# Estimating Dynamic Oligopoly Models of Imperfect Competition

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# Introduction

**Why are we interested in dynamic oligopoly?**

1. **Effects of policy/environmental change on industry structure and welfare**, e.g.
    - Mergers and antitrust
    - Environmental policy change
    - Removal of barriers to trade
    - etc.

# Introduction

**Why are we interested in dynamic oligopoly?**

1. Effects of policy/environmental change on industry structure and welfare
2. **Some parameters can only be inferred through dynamic equilibrium**
   - Sunk costs of entry/exit
   - Investment/adjustment costs
   - Learning by doing spillovers

# Introduction

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1. Effects of policy/environmental change on industry structure and welfare
2. Some parameters can only be inferred through dynamic equilibrium
3. **Study dynamic competition**
    - Collusion, testing for collusion
    - Entry
    - Dynamic competition: R&D/investment, learning by doing, durable goods, network effects, experience goods, etc.

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2. Some parameters can only be inferred through dynamic equilibrium
3. Study dynamic competition
4. **Further understanding of industry dynamics**
    - Why are some industries concentrated and others not?
    - How can an industry be highly concentrated and still have many small firms?
    - What explains the stability/instability of industry structure over time?

**Hurdles in working with dynamic oligopoly models:**

1. Computational burden (curse of dimensionality).
2. Multiple equilibria.
3. Other issues:
   - Model complexity.
   - Heavy computer programming burden.
   - Data requirements/Identification

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**Model and Notation:**

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- **State Transitions:** $P(\mathbf{s}_{t+1}|\mathbf{a}_t, \mathbf{s}_t)$.
- **Discount Factor:** $\beta$
- **Objective Function:** Agent maximizes EDV,

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \pi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}; \theta). \quad (1)$$

# General Framework

**Equilibrium Concept:** Markov Perfect Equilibrium [MPE]

**Strategies:** $\sigma_i : S \times \mathbb{R} \to A_i$.

I.e., $a_i = \sigma_i(\mathbf{s}, \nu_i)$ (could be vector valued)

# General Framework

**Recursive Formulation:**

$$V_i(\mathbf{s}|\sigma) = \mathbb{E}_\nu \left[ \pi_i(\sigma(\mathbf{s}, \nu), \mathbf{s}_t, \nu_i; \theta) + \beta \int V_i(\mathbf{s}'|\sigma(\mathbf{s}, \nu)) dP(\mathbf{s}'|\sigma(\mathbf{s}, \nu), \mathbf{s}) \right]$$

**Equilibrium Definition:**
A MPE is given by a Markov profile, $\sigma$, such that for all $i$, $\mathbf{s}$, $\sigma_i'$,

$$V_i(\mathbf{s}|\sigma_i, \sigma_{-i}) \geq V_i(\mathbf{s}|\sigma_i', \sigma_{-i}). \quad (1)$$

# Example

**Dynamic Oligopoly w/ Investment, Entry, Exit**
(cf. Ericson and Pakes (1995), Pakes and McGuire (1994))

- Period return function:

$$\pi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}; \theta) = q_{it}(\mathbf{s}_t, \mathbf{p}_t; \theta_1)\,(p_{it} - \mathrm{mc}(s_{it}, q_{it}; \theta_2)) - C(I_{it}, \nu_{it}; \theta_3),$$

- $s$ is product quality
- $I$ is investment or advertising that improves quality
- $q(\cdot)$ - quantities (demand system),
- $p$ - prices (choice variable),
- $mc(\cdot)$ - marginal cost function,
- $C(\cdot)$ - cost of investment function.
- $\nu$ - private shock to cost of investment,
- $\theta = (\theta_1, \theta_2, \theta_3)$ - parameters to be estimated,

# Example

- Entry:
    - One short-lived potential entrant per period
    - $F_e(x^e)$ - distribution of privately known entry cost.
    - Enter if EDV of entering is greater than entry cost.
- Exit:
    - Each incumbent can exit in any period and receive $\Psi$

Assuming they are not observed directly, need dynamic model to estimate entry/exit costs as well as cost of investment function.

