Markov Perfect Industry Dynamics: Recent Advances in Applications of Dynamic Oligopoly Models

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Introduction

Why are we interested in dynamic oligopoly?

1. **Effects of policy/environmental change on industry structure, innovation, and consumer welfare**, e.g.
    - Mergers and antitrust
    - Environmental policy change
    - Removal of barriers to trade
    - etc.

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1. Effects of policy/environmental change on industry structure, innovation, and consumer welfare
2. **Some parameters can only be inferred through dynamic equilibrium**
    - Sunk costs of entry/exit
    - Investment/adjustment costs
    - Learning by doing spillovers

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2. Some parameters can only be inferred through dynamic equilibrium
3. **Study dynamic competition**
    - Collusion, testing for collusion
    - Entry
    - Dynamic competition: R&D/investment, learning by doing, durable goods, network effects, experience goods, etc.

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3. Study dynamic competition
4. **Further understanding of industry dynamics**
    - Why are some industries concentrated and others not?
    - How can an industry be highly concentrated and still have many small firms?
    - What explains the stability/instability of industry structure over time?

Hurdles in working with dynamic oligopoly models:

1. Computational burden (curse of dimensionality).
2. Multiple equilibria.
3. Other issues:
   - Model complexity.
   - Heavy computer programming burden.
   - Data requirements/Identification

Introduction

Consider a simple discrete time discrete state dynamic oligopoly framework (like Ericson and Pakes (1995)):

- Small number of firms in a dynamic industry
- Firm heterogeneity: state variable that represents ability to compete
- Dynamics driven by entry, exit and investment
- Can also have dynamics in price/quantity
- No analytic solution: compute equilibrium on a computer

Model of Imperfect Competition

our firm

state = quality
$x_{it} \in \mathbb{N}$

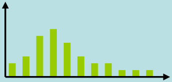
market
(at every t)

profit
$\pi_m(x, s)$

Increasing in x
m: market size

competitors

state =
$s_{-i,t} \in \mathbb{N}^{\infty}$



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Problem: complexity of strategies in EP models:

- Suppose:
  - each firm can take on any one of K states
  - N firms in market
- Then there are K^N points in state space.
- Assuming symmetric strategies, still $\binom{N+K-1}{N}$.

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Example:

- Industry with 20 firms and 40 state points per firm.
- Would require >20 *million GB* of RAM to store policy function once in computer memory.
- $\Rightarrow$ places substantial limitations on applied literature.

Introduction

- **Conclude:** for most real world industries, computing equilibria *exactly* is impossible.

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- Poses two separate sets of challenges for empirical work:
  1. Estimation
  2. Counterfactual exercises
- Will talk briefly about first issue, longer about second issue. (More details on estimation in tomorrow's session.)

General Framework

Model and Notation:

Notation of game is discrete state space and discrete action space:

- **Agents:** $i = 1, ..., N$

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- **State Transitions:** $P(\mathbf{s}_{t+1}|\mathbf{a}_t, \mathbf{s}_t)$.
- **Discount Factor:** β
- **Objective Function:** Agent maximizes EDV,

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \pi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}; \theta). \quad (1)$$

General Framework

Equilibrium Concept: Markov Perfect Equilibrium [MPE]

Strategies: $\sigma_i : S \times \mathbb{R} \to A_i$.

I.e., $a_i = \sigma_i(\mathbf{s}, \nu_i)$ (could be vector valued)

General Framework

Recursive Formulation:

$$V_i(\mathbf{s}|\sigma) = \mathbb{E}_\nu \left[\pi_i(\sigma(\mathbf{s},\nu), \mathbf{s}_t, \nu_i; \theta) + \beta \int V_i(\mathbf{s}'|\sigma(\mathbf{s},\nu)) dP(\mathbf{s}'|\sigma(\mathbf{s},\nu), \mathbf{s})\right]$$

Equilibrium Definition:
A MPE is given by a Markov profile, σ, such that for all i, $\mathbf{s}$, σ_i',

$$V_i(\mathbf{s}|\sigma_i, \sigma_{-i}) \geq V_i(\mathbf{s}|\sigma_i', \sigma_{-i}). \quad (1)$$

