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Storability, market structure, and demand-shift incentives

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We consider a two-period model in which buyers can store a good by purchasing in advance of consumption so as to realize potential gains from intertemporal arbitrage. We find that storability introduces a kink in the aggregate period-1 demand. When supply is oligopolistic (quantity setting) and consumers are sufficiently patient (storage cost is relatively low), each firm has a strong current incentive to capture future market share from a rival. As a result, in equilibrium, the price path is increasing and there is rational in-advance purchase by buyers. In contrast, the monopoly and perfectly competitive markets exhibit no such price dynamics. Intermediate storage costs result in multiple equilibria, with at least one that involves advance purchase and one that does not.

1. Introduction

■ We frequently observe sellers offering a storable good at reduced prices relative to those in past or future market periods. Moreover, these price variations are often anticipated by buyers of the good. Familiar examples include weekly supermarket advertisements that encourage buyers to "stock up" on items and the common practice of having a "sale" at the same points in time across the year. In addition to consumers and retail markets, we also observe manufacturers in wholesale markets who offer incentives to purchase in advance. Retailers often take advantage of the low prices offered by a manufacturer during a trade-deal promotion to engage in forward buying and build inventory (see Blattberg and Neslin, 1990). This raises the question of why sellers would want to shift the sales of goods consumed repeatedly by buyers to periods in which prices are relatively low.

In this article we argue for the necessity of price dynamics in the market for a storable good. When supply is imperfectly competitive, we find that market outcomes are characterized by endogenous dynamics even when the underlying cost and demand structure is unchanging

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over time. The economic force behind this result is that an oligopolistic (quantity-setting) firm has a strong incentive to shift demand for a storable good from the future, where it is shared with rivals, into the present by inducing buyers to store the good.[1] In contrast, demand shifting does not arise under perfect competition or pure monopoly, where a repetition of the static single-period equilibrium is stable. Because oligopolistic competitors find it profitable to capture future market share from rivals, the single-period outcome is unstable. Instead, the market has an initial highly competitive or low-price phase in which buyers store the good and then a high-price phase in which buyers draw down inventories for consumption.[2]

Recent empirical work by Erdem, Imai, and Keane (2003) and Hendel and Nevo (2005) focuses on identifying the extent of buyer storage by examining dynamic structural models of consumer demand. Both find that buyers are forward-looking and respond when the current price is low relative to expected future prices. Furthermore, they report that static estimates, which ignore dynamic components, overstate the price elasticity of consumption demand (by at least a factor of 2).

The empirical distinction between static and dynamic price elasticities is helpful for understanding the equilibrium structure and predictions of our model. In Figure 1 we exhibit the static consumption demands, which we take for now to be the same in each period. Incorporating storage demand, which is derived as part of the equilibrium, we have the solid lines in Figure 1 (linearity is convenient for the graph; the model requires only downward-sloping marginal revenue). Demand in period 1 coincides with consumption demand until the price is low enough to trigger storage by buyers (the kink at K). Demand in period 2 is then shifted in from consumption demand, much like a residual-demand curve.

If the good were not storable, the equilibrium would be at point C (static Cournot outcome) in each period and no dynamics arise. When the good is storable, however, the equilibrium is at A in period 1 and then B in period 2, with prices rising and quantity purchased falling. In period 1, we have a low price and buyers are induced to store the good in the amount x as firms compete aggressively, reflecting the demand-shift incentive. Because of storage, residual demand in period 2 is shifted inward from the static consumption demand. Calculating a simple price elasticity, $(q_A - q_B)/q_B$ divided by $(p_A - p_B)/p_B$, we see that a static interpretation overstates the elasticity for two reasons. First, demand at A includes a storage component, which means q_A is above the static level for period 1. Second, the demand q_B at B includes a residual-demand effect and is thus below the static level for period 2. Both effects work toward providing an equilibrium account that is consistent with the dynamic empirical demand elasticity findings noted above.

Our basic model is a standard two-period quantity-setting duopoly market in which we introduce only one structural economic change: the good is storable by the buyers between periods. The firms sell a homogeneous output, which avoids the complication of differentiation issues, and produce at a constant marginal cost. Each buyer seeks to consume a single unit of the good in each period. Buyers are heterogeneous in their valuation of the good, and valuations satisfy the decreasing inverse hazard rate property (a similar demand structure arises for homogeneous consumers with concave utility over continuous quantities). This implies decreasing marginal revenue and, consequently, the static (one-period) version of the game has a unique equilibrium at the standard Cournot outcome.

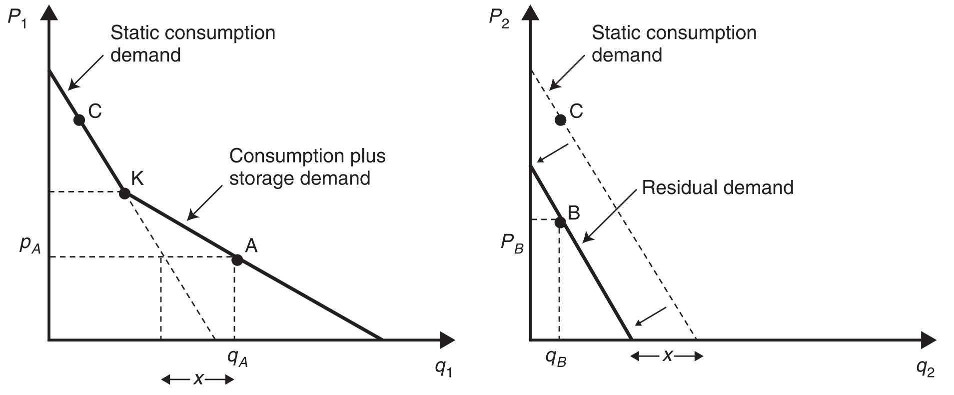
Buyers are allowed to store the good (at no direct cost) by purchasing today for future consumption. All agents are risk neutral and discount future payoffs at the same rate. While only the firms have market power and buyers act as price takers, we want to capture the notion that each buyer makes an optimal storage decision relative to an accurate prediction of how prices will

[1] A similar incentive arises for price setting with differentiated products; quantity setting allows us to study the simpler case of a homogeneous good. With differentiated products, we must keep track of storage for each good, and storage behavior by the buyers becomes more intricate as it involves comparisons of surplus for each good across different prices (leading to regions rather than simple inequalities to determine when storage occurs for one or more of the goods).

[2] Demand shifting also arises with durable goods and with forward contracting, as discussed later in the Introduction.

FIGURE 1

ENDOGENOUS DEMANDS



change over time. Thus, buyers have rational expectations (perfect foresight) regarding equilibrium prices and storage demand is positive only when the current price is below the discounted expected price of the good next period. We also examine comparative statics with respect to demand growth or contraction.

The equilibrium exhibits three important properties: the demand-shift incentive, price arbitrage restrictions generated by storage demand, and the resulting intertemporal price and quantity variation. To gain intuition, consider a repetition of the static Cournot outcome. This is, in fact, a local profit maximum for each firm, and buyers have a strict incentive not to store the good, since the price is expected to remain constant. In Figure 1, point C is a discrete distance from the kink where storage demand becomes positive. This outcome, however, is not a global best response for either firm.

A sufficiently large deviation from the Cournot output will drive the current price down and trigger storage by buyers. Although this depresses prices, the deviating firm is able to capture fully the resulting storage demand rather than share it equally with the rival in the next period. This is necessarily profitable if the discount factor is sufficiently high. As a result, the demand-shift incentive leads to an equilibrium in which buyers are induced to store the good initially at a relatively low price. The price in period 2 must then be higher, consistent with equilibrium storage, in inverse proportion to the discount factor (the price difference will also reflect any physical depreciation or spoilage of the good as well as any storage costs incurred by buyers). On the other hand, if the discount factor is very low, then buyers require a large discount off of the expected future price in order to be induced to store. The associated loss in revenue from selling to the period-1 consumption demand at this discounted price makes it unprofitable for a firm to shift demand. Interestingly, for intermediate values of the discount factor, both storage and no-storage equilibria exist.

The welfare consequences of storage in our analysis are straightforward. Consumers benefit unambiguously because prices are lower and consumption is higher on a period-by-period basis. For firms, profits are lower because the demand-shift incentive leads to aggressive period-1 competition for storage demand and the firms would earn higher overall profits in the absence of storage (a repetition of the static Cournot outcome). It is also clear that a dynamic inefficiency is present, since production costs are incurred prior to consumption for the amount that is stored.

Imperfect competition (oligopoly) is crucial for our results on market dynamics. Under perfect competition the market would remain at a price equal to marginal cost. Similarly, a monopolist would charge the static monopoly price in each period, since inducing storage would involve shifting future demand back to the current period where the price is lower. The absence of a monopoly incentive highlights the role of market structure in our explanation, and potentially constitutes a testable prediction. A second set of predictions involves countercyclical prices.

First, on the endogenous side, Figure 1 illustrates that prices are low and quantities are high in period 1, where storage demand shifts out from consumption demand, while prices are high and quantities are low in period 2, where residual-demand shifts in from consumption demand. Second, exogenous contractions in consumption demand still involve equilibrium storage outcomes and, hence, rising prices.[3]

Demand shifting also occurs with durable goods in oligopoly market settings, where firms have an incentive to sell rather than rent the good (Bucovetsky and Chilton, 1986; Bulow, 1986; Carlton and Gertner, 1989). Equilibrium in these models, however, differs from that in our storable-good model. Consider, for example, the two-period rental and sale model in Bucovetsky and Chilton (1986). Aggregate output (rentals plus sales) and the rental rate for period 1 always lie on the static consumption demand curve; with a durable good, arbitrage involves only the sale price relative to the present discounted value of rental prices. Demand shifting is then costless in the sense that the rental rate in period 1 is not reduced when a unit is sold rather than rented. In contrast, demand for a storable good exhibits a kink, departing from static demand, and this reflects a storage arbitrage condition on the price of the good across periods (recall Figure 1). This makes demand shifting (inducing storage) costly for a seller in a storable-goods market because it reduces the period-1 price. While the durable-goods market has a positive level of sales for any positive discount factor, we find that a storable-goods market will have a no-storage equilibrium as well as a storage one at intermediate discount factors. The contrasting demand structures thus create different strategic incentives and result in qualitatively different market outcomes (for common underlying consumption demands).

Demand shifting also arises when sellers can write forward contracts with buyers. Allaz and Vila (1993) show that if each of two sellers can forward contract with buyers arbitrarily many times before the consumption date, then the equilibrium price must converge to marginal cost. Intuitively, buyers with perfect foresight will not buy forward unless the contractual price is no higher than the expected spot market price. As long as there is a residual demand above the marginal cost, the sellers will find it profitable to contract to sell more. However, this equilibrium outcome differs from that in our storable-good model. A key difference between forward contracting and storage is that the former does not involve purchase in advance—all exchanges take place at the date of consumption. Thus, buyers require no discount from the expected spot market price to enter into a forward contract. In this sense, demand shifting via forward contracting is costless (as in a durable-good market with rentals and sales) and, in equilibrium, there will always be a positive level of forward contracting.[4]

The recent contribution of Hong, McAfee, and Nayyar (2002, HMN hereafter) also examines buyer inventories and imperfect competition in a theoretical framework.[5] In their model, all buyers have the same valuation for the good and the focus is on heterogeneity with respect to price searching and storage abilities. We focus on the heterogeneity of buyers with respect to valuation for the good and, hence, variations in the extent of storage; all buyers in our model are able to store the good. HMN model consumer storage and intratemporal price dispersion where, as in Varian (1980), each oligopolist owns a captive monopoly market (captives) while competing to attract additional buyers (shoppers) who purchase only from the lowest-priced firm and can purchase for storage. Equilibrium involves price dispersion (mixed strategies). Mean prices exhibit intertemporal cyclicality, depending on whether shoppers purchase for inventory at a low realized price in the previous period, a property we also find for the price path in our two-period model.

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[3] Demand-accumulation models of durable goods (e.g., Sobel, 1991) also provide an explanation for price dynamics and intertemporal demand movements. Empirical work seeking to uncover evidence of such pricing, based on a model of imperfect competition, is provided by Pesendorfer (2002).

[4] Distinct from the idea of *demand shifting* is that of *demand building*, which arises in models of dynamic competition with customer switching costs (see Beggs and Klemperer, 1992) or network externalities (see Katz and Shapiro, 1985) where an increase in current market share improves *future* profits.

[5] Trade deals and reduced wholesale prices are examined in the marketing literature by Lal, Little, and Villas-Boas (1997). They find a mixed-strategy equilibrium for wholesale prices in which the retailer can forward buy and hold inventory for later sale to brand-loyal customers. See also Jeuland and Narasimhan (1985).

The fundamental economic assumptions, however, differ significantly across the two models.[6] As a result, the equilibrium forces driving several critical dimensions, such as the level of storage, the effect of storage on future prices, and the existence of arbitrage gains to storage, reflect different economic forces and result in differences in the outcomes. The contrast in model structures is helpful for understanding the economic forces at work, and we discuss these points in more detail when we study storage demand.

