Sovereign Debt Default: The Impact of Creditor Composition *

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Abstract

The main motivation of this paper is to study the impact of the composition of creditors on the probability of default and the corresponding default risk premium on sovereign bonds, when there is debtor moral hazard. In the absence of any legal enforcement, relational contracts work only when there are creditors who have a repeated relationship with the borrower. We show that ownership structures with a larger fraction of long term lenders are associated with a lower default probability and lower risk premia, except when the country is faced with very productive investment opportunities.

JEL: F34, G34, H63, D82, D86

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1 Introduction

In recent years countries have turned increasingly from bank loans to bond issues to raise capital. As a result, the international capital markets are more diversified and function more efficiently. Specifically, there is a broader investor base available to provide financing for emerging market sovereigns, which has helped diversify risk. But there is a serious downside if a country faces unsustainable debt. Private creditors have become increasingly numerous, anonymous and difficult to coordinate. (IMF (2003))

Lending in sovereign debt markets in the 1980s was very different from what it is now: there were syndicated bank loans and a small number of banks which operated on a common set of assumptions that tended to avoid legal action. In contrast, Brady bonds and subsequent new debt issues in the 1990s were purchased by thousands of new investors, including institutional hedge funds (see Figure 2 in Appendix).

This greater diversity among creditors meant that they were less likely to be constrained by tacit understandings about a shared collective interest[1]. Is this good or bad for creditors and/or debtors? Reinhart and Rogoff (2009) suggest that the thinking behind the debt crises of the 1990s and early 2000s in Latin America was that bonds were much safer as a source of financing than loans. The reasoning was that debtor countries will be much more hesitant to default given that un-coordinated small creditors are bad at re-negotiating debt, and thus there would be no repeat of the 1980s in which banks had rescheduled debt. Thus the lack of coordination of creditors was more costly ex-post, ensuring a lower ex-ante probability of default. Sachs (1984) argues that small investors are less likely to be co-ordinated in negotiations leading to *more* defaults due to loss of confidence in rolling over debt. Diamond (1991), too, comes out in

[1]For example, let us examine the events after the Argentine default of 2001. The ex-post default evidence on bond issues has been studied by e.g. Dhillon, García-Fronti, Ghosal, and Miller (2006) and Sgard (2005). It reveals an interesting pattern of creditor composition post default: a large number of small creditors (more than 1/4 million) and a few very big lenders. This creditor heterogeneity has been cited as one of the causes for delay in the post default renegotiation. The coordination problem was resolved just before the 2005 swap: a significant fraction of the small lenders sold their bonds cheap to big lenders allowing them to start bargaining with Argentina.

favour of banks in the context of corporate finance- banks provide monitoring services which directly issued bonds do not, leading to lower probability of default. None of these mentions however the trade off between lower prices when competition is higher[2] (leading to higher welfare for the debtor country) and the lack of coordination of creditors, though implicitly, this must have been part of the reason for the broadening of the investor base. In this paper we focus on the trade off between lower prices and lower co-ordination when the market structure becomes more competitive and analyse one particular mechanism that affects this trade-off.

In sovereign debt markets it is well known (Eaton and Gersovitz (1981), Eaton (1996)) that debt cannot be contractually enforced, and reputational mechanisms are crucial for creating incentives to repay. When debt cannot be contractually enforced in the debtor country, creditors who can coordinate punishments for default or rewards for good behaviour may achieve much better outcomes in terms of default incentives than un-coordinated creditors. Coordination problems can arise at various stages: e.g. at the level of the contracts that are signed: how favourable they are to creditors, at the level of punishment in case of default and at the level of the re-negotiation of debt in case of partial default. This paper focuses on the coordination problems that arise at the level of the contracts: the more coordinated creditors are, the better the terms of the contract to creditors, but also the better is the contract design when debt repayment has to be self enforcing and there is moral hazard on the part of debtors. We study the question of how the composition of debt, in particular, the presence of large creditors who have a repeated relationship with the debtor affects (ex-ante) the probability of default and the default risk premium on bonds. Our main result is that higher competition leads to higher probability of default and higher risk premia on bonds. Two key assumptions on large creditors are, first, that they are able to coordinate in contract design and second, that they are able to commit to the contract.

Our formal model is a stochastic dynamic game where there is a fraction of "large"

[2]Kovrijnykh and Szentes (2007) however do consider the price outcomes of different market structures in a different setting.

lenders and a fraction of “small” lenders all of whom buy one period bonds from the borrower country. The only difference between the two types of lenders is that large borrowers internalize the effects of their own actions on the rate of interest and on the probability of default, and have access to a commitment technology for contracts while small lenders do not. Small lenders always offer to buy bonds at any rate of interest higher than the risk free rate. Large lenders however have bargaining power in setting the rate of interest while small lenders take it as given and only decide whether to buy bonds or not. The borrower country chooses the level of effort (or investment): the higher the effort the higher the probability of a higher output realisation. In case there is a low output realisation, the country must default on its debt. In this setting, we look for the optimal dynamic contract for the borrower country which must take into account two incentive problems: moral hazard and repudiation of debt servicing in the good state and must take account of the participation constraint of the debtor country as well.

Large lenders choose their profit maximizing rate of interest subject to the incentive compatibility conditions, and the participation constraint of the debtor. Small lenders exert a negative externality on large lenders since they free ride on providing incentives to repay. We show that the higher the fraction of bonds held by small creditors, the higher is the probability of default and the higher are the corresponding default risk premia, except when opportunities for investment have a very high expected return. This result has a similar flavor to a phenomenon pointed out by Hellman, Murdock, and Stiglitz (2000) that banks have tendencies to gamble on investments much more the higher is the competition they face in the market. In contrast to models of relationship banking where lenders have an advantage in monitoring debtors when they have a ”relationship” with them (see Boot (2000)), our paper is predicated on the premise that moral hazard problems are solved using contract design rather than monitoring: instead of the cost of monitoring, here it is costly for lenders to incentivize debtors through lowering the rate of interest on the debt.

