Sovereign Debt Default: The Impact of Creditor Composition *

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Abstract

The main motivation of this paper is to study the impact of the composition of creditors on the probability of default and the risk premium on sovereign bonds. In the absence of any legal enforcement, relational contracts work only when there are creditors who have a repeated relationship with the borrower. We show that ownership structures with a larger fraction of long term lenders are associated with a lower default probability and lower risk premia.

JEL:

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1 Introduction

In recent years countries have turned increasingly from bank loans to bond issues to raise capital. As a result, the international capital markets are more diversified and function more efficiently. Specifically, there is a broader investor base available to provide financing for emerging market sovereigns, which has helped diversify risk. But there is a serious downside if a country faces unsustainable debt. Private creditors have become increasingly numerous, anonymous and difficult to coordinate. (IMF, 2003)

"The Costs and Benefits of Arrears Accumulation are assessed creditor by creditor, taking into account such factors as the expected likelihood, terms and timeline of new debt inflows, the financial and reputational damage of default and vulnerability to litigation. Overlapping political, strategic and legal considerations come together in the formulation of a selective default plan favouring some creditors over others. Sovereign default comes not as an accident, but as a willful policy decision, viewed by the debtor government as a politically less onerous strategy than the other alternatives available. " IMF Working Paper WP/02/170 by Ashok Vir Bhatia.

During the 1980s, lending in sovereign debt markets was very different from what it is now: there were syndicated bank loans and a small number of banks which operated on a common set of assumptions that tended to avoid legal action. In contrast, Brady bonds and subsequent new debt issues in the 90s were purchased by thousands of new investors, including institutional hedge funds – see Figure 1 (WRIGHT 2004). This greater diversity among creditors meant that they were less likely to be constrained by tacit understandings about a shared collective interest.

One example of the new market structure was the Argentine default of 2001. The ex-post default evidence on bond issues has been studied by e.g. Dhillon, García- Brighton, Ghosal, and Miller (2006) Sgard (2005). It reveals an interesting pattern of creditor composition post default: a large number of small creditors (more than 1/4 million) and few very big lenders. This creditor heterogeneity has been cited as one of the causes for delay in the post default renegotiation. The coordination problem was resolved just before the 2005 swap: a significant fraction of the small lenders sold their bonds cheap to big lenders allowing them to start bargaining with Argentina(CITATION??).

In general, besides re-negotiation of debt, large creditors can have co-ordinating effects on the market for sovereign debt. Underlying these issues of creditor co-ordination is the fact that enforcing contracts is problematic in sovereign market so that informal or relational contracts become important.

In this paper we study the question of how the composition of debt, in particular, the presence of large creditors who have a repeated relationship with the debtor affects the probability of default and the risk premium on bonds.

Our formal model is a stochastic dynamic game where there is a fraction 1 − α of ”large” lenders and a fraction α of ”small” lenders all of whom buy one period bonds from the borrower country. The only difference between the two types of lenders is that large borrowers internalize the effects of their own actions on the rate of interest and on the probability of default, while small lenders do not.
Small lenders always offer to buy bonds at any rate of interest higher than the risk free rate. Large lenders however have bargaining power in setting the rate of interest while small lenders take it as given and only decide whether to buy bonds or not. The borrower country chooses the level of effort: the higher the effort the higher the probability of a good outcome. In case there is a bad outcome, the country must default on it’s debt. In this setting, the optimal contract for the borrower country must take into account two incentive problems: moral hazard and repudiation of debt servicing in the good state. Large lenders choose their profit maximizing rate of interest subject to the presence of small creditors, and the incentive compatibility conditions. Hence small lenders exert a negative externality on large lenders through the lowering of the rate of interest as well as through the free riding on providing incentives to repay. There is an indirect effect on incentives by small lenders: lowering the rate of interest encourages repayment. The direct effect on incentives is however negative since they free ride on rewarding the borrower for repayment or punishing for default. We show that the effect of small creditors on the probability of default is positive: the higher the fraction of bonds held by small creditors, the higher is the probability of default and the higher are the risk premia. This result has a similar flavor to a phenomenon pointed out by Hellman and Stiglitz (2000) that banks have tendencies to gamble on investments much more the higher is the competition they face in the market.

1.1 Related Literature

One of the first questions to understand in the absence of well developed enforcement mechanisms in a sovereign debt market is: why do countries repay debt? One possible explanation is that they repay because they are worried about sanctions in case of default. There is a literature (Bulow and Rogoff (1989) Fernandez and Rosenthal (1990)) focusing on direct punishment in the form of sanctions to the defaulter country. The problem with this interpretation is that due to the nature of the sovereign debt market, there is no clear evidence of effective sanctions.