We describe the oligopoly storage model in Section 2. We then examine equilibrium storage demand and price arbitrage in Section 3. In Section 4 we prove existence and then characterize equilibria. In Section 5 we derive the monopoly outcome, and in Section 6 we provide the equilibrium foundation for our residual-demand (rationing) approach. Section 7 concludes, and all proofs are in the Appendix.

2. The model

■ Consider the following two-period game, with time indexed by $t = 1, 2$. Two firms, i and j, each choose a quantity q_t^i and q_t^j, respectively, to produce in period t. Let p_t denote the price of the good and $q_t = q_t^i + q_t^j$ the total output in t. Each firm has an identical constant marginal cost of production c, where $c < 1$. Both firms, as well as all buyers, discount future payoffs at $\delta < 1$ per period.

Buyers act as *price takers,* and each buyer consumes either one unit of the good or zero per period. The set of buyers seeking to consume in $t = 1$ is given by a continuum of total mass 1, and buyer valuations for consumption at $t = 1$ are distributed as $F(v)$. Thus, a buyer with value v will purchase for consumption in $t = 1$ whenever $v \geq p_1$, and hence consumption demand for period 1 is

$$Q_1^C(p_1) = 1 - F(p_1). \tag{1}$$

We assume that F has support $[0, 1]$ with $F(0) = 0$, $F(1) = 1$, and that F is twice continuously differentiable with a strictly positive density f on $[0, 1]$. To ensure that marginal revenue is strictly decreasing, we make the standard assumption that the inverse hazard rate, $h(v) \equiv [1-F(v)]/f(v)$, is strictly decreasing in v.

Demand for consumption in $t = 2$ is similar, with the exception that we allow for an expansion or contraction in overall market demand, via a demand parameter λ, in order to examine the impact of market growth on equilibrium. When $\lambda = 1$, consumption demand is time invariant (constant across $t = 1, 2$). $\lambda > 1$ corresponds to a growing market (outward demand shift) and $\lambda < 1$ to a shrinking market (inward shift). To avoid trivial cases (a degenerate market at $t = 2$), we assume $\lambda > c$.

Buyers seeking to consume in $t = 2$ form a continuum of total mass λ, where $\lambda F(v/\lambda)$ buyers have a valuation for period-2 consumption that is less than or equal to v, where $v \in [0, \lambda]$. In the absence of storage opportunities, a buyer with value v will purchase for consumption in $t = 2$ whenever $v \geq p_2$ and period-2 consumption demand will be $\lambda[1 - F(p_2/\lambda)]$. Note that $\lambda[1-F(p_2/\lambda)] \gtrless 1 - F(p_2)$ as $\lambda \gtrless 1$ so that λ corresponds unambiguously to an outward or inward shift of consumption demand relative to period 1. For example, when F is the uniform distribution, λ corresponds to a shift in the intercept of the (linear) demand function, since $\lambda[1 - F(p_2/\lambda)] = \lambda - p_2$.

The model admits several interpretations for demand growth and the set of buyers. For the benchmark case of $\lambda = 1$, simply imagine that all buyers participate in both of the $t = 1, 2$ markets and that an individual buyer with values v_1 and v_2 for $t = 1, 2$ consumption has preferences $v_1 - p_1 + \delta(v_2 - p_2)$. Alternatively, $\lambda = 1$ corresponds to each buyer having a time-invariant

[6] In many respects, the models are complementary (beyond clear differences such as price versus quantity setting). For example, one could introduce captive buyers into our model. Similarly, one could introduce valuation heterogeneity in HMN. On the empirical side, both forms of buyer heterogeneity, valuation and storage/search, appear to be important; see Hendel and Nevo (2005, forthcoming) and Erdem, Imai, and Keane (2003) for more on this point.

valuation, $v_1 = v_2 = v$, where $v \sim F$. To interpret demand growth ($\lambda > 1$), set $v_1 = v$ and $v_2 = \lambda v$ for these buyers and suppose that an additional mass $\lambda - 1$ of buyers, with $v \sim (\lambda - 1)F$, only value period-2 consumption (i.e., the additional buyers have $v_1 = 0$ and $v_2 = \lambda v$). For a demand contraction, suppose that a fraction $1 - \lambda$ of the original buyers exit the market after period 1 (equivalently, this fraction has $v_2 = 0$). Alternatively, we can view the buyers for period-1 and period-2 consumption as distinct populations for which period-1(2) buyers have no value for period-2(1) consumption. Finally, as noted below, the demand structure can also be generated with consumers who have concave utility over continuous quantities and additively separable two-period preferences.

Now consider storage. We make the critical assumption that buyers have the option of purchasing the good in period 1 and storing it for consumption in period 2. Alternatively, a buyer can wait and purchase in period 2. Let $x \geq 0$ denote the total amount of the good stored by all buyers in period 1. Period-2 demand must then account for the fact that any buyer who stored the good will not have a consumption demand in period 2. The standard modelling construction for residual demand in period 2 is to assume that x was purchased by buyers with values at the high end of the distribution. We adopt this residual-demand approach, since it simplifies the analysis relative to working with a more general rationing rule. We show in Section 6 that this is without loss of generality and that it is the definition of equilibrium rather than the rationing rule that determines the extent of storage.

Thus, we assume that storage x is held by buyers with high values for period-2 consumption. Consequently, consumption demand in period 2 is from buyers with $v \leq \lambda F^{-1}(1 - x/\lambda)$ and we have

$$Q_2^C(p_2, x) = \lambda - x - \lambda F(p_2/\lambda), \tag{2}$$

whereby inverse-consumption demand for period 2 is given by

$$P_2(q_2 + x) = \lambda F^{-1}(1 - (q_2 + x)/\lambda). \tag{3}$$

The timing of events is as follows. At each t, the two firms simultaneously choose that period's output level. The current price is then determined by market clearing: aggregate demand from buyers must equal total supply from the firms. To avoid information issues, suppose that each firm observes the period-1 output of the other firm before producing for period 2; thus, we have a proper subgame at the start of period 2.

□ **Period-2 outcomes.** The (subgame-perfect) equilibrium outcome in period 2, given storage $x \geq 0$, follows from standard Cournot analysis for the residual-demand curve (3). We record these results now in order to streamline the subsequent analysis. In period 2, firm i chooses q_2^i to maximize

$$\pi_2(q_2^i, q_2^j + x) \equiv \left[P_2\left(q_2^i + q_2^j + x\right) - c\right] q_2^i. \tag{4}$$

Then, a symmetric interior solution for i (and similarly for j) requires

$$q_2^{i*} P_2'(q_2^* + x) + P_2(q_2^* + x) = c, \tag{5}$$

where $q_2^{i*} = q_2^*/2$. Define a threshold level of storage by $\bar{x} = \lambda[1 - F(c/\lambda)]$. We have the following.

Lemma 1. For each $x \geq 0$, there exists a unique equilibrium outcome for period 2 and the equilibrium is symmetric across firms with $q_2^{i*} = q_2^{j*}$. For $x < \bar{x}$, each firm produces a positive output and price exceeds marginal cost. For $\bar{x} \leq x \leq \lambda$, each firm produces a quantity of zero.

The lemma follows from standard techniques for Cournot analysis (see Vives, 1999). The inverse-hazard assumption implies that the marginal revenue of firm i is strictly decreasing in i's output, and this implies existence. Uniqueness follows from the best reply of each firm being downward sloping with a slope greater than -1 (this follows from a standard monotone comparative statics argument). Symmetry follows from the symmetry of best replies across players.

Period-2 output, $q_2^*(x)$, is given by (5) upon noting that $q_2^{i*} = q_2^*/2$. We then have $P_2^*(x) \equiv P_2(q_2^*(x) + x)$ for the equilibrium period-2 price as a function of the storage level x. Each firm earns $\Pi_2(x) = [P_2^*(x) - c]q_2^*(x)/2$. Of course, when storage is sufficiently large (above $\bar{x}$), no buyer with a valuation above marginal cost is in the market in period 2. In this event, any price above $\lambda F^{-1}(1 - x/\lambda)$ will clear the market, since the resulting demand of zero balances the choice of zero output by the firms. Whenever the period-2 market is active, we have the following.

Lemma 2. Suppose $x < \bar{x}$. Then $P_2^*(x)$ is differentiable in x, with $P_2^{*\prime}(x) < 0$.

Thus, greater storage by buyers in period 1 results in a lower equilibrium price for period 2. We also note, for later reference, that $P_2^*(x)$ increases with λ and, intuitively, an exogenous increase in period-2 consumption demand must result in a higher price (see the proof of Lemma 2). With period-2 outcomes characterized relative to storage, we are ready to specify storage decisions.

□ **Storability and the buyer's problem.** Aggregate demand for period 1 must be derived from the underlying purchase decisions of individual buyers in response to the current market price and the expected future price. Letting p_2^e denote the buyers' (common) expectation of the period-2 market price, consider a buyer with value $v \in [0, \lambda]$ for period-2 consumption and the storage decision. Purchasing at $t = 1$ for p_1 and then consuming at $t = 2$ has payoff $\delta v - p_1$. By waiting until $t = 2$, the buyer can purchase at p_2^e for a payoff of $\delta(v - p_2^e)$. Thus, whenever $p_1 < \delta p_2^e$ the buyer strictly prefers storage to waiting. Of course, v must satisfy $\delta v - p_1 \geq 0$ for the buyer to be willing to make the storage purchase. Thus, the storage decision rule for a buyer with value v is given by

$$D_S(p_1, p_2^e, v) = \begin{cases} 1 & \text{if } p_1 < \delta p_2^e \text{ and } \delta v \geq p_1 \\ \{0, 1\} & \text{if } p_1 = \delta p_2^e \text{ and } \delta v \geq p_1 \\ 0 & \text{if } p_1 > \delta p_2^e. \end{cases} \tag{6}$$

When $p_1 > \delta p_2^e$, the current price is greater than the discounted expected future price, and storage is strictly dominated. On the other hand, when p_1 is below δp_2^e, all buyers with $v > p_1/\delta$ strictly prefer to store the good; implicitly, no buyer expects to purchase the good in period 2 in this case. When equality holds, and $p_1 = \delta p_2^e$, buyers with $v > p_1/\delta$ are indifferent between storing the good and waiting to purchase. Effectively, discounting is a simple device for capturing an economic "cost" to storage, as, for example, buyers will postpone purchases whenever prices are expected to remain constant (or rise only a small amount relative to δ).

The storage decision rule, (6), also emerges in a continuous choice framework. Suppose, for instance, that a large number of identical buyers each have a concave utility function over consumption and can purchase any desired quantities. Then, the storage versus future consumption decision rule follows the same price comparison as above. Further, if we interpret F in terms of the (inverse) marginal utility of consumption, then aggregate demand will also coincide.[7] We note in passing that the model can accommodate a direct (additive) storage cost: if a buyer incurs cost s to store the good for one period, then $p_1 + s < \delta p_2^e$ becomes the threshold for storage. Also, we can allow for a spoilage rate over time: if 1 unit in period 2 requires the storage of $1/\theta$ units, then θ is equivalent to discounting from the buyer's point of view.

We emphasize that aggregate storage demand, denoted by $Q_S(p_1)$, is endogenous in this model. Buyers respond to current and expected future prices when making their storage decision.

[7] Suppose the typical buyer chooses current consumption and storage and plans future consumption in response to p_1 and p_2^e to maximize $U(c_1) - p_1[c_1 + d] + \delta[U(c_2 + d) - p_2^e c_2]$. Then, only one of d and c_2 is positive at an optimum (unless $p_1 = \delta p_2^e$, in which case only the sum of d and c_2 is determined) and the choice depends on p_1 versus δp_2^e. Further, since the optimal choices are characterized by $U'(c_1^*) = p_1$ and $U'(d^* + c_2^*) = \min\{p_1/\delta, p_2^e\}$, the same aggregate consumption and storage demands emerge when we view F in terms of the inverse marginal utility.

To be rational, the period-2 price expectations must be linked to how period-2 market outcomes are affected by the effect of storage on future demand. We will derive $Q_S(p_1)$ once we have introduced the definition of equilibrium.

□ **Firms' profit-maximization problem.** Each firm maximizes the sum of current and future discounted profits. In doing so, they must recognize that greater aggregate sales today imply a smaller size for the residual market tomorrow. By encouraging buyers to store the good (producing more today to drive prices lower), however, an individual firm can capture future market share from its opponent. In this sense, higher individual sales today generate a greater share of the intertemporal market.

In period 1, firm i chooses output q_1^i to maximize

$$\max_{q_1^i} [P_1(q_1^i + q_1^j) - c]q_1^i + \delta \Pi_2(x), \tag{7}$$

for aggregate inverse demand $P_1(q_1)$ and $x = Q_S(P_1(q_1^i + q_1^j))$. Similarly, firm j seeks to maximize its total profit.

□ **Definition of equilibrium.** We can now define equilibrium, taking care to specify the requirements on expectation formation for buyers, as well as period-1 storage demand and market clearing. Since the period-2 outcomes are fully described above for any amount of period-1 storage, x, we omit their explicit specification from the equilibrium requirements.