Introduction

E.g. Commercial Aircraft Manufacturing (Benkard (2004))

- Two firms, each with a small number of products
- Product differentiation: Plane type, and Plane "quality" state variables
- Learning-by-doing: Experience state variable
- I.e., three state variables (**s**) per product
- In addition, aggregate demand state variable
- Prices set in dynamic equilibrium (**a**)
- Also entry and exit of products (**a**)
- Random shocks (ν) to product quality, cost of entry, and scrap value

An Incomplete List of Recent Applications

- Advertising (Doraszelski & Markovich 2003).
- Auctions (Jofre-Benet & Pesendorfer 2003).
- Capacity accumulation (Besanko & Doraszelski 2004).
- Collusion (Fershtman & Pakes 2000, de Roos 2004).
- Competitive convergence (Langohr 2003).
- Consumer learning (Ching 2002).
- Environmental Policy (Ryan 2009).
- Firm size and growth (Laincz & Domingues Rodrigues 2004).
- Learning by doing (Benkard 2000, 2004, Besanko, Doraszelski, Kryukov & Satterthwaite 2004).
- Mergers (Berry & Pakes 1993, Gowrisankaran 1999, Jeziorski (2009), Stahl (2009), Benkard, Bodoh-Creed and Lazarev (2010)).
- Product Repositioning (Sweeting 2009)
- Network externalities (Markovich 1999, Jenkins, Liu, Matzkin, and McFadden (2004)).
- R&D (Gowrisankaran & Town 1997, Goettler 2009).
- International trade (Erdem & Tybout 2003).
- Finance (Goettler, Parlour & Rajan 2004).
- Entry/sunk costs (Pesendorfer and Schmidt-Dengler 2003, Aguirregabiria and Mira 2006, Collard-Wexler 2006, Beresteanu and Ellickson 2007)

Estimation (1)

Benkard (2004):

- Observe all costs (production, sunk, fixed) directly.
- Estimate parameters “offline” (without imposing equilibrium).
- Compute equilibria only to evaluate counterfactuals.
- Rarely feasible.

Estimation (2)

Rust (1987), Gowrisankaran and Town (1997):
(nested fixed point algorithm)

- For each value of parameters, θ,
  1. Compute equilibrium ($V(\mathbf{s};\theta)$ and $\sigma(\mathbf{s},\nu;\theta)$).
  2. Construct likelihood/GMM objective.
  3. Repeat until objective maximized.
  4. (Also can do MPEC – Su and Judd (2009).)
- Difficulties:
  - computational burden
  - programming burden
  - multiple equilibria
  - essentially infeasible in real world oligopoly problems (without major modelling compromises)

Estimation (3)

Bajari, Benkard, Levin (2007) (Hotz and Miller (1993))

- Use data on $(a, \mathbf{s})$ to construct nonparametric estimates of strategy functions, $a_i = \sigma_i(\mathbf{s}, \nu_i)$.

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  - data chooses equilibrium (under some assumptions),
  - computationally simple,
  - but, stronger data requirements

Some Recent Applications of “BBL”

- Collard-Wexler (2009) – Estimating costs of adjustment in the concrete industry
- Jeziorski (2009) – Estimating merger synergies for radio stations
- Ryan (2009) – Effect of environmental regulations on industry structure in cement
- Sweeting (2009) – Estimating the costs of changing a radio station’s format
- Stahl (2009) – Estimating the incentives to merge in broadcast television
- Benkard, Bodoh-Creed, and Lazarev (2010) – Estimating the effects of a particular proposed U.S. airline merger on industry structure over time

Unsolved Problems in Estimation

Remaining issues:

1. Main issue: unobserved serially correlated state variables
2. Other technical issues such as efficiency issues

Second Half of Talk: Counterfactual Modelling

- Even if we can estimate the model, we still need to compute equilibria to evaluate alternative policies. No way around this.
- e.g., Benkard (2004). In counterfactuals I use single product firms.
- e.g. Ryan (2009). Cement industry markets contain from 1-25 firms. In counterfactuals he is forced to use "toy" version of model with 3 firms.
- Do results extend to real industries?
- We saw earlier that computation is impossible for real industry
- Options: give up, or use approximations of some kind

Introduction, Part 2

Another totally separate but related issue:

- MPE models are so complicated that firms can't possibly be solving them

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- Rules of thumb/behavioral models — could use intuition to guide us in choosing approximations to the MPE solution
- In general would think that firms follow strategies that are simpler than MPE strategies
- Aside: might we be able to find a model that is both easier to solve and perhaps even a better model of behavior?