A second interpretation argues that countries repay because they are worried about the impact of a default on their reputation Eaton and Gersovitz (1981). These papers ensure a country’s commitment to repay through the threat of no future credit in the market. Following a similar argument, Eaton (1996) describes a model in which there are two types of borrowers. The bad one always defaults if it is optimal and the good one always try to repay if they can. In any case, at low levels of debt, bad borrowers have a reputation incentive to repay. After 20 years of the original reputation paper, Kletzer and Wright (2000) describe how informal relational contracts can sustain lending allowing for partial market exclusion in case of default with the creditor extracting all future gains from the debtor. However, in equilibrium there is no default because in the hypothetical case of default, the equilibrium in the subgame is exactly as painful as autarky. The main result is that they have a model where

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\(^1\)For a detailed description of the issue, please see Sturzenegger and Zettelmeyer (2006)
without insurance, without sanctions and without international institutions; sovereign debt markets still exist. The common feature of the reputation models is that the only way that creditors could punish a country is by denial of future credit. An important assumption of the reputational models is that creditors are able to fully coordinate on punishments. Wright (2002) describes how market could be sustained if lenders collude in the lending condition and punishing in case of default. Finally, Wright (2005) analyses how the developing country sovereign debt market has became progressively more competitive and how this harm coordination among creditors. He describes how this affect welfare and concludes that reduced competition in the market is welfare improving.

This new market structure described above could be modeled introducing small creditors, leading to a 2 tier market structure: One big lender and a bunch of small creditors.\(^2\) The main motivation of this paper is to study the impact of the distribution of sovereign debt among creditors on the probability of default and the risk premium on sovereign bonds in the absence of international institutions. This assumption can also be justified by using the Kletzer and Wright (2000) seniority equilibrium where new lending to a country depends on satisfactory renegotiations with existing lenders in case of default. We show that when the debtor has moral hazard then the anticipated creditor composition is going to affect the probability of default as well as the risk premium needed for lending to be positive. In our set up there are two driving forces that are important determinants of the probability of country default: the larger the fraction of small creditors the bigger the externality imposed through the determination of market interest rate and also the larger the cost of ensuring repayment and effort incentives. We also discuss the option for small creditors to free ride.\(^3\) We show that the competitive effect leads to falling risk premia but the externalities imposed on incentives leads to increasing risk premia to compensate large lenders who pay the cost. A related paper by Kovrijnykh and Szentes (2007) analyzes the effects of default (debt overhang) on market structure. They show that when debt overhang occurs then seniority rules imply that only the incumbent lender is willing to lend to the country and so he exploits his monopoly power. After a series of good shocks however the lender finds it optimal to allow access to other lenders again. Our paper, in contrast, looks at how market structure affects the probability of default.

Evidence that supports are theoretical results is provided in a paper by Drelichman and Voth (2008) that looks at defaults in the age of Philip II, in the 16th century. Philip II of Spain accumulated huge debts and defaulted four times but yet was able to get access to funds. They show that in fact the lenders were a highly coordinated group with the ability to cut off Philip II’s access to smoothing services. Lending morataria were sustained by a “cheat the cheater” strategy as in Kletzer and Wright

\(^2\)This captures the intuition that the debt dynamics is driven by presence of non-anonymous repeated ”large” creditors. These are coordinated. Our results would hold for homogenous creditors with free riding, however we simplify by thinking of large and small and assume that large never free ride while small always do.

\(^3\)While the presence of large creditors ensure a lower probability of default, the small creditors gain by getting a repayment at no cost.
(2000). We assume that when syndicated bank loans give way to bonds then there is a reduced coordination among lenders making it more difficult to sustain lending moratoria: this in turn leads to higher defaults and market break-downs.

The paper is organised in two sections. The next section introduces the setup of the model. Section three analyses Sequential equilibria of the repeated game. Finally, we draw some conclusions.

2 The Model

The model investigates the link between the composition of bond ownership, the probability of default and the risk spread on the debt. What we have in mind here is that there are some creditors in the market who are more "powerful" in the sense that (a) they internalize the effect of their actions on the repayment rate and and (b) they can influence the interest rate of the bonds even though they are issued by the borrower country. Formally, we assume that a fraction \(1 - \alpha\) of creditors are "large" creditors who do influence the rate of interest and who act strategically. All large creditors are assumed to be identical and own \(\alpha_l\) shares in the total bond issue while small creditors are identical and own \(\alpha_s < \alpha_l\) shares of the total bond issue each. Let \(n_l, n_s\) be the numbers of large and small creditors who own bonds respectively: then \(\alpha = n_s \alpha_s\) and \(1 - \alpha = n_l \alpha_l\). Small creditors are assumed to be non-strategic: they are willing to buy bonds as long as the bonds pay in expected terms at least the risk free rate, normalized to 1.

Large creditors use bargaining with the borrower to set the rate of interest. To represent the outcome of this bargaining game we simply assume a very reduced form where the rate chosen depends on a weighted sum of the offer by small creditors which we assume to be the risk free rate and the offer by large creditors \(R_l\). Hence \(R = \alpha R_s + (1 - \alpha) R_l\). Large creditors choose \(R_l\) to maximize joint utility. Moreover small creditors only participate if they get an expected return of at least 1, i.e. \(PR_s = 1\) which implies that \(R_s = \frac{1}{P}\). Hence \(R = (1 - \alpha) R_l + \alpha \frac{1}{P}\), so that \(R_l = \frac{1}{1 - \alpha} (R - \frac{\alpha}{P})\). Large creditors therefore get a gross return of \(R_L = (1 - \alpha) R\) so that \(R_L = (1 - \alpha)^2 R_l + \frac{\alpha}{P} (1 - \alpha)\).