Definition. A subgame-perfect rational-expectations equilibrium (henceforth, an equilibrium) consists of period-2 price expectations, $P_2^e(p_1)$, storage demand, $Q_S(p_1)$, inverse demand $P_1(q_1)$, and outputs $\{q_1^{i*}, q_1^{j*}\}$, together with the associated period-2 outcomes, such that

(i) *prices are market clearing:* for any $q_1 \in [0, 1 + \lambda]$, at $p_1 = P_1(q_1)$ we have $Q_1^C(p_1) + Q_S(p_1) = q_1$;

(ii) *buyer storage choices are optimal:* $Q_S(p_1)$ aggregates individual buyers' storage decisions, as described in (6), at $(p_1, P_2^e(p_1))$ for all $p_1 \in [0, 1]$;

(iii) *buyer expectations are rational:* $P_2^e(p_1) = P_2^*(x)$ at $x = Q_S(p_1)$ for all $p_1 \in [0, 1]$;

(iv) *firm profit maximization:* given q_1^{j*}, q_1^{i*} solves the profit-maximization problem in (7), and similarly for j.

Condition (i) states that the price in period 1 is set to clear the market in the sense that if firms produce q_1 in total, then buyers are willing to purchase this quantity at the price p_1. Thus, each firm can employ $P_1(q_1)$ to assess the profit consequences of different output levels. Condition (ii) specifies that whenever there are gains to buying in advance of consumption, buyers will act to realize such gains. This storage choice depends critically on buyer price expectations, and condition (iii) requires that buyers have rational beliefs about how future price depends on current price. It is important to note that this condition applies not only for a realized equilibrium price p_1^* but also at alternative period-1 prices that are off of the equilibrium path. Thus, if a firm overproduces relative to equilibrium, then buyers accurately assess the implications for future prices.[8] Finally, condition (iv) is a standard duopoly (Nash) equilibrium requirement relative to the payoff structure (which incorporates period-2 outcomes).

3. Storage demand and price arbitrage

■ In this section we characterize aggregate storage demand and the resulting restrictions on prices. Given a price p_1, storage demand is endogenous because storage will affect the equilibrium

[8] Buyer price expecations can equivalently be described directly in terms of $P_2^*(x)$, if buyers observe the storage level. The equilibrium definition implies that buyers can make the correct inference observing only the period-1 price.

price in period 2. The resulting price in period 2 must, however, be consistent with the initial choice of buyers to store the good in period 1. We begin with arbitrage restrictions on equilibrium prices.

□ **Price arbitrage.** We know, from (6), that buyers are indifferent about buying in advance when $p_1 = \delta p_2^e$. Given this, quantity is indeterminate and optimal storage requires only that x lies between 0 and $\lambda[1 - F(p_1/\lambda\delta)]$. Equilibrium, however, implies a tighter characterization of storage behavior.

Lemma 3. In equilibrium, if $x > 0$, then $p_1 = \delta P_2^e(p_1)$; further, $x < \bar{x}$ and $q_2^*(x) > 0$ also hold.

Intuitively, Lemma 3 states that whenever buyers choose to store the good in equilibrium, prices leave them indifferent. Effectively, the option to store brings the discounted period-2 price into equality with the current price and eliminates any strictly positive arbitrage gains. The proof of Lemma 3 is also instructive as it implies that, in equilibrium, it can never be the case that all period-2 consumption is bought in advance in period 1. In other words, there must be positive sales in period 2.

□ **Storage demand.** We can now derive aggregate storage and consumption demand for period 1. Formally, for any given price p_1, we must find the aggregate storage quantity $Q_S(p_1)$ that is consistent with an optimal storage choice (6) when buyers hold rational expectations about the consequences of storage for period-2 prices, as required by condition (iii) for equilibrium.

To begin, recall from above (just after Lemma 2) that stronger period-2 demand (larger λ) will increase the period-2 price. Intuitively, we can expect that extremely large or small values for λ will lead to a trivial storage outcome. To rule out such cases, which are easily dealt with as limiting cases of the analysis, assume that

$$c < \delta p_2^C < 1,$$

where $p_2^C \equiv P_2^*(0)$ is simply the (static) equilibrium Cournot price for period 2 (i.e., no storage and demand of $q = \lambda[1 - F(p/\lambda)]$ in period 2); implicitly, λ is in an open interval containing 1.

Now consider storage demand when $p_1 > \delta p_2^C$. We claim buyers will not store at such a price. The reason is that when $x = 0$, we will have a relatively low period-2 price. Formally, $P_2^*(0) = p_2^C = P_2^e(p_1)$, and we then have $p_1 > \delta P_2^e(p_1)$, which means that waiting to purchase in period 2 is optimal. Any $x > 0$ would only depress the period-2 price and reinforce the decision to wait. Hence, we have $Q_S(p_1) = 0$ at any such p_1.

Now suppose $p_1 < \delta p_2^C$. Then, if $x = 0$, we have $P_2^*(0) = p_2^C = P_2^e(p_1)$. Hence, $p_1 < \delta P_2^e(p_1)$, and so all buyers with a period-2 valuation above p_1/δ have a strict incentive to purchase in period 1 and store the good. Consequently, $Q_S(p_1) > 0$ at any such price. In equilibrium, the extent of storage must be sufficiently large to pull the period-2 price down to p_1/δ. To find this level of storage, recall that by no-arbitrage (Lemma 3), we have $p_1 = \delta P_2^*(x)$ whenever $x > 0$. Then, in order for the aggregate storage behavior of buyers to be consistent with equilibrium expectations for period 2, we must have

$$Q_S(p_1) \equiv x = [P_2^*]^{-1}(p_1/\delta). \tag{8}$$

Recall that $P_2^*(x)$ is well defined and invertible (as it is strictly decreasing) for all $x \leq \bar{x}$. If $x > \bar{x}$ (i.e., when $p_1 < \delta c$), then residual period-2 demand is such that no firm can profitably produce and sell in period 2. As noted after Lemma 1, this market outcome with no sales is consistent with any period-2 price that is sufficiently high. To have $Q_S(p_1)$ well defined for all p_1, it is convenient to adopt the convention that if $p_1 < \delta c$, then $[P_2^*]^{-1}(p_1/\delta) \equiv \lambda[1 - F(p_1/\lambda\delta)]$. Intuitively, if $p_1 < \delta c$, then all period-2 buyers who value the good at or above p_1/δ will purchase in period 1, knowing that in period 2 there will be no market transactions. We have thus established the following.

Lemma 4. In equilibrium, $Q_S(p_1)$ is nonincreasing and differentiable everywhere (except at $p_1 = \delta p_2^C$) for all $p_1 \geq 0$. Further,

$$Q_S(p_1) = \begin{cases} 0 & \text{if } p_1 > \delta p_2^C \\ [P_2^*]^{-1}(p_1/\delta) & \text{if } p_1 \leq \delta p_2^C. \end{cases} \quad (9)$$

Aggregate demand in period 1, which is the sum of consumption and storage demand, is then

$$Q_1(p_1) = \begin{cases} 1 - F(p_1) & \text{if } p_1 > \delta p_2^C \\ 1 - F(p_1) + [P_2^*]^{-1}(p_1/\delta) & \text{if } p_1 \leq \delta p_2^C. \end{cases} \quad (10)$$

Since F and $[P_2^*]^{-1}$ are each differentiable (except at $p_1 = \delta p_2^C$), the inverse aggregate-demand function, $P_1(q_1)$, is differentiable everywhere except at the point $q_K \equiv 1 - F(\delta p_2^C)$. Moreover, since $P_2^*(x)$ is strictly decreasing, we see that inverse aggregate period-1 demand decreases everywhere and has a kink at the point q_K. Intuitively, at prices above δp_2^C, buyers do not seek to buy in advance because the period-2 price will be sufficiently low (at p_2^C). In this case, aggregate period-1 demand consists only of consumption demand for period 1. At prices below δp_2^C, however, some buyers purchase their period-2 consumption in advance and store the good. Consequently, at any price below δp_2^C, the aggregate period-1 demand lies above the level of period-1 consumption demand. Graphically, this corresponds to a kink (see Figure 1 above) at the price δp_2^C and quantity q_K where demand rotates outward, becoming "flatter" in the sense that the slope is less negative, once buyers start augmenting their consumption demands with storage demand.

Storage by buyers thus provides an equilibrium rationale for a kinked-demand function in oligopoly markets. The economic logic for the kink, however, extends beyond the oligopoly setting. As we will see in Section 5, the period-2 pricing behavior of a monopoly supplier also generates a kink in period-1 demand, albeit at a higher price. In both of the oligopoly and monopoly settings, period-2 prices are determined via the residual-demand structure given prior storage. Thus, the kink in demand arises when current and future prices are related via the arbitrage condition and the future price is negatively related to the extent of storage.

The contrast in storage demand between our model and that in HMN is helpful for understanding the economic forces created by the underlying source of buyer heterogeneity. In our model, period-2 prices fall with the extent of storage (recall Lemma 2) because buyers have different valuations for the good. Thus, whenever prices induce storage, it is necessarily the case that buyers with valuations that are high relative to current prices are the ones who store the good. Buyers from the lower end of the valuation distribution necessarily remain in the market and seek to purchase for consumption in period 2. In contrast, in HMN, heterogeneity takes the form of captives versus shoppers. There, storage implies that shoppers are removed from future consumption demand. Storage then leads to higher prices (a higher mean price in the equilibrium dispersion) in the future period, since each firm has a stronger incentive to price high and extract surplus from their captive buyers.

Two further points on heterogeneity now follow directly. First, with valuation heterogeneity, the extent of storage can vary smoothly and it is part of the equilibrium determination. With a common valuation, all buyers tend to be on the same side of the storage decision. Thus, the storage level in HMN coincides with the number of shoppers when storage occurs, and it is only the price dispersion that adjusts in equilibrium. A common property of both models, however, is the presence of an externality among buyers: storage at the aggregate level, as discussed by HMN, influences future prices. Second, valuation heterogeneity leads to a tight arbitrage linkage in prices across periods when storage occurs (recall Lemma 3), for the same reason that the extent of storage demand varies as buyers with different valuations become active.

□ **A linear-demand example.** Suppose F is uniform on [0, 1]. Then (3) reduces to $P_2(q_2+x) = \lambda - (q_2 + x)$. The corresponding period-2 price is easily verified to be $P_2^*(x) = (1/3)[\lambda - x + 2c]$. With $p_1 = \delta P_2^*(x)$ from Lemma 3, we have, for $0 < x < (\lambda - c)$,

$$p_1 = \delta P_2^*(x) = \frac{\delta}{3}[\lambda - x + 2c]. \tag{11}$$

Upon rearranging, we have storage demand of

$$Q_S(p_1) \equiv x = \lambda + 2c - 3p_1/\delta. \tag{12}$$

Aggregate demand in period 1 is then given by

$$Q_1(p_1) = \begin{cases} 1 - p_1 & \text{if } p_1 > \delta(\lambda + 2c)/3 \\ 1 + \lambda + 2c - (1 + 3/\delta)p_1 & \text{if } \delta(\lambda + 2c)/3 \geq p_1 \geq \delta c \\ 1 + \lambda - (1 + 1/\delta)p_1 & \text{if } \delta c > p_1. \end{cases} \tag{13}$$

The period-1 demand panel in Figure 1 exhibits the main qualitative features of this example.

4. Equilibria

■ The basic economic forces that will determine equilibrium outcomes can now be identified. On the one hand, the static Cournot outcome is a natural candidate. Since this outcome lies above the kink in the demand function, it is a local simultaneous best response for the firms: small changes in quantity will not trigger storage, and each firm is at a local profit maximum. The kink in demand, on the other hand, provides a market "carrot": a sufficiently large output increase will trigger storage, and the resulting increase in sales (above period-1 consumption demand) may make the deviation profitable. We refer to this as the "demand-shift incentive," and our main result, established below, is that this incentive is sufficiently strong that equilibrium always involves storage whenever δ is not too small.

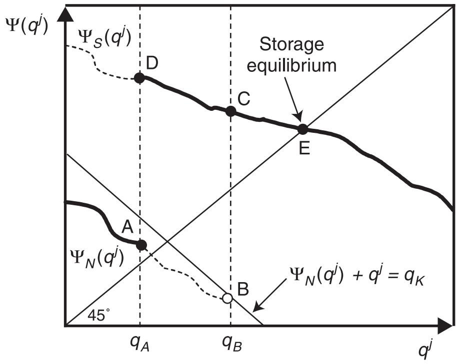
Thus, our goal in this section is to establish the existence of equilibrium and characterize when storage occurs. The results apply to a fairly broad set of demand settings (value distributions) and only rely on the assumption of downward-sloping marginal revenue (the inverse-hazard property). While examples, including the linear-demand example presented later in this section, have best-response functions with a simple structure, this cannot be expected in general. Consequently, this section has a more prominent technical component.