Wright (2005) shows that when contracts are not enforceable, then the constrained efficient allocation cannot be sustained since creditors cannot credibly commit to punishing after default.[3] In our paper, we focus on a more positive question in the context of unenforceable contracts and unobserved effort: default probabilities and default risk premia when the punishment is fixed but the rewards for repayment are allowed to be flexible. We then ask what is the best outcome possible when the fraction of creditors who can coordinate is allowed to vary and then we compare the different cases. In contrast to Wright (2005), we look for the properties of the optimal dynamic contract for an exogenously given degree of creditor coordination. We find that the optimal dynamic contract is stationary.

The paper is organized in two sections. Section 2 introduces the setup of the model. Section 3 analyses Sequential equilibria of the repeated game. Sections 4 and 5 build intuition by using a simple binary effort model and Section 6 generalizes the conclusions to the continuous effort case. Finally Section 7 concludes.

2 The Model

The model is set up to investigate in a simple way, the link between the composition of bond ownership, the probability of default and the default risk premium on bonds.

There is one debtor country which has a fixed endowment of labour. There are two goods. The borrower country has no endowment of good 1 (capital) and no storage technology. The debtor borrows 1 unit of good 1 in every period to invest in a risky technology which yields the end of period payoffs of Y_H of good 2 with probability P and $Y_L = 0$ with $1 - P$. The probability is assumed to be an increasing and concave function of the level of effort (or investment measured in units of good 2), e. The cost of effort is also e. The rate of transformation of good 1 into good 2 is one to one, and prices are normalized at good 1 prices. The debtor country cares about the quantity of good 2 that is available to consume in each period. The debtor country issues a one period bond with a face value of R. The bond can be bought by two types of creditors:

[3] Of course whether creditors lack of coordination means that the debtor gains ex-post as in his paper (thus has more reasons to default), or the debtor loses out ex-post (and thus has lower incentives to default ex-ante) because re-negotiation is difficult and some creditors hold out for better deals is ultimately an empirical question.

large creditors and small creditors. As mentioned before the difference between the two is that large creditors can commit to future rewards and have full bargaining power on the rate of interest on the bonds, while small creditors cannot. We analyze three different types of market structure: (1) Monopoly (or completely coordinated large lenders who own all the bonds) (2) Perfect competition - there are no large lenders (3) Intermediate market structure with a fraction α of bonds owned by small lenders. There is a credit constraint on large lenders so that they cannot buy more than $1 - \alpha$ fraction of the bonds. Small creditors are assumed to be non-strategic: they are willing to buy bonds as long as the bonds pay at least the risk free rate which is normalized to 1. All creditors are assumed to be risk neutral, as is the borrower country.

The time line for the stage game is given in Figure 1 below. First the debt contract is signed with the borrower receiving one unit of lending (in good 1) and promising to pay R (in good 2) a period later. Given this contract and expected future payoffs, the borrower then determines his optimal effort. Depending on effort $e \in \mathbb{R}_{+}$, the probability of getting high output Y_H is $P(e)$. Otherwise there is low output Y_L.

Finally, outcomes of the borrower's investment are realized. Output is not observed by creditors, so contracts are not state contingent. If high output occurs, the borrower could pay off the debt and engage in a new round of borrowing; or could default. If the bad state is realized we assume that there is 0 output so that there is no option but to default and pay nothing when there is no possibility of legal enforcement. In this case the borrower is excluded from the market forever.[4]

Figure 1 shows the stage game of a discrete time, infinite horizon repeated game with imperfect monitoring and we investigate the properties of the sequential equilibria of this game (for simplicity we have not shown that effort is unobservable in the figure). The effort is unobservable to creditors, and is costly, so the debtor is subject to moral hazard. Large creditors are able to condition the rewards for repayment on the full history of repayment.

[4]Another important mechanism by which creditor coordination can affect the probability of default is the credibility of ex-post punishment. This is tackled in Wright (2005).

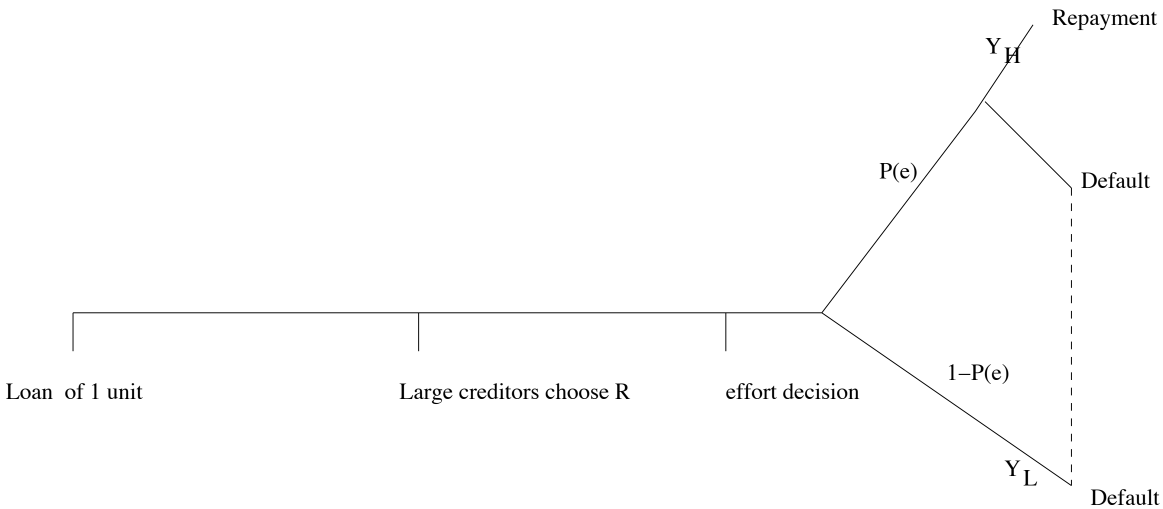

Figure 1: Time Line

3 Sequential equilibria of the repeated game

This section analyses the sequential equilibrium resulting from repeated interactions between the borrower, the large creditor(s) (treated essentially as one fully coordinated group) and a fringe of small creditors. Given that the borrower's output is not observable by the creditor, we look for a non-state-contingent optimal contract. By observing the past payment history of the borrower, the (representative) large creditor decides R on the current loan. We first describe the sequential problem facing the large lenders. Since we restrict attention to maximum punishment contracts, once there is default, the game ends. The publicly observable history is the sequence of repayments if there has been no default till time t. The optimal dynamic contract is a history dependent sequence of repayments $\{R(h_t)\}_{t=1}^{\infty}$ which maximizes the large lenders' payoffs.

