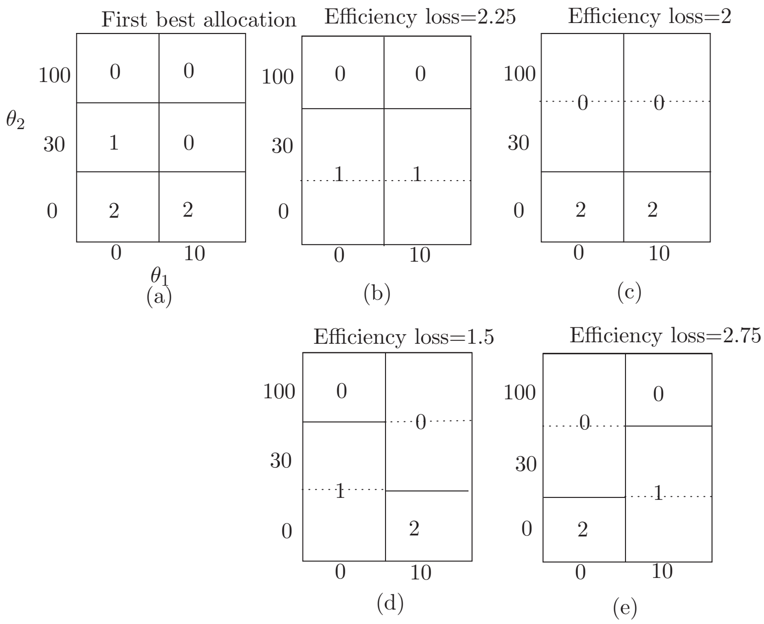

Figure 1: Example 1

The latter turns out to generate a smaller loss of efficiency compared with the first-best allocation.

Now consider a sequential protocol where Agent 1 sends a binary report first, to which Agent 2 responds with a binary report. Agent 2 can now condition his threshold on the report sent by Agent 1. Cases (d) and (e) show two different reporting strategies used by Agent 2 where the threshold does vary with Agent 1's report. Note also that cases (b) and (c) continue to be available here, since Agent 2 can also use a strategy in which the threshold does not vary with Agent 1's message. It turns out the smallest efficiency loss is incurred in case (d). Compared to the simultaneous reporting protocol, a sequential protocol allows more information to be communicated to the Principal, within the constraints allowed by the communication technology. Of course this requires Agent 2 be granted access to the reports filed by Agent 1. If there are no incentive problems there is no loss associated with letting Agent 2 acquire information about Agent 1's type, while production assignments are chosen on the basis of better information.

If there were more rounds of communication, the agents would be able to report their true types. But agents could have more types as well. In general, if the type space is rich enough relative to the communication channels available, agents would not be able to report everything they know. If there is also a constraint on the maximum number of rounds of binary communication which prevents agents from reporting everything they know (say because of constraints on the total amount of time taken up in communication), it is easy to see that the greatest amount of information could be communicated to the Principal if the agents were to release information gradually to one other, which would enable them to tailor their reporting strategies in later rounds on the messages exchanged in previous rounds.

Providing more information to agents regarding messages sent by other agents in previous rounds may of course generate incentive problems. This is the key problem that we address in this paper. To illustrate the nature of the incentive problem in dynamic communication protocols, we turn to the next example.

Example 2

We now present a different example which illustrates in the simplest possible way the special problems that arise in analyzing incentives in dynamic protocols.

Suppose that Agent 1's type θ_1 takes three possible values: 0, 1 and 2 with equal probability, while Agent 2's type θ_2 is takes two possible values 0 and 1 with equal probability where θ_i denotes unit cost of production for $i \in \{1, 2\}$. Consider a communication protocol, which is an extensive form of communication with three rounds of binary messages. We abstract from Agent 2's reporting incentives by assuming he reports truthfully, and focus on Agent 1's reporting incentives.

Round 1 Agent 1 sends a binary message $m_{11} \in \{0, 1\}$ in the first round. If he sends 0, the communication ends and the mechanism specifies allocation $(t_1(0), q_1(0))$ for agent 1 where t_1 is a transfer to Agent 1 and q_1 is a quantity to be supplied by Agent 1. If he chose $m_{11} = 1$, the game proceeds to Round 2.

Round 2 Agent 2 sends a binary message $m_{22} \in \{0, 1\}$, and the game proceeds to Round 3.

Round 3 Agent 1 sends a binary message $m_{13} \in \{0, 1\}$. The mechanism then specifies a transfer and quantity to be supplied by agent 1 as a function of messages exchanged at rounds 2 and 3: $(t_1(1, m_{22}, m_{13}), q_1(1, m_{22}, m_{13}))$.

At the beginning of each round, each agent knows messages sent by the other agent until the end of the previous round. Figure 2 describes the extensive form, which has five terminal nodes $h \in H \equiv \{0, 100, 101, 110, 111\}$.

Consider the question of incentive compatibility of the following communication strategies $(c_1(\theta_1), c_2(\theta_2))$, which specifies messages to be taken by agents for each node, recommended by the Principal:

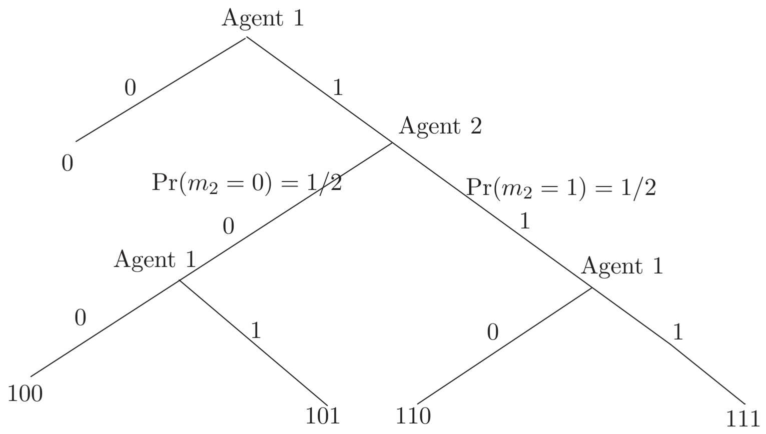

Figure 2: Example 2 (Extensive Form of Communication)

Round 1 Agent 1 sends $m_{11} = 0$ if $\theta_1 = 0$, and $m_{11} = 1$ if $\theta_1 \in \{1, 2\}$.

Round 2 Agent 2 sends $m_{22} = 0$ if $\theta_2 = 0$, and $m_{22} = 1$ if $\theta_2 = 1$

Round 3 Agent 1 sends $m_{13} = 0$ if $\theta_1 = 1$ and $m_{13} = 1$ if $\theta_1 = 2$, regardless of messages in previous rounds.

Specifically, given a quantity allocation $\{q_1(h) : h \in H\}$, the question is whether there exist transfers $\{t(h) : h \in H\}$ that ensure Agent 1's incentives to follow these recommendations, assuming Agent 2 follows them. Note that the message choices of type 0 of agent 1 in Round 3 (if he happened to have chosen $m_{11} = 1$ in Round 1) have not been specified. It will be necessary to consider possible choices that this type could make, in order to determine whether the preceding strategies are incentive compatible for agent 1,. We discuss this below.

The recommended strategies determine the probability with which various terminal nodes are achieved. Let $P(h)$ denote the probability that terminal node h is reached. Figure 3 describes these probabilities for each type. Note that every terminal node is reached with positive probability. In other words, all message options provided at various rounds are used with positive probability. The protocol is *parsimonious* relative to communication strategies $(c_1(\theta_1), c_2(\theta_2))$, using the terminology of Van Zandt (2007).

A necessary condition for incentive compatibility of these strategies for Agent 1 (for some set of transfers) is that no type should want to deviate to the strategy prescribed for another type. Had the Principal been able to use a one-shot revelation mechanism in which each agent independently sends a type report, these conditions would also have

	$(P(0), P(100), P(101), P(110), P(111))$
type 0	$(1, 0, 0, 0, 0)$
type 1	$(0, 1/2, 0, 1/2, 0)$
type 2	$(0, 0, 1/2, 0, 1/2)$

Figure 3: Example 2 (Parsimonious Protocol)

been sufficient to ensure incentive compatibility. The reason is that the size of the message space is equal to the set of possible types, so the range of possible deviations available to any type is precisely the range of strategies utilized by other types. Following standard arguments, these conditions reduce to a single monotonicity condition on the expected quantity produced by Agent 1 with respect to his type report (when using Bayesian equilibrium as the solution concept):

$$q_1(0) \geq (1/2)q_1(100) + (1/2)q_1(110) \geq (1/2)q_1(101) + (1/2)q_1(111). \quad (1)$$

In the dynamic mechanism above, however, each type has a larger range of deviations available. Agent 1 has eight possible communication strategies to choose from, corresponding to various combinations of messages sent at Round 1, and those sent in Round 3 following the message sent by Agent 2 in Round 2. The question is whether the necessary condition for incentive compatibility described above suffices to ensure that no type of Agent 1 has a profitable deviation.

Van Zandt (2007) poses this question as the converse of the Revelation Principle: does incentive compatibility in the static revelation mechanism ensure incentive compatibility in the dynamic mechanism (after the latter has been pruned to eliminate unused messages, i.e., the dynamic mechanism is parsimonious)? He points out the answer is yes, if the solution concept used is *ex post incentive compatibility (EPIC)*. This concept imposes the requirement that no type should regret his strategy choice at the end of the game, after learning the messages sent by the other agent. In the static revelation mechanism, in our example this requires type 0 to prefer $h = 0$ to both $h = 100$ and $h = 101$, nodes that could have been reached upon mimicking type 1's strategy. It also requires type 0 to prefer $h = 0$ to $h = 110$ and $h = 111$, nodes that it could have reached upon mimicking type 2's strategy. Hence type 0 weakly prefers the node $h = 0$ to all the four other terminal nodes. Similarly type 1 should not prefer either $h = 0$ or $h = 101$ to the node reached $h = 100$ when agent 2 sends message $m_{22} = 0$ at round 2. Nor should he prefer either $h = 0$ or $h = 111$ to the node $h = 110$ reached when agent 2 sent message $m_{22} = 1$

instead. An analogous set of inequalities for type 2 completes the set of requirements for incentive compatibility in the revelation mechanism.

It is obvious that these suffice to ensure incentive compatibility as well in the dynamic protocol. Essentially, the condition of 'no-regret after learning messages sent by other types' implies that letting agents learn about messages sent by other agents does not disturb their incentive to follow the recommended strategies. Hence dynamic protocols do not entail additional incentive constraints when the solution concept is EPIC. In our context there is a simple necessary and sufficient condition for EPIC-incentivizability of a given quantity allocation, involving monotonicity of the assigned quantity with respect to the reported type of the agent, for every possible type reported by the other agent:

$$q_1(0) \geq q_1(100) \geq q_1(101) \tag{2}$$

and

$$q_1(0) \geq q_1(110) \geq q_1(111). \tag{3}$$

This argument does not extend to the case of Bayesian incentive compatibility. Here the incentives of agents to follow the recommended strategies may depend on their lack of knowledge of messages sent by other agents. In our example each type of agent 1 has available deviations which do not constitute strategies chosen by any other type. This is despite the fact that the protocol is parsimonious. For instance, type 0 could deviate to selecting $m_{11} = 1$ in Round 1, followed by $m_{13} = 0$ if $m_{22} = 0$ and $m_{13} = 1$ if $m_{22} = 1$. This is a strategy not selected by any type of Agent 1. The condition that type 0 does not benefit by deviating to a strategy chosen by some other type, does not automatically ensure that it would not benefit by deviating to some other strategy.