Figure 2 provides an overview of the main equilibrium issues. The solid curve shows the global best response of i to output q^j from j. The first feature to note is how i responds to the output level of q_B by j. Based purely on static profit incentives (period-1 consumption demand), i would respond to q_B such that aggregate output is at the kink q_K; thus, point B lies on the familiar static Cournot best response (labelled Ψ_N). But marginal revenue jumps upward at q_K, so this cannot be i's global best response. Instead, the best response lies directly above at point C, on the Ψ_S curve, where aggregate output involves positive storage. The higher output for i reflects the demand-shift incentive. Intuitively, higher output from j always moves the market closer to the demand kink and increases the incentive for i to respond with an output that induces storage.

Point A in Figure 2 is where the demand-shift incentive for i first dominates the static incentive. Thus, the best response jumps from point A, the static Cournot response, to the higher level at point D. Since the jump occurs before Ψ_N hits the 45° line, the static Cournot outcome is not an equilibrium. Instead, the equilibrium is at point E in Figure 2, where Ψ_S hits the 45° line. In general, however, the Ψ_S branch of the best response need not be as well behaved as in Figure 2. Further, the location of the jump point q_A is critical for when equilibrium involves storage, and we must relate it to the effects of demand growth (λ) and the implicit cost of storage (δ).

FIGURE 2

BEST-RESPONSE AND STORAGE EQUILIBRIUM



□ **Existence.** To begin the equilibrium analysis, recall the discounted sum of profits over both periods from (7):

$$\Pi(q^i, q^j) \equiv [P_1(q^i + q^j) - c]q^i + \delta\Pi_2(x), \tag{14}$$

where $x = Q_S(P_1(q^i + q^j))$; we drop the t subscript when it is clear that we are considering period-1 outputs. In period 1, each firm chooses quantity to maximize this discounted profit sum. Given the kink at $q_K \equiv 1 - F(\delta p_2^C)$, the best-response quantity choice will exhibit discontinuities and may also be multivalued. Consequently, we work with the best-response correspondence

$$\Psi(q^j) = \operatorname*{argmax}_{0 \le q^i \le 1+\lambda} \Pi(q^i, q^j). \tag{15}$$

Quantity choices above $1+\lambda$ imply a price of zero in period 1. Since $\Pi(q^i, q^j)$ is continuous on a compact set, we know that $\Psi(q^j)$ is nonempty for all $q^j \in [0, 1+\lambda]$. We must identify when a firm will find it optimal to induce buyers to store the good. To this end, we can examine the best-response problem in terms of two subproblems. First, consider the best-response problem when i is restricted to choosing a quantity that induces storage by buyers, as given by

$$\Psi_S(q^j) = \operatorname*{argmax}_{\max\{q_K - q^j, 0\} \le q^i \le 1+\lambda} \Pi(q^i, q^j), \tag{16}$$

where $q^j \in [0, 1+\lambda]$. By construction, the domain for the q^i choice implies that we are on the storage region of the period-1-demand function. As with Ψ, we know Ψ_S is nonempty. Next, consider the best-response problem when i can feasibly choose a quantity that does not induce storage, as given by

$$\Psi_N(q^j) = \operatorname*{argmax}_{0 \le q^i \le q_K - q^j} [F^{-1}(1 - (q^i + q^j)) - c]q^i, \tag{17}$$

where $0 \le q^j \le q_K$. The specified quantity ranges imply that we are in the no-storage region of period-1 demand. Consequently, the payoff $\Pi(q^i, q^j)$ reduces to the period-1 profit flow plus $\delta\Pi_2(0)$. In fact, it is clear that $\Psi_N(q^j)$ must be a singleton set, since it is a constrained version of the best-response problem for a firm that faces the (static) period-1 consumption-demand function. This best-response function is (static period-1 Cournot market)

$$\varphi_C(q^j) = \operatorname*{argmax}_{0 \le q^i \le 1+\lambda} [F^{-1}(1 - (q^i + q^j)) - c]q^i. \tag{18}$$

It is routine to verify (a special case of Lemma 1) that $\varphi_C(q^j) = 0$ when $q^j \ge 1 - F(c)$.

Further, when $q^j < 1 - F(c)$, we find $\varphi_C(q^j) = 1 - F(p) - q^j$, at the unique p that satisfies $p - c = [1 - F(p) - q^j]/f(p)$; in this case, $\varphi_C(q^j)$ is positive, differentiable, and strictly decreasing with a slope greater than -1 (under the hazard assumption on F).

We employ these best responses to identify when storage arises. For $q^j > q_K$, the opposing firm is at a high output level and we have $\Psi(q^j) = \Psi_S(q^j)$, since storage necessarily occurs at these output levels. On the other hand, when $q^j \leq q^K$, and the opposing firm is producing at a relatively low level, we have $\Psi(q^j) \subset \Psi_S(q^j) \cup \Psi_N(q^j)$. Intuitively, if the period-1 output of firm j is relatively low, i can restrict output so as not to induce storage. Each firm then earns the per-firm Cournot profit in period 2 corresponding to the full consumption demand. Alternatively, i can expand output to induce storage by buyers, reducing the period-1 price as well as the per-firm period-2 profits (due to the reduced consumption demand in period 2). However, in expanding output, i alone sells to the entire shifted period-2 demand, which if not shifted to period 1 would have been shared equally between i and j in period 2. As a result, the higher share of the intertemporal output that i captures in period 1 (at the expense of j's period-2 sales) may make it profitable for i to expand output and induce storage.

The next lemma relates the best responses and shows that an equilibrium exists.

Lemma 5. The following properties hold for the best-response correspondence:

(i) $\Phi(q^j) \equiv \{q^i + q^j \mid q^i \in \Psi(q^j), q^j \in [0, 1 + \lambda]\}$ is a strongly increasing correspondence;

(ii) if $\Psi_N(\hat{q}^j) \subset \Psi(\hat{q}^j)$ for some $\hat{q}^j$, then $\Psi(q^j) = \Psi_N(q^j)$ for all $q^j < \hat{q}^j$; further, if $\hat{q}^i + \hat{q}^j < q_K$, where $\hat{q}^i \in \Psi_N(\hat{q}^j)$, then $\Psi_S(q^j) \cap \Psi(q^j) = \emptyset$;

(iii) if $\Psi_S(\hat{q}^j) \subset \Psi(\hat{q}^j)$ for some $\hat{q}^j$, then $\Psi(q^j) = \Psi_S(q^j)$ for all $q^j > \hat{q}^j$; further, if $\hat{q}^i + \hat{q}^j > q_K$, where $\hat{q}^i \in \Psi_S(\hat{q}^j)$, then $\Psi_N(q^j) \cap \Psi(q^j) = \emptyset$.

Finally, there exists an equilibrium.

In a static Cournot game, the aggregate quantity analog of Φ is strongly increasing whenever aggregate inverse demand is strictly decreasing (see Vives, 1999). Even though our model is dynamic, Φ continues to be strongly increasing because equilibrium period-2 profit is fully determined by the aggregate period-1 output, q_1, via x. Existence of (pure-strategy) equilibrium follows from the fact that reaction functions do not have discontinuities of the "jump down" variety, and therefore must intersect the 45° line (Tarski's intersection point theorem; see also Roberts and Sonnenschein (1976)).

□ **Storage equilibria.** When is the demand-shift incentive sufficiently strong that a firm will induce storage? Of course, this is trivial if q^j is sufficiently large (above q_K). The important case arises when q^j is relatively low and firm i has the option to produce at a low level where storage is not induced, as well as the option to produce at a high level and induce storage. We find that it is typically optimal to induce storage well before j's output level reaches q_K. To this end, let q_B denote the unique output for j at which $\varphi_C(q_B) + q_B \equiv q_K$. That is, q_B induces a static Cournot best response from i such that we are at the kink in the period-1 demand function. Recalling that $q_K \equiv 1 - F(\delta p_2^C) \in [0, 1]$, we see that $q_B = q_K$ whenever $q_K \geq 1 - F(c)$ or, equivalently, when $\delta p_2^C \leq c$. Otherwise, we have $0 < q_B < q_K < 1 - F(c)$.

Now consider the first time Ψ can jump from inducing no storage to storage. At $q^j = q_B$, we know from above that $\varphi_C(q_B) + q_B = q_K$. Hence, a global best response for i to $q^j = q_B$ will necessarily induce storage, since the upward jump in marginal revenue for i when total output is at the kink implies that i can increase profit by choosing $q^i > \varphi_C(q_B)$. By Lemma 5 (iii), we then have $\Psi(q^j) = \Psi_S(q^j)$ for all $q^j \geq q_B$, so the first time the best response for i jumps from no storage to storage must occur at or below $q^j = q_B$. Defining $q_A \equiv \inf\{q^j \mid \Psi_S(q^j) \subset \Psi(q^j)\}$, we have $q_A \leq q_B$, and, by Lemma 5 (ii), q_A is the first and only time the best response Ψ jumps from no storage to storage. We then have the following.

Lemma 6. Let $\alpha \equiv 1 + (1/2)q_2^{*\prime}(0)$. Then $q_A < q_B$ if (DS) holds, where (DS) is the inequality

$$\delta > \frac{c}{\alpha c + (1-\alpha)p_2^C}.$$

Recall that the period-1 demand function has a kink at the price δp_2^C and quantity q_K and "flattens out" at lower prices where storage demand is positive. The kink implies that there is a jump in marginal revenue at q_K. Lemma 6 establishes a threshold (DS) for demand storage. Since the demand function "flattens" at q_K, there is a jump in marginal revenue at q_K, implying that, purely from the revenue standpoint, i is better off increasing its own output. (Thus, aggregate output can never be equal to q_K in equilibrium.) This gain is tempered by the reduced period-2 profit for i as a result of the consequent reduction in residual demand. Therefore, the net effect favors demand shifting only if the jump in marginal revenue is sufficiently large. To gain intuition for (DS), consider the special case of $\lambda = 1$ (identical consumption demand in both periods). The kink in demand is located at price δp_2^C and quantity $q_K = 1 - F(\delta p_2^C)$. But with $\lambda = 1$, the static Cournot equilibrium outcomes for periods 1 and 2 coincide at p_2^C. Thus, as δ increases, the kink in period-1 demand is moving closer to the static Cournot outcome for period 1 (reaching it in the limit when $\delta = 1$). Similarly, q_B is approaching the per-firm static Cournot output. Thus, as δ becomes large, if j produces at q_B, then a very small increase in output by i above the Cournot level will push the period-1 outcome beyond the demand kink, and, as a result, firm i will benefit from the associated upward jump in marginal revenue. Of course, this logic continues to apply if q^j is sufficiently close to q_B, and, therefore, the best response of i will jump from no storage to storage before j's output reaches q_B (i.e., $q_A < q_B$ must hold). Thus, we see that (DS) is a sufficient condition on δ and, implicitly, λ such that q_A is strictly below q_B. Since $p_2^C > c$, it follows directly that $1 > c/[\alpha c + (1-\alpha)p_2^C]$, thereby establishing the validity of the threshold for δ in (DS).

We then have the following.

Proposition 1. There exist $\delta^* < 1$ and $\lambda^* < 1$ such that, for any discount factor $\delta > \delta^*$ and any demand growth parameter $\lambda > \lambda^*$, all equilibria have strictly positive storage.

With $\lambda = 1$ and δ sufficiently large, equilibrium storage implies that price increases from periods 1 to 2 even though the exogenous periods-1 and -2 consumption demands are identical. More subtly, we see that the equilibrium price increases even though the endogenous period-2 market demand, reflecting positive storage, is less than the period-1 market demand (in equilibrium, quantity demanded in period 2, at any given price, is strictly less than that for period 1).

Of course, a storage equilibrium will continue to exhibit an increasing price path when $\lambda > 1$ (consumption demand is growing). Note, however, that storage equilibrium prices are still below the prices corresponding to the static Cournot equilibrium outcomes for each period (this always holds for period 2 and will also hold for period 1 except when λ is very large). A more surprising property is the prediction that prices will increase even in market settings where $\lambda < 1$ and consumption demand is declining. Formally, for $\delta > \delta^*$, there exist values of λ strictly less than one, such that any equilibrium must involve strictly positive storage and, therefore, an increasing price path. In other words, an exogenous decline in consumption demand is accompanied by an increase in the market price.[9]

□ **No-storage and multiple equilibria.** We also want to examine how demand growth and discounting influence the set of equilibria. The key to this involves understanding how q_A, the

[9] A two-period setting is limited with respect to the dynamics of exogenous shifts in consumption demand in that only one of the market responses (in anticipation of the shift or subsequent to the shift) can be examined. The recent empirical work by Chevalier, Kashyap, and Rossi (2003) examines pricing with respect to imperfect competition and finds countercyclical prices during demand peaks. To the extent that normal periods of demand correspond to no-storage (static) outcomes, a shift to storage outcomes at peak demand times is then consistent with lower prices and higher quantities. Clearly, however, a multiperiod model is needed to address these dynamic transitions.

best-response jump point, moves as δ and λ vary. We will focus on δ; to this end, let us fix $\lambda = 1$ for simplicity.[10] We then have the following.