The sovereign bond market consists of one borrower, large creditors and small creditors. The sovereign borrows 1 unit to invest in a risky technology which yields the end of period payoffs of \(Y_H\) with probability \(P\) and \(Y_L = 0\) with \(1 - P\). The probability is assumed to be an increasing and concave function of the level of effort, \(e\). All agents are risk neutral.

The time line is given in Figure 2 below. First the debt contract is signed with the borrower receiving one unit of lending and promising to pay \(R\) a period later. Given this contract and expected future payoffs, the borrower then determines his optimal effort. Finally, outcomes of the borrower’s investment are realized. Output is not observed by creditors, so contracts are not state contingent.

\[\text{We can justify this assumption with the following interpretation: large creditors are able to perfectly co-ordinate their actions: it is worthwhile for them to do so only if they can cover the fixed costs of coordination by owning a large enough fraction of bonds.}\]
If high output occurs, the borrower could pay off the debt and engage in a new round of borrowing; or could default. If the bad state is realized we assume that \( Y_H > R > 0 \), so that when output is low there is no option but to default and pay nothing when there is no possibility of legal enforcement. In this case the borrower is excluded from the market forever.

If the game is a one shot game then defaulting may be deterred if there is a sufficiently high and enforceable output penalty imposed by the courts for default. This implies that in the absence of enforcement the one shot game leads to sure default and anticipating this, no lenders will be willing to lend. In order to get an equilibrium with positive lending, when there are no other punishment mechanisms, we therefore need at least some creditors who have a repeated interaction with the borrower country. We assume that large creditors have a repeated interaction, while small creditors do not. This is captured by introducing a participation constraint for small creditors. The extensive form game (2) is the stage game of an infinite horizon repeated game with imperfect monitoring and we investigate the properties of the sequential equilibria of this game. The effort is unobservable to creditors, and is costly so the debtor is subject to moral hazard. Large creditors interact with the borrower repeatedly while small creditors only interact for one period. Large creditors are able to condition the rewards for not defaulting on the full history of repayment. The effort is unobservable to creditors, and is costly so the debtor is subject to moral hazard. Large creditors interact with the borrower repeatedly while small creditors only interact for one period. Large creditors are able to condition the rewards for not defaulting on the full history of repayment.

A summary of the variables is provided in Table 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R )</td>
<td>Full repayment</td>
<td>( R &gt; 1 )</td>
</tr>
<tr>
<td>( e )</td>
<td>Borrower’s effort</td>
<td>( e \geq 0 )</td>
</tr>
<tr>
<td>( P )</td>
<td>Probability of good state</td>
<td>( P'(e) &gt; 0, P''(e) &lt; 0, P'''(e) &lt; 0 ), ( P(0) = 0, P'(0) \to \infty )</td>
</tr>
<tr>
<td>( Y_H )</td>
<td>Output in the good state</td>
<td>( Y_H &gt; R )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Small creditors’ bonds share</td>
<td>( 0 \leq \alpha \leq 1 )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Judicial efficiency probability</td>
<td>( 0 \leq \phi \leq 1 )</td>
</tr>
<tr>
<td>( R_F )</td>
<td>Repayment at risk free rate</td>
<td></td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor for large creditors</td>
<td></td>
</tr>
</tbody>
</table>
Borrower asks for 1

Large Creditor
Chooses R

small creditors

Borrower chooses effort e

Y_H

P(e)

1-P(e)

DEFAULT

Borrower pays max\{X, (1-a)X+a\phi R\}

Repayment in Full R
In this setup there are two driving forces that are important determinants of the probability of default: the larger the fraction of small creditors the bigger the externality imposed through the determination of $R$ (i.e. the lower is the monopoly power of large lenders in setting price) and also the larger the per capita cost of ensuring repayment and effort incentives: since by definition small creditors get the returns $R$ per unit, without incurring any of the costs of ensuring repayment. Small creditors therefore free ride: if the actual $R$ is bigger than 1 (the risk free rate) then they gain by getting a higher repayment while the presence of large creditors ensures a lower probability of default since they choose $R_l$ (which affects $R$) anticipating the effect on the incentive to repay. What is the combined effect of these two forces? The direct competitive effect on $R$ is to lower it, thus improving incentives to repay. The indirect effect of high $\alpha$ is to increase the net per capita cost to large lenders of ensuring repayment by large creditors since they get only $(1-\alpha)R$ of the total repayment but they provide all the promised rewards in terms of lower $R_l$ for repayment. The competitive effect leads to falling risk premia but the externalities imposed on incentives leads to increasing risk premia to compensate large lenders. This is what we show in the next section.