To see that the dynamic protocol imposes additional incentive compatibility constraints, consider the question of characterizing quantity assignments $\{q_1(h)\}$ that are Bayesian-incentivizable by some set of transfers. In a static revelation mechanism, incentivizability requires only (1), whereas in the dynamic protocol, the following additional constraints are required:

$$q_1(100) \geq q_1(101) \tag{4}$$

and

$$q_1(110) \geq q_1(111) \tag{5}$$

to ensure that in Round 3 types 1 and 2 do not want to imitate one another's message, having heard the message reported by Agent 2 in Round 2.

Hence the added flexibility of production assignments in a dynamic protocol may come at the expense of increasing the number of incentive constraints. This is the key trade-off involved in comparing dynamic with static protocols. In order to make progress with studying design of optimal mechanisms with limited communication, we need to characterize the precise set of incentive constraints associated with any given dynamic communication protocol.

Such a characterization is not available in existing literature. Fadel and Segal (2009) provide a set of sufficient conditions for Bayesian incentive compatibility in the dynamic protocol, which are stronger than the necessary conditions represented by the combination of (1), (5) and (4). Their Proposition 6 observes that EPIC-incentivizability implies BIC-incentivizability. Hence conditions (2) and (3) are sufficient for BIC-incentivizability. But these are stronger than the combination of (1), (5) and (4). These happen to be automatically satisfied in the informationally efficient allocation (incorporating only the communicational constraints) in contexts involving a single round of communication in many contexts (Melumad, Mookherjee and Reichelstein (1992, 1997), Blumrosen, Nisan and Segal (2007), Blumrosen and Feldman (2006)), as well as in some dynamic communication contexts (Kos (2011b)). In such contexts, thus, the informationally efficient allocation ends up being incentivizable, so the incentive constraints do not impose any additional cost. But this property is not true in general. For instance, it is not satisfied in the constrained efficient allocation in case (d) of Example 1.

In this paper we show that conditions (1), (5) and (4) are collectively both necessary *and* sufficient for Bayesian-incentivizability in the dynamic protocol. This property holds in general. It is a key step that allows us to pose the mechanism design problem as selection of a communication protocol and a contract (represented by quantities and transfers) to maximize the Principal's payoff subject to the constraints on the protocol and these incentive compatibility constraints.

The idea for the sufficiency argument can be illustrated as follows. Given any quantity allocation satisfying (1), (5) and (4), we construct incentivizing transfers $\{\hat{t}_1(h) \mid h \in H\}$ as follows.

(i) Choose $q_1(10d)$ and $q_1(11d)$ such that $q_1(10d) \geq q_1(100) \geq q(101)$, $q_1(11d) \geq q_1(110) \geq q_1(111)$ and

$$q_1(0) = (1/2)q_1(10d) + (1/2)q_1(11d).$$

(ii) Choose $(\hat{t}_1(10d), \hat{t}_1(100), \hat{t}_1(101))$ and $(\hat{t}_1(11d), \hat{t}_1(110), \hat{t}_1(111))$ such that

- Among $\{(\hat{t}_1(10d), q_1(10d)), (\hat{t}_1(100), q_1(100)), (\hat{t}_1(101), q_1(101))\}$, type 0 prefers $(\hat{t}_1(10d), q_1(10d))$, type 1 prefers $(\hat{t}_1(100), q_1(100))$ and type 2 prefers $(\hat{t}_1(101), q_1(101))$.
- Among $\{(\hat{t}_1(11d), q_1(11d)), (\hat{t}_1(110), q_1(110)), (\hat{t}_1(111), q_1(111))\}$, type 0 prefers $(\hat{t}_1(11d), q_1(11d))$, type 1 prefers $(\hat{t}_1(110), q_1(110))$ and type 2 prefers $(\hat{t}_1(111), q_1(111))$.

(iii) Choose $\hat{t}_1(0) = (1/2)\hat{t}_1(10d) + (1/2)\hat{t}_1(11d)$.

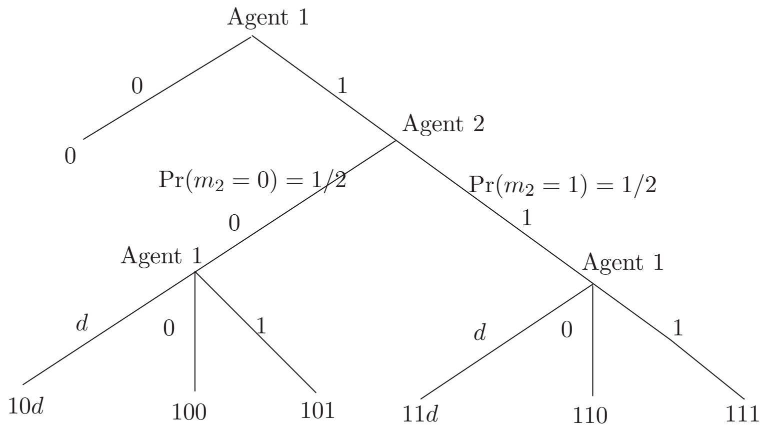

Figure 4: Example 2 (Augmented Communication Protocol)

Since $q_1(0) \geq (1/2)q_1(100) + (1/2)q_1(110)$, it is easy to check existence of $q_1(10d)$ and $q_1(11d)$ satisfying (i). The monotonicity conditions ($q_1(10d) \geq q_1(100) \geq q(101)$ and $q_1(11d) \geq q_1(110) \geq q_1(111)$) guarantee the existence of $(\hat{t}_1(10d), \hat{t}_1(100), \hat{t}_1(101))$ and $(\hat{t}_1(11d), \hat{t}_1(110), \hat{t}_1(111))$ satisfying (ii).

This can be interpreted as follows. Consider a hypothetical augmentation of the communication protocol, where agent 1 is provided with an additional choice d at each of the two nodes in Round 3 (See Figure 4). This adds two new terminal nodes, with assigned quantities and transfers that are designed to be selected by type 0 in Round 3 if that type would have deviated at Round 1 by choosing $m_{11} = 1$ instead of 0. Condition (ii) ensures that type 0 selects the newly added terminal nodes, while the other two types are not motivated to deviate to these in Round 3. Condition (iii) then constructs a transfer corresponding to terminal node $h = 0$ which provides the same expected payoff to type 0 at this node, as the expected payoff from deviating to $m_{11} = 1$ and then proceeding to Round 3. The monotonicity conditions on the quantity allocation ensures that types 1 and 2 do not want to deviate to $m_{11} = 0$, while by construction type 0 does not have an incentive to deviate to $m_{11} = 1$.

Finally, we go back to the original protocol where the two new nodes $10d$ and $11d$ are not added, and we use the constructed transfers at all the remaining nodes. It is evident that each type now has an incentive to follow the recommended communication strategies.

We show in the succeeding sections that this method works generally. Hence we are able to obtain a set of conditions that are both necessary and sufficient for Bayesian-incentivizability of a given quantity allocation combined with a set of supporting communication strategies. This enables us to extend standard methods (based on the Revenue

Equivalence Theorem) of posing the mechanism design problem in terms of the quantity allocation alone.

3 Model

There is a Principal (P) and two agents 1 and 2. Agent $i = 1, 2$ produces a one-dimensional nonnegative real valued input q_i at cost $\theta_i q_i$, where θ_i is a real-valued parameter distributed over an interval $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ according to a positive-valued, continuously differentiable density function f_i and associated c.d.f. F_i. The distribution satisfies the standard monotone hazard condition that $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ is nondecreasing, implying that the 'virtual cost' $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is strictly increasing. θ_1 and θ_2 are independently distributed, and these distributions F_1, F_2 are common knowledge among the three players. The inputs of the two agents combine to produce a gross return $V(q_1, q_2)$ for P. In some contexts there may be technological restrictions on the relation between the outputs of the two agents. For instance the agents may work in a team that produce a joint output q, in which case $q_1 = q_2 = q$. Or the Principal may organize a procurement auction between two suppliers that produce perfect substitutes, to procure a given total quantity of the common good. Normalizing this desired quantity to 1, we can set $q_2 = 1 - q_1 \in [0, 1]$. In this case of course the constraint $q_2 = 1 - q_1$ is not a technological restriction, and the production function can be written as $V = \min\{q_1 + q_2, 1\}$. To accommodate the possibility of joint production, let $Q \subset R_+ \times R_+$ denote a set of technologically feasible combination of input supplies (q_1, q_2).

The Principal makes transfer payments t_i to i. The payoff of i is $t_i - \theta_i q_i$. Both agents are risk-neutral and have autarkic payoffs of 0. The Principal's objective takes the form

$$V(q_1, q_2) - \lambda_1(t_1 + t_2) - \lambda_2(\theta_1 q_1 + \theta_2 q_2) \tag{6}$$

where $\lambda_1 \geq 0$, $\lambda_2 \geq 0$ respectively represent welfare weights on the cost of transfers incurred by the Principal and cost of production incurred by the agents. Applications include:

Application 1: Internal organization/procurement

The Principal is the owner of a firm composed of two divisions whose respective outputs combine to form revenues $V = V(q_1, q_2)$. If there is a single joint output, $Q = \{(q_1, q_2) | q_1 = q_2\}$. If the two divisions produce different inputs independently, $Q = R_+ \times R_+$, while the need to coordinate their activities arises if V is non-separable in q_1 and q_2. The principal seeks to maximize profit, hence $\lambda_1 = 1$ and $\lambda_2 = 0$. The same applies when the two agents correspond to external input suppliers.

Application 2: Environmental regulation

The Principal is an environmental regulator seeking to control abatements q_i of two firms $i = 1, 2$ with $Q = R_+ \times R_+$. $V(q_1+q_2)$ is the gross social benefit from total abatement, and θ_i is the firm i's unit cost for abatement activity. Consumer welfare equals $V - (1 + \lambda)F$ where F is the total tax revenue collected from consumers and λ is the deadweight loss involved in raising these taxes. The revenue is used to reimburse transfers t_1, t_2 to the firms. Social welfare equals the sum of consumer welfare and firm payoffs, which reduces to (6) with $\lambda_1 = \lambda, \lambda_2 = 1$. The same approach can be used for regulation of public utilities, where q_i denotes the output supplied by utility company i, and t_i is a payment made to company i from revenues raised from consumers or taxpayers.

Application 3: Allocating private goods

The Principal seeks to allocate a fixed quantity q of a private good. In this environment, θ_i is negative, with $-\theta_i$ representing agent i's valuation of one unit of product, and $-t_i$ the amount paid by agent i. $Q = \{(q_1, q_2) \in R_+ \times R_+ \mid q_1 + q_2 \leq q\}$. $V(q_1, q_2) = 0$, $\lambda_1 = 1$ and $\lambda_2 = 0$ represents the case where the Principal is solely concerned about revenue, while $V(q_1, q_2) = 0$, $\lambda_1 = 0$ and $\lambda_2 = 1$ represents the case where the Principal is concerned about efficiency. The auction of a single item corresponds to the case of $Q = \{(q_1, q_2) \in \{0, 1\} \times \{0, 1\} \mid q_1 + q_2 \leq 1\}$.