Lemma 7. Let $\lambda = 1$. Then, for $\delta \in [0, 1]$, the best-response jump point q_A is continuous and nonincreasing in δ. Further, q_A is strictly decreasing whenever $q_A > 0$.

Lemmas 6 and 7 share a common intuition. As δ increases, storage demand shifts up and the kink occurs at a higher price. Thus, holding q^j fixed, it becomes more profitable for i to increase output. Since period-1 consumption demand is independent of δ, this increase in i's output makes equilibrium storage more likely. As a result, whenever positive, q_A decreases with an increase in δ. We then have the following.

Proposition 2. Let $\lambda = 1$. Then there exists a discount factor δ^n with $c/p_1^C < \delta^n < 1$ such that the equilibrium with no storage exists if and only if $\delta \leq \delta^n$.

Propositions 1 and 2 together with equilibrium existence (Lemma 5) imply that for some values of δ strictly less than δ^n, there may exist both storage and no-storage equilibria. Recall that the no-storage equilibrium is simply the static Cournot equilibrium. Further, since storage is induced only at period-1 prices that are lower than the discounted static Cournot price from period 2, this implies that from the seller's perspective, the no-storage equilibrium is Pareto superior to any storage equilibrium. Interest in such multiplicity of equilibria is then natural. As a comparative static exercise, we wish to identify the set of δ values for which there exist both no-storage and storage equilibria. Toward that end, observe that storage equilibria correspond to points of intersection between the 45° line and the correspondence $\Psi \cap \Psi_S$. Naturally, in order to identify the full set of values of δ for which storage equilibria exist, we need to know how Ψ_S behaves as we vary δ. It turns out that, without imposing stronger assumptions on demand, little can be said about whether or not Ψ_S is monotonic with respect to δ. However, the case in which F is uniform has a closed-form solution.

Proposition 3. Suppose F is uniform and $\lambda = 1$. Then there exist discount factors δ^s and δ^n with $c/p_1^C < \delta^s < \delta^n < 1$ such that

(i) if $\delta^n < \delta$, then there exists a unique equilibrium and it has strictly positive storage;

(ii) if $\delta^s \leq \delta \leq \delta^n$, then there exist exactly two equilibria, one with no storage (the repeated static Cournot outcome) and one with strictly positive storage;

(iii) if $\delta < \delta^s$, then there exists a unique equilibrium and it has no storage (the repeated static Cournot outcome).

Recall that $1/\delta$ serves as the storage cost to buyers. When δ is sufficiently high, buyers require a small discount off the expected future price in order to be induced to store. Then, inducing storage is relatively inexpensive to firms, and each firm, in an effort to shift future demand from its rival, overproduces in period 1 (relative to its static Cournot output), resulting in buyer storage. When δ is sufficiently low, buyers require a large discount in order to store. As a result, each firm finds it too costly to capture future market share from its rival and, in equilibrium, each produces its static Cournot output so that no storage occurs.

Finally, for storage costs in the intermediate range, there are two equilibria. One (static Cournot) has no storage. In this case, the rival firm is producing a relatively small output, and inducing storage involves such a large price reduction that a firm finds it too costly to do so. The other equilibrium (possibly multiple when F is not uniform) is one in which both firms overproduce, thus inducing storage. In this case, the rival firm is producing a relatively high output—as a result, the kink in the demand function is sufficiently close that it is profitable for a firm to overproduce and induce storage.

[10] The argument that follows can be applied at λ above and below one. The added complication is that we must also introduce corner conditions on the critical discount factors, since more extreme λ values will necessarily eliminate storage or no-storage equilibria across all discount factors. Even with $\lambda = 1$, the proof of Lemma 7 is fairly involved.

With regard to our demand kink, one might expect buyer heterogeneity in storage costs to "smooth out" the kink. However, for a simple example in which buyers differ in the discount (or spoilage) factor and $\delta \sim$ uniform $[0, \overline{\delta}]$, the kink persists but the location moves to $\overline{\delta} p_2^C$. Thus, when $\overline{\delta} = 1 = \lambda$, the demand kink coincides with the Cournot price and only storage equilibria remain. (We thank the Editor for suggesting that we examine this point.)

□ **A numerical example.** Extending our linear-demand example, consider $\lambda = 1$ and $c = 0$. From Proposition 3, a no-storage (i.e., Cournot) outcome and a storage outcome are the two candidates for equilibria. Thus, we solve explicitly to identify the critical discount factors, δ^s and δ^n, that characterize existence. Solving for the no-storage outcome yields $q_c^{i*} = 1/3$ (the Cournot outcome). Next, from (16), the first-order condition for the storage best response of i is

$$2 - [q^j + 2\Psi_S(q^j)] - \frac{2}{9}(1-x)\frac{3}{3+\delta} = 0.$$

Storage is, from (12), $x \equiv Q_S(P_1(q_1)) = [3q_1 - (3-\delta)]/(3+\delta)$, and then we find $\Psi_S(q^j) = [(1+\delta)(2-q^j)]/[2(2+\delta)]$. Solving for the symmetric outcome, we find that $q_s^{i*} = 2(1+\delta)/(5+3\delta)$ is the candidate equilibrium storage outcome.

We find the critical discount factors, δ^s and δ^n, by determining when each of q_c^{i*} and q_s^{i*} is a global best response and, hence, an equilibrium output. For the no-storage outcome, we employ (5) to calculate the profit $\Pi(q_c^{i*}, q_c^{i*})$ and compare this to the profit $\Pi(\Psi_S(q_c^{i*}), q_c^{i*})$ for an optimal deviation by i that involves storage (taking care to note that the corner solution for Ψ_S will apply at small δ). Solving, we find $\delta^n = .825$. Similarly, for the storage outcome, we compare $\Pi(q_s^{i*}, q_s^{i*})$ to the deviation profit $\Pi(\Psi_N(q_s^{i*}), q_s^{i*})$ and then solve to find $\delta^s = .626$. For a graph of these outputs relative to period-1 demand, refer back to Figure 1.

5. Monopoly prices

■ Consider the dynamic pricing problem confronting a monopolist who faces buyers with rational storage behavior. We are interested in settings where the monopolist cannot commit to period-2 prices in advance. Then, given storage of $x \geq 0$, the monopoly problem in period 2 is to choose p_2 to maximize

$$\pi_2^M(p_2, x) \equiv (p_2 - c)Q_2^C(p_2, x),$$

with $Q_2^C(p_2, x)$ given by (2). For large storage, $x \geq \bar{x}$, it is optimal to shut down in period 2. When $x < \bar{x}$, the monopolist chooses to price at the unique $P_2^M(x)$ at which marginal revenue equals marginal cost, namely, the solution to

$$p_2 - \frac{\lambda[1 - F(p_2/\lambda)] - x}{f(p_2/\lambda)} = c.$$

(These results follow directly from the proof of Lemma 1 for a rival firm output of zero.) Of course, quantity then follows from demand, $Q_2^M(x) = Q_2^C(P_2^M(x), x)$. The period-2 monopoly payoff is then $\Pi_2^M(x) = (P_2^M(x) - c)Q_2^M(x)$, and, by the envelope theorem, $\Pi_2^{M\prime}(x) = -(P_2^M(x) - c)$. Finally, note that we can obtain the static period-1 monopoly solution simply by setting $x = 0$ and $\lambda = 1$ in the above period-2 problem; for later reference, let P_1^M denote this monopoly price.

In full analogy to the duopoly case, we find that rational storage behavior by buyers implies

$$Q_1^S(p_1) \equiv x = \begin{cases} 0 & \text{if } p_1 > \delta P_2^M(0) \\ [P^M]^{-1}(p_1/\delta) & \text{if } p_1 \leq \delta P_2^M(0), \end{cases} \qquad (19)$$

where $P_2^M(0)$ is the optimal monopoly price in period 2 when $x = 0$ (for the full-consumption demand of $P_2(q_2) = \lambda F^{-1}(1 - q_2/\lambda)$). Also, as with Lemma 3, we find that $x > 0$ implies

$p_1 = \delta P_2^M(x)$. We can now examine the period-1 problem for the monopolist in terms of two components: (i) whether to induce storage or not and then (ii) what price to charge. Choosing no storage ($x = 0$) constrains the price choice to be high, and we have the no-storage (NS) problem

$$\max_{p_1 \geq \delta P_2^M(0)} (p_1 - c) Q_1^C(p_1) + \delta \Pi_2^M(0).$$

Choosing storage ($x > 0$) necessarily fixes $p_1 = \delta P_2^M(x)$, and we have the storage (S) problem

$$\max_{x>0} [\delta P_2^M(x) - c][Q_1^C(\delta P_2^M(x)) + x] + \delta \Pi_2^M(x).$$

(The payoffs for the two problems coincide at $x = 0$ and $p_1 = \delta P_2^M(0)$.) We then have the following.

Proposition 4. Suppose that $P_1^M > \delta P_2^M(0)$. Then the monopolist maximizes profit by charging the static monopoly price in period 1, $p_1^* = P_1^M$, and induces no storage in equilibrium.

On the equilibrium path the monopolist then charges $P_2^M(0)$ in period 2. The condition on static monopoly prices in Proposition 4 is a fairly weak requirement. In the benchmark case of constant demand across periods, the condition holds for all discount rates, since $P_1^M = P_2^M(0)$ when $\lambda = 1$. It also holds for a range of λ above one when $\delta < 1$, and it clearly holds for all $\lambda < 1$.[11]

Proposition 4 is intuitive: a monopolist has no strategic incentive to try to shift demand. The only potential motivation for demand shifting by the monopolist is discounting, as each dollar of period-2 profit is worth less than a dollar of period-1 profit. However, if buyers have the same discount factor as the monopolist, then rational storage requires that the period-1 price be exactly equal to the discounted period-2 price. As a result, on the marginal unit that the monopolist sells toward storage, the lower period-1 price exactly offsets the gain from demand shifting. On the other hand, if $P_1^M > \delta P_2^M(0)$, then any pair of period-1 and -2 prices that induce buyer storage must be less than the static (per-period) profit-maximizing monopoly prices. Therefore, any pair of prices that induces buyer storage in a monopoly will result in a profit that is strictly less than the profit from charging the optimal static monopoly prices.

6. Equilibrium demand rationing

■ We have assumed that the period-2 demand curve follows a standard residual-demand construction, with the storage quantity x always being held by buyers at the high end of the valuation distribution. Different rationing rules, however, imply different period-2 outcomes. In this section we show that the definition of equilibrium, including perfect foresight, determines a unique storage demand quantity that is independent of the rationing rule. The residual-demand rule can then be regarded as a convenient simplifying assumption.

A rationing rule is a way of assigning willing buyers to storage. This situation arises only when $p_1 = \delta p_2^e$; otherwise, storage demand is uniquely determined by (6). A general rationing rule, $R(v, p_1)$, maps buyer valuations into $\{0, 1\}$. The interpretation is that when $p_1 = \delta p_2^e$, a buyer with valuation v obtains the good for storage if $R(v, p_1) = 1$. Consistency requires that $R(v, p_1) \equiv 0$ for all $v < p_1/\delta$, since these buyers strictly prefer not to store at p_1. For period-2 demand, let $D_0(v; p_1) \equiv \{\tilde{v} \mid p_1/\delta \leq \tilde{v} \leq v \text{ and } R(\tilde{v}, p_1) = 0\}$ denote the set of buyers with valuation at or below v who were willing to store the good but did not obtain it under R. Since these buyers remain in the market (set $\lambda = 1$ for simplicity), the buyer distribution for period 2 is given by

$$G(v, p_1) = \begin{cases} F(v) & \text{if } 0 \leq v < p_1/\delta \\ F(p_1/\delta) + F[D_0(v, p_1)] & \text{if } p_1/\delta \leq v \leq 1, \end{cases}$$

[11] Even when λ is sufficiently above one that $P_1^M < \delta P_2^M(0)$, a monopolist may still find it optimal to remain at the "corner" and keep $p_1 = \delta P_2^M(0)$ to avoid storage. This is true for all λ when F is uniform [0, 1].

where $F[D_0(v, p_1)] \equiv \int_{D_0(v,p_1)} dF$ denotes the measure of buyers in $D_0(v, p_1)$ under F. Since F is atomless, we have $F[D_0(p_1/\delta, p_1)] = 0$ and $G(v, p_1)$ is continuous at $v = p_1/\delta$. Storage is then given by $x = G(1, p_1) - G(p_1/\delta, p_1)$, and period-2 demand under R is

$$Q_2^R(p_2, p_1) = G(1, p_1) - G(p_2, p_1).$$

Note that p_1 enters because rationing arises when $p_1 = \delta p_2^e$ and we need to consider period-2 demand in out-of-equilibrium situations where $p_2 \neq p_2^e$.

Now consider how storage and equilibrium are related. The analog of Lemma 3 is the following.

Lemma 8. Consider any rationing rule R. If $x > 0$, then $p_1 = \delta P_2^e(p_1)$. For the monopoly market, $x > 0$ if and only if $p_1 < \delta P_2^M(0)$. For the duopoly market, $x > 0$ if and only if $p_1 < \delta p_2^C$.