How do we choose what approximations to use?
Some ideas:

1. **Approximate the value and/or policy functions with simpler functions.**
    - "Neural Dynamic Programming"
    - Judd has some older work on this
    - Some technical issues in games, but might work okay
    - Would still require a large computational and programming burden

How do we choose what approximations to use?

Some ideas:

1. Approximate the value and/or policy functions with simpler functions.
2. **Rules of Thumb**
   - Suppose we just assume that strategies are simpler functions, such as functions of only the first K moments of the states (mean, variance, etc)
   - Maybe largest firm(s)' states should be in there too?
   - Major problem with this: state variable is not Markov. Makes it tough to prove anything (can't use DP tools) and firms can't have "correct" beliefs so now we have to specify something else instead – more on this later.
   - Could in principle do this, and could in principle compute a solution, but how do we know if it worked (in the sense of approximating MPE behavior)? Especially when we cannot even compute an MPE to compare it to? Better have "right model", but how can we know what is right?
   - Still, approach has potential

How do we choose what approximations to use?
Some ideas:

1. Approximate the value and/or policy functions with simpler functions.
2. Rules of Thumb
3. **Question: as state space gets bigger, what happens?**
    - Problem starts simple: single agent DP problem, monopoly problem.
    - Then things get more complex as you add firms
    - Eventually, shouldn't things get simple again? Does largest firm really care what the exact state variable of the bottom half of the industry is if there are 100 firms?

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These thoughts represent our starting point for OE

References

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Introduction, Part 2

The papers I listed above explore an approximation method that reduces computational complexity:

1. Define Oblivious Equilibrium (OE) – notion of equilibrium that is simple to compute.
2. Give precise conditions under which OE approximates MPE as the market size grows.
3. Provide error bounds that provide a numerical measure of the accuracy of the approximation for a given industry.
4. Do numerical experiments to evaluate usefulness of the approximation.
5. Provide extensions to the basic idea.

Model

Consider a simple version of model above in which there is only one state variable per firm (like Ericson and Pakes (1995))

Key differences from Ericson and Pakes:

- No aggregate shocks (see extensions).
- More general entry, exit and investment processes.
- No restrictions on state space, number of firms, number of potential entrants, ...

Elements of the Model:
State Space and Single-Period Profits

Infinite horizon: $t = 1,2,3,\ldots$

our firm i

state: $x_{it} \in \mathbb{N}$
(e.g. quality)

at every t: market

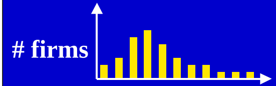
profits

$\pi_m(x, s)$

Increasing in x
m: market size

competitors

state: $s_{-i,t} \in \mathbb{N}^{\infty}$



Oblivious Equilibrium (OE)

Intuition:

- Consider industry with moderate to large number of firms
- Each firm's quality may change quite a lot over time
- Identity of top firms may change periodically
- But, ignoring aggregate shocks, distribution of firm sizes will be relatively stable.

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Oblivious strategies:

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- Equivalent to firms having no current information about rivals' states.
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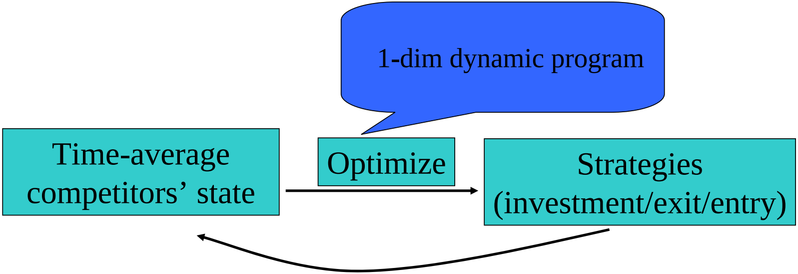
Oblivious equilibrium is equilibrium in oblivious strategies.