3 Sequential equilibria of the repeated game

This section analyses the sequential equilibrium resulting from repeated interactions between the borrower, the large creditor and a fringe of small creditors. Given that borrower’s output is not observable by the creditor, we look for non-state-contingent optimal relational contract. By observing the past payment history of the borrower, the large creditor decides $R_l$ on the current loan. We first describe the sequential problem facing the large lenders. The state of the economy at time $t$ is indexed by a state $\omega \in \{G, B\}$ where $G$ stands for the good state and $B$ for the bad state. The state is only observed by the borrower. Notice that we restrict attention to maximum punishment contracts, so once there is default the game ends. We also assume that the borrower always defaults in the bad state. The publicly observable history is the sequence of repayments if there has been no default till time $t$. (Note that we do not need to formulate the decision of the borrower once a bad state has occurred). The optimal dynamic contract is a history dependent sequence of repayments $\{R(h_t)\}_{t=1}^\infty$ which maximizes the large lenders’ payoffs. In the equilibrium we must have the following:

(1) The borrower’s choose $e$ in period $t$ to maximize:

$$U_t = \max_e \{P(e)[Y_H - R(h_t) + \beta U_{t+1}] - e\}$$

having observed the contract $R(h_t)$. Observe that the price setting process implies that $R_L = (1-\alpha)^2 R_l + \frac{\alpha}{2}(1-\alpha)$. (2) Large lenders then choose the contract to maximize their payoff:

$$V_t = \max_{\{R_l(h_t)\}} \{P(e)[(1-\alpha)^2 R_l(h_t) + \beta V_{t+1}] - (1-\alpha)^2\},$$

subject to the following constraints:
(i) Borrower’s participation constraint:

\[ U_t(e^*) \geq 0 \]  

where the autarky payoff is normalized to 0.

(ii) No Default Constraint of Borrower in the good state:

\[ Y_H - R(h_t) + \beta U_t + 1(R(h_{t+1}) \geq Y_H \]  

(iii) Small creditors’ participation constraint:

\[ P(e)R = 1 \]

This problem has a recursive formulation, using lifetime utility of the borrower as a state variable, following Spear and Srivastava (1987). This variable is enough to summarize information about a borrowers’ default history. We consider the space of contracts where incentive problems can be partially overcome using memory and future promises. Contracts are restricted to depend only on publicly observable outcomes, which in this case is just whether there is default or not. We follow the previous literature (Spear and Srivastava (1987), Thomas and Worrall (1988), Abreu, Pearce and Stachetti (1990), Phelan and Townsend(1991) among others) in formulating the contracting promise recursively using a “promised value”. The contracts specify moreover that upon default, the borrower will be permanently excluded from the future credit market. The contract design problem of the above setup needs to take into account two basic elements: one is the lack of commitment mechanism on the part of the borrower i.e. that debt can be repudiated, the other is the private information concerning the actual realised states. One-sided commitment problem in a similar context has been studied by Kocherlakota (1996) who looks at self-sustaining insurance contract in a village economy where villagers face idiosyncratic endowment shocks. There, full insurance is not possible as the optimal contract has to take into account the lack of commitment on the part of villagers even if their endowments are public information. Instead, the optimal relational contract derived exhibits history dependence, where history summarizes all past endowments. Such history dependence is dealt with in a recursive manner by using a “promised value”. A similar problem with asymmetric information has been investigated by Thomas and Worrall (1988).

To induce the borrower to repay the loan in the good state, the large creditor has to promise more favorable terms in future loan contract. To make things simple, we assume that any promise made by the large creditor is fully credible. Notice that these large creditors are like the “swiss banks” in the Cole-Kehoe model. How does this “promised value” capture history dependence in our model? Let \( h_t \)

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For other applications using “promised value” approach, see Ljungqvist and Sargent (2000, Chapters 15 and 16).

Atkeson (1991) looked at a problem of lending to a sovereign in the absence of complete information and enforcement, where distribution of future states depends on investment.

Note that our setup implies that the borrower always defaults in the bad state.
track borrower’s past output realizations up to time $t$. As the loan contract is not state contingent, the borrower will default in the bad state. So $h_t$ simply counts the number of times that the borrower have made full repayments (or a string of realisations of good state).

Let $\delta_t$ be the “promised value” (present value of lifetime utility in $t$) made by the large creditor in the period $t-1$ for the delivery in period $t$. Given $\delta_t$, the current period interest rate is determined, $R_t(\delta_t)$. Conditioned on this interest rate and the future promise $\delta_{t+1}$, the borrower decides on the optimal effort, $e^*(R_t, \delta_{t+1})$. The future promise also affects whether the borrower would repay the loan in the good state. So when period $t+1$ arrives, $\delta_{t+1}$ would be delivered only if there was a good state in period $t$. This implies that $\delta_{t+1}$ depends on $\delta_t$ and a realisation of $Y_H$ at $t$, $\delta_{t+1} = f(\delta_t, Y_H)$. Iterating this relationship forward from the initial $\delta$ implies that $\delta_{t+1}$ depends on $h_t$. In what follows, we denote $\delta$ the promise made by the large creditor in the last period, and $\delta'$ the promise made in the current period. In the optimisation faced by the large creditor, $\delta$ serves as a state variable.