Thus, the particular rationing scheme is irrelevant to the (arbitrage) equilibrium-price relationship: whenever storage is positive, the period-1 price equals the discounted period-2 price. In addition, equilibrium storage occurs in exactly the same price regions as before, namely, when the period-1 price is below the discounted static equilibrium price.

The final step is to consider the exact level of storage that can occur in equilibrium. We have the following.

Proposition 5. Consider any rationing rule R. If $p_1 \leq \delta P_2^M(0)$, then storage demand in equilibrium for the monopoly market is given by $Q_S(p_1) = [P_2^M]^{-1}(p_1/\delta)$ and zero for higher p_1. For the duopoly market, replace $[P_2^M]^{-1}$with $[P_2^*]^{-1}$.

Again, this is exactly what we found for the residual-demand rule. Thus, the equilibrium requirements for storage by buyers and seller optimization are sufficient to pin down the level of storage demand, independent of the particular rationing rule. It is important to realize that, in equilibrium, positive storage implies that rationing is always occurring: when $p_1 = \delta P_2^e(p_1)$, all buyers with $v \geq \delta p_1$ are indifferent. All of these buyers, however, cannot store the good in equilibrium, as the resulting lack of demand would lead to a period-2 price below p_1/δ. In equilibrium the seller(s) must find it optimal to behave in period 2 such that a market price of $p_2 = p_1/\delta$ prevails. In turn, this implies that among the indifferent buyers, sufficiently many of the relatively low-value ones must remain in the market. This rules out, for example, a proportional or low-end rationing rule. The residual-demand rationing rule, however, is fully consistent with this requirement.

7. Conclusion

■ The rising price path in the storage equilibrium of our two-period model raises the important issue of long-run market dynamics. As we have argued, equilibrium storage implies rising prices, which cannot continue indefinitely. Thus, equilibrium price and associated storage (inventory) cycles become a distinct possibility. This is an important topic for future research on storable goods.

Appendix

■ Proofs of Lemmas 1-3, 5-8 and Propositions 1-5 follow.

Proof of Lemma 1. To prove the lemma, it is enough to establish that the period-2 marginal revenue of each firm is decreasing in its own output (see the discussion following Lemma 1 in the main text). With constant marginal cost, this implies that each firm's period-2 payoff function (symmetric across the firms) is quasi-concave in its own output. It then follows that each firm has a unique optimal output, and, furthermore, these two outputs are equal in equilibrium.

Define the period-2 revenue of firm i by $R(q_2^i, q_2^j + x) \equiv P_2(q_2^i + q_2^j + x)q_2^i$. Note that the period-2 price falls to zero when $x^j \equiv q_2^j + x \geq \lambda$ and it falls below c when $x^j \geq \bar{x}$. Fixing $x^j < \lambda$, we have

$$MR_i \equiv \frac{\partial}{\partial q_2^i} R(q_2^i, x^j) = p - \frac{q_2^i}{f(p/\lambda)},$$

for any $q_2^i \leq \lambda - x^j$, where the MR_i expression is evaluated at $p = P_2(q_2^i + x^j)$. Differentiating again, we have

$$MR_i' \equiv \frac{\partial}{\partial q_2^i} MR_i = -\frac{2}{f(p/\lambda)} - \frac{q_2^i f'(p/\lambda)}{\lambda f(p/\lambda)^3}.$$

Clearly, $MR_i' < 0$ holds at $q_2^i = 0$. Taking $q_2^i > 0$ and noting that $\lambda[1 - F(p/\lambda)] - x^j = q_2^i$, we have $MR_i' < 0 \iff$

$$0 < \frac{2\lambda}{\lambda[1 - F(p/\lambda)] - x^j} f(p/\lambda)^2 + f'(p/\lambda).$$

For the inverse hazard, we have $h'(p/\lambda) < 0$, and this implies $f(p/\lambda)^2/[1 - F(p/\lambda)] + f'(p/\lambda) > 0$. Noting that $2\lambda/[\lambda[1 - F(p/\lambda)] - x^j] > 1/[1 - F(p/\lambda)]$ is implied by $q_2^i > 0$, we see that the above condition for $MR_i' < 0$ is satisfied.

Finally, note that $MR_i = c$ has a unique solution at $q_2^i > 0$ when $x^j < \bar{x}$; when $x^j \geq \bar{x}$, the best response is $q_2^i = 0$. To see that the best response (when positive) has a slope strictly greater than -1, we can apply a standard monotone comparative statics argument to the equivalent problem of choosing $q \geq x^j$ to maximize $[P_2(q) - c](q - x^j)$. The cross-partial in q and x^j of this objective is just MR_i, which is positive (and equal to c) at the solution. Thus, q is strictly increasing in x^j, and from $q_2^i = q - x^j$ we obtain the desired slope property. *Q.E.D.*

Proof of Lemma 2. By Lemma 1, the period-2 equilibrium is symmetric and $q_2^{i*}/q_2^* = 1/2$. Then (5) becomes

$$P_2^*(x) \equiv P_2(q_2^* + x) = c - \frac{1}{2} q_2^* P_2'(q_2^* + x). \tag{A1}$$

Therefore,

$$P_2^*(x) = c + \frac{1}{2} \frac{\lambda - x - \lambda F(P_2^*(x)/\lambda)}{f(P_2^*(x)/\lambda)}, \tag{A2}$$

upon substituting for $P_2'(q_2 + x)$ as implied by (3) and for q_2^* as implied by (2). $P_2^*(x)$ is the fixed point to (A2); existence follows directly from the parameter assumptions on λ and c and the inverse-hazard property. Since the inverse hazard $h(p)$ is continuous and differentiable, the Implicit Function Theorem implies that $P_2^*(x)$ is continuous and differentiable. Differentiating both sides of (A2) with respect to x, and evaluating at $p = P_2^*(x)/\lambda$, we have

$$P_2^{*\prime}(x) = \frac{-1}{2f(p)^2} \left\{ f(p) + f(p)^2 P_2^{*\prime}(x) + \lambda^{-1}[\lambda - x - \lambda F(p)] f'(p) P_2^{*\prime}(x) \right\}. \tag{A3}$$

Collecting terms and rearranging, we find

$$P_2^{*\prime}(x) \left[\frac{\lambda - x - \lambda F(p)}{\lambda f(p)^2} \right] \left[\frac{3\lambda f(p)^2}{\lambda - x - \lambda F(p)} + f'(p) \right] = \frac{-1}{f(p)}. \tag{A4}$$

The first bracketed term on the left is clearly positive. As in the proof of Lemma 1, $h'(p) < 0$ implies that the second term is also positive. Therefore, we have $P_2^{*\prime}(x) < 0$.

It is a straightforward comparative static exercise, employing (A2) to make explicit the dependence of $P_2^*(x)$ on the parameter λ, to verify that $P_2^*(x)$ is differentiable and strictly increasing in λ. *Q.E.D.*

Proof of Lemma 3. We know from condition (ii) in the Definition for equilibrium that if $x > 0$, then $p_1 \leq \delta P_2^e(p_1)$. Now suppose $p_1 < \delta P_2^e(p_1)$. We claim that $x = Q_S(p_1) \geq \bar{x}$ must hold. If not, we have $q_2^*(x) > 0$ and $P_2^*(x) > c$, by Lemma 1. Consequently, buyers with v above $P_2^*(x)$ will purchase the quantity $q_2^*(x)$ in period 2. However, with $p_1 < \delta P_2^e(p_1) = P_2^*(x)$, all of these buyers strictly prefer to purchase the good in period 1, in contradiction of optimal storage choices. Thus, $x \geq \bar{x}$ and all units consumed in period 2 are purchased in period 1, with no further sales in period 2. From x, we calculate that all buyers with $v \geq v_x \equiv \lambda F^{-1}(1 - x/\lambda)$ purchase the good for storage in period 1, and $x \geq \bar{x}$ implies $c \geq v_x$. Since $v_x \geq p_1/\delta$ is implied by optimal storage, we see that $c \geq p_1/\delta$ and the period-1 price is below marginal cost. Clearly, this cannot hold in equilibrium; it implies an overall negative profit for each firm, as there are positive sales in period 1 ($x > 0$) and no further sales in period 2, so either firm would be strictly better off not producing.

The second part of Lemma 3 follows directly from the above argument and Lemma 1. *Q.E.D.*

Proof of Lemma 5. (i) Given q^j, the choice of q^i is equivalent to the problem of choosing a $q \geq q^j$. Rewriting the objective function in terms of q and q^j, $\Pi(q^i, q^j)$ becomes

$$\hat{\Pi}(q, q^j) = [P_1(q) - c](q - q_j) + \delta \Pi_2(Q_S(P_1(q))).$$

Since $\hat{\Pi}(q, q^j)$ has strictly increasing differences in q and q^j, as follows from strictly decreasing period -1 demand, we can apply the standard monotone comparative static result for supermodular functions since the domains for q and q^j are each closed real intervals. Thus, the set of best responses for the q choice, $\Phi(q^j)$, is strongly increasing: for $\hat{q}^j > q^j$, we have $\hat{q} \geq q$ for each $\hat{q} \in \Phi(\hat{q}^j)$ and $q \in \Phi(q^j)$.

(ii) Suppose, to the contrary, that $\Psi(q^j) \not\subset \Psi_N(q^j)$. Then $\Psi_S(q^j) \subset \Psi(q^j)$ for some such q^j. By construction of Ψ_S, we have $q^i + q^j \geq q_K$ for any $q^i \in \Psi_S(q^j)$. Similarly, for Ψ_N we have $\hat{q}^i + \hat{q}^j \leq q_K$ for any $\hat{q}^i \in \Psi_N(\hat{q}^j)$. Since Φ is strongly increasing, we must also have $q^i + q^j \leq \hat{q}^i + \hat{q}^j$. Therefore, $q^i + q^j = q_K = \hat{q}^i + \hat{q}^j$. Further, $q^i > \hat{q}^i$ must hold, since $q^j < \hat{q}^j$. Thus, aggregate output is at the date-1 demand kink in both cases. Now, observe that $\Psi_S(q^j)$ can have no element greater than $q^i = q_K - q^j$, or else Φ would fail to be strongly increasing. By the feasible set for Ψ_S, we then have $\Psi_S(q^j) = \{q_K - q^j\}$. But $q^i = q_K - q^j$ is necessarily feasible for the Ψ_N problem at q^j. Therefore, the payoff from any choice in $\Psi_N(q^j)$ is at least as large as that from $\Psi_S(q^j)$, and we must then have $\Psi_N(q^j) \subset \Psi(q^j)$, which contradicts the initial hypothesis. The later claim in part (ii) of the Lemma follows directly, since the inequalities become strict in the preceding argument.

(iii) This is analogous to the proof of property (ii).

Existence of an equilibrium follows directly from Tarski's intersection point theorem (see, e.g., Theorem 2.6 in Vives (1999)), since Φ is strongly increasing. *Q.E.D.*

Proof of Lemma 6. We will show that any best response in Ψ for i to $q^j = q_B$ necessarily involves an output strictly above q_K, implying strictly positive storage, when the (DS) condition in the lemma holds. Thus, by continuity of the payoff function Π, this remains true for an interval where $q^j < q^B$. Therefore, we have $q_A < q_B$.

First, rewrite $\Pi(q^i, q^j)$ with price as the choice variable for firm i: choose $p \leq \delta p_2^C$ to maximize

$$\Pi_S(p, q^j) \equiv (p - c)[Q_1(p) - q^j] + \delta \Pi_2(Q_S(p)).$$

Clearly,

$$\begin{aligned}\frac{\partial}{\partial p}\Pi_S(p, q^j) &= (p - c)Q_1'(p) + [Q_1(p) - q^j] + \delta \Pi_2'(Q_S(p))Q_S'(p) \\ &= [1 - F(p) + Q_S(p) - q^j] - (p - c)f(p) + Q_S'(p)[(p - c) + \delta \Pi_2'(Q_S(p))],\end{aligned}$$

upon substituting for period-1 demand from (10). Next, from $\Pi_2(x) = [P_2^*(x) - c]\frac{1}{2}q_2^*(x)$, we have

$$\Pi_2'(x) = [P_2^*(x) - c]\frac{1}{2}q_2^{*\prime}(x) + P_2^{*\prime}(x)\frac{1}{2}q_2^*(x) = -[P_2^*(x) - c]\left\{1 + \frac{1}{2}q_2^{*\prime}(x)\right\},$$

where the last step follows from the definition $P_2^*(x) = P_2(q_2^*(x) + x)$, which implies $P_2^{*\prime}(x) = P_2'(q_2^*(x) + x)[q_2^{*\prime}(x) + 1]$, and from (5) for the period-2 equilibrium, which implies $P_2^*(x) - c = -(1/2)q_2^* P_2'(q_2^*(x) + x)$. Noting that $P_2^*(x) = p/\delta$ by Lemma 3 for this price range, collecting terms and simplifying yields

$$\frac{\partial}{\partial p}\Pi_S(p, q^j) = [1 - F(p) + Q_S(p) - q^j] - (p - c)f(p) + Q_S'(p)\left[(p - c) - (p - \delta c)\left\{1 + \frac{1}{2}q_2^{*\prime}(x)\right\}\right].$$

Now, consider $q^j = q_B$ and the choice $\hat{p} \equiv \delta p_2^C$; note that this corresponds to a quantity choice by i of $q^i = q_K - q_B$ (the kink in period-1 demand). By construction, $\varphi_C(q_B) + q_B = q_K$ and we have $f(\hat{p})(\hat{p} - c) = 1 - F(\hat{p}) - q^j$ and $Q_S(\hat{p}) = 0$. Hence,

$$\left(\frac{\partial}{\partial p}\Pi_S(p, q^j)\right|_{(\hat{p}, q_B)} = Q_S'(\hat{p})\left[(\hat{p} - c) - (\hat{p} - \delta c)\left\{1 + \frac{1}{2}q_2^{*\prime}(0)\right\}\right].$$

Since $Q_S'(p) < 0$, this expression is negative if and only if the second term is positive. Simplifying with α as defined in the lemma yields the (DS) condition.