Oblivious Equilibrium



- Efficient algorithm: couple of minutes running time.
- Can solve for many firms and many state points per firms (hundreds/thousands).

Comments

Is/are OE model/strategies too simple to be realistic? Does it only work when industry is close to perfect competition?

1. OE is a subtle concept
2. Strategies not as simple as they appear because they optimize over equilibrium industry distribution
3. OE is a building block much like DP (monopoly problem) is the building block used by previous literature
4. OE is so simple that we can easily afford to improve upon it

Notation

We have multi-dimensional strategy (σ) so we will separate into two parts:

- μ is incumbent firm's strategy functions, includes an exit function and an investment function
- λ is entrants' strategy function – determines the entry distribution at each state **s**.

Oblivious Value Functions

- $\tilde{s}_{\mu,\lambda} =$ expected industry state (vector) in the long-run when firms use an oblivious strategy μ and the oblivious entry rate is λ.
- Oblivious value function:

$$\tilde{V}(x|\mu', \mu, \lambda) = E_{\mu'}\left[\sum_{k=t}^{\tau_i} \beta^{k-t}\left(\pi(x_{ik}, \tilde{s}_{\mu,\lambda}) - d\iota_{ik}\right) + \beta^{\tau_i - t}\phi_{i,\tau_i}\middle| x_{it} = x\right].$$

- Note: $\tilde{V}$ depends on competitors' strategy through $\tilde{s}$.

Oblivious Equilibrium (OE)

Oblivious equilibrium: oblivious (μ, λ) such that:

1. Firm strategies optimize an oblivious value function:

$$\sup_{\mu' \in \tilde{\mathcal{M}}} \tilde{V}(x|\mu', \mu, \lambda) = \tilde{V}(x|\mu, \lambda), \qquad \forall x \in \mathbb{N}.$$

2. Either the oblivious expected value of entry is zero or the entry rate is zero (or both):

$$\lambda \left(\beta \tilde{V}(x^e|\mu, \lambda) - \kappa \right) = 0$$
$$\beta \tilde{V}(x^e|\mu, \lambda) - \kappa \leq 0 \qquad \lambda \geq 0.$$

Theoretical Justification

Theorem: Under a "light tail" condition, OE approximates MPE asymptotically in large markets.

Note:

- Asymptotics on market size
- It would seem that OE should yield good approximation if average number of firms becomes large
- However, also need "light tail condition" that limits average size of firms

Computational Approach

Additional questions:

1. How well does OE approximate MPE in practice?
2. How many firms are required for a good approximation?
3. How can we know if the approximation is good in a particular application?

Answer: computable error bounds

Error Bounds

Error bounds ask question: how much better could a firm do by deviating from OE to a Markov best response?

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Simplest error bound:

$$E[\sup_{\mu' \in \mathcal{M}} V(x, s|\mu', \tilde{\mu}, \tilde{\lambda}) - V(x, s|\tilde{\mu}, \tilde{\lambda})] \leq \epsilon \,, \ \forall x \in \mathbb{N} \ .$$

where

$$\epsilon = \frac{2}{1-\beta} E\left[\max_{y} |\pi_m(y, s) - \pi_m(y, \tilde{s})|\right] \ ,$$

Expectation is with respect to s, a random vector sampled according to the invariant distribution. $\tilde{s} = E[s]$.

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- Does not require knowledge of MPE or even Markov best response.
- Bound is very general to changes in the model.
- Bound is not tight. Paper has more complex but tighter bounds