Application 4: Public good decisions

Here $q = q_1 = q_2$ represents the level of a public good, whose valuation by agent i is $-\theta_i > 0$. Here nonrivalry and nonexcludability implies $Q = \{(q_1, q_2) \mid q_1 = q_2\}$. $V(q, q) = -C(q)$ is interpreted as the cost of producing the public good. These costs are covered by taxes raised from consumers, which involve a deadweight loss of λ. Social welfare corresponds to (6) with $\lambda_1 = 0, \lambda_2 = \frac{1}{1+\lambda}$.

4 Communication and Contracting

4.1 Timing

The mechanism is designed by the principal at an ex-ante stage ($t = -1$). It consists of a *communication protocol* (explained further below) and a set of contracts to each agent. There is enough time between $t = -1$ and $t = 0$ for all agents to read and understand the offered contracts.

At $t = 0$, each agent i privately observes the realization of θ_i, and independently decides whether to participate or opt out of the mechanism. If either agent opts out the game ends; otherwise they enter the planning or communication phase which lasts until $t = T$.[9]

At $t = T$, each agent $i = 1, 2$ or the principal selects production level q_i, depending on whose choice variable q_i is (an issue discussed further below).

Finally, after production decisions have been made, payments are made according to the contracts signed at the *ex ante* stage, and verification by the Principal of messages exchanged by agents and outputs produced by them.

4.2 Communication Technology

Once both agents have agreed to participate, communication takes place in a number of successive rounds $t = 1, \ldots, T$. We consider the case where receiving (reading or listening) a message does not entail any time delay and any cost, as in verbal exchanges where speaking is time-consuming and the audience listens at the same time that the speaker speaks. This is the implicit assumption in most existing literature on costly communication.

Let m_i denote a message sent by i. For simplicity, we assume that m_i is received by agent j. Later we argue that the optimal allocation is implemented with this communication structure, i.e., it is not profitable to hide some messages from specific agents. Hence this restriction is without loss of generality, and enables us to simplify the exposition.

Given that agents exchange messages directly with one another (as well as sending these to the Principal), there is no rationale for the Principal to send any messages to the agents.

The *communication capacity* of any agent $i \in I \equiv \{1, 2\}$ is a message set $\mathcal{R}^i$, which contains all messages m_i that i can feasibly send in any given round of communication. Let $l(m_i)$ denote *length* of m_i, which is an integer. This represents the time taken by the sender to compose or send the message. If messages are binary-encoded, $l(m_i)$ is the number of 0-1 bits needed to communicate m_i. Communicating no message at all may also convey some information. If this is costless, it can be possible for agents to communicate some information costlessly. We exclude this possibility by assuming that every message in the message set has positive length, unless the message set is empty.

The following assumption imposes the fundamental limitation on communication, stating that the number of messages available to any agent not exceeding any given length is finite:

[9] Alternatively, the decision to participate can be represented as the initial round (round 0) of the communication phase.

Assumption 1 *For any $k < \infty$, there exists an integer $n < \infty$ such that $^{\#}\{m_i \in \mathcal{R}^i \mid l(m_i) < k\} < n$.*

In addition, we impose either one of the following three communication constraints. Let m_{it} be message sent by i in round t.

Communication Constraint 1

For any $i \in \{1, 2\}$, there exists $k_i < \infty$ such that a sequence of i's messages $(m_{i1}, m_{i2}, ..., m_{iT})$ is feasible if and only if it satisfies

$$\Sigma_{t=1}^{T} l(m_{it}) \leq k_i$$

and $m_{it} \in \mathcal{R}^i$ for any i and any t. This is a constraint on the aggregate length of messages sent by each agent.

Communication Constraint 2

There exists $k < \infty$ such that a sequence of messages $\{(m_{i1}, m_{i2}, ..., m_{iT})\}_{i\in\{1,2\}}$ is feasible if and only if it satisfies

$$\Sigma_{i\in\{1,2\}} \Sigma_{t=1}^{T} l(m_{it}) \leq k$$

and $m_{it} \in \mathcal{R}^i$ for any i and any t. This represents a constraint on the aggregate length of messages sent by all agents.

Communication Constraint 3

There exists $D < \infty$ such that a sequence of messages $\{(m_{i1}, m_{i2}, ..., m_{iT})\}_{i\in\{1,2\}}$ is feasible if and only if it satisfies

$$\Sigma_{t=1}^{T} \max\{l(m_{1t}), l(m_{2t})\} \leq D$$

and $m_{it} \in \mathcal{R}^i$ for any i and any t. Here the delay in any round is the maximal length of messages sent by any agent in that round. The total delay across different rounds is constrained.

4.3 Communication Protocol

Given a communication technology represented by either of the above constraints, the Principal can select a communication protocol, which is a rule defining T the number of rounds of communication, and in any given round the message set for any agent, which may depend on the history of messages exchanged in previous rounds. If some agents are not supposed to communicate anything in any round, their message sets are null in those rounds. This allows us to include protocols where agents take turns in sending messages in different rounds. Other protocols may involve simultaneous reporting by all agents in each round.

Message histories and message sets are defined recursively as follows. Let m_{it} denote a message sent by i in round t. Given a history h_{t-1} of messages exchanged (sent and received) by i until round $(t-1)$, it is updated at round t to include the messages exchanged at round t: $h_t = (h_{t-1}, \{m_{it}\}_{i\in I})$. And for every i, $h_0 = \emptyset$. The message set for i at round t is then a subset of $\mathcal{R}^i$ which depends on h_t, unless it is null.

Formally, the *communication protocol* specifies the number of rounds T, and for every round $t \in \{1, \ldots, T\}$ and every agent i, a message set $M_i(h_{t-1}) \subseteq \mathcal{R}^i$ or $M_i(h_{t-1}) = \emptyset$ for every possible history h_{t-1} until the end of the previous round.[10] It must satisfy the relevant constraint on the communication technology: the relevant inequality must be satisfied for any sequence of messages $\{(m_{i1}, m_{i2}, ..., m_{iT})\}_{i\in I}$ satisfying $m_{i1} \in M_i(h_0)$ and $m_{it+1} \in M_i(h_{t-1}, \{m_{jt}\}_{j\in I})$ for any i and t. Let a protocol be denoted by p, and the set of possible protocols given communication technology be denoted by $\mathcal{P}$.

4.4 Communication Plans and Strategies

Given a protocol $p \in \mathcal{P}$, a *communication plan* for agent i specifies for every round t a message $m_{it}(h_{t-1}) \in M_i(h_{t-1})$ for every possible history h_{t-1} that could arise for i in protocol p until round $t-1$. The set of communication plans for i in protocol p is denoted $C_i(p)$. For communication plan $c = (c_1, c_2) \in C(p) \equiv C_1(p) \times C_2(p)$, let $h_t(c)$ denote the history of messages generated thereby until the end of round t. Let $H_t(p) \equiv \{h_t(c) \mid c \in C(p)\}$ denote the set of possible message histories in this protocol until round t. For a given protocol, let $\mathcal{H} \equiv H_T(p)$ denote the set of possible histories at the end of round T. It is evident from our assumption about communication technology that the number of elements in $C_i(p)$ has an upper bound.[11]

[10] We depart from Fadel and Segal (2009) and Van Zandt (2007) insofar as their definition of a protocol combines the extensive form game of communication as well as the communication strategy of each agent.

[11] The proof that ${}^{\#}C_i(p)$ has an upper bound is sketched as follows. When a sequence of messages $\{(m_{i1}, ..., m_{iT})\}_{i\in I}$ satisfies communication constraint 1 and 3 in Section 4.2, it also satisfies 2. So it suffices to prove that ${}^{\#}C_i(p)$ has an upper bound for any p satisfying 2. Without loss of generality, we can focus our attention to p such that ${}^{\#}M_1(h_t) +^{\#} M_2(h_t) \geq 2$ for any $h_t \in \cup_{\tau=0}^{T-1} H_\tau$, since we can delete any round in which message sets for both agents are null. Then by our assumption that non-null message set does not include null message, $l(m_{1t}) + l(m_{2t}) \geq 1$ for any $(m_{1t}, m_{2t}) \in M_1(h_{t-1}) \times M_2(h_{t-1})$ and any $h_{t-1} \in \cup_{\tau=0}^{T-1} H_\tau$. Constraint 2 implies $T \leq \Sigma_{t=1}^{T}[l(m_{1t}) + l(m_{2t})] \leq k$ or T has an upper bound. Assumption 1 implies that there exists $A < \infty$ such that ${}^{\#}M_i(h_t) < A$ for any $h_t \in \cup_{\tau=0}^{T-1} H_\tau$ and any $i \in I$, since otherwise, $l(m_{it})$ does not have an upper bound for some $m_{it} \in M_i(h_t)$ and constraint 2

Given a protocol $p \in \mathcal{P}$, a *communication strategy* for agent i is a mapping $c_i(\theta_i) \in C_i(p)$ from the set $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ of types of i to the set $C_i(p)$ of possible communication plans for i. In other words, a communication strategy describes the dynamic plan for sending messages, for every possible type of the agent. The finiteness of the set of dynamic communication plans implies that it is not possible for others in the organization to infer the exact type of any agent from the messages exchanged. Perforce, non-negligible sets of types will be forced to pool into the same communication plan.

4.5 Production Decisions and Contracts

Many authors in previous literature (Blumrosen, Nisan and Segal (2007), Blumrosen and Feldman (2006) and Kos (2011a, 2011b)) have limited attention to mechanisms where output assignments and transfers are specified as a function of the information communicated by the agents. Decision-making authority is effectively retained by the Principal in this case. This is natural in settings involving auctions or public goods. We shall refer to such mechanisms as *centralized.* A *contract* in this setting specifies a quantity allocation $(q_1(h), q_2(h)) : \mathcal{H} \to Q$, with corresponding transfers $(t_1(h), t_2(h)) : \mathcal{H} \to \Re \times \Re$. A *centralized mechanism* is then a communication protocol $p \in \mathcal{P}$ and an associated contract $(q(h), t(h)) : \mathcal{H} \to Q \times \Re \times \Re$.

Some authors (Melumad, Mookherjee and Reichelstein (1992, 1997)) have explored mechanisms where the Principal delegates decision-making to one of the two agents, and compared their performance with centralized mechanisms. This is a pertinent question in procurement, internal organization or regulation contexts. They consider mechanisms where both contracting with the second agent as well as production decisions are decentralized (while restricting attention to communication protocols involving a single round of communication). Here we focus attention on mechanisms where the Principal retains control over the design of contracts with both agents, while decentralizing decision-making authority to agents concerning their own productions. This is feasible only if the production decisions of the two agents can be chosen independently, i.e., there are no technical complementarities or jointness restrictions on their outputs. We refer to such mechanisms as *decentralized.* The potential advantage of decentralizing production decisions to agents is that these decisions can then be based on information possessed by the agents which is richer than what they can communicate to the Principal. Transfers can then be based on output decisions as well as messages exchanged.