# An Incomplete List of Recent Applications

- Advertising (Doraszelski & Markovich 2003).
- Auctions (Jofre-Benet & Pesendorfer 2003).
- Capacity accumulation (Besanko & Doraszelski 2004).
- Collusion (Fershtman & Pakes 2000, de Roos 2004).
- Competitive convergence (Langohr 2003).
- Consumer learning (Ching 2002).
- Environmental Policy (Ryan 2009).
- Firm size and growth (Laincz & Domingues Rodrigues 2004).
- Learning by doing (Benkard 2000, 2004, Besanko, Doraszelski, Kryukov & Satterthwaite 2004).
- Mergers (Berry & Pakes 1993, Gowrisankaran 1999, Jeziorski (2009), Stahl (2009), Benkard, Bodoh-Creed and Lazarev (2010)).
- Product Repositioning (Sweeting 2009)
- Network externalities (Markovich 1999, Jenkins, Liu, Matzkin, and McFadden (2004)).
- R&D (Gowrisankaran & Town 1997, Goettler 2009).
- International trade (Erdem & Tybout 2003).
- Finance (Goettler, Parlour & Rajan 2004).
- Entry/sunk costs (Pesendorfer and Schmidt-Dengler 2003, Aguirregabiria and Mira 2006, Collard-Wexler 2006, Beresteanu and Ellickson 2007)

# Estimation (1)

**Benkard (2004):**

- Observe all costs (production, sunk, fixed) directly.
- Estimate parameters "offline" (without imposing equilibrium).
- Compute equilibria only to evaluate counterfactuals.
- Rarely feasible.

# Estimation (2)

**Rust (1987), Gowrisankaran and Town (1997):**
(nested fixed point algorithm)

- For each value of parameters, $\theta$,
  1. Compute equilibrium ($V(\mathbf{s};\theta)$).
  2. Construct likelihood/GMM objective.
  3. Repeat until objective maximized.
  4. (Also can do MPEC – Su and Judd (2009).)
- Difficulties:
  - computational burden
  - programming burden
  - multiple equilibria
  - essentially infeasible in real world oligopoly problems (without major modelling compromises)

# Estimation (3)

**Bajari, Benkard, Levin (2007)** (Hotz and Miller (1993))

- Use data on $(a, \mathbf{s})$ to construct nonparametric estimates of strategy functions, $a_i = \sigma_i(\mathbf{s}, \nu_i)$.
- Along with the transition probabilities, the strategy functions can be used to simulate industry sample paths in observed equilibrium
- For each value of $\theta$,
    1. Use simulated paths to estimate EDV at each state, $\hat{V}(\mathbf{s}; \theta)$.
    2. Construct likelihood/GMM objective.
    3. Repeat until objective maximized
- Comments:
    - data chooses equilibrium (under some assumptions),
    - computationally simple,
    - but, stronger data requirements

# Dynamic oligopoly example:

- Estimate $(\theta_1, \theta_2)$ using standard techniques (e.g. BLP).
- (Unobserved states could also be recovered here.)
- Project investment onto state variables nonparametrically:

$$I_{it} = f_I(\mathbf{s}_t, \nu_{it}).$$

  Since investment is monotonic in $\nu_{it}$ this amounts to recovering $F(I_i|\mathbf{s})$ for each $\mathbf{s}$.
- (Investment is only observed if firm does not exit but that doesn't matter because of the monotonicity.)
- Project entry and exit onto state variables:

$$\chi_{it}^e = f_e(\mathbf{s}_t, \nu_{it}), \ \chi_{it} = f_x(\mathbf{s}_t, \nu_{it})$$

- Estimate state transition function, $P : S \times A \to \Delta(S)$, using MLE.

# First Step (cont.):

Construct $\hat{V}(\mathbf{s}_0, \nu_0; \theta)$ using "forward simulation":

1. Policy in initial period is $a_{i0} = \widehat{\sigma}(\mathbf{s}_0, \nu_{i0})$.
2. Draw $\mathbf{s}_{t+1}$ from $\widehat{P}(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$, $\nu_{t+1}$ from $G(\nu_{t+1} | \mathbf{s}_{t+1})$.
3. Repeat 1 & 2 to obtain one simulated path.
4. Profits at each $(\mathbf{a}_t, \mathbf{s}_t, \nu_t)$ are given by $\pi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}; \theta)$.
5. Use many simulated paths to construct $\hat{V}(\mathbf{s}_0, \nu_0; \theta)$.