In the equilibrium we must have the following:

(1) The borrower chooses the effort level e, in period t to maximize:

$$U_t = max_{e_s} \sum_{s=t}^{\infty} \beta^{s-t} [\Pi_{j=0}^{s-t-1} P(e_j)] \{P(e_s)[Y_H - R(h_s)] - e_s\} \tag{1}$$

having observed the contract $R(h_t)$, where $\Pi_{j=0}^{-1} P(e_j) = 1$.

(2) Large lenders choose the contract to maximize their payoff:

$$V_t = \max_{\{R(h_s)\}} \sum_{s=t}^{\infty} \beta^{s-t} [\Pi_{j=0}^{s-t-1} P(e_j)] \{P(e_s)[(1-\alpha)R(h_s) - (1-\alpha)\}, \tag{2}$$

subject to the following constraints:

(i) Borrower's participation constraint:

$$U_t(e^*(h_t)) \geq 0 \tag{3}$$

where the autarky payoff is normalized to 0.

(ii) Non-repudiation Constraint (or Self Enforcement constraint) of the borrower in the good state:

$$Y_H - R(h_t) + \beta U_{t+1}(R_s(h_{t+1})) \geq Y_H. \tag{4}$$

In equilibrium we check that the following is satisfied as well:

(iii) Small creditors' participation constraint:

$$P(e^*(h_t))R(h_t) \geq 1. \tag{5}$$

Following Spear and Srivastava (1987), this problem can be formulated recursively, using lifetime utility of the borrower as a state variable which summarizes information about a borrowers' default history. We consider the space of contracts where incentive problems can be partially overcome using memory and future promises. Contracts are restricted to depend only on publicly observable outcomes, which in this case is just whether there is default or not. We follow the previous literature (Spear and Srivastava (1987), Thomas and Worrall (1988), Abreu, Pearce, and Stachetti (1990), Phelan and Townsend (2000) among others) in formulating the contracting promise recursively using a "promised value". The contracts specify moreover that upon default, the borrower will be permanently excluded from the future credit market.

The contract design problem of the above setup needs to take into account two basic elements: one is the lack of commitment mechanism on the part of the borrower i.e. that debt can be repudiated, the other is the private information concerning the actual realised states. The one-sided commitment problem in a similar context has been studied by Kocherlakota (1996) who looks at self-sustaining insurance contracts in a village economy where villagers face idiosyncratic endowment shocks. There, full insurance is not possible as the optimal contract has to take into account the lack of commitment on the part of villagers even if their endowments are public information.

Instead, the optimal relational contract derived exhibits history dependence, where history summarizes all past endowments. Such history dependence is dealt with in a recursive manner by using a "promised value".[5] A similar problem with asymmetric information has been investigated by Thomas and Worrall (1988).

To induce the borrower to put in higher effort and to repay the loan when output is high, the large creditor has to promise more favorable terms in future loan contracts. How does this "promised value" capture history dependence in our model? Let h_t track the borrower's past output realizations up to time t. As the loan contract is not state contingent, the borrower will default in the bad state. So h_t simply counts the number of times that the borrower has made full repayments (or a string of realizations of the good state).

Let δ_t be the "promised value" (present value of lifetime utility in t) made by the large creditor in the period $t-1$ for delivery in period t. Given δ_t, the current period interest rate is determined, $R_t(\delta_t)$. Conditioned on this interest rate and the future promise δ_{t+1}, the borrower decides on the optimal effort, $e^*(R_t, \delta_{t+1})$. The future promise also affects whether the borrower would repay the loan in the good state. So when period $t+1$ arrives, δ_{t+1} would be delivered only if there was a good state in period t. This implies that δ_{t+1} depends on δ_t and a realisation of Y_H at t, $\delta_{t+1} = f(\delta_t, Y_H)$. Iterating this relationship forward from the initial δ implies that δ_{t+1} depends on h_t. In what follows, we denote δ the promise made by the large creditor in the last period, and δ' the promise made in the current period. In the optimisation faced by the large creditor, δ serves as a state variable.

We solve now for the dynamic optimal contract among those with maximum punishment for this problem with one sided commitment and moral hazard.

We first build intuition by analyzing equilibria for the discrete effort case and then generalize it to continuous effort. In the next section we assume binary effort levels: $e \in \{0, 1\}$. With $e = 1$, the probability of high state is P_1 and the cost of effort, measured in units of the investment good, is $c > 0$. With $e = 0$, the probability of high

[5] For other applications using "promised value" approach, see Ljungqvist and Sargent (2004), Chapters 15 and 16.

state is P_0 and the cost of effort is normalized to zero. We assume $1 > P_1 > P_0 > 0$. As before, we assume that the borrower will default if the low state is realized.

4 The Discrete Effort case

4.1 Efficiency

First we specify the associated investment problem by a central planner who combines utilities of the lenders and the borrower and finances the investment herself. The value function of the planner is

$$V_t^P = max_{e\in\{0,1\}}\{P_e(t)(Y_H + \beta V_{t+1}^P) - 1 - ce\}, \tag{6}$$

where $e = 1$ indicates the high effort level and $e = 0$ the low effort level. Using stationarity, this translates into:

$$V^P(e) = \begin{cases} (P_1 Y_H - 1 - c)/(1 - \beta P_1) & \text{if} \quad Y_H - \beta \geq c(1 - \beta P_0)/(P_1 - P_0), \\ (P_0 Y_H - 1)/(1 - \beta P_0) & \text{otherwise.} \end{cases} \tag{7}$$

The first line of equation (7) represents the value function with high effort, and the second line that of low effort.

The feasibility for high or low effort is simply $V^P(e) \geq 0$, i.e.,

$$Y_H \geq (1 + c)/P_1 \qquad \text{for high effort,} \tag{8}$$

$$Y_H \geq 1/P_0 \qquad \text{for low effort.} \tag{9}$$

In addition to (8) and (9), the condition for choosing high effort is given in the upper line of (7), re-written below

$$Y_H - \beta \geq \frac{c(1 - \beta P_0)}{P_1 - P_0}. \tag{10}$$

Note that (7) provides the bench-mark of efficiency: given (8) and (10), it is efficient to choose high effort; given (9) and the complement of (10), it is efficient to choose low effort. In what follows, we solve for the optimal stationary contract and analyse the properties of the optimal contract.