Formally a *decentralized mechanism* is a communication protocol p, a feasible output space $Q = \Re_+ \times \Re_+$, and a pair of contracts for the two agents, where the contract for

is violated. Then ${}^{\#}H_T$, which is the total number of terminal nodes, has an upper bound A^{2k}. Since ${}^{\#}H_t \leq {}^{\#} H_T$ for any $t \leq T-1$, H_t has an upper bound $B < \infty$ for any $t \leq T-1$. Then since ${}^{\#}C_i(p) = \Pi_{t=0}^{T-1} \Pi_{h_t \in H_t} [{}^{\#}M_i(h_t)]$ (with ${}^{\#}M_i(h_t) \equiv 1$ if $M_i(h_t)$ is null set), ${}^{\#}C_i(p) < A^{Bk} < \infty$.

agent i is a transfer rule $t_i(q_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re$. Such a mechanism induces a quantity allocation $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$ which maximizes $[t_i(q, h) - \theta_i q]$ with respect to choice of q.[12] To simplify exposition we specify the quantity allocation as part of the decentralized mechanism itself.

A centralized mechanism can then be viewed as a special case of a decentralized mechanism in which $q_i(\theta_i, h)$ is measurable with respect to h, i.e., does not depend on θ_i conditional on h. It corresponds to a mechanism in which the Principal sets an output target for each agent (based on the messages communicated) and then effectively forces them to meet these targets with a corresponding incentive scheme. We can therefore treat every mechanism as decentralized, in a formal sense.

In view of this, say that a mechanism is *truly decentralized* if it is not centralized. We shall in due course evaluate the relative merits of centralized and truly decentralized mechanisms.

4.6 Feasible Production Allocations

The standard way of analysing the mechanism design problem with unlimited communication is to first characterize production allocations that are feasible in combination with some set of transfers, and then use the Revenue Equivalence Theorem to represent the Principals objective in terms of the production allocation alone, while incorporating the cost of the supporting transfers. To extend this method we seek to characterize feasible production allocations.

A *production allocation* is a mapping $\tilde{q}(\theta) \equiv (q_1(\theta), q_2(\theta)) : \Theta_1 \times \Theta_2 \rightarrow Q$. Restrictions are imposed on production allocations owing both to communication and incentive problems.

Consider first communication restrictions. A production allocation $\tilde{q}(\theta)$ is said to be *communication-feasible* if: (a) the mechanism involves a communication protocol p satisfying the specified constraints on communication, and (b) there exist communication strategies $c(\theta) = (c_i(\theta_i), c_j(\theta_j)) \in C(p)$ and output decisions of agents $q_i(\theta_i, h) : \Theta_i \times \mathcal{H} \rightarrow \Re_+$, such that $\tilde{q}(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$. Here $h(c)$ denotes the message histories generated by the communication strategies c in this protocol.

The other set of constraints pertain to incentives. A communication-feasible production allocation $\tilde{q}(\theta)$ is said to be *incentive-feasible* in a mechanism if there exists a Perfect Bayesian Equilibrium (PBE) of the game induced by the mechanism which implements the production allocation. In other words, there must exist a set of communication

[12]Since i infers the other's output q_j $(j \neq i)$ only through h, we can restrict attention to contracts where the payments to any agent depends only on his own output without loss of generality. Specifically, if t_i were to depend on q_j, the expected value of the transfer to i can be expressed as a function of q_i and h, since agent i's information about q_j has to be conditioned on h.

strategies and output decision strategies satisfying condition (b) above in the requirement of communication-feasibility, which constitutes a PBE.

4.7 Characterization of Incentive Feasibility

We now seek to characterize incentive-feasible production allocations. Using the single-dimensional output of each agent and the single crossing property of agent preferences, we can obtain as a necessary condition a monotonicity property of expected outputs with respect to types at each decision node. To describe this condition, we need the following notation.

It is easily checked (see Lemma 1 in the Appendix) that given any strategy configuration $(c_1(\theta_i), c_2(\theta_2))$ and any history h_t until the end of round t in a communication protocol, the set of types (θ_1, θ_2) that could have generated the history h_t can be expressed as the Cartesian product of subsets $\Theta_1(h_t), \Theta_2(h_t)$ such that

$$\{(\theta_1, \theta_2) \mid h_t(c(\theta_1, \theta_2)) = h_t\} = \Theta_i(h_t) \times \Theta_j(h_t). \tag{7}$$

A necessary condition for incentive-feasibility of a production allocation $q(\theta)$ which is communication-feasible in a protocol p and supported by communication strategies $c(\theta)$ is that for any $t = 1, \ldots, T$, any $h_t \in H_t$ and any $i = 1, 2$:

$$E[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \text{ is non-increasing in } \theta_i \text{ on } \Theta_i(h_t), \tag{8}$$

where H_t denotes the set of possible histories until round t generated with positive probability in the protocol when $c(\theta)$ is played, and $\Theta_i(h_t)$ denotes the set of types of i who arrive at h_t with positive probability under the communication strategies $c(\theta)$.

The necessity of this condition follows straightforwardly from the dynamic incentive constraints which must be satisfied for any history h_t on the equilibrium path. Upon observing h_t, i's beliefs about θ_j are updated by conditioning on the event that $\theta_j \in \Theta_j(h_t)$. Any type of agent i in $\Theta_i(h_t)$ will have chosen the same messages up to round t. Hence any type $\theta_i \in \Theta_i(h_t)$ has the opportunity to pretend to be any other type in $\Theta_i(h_t)$ from round $t+1$ onward, without this deviation being discovered by anyone. A PBE requires that such a deviation cannot be profitable. The single-crossing property then implies condition (8).

As noted earlier, the existing literature has provided sufficient conditions for incentive-feasibility that are considerably stronger than (8). Fadel and Segal (2009) in a more general framework (with abstract decision spaces and no restrictions on preferences) provide two sets of sufficient conditions. One set (provided in their Proposition 6) of conditions is based on the observation that the stronger solution concept of ex post incentive compatibility implies Bayesian incentive compatibility. In our current context ex post incentive compatibility is assured by the condition that for each $i = 1, 2$:

$$q_i(\theta_i, \theta_j) \text{ is globally non-increasing in } \theta_i \text{ for every } \theta_j \in \Theta_j \tag{9}$$

Another set of sufficient conditions (Proposition 3 in Fadel-Segal (2009)) imposes a no-regret property with respect to possible deviations to communication strategies chosen by other types following every possible message history arising with positive probability under the recommended communication strategies. This is applied to every pair of types for each agent at nodes where it is this agent's turn to send a message. In the context of centralized mechanism (which Fadel and Segal restrict attention to), this reduces to the condition that for any any $i = 1, 2$ and any $h_t \in H_t, t = 1, \ldots, T$ where it is i's turn to move (i.e., $M_i(h_t) \neq \emptyset$):[13]

$$E[q_i(\theta_i, \theta_j)|\theta_j \in \Theta_j(h_t)] \text{ is globally non-increasing in } \theta_i. \tag{10}$$

Our first main result is that the necessary condition (8) is also sufficient for incentive feasibility, provided the communication protocol prunes unused messages. Suppose that p is a communication protocol in which communication strategies used are $c(\theta)$. Then p is *parsimonious relative to communication strategies* $c(\theta)$ if every possible history $h \in H$ in this protocol is reached with positive probability under $c(\theta)$.

Proposition 1 *Condition (8) is sufficient for incentive-feasibility of a production allocation $q(\theta)$ which is communication-feasible in a protocol p and supported by communication strategies $c(\theta)$, provided the protocol is parsimonious with respect to $c(\theta)$.*

Any protocol can be pruned by deleting unused messages under any given set of communication strategies, to yield a protocol which is parsimonious with respect to these strategies. Hence it follows that condition (8) is both necessary and sufficient for incentive-feasibility.

The proof of Proposition 1 is provided in the Appendix. The main complication arises for the following reason. In a dynamic protocol with more than one round of communication, no argument is available for showing that attention can be confined to communication strategies with a threshold property. Hence the set of types $\Theta(h_t)$ pooling into message history h_t need not constitute an interval. The monotonicity property for output decisions in (8) holds only 'within' $\Theta(h_t)$, which may span two distinct intervals. The monotonicity property may therefore not hold for type ranges lying between the two

[13] As Fadel and Segal point out, it suffices to check the following condition at the last node of the communication game at which it is agent i's turn to move.

intervals. This complicates the conventional argument for construction of transfers that incentivize a given output allocation.

The proof is constructive. Given a production allocation satisfying (8) with respect to set of communication strategies in a protocol, we first prune the protocol to eliminate unused messages. Then incentivizing transfers are constructed as follows. We first construct a set of functions representing expected outputs of each agent following any given history h_t at any stage t, as a function of the type of that agent. Condition (8) ensures the expected output of any agent i is monotone over the set $\Theta_i(h_t)$. These are the types of i that actually arrive at h_t with positive probability on the equilibrium path. The proof shows it is possible to extend this function over all types of this agent (not just those that arrive at h_t on the equilibrium path) which is globally monotone, in a way that agrees with the actual expected outputs on the set $\Theta_i(h_t)$, and which maintains consistency across histories reached at successive dates. This amounts to assigning outputs for types that do not reach h_t on the equilibrium path, which can be thought of as outputs they would be assigned if they were to deviate somewhere in the game and arrive at h_t. Since this extended function is globally monotone, transfers can be constructed in the usual way to incentivize this allocation of expected output. The construction also has the feature that the messages sent by the agent after arriving at h_t do not affect the expected outputs that would thereafter be assigned to the agent, which assures that the agent does not have an incentive to deviate from the recommended communication strategy.

Consider the following example which illustrates the construction of transfers that incentivize an allocation satisfying the necessary condition (8). Agent i's cost is uniformly distributed over $[0,1]$. There are three rounds of communication. In round 1, agent 1 reports $m_{11} \in \{L, R\}$. In round 2, agent 2 reports $m_{22} \in \{U, D\}$. In round 3, agent 1 reports $m_{13} \in \{0, 1\}$. The mechanism is decentralized, with each agent choosing their respective outputs at the end of round 3. The recommended communication strategies (on the equilibrium path) are the following. In round 1, agent 1 reports $m_{11} = L$ if $\theta_1 \in [0, 1/3] \cup [2/3, 1]$, and R otherwise. In round 2, agent 2 reports $m_{22} = U$ if $\theta_2 \in [1/2, 1]$ and D otherwise. In round 3, agent 1 has a null message set if he reported R in round 1. If he reported L in round 1, in round 3 he reports $m_{13} = 0$ if $\theta_1 \in [0, 1/3]$ and $m_{13} = 1$ if $\theta_1 \in [2/3, 1]$.

We focus on constructing transfers for agent 1 so as to induce this agent to follow the recommended strategy, while assuming that agent 2 follows his. Hence we check only the reporting incentives for agent 1 in rounds 1 and 3. These depend on outputs that agent 1 is expected to select at the end of round 3, as a function of messages sent in the first three rounds, besides the true type of agent 1. These outputs are represented by functions $l(\theta_1, m_{22})$ and $r(\theta, m_{22})$ corresponding to first round announcements of L and R respectively, and are shown in Figure 5.