We want to focus on the trade off between the competitive effect of having more creditors in the game (captured by a higher fraction of small creditors) and the free riding on the provision of reputational incentives for repayment to the borrower. We do this by introducing a bargaining game between creditors and the borrower where the repayment $R$ is determined. Large creditors choose $R_l$ taking as given the beliefs about $\alpha$. Small creditors offer the risk free rate. We solve now for the dynamic optimal contract among those with maximum punishment for this problem with one sided commitment.

Let us first focus on the borrowers optimization problem. Let $x = \beta \delta' - R$. Given the contract $(R, \delta')$ the borrower chooses optimal effort to maximize:

$$u(e; R, \delta') \equiv P(e)(Y_H + x) - e$$

where we assume that the discount factor of the borrower, $\beta$ is identical to that of the large creditor, and the probability of the good state, $P(e)$, is increasing and concave in effort, $e$. In addition, we assume that $P''(e) < 0$, $P(0) = 0$, $P'(0) \to \infty$, $P(\infty) = 1$ and $P'(\infty) = 0$. This yields the following first order condition of the borrower:

$$P'(e) = \frac{1}{Y_H + x}$$

(7)

We define $e^*$ as the optimal effort:

$$e^*(x) = \arg\max_e u(e; x).$$

(8)

Hence the participation constraint of the borrower becomes:

$$u(e^*; x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq 0.$$  

(9)

In addition, for the borrower to have incentive to make full repayment in the good state, he needs to be “rewarded” when honouring the current contract. This is reflected in the following “no-default”
constraint  
\[ Y_H + x \geq Y_H. \]  
(10)

This constraint says that conditional on the good state, the borrower will prefer to honour the contract than to default. Clearly it is equivalent to asking \( \beta \delta' - R \geq 0 \): this is asking that the continuation value of the future relationship is large enough that the borrower prefers to pay whenever the good state is realized rather than default and terminate the relationship. Comparing (9) and (10), we have \( \delta' \geq 0 \).

In the following we assume first that only one type of bond contract can be written for all creditors. The bonds are one period contracts. Large creditors are players who are find it profitable to be committed to re-lending in every subsequent period conditional on no default. We also assume that large creditors are fully coordinated: they decide both \( R \) and \( \delta' \) as a group. In the absence of this assumption, there would be free riding even among large creditors leading to suboptimal contracts. Although our interest is in the free riding issue, we use the simplifying assumption that there are only two types of creditors, one who always free rides and one who never does. The only parameter that matters for the analysis is therefore the exogenously given \( \alpha \) which captures how the extent of free riding affects the equilibrium.

Solving now for the optimal contract of large lenders: Let \( V(\delta) \) be the maximum expected present value to the large creditor conditional on the state \( \delta \). Then \( V(\delta) \) at any given time must satisfy the following Bellman equation

\[
V(\delta) = \max \{ R_l, \delta' \} 
\]

\[
- \{ P[(1-\alpha)R + \beta V(\delta')] - (1-\alpha) \},
\]

(11)

where \( 0 < \beta < 1 \) is the large creditor’s discount factor, \( R = \alpha R_F + (1-\alpha)R_l \) and \( \delta' \) denote the current period interest rate and promise respectively, and \( P \) is the probability of good state. Equation (11) simply specifies \( V(\delta) \) as the expected payoffs under the optimal contract. If the good state is realised, the large creditor receives \( (1-\alpha) \) fraction of the full repayment and the discounted continuation value associated with future relationship. In the bad state, the borrower defaults, creditors obtains nothing and the relationship is terminated. Since the promise made by the large creditor is on the total amount of lending, while the return is only on \( (1-\alpha) \), small creditors will free ride on borrower’s full repayment in good state. Moreover, notice that the presence of small lenders depresses the repayment \( R \) as well. As \( \delta \) reflect the “transfer” from the large creditor to the borrower, we must have \( V'(\delta) < 0 \). We show this later.

Given that the large creditor is engaged in current period lending, the assumption that creditors are committed to paying their “promised value” made in the previous period, \( \delta \), implies the following constraint:

\[
u(e^*; x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq \delta,
\]

(12)

where \( Y_H \) is output realised in the good state respectively, and \( e^* \) is the optimal effort chosen by the borrower. Equation (12) is the so-called “promise-keeping” constraint on the part of the large
It is clear that $\delta$ is measured in terms of borrower’s utility. The promise keeping constraint reflects the set of $(R, \delta')$ that are consistent with the borrowers participation constraint, given that lenders are committed to honoring their promise. If this is violated the borrower prefers not to accept the contract and then get its reservation value $\delta$.

This maximization is done subject to the No repudiation constraint of the borrower (10) and the participation constraint (12).

Finally the contract must also satisfy the participation constraints of large and small creditors. Note that the rationality condition of the borrower requires $\delta \geq 0$. The rationality condition of the large creditor, $V(\delta) \geq 0$, implies $\delta \leq \delta_{max}$ as $V(\delta)$ is decreasing in $\delta$. So the domain of the value function we consider will be in $\delta \in [0, \delta_{max}]$. The participation of small creditors is satisfied by assumption in the definition of $R$. Generally speaking, the optimal contract of this repeated game may possess non-stationary equilibrium.