5 Stationarity of equilibria

Recall that in our model we assume that large creditors are able to commit to financing the borrower in the next period if the borrower repays. We capture this commitment using the next periods continuation value that is promised to the borrower. Let large creditor's last period promised-value (delivered in the current period) to the borrower be δ_t, and the current promised value (to be delivered in the next period) be δ_{t+1}. Both δ_t and δ_{t+1} are non-negative. A contract is therefore a pair (R_t, δ_{t+1}) that is conditioned on δ_t which summarizes past history.

Conditional on the current high effort, the value function of the borrower is

$$V_1 = P_1(Y_H - R) - c + \beta P_1 \delta'. \tag{11}$$

The borrower is willing to participate if $V_1 \geq \delta$, i.e.,

$$Y_H - R + \beta\delta' \geq (c + \delta)/P_1. \tag{12}$$

Similarly, conditional on the current low effort, the borrower's value function is

$$V_0 = P_0(Y_H - R) + \beta P_0 \delta'. \tag{13}$$

The borrower is willing to participate if $V_0 \geq \delta$, i.e.,

$$Y_H - R + \beta\delta' \geq \delta/P_0. \tag{14}$$

The no-repudiation-constraint for both cases is simply

$$Y_H - R + \beta\delta' \geq Y_H. \tag{15}$$

Given the contract (R, δ'), from (11) and (13), one can show that the borrower will exert effort as long as

$$Y_H - R + \beta\delta' \geq c/(P_1 - P_0). \tag{16}$$

And low effort is selected if

$$Y_H - R + \beta\delta' \leq c/(P_1 - P_0). \tag{17}$$

Now consider the large creditor's problem. The large creditor chooses the optimal contract such that

$$W = \max_{R,\delta'}\{(1-\alpha)(PR-1) + \beta P W'\}, \tag{18}$$

subject to (12), (15 and (16) or (14), (15) and (17), where W' represents the large creditor's future expected payoffs.

Assuming that a stationary equilibrium is reached in the next period, W' can take one of the two forms specified below. Conditional on high effort, the large creditor's future value function is

$$W_1' = (1-\alpha)\frac{P_1 Y_H - c - 1}{1-\beta P_1} - \delta_1'. \tag{19}$$

This is clear from (6). Given the structure of the contract, the large creditor can extract $(1-\alpha)$ fraction of the total benefit (while small creditors obtain α fraction), and transfers δ_1' to the borrower. Since the transfers only come from the large creditor, small creditors free ride.

Similarly, the future value function for the large creditor given low effort by the borrower is:

$$W_0' = (1-\alpha)\frac{P_0 Y_H - 1}{1-\beta P_0} - \delta_0'. \tag{20}$$

We now consider stationary equilibria: this requires the imposition of the stationarity condition

$$\delta_i = \delta_i'. \tag{21}$$

The properties of these stationary equilibria are presented in the following Proposition:

Proposition 1 *Assume $c \geq P_1/P_0 - 1$,*

(i) *For $c/(P_1 - P_0) \geq Y_H \geq \beta + \frac{(1-\beta P_0)c}{P_1 - P_0}$, there is a stationary high effort equilibrium if $\alpha \leq \tilde{\alpha}$ and a stationary low effort equilibrium if $\alpha > \tilde{\alpha}$, where*

$$\tilde{\alpha} = 1 - \frac{P_0(1-\beta P_0)(1-\beta P_1)[c/(P_1-P_0) - Y_H]}{(P_1-P_0)[Y_H - \beta - c(1-\beta P_0)/(P_1-P_0)]}. \tag{22}$$

In the high effort equilibrium, the optimal contract takes the form:

$$R_1 = Y_H - \frac{c(1-\beta P_0)}{P_1 - P_0}, \quad (23)$$

$$\delta_1 = \delta_1' = \frac{cP_0}{P_1 - P_0}. \quad (24)$$

In the low effort equilibrium, the optimal contract takes the form:

$$R_0 = \beta P_0 Y_H, \quad (25)$$

$$\delta_0 = \delta_0' = P_0 Y_H, \quad (26)$$

$$(27)$$

Furthermore, we have:

$$R_1 \leq R_0, \quad (28)$$

$$\delta_1 \geq \delta_0, \quad (29)$$

(ii) *For $c/(P_1 - P_0) < Y_H$, there is only a high effort stationary equilibrium. In this case, the optimal contract takes the form:*

$$R_1 = \beta(P_1 Y_H - c), \quad (30)$$

$$\delta_1 = \delta_1' = (P_1 Y_H - c). \quad (31)$$

(iii) *For $\frac{(1-\beta P_0)c}{P_1 - P_0} > Y_H$, there is only a low effort stationary equilibrium. In this case, the optimal contract takes the form:*

$$R_0 = \beta P_0 Y_H, \quad (32)$$

$$\delta_0 = \delta_0' = P_0 Y_H, \quad (33)$$

Proof: See Appendix.

To see the effect of α, note that from the proposition increasing α above the threshold $\tilde{\alpha}$ would shift the large creditor's choice to the inefficient solution- the contract inducing low effort. Increasing α even further would mean that no debt contract would be offered.

Results in Proposition 1(i) are quite interesting and intuitive. When Y_H is not very large ($Y_H \leq c/(P_1 - P_0)$), the contract has to be more attractive to induce high effort from the borrower than to prevent repudiation, while in the low effort equilibrium, the large creditor only has to ensure that the borrower will not repudiate. This is why the interest rate in the high effort equilibrium is lower and the promised value is higher.

When α increases, by maintaining high effort, large creditors extract less from the borrower when they have to make the contract sufficiently attractive to induce high effort from the borrower. This lowers profits. To maintain low effort, on the other hand, the above effort inducing effect is absent, so increasing α reduces large creditors' profits more in the high effort case than in the low effort case. When α is sufficiently large, low effort is chosen as an equilibrium. Thus, when $\alpha > \tilde{\alpha}$ then we obtain the inefficient low effort solution. the main message is that free riding by the fringe of short term creditors prevents the efficient high effort solution. In the absence of enforcement and participation constraints high effort would always be chosen and creditors would extract all surplus from the borrower. The borrower is indifferent between high and low effort.