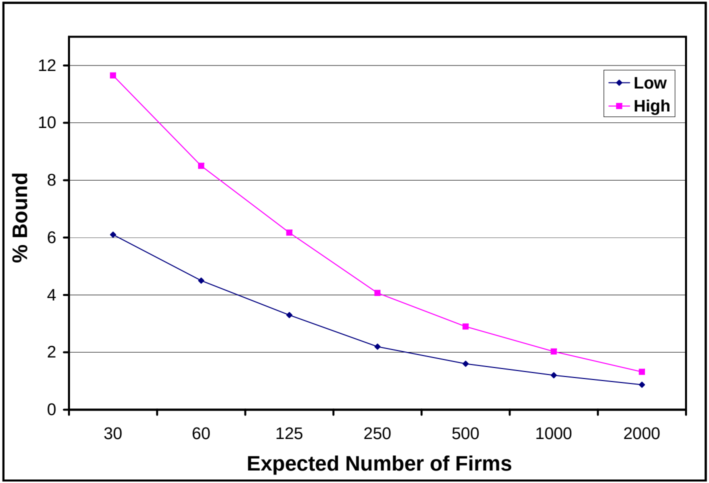
Numerical Evidence

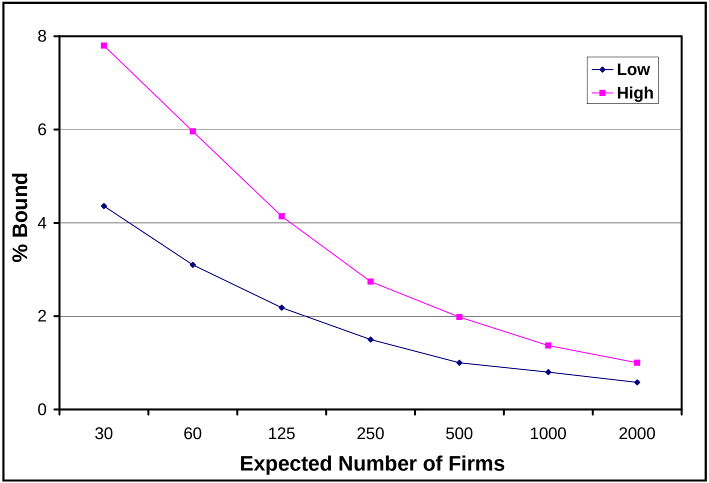
Model summary:

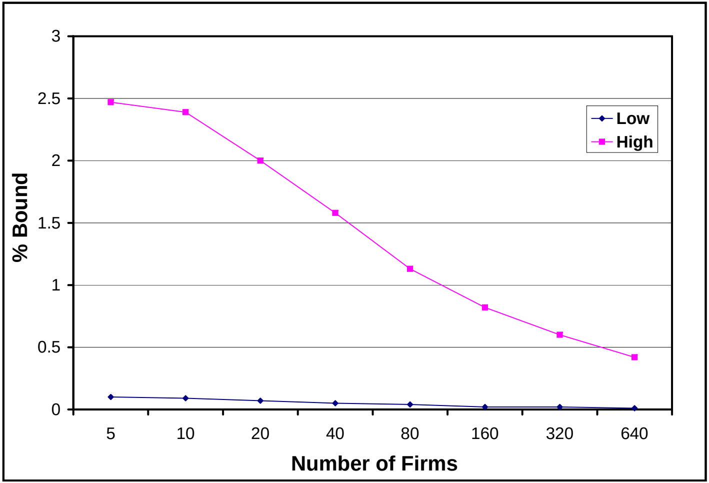
- Model described earlier with logit demand and price setting
- Product utility is

$$\theta_1 \log(\text{quality}) + \theta_2 \log(\text{income} - \text{price}) + \text{logit error}$$

- Most important parameter is θ_1
- (Next three charts are base/deterministic entry, exit/no entry, exit.)


12
10
8
6
4
2
0
% Bound
Low
High
30
60
125
250
500
1000
2000
Expected Number of Firms


8
6
4
2
0
% Bound
Low
High
30
60
125
250
500
1000
2000
Expected Number of Firms


3
2.5
2
1.5
1
0.5
0
% Bound
Low
High
5
10
20
40
80
160
320
640
Number of Firms

Computational Experiments

- Experiment 2: Comparison with MPE
  - Restrict state space: 4 firms, 15 states per firm (to compute MPE).
  - No entry/exit case (because bound is small only for that case).
  - Compare investment, prod. surplus, cons. surplus, C1, C2.
  - Bound is small $\Rightarrow$ these stats are close to actual.
  - Policies close for larger bounds if distribution symmetric (Fig 4).
  - Generally bound is 10-20 times bigger than actual difference.