The Appendix shows that the all equilibria in this game are stationary, except for the first period. This is quite intuitive since the game itself has a stationary structure: maximum punishment ensures that whenever there is default, all lending is stopped. Since the only observable variable is the number of defaults, punishment is fixed, and there is no re-negotiation, there is no other way that history could matter. However, the initial $\delta$ captures history so that the predicted pattern of default would depend on history. Now we look at the properties of the stationary equilibria.

**Proposition 1** Let $V(\delta)$ be continuously differentiable, and $Y_H$ sufficiently large. Then (a) there exists a unique solution, $V(\cdot)$ to the Bellman equation (11) subject to constraints (10) and (12); (b) $V(\delta)$ is decreasing and concave; and (c) $V(\cdot) \geq 0$.

**Proof**: Appendix 1.

**Proposition 2** Let $Y_H$ be sufficiently large. Then there exists a unique optimal contract which is always stationary after the first period.

**Proof**: Appendix 2.

**Proposition 3** In the stationary optimal contract, $\partial R/\partial \alpha > 0$, and $\partial P/\partial \alpha < 0$.

**Proof**: Appendix 3.

The intuition for Proposition 3 is quite simple. Note that from the Bellman equation (11), the contract has two different effects on the value function. On the one hand, given the probability $P$, the large creditor would like to choose largest $R$ and smallest $\delta'$ so as to increase its payoff in the good state. We term this as the payoff increasing effect. This means that it will have incentive to choose the smallest possible $x$. On the other, the large creditors also have the incentive to have high $P$. We term this as the probability increasing effect. This requires high $x$. The optimal contract depends
which of these two effects dominates. The probability increasing effect depends on $\alpha$ since the higher is $\alpha$ the greater is the free riding by small creditors on large.

When $\alpha$ is large, the presence of the free-riding small creditors will decrease the payoff to the large creditor in the good state. So the large creditors will have more incentive to decrease $x$, leading to this payoff increasing effect dominating the probability increasing effect. In this case, the participation constraint of the borrower, (9) is binding. From Proposition 3, it is also clear that as $\alpha$ increases, optimal $x$ decreases. This implies that large fraction of small creditors increase the probability of default but lead to higher risk premia.

4 Conclusions

In this paper we presented a stylized model to analyze the effects of creditor composition on the probability of default by sovereign governments. In the model we assumed that small creditors own a fixed proportion $\alpha$ of the total bond issue. Small creditors have a direct effect on the price of bonds through increasing the competition in the market, however they free ride on large creditors by not taking the impact of their chosen $R$ on the incentives of the borrower to decrease the probability of default. The net effect according to our model is the opposite of increasing competition in the market. Indeed, an increasing share of small creditors in the bond market increases the probability of default and increases the risk premium. Our model can explain why a shift from syndicated loans to bonds might lead to more volatility in the market than before.

Obviously, a lot remains to do. We are interested in endogenizing the entry of large and small creditors: if there are secondary markets in bonds then small creditors might have incentives to sell to large creditors so that the ownership structure in the end may be no different from syndicated loans! Second, we would like to relax the maximum punishment rule we imposed: it would be interesting to analyze the case where instead of only rewards to the borrower the lenders can use reductions in access to the market as punishment. The result of negotiations after default depend on the ownership structure and that in itself would alter the default rates. We might also consider allowing repayments to some creditors and not others: an endogenous seniority rule that emerges in response to reputational concerns.
References


A Proofs

Appendix 1: Proposition 1

Proof.
In what follows we assume that $V(\cdot)$ is continuously differentiable. We first illustrate that the large creditor’s value function and its promised value are bounded from above so a metric can be defined, we then show there exists a unique value function to the Bellman equation for some $\delta$ subject to constraints (10) and (12). Finally, we show that the value function is decreasing and concave.

(a) Value function and promised value are bounded from above

Suppose that we can implement some contract to generate the Pareto frontier to the problem, the combined benefits to all players are given by the following value function

$$W = \max_e \{ P(e)Y_H - e - 1 + P(e)\beta W \}$$ (13)

where $W$ defines the Pareto frontier. Given $Y_H < \infty$, then

$$W = \max_e \{ P(e)Y_H - e - 1 \} / [1 - P(e)\beta] \equiv \bar{W} < \infty.$$ (14)

For any allocation of this benefits between creditors and the borrower, we must have $V \leq \bar{W}$. So for any bounded pair of $V$, we can define a metric. For the same reason, the borrower’s utility must also be bounded from above by $\bar{W}$. This implies that the promised value by the large creditor must be bounded from above. Denote the highest promised value by $\bar{\delta}$, then

$$\delta, \delta' \leq \bar{\delta}.$$ (15)

(a) We use Blackwell’s sufficient condition to show that there exists a unique solution to the Bellman equation (11).