Contrasting the two cases $\alpha = 0$ and $\alpha = 1$ is easy since equilibrium contracts (R, δ') do not depend on α except through the threshold $\tilde{\alpha}$. It is obvious that when $\alpha = 0$ there is a unique high effort equilibrium, and as $\alpha \to 1$ there is a low effort equilibrium when $c \geq \frac{P_1}{P_0 - 1}$ or when $Y_H < \frac{(1-\beta P_0)c}{P_1 - P_0}$ but when $Y_H > \frac{c}{P_1 - P_0}$ then there exists a high effort equilibrium. So when the market is very competitive, and the rewards from putting in high effort are sufficiently high (Y_H sufficiently high) then the incentives to put in effort become more high powered due to the debtor being the residual claimant.

The limiting case of perfect competition is when $\alpha = 1$. In this case competition between lenders reduces R_1 to $\frac{1}{P_1}$ and $R_0 = \frac{1}{P_0}$. The debtor chooses high effort iff $\frac{Y_H - \frac{1}{P_1} - c}{1 - \beta P_1} \geq \frac{Y_H - \frac{1}{P_1} - c}{1 - \beta P_0}$, i.e. if $Y_H - c \geq \frac{\beta(P_1 + P_0) - (P_1 - P_0)}{\beta P_1 P_0}$ then it is possible to have the efficient solution.

The intuition behind this result is that the incentives to put in high effort depend both on Y_H and on the promised value, δ'. While the market structure affects δ', clearly Y_H is exogenous to market structure. Hence even with perfect competition, when $\delta' = 0$ we still get the efficient outcome when Y_H is sufficiently high.

Finally, we cannot show that these two types of stationary equilibria are unique for the different parameter configurations. There may also be non-stationary equilibria. However in the general case, with continuous effort we show that the stationary equilibrium is unique. We conjecture that the stationary contract is the unique dynamic optimal contract even in the discrete case. The intuition can be seen clearly in the discrete case: when $c/(P_1 - P_0) \geq Y_H$ then we know from Proposition 1 that constraint (15) is never binding, while if $c/(P_1 - P_0) < Y_H$ then constraint (16) is never binding. Hence given Y_H the same constraints determine R, δ' throughout. If Y_H varied in different states (i.e. if there was more than one state) then we may not have a unique stationary solution.

5.1 Market structure and stability

In the previous section we showed that if $\alpha \in (0, 1)$ then large creditors are worse off with higher α. However, if there is a secondary market for bonds it may be possible to buy or sell them and change α. While we do not model a secondary market, in this section we check which levels of α are stable in the sense that there is no incentive to change the bond holdings. Small creditors never want to change since they are better off (in terms of the present value) than large creditors for any given equilibrium. What about large creditors? Since large creditors are assumed to be fully co-ordinated, the only way they can divest their bonds is to sell them all together, i.e. till the point when $\alpha = 1$. In this case R changes to the perfectly competitive level $\frac{1}{P_0}$. Since $\delta = 0$ by assumption in the perfectly competitive case (small creditors have no commitment technology), the maximum present value to the creditors per bond is $\frac{1}{1-\beta P_0}$. Given this, under what conditions does the high effort equilibrium survive? In the following proposition we show the conditions under which the large creditors stay in the market rather than divesting all their bond holdings. The proposition below shows that there exist thresholds of $\alpha < 1$ below which the intermediate market structure is stable in equilibrium.

Let $\hat{\alpha}_H = 1 - \frac{cP_0}{P_1 - P_0} \frac{(1-\beta P_1)(1-\beta P_0)}{(1-\beta P_0)(P_1 Y_H - c - 1) - (1-\beta P_1)}$, $\hat{\alpha}_{H1} = 1 - \frac{(P_1 Y_H - c)(1-\beta P_1)(1-\beta P_0)}{(P_1 Y_H - c - 1)(1-\beta P_0) - (1-\beta P_1)}$ and $\hat{\alpha}_L = 1 - \frac{(P_0 Y_H)(1-\beta P_0)}{(P_0 Y_H - 2)}$

Proposition 2 *Assume $c \geq P_1/P_0 - 1$, and*

(i) *Suppose $c/(P_1 - P_0) \geq Y_H \geq \beta + \frac{(1-\beta P_0)c}{P_1 - P_0}$ and $\alpha \leq \tilde{\alpha}$ the stationary high effort equilibrium is stable if $\alpha \leq \hat{\alpha}_H$ and the optimal contract takes the form:*

$$R_1 = Y_H - \frac{c(1-\beta P_0)}{P_1 - P_0}, \tag{34}$$

$$\delta_1 = \delta_1' = \frac{cP_0}{P_1 - P_0}. \tag{35}$$

Otherwise, if $\hat{\alpha}_H < \alpha$ the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

(ii) *Suppose $c/(P_1 - P_0) \geq Y_H \geq \beta + \frac{(1-\beta P_0)c}{P_1 - P_0}$ and $\alpha > \tilde{\alpha}$ or $\frac{(1-\beta P_0)c}{P_1 - P_0} > Y_H$ the stationary low effort equilibrium is stable if $\hat{\alpha}_L \geq \alpha$ and the optimal contract takes the form:*

$$R_0 = \beta P_0 Y_H, \tag{36}$$

$$\delta_0 = \delta_0' = P_0 Y_H, \tag{37}$$

Otherwise the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

(iii) *Suppose $c/(P_1 - P_0) < Y_H$, the high effort stationary equilibrium is stable if $\alpha \leq \hat{\alpha}_{H1}$ In this case, the optimal contract takes the form:*

$$R_1 = \beta(P_1 Y_H - c), \tag{38}$$

$$\delta_1 = \delta_1' = (P_1 Y_H - c). \tag{39}$$

Otherwise the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

Proof: See Appendix.