**Note:**

- Under the assumptions, "correct" equilibrium consistently estimated by $\widehat{\sigma}$.

# Second Step:

**Idea:** Find the set of parameters that *rationalize* the observed policies. I.e., conditional on $P$ and $\sigma$, find the set of parameters that satisfy the requirements for equilibrium.

**Optimality inequalities defining MPE:**

For all $i$, $\sigma_i'$, and initial state, $s_0$, it must be that

$$V_i(\mathbf{s}|\sigma_i, \sigma_{-i}) \geq V_i(\mathbf{s}|\sigma_i', \sigma_{-i}).$$

This system of inequalities contains all information available from the definition of equilibrium. To learn $\theta$ we simply plug in the estimated $\hat{V}$'s and find the $\theta$ that best satisfies these inequalities.

# Important computational trick:

$$V_i(\mathbf{s}|\sigma_i, \sigma_{-i}) = \mathbb{E}_{\sigma_i, \sigma_{-i}|\mathbf{s}_0} \sum_{t=0}^{\infty} \beta^t \pi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}; \theta)$$

**Suppose** period return function is linear in the parameters:

$$\pi_i(\mathbf{a}, \mathbf{s}, \nu_i; \theta) := \Phi_i(\mathbf{a}, \mathbf{s}, \nu_i) \cdot \theta.$$

Let

$$W(\mathbf{s}_0; \sigma_i, \sigma_{-i}) := \mathbb{E}_{\sigma_i, \sigma_{-i}|\mathbf{s}_0} \sum_{t=0}^{\infty} \beta^t \Phi_i(\mathbf{a}_t, \mathbf{s}_t, \nu_{it}).$$

Then the value function can be computed as

$$V_i(\mathbf{s}|\sigma_i, \sigma_{-i}) = W(\mathbf{s}_0; \sigma_i, \sigma_{-i}) \cdot \theta$$

where $W$ is a function only of things that are known.

System of inequalities defining equilibrium becomes:

$$W(\mathbf{s}_0; \sigma_i, \sigma_{-i}) \cdot \theta \geq W(\mathbf{s}_0; \sigma_i', \sigma_{-i}) \cdot \theta \quad (6)$$

for all $i$, $\sigma_i'$, $s_0$.

**Comments:**

- This system contains all information that the model provides about the unknown parameter vector, $\theta$.
- System is linear in $\theta$.
- Easy to compute/implement.
- Makes no difference if policies are discrete (entry,exit) or continuous (investment, quantity, price, etc) or both.

# Second Step (cont.):

Given $\sigma$ and $P$, let $\Theta_0$ be the set of parameters that rationalize the observed data,

$$\Theta_0(\sigma, P) := \{\theta : \theta, \sigma, P \text{ satisfy (6) for all } s_0, i, \sigma_i'\}.$$

where (6) is the system of optimality inequalities,

$$W(\mathbf{s}_0; \sigma_i, \sigma_{-i}) \cdot \theta \geq W(\mathbf{s}_0; \sigma_i', \sigma_{-i}) \cdot \theta, \tag{6}$$

for all $i$, $\sigma_i'$, $s_0$.

The goal of estimation is to learn $\Theta_0$.

# Implementation:

- Randomly pick a small subset of inequalities denoted $\{x_k\}$ (so $x_k$ refers to an $(i, \mathbf{s}_0, \sigma')$ triple).
- We use alternative policies of the form,

$$\sigma'(\mathbf{s})_k = \hat{\sigma}(\mathbf{s}) + \epsilon_k,$$

  where $\epsilon_k \sim N(0, \sigma_\epsilon^2)$.
- Simulate $W$'s for these inequalities using forward simulation.