1) Monotonicity (for a given $\delta'$)

Define the right hand side of (11) as a function of $x$ and $V$:

$$T(V(\delta')) = \max_{R,\delta'} \{ P(e)(1 - \alpha)^2 R_l + P(e)\beta V(\delta') \} - (1 - \alpha)^2$$ (16)

subject to constraints (10) and (12), where $T$ is an operator in a metric space. For monotonicity, we have to show that $V \geq V$ implies $T(V) \geq T(V)$.

Construct the Lagrangean

$$L = \max_{R,\delta'} \{ P[(1 - \alpha)^2 R_l + \beta V(\delta')] - (1 - \alpha)^2 + \lambda[P(e^*)(Y_H + x) - e^*(x) - \delta] + \mu x + \phi(\beta R_l - 1) \}$$ (17)

where $\lambda, \mu$ are Lagrange multipliers. The FOCs to (17) determines $R$ and $\delta'$. Using the envelope theorem

$$\partial L / \partial V = \beta P(e) > 0$$ (18)

So $T(V)$ is monotonic.

2) Discounting

We have to show that $T(V + c) \leq T(V) + \beta c$ for $\beta \in (0, 1)$. It is straightforward to show that

$$T(V + c) = \max_x \{ P(x)(1 - \alpha)^2 R_l + P(x)\beta [V(\delta') + c] \} - (1 - \alpha)^2$$

$$= T(V) + P\beta c \leq T(V) + \beta c$$ (19)
Hence the operator \(T\) satisfies monotonicity and discounting, so by the Blackwell's sufficient condition, \(V\) exists and is unique.

(b): Now we prove that the value function is decreasing and concave. Using the envelope theorem to (17) we have: \(V' (\delta) = \partial L/\partial \delta = -\lambda\), so \(V (\delta)\) is decreasing.

Let the iteration of the value function be given by

\[
V_{i+1} = T (V_i) = \max_{R_l, \delta'} \{ P[ (1 - \alpha) \alpha R_F + (1 - \alpha)^2 R_l - 1 + \alpha + \beta V_i (\delta')] \\
+ \lambda [P(e^*) (Y_H + x) - e^* (x) - \delta] + \mu x + \eta V_i \} \tag{20}
\]

Assume that \(V_0\) is the initial value function and w.l.o.g assume it to be concave. Since \(P[(1 - \alpha)^2 R_l + \beta V_i (\delta')]\) is concave in \(R_l\) and \(\delta'\), constraints (10), (12) form a convex set of \(R_l\) and \(\delta'\), so \(V_1\) is a concave function of \(\delta\). Through this consecutive iteration process, \(\lim_{i \to \infty} V_i\) must converge to a concave function.

Clearly, \(V (\delta)\) is a decreasing and concave function in the restricted domain \(\delta \in [0, \delta_{\text{max}}]\).

To show that \(V (\cdot) \geq 0\), we only have to show that the feasible set used in optimising the RHS of (17) is not empty. This is because if there is a solution, then \(PR_l \geq 1\), so period utility and the value function of the large lender is non-negative.

Note first that the small creditors participation constraint, \(P(e^* (x)) R = 1\) generates a decreasing convex function in \((R, x)\) space, i.e., \(S = \{(R, x) : P(e^* (x)) R \geq 1\}\) is a convex set. Observe that the LHS of the borrower’s participation constraint (9) is strictly increasing in \(x\), so that the constraint is equivalent to \(x \geq \bar{y}\). So the intersection of these two sets is a non-empty convex set.

It remains to check this intersection is non-empty when we add the constraint (15). The binding constraint (15) translates into a downward sloping straight line in \((R, x)\) space. The set associated constraint (15) is simply a triangle formed by the two axes and the downward sloping line mentioned above. This set is clearly convex. When \(Y_H\) increases, this set expands. Therefore, there exists a \(Y_H\) such that the feasible set is convex and non-empty.

**Appendix 2: Proposition 2**

**Proof.**

In what follows, we first show the existence of a stationary equilibrium when \(Y_H\) is sufficiently large and \(\alpha \in (0, \alpha_{\text{max}})\). Then we look at the properties of the stationary equilibrium.

Given \(\delta\), the optimal choice of \(\delta'\) is determined by the following FOC:

\[
V' (\delta') = -(1 - \alpha)^2. \tag{21}
\]

If such a stationary equilibrium, \(\delta'\), exists, then it must be reached in just one period.

Note that without imposing \(\delta \geq 0\) and \(V (\delta) \geq 0\), Proposition 1 ensures that there always exists some \(\delta'\) such that (21) can be satisfied. However, from Proposition ??, the rationality conditions
\( \delta \geq 0 \) and \( V(\delta) \geq 0 \) imply that the value function is restricted to some domain \( \delta \in [0, \delta_{\text{max}}] \) where \( V(\delta_{\text{max}}) = 0 \). To show the existence of the stationary equilibrium \( \delta' \), we only need to show that \( \delta' \in [0, \delta_{\text{max}}] \).

From Proposition 1, if \( Y_H \) is large enough, \( V(\cdot) \geq 0 \). Consider the case where \( \delta = 0 \). Note that the borrower’s period utility \( u(e^*; x) = P(e^*)(Y_H + x) - e^*(x) \) is increasing in \( Y_H \), so for large enough \( Y_H \), \( u(e^*; x) > 0 \). In this case constraint (10) is binding and (12) is not (for \( \delta = 0 \)). Using the envelope theorem, it is clear that \( \partial V(0)/\partial \delta = 0 \).