6 The continuous effort case

Let us first focus on the borrowers optimization problem, and converting it to the recursive form. Let $x = \beta\delta' - R$. Given the contract (R, δ') the borrower chooses optimal effort to maximize:

$$u(e; R, \delta') \equiv P(e)(Y_H + x) - e \tag{40}$$

where we assume that the discount factor of the borrower, β is identical to that of the large creditor, and the probability of the good state, $P(e)$, is increasing and concave in effort, e. In addition, we assume that $P'''(e) < 0$, $P(0) = 0$, $P'(0) \to \infty$, $P(\infty) = 1$ and $P'(\infty) = 0$. This yields the following first order condition of the borrower:

$$P'(e) = \frac{1}{Y_H + x} \tag{41}$$

We define e^* as the optimal effort:

$$e^*(x) = arg\max_e u(e; x). \tag{42}$$

Hence the participation constraint of the borrower becomes:

$$u(e^*; x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq 0. \tag{43}$$

In addition, for the borrower to have incentive to make full repayment in the good state, he needs to be "rewarded" when honouring the current contract. This is reflected in the following "non-repudiation" constraint

$$Y_H + x \geq Y_H. \tag{44}$$

This constraint says that conditional on the good state, the borrower will prefer to honour the contract than to default. Clearly it is equivalent to asking $\beta\delta' - R \geq 0$: this is asking that the continuation value of the future relationship is large enough that the borrower prefers to pay whenever the good state is realized rather than default and terminate the relationship. Notice that the participation constraint for the borrower must be satisfied in every period so that we have $\delta' \geq 0$.

Solving now for the optimal contract of large lenders: Let $V(\delta)$ be the maximum expected present value to the large creditor conditional on the state δ. Then $V(\delta)$ at any given time must satisfy the following Bellman equation

$$V(\delta) = \max_{\{R,\delta'\}}\{(PR-1)(1-\alpha) + \beta PV(\delta')\}, \tag{45}$$

where $0 < \beta < 1$ is the large creditor's discount factor, δ' denotes the current period interest rate and promise respectively, and P is the probability of good state. Equation (45) simply specifies $V(\delta)$ as the expected payoffs under the optimal contract. If the good state is realized, the large creditor receives $(1-\alpha)$ fraction of the full repayment and the discounted continuation value associated with the future relationship. In the bad state, the borrower defaults, creditors obtain nothing and the relationship is terminated. Since the promise made by the large creditor is on the total amount of lending, while the return is only on $(1-\alpha)$, small creditors will free ride on borrower's full repayment in good state. As δ reflects the "transfer" from the large creditor to the borrower, we must have $V'(\delta) < 0$. We show this later.

Given that the large creditor is engaged in current period lending, the assumption that creditors are committed to paying their "promised value" made in the previous period, δ, implies the following constraint:

$$u(e^*;x) \equiv P(e^*)(Y_H + x) - e^*(x) \geq \delta, \tag{46}$$

where Y_H is output realised in the good state respectively, and e^* is the optimal effort chosen by the borrower. Equation (46) is the so-called "promise-keeping" constraint on the part of the large creditor. It is clear that δ is measured in terms of borrower's utility. The promise keeping constraint reflects the set of (R, δ') that are consistent with the borrowers participation constraint, given that lenders are committed to honoring their promise. If this is violated the borrower prefers not to accept the contract and then get its reservation value δ.

This maximization is done subject to the No repudiation constraint of the borrower (44) and the participation constraint (46).

Finally the contract must also satisfy the ex-ante participation constraint of large creditors (and small creditors participation must be satisfied in equilibrium) Note that the rationality condition of the borrower requires $\delta \geq 0$. The ex-ante participation constraint of the large creditor, $V(\delta) \geq 0$, implies $\delta \leq \delta_{max}$ as $V(\delta)$ is decreasing in δ. So the domain of the value function we consider will be in $\delta \in [0, \delta_{max}]$.

Generally speaking, the optimal contract of this repeated game may possess non-stationary equilibrium. The Appendix shows that the all equilibria in this game are stationary, except for the first period. Existence of a stationary equilibrium is a bit puzzling since intuitively it would seem that creditors are better off with a scheme that provides discounts in the future as a reward for repayments now. We believe that the unique equilibrium is an artefact of the assumption of only one state of nature, since the game ends once default occurs.

Now we look at the properties of the stationary equilibria.

Proposition 3 *Let $V(\delta)$ be continuously differentiable, and Y_H sufficiently large. Then (a) there exists a unique solution, $V(\cdot)$ to the Bellman equation (45) subject to constraints (44) and (46); (b) $V(\delta)$ is decreasing and concave; and (c) $V(\cdot) \geq 0$.*

Proof: See Appendix.

Proposition 4 *Let Y_H be sufficiently large. Then there exists a unique optimal contract which is always stationary after the first period.*

Proof: See Appendix.

A sufficient condition for existence of the optimal contract is that Y_H is sufficiently large.

Proposition 5 *In the stationary optimal contract, $\frac{\partial R}{\partial \alpha} > 0, \frac{\partial P}{\partial \alpha} < 0$ and $\frac{\partial e^*}{\partial \alpha} < 0$.*

Proof: See Appendix.

The intuition for Proposition 5 is quite simple. Note that from the Bellman equation (45), the contract has two different effects on the value function. On the one hand, given the probability P, the large creditors would like to choose the largest R and smallest δ' so as to increase its payoff in the good state. We term this as the collusive (the extreme case is when $\alpha = 0$) effect. This means that creditors will have incentives to choose the smallest possible x. On the other, the large creditors also have the incentive to have high P. We term this as the probability increasing effect. This requires high x. The optimal contract depends which of these two effects dominates. The probability increasing effect depends on α since the higher is α the greater is the free riding by small creditors on large so that the participation constraint of large creditors is affected.

When α is large, the presence of the free-riding small creditors will decrease the payoff to the large creditor in the good state. So the large creditors will have more incentive to decrease x, leading to this collusive effect dominating the probability increasing effect. In this case, the participation constraint of the borrower,(43) is binding. From Proposition 5, it is also clear that as α increases, optimal x decreases. This implies that large fraction of small creditors increase the probability of default but lead to higher risk premia.

In terms of efficiency Proposition 5 shows that as α increases, the effort decreases. So competition has a bad effect on incentives to repay when there are moral hazard and enforceability problems.