# Implementation (cont):

- Let $g_{n_s}(x,\theta;\alpha) =$

$$\left[\widehat{W}_{n_s}(\mathbf{s};\sigma_i(\alpha),\sigma_{-i}(\alpha)) - \widehat{W}_{n_s}(\mathbf{s};\sigma_i',\sigma_{-i}(\alpha))\right]\cdot\theta,$$

- and find $\theta$ that minimize violations of the inequalities,

$$Q_n(\widehat{\theta},\widehat{\alpha}_n) = \inf_{\theta\in\Theta^*} Q_n(\theta,\widehat{\alpha}_n).$$

where

$$Q_n(\theta,\alpha) = \frac{1}{n_I}\sum_{k=1}^{n_I} 1\{g_{n_s}(X_k,\theta;\alpha) < 0\} g_{n_s}(X_k,\theta;\alpha)^2.$$

# Implementation, Identified Case:

- Right now we assume first stage is indexed by a parameter $\alpha$.
- In that case, (if $\frac{n_s}{n} \to 0$ and $\frac{n_l}{n} \to r$),

$$\widehat{\theta} \xrightarrow{p} \theta_0$$

and

$$\sqrt{n}(\widehat{\theta} - \theta_0) \xrightarrow{d} N(0, H_0^{-1} \Lambda_0 V_\alpha \Lambda_0' H_0^{-1}).$$

where,

$$H(\theta) \equiv -\mathbb{E} \frac{\partial^2 \{g(X_k, \theta_0; \alpha_0) < 0\} g^2(X_k, \theta_0; \alpha_0)}{\partial\theta\partial\theta'},$$

$H_0 = H(\theta_0)$, and

$$\Lambda_0 \equiv \mathbb{E} \frac{\partial^2 \{g(X_k, \theta_0; \alpha_0) < 0\} g^2(X_k, \theta_0; \alpha_0)}{\partial\theta\partial\alpha'}.$$

# Implementation (cont):

- Easiest to compute standard errors using subsampling.
- If model is only set-identified:
  1. Use same objective function.
  2. Compute standard errors via Chernozhukov, Hong, and Tamer (2004).
  3. (Identified set is a convex polyhedron.)

# Dynamic Oligopoly Monte Carlo

**States:** $s_j =$ quality of firm $j$'s product.
**Demand:**

$$U_{rj} = \gamma h(s_j) + \alpha \ln(y_r - p_j) + \epsilon_{rj}$$

**Investment:** Probability of successful investment is:

$$aI_j/(1 + aI_j),$$

**Cost of investment function:**

$$c(I) = \theta_{3,1} * I.$$

**Entry Costs:** $U[x^l, x^h]$.
**Scrap Value:** $\Phi$.

# Second Stage

For every initial state, $s_0$, and every alternative investment policy, $\sigma'(\mathbf{s}) = (I'(\mathbf{s}), \chi'(\mathbf{s}))$,

$$\left[\widehat{\mathbb{E}}_{\sigma_i,\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t \tilde{\pi}_i(\mathbf{a}_t, \mathbf{s}_t) - \widehat{\mathbb{E}}_{\sigma_i',\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t \tilde{\pi}_i(\mathbf{a}_t, \mathbf{s}_t)\right]$$
$$+\theta_{3,1}\left[\widehat{\mathbb{E}}_{\sigma_i,\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t I_{it} - \widehat{\mathbb{E}}_{\sigma_i',\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t I_{it}\right]$$
$$+\Psi\left[\widehat{\mathbb{E}}_{\sigma_i,\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t \{\chi(\mathbf{s}_t) = 1\} - \widehat{\mathbb{E}}_{\sigma_i',\sigma_{-i}} \sum_{t=0}^{\infty} \beta^t \{\chi'(\mathbf{s}_t) = 1\}\right] \geq 0$$

where $\tilde{\pi}(a, s)$ represents static profits, $q(p - mc)$.

Use MD to estimate $\theta_{3,1}$ and $\Psi$.

Also straightforward to estimate sunk cost of entry distribution (parametrically or nonparametrically) – see paper for details.