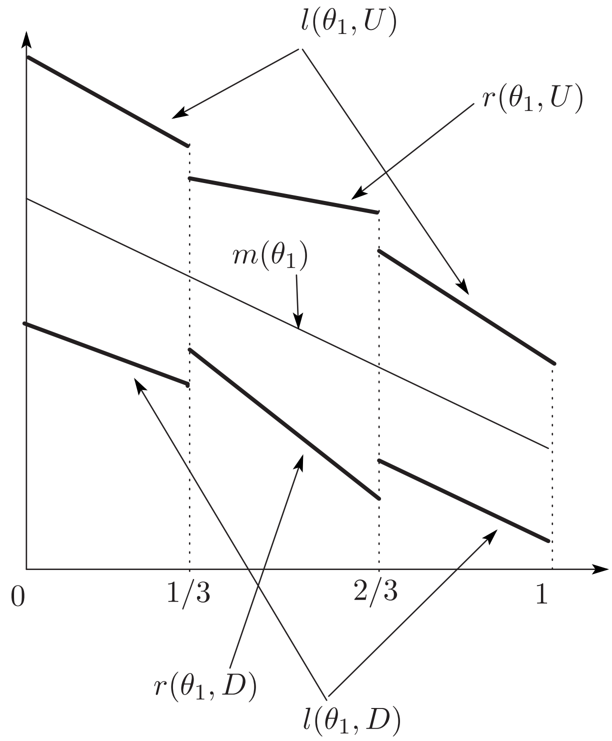

Figure 5: Outputs for Agent 1

Define first-round expected outputs $m(\theta_1)$ as follows:

$$m(\theta_1) \equiv (1/2)l(\theta_1, D) + (1/2)l(\theta_1, U)$$

for $\theta_1 \in [0, 1/3] \cup [2/3, 1]$, and

$$m(\theta_1) \equiv (1/2)r(\theta_1, D) + (1/2)r(\theta_1, U)$$

for $\theta_1 \in [1/3, 2/3]$. This is also shown in Figure 5.

Then the necessary condition for incentive feasibility is that the following monotonicity properties of first-round and third-round expected outputs of agent 1 are satisfied:

- $m(\theta_1)$ is non-increasing in $\theta_1 \in [0, 1]$
- $l(\theta_1, D)$ and $l(\theta_1, U)$ are non-increasing in θ_1 on $[0, 1/3] \cup [2/3, 1]$
- $r(\theta_1, D)$ and $r(\theta_1, U)$ are non-increasing in θ_1 on $[1/3, 2/3]$

As is evident from Figure 5, these are not globally monotone in θ_1, given messages sent by Agent 2.

As a first step in our construction, we extend the third round output functions l, r to $\tilde{l}, \tilde{r}$ to the entire type space $[0, 1]$ as shown in Figures 6, 7 respectively. This can be done to maintain the following two properties: (a) the extended functions are globally monotone in θ_1, and (b) their average equals $m(\theta_1)$ for all θ_1. These can be interpreted as outputs corresponding to off-equilibrium messages.

Let $q^*(\theta_1, h_3)$ denote the round-3 output of agent 1 corresponding to type θ_1 and message vector h_3 consisting of messages sent in the first three rounds, based on these extended functions. The function q^* is well-defined for all θ_1 and all possible message histories till round 3, both on and off the equilibrium path. Now define transfers to agent 1 as a function of the round-3 expected output function as follows:

$$t(q^*(\theta_1, h_3), h_3) = \theta_1 q^*(\theta_1, h_3) + \int_{\theta_1}^{1} q^*(y, h_3)dy \tag{11}$$

This ensures that every type θ_1 will optimally select expected output of $q^*(\theta_1, h_3)$ at the end of the communication phase following any given history h_3, both on and off the equilibrium path. Moreover, by construction, after conditioning on the output q^* that agent 1 expects to choose at the end of round 3, its transfer does not depend on the message m_{13} sent in that round. Hence no type of Agent 1 has an incentive to deviate from the recommended communication strategy on the equilibrium path in round 3.

Now consider reporting incentives in round 1. At this stage Agent 1 does not yet know the message to be sent by Agent 2 in round 2. By construction of the extended output

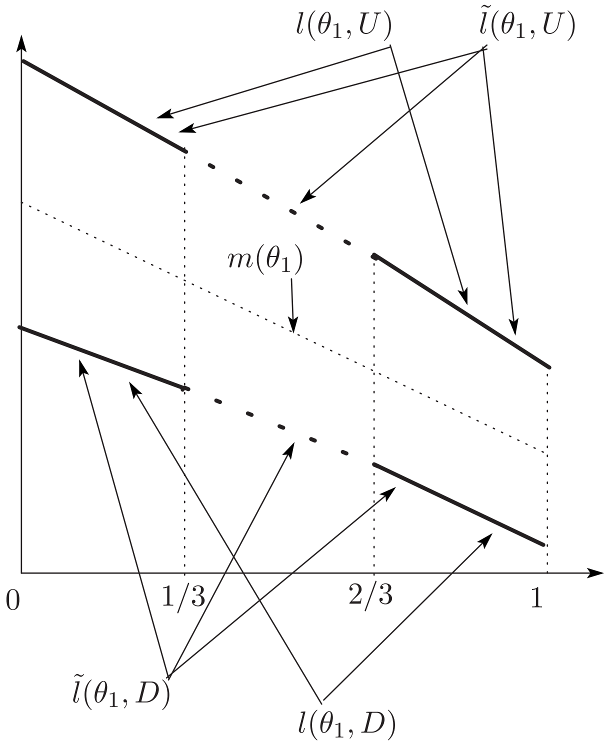

Figure 6: Construction of $\tilde{l}$

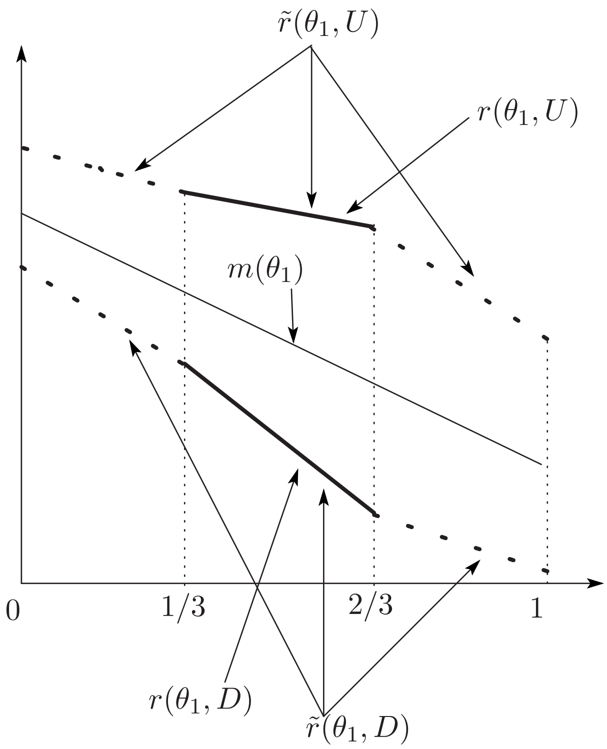

Figure 7: Construction of $\tilde{r}$

functions, the expected output for an agent with true type θ_1 is $m(\theta_1)$, irrespective of whether or not it deviates from the recommended reporting strategy in round 1. Its expected transfer in round 1 therefore do not depend on the messages it sends in round 1, implying it has no incentive to deviate in round 1 either.

Note finally that taking expectations of (11) with respect to types of Agent 2 combined with the equilibrium reporting strategies, the ex ante expected transfers paid by the Principal to Agent 1 depends only on the first round expected output function $m(\theta_1)$, i.e., do not depend on the particular way in which the round 3 output functions were extended.

5 Characterizing Optimal Mechanisms

Having characterized feasible allocations, we can now restate the mechanism design problem as follows.

Note to start with that the interim participation constraints imply that every type of each agent must earn a non-negative expected payoff from participating. Agents that do not participate do not produce anything or receive any transfers. Hence by the usual logic it is without loss of generality that all types participate in the mechanism. The single crossing property ensures that expected payoffs are nonincreasing in θ_i for each agent i. Since $\lambda_1 \geq 0$ it is optimal to set transfers that incentivize any given output allocation rule $q(\theta)$ satisfying (8) such that the expected payoff of the highest cost type $\bar{\theta}_i$ equals zero for each i. The expected transfers to the agents then equal (using the arguments in Myerson (1981) to establish the Revenue Equivalence Theorem):

$$\Sigma_{i=1}^2 E[v_i(\theta_i) q_i(\theta_i, \theta_j)]$$

where $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$. Consequently the expected payoff of the Principal is

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i) q_i(\theta_i, \theta_j) - w_j(\theta_j) q_j(\theta_i, \theta_j)] \tag{12}$$

where $w_i(\theta_i) \equiv (\lambda_1 + \lambda_2)\theta_i + \lambda_1 \frac{F_i(\theta_i)}{f_i(\theta_i)}$.

This enables us to state the problem in terms of selecting an output allocation in combination with communication protocol and communication strategies. Given the set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, the problem is to select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p and output allocation $q(\theta)$ to maximize (12), subject to the constraint that (i) there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that $q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta))))$ for all $\theta \in \Theta$, and (ii) the output allocation satisfies condition (8).

Condition (i) is essentially a constraint of communication-feasibility, which applies even in the absence of incentive problems. Condition (ii) is the additional constraint represented by incentive problems. Note that the above statement of the problem applies since attention can be confined without loss of generality to protocols that are parsimonious with respect to the assigned communication strategies. To elaborate, note that conditions (i) and (ii) are both necessary for implementation. Conversely, given an output allocation, a communication protocol, and communication strategies in the protocol that satisfy conditions (i) and (ii), we can prune that protocol by deleting unused messages to obtain a protocol that is parsimonious with respect to the given communication strategies. Then Proposition 1 ensures that the output allocation can be implemented as a PBE in the pruned protocol with suitably constructed transfers, which generate an expected payoff (12) for the Principal while ensuring all types of both agents have an incentive to participate.

Now observe that constraint (ii) is redundant in this statement of the problem. If we consider the relaxed version of the problem stated above where (ii) is dropped, the solution to that problem must automatically satisfy (ii), since the monotone hazard rate property on the type distributions F_i ensure that $w_i(\theta_i)$ is an increasing function for each i. This generates our main result.

Proposition 2 *The mechanism design problem can be reduced to the following. Given the set $\mathcal{P}$ of feasible communication protocols defined by the communication constraints, select a protocol $p \in \mathcal{P}$, communication strategies $c(\theta)$ in p and output allocation $q(\theta)$ to maximize (12), subject to the constraint of communication feasibility alone, i.e., there exists a set of output decision strategies $q_i(\theta_i, h), i = 1, 2$ such that*

$$q(\theta) = (q_1(\theta_1, h(c(\theta))), q_2(\theta_2, h(c(\theta)))), \forall \theta \in \Theta. \tag{13}$$

In the case of unlimited communication, this reduces to the familiar property that an optimal output allocation can be computed on the basis of unconstrained maximization of expected payoffs (12) of the Principal which incorporate incentive rents earned by the agents. With limited communication additional constraints pertaining to communication feasibility have to be incorporated. In the absence of incentive problems, the same constraint would apply: the only difference would be that the agents would not earn incentive rents and the objective function of the Principal would be different (w_i would be replaced by $\tilde{w}_i = (\lambda_1 + \lambda_2)\theta_i$).