Table: Comparison of MPE and OE strategies (4 firms, no entry and exit)

Parameters		Long Run Statistics (% Diff)					Perf Bound (% Diff)		Actual (% Diff)	
θ_1	d	Inv.	Prod Surp	Cons Surp	C1	C2	Max Diff	Weighted Avg	Max Diff	Weighted Avg
0.10	0.10	−0.26	−0.01	−0.02	0.03	0.03	0.14	0.13	0.08	0.07
0.30	0.30	−0.13	0.06	0.08	0.08	0.16	1.67	1.22	0.04	0.01
0.50	0.50	−0.11	0.20	0.28	0.18	0.50	6.64	3.61	0.21	0.06
0.70	0.70	−2.21	0.40	0.15	1.08	2.09	18.85	8.35	1.60	0.67
0.85	0.70	−2.19	0.23	−0.28	1.37	2.10	30.80	9.64	1.80	0.20
0.15	0.27	3.54	0.14	0.20	1.22	0.46	0.36	0.35	0.10	0.10
0.20	0.35	4.18	0.29	0.42	1.93	1.03	0.81	0.77	−0.09	−0.05
0.30	0.55	9.28	0.93	1.31	5.10	2.45	1.96	1.85	0.26	0.25
0.40	0.80	21.02	2.10	2.93	11.58	4.12	3.01	2.92	0.30	0.29
0.50	1.00	18.62	3.30	4.33	15.69	5.94	6.29	5.86	0.32	0.30

Long run statistics and value functions simulated with a relative precision of 1.0% and a confidence level of 99%. Error bound simulated with a relative precision of at most 10% and a confidence level of 99%.

Table: Comparison of MPE and OE Investment (4 firms, no entry and exit)

Parameters		Investment		
θ_1	d	MPE	OE	% Diff
0.10	0.10	0.752	0.754	−0.26
0.30	0.30	0.754	0.755	−0.13
0.50	0.50	0.741	0.742	−0.11
0.70	0.70	0.694	0.709	−2.21
0.85	0.70	0.748	0.765	−2.19
0.15	0.27	0.192	0.185	3.54
0.20	0.35	0.261	0.250	4.18
0.30	0.55	0.238	0.216	9.28
0.40	0.80	0.168	0.133	21.02
0.50	1.00	0.195	0.158	18.62
Investment simulated with a relative precision of 1.0% and a confidence level of 99%.				

Pause: The Big Picture

In OE we have a model that:

- is simple to compute
- is exactly correct if only one firm
- is exactly correct if many firms
- is a not unrealistic behavioral model for markets with more than a few firms
- has nice theoretical properties

However, clearly in some industries we might think that firms would use additional information in their strategies. Can we use OE as the basis for models of such industries as well?

Extensions

Idea:

- OE is trivial to compute
- Can we increase computation and improve model?

Extensions:

1. Transitional dynamics
2. Strategic interaction in concentrated industries
3. Aggregate shocks

Transitional Dynamics

"Nonstationary OE":

- Firms now know the starting state, s_0, but will not update this knowledge
- and follow sequence of oblivious strategies, $\tilde{\mu}_t$, $t = 0, 1, ...$
- Sequence of oblivious entry rates, λ_t, $t = 0, 1, ...$
- This generates a sequence of expected states, $\tilde{s}_t$, $t = 0, 1, ...$ (where $\tilde{s}_0 = s_0$)
- In nonstationary OE, firms are optimizing against the sequence expected in equilibrium.
- This is a sequence of one-dimensional problems.