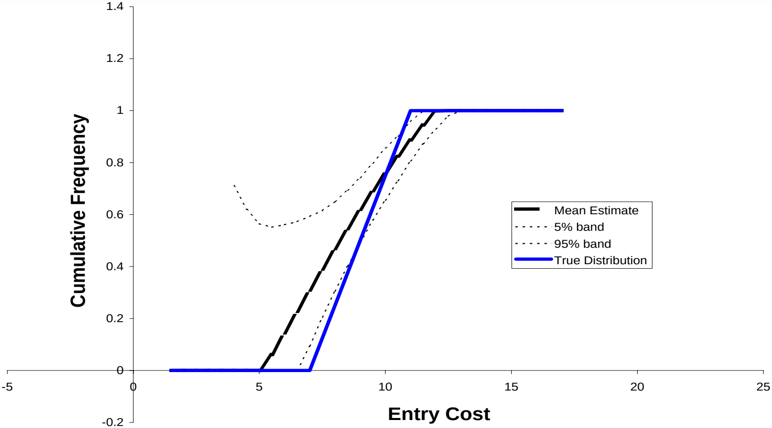
Table: Dynamic Oligopoly Monte Carlo Parameters

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| Demand: | | Investment Cost: | |
| $\alpha$ | 1.5 | $\theta_{3,1}$ | 1 |
| $\gamma$ | 0.1 | | |
| $M$ | 5 | Marginal Cost: | |
| $y$ | 6 | $mc$ | 3 |
| | | | |
| Investment Evolution | | Entry Cost Distribution | |
| $\delta$ | 0.7 | $x^l$ | 7 |
| $a$ | 1.25 | $x^h$ | 11 |
| | | | |
| Discount Factor | | Scrap Value: | |
| $\beta$ | 0.925 | $\Psi$ | 6 |

# Table: Dynamic Oligopoly With Nonparametric Entry Distribution

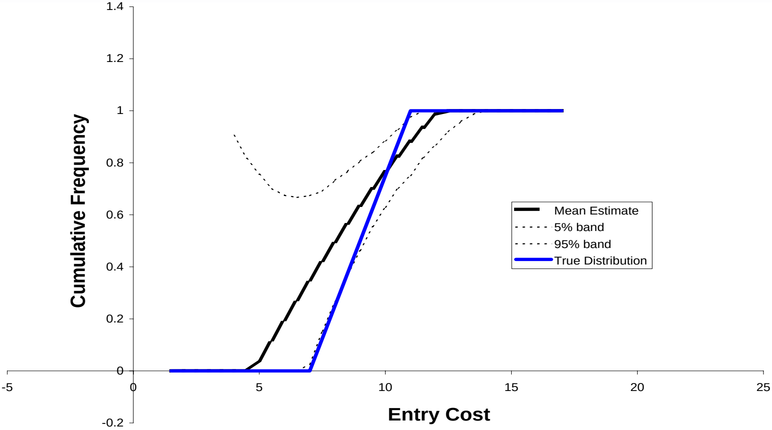
| | Mean | SE(Real) | 5%(Real) | 95%(Real) | SE(Subsampling) |
|---|---|---|---|---|---|
| $n = 400$, $n_I = 500$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.05 | 0.91 | 1.10 | 0.03 |
| $\Psi$ | 5.38 | 0.43 | 4.70 | 6.06 | 0.39 |
| $n = 200$, $n_I = 500$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.08 | 0.89 | 1.14 | 0.05 |
| $\Psi$ | 5.32 | 0.56 | 4.45 | 6.33 | 0.53 |
| $n = 100$, $n_I = 300$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.10 | 0.84 | 1.17 | 0.06 |
| $\Psi$ | 5.30 | 0.72 | 4.15 | 6.48 | 0.72 |