Proposition 2 neatly separates costs imposed by incentive considerations, from those imposed by communicational constraints. The former is represented by the replacement of production costs of the agents by their incentive-rent-inclusive virtual costs in the objective function of the Principal, in exactly the same way as in a world with costless, unlimited communication. The costs imposed by communicational constraints are represented by the restriction of the feasible set of output allocations, which must now vary more coarsely with the type realizations of the agents. This can be viewed as the natural extension of Marschak-Radner (1972) characterization of optimal team decision problems to a setting with incentive problems. In particular, the same computational techniques can be used to solve these problems both with and without incentive problems: only the form of the objective function needs to be modified to replace actual production costs by virtual costs. The 'desired' communicational strategies can be rendered incentive compatible at zero additional cost.

Van Zandt (2007) and Fadel and Segal (2009) discuss the question of 'communication cost of selfishness', which pertains to the related notion of separation between incentive and communicational complexity issues. In their context they take an arbitrary social choice function (allocation in our notation) and examine whether the communicational complexity of implementing it is increased by the presence of incentive constraints. In our context we fix communicational complexity and select an allocation to maximize the Principal's expected payoff (the exact representation of which depends on whether or not incentive problems are present). Van Zandt shows that the separation property applies quite generally when we use the solution concept of ex post incentive compatibility. This also turned out to be true for Bayesian implementation for a class of problems involving one round limited communication protocols, one dimensional outputs and single-crossing preferences (Melumad, Mookherjee and Reichelstein (1992, 1997), Blumrosen and Feldman (2006), Blumrosen, Nisan and Segal (2007)). Proposition 2 shows this result extends to arbitrary dynamic communication protocols, provided we continue to operate in a world with one dimensional outputs and single crossing preferences for every agent.

6 Implications for Decentralization versus Centralization of Production Decisions

We now examine implications of Proposition 2 for the value of truly decentralized mechanisms compared with centralized ones. Consider a setting where there is no technological restriction on the relation between outputs of the two agents, i.e., $Q = \Re_+ \times \Re_+$, so their outputs can be chosen independently. If production decisions are made by the Principal, outputs are measurable with respect to the history of exchanged messages. If production decisions are delegated to the agents, this is no longer true, since they can be decided by the agents on the basis of information about their own true types, which is richer than what they managed to communicate to the Principal. Unlike settings of unlimited communication, centralized mechanisms can no longer replicate the outcomes of decentralized ones. Contracts are endogenously incomplete, thus permitting a nontrivial comparison of centralized and decentralization decision rights.

The typical tradeoff associated with delegation of decision rights to better informed agents compares the benefit of increased flexibility of decisions with respect to the true state of the world, with the cost of possible use of discretion by the agent to increase his own rents at the expense of the Principal. Proposition 2 however shows that once the incentive rents that agents will inevitably earn have been factored in the Principal's objective, incentive considerations can be ignored. The added flexibility that delegation allows then ensures that delegation is the superior arrangement. We show below that delegation strictly outperforms centralization provided the gross benefit function of the Principal is smooth, strictly concave and satisfies Inada conditions.

Proposition 3 *Suppose that (i) outputs of the two agents can be chosen independently ($Q = \Re_+ \times \Re_+$); and (ii) $V(q_1, q_2)$ is twice continuously differentiable, strictly concave and satisfies the Inada condition $\frac{\partial V}{\partial q_i} \to \infty$ as $q_i \to 0$. Then given any feasible centralized mechanism there exists a corresponding truly decentralized mechanism which generates a strictly higher payoff to the Principal.*

The outline of the argument is as follows. The finiteness of the set of feasible communication plans for every agent implies the existence of non-negligible type intervals over which communication strategies and message histories are pooled. Consequently if decisions are centralized, the production decision for i must be pooled in the same way. Instead if production decisions are left to agent i, the production decision can be based on the agent's knowledge of its own true type. Under the assumptions of Proposition 3 which ensure that the optimal outputs are always interior, this added 'flexibility' will allow an increase in the Principal's objective (12) while preserving communication feasibility. The result then follows upon using Proposition 2.

7 Implications for Choice of Communication Protocol

Proposition 2 also has useful implications for the ranking of different communication protocols. Given any set of communication strategies in a given protocol, in state (θ_i, θ_j) agent i learns that θ_j lies in the set $\Theta_j(h(c_i(\theta_i), c_j(\theta_j)))$, which generates an information partition for agent i over agent j's type.

Say that a protocol $p_1 \in \mathcal{P}$ is *more informative* than another $p_2 \in \mathcal{P}$ if for any set of communication strategies in the former, there exists a set of communication strategies in the latter which yields (at round T) an information partition to each agent over the type of the other agent which is more informative in the Blackwell sense in (almost) all states of the world.

It then follows that a more informative communication protocol permits a wider choice of communication feasible output allocations. Hence Proposition 2 implies that the Principal prefers more informative protocols. She has no interest in blocking the flow of communication among agents.

This is the reason we have assumed that all messages are addressed to everyone else in the organization. If the transmission and processing of messages entail no resource or time costs, this ensures maximal flow of information between agents. In contrast much of the literature on informational efficiency of resource allocation mechanisms (in the tradition of Hurwicz (1960, 1972) or Mount and Reiter (1974)) has focused exclusively on centralized communication protocols where agents send messages to the Principal rather than one another. Such protocols restrict the flow of information among agents, presumably in order to economize on costs of transmitting communication or of processing of information. Our approach is based instead on the notion that the main cost of communication involves the writing of messages.

Maximizing the flow of information among agents has implications for the nature of optimal protocols. These will depend on the precise nature of the communication constraint. The following result describes implications of the three different kinds of constraints described in Section 4. We assume that messages are sent in 0-1 bits.

Proposition 4 *(i) Suppose either Communication Constraint 1 or 2 applies. Then an optimal protocol has the feature that only one agent sends messages in any given communication round.*

(ii) Suppose Communication Constraint 3 applies, limiting the total delay of the communication phase to the time taken by any single agent to send D bits. Then the optimal protocol involves D rounds of communication with both agents simultaneously sending one bit of information in each round.

These results follow from the objective of maximizing the amount of information communicated within the allowed constraints. If the constraint is on the total number of messages sent by either or both agents, an optimal protocol features message sending by a single agent in any given round. Simultaneous reporting is not optimal, as it is dominated by a sequential reporting protocol in which one of the agents waits to hear messages sent by the other before sending his own. We have not able to establish any general result concerning the number of communication rounds in this case.

When the constraint instead applies to the total delay of the communication phase, the nature of optimal protocols is quite different. Now sequential reporting is not optimal. Any agent who is silent in any given round could send some messages in that round without adding to the total delay of the mechanism. Indeed, in any given round both agents must send the same number of messages. Moreover, it is optimal for them both to send one bit of information in any round. Otherwise if they both send k bits of information in a single round, they can instead send 1 bit each in k rounds, which will increase the amount of information exchanged without adding to the total delay. Hence an optimal protocol must involve exactly D rounds of information with one bit sent per round by both agents.

8 Concluding Comments

Our theory can be extended in a number of directions.

One is to allow for a larger range of communication networks, where agents may selectively send messages to others. While the costs of transmitting messages to multiple receivers may be trivial with contemporary information technology, the costs of processing information by receivers may be substantial. Our approach is based on the notion that the costs of information only incorporate the costs of writing messages. Incorporating information processing limitations will be an important step in future research.

We also ignored the possibility of delegating the responsibility of contracting with other agents to some key agents. A broader concern is that we ignored the communicational requirements involved in contracting itself, by focusing only on communication in the process of implementation of the contract, which takes place after parties have negotiated and accepted a contract. Under the assumption that pre-contracting communication is costless, and messages exchanged between agents are verifiable by the Principle, it can be shown that delegation of contracting cannot dominate centralized contracting if both are equally constrained in terms of communicational requirements. Subcontracting may thus be potentially valuable in the presence of costs of pre-contract communication, or if agents can directly communicate with one another in a richer way than how they communicate with the Principal. Exploring such extensions is an interesting question for future research.

Appendix: Proofs

Lemma 1 *Consider any communication protocol $p \in \mathcal{P}$. For any $h_t \in H_t(p)$:*

$$\{c \in C(p) \mid h_t(c) = h_t\}$$

is a rectangle set in the sense that if $h_t(c_i, c_{-i}) = h_t(c_i^{'}, c_{-i}^{'}) = h_t$ for $(c_i, c_{-i}) \neq (c_i^{'}, c_{-i}^{'})$, then

$$h_t(c_i^{'}, c_{-i}) = h_t(c_i, c_{-i}^{'}) = h_t$$

Proof of Lemma 1: The proof is by induction. Note that $h_0(c) = \phi$ for any c, so it is true at $t = 0$. Suppose the result is true for all dates up to $t - 1$, we shall show it is true at t.

Note that

$$h_t(c_i, c_{-i}) = h_t(c_i^{'}, c_{-i}^{'}) = h_t \tag{14}$$

implies

$$h_\tau(c_i, c_{-i}) = h_\tau(c_i^{'}, c_{-i}^{'}) = h_\tau \tag{15}$$

for any $\tau \in \{0, 1, .., t-1\}$. Since the result is true until $t - 1$, we also have

$$h_\tau(c_i^{'}, c_{-i}) = h_\tau(c_i, c_{-i}^{'}) = h_\tau \tag{16}$$

for all $\tau \leq t - 1$. So under any of the configurations of communication plans (c_i, c_{-i}), $(c_i^{'}, c_{-i}^{'})$, $(c_i^{'}, c_{-i})$ or $(c_i, c_{-i}^{'})$, member i experiences the same message history h_{t-1} until $t - 1$. Then i has the same message space at t, and (14) implies that i sends the same messages to others at t, under either c_i or $c_i^{'}$.

(15) and (16) also imply that under either c_{-i} or $c_{-i}^{'}$, others send the same messages to i at all dates until $t-1$, following receipt on the (common) messages sent by i until $t-1$ under these different configurations. The result now follows from the fact that messages sent by others to i depend on the communication plan of i only via the messages they receive from i. So i must also receive the same messages at t under any of these different configurations of communication plans. ■

Proof of Proposition 1:

Let $q_i(\theta_i, \theta_j)$ be a production allocation satisfying (8), which is supported by a communication strategy vector $\tilde{c}(\theta)$ in a protocol $\tilde{p}$ which is parsimonious with respect to these strategies. In this protocol all histories are reached with positive probability on the equilibrium path, hence beliefs of every agent with regard to the types of the other agent are obtained by applying Bayes rule.