Transitional Dynamics

Nonstationary oblivious value function:

$$\tilde{V}_t(x|\mu', \mu, \lambda, s) = E_{\mu'}\Big[\sum_{k=t}^{\tau_i} \beta^{k-t}\left(\pi(x_{ik}, \tilde{s}_{(\mu,\lambda,s),k}) - d\iota_{ik}\right) + \beta^{\tau_i - t}\phi_{i,\tau_i}\Big| x_{it} = x\Big].$$

Transitional Dynamics

Nonstationary OE satisfies two conditions:

1. Strategies maximize a nonstationary oblivious value function:

$$\sup_{\mu' \in \tilde{\mathcal{M}}_{ns}} \tilde{V}_0(x|\mu', \mu, \lambda, s) = \tilde{V}_0(x|\mu, \lambda, s), \qquad \forall x \in \mathbb{N}.$$

2. For all $t \geq 0$,

$$\lambda_t \left(\beta \tilde{V}_{t+1}(x^e|\mu, \lambda, s) - \kappa \right) = 0$$
$$\beta \tilde{V}_{t+1}(x^e|\mu, \lambda, s) - \kappa \leq 0$$
$$\lambda_t \geq 0.$$

Transitional Dynamics

Comments:

- We prove existence of a NOE that converges to an OE
- In practice, enforce this convergence in computation.
- Computation similar to before: sequence of one dimensional problems.
- Similar asymptotics should hold.
- New bound (for period 0).
- Computations show that model is not only useful for capturing transitional dynamics of a policy change, but also helps in matching behavior in concentrated industries because starting point is now known to firms.

Dominant Firms

"Dominant Firm OE"

- One or more "dominant" firms.
- Firms are either "dominant" or "fringe" (**forever**)
- (Can't have switching because then state is not Markov.)
- All firms track own state, states of all dominant firms
- Firms maximize EDV subject to expected fringe firm distribution conditional on the dominant firms' state
- Old bounds still hold for "fringe" firms.
- (Bounds for dominant firms difficult because evolution of fringe is not Markov.)
- Find that dominant firms behave differently from fringe firms and in fact use their informational advantage to reinforce their dominance
- Bounds get smaller when add dominant firms
- Could be a good model for industries with leading firms

Dominant Firms

- Exp 1: Convex inv cost (higher for high states), low θ_1
  - investment not valuable and costly at high states)
  - close to OE
- Exp 2: High inv cost, high θ_1
  - investment valuable and always costly
  - entry deterrence but not inv deterrence
- Exp 3: Low inv cost, high depreciation rate
  - OE has some big firms
  - dom firm always big
  - entry deterrence (but not inv deterrence)

Dominant Firms

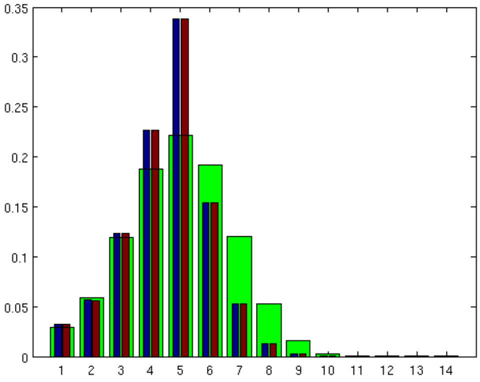


Figure: Distribution of firms in Experiment 1: blue - pure OE, red - dom. firm OE fringe, green - dom. firm OE leader

Dominant Firms

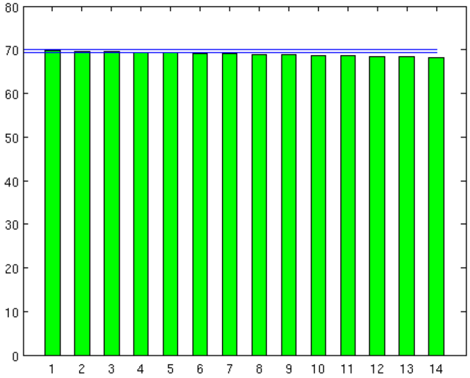


Figure: Expected number of fringe firms — dom firm state in Experiment 1

Dominant Firms

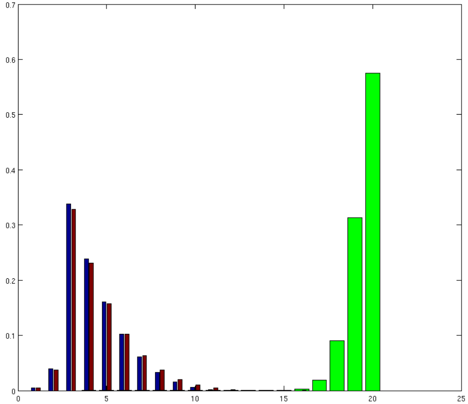


Figure: Distribution of firms in Experiment 2: blue - pure OE, red - dom. firm OE fringe, green - dom. firm OE leader