7 Conclusions

In this paper we presented a stylized model to analyze the effects of creditor composition on the probability of debtor default when there is moral hazard. In the model we assumed that small creditors own a fixed proportion α of the total bond issue. Small creditors free ride on large creditors by not being able to commit to future discounts to decrease the probability of default. The net effect according to our model is that increasing competition in the market is bad unless the realization Y_H is very high. If the returns on investment are sufficiently large then a perfectly competitive market may be better for debtors and creditors. However, if that is not the case then an increasing share of small creditors in the bond market increases the probability of default and increases the default risk premium. Our model can explain why a shift from syndicated loans to bonds might lead to more volatility in the market than before. Obviously, a lot remains to do. We are interested in endogenizing the entry of large and small creditors: if there are secondary markets in bonds then small creditors might have incentives to sell to large creditors so that the ownership structure in the end may be no different from syndicated loans. In the paper we assume that all large creditors can coordinate as one. However when the number of such creditors increases (or their size becomes smaller) there may be conflicts within them. The oligopolistic situation would be different from the monopoly we have assumed here.

Second, we would like to relax the maximum punishment rule we imposed: it would be interesting to analyze the case where instead of only rewards to the borrower the lenders can use reductions in access to the market as punishment. The result of negotiations after default depend on the ownership structure and that in itself would alter the default rates. We might also consider allowing repayments to some creditors and not others: an endogenous seniority rule that emerges in response to reputational concerns.

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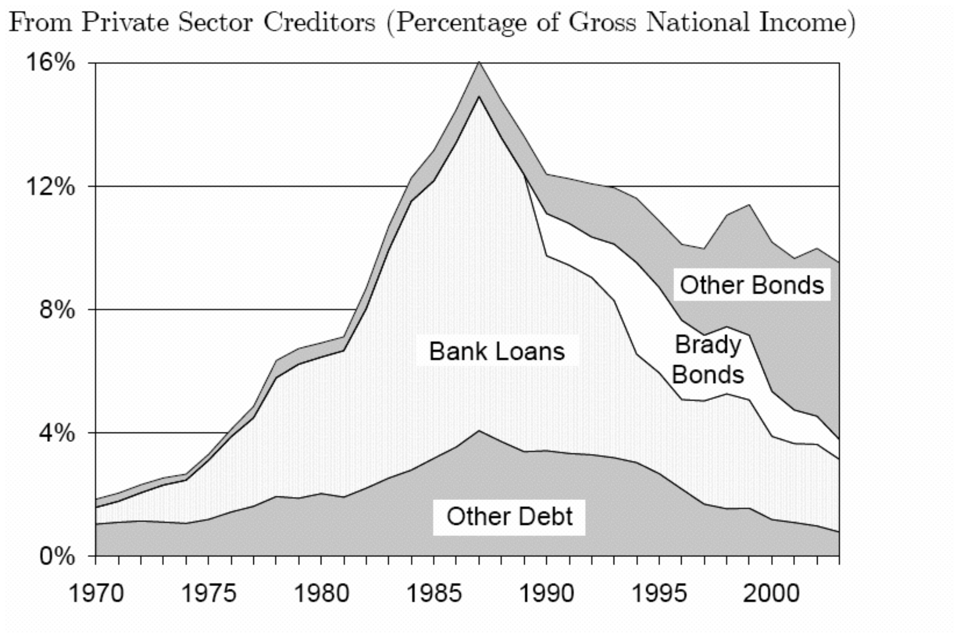

Figure 2: Taken from Wright (2005)

A Proof of Proposition 1

The assumption $c \geq P_1/P_0 - 1$ ensures (7), (9) and (10) are satisfied. So both low and high effort equilibria are feasible.

(i) The optimal contract in the stationary high effort equilibrium is a solution to

$$W_1 = \max_{R_1, \delta_1'} \{(1-\alpha)(P_1 R_1 - 1) + \beta P_1 W_1'\}, \tag{A.1}$$

subject to (12), (15), (16) and (21); and where W_1 is given in (19). Under the condition $c/(P_1 - P_0) \geq Y_H$, (16) implies (15). So the relevant constraints are (12), (16) and (21). Since the RHS of (A.1) is linear in R_1 and δ_1', its optimum must be attained on the boundary. This implies that all three constraints (12), (16) and (21) are binding, giving the results as in (23) and (24). The resulting value function for the large creditor then becomes

$$W_1 = (1-\alpha)\frac{P_1 Y_H - c - 1}{1 - \beta P_1} - \frac{c P_0}{P_1 - P_0}. \tag{A.2}$$

Similarly, the optimal contract for the stationary low effort equilibrium is obtained when constraints (14),(17) and (21) are binding. Solving them yields (32) and (33). In this case, the resulting value function for the large creditor becomes

$$W_0 = (1-\alpha)\frac{P_0 Y_H - 1}{1 - \beta P_0} - P_0 Y_H. \tag{A.3}$$

The threshold $\tilde{\alpha}$ is determined when $W_0 = W_1$. Below this threshold, $W_0 < W_1$, so stationary high effort equilibrium is chosen.

Let $\alpha_H = \{\alpha : W_1 = 0\}$, and $\alpha_L = \{\alpha : W_0 = 0\}$. Given the parameter restrictions, it is straight forward to show $1 - \tilde{\alpha} \geq 1 - \alpha_H \geq 1 - \alpha_L$. Increasing α above $\tilde{\alpha}$ shifts the high effort equilibrium to the low effort equilibrium, and increasing α further above α_L means no debt contract would be offered.

(ii) For $c/(P_1 - P_0) < Y_H$, constraint (15) and (17) cannot be satisfied at the same time. This implies that the feasible set for stationary low effort solution is empty. So, there is no low effort equilibrium.

(iii) For $Y_H < \frac{(1-\beta P_0)c}{P_1 - P_0}$, high effort yields a negative R_1, hence it is not feasible. The optimal contract for the low effort equilibrium is determined when (12), (15) and (21) are binding.

QED

B Proof of Proposition 2

Proof.

(i) Suppose $c/(P_1 - P_0) \geq Y_H \geq \beta + \frac{(1-\beta P_0)c}{P_1-P_0}$ and $\alpha \leq \tilde{\alpha}$

The following equation provides the conditions under which the per bond returns are higher:

$$\frac{P_1 Y_H - c - 1}{1 - \beta P_1} - \frac{1}{1-\alpha}\frac{cP_0}{P_1 - P_0} \geq \frac{1}{1 - \beta P_0} \tag{B.1}$$

The LHS is the Present Value of a unit bond held when large creditors together hold $1-\alpha$ of the bonds vs the Present Value of a bond if they were to give up all their bonds and enter the market as a small creditor (in a perfectly competitive lending market). Solving for α which satisfies this equation with equality we get $\hat{\alpha}_H$.