Define $\hat{q}_i(\theta_i, h_t)$ by

$$\hat{q}_i(\theta_i, h_t) \equiv E[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)].$$

for any $h_t \in H_t$ and any t. Condition (8) requires $\hat{q}_i(\theta_i, h_t)$ to be non-increasing in θ_i on $\Theta_i(h_t)$. Note that

$$\hat{q}_i(\theta_i, h(\tilde{c}(\theta_i, \theta_j))) = E_{\tilde{\theta}_j}[q_i(\theta_i, \tilde{\theta}_j) \mid \tilde{\theta}_j \in \Theta_j(h(\tilde{c}(\theta_i, \theta_j)))] = q_i(\theta_i, \theta_j),$$

since $q_i(\theta_i, \tilde{\theta}_j) = q_i(\theta_i, \theta_j)$ for any $\tilde{\theta}_j \in \Theta_j(h(\tilde{c}(\theta_i, \theta_j)))$.

Step 1: The relationship between $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$

Suppose that i observes h_t at round t. Given selection of $m_{i,t+1} \in \tilde{M}_i(h_t)$ where $\tilde{M}_i(h_t)$ is the message set for h_t in protocol $\tilde{p}$, agent i's history at round $t+1$ is subsequently determined by messages received by i in round t. Let the set of possible histories h_{t+1} at $t+1$ be denoted by $H_{t+1}(h_t, m_{i,t+1})$. Evidently for $j \neq i$, $\{\Theta_j(h_{t+1}) | h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})\}$ constitutes a partition of $\Theta_j(h_t)$:

$$\cup_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Theta_j(h_{t+1}) = \Theta_j(h_t)$$

and

$$\Theta_j(h_{t+1}) \cap \Theta_j(h'_{t+1}) \neq \phi$$

for $h_{t+1}, h'_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ such that $h_{t+1} \neq h'_{t+1}$. The probability of $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ conditional on $(h_t, m_{i,t+1})$ is represented by

$$\Pr(h_{t+1} \mid h_t, m_{i,t+1}) = \Pr(\Theta_j(h_{t+1})) / \Pr(\Theta_j(h_t)).$$

From the definition of $\hat{q}_i(\theta_i, h_t)$ and $\hat{q}_i(\theta_i, h_{t+1})$, for any $m_{i,t+1} \in \tilde{M}_i(h_t)$ and any $\theta_i \in \Theta_i$,

$$\Sigma_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} \mid h_t, m_{i,t+1}) \hat{q}_i(\theta_i, h_{t+1}) = \hat{q}_i(\theta_i, h_t).$$

Step 2: For any $h_{t+1}, h'_{t+1} \in H_{t+1}(h_t, m_{it+1})$, $\Theta_i(h_{t+1}) = \Theta_i(h'_{t+1}) \subset \Theta_i(h_t)$

By definition

$$\Theta_i(h_{t+1}) = \{\theta_i \mid \tilde{m}_{i,t+1}(\theta_i, h_t) = m_{i,t+1}\} \cap \Theta_{it}(h_t)$$

where $\tilde{m}_{i,t+1}(\theta_i, h_t)$ denotes i's message choice corresponding to the strategy $\tilde{c}_i(\theta_i)$. The right hand side depends only on $m_{i,t+1}$ and h_t. It implies that the set $\Theta_i(h_{t+1})$ does not vary across different $h_{t+1} \in H_{t+1}(h_t, m_{it+1})$. To simplify exposition, we denote this set henceforth by $\Theta_i(h_t, m_{it+1})$.

Step 3: Construction of $\tilde{q}_i(\theta_i, h_t)$

We construct $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ based on the following Claim 1.

Claim 1:

For arbitrary $q_i(\theta_i, \theta_j)$ satisfying (8), there exists $\tilde{q}_i(\theta_i, h_t)$ for any $h_t \in H_t$ and any t so that

(a) $\tilde{q}_i(\theta_i, h_t) = \hat{q}_i(\theta_i, h_t)$ for $\theta_i \in \Theta_i(h_t)$

(b) $\tilde{q}_i(\theta_i, h_t)$ is non-increasing in θ_i on Θ_i

(c) $\Sigma_{h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})} \Pr(h_{t+1} \mid h_t, m_{i,t+1}) \tilde{q}_i(\theta_i, h_{t+1}) = \tilde{q}_i(\theta_i, h_t)$ for any $\theta_i \in \Theta_i$ and any $m_{i,t+1} \in \tilde{M}_i(h_t)$ where $\tilde{M}_i(h_t)$ is the message set for h_t in protocol $\tilde{p}$.

Claim 1 states that there exists an 'auxiliary' output rule $\tilde{q}_i$ as a function of type θ_i and message history which is globally non-increasing in type (property (b)) following any history h_t, and $\tilde{q}_i(\theta_i, h_t)$ equals the expected value of $\tilde{q}_i(\theta_i, h_{t+1})$ conditional on (h_t, m_{it+1}) for any $m_{it+1} \in \tilde{M}_i(h_t)$ (property (c)).
In order to establish Claim 1, the following Lemma is needed.

Lemma 2 *For any $B \subset R_+$ which may not be connected, let A be an interval satisfying $B \subset A$. Suppose that $F_i(a)$ for $i = 1, ..., N$ and $G(a)$ are real-valued functions defined on A, each of which has the following properties:*

- *$F_i(a)$ is non-increasing in a on B for any i.*
- *$\Sigma_i p_i F_i(a) = G(a)$ for any $a \in B$ and for some p_i so that $p_i > 0$ and $\Sigma_i p_i = 1$.*
- *$G(a)$ is non-increasing in a on A.*

Then we can construct real-valued function $\bar{F}_i(a)$ defined on A for any i so that

- *$\bar{F}_i(a) = F_i(a)$ on $a \in B$ for any i.*
- *$\Sigma_i p_i \bar{F}_i(a) = G(a)$ for any $a \in A$ and for the same p_i*
- *$\bar{F}_i(a)$ is non-increasing in a on A for any i.*

This lemma says that we can construct functions $\bar{F}_i(a)$ so that the properties of functions $F_i(a)$ on B are also maintained on the interval A which covers B.

Proof of Lemma 2:

If this statement is true for $N = 2$, we can easily show that this also holds for any $N \geq 2$. Suppose that this is true for $N = 2$.

$$\Sigma_{i=1}^{N} p_i F_i(a) = p_1 F_1(a) + (p_2 + ... + p_N) F^{-1}(a)$$

with

$$F^{-1}(a) = \Sigma_{i \neq 1} \frac{p_i}{p_2 + ... + p_N} F_i(a).$$

Applying this statement for $N = 2$, we can construct $\bar{F}_1(a)$ and $\bar{F}^{-1}(a)$ which keeps the same property on A as on B. Next using the constructed $\bar{F}^{-1}(a)$ instead of $G(a)$, we can apply the statement for $N = 2$ again to construct desirable $\bar{F}_2(a)$ and $\bar{F}^{-2}(a)$ on A based on $F_2(a)$ and $F^{-2}(a)$ which satisfy

$$\frac{p_2}{p_2 + ... + p_N} F_2(a) + [1 - \frac{p_2}{p_2 + ... + p_N}] F^{-2}(a) = F^{-1}(a).$$

on B. We can use this method recursively to construct $\bar{F}_i(a)$ for all i.

Next let us show that the statement is true for $N = 2$. For $a \in A \backslash B$, define $\underline{a}(a)$ and $\bar{a}(a)$, if they exist, so that

$$\underline{a}(a) \equiv \sup\{a' \in B \mid a' < a\}$$

and

$$\bar{a}(a) \equiv \inf\{a' \in B \mid a' > a\}.$$

It is obvious that at least one of either $\underline{a}(a)$ or $\bar{a}(a)$ exists for any $a \in A \backslash B$.

Let's specify $\bar{F}_1(a)$ and $\bar{F}_2(a)$ so that $\bar{F}_1(a) = F_1(a)$ and $\bar{F}_2(a) = F_2(a)$ for $a \in B$, and for $a \in A \backslash B$ as follows.

(i) For $a \in A \backslash B$ so that only $\underline{a}(a)$ exists,

$$\bar{F}_1(a) = F_1(\underline{a}(a))$$

$$\bar{F}_2(a) = \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}$$

(ii)For $a \in A \backslash B$ so that both $\underline{a}(a)$ and $\bar{a}(a)$ exist,

$$\bar{F}_1(a) = \min\{F_1(\underline{a}(a)), \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}\}$$

$$\bar{F}_2(a) = \max\{F_2(\bar{a}(a)), \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}\}$$

(iii)For $a \in A\backslash B$ so that only $\bar{a}(a)$ exists,

$$\bar{F}_1(a) = \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}$$

$$\bar{F}_2(a) = F_2(\bar{a}(a))$$

It is easy to check that $\bar{F}_i(a)$ is non-increasing in a on A for $i = 1, 2$ and

$$p_1 \bar{F}_1(a) + p_2 \bar{F}_2(a) = G(a)$$

for $a \in A$. This completes the proof of the lemma. ■

Proof of Claim 1:

Choose arbitrary $t \in \{1, ..., T\}$ and $h_t \in H_t$. Suppose that $\tilde{q}_i(\theta_i, h_t)$ satisfies (a) and (b) in Claim 1. Then for any $m_{i,t+1} \in \tilde{M}_i(h_t)$, we can construct a function $\tilde{q}_i(\theta_i, h_{t+1})$ for any $h_{t+1} \in H_t(h_t, m_{it+1})$ so that (a), (b) and (c) are satisfied. This result is obtained upon applying Lemma 2 with

$$B = \Theta_i(h_t, m_{i,t+1})$$

$$A = \Theta_i$$

$$a = \theta_i$$

$$G(\theta_i) = \hat{q}_i(\theta_i, h_t)$$

$$F_{h_{t+1}}(\theta_i) = \hat{q}_i(\theta_i, h_{t+1})$$

$$p_{h_{t+1}} = \frac{\Pr(\Theta_j(h_{t+1}))}{\Pr(\Theta_j(h_t))}$$

for any $h_{t+1} \in H_{t+1}(h_t, m_{i,t+1})$ where each element of the set $H_{t+1}(h_t, m_{i,t+1})$ corresponds to an element of the set $\{1, ..., N\}$ in Lemma 2. This means that for $\tilde{q}_i(\theta_i, h_t)$ which satisfies (a) and (b) for any $h_t \in H_t$, we can construct $\tilde{q}_i(\theta_i, h_{t+1})$ which satisfies (a)-(c) for any $h_{t+1} \in H_{t+1}$.

With $h_0 = \phi$, since $\tilde{q}_i(\theta_i, h_0) = \hat{q}_i(\theta_i, h_0)$ satisfies (a) and (b), $\tilde{q}_i(\theta_i, h_1)$ is constructed so that (a)-(c) are satisfied for any $h_1 \in H_1$. Recursively $\tilde{q}_i(\theta_i, h_t)$ can be constructed for any $h_t \in \cup_{\tau=0}^{T} H_\tau$ so that (a)-(c) are satisfied. ■

Step 4

We are now in a position to complete the proof of sufficiency. We focus initially on the case where $Q = \Re_+ \times \Re_+$ and the mechanism is decentralized so agents select their own outputs independently. Later we show how to extend the proof to other contexts.