Now, we look at the local behaviour of the value function \( V(\delta) \) near \( \delta_{\text{max}} \). Since \( V(0) > 0 \), so \( V(\delta_{\text{max}}) = 0 \) only if \( V'(\delta_{\text{max}}) < 0 \). From the envelope theorem, this is the case where \( \lambda > 0 \), so constraint (12) is binding, i.e., \( P(e^*)(Y_H + x) - e^*(x) = \delta_{\text{max}} \). Differentiating both sides of the constraint with respect to \( \delta \) and incorporating the FOC (7) yield

\[
P(e^*) \frac{\partial x^*}{\partial \delta} = 1. \tag{22}
\]

Using the Bellman equation (11) at \( \delta_{\text{max}} \),

\[
V(\delta_{\text{max}}) = P(e^*)[(1 - \alpha)^2 R_i^* + \beta V'(\delta')] - (1 - \alpha)^2. \tag{23}
\]

one can differentiate both sides with respect to \( \delta \) to obtain

\[
\partial V(\delta_{\text{max}})/\partial \delta = -(1 - \alpha) + P'(e^*)[de^*(x)/dx]dx/d\delta[(1 - \alpha)\tilde{R} + \beta V'(\delta')] + \alpha(1 - \alpha)\beta P d\delta'/d\delta. \tag{24}
\]

Assume that the future equilibrium \( \delta' \) exists and is stationary (which will be validated later), and let \( Y_H \) to be very large so \( P(e^*) \rightarrow 0 \), then

\[
\partial V(\delta_{\text{max}})/\partial \delta = -(1 - \alpha) < V'(\delta') = -(1 - \alpha)^2. \tag{25}
\]

Since the value function is continuously differentiable and concave, there must be an interior solution such that \( V'(\delta') = -(1 - \alpha)^2 \) which is stationary.

**Appendix 3: Proposition 3**

Before proving this Proposition we need the following lemma:

**Lemma 1** Suppose there exists an optimal contract which is stationary. Then in equilibrium, the participation constraint of borrowers (9) is always binding.

**Proof.**

By Proposition 2, in a stationary equilibrium we have \( V'(\delta') = -(1 - \alpha)^2 \). Moreover by the Lagrangean equation (??), \( V'(\delta) = \lambda \). By Stationarity, \( \delta' = \delta \) so \( V'(\delta) \neq 0 \) implies that \( \lambda \neq 0 \). Hence the constraint (9) is always binding in equilibrium.

**Proof.** First notice that at equilibrium we have
\[ V'(\delta') = -(1 - \alpha)^2. \]  

(26)

Hence, by the implicit function theorem we have \( \frac{d\delta'}{d\alpha} = \frac{2(1 - \alpha)}{V''(\delta')} < 0 \) since \( V'' < 0 \).

From Lemma (1), we know that the participation constraint of the borrower, (9) is binding, so:

\[ P(e^*)(Y_H + x) - e^*(x) = \delta \]  

(27)

By the implicit function theorem:

\[ P'(e^*) \frac{\partial e}{\partial x} \frac{\partial x}{\partial \alpha} (Y_H + x) + P(e^*) \frac{\partial x}{\partial \alpha} - \frac{\partial e}{\partial x} \frac{\partial x}{\partial \alpha} = \frac{\partial \delta}{\partial \alpha} \]  

(28)

From the first order condition of the borrower, (7), we know that \( P'(e^*) = \frac{1}{Y_H + x} \). Substituting this in equation (28) above, we get:

\[ P(e^*) \frac{\partial x}{\partial \alpha} = \frac{\partial \delta}{\partial \alpha} \]  

(29)

Now, \( x = \beta \delta - R \), in the stationary equilibrium, so again, using the Implicit Function theorem:

\[ \frac{\partial R}{\partial \alpha} = \beta \frac{\partial \delta}{\partial \alpha} - \frac{\partial x}{\partial \alpha} \]  

(30)

Using equation (29) above, we get:

\[ \frac{\partial R}{\partial \alpha} = \beta \frac{\partial \delta}{\partial \alpha} - \frac{1}{P} \frac{\partial \delta}{\partial \alpha} = \frac{\beta P - 1}{P} \frac{\partial \delta}{\partial \alpha} > 0. \]  

(31)

Consider the first order condition for the borrower equation (7): By the Implicit Function theorem, we have that

\[ \frac{\partial e}{\partial x} = -\frac{1}{(Y_H + x)^2} \frac{1}{P^2} > 0 \]  

(32)

Also by assumption \( P' > 0 \) so \( \frac{\partial P}{\partial \alpha} = P' \frac{\partial e}{\partial x} \frac{\partial x}{\partial \alpha} < 0 \). Hence, when the constraint (9) is binding then as \( \alpha \) increases, \( R \) increases and \( P \) decreases. 

\[ \blacksquare \]