To complete the argument, suppose now that (DS) holds. This directly implies that all elements of $\Psi(q_B)$ are strictly greater than $q_K - q_B$. By construction, $\Psi_N(q_B) = \{q_K - q_B\}$ is the optimal choice for i subject to $q^i \leq q_K - q_B$. Every optimal choice subject to $q^i \geq q_K - q_B$ is strictly greater than $q_K - q_B$, since $\Pi(q^i, q_B)$ is strictly increasing in q^i at $q^i = q_K - q_B$, as follows from

$$\left(\frac{\partial}{\partial p}\Pi_S(p, q^j)\right|_{(\hat{p}, q_B)} < 0.$$

Further, since $\Pi(q^i, q^j)$ is continuous, the value of $\Pi(q^i, q^j)$ at $q^i \in \Psi(q_B)$ and $q^j = q_B - \varepsilon$, for small enough $\varepsilon > 0$, must strictly exceed that value of $\Pi(q^i, q^j)$ at $q^i \in \Psi_N(q_B - \varepsilon)$ and $q^j = q_B - \varepsilon$. This establishes that $q_A < q_B$. *Q.E.D.*

Proof of Proposition 1. Consider the set of δ and λ such that (DS) is satisfied. From the definition $P_2^*(x) = P_2(q_2^* + x)$, we have $P_2^{*\prime}(x) = -[1 + q_2^{*\prime}(x)]/f(P_2^*(x)/\lambda)$. Then, $P_2^{*\prime}(x)$ from (A4) directly implies that $1/2 < \alpha < 3/4$. Hence, a sufficient condition for (DS) is $\delta > (4c)/[p_2^C + 3c]$. This holds with strict inequality at $\delta = \lambda = 1$, since the Cournot price satisfies $p_2^C > c$. Since the Cournot price varies continuously with λ, (DS) continues to hold for a range of δ and λ values strictly below one.

Now consider possible equilibria when $\delta = \lambda = 1$. Note that $p_1^C = \delta p_2^C$ in this case and the static Cournot outcome for period 1 coincides with the kink in period-1 demand. Further, this is the only possible equilibrium with no storage. We know, however, that $q_A < q_B = q_1^C/2$ holds and so either firm has a strict incentive to increase output and induce storage, as Lemma 5 implies $q_1^C/2 = \varphi_C(q_1^C/2) \notin \Psi(q_1^C/2)$. By Lemma 5, however, an equilibrium exists and, hence, it must have strictly positive storage. *Q.E.D.*

Proof of Lemma 7. This applies for all $\delta \in [0, 1]$. First, we dispense with the trivial case of extremely low discount factors, $\delta \leq c/p_2^C$, where the demand kink occurs at or below marginal cost. This implies $q_B(\delta) = q_K(\delta) = 1-F(\delta p_2^C) \geq 1-F(c)$. A simple dominance argument for the best response of i to q^j then shows that Ψ reduces to the static Cournot best response, φ_C, since it is never optimal to produce beyond the demand kink. Thus, $q_A(\delta) = q_K(\delta)$ in this region.

Henceforth, we take $\delta \geq c/p_2^C$. To make explicit the dependence of the problem on δ, we proceed as follows. Define the value function $V_N(q_j, \delta) = \max_{q^i \in D_N(q^j,\delta)} \Pi(q^i, q^j, \delta)$, where the constraint set $D_N(q^j, \delta) \equiv [0, q_K(\delta) - q^j]$; we have $q^j \in [0, q_K(\delta)]$ and $\delta \in [c/p_2^C, 1]$ for this problem. Similarly, $V_S(q_j, \delta) = \max_{q^i \in D_S(q^j,\delta)} \Pi(q^i, q^j, \delta)$ where the constraint set $D_S(q^j, \delta) \equiv [\max\{q_K(\delta) - q^j, 0\}, 1 + \lambda]$; we have $q^j \in [0, 1 + \lambda]$ and $\delta \in [c/p_2^C, 1]$. By the Maximum Theorem (see, e.g., Sundaram, 1996), each value function is continuous over the associated set of (q_j, δ).

Further, the standard envelope theorem applies whenever the optimal q^i choice is interior to the constraint interval. Thus, whenever interior choices are optimal we have

$$\frac{\partial}{\partial\delta} V_N(q_j, \delta) = \Pi_2(0),$$
$$\frac{\partial}{\partial\delta} V_S(q_j, \delta) = \Pi_2(Q_S(p)) - \frac{p}{\delta}\{(p - c)f(p) - [1 - F(p) + Q_S(p) - q^j]\},$$

where p in the latter expression is the resulting price at an optimal interior quantity choice.

Case 1. Suppose we have $0 < q_A(\delta) < q_B(\delta)$ at δ. From above, $\delta > c/p_2^C$ necessarily holds. By definition of q_A and the continuity of V_S and V_N, we see that

$$V_S(q_A(\delta), \delta) = V_N(q_A(\delta), \delta)$$
$$\Rightarrow \quad \delta[\Pi_2(Q_S(p^S)) - \Pi_2(0)] = (p^N - c)q^N - (p^S - c)q^S,$$

where (p^S, q^S) and (p^N, q^N) refer to price and an optimal quantity choice for each of V_S and V_N, respectively. Since $q_A(\delta) < q_B(\delta)$, we know that $\Psi_N(q^j, \delta) = \{\varphi_C(q^j)\}$, and this optimal choice is interior for any $q^j < q_B(\delta)$. Hence, the $\Psi_N(q_A(\delta), \delta)$ choice is interior and $V_N(q_A(\delta), \delta) > \Pi(q_K(\delta) - q_A(\delta), q_A(\delta), \delta)$. Any optimal choice in $\Psi_S(q_A(\delta), \delta)$ is therefore interior, since the corner choices for q^i in $D_S(q^j, \delta) \equiv [q_K(\delta) - q_A(\delta), 1 + \lambda]$ cannot yield $V_S(q_A(\delta), \delta) = V_N(q_A(\delta), \delta)$, as required. Then

$$\delta \frac{\partial}{\partial\delta}\left[V_S(q^j, \delta) - V_N(q^j, \delta)\right]\Big|_{q^j = q_A(\delta)} = (p^N - c)q^N - (p^S - c)q^S + p^S[q^S - (p^S - c)f(p^S)],$$

upon substituting with the expression for $V_S(q_A(\delta), \delta) = V_N(q_A(\delta), \delta)$ and with $q^S + q_A = Q_1(p^S) = 1 - F(p^S) + Q_S(p^S)$. By interior choices, we know $p^N > \delta p_2^C > p^S$, $Q_S(p^S) > 0$, and $q^N < q^S$.

We show first that $q^S - (p^S - c)f(p^S) > Q_S(p^S) > 0$. To see this, note that $[1 - F(p) - q_A]/f(p) - (p - c)$ is strictly decreasing in p, by the hazard assumption, and that it equals zero at the price $p^N = F^{-1}(1 - \varphi_C(q_A) - q_A)$. Since $p^N > \delta p_2^C > p^S$, we then have $[1 - F(p^S) - q_A]/f(p^S) - (p^S - c) > 0$. Noting that $q^S = 1 - F(p^S) + Q_S(p^S) - q_A$ and that $Q_S(p^S) > 0$, the claim follows.

Next, we show $(p^N - c)q^N - (p^S - c)q^S > -(p^S - c)Q_S(p^S)$. Since $q^N = \varphi_C(q_A)$, we know

$$(p^N - c)q^N = \max_p (p - c)[1 - F(p) - q_A] > (p^S - c)[1 - F(p^S) - q_A] = (p^S - c)[q^S - Q_S(p^S)],$$

and the claim follows directly. Combining these two claims, we then have

$$\frac{\partial}{\partial\delta}[V_S(q_A(\delta), \delta) - V_N(q_A(\delta), \delta)] > cQ_S(p^S) > 0.$$

To see that $q_A(\delta)$ is continuous and strictly decreasing at a δ where $0 < q_A(\delta) < q_B(\delta)$, simply note that we can apply the Implicit Function Theorem to $V_S(q_A(\delta), \delta) - V_N(q_A(\delta), \delta)$ since, by interior choices, we have

$$\frac{\partial}{\partial q^j}\left[V_S(q^j, \delta) - V_N(q^j, \delta)\right]\Big|_{q^j = q_A(\delta)} = p^N - p^S > 0,$$

and, therefore, $q_A(\delta)$ is differentiable with $q_A'(\delta) < -cQ_S(p^S)/(p^N - p^S) < 0$.

Case 2. Suppose $q_A(\delta) = 0$ at some δ. Again, this implies $\delta > c/p_2^C$. By definition of q_A and continuity of $(V_S - V_N)$, we must have $V_S(0, \delta) - V_N(0, \delta) \geq 0$. If this is strict, then continuity also implies $q_A(\hat{\delta}) = 0$ for any $\hat{\delta}$ sufficiently close to δ, as implied by strict inequality of $V_S - V_N$ and Lemma 5. Thus $q_A(\delta)$ is continuous and weakly decreasing.

If equality holds, then we can argue as above that we must have interior choices for each of V_S and V_N at $(0, \delta)$. Applying the envelope theorem as above, we conclude that $(V_S - V_N)$ is increasing in δ at $(0, \delta)$ and, hence, for $\hat{\delta}$ sufficiently close to δ that $q_A(\hat{\delta}) = 0$ for $\hat{\delta} > \delta$ and that $q_A(\hat{\delta}) > 0$ for $\hat{\delta} < \delta$. Thus, $q_A(\delta)$ is weakly decreasing in δ.

For continuity as $\hat{\delta} < \delta$ approaches δ, consider $\lim_{\hat{\delta}\uparrow\delta} q_A(\hat{\delta})$. By the interior case, $q_A(\hat{\delta})$ is positive and strictly decreasing in $\hat{\delta}$, so the limit exists. Suppose the limit is positive and equals $\bar{q} > 0$. Since $V_S - V_N$ is strictly increasing in q^j for the interior case, we know $V_S(\bar{q}, \delta) - V_N(\bar{q}, \delta) \equiv \varepsilon > 0$ must hold. By continuity of $V_S - V_N$, we can find a $\gamma > 0$ for ε such that if $(q^j, \hat{\delta})$ is within a radius of γ of $(\bar{q}, \delta)$, then the corresponding $V_S - V_N$ differences must be within ε of each other. But then $[V_S(q_A(\hat{\delta}), \hat{\delta}) - V_N(q_A(\hat{\delta}), \hat{\delta})]$ must be positive for $\hat{\delta}$ near δ. But since $q_A(\hat{\delta})$ is positive, we know by construction that the difference is identically zero. Hence, $\bar{q} = 0$ must hold and we have continuity.

Case 3. Suppose $q_A(\delta) = q_B(\delta)$ at δ. Consider continuity. If $q_A(\delta)$ is not continuous at δ, then we can find a sequence (δ_n) converging to δ and an associated sequence (q_n) where $q_n \equiv q_A(\delta_n)$ such that (q_n) converges to some $\bar{q} < q_A(\delta)$. Note that we can only have $\bar{q} < q_A(\delta)$, since $q_A(\delta_n) \leq q_B(\delta_n)$ holds for all n. By definition of $q_A(\delta)$, we must have $V_S(\bar{q}, \delta) - V_N(\bar{q}, \delta) < 0$, since $\bar{q} < q_A(\delta)$. Also, since $q_n \equiv q_A(\delta_n)$, we must have $V_S(q_n, \delta_n) - V_N(q_n, \delta_n) \geq 0$ for each n and, therefore, $\lim_{n\to\infty}[V_S(q_n, \delta_n) - V_N(q_n, \delta_n)] \geq 0$. But this limit must coincide with $V_S(\bar{q}, \delta) - V_N(\bar{q}, \delta)$, which is negative, by continuity of $V_S - V_N$ and the convergence of (δ_n) to δ and (q_n) to $\bar{q}$. Hence, we have a contradiction and, therefore, $q_A(\delta)$ is continuous at δ.