Dominant Firms

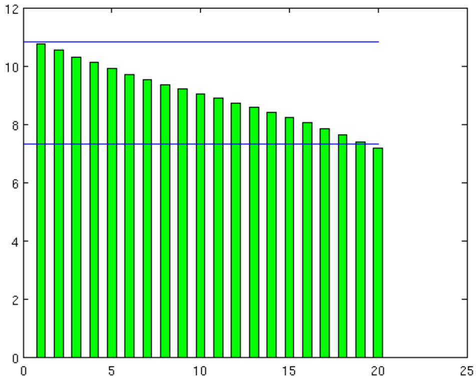


Figure: Expected number of fringe firms — dom firm state in Experiment 2

Dominant Firms

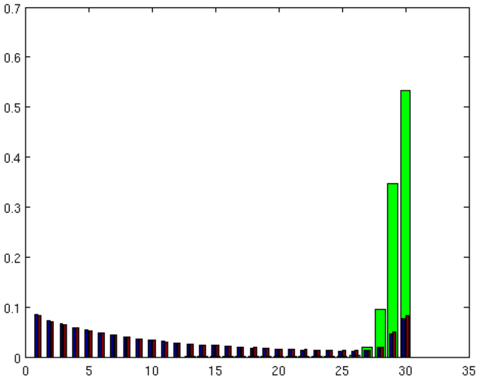


Figure: Distribution of firms in Experiment 3: blue - pure OE, red - dom. firm OE fringe, green - dom. firm OE leader

Dominant Firms

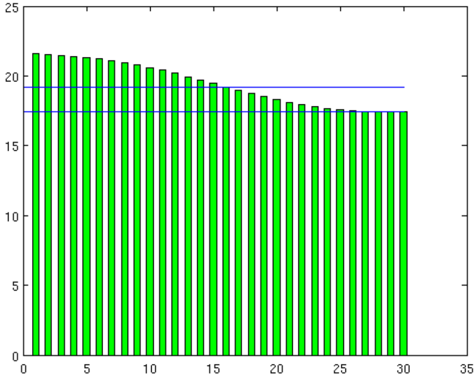


Figure: Expected number of fringe firms — dom firm state in Experiment 3

Aggregate shocks model

New model:

- Let z_t be some aggregate states that follow a first order Markov process.
- $\pi(x, s, z)$
- Let w_t represent a vector of functions of $(z_0, ..., z_t)$.
- Firms use w to form expectations on industry state, $\tilde{s}(w)$.
- I.e., they maximize EDV subject to expected distribution of firm states conditional on w (not Markov)
- Extended oblivious strategies: $\mu(x, w)$, $\lambda(x, w)$.

Aggregate shocks model

Extended oblivious value function:

$$\tilde{V}(x, \mathbf{w}|\mu', \mu, \lambda) = E_{\mu'}\Big[\sum_{k=t}^{\tau_i} \beta^{k-t}\left(\pi(x_{ik}, \tilde{s}_{\mu,\lambda}(\mathbf{w}_k), z_k) - d\iota_{ik}\right) + \beta^{\tau_i - t}\phi_{i,\tau_i}\Big| x_{it} = x, \mathbf{w}_t = \mathbf{w}\Big] . \quad (2)$$

Aggregate shocks model

Comments

- New equilibrium and bounds.
- If w contains past values of z, no longer strictly “Markov”
- Instead we assume that firms only track a finite history
- Model is not “air tight” in the same way that “rules of thumb” models would not be.
- However, we do find that we get realistic behavior from this model
- Asymptotics different.
- Can do nonstationary OE.

Conclusions

1. Estimation issues largely worked out – one remaining hurdle is serially correlated unobserved states
2. OE as a computational approach
   - Algorithms are simple to program and computationally light.
   - Computable bounds on the approximation error.
   - Asymptotically correct
   - Basis for extended methods that use more information (work in progress)
3. Alternative area to explore: carefully chosen behavioral models