## Figure: Entry Cost Distribution for $n = 400$



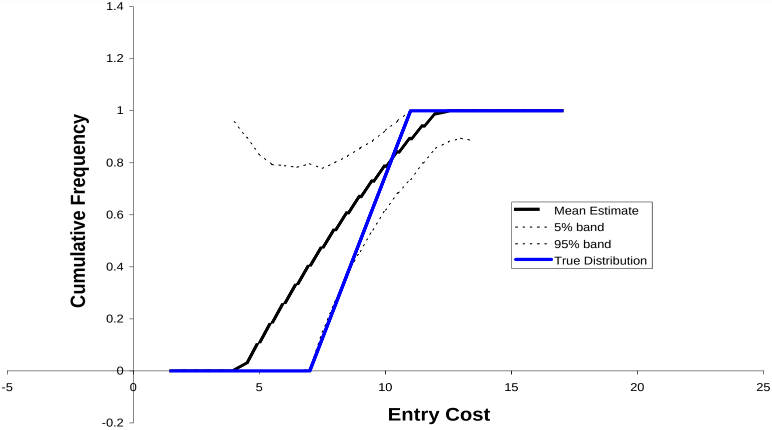
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
Cumulative Frequency
-5
0
5
10
15
20
25
Entry Cost
Mean Estimate
5% band
95% band
True Distribution

Figure: Entry Cost Distribution for $n = 200$



Cumulative Frequency
Entry Cost
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
0
5
10
15
20
25
Mean Estimate
5% band
95% band
True Distribution

## Figure: Entry Cost Distribution for $n = 100$



1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-5
0
5
10
15
20
25
Cumulative Frequency
Entry Cost
Mean Estimate
5% band
95% band
True Distribution

## Table: Dynamic Oligopoly With Parametric Entry Distribution

| | Mean | SE(Real) | 5%(Real) | 95%(Real) | SE(Subsampling) |
|---|---|---|---|---|---|
| $n = 400$, $n_l = 500$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.06 | 0.92 | 1.10 | 0.04 |
| $\Psi$ | 5.38 | 0.42 | 4.68 | 6.03 | 0.41 |
| $x^l$ | 6.21 | 1.00 | 4.22 | 7.38 | 0.26 |
| $x^h$ | 11.2 | 0.67 | 10.2 | 12.4 | 0.30 |
| $n = 200$, $n_l = 500$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.07 | 0.89 | 1.13 | 0.05 |
| $\Psi$ | 5.28 | 0.66 | 4.18 | 6.48 | 0.53 |
| $x^l$ | 6.20 | 1.16 | 3.73 | 7.69 | 0.34 |
| $x^h$ | 11.2 | 0.88 | 9.99 | 12.9 | 0.40 |
| $n = 100$, $n_l = 300$ | | | | | |
| $\theta_{3,1}$ | 1.01 | 0.10 | 0.84 | 1.17 | 0.06 |
| $\Psi$ | 5.43 | 0.81 | 4.26 | 6.74 | 0.75 |
| $x^l$ | 6.38 | 1.42 | 3.65 | 8.43 | 0.51 |
| $x^h$ | 11.4 | 1.14 | 9.70 | 13.3 | 0.58 |

# Alternative Second Step Estimators

- The first step estimation, estimating $V$ from the data is the most important innovation.
- Could do many things in second step: MLE, GMM, etc.
- E.g., can put estimated $V$'s on RHS of Bellman equation and solve for optimal policy at each state, then form moment conditions based on the expected value of the policy at each state.

# Alternative Second Step Estimators

- Because the estimated $V$'s have sampling error in finite samples, there could potentially be a large finite sample bias in the estimators described above.
- One potential solution to this is to aggregate moments
- E.g. Base your moment conditions on expected value of the policy across all states.
- This is asymptotically inefficient, but in finite samples the aggregation aggregates out noise in the estimated $V$'s at each state.

# Some Recent Applications of "BBL"

- Collard-Wexler (2009) – Estimating costs of adjustment in the concrete industry
- Jeziorski (2009) – Estimating merger synergies for radio stations
- Ryan (2009) – Effect of environmental regulations on industry structure in cement
- Sweeting (2009) – Estimating the costs of changing a radio station's format
- Stahl (2009) – Estimating the incentives to merge in broadcast television
- Benkard, Bodoh-Creed, and Lazarev (2010) – Estimating the effects of a particular proposed U.S. airline merger on industry structure over time

# Unsolved Problems in Estimation

Remaining issues:

1. Main issue: unobserved serially correlated state variables
2. Other technical issues such as efficiency issues