Consider monotonicity of $q_A(\delta)$ at δ. If $\hat{\delta} > \delta$, then $q_A(\hat{\delta}) \leq q_B(\hat{\delta})$ together with q_B strictly decreasing imply $q_A(\hat{\delta}) < q_A(\delta)$. On the other side, suppose there is some $\hat{\delta} < \delta$ with $q_A(\hat{\delta}) \leq q_A(\delta)$. We claim this implies that q_A is strictly decreasing on the interval $[\hat{\delta}, \delta)$ and, hence, that q_A is not continuous at δ. To see why, note that $q_A(\hat{\delta}) < q_B(\hat{\delta})$ must hold, since q_B is strictly decreasing. Then for any $\delta' \in (\hat{\delta}, \delta)$ we must have $q_A(\delta') < q_A(\hat{\delta})$, since we can apply the above monotonicity argument to $\delta' > \hat{\delta}$. Further, q_A must be strictly decreasing at any such δ', since we have $q_A(\delta') < q_A(\hat{\delta}) \leq q_A(\delta) = q_B(\delta) < q_B(\delta')$. Thus, q_A is strictly decreasing on $[\hat{\delta}, \delta)$ and we have $\lim_{\delta'\uparrow\delta} q_A(\delta') < q_A(\hat{\delta}) \leq q_A(\delta)$, contradicting continuity of q_A at δ. Hence, we must have $q_A(\hat{\delta}) > q_A(\delta)$ for $\hat{\delta} < \delta$ and q_A is strictly decreasing. *Q.E.D.*

Proof of Proposition 2. Note that a no-storage equilibrium corresponds to the intersection of the 45° line with $\Psi \cap \Psi_N$. Recall also that the only candidate for a no-storage equilibrium is the static Cournot equilibrium. We know from Proposition 1 that, with $\lambda = 1$ and δ sufficiently large, we have $q_A(\delta) < q_B(\delta)$. Thus, an equilibrium with no storage cannot exist if δ is sufficiently close to one. Also, from the proof of Lemma 7, we see that $q_A(\delta) = q_K(\delta) > q_1^C/2$ when $\delta < c/p_1^C$. Define $\delta^n \equiv [q_A]^{-1}(q_1^C/2)$. Observe that since $q_A(\delta)$ is continuous and strictly decreasing in δ whenever it is positive, δ^n as defined above is unique. To prove the proposition, note that for any $\delta < \delta^n$, $q_A(\delta) > q_1^C/2$, implying an intersection of $\Psi \cap \Psi_N$ with the 45° line. Therefore, for any $\delta < \delta^n$, the no-storage equilibrium exists. Now consider any $\delta > \delta^n$, which implies $q_A(\delta) < q_1^C/2$. This implies that $\Psi(q_1^C/2) \cap \Psi_N(q_1^C/2) = \emptyset$: the global best response to a rival's output must necessarily induce storage, when the latter is equal to the per-firm static Cournot (no storage) output. Therefore, the no-storage equilibrium does not exist. *Q.E.D.*

Proof of Proposition 3. Recall that

$$\Psi_S(q^j) = \underset{\max\{q_K - q^j, 0\} \leq q^i \leq 1+\lambda}{\operatorname{argmax}} \left[P_1(q^i + q^j) - c\right] q_i + \delta \Pi_2(x), \tag{A5}$$

where P_1 is the inverse of the aggregate-demand function, as given by the second line in (13), and x is as given by (12). We know the storage best response, $\Psi_S(q^j)$, must have $p_1 > c$, or else i would be better off reducing output. Restricting attention to interior solutions of (A5), simplifying the first-order condition yields

$$\Psi_S(q^j) = -\left(\frac{1+\delta}{2}\right) q^j + \left(\frac{1+\delta}{2}\right)(1 + \lambda + 2c) + \frac{c}{2\delta}(2\delta - 1)(3 + \delta).$$

By inspection, $\Psi_S(q^j)$ is single valued and strictly decreasing in q^j. Furthermore, the partial derivative with respect to δ is equal to

$$(1 + \lambda + 2c - q^j)/2 + c\frac{d}{d\delta}\{(1 - 1/2\delta)(3 + \delta)\}.$$

With $q^j < 1 + \lambda$, this expression is positive. Hence $\Psi_S(q^j)$ is strictly increasing in δ.

Let $\varphi_S(q^j, \delta)$ denote the single-valued correspondence Ψ_S. Using φ_S, define $\delta^s \equiv \inf\{\delta \mid q_A(\delta) = \varphi_S(q_A(\delta), \delta)\}$. We claim that a storage equilibrium exists *if and only if* $\delta \geq \delta^s$. Recall that storage equilibria correspond to points of intersection of the 45° line with $\Psi \cap \Psi_S$. From above, we have that $\varphi_S(q_A(\delta), \delta)$ is strictly increasing in its second argument. Furthermore, it is strictly decreasing in its first argument. Since $q_A(\delta)$ is decreasing in δ, we have that $\varphi_S(q_A(\delta), \delta)$ is strictly increasing in δ. To prove the claim, note then that for any $\delta < \delta^s$, there cannot be a point of intersection between

$\Psi \cap \Psi_S$ and the 45° line. On the other hand, for any $\delta \geq \delta^s$, there must exist such a point of intersection. Since φ_S is strictly decreasing with respect to q^j, storage equilibrium must be unique.

Thus far, we have proved (i) and (iii). Since any equilibrium must be characterized by either storage or no storage, by Lemma 5 (existence), we must have that $\delta^s \leq \delta^n$. Then it remains to show that $\delta^s < \delta^n$. Now, $q_A(\delta^n) \equiv q_1^C/2 < \varphi_S(q_1^C/2, \delta^n) = \varphi_S(q_A(\delta^n), \delta^n)$, where the inequality follows from the fact that for any $\delta > c/p_1^C$, $q_1^C = q_1^C/2 + \varphi_C(q_1^C/2) < q_K(\delta) < q_1^C/2 + \varphi_S(q_1^C/2, \delta)$. But, as argued earlier, $\varphi_S(q_A(\delta), \delta)$ is strictly increasing and continuous in δ, whereas $q_A(\delta)$ is decreasing and continuous in δ. Therefore, the two functions must cross at some value of δ (defined δ^s), which is strictly below δ^n. *Q.E.D.*

Proof of Proposition 4. To begin, note that the (NS) problem reduces to a static monopoly choice with price constrained to be above $\delta P_2^M(0)$. Since P_1^M satisfies this constraint (by assumption), it is the optimal choice and the payoff is $\Pi_1^M = (P_1^M - c)Q_1^M$. Now consider the (S) problem. Let $\pi^M(x)$ denote the objective function. First, note that for any $x > 0$ we have

$$\Pi_1^M = \left(P_1^M - c\right) Q_1^M > \left[\delta P_2^M(x) - c\right] Q_1^C\left(\delta P_2^M(x)\right),$$

since $p_1 = \delta P_2^M(x)$ is feasible but strictly suboptimal for the unconstrained static period-1 monopoly problem. Next, noting that $\delta < 1$, we clearly have $[\delta P_2^M(x) - c] < \delta[P_2^M(x) - c]$. Hence, we see that

$$\pi^M(x) < \Pi_1^M + \delta\left[P_2^M(x) - c\right]x + \delta\Pi_2^M(x).$$

Next, we claim that $\delta[P_2^M(x) - c]x + \delta\Pi_2^M(x) \leq \delta\Pi_2^M(0)$. Consider the period-2 monopoly problem when we set storage to zero. Note that $q = x + Q_2^M(x)$ is a feasible quantity choice for that problem. Also, from (3), the expression for residual demand in period 2, we know $P_2^M(x) = P_2(x + Q_2^M(x))$. Thus,

$$\begin{aligned}\Pi_2^M(0) &= \left[P_2^M(0) - c\right] Q_2^M(0)\\ &\geq \left[P_2\left(x + Q_2^M(x)\right) - c\right][x + Q_2^M(x)]\\ &= \Pi_2^M(x) + \left[P_2^M(x) - c\right]x,\end{aligned}$$

as claimed. Combining the above bounds, static monopoly prices are optimal since

$$\begin{aligned}\pi^M(x) &< \Pi_1^M + \delta\left[P_2^M(x) - c\right]x + \delta\Pi_2^M(x)\\ &\leq \Pi_1^M + \delta\Pi_2^M(0).\end{aligned}$$

Q.E.D.

Proof of Lemma 8. Consider monopoly. If storage is positive in equilibrium, then we necessarily have $p_1 \leq \delta p_2^e$. If this is strict, then storage demand must be $1 - F(p_1/\delta)$. In equilibrium, $p_2^* = p_2^e$ must hold and, thus, the monopolist would sell a quantity of zero and earn zero profit in period 2. But period-2 demand at prices below p_1/δ follows $Q_2^C(p_2) = F(p_1/\delta) - F(p_2)$, and such prices yield a positive profit in period 2. Thus, the inequality cannot be strict and we must have $p_1 = \delta P_2^e(p_1) = \delta p_2^*$ when storage is positive.

Suppose $p_1 < \delta P_2^M(0)$ and storage is zero. Then period-2 demand reduces to $Q_2^C(p_2) = 1 - F(p_2)$, which coincides with static demand, and the monopolist will set $p_2 = P_2^M(0)$. Since $p_2^e = P_2^M(0)$ must hold in equilibrium, we then have $p_1 < \delta p_2^e = \delta P_2^M(0)$ and storage demand must be positive, which contradicts the hypothesis.

Now suppose $p_1 > \delta P_2^M(0)$ and storage is positive. Since storage is positive, we know that $p_2^* = p_1/\delta$ must be an optimal choice for the monopolist. Hence, $p_2^* > P_2^M(0)$. Consider the period-2 payoff function for the monopolist (suppress the dependence on p_1):

$$\pi_2^M(p_2) = Q_2^R(p_2)[p_2 - c] = \begin{cases}[G(1) - F(p_2)](p_2 - c) & \text{for } p_2 \leq p_1/\delta\\ [G(1) - G(p_2)](p_2 - c) & \text{for } p_2 > p_1/\delta.\end{cases}$$

For prices $p_2 \leq p_1/\delta$, we see that $\pi_2^M(p_2) = -[1 - G(1)](p_2 - c) + [1 - F(p_2)](p_2 - c)$. The first term is linear and strictly decreasing in p_2. The second term is the static monopoly payoff function and we know it is strictly quasi-concave with a unique maximum at $P_2^M(0)$. Since $p_1/\delta > P_2^M(0)$, the second term is strictly decreasing at $p_2 = p_1/\delta$. Combining, we see that reducing p_2 from p_1/δ must increase profit. Hence, it cannot be optimal for the monopolist to charge $p_2^* = p_1/\delta$ and, therefore, storage must be zero when $p_1 > \delta P_2^M(0)$.

The proof for the duopoly case is analogous and therefore omitted. *Q.E.D.*

Proof of Proposition 5. From Lemma 8, the result is established if we can show that $Q_S(p_1) = [P^M]^{-1}(p_1/\delta)$ for

$p_1 \leq \delta P_2^M(0)$. In equilibrium, it must be optimal for the monopolist to set a period-2 price of p_1/δ for period-2 demand of $Q_2^R(p_2)$ under the rationing rule R. Recall $\pi_2^M(p_2)$ from the proof of Lemma 8. We know that $\pi_2^M(p_2)$ is differentiable for $p_2 \leq p_1/\delta$, and direct calculation yields

$$\frac{d}{dp_2}\pi_2^M(p_2) = -[1 - G(1)] + [1 - F(p_2)] - f(p_2)](p_2 - c) \Rightarrow$$

$$\left.\frac{d}{dp_2}\pi_2^M(p_2)\right|_{p_2=p_1/\delta} = -[1 - G(1)] + [1 - F(p_1/\delta)] - f(p_1/\delta)](p_1/\delta - c) \geq 0,$$

which is necessary for an optimum. With $p_1 \leq \delta P_2^M(0)$, we know that the static monopoly payoff is strictly increasing in this price range. Now consider $p_2 \geq p_1/\delta$ and note that we now have

$$\begin{aligned}\pi_2^M(p_2) &= [G(1) - G(p_2)](p_2 - c)\\ &= -[1 - G(1)](p_2 - c) + [1 - F(p_2)](p_2 - c) + [F(p_2) - G(p_2)](p_2 - c).\end{aligned}$$

Optimality requires that $\pi_2^M(p_2)$ be nonincreasing at $p_2 = p_1/\delta$. The third term is always nonnegative for any R and it is zero at $p_2 = p_1/\delta$. Thus, $\pi_2^M(p_2)$ would be strictly increasing if the first two terms, which are differentiable, were strictly increasing. But these terms are the same as in the previous case and we can immediately conclude that optimality of $p_2 = p_1/\delta$ requires $-[1 - G(1)] + [1 - F(p_1/\delta)] - f(p_1/\delta)](p_1/\delta - c) \leq 0$. Thus, in equilibrium we must have

$$G(1) = 1 - F(p_1/\delta) - f(p_1/\delta)(p_1/\delta - c),$$

for any rationing rule. But this is exactly the definition of $[P^M]^{-1}(p_1/\delta)$ and, thus, we are done.

The duopoly case is again analogous and the proof is omitted. *Q.E.D.*

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