Given $\tilde{q}_i(\theta_i, h)$ (with $h = h_T$) constructed in Claim 1, construct transfer functions $t_i(q_i, h)$ as follows:

$$t_i(q_i, h) = \hat{\theta}_i(q_i, h)q_i + \int_{\hat{\theta}_i(q_i,h)}^{\bar{\theta}_i} \tilde{q}_i(x, h)dx.$$

for $q_i \in Q_i(h) \equiv \{\tilde{q}_i(\theta_i, h) \mid \theta_i \in \Theta_i\}$, and $t_i(q_i, h) = -\infty$ for $q_i \notin Q_i(h)$ where $\hat{\theta}_i(q_i, h)$ is defined as follows:

$$\hat{\theta}_i(q_i, h) \equiv \sup\{\theta_i \mid \tilde{q}_i(\theta_i, h) \geq q_i\}.$$

We show that the specified communication strategies ($\tilde{c}(\theta)$ and output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$ constitute a PBE (combined with beliefs obtained by applying Bayes rule at every history). By construction, $\tilde{q}_i(\theta_i, h)$ maximizes $t_i(q_i, h) - \theta_i q_i$ for any $h \in H_T$ and any $\theta_i \in \Theta_i$, where

$$t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h) = \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h)dx.$$

Now turn to the choice of messages. Start with round T. Choose arbitrary $h_{T-1} \in H_{T-1}$ and arbitrary $m_{iT} \in \tilde{M}_i(h_{T-1})$. The expected payoff conditional on $\theta_j \in \Theta_j(h_{T-1})$ (i.e., conditional on beliefs given by $\Pr(h \mid h_{T-1}, m_{iT}) = \frac{\Pr(\Theta_j(h))}{\Pr(\Theta_j(h_{T-1}))}$ for $h \in H_T(h_{T-1}, m_{iT})$) is

$$\begin{aligned} & E_h[t_i(\tilde{q}_i(\theta_i, h), h) - \theta_i \tilde{q}_i(\theta_i, h) \mid h_{T-1}, m_{iT}] \\ = & \int_{\theta_i}^{\bar{\theta}_i} E_h[\tilde{q}_i(x, h) \mid h_{T-1}, m_{iT}]dx \\ = & \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(x, h_{T-1})dx. \end{aligned}$$

This does not depend on the choice of $m_{iT} \in \tilde{M}_i(h_{T-1})$. Therefore agent i does not have an incentive to deviate from $m_{iT} = \tilde{m}_{iT}(\theta_i, h_{T-1})$.

The same argument can recursively be applied for all previous rounds t, implying that $m_{i,t+1} = \tilde{m}_{i,t+1}(\theta_i, h_t)$ is an optimal message choice for any $h_t \in H_t$ and any t. This establishes that the communication strategies $\tilde{c}(\theta)$ combined with output choices $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h))$ constitute a PBE.

The argument extends in an obvious way to the case of a centralized mechanism, since this is a special case of the previous mechanism where the assigned outputs $\hat{q}_i(\theta_i, h)$ are measurable with respect to h, i.e., are independent of θ_i conditional on h. Agent i can effectively be forced to choose output $\hat{q}_i(h)$ following history h at the end of the communication phase. The same argument as above ensures that the assigned communication strategies constitute a PBE.

A similar extension works for the decentralized mechanism where the feasible outputs are constrained to lie in some set Q which is a subset of $\Re_+ \times \Re_+$. From the construction of $\tilde{q}_i(\theta_i, h)$ (with $h = h_T$) in Claim 1, $(\tilde{q}_i(\theta_i, h), \tilde{q}_j(\theta_j, h)) \in Q$ holds for any $(\theta_i, \theta_j) \in \Theta_i(h) \times \Theta_j(h)$. On the other hand, it may not hold for $(\theta_i, \theta_j) \notin \Theta_i(h) \times \Theta_j(h)$. Define $\tilde{Q}_i(h) \equiv \{\tilde{q}_i(\theta_i, h) \mid \theta_i \in \Theta_i(h)\}$, the set of possible outputs for i on the equilibrium path, following h. Then $\tilde{Q}_i(h) \times \tilde{Q}_j(h) \subset Q$, implying that as long as agent i chooses an output in $\tilde{Q}_i(h)$ following message history h, the allocation constraint is not violated. Now construct a new set of transfers $\hat{t}_i(q_i, h)$ as follows: $\hat{t}_i(q_i, h) = t_i(q_i, h)$ for $q_i \in \tilde{Q}_i(h)$, and $\hat{t}_i(q_i, h) = -\infty$ for $q_i \notin \tilde{Q}_i(h)$ where $t_i(q_i, h)$ is the transfer function constructed in the previous argument. In this new mechanism, the agent's expected payoff is preserved on the equilibrium path, while they are not increased off the equilibrium path. Hence the postulated strategies constitute a PBE. ■

Proof of Proposition 2:

We show that the solution of the relaxed problem where (ii) is dropped satisfies (ii). Suppose not. Let the solution of the relaxed problem be represented by a (parsimonious) communication protocol p, communication strategies $c(\theta)$ and output allocation $(q_1(\theta_1, \theta_2), q_2(\theta_1, \theta_2))$. H_t, $\Theta_i(h_t)$ and $\Theta_j(h_t)$ are well defined for $(p, c(\theta))$. Then there exists t, $h_t \in H_t$ and $\theta_i, \theta_i' \in \Theta_i(h_t)$ with $\theta_i > \theta_i'$ so that

$$E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] > E_{\theta_j}[q_i(\theta_i', \theta_j) \mid \theta_j \in \Theta_j(h_t)].$$

This implies that at least either one of

$$\begin{aligned} & E[V(q_i(\theta_i', \theta_j), q_j(\theta_i', \theta_j)) - w_i(\theta_i)q_i(\theta_i', \theta_j) - w_j(\theta_j)q_j(\theta_i', \theta_j) \mid \theta_j \in \Theta_j(h_t)] \\ > \; & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i)q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \end{aligned}$$

or

$$\begin{aligned} & E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - w_i(\theta_i')q_i(\theta_i, \theta_j) - w_j(\theta_j)q_j(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_t)] \\ > \; & E[V(q_i(\theta_i', \theta_j), q_j(\theta_i', \theta_j)) - w_i(\theta_i')q_i(\theta_i', \theta_j) - w_j(\theta_j)q_j(\theta_i', \theta_j) \mid \theta_j \in \Theta_j(h_t)] \end{aligned}$$

holds. This means that if at least one type of either θ_i or θ_i' takes other type of communication plan and output decision rule, P's payoff is improved. This is a contradiction. ■

Proof of Proposition 3:

Consider any communication-feasible centralized mechanism with protocol p and communication strategies $c(\theta)$ that result in an output allocation $q^*(\theta) = q(h(c(\theta)))$. Consider any history h that arises from these communication strategies with positive probability, and let the corresponding set of types be $\Theta_i(h) \times \Theta_j(h)$. Then q^* must be constant over $\Theta_i(h) \times \Theta_j(h)$.

For arbitrary q_i, denote

$$E[V(q_i, q_j^*(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h)]$$

by $\hat{V}(q_i, h)$. Consider the problem of choosing q_i to maximize $\hat{V}(q_i, h) - w_i(\theta_i)q_i$ for any $\theta_i \in \Theta_i(h)$. It is evident that the function $\hat{V}(q_i, h_i)$ is strictly concave in q_i, and satisfies the Inada condition. Given the monotonicity of $w_i(\theta_i)$, the optimal solution to this problem, denoted by $\hat{q}_i(\theta_i, h)$, is strictly decreasing in θ_i on $\Theta_i(h)$. Hence

$$\begin{aligned} & E[V(\hat{q}_i(\theta_i, h), q_j^*(\theta)) - w_i(\theta_i)\hat{q}_i(\theta_i, h) - w_j(\theta_j)q_j^*(\theta) \mid (\theta_i, \theta_j) \in \Theta_i(h) \times \Theta_j(h)] \\ > \; & E[V(q^*(\theta)) - w_i(\theta_i)q_i^*(\theta) - w_j(\theta_j)q_j^*(\theta) \mid (\theta_i, \theta_j) \in \Theta_i(h) \times \Theta_j(h)] \end{aligned}$$

Now replace the output allocation $((q_i^*(\theta), q_j^*(\theta))$ by $(\hat{q}_i(\theta), q_j^*(\theta))$ over $\Theta_i(h) \times \Theta_j(h)$, while leaving it unchanged everywhere else. This is a decentralized mechanism which is communication-feasible, which attains a strictly higher expected payoff for the Principal compared with the centralized mechanism. ■

Proof of Proposition 4:

We compare two communication protocols p and $\tilde{p}$. Let $\{(\Theta_i(h, c), \Theta_j(h, c)) \mid h \in H_T(p)\}$ be an information partition on $\Theta_i \times \Theta_j$ induced by communication protocol p and communication strategies $c(\theta)$ in p. We say that $\tilde{p}$ is *more informative* than p, if for any $(p, c(\theta))$, there exists $\tilde{c}(\theta)$ in $\tilde{p}$ such that the partition induced by the latter is finer than the one induced by the former. When this is true, it is evident that protocol $\tilde{p}$ dominates $\tilde{p}$ in terms of the maximum value of the Principal's payoff in the problem described in Proposition 2.

For (i), suppose there exists $t-1$ and $h_{t-1} \in H_{t-1}$ such that $M_i(h_{t-1})$ and $M_j(h_{t-1})$ are both nonempty for some protocol p and $c(\theta)$. Then consider a new communication protocol $\tilde{p}$ where round t (with history h_{t-1}) is divided into two steps with sequential communication: in the first step, i selects message m_{it} from $M_i(h_{t-1})$, and upon observing m_{it}, j sends message m_{jt} from $M_j(h_{t-1})$ in the second step. All other components of the protocol are preserved. Evidently this modification does not violate either communication constraint 1 or 2. In this protocol $\tilde{p}$, j can send messages which can depend on m_{it}, something that is not possible in p. Hence $\tilde{p}$ is more informative than p.

For (ii), for any $(p, c(\theta))$, suppose that either $M_i(h_{t-1})$ or $M_j(h_{t-1})$ includes message with $l(m) \geq 2$ for some $h_{t-1} \in H_{t-1}$. Suppose that $M_i(h_{t-1})$ (or $M_j(h_{t-1})$) includes n_i (or n_j)

bits of messages with $n_i \geq \max\{2, n_j\}$. Now construct a new communication protocol $\tilde{p}$ where round t (with history h_{t-1}) is divided into n_i steps where message space for Step h is $\tilde{M}_{ih} = \tilde{M}_{jh} = \{0, 1\}$ for $h \leq n_j$ and $\tilde{M}_{ih} = \{0, 1\}$ and $\tilde{M}_{jh} = \emptyset$ for $n_j < h \leq n_i$. All other components of the communication protocol are preserved. In $\tilde{p}$, both agents can send the same messages as in $c(\theta)$, and can also send messages which are contingent on those observed in previous steps. Therefore we can find $\tilde{c}(\theta)$ which generates a finer partition than $c(\theta)$ did in the original protocol. It implies that it is optimal that either one or both agents send no more than one binary message per round. Moreover any agent who is silent in any given round could send some messages in that round without adding to the total delay of the mechanism, increasing amount of information exchanged between agents. Hence it is optimal for both agents to simultaneously send one binary message in each round. ■

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