To check that $\hat{\alpha}_H > 0$, we can re-write equation (B.1) as follows:

$$\frac{P_1 Y_H - c - 1}{1 - \beta P_1} - \frac{1}{1 - \beta P_0} > \frac{1}{1-\alpha}\frac{cP_0}{P_1 - P_0} \tag{B.2}$$

The LHS is a constant while the RHS is an increasing function of α which goes to infinity as $\alpha \to 1$. When $\alpha = 0$, the RHS is $\frac{cP_0}{P_1-P_0}$. So to get a positive solution for α we need that $\frac{P_1 Y_H - c - 1}{1-\beta P_1} - \frac{1}{1-\beta P_0} > \frac{cP_0}{P_1-P_0}$

Otherwise, if $\hat{\alpha}_H < \alpha \leq \tilde{\alpha}$ the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

(ii) Suppose $c/(P_1 - P_0) \geq Y_H \geq \beta + \frac{(1-\beta P_0)c}{P_1-P_0}$ and $\alpha > \tilde{\alpha}$ or $\frac{(1-\beta P_0)c}{P_1-P_0} > Y_H$. The following equation provides the conditions under which the per bond returns are higher with the large creditors than with a perfectly competitive creditor market:

$$\frac{P_0 Y_H - 1}{1 - \beta P_0} - \frac{P_0 Y_H}{1-\alpha} \geq \frac{1}{1 - \beta P_0} \tag{B.3}$$

The LHS is the Present Value of a unit bond held when large creditors together hold $1-\alpha$ of the bonds vs the Present Value of a bond if they were to give up all their bonds and enter the market as a small creditor (in a perfectly competitive lending market).

Solving for α which satisfies this equation with equality we get $\hat{\alpha}_L$.

Otherwise, the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

(iii) Suppose $c/(P_1 - P_0) < Y_H$.

The following equation provides the conditions under which the per bond returns are higher with the large creditors and high effort than with a perfectly competitive creditor market:

$$\frac{P_1 Y_H - c - 1}{1 - \beta P_1} - \frac{1}{1-\alpha}(P_1 Y_H - c) \geq \frac{1}{1 - \beta P_0} \tag{B.4}$$

Solving for α which satisfies this equation with equality we get $\hat{\alpha}_{H1}$.

Otherwise the only equilibrium for these parameters is the perfectly competitive low effort equilibrium.

■

C Proof of Proposition 3

Proof.

Claim 6 *The feasible set is convex and compact.*

Proof. The large creditors' value function is given by:

$$V(\delta) = \max_{R,x}\{[P(e^*)R - 1](1-\alpha) + P(e^*)\beta V(\delta')\} \tag{C.1}$$

subject to the constraints:

$$u(e^*; x) \equiv \max_{e^*}[P(e^*)(Y_H + x) - e^*(x)] \geq \delta \geq 0, \tag{C.2}$$

$$x \geq 0. \tag{C.3}$$

where $\delta' = (R + x)/\beta$. Note that (C.2) is the combination of (43) and (46), and (C.3) is a simplification of (44).

In addition we need to satisfy the small creditors participation constraint:

$$P(e^*)R \geq 1 \tag{C.4}$$

and large creditors ex-ante participation:

$$V(\delta) \geq 0. \tag{C.5}$$

Let the set of R, x satisfying constraints (C.2)–(C.5) be denoted by $G(R, x)$. To show the set $G(R, x)$ is convex, compact and non-empty for Y_H sufficiently large, we use Figure 3 where the horizontal axis represents x and the vertical R.

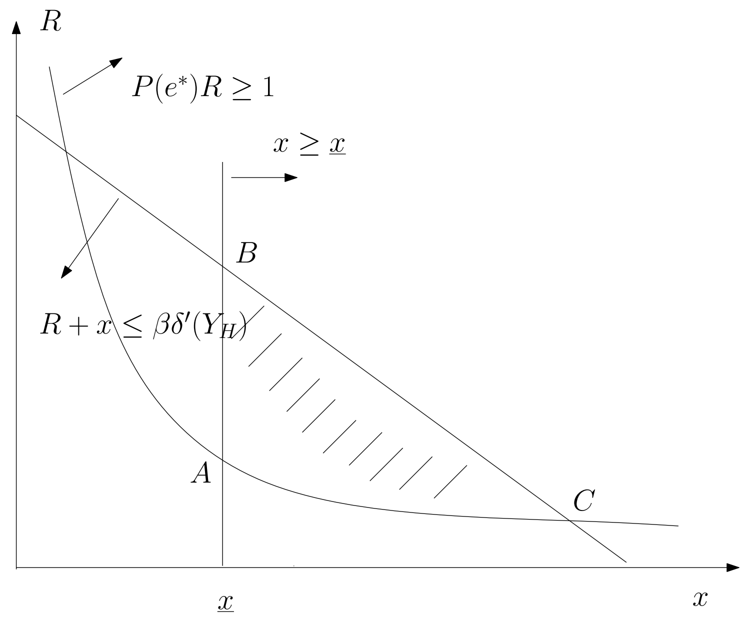

Figure 3: The feasible set.

Note first that $u(e^*; x)$ is strictly increasing in x. Setting (C.2) as an equality implies that there exists a lower bound $\underline{x}$ for x. Combining (C.2) and (C.3) implies $x \geq \max\{0, \underline{x}\}$. The feasible (R, x) given by (C.2) and (C.3) are represented by the area to the right of the vertical straight line AB (assuming $\underline{x} > 0$).

Second, note that $P(e^*)$ is increase and concave in e^* and $e^*(x)$ is strictly increasing in x, so $P(e^*)R = 1$ traces a decreasing and convex schedule in R, x space. Constraint $P(e^*)R \geq 1$ then determines the area above the convex schedule AC in the figure.

Third, there is a natural restriction on the feasibility of (R, x). Suppose the investment project is self-funded, the net-present-value for given level of effort e^* is $(P(e^*)Y_H - 1)/[1 - \beta P(e^*)]$. This would be the maximum rent that the creditors can extract. So the future promised value δ' must have an upper bound generally smaller than $(P(e^*)Y_H - 1)/[1 - \beta P(e^*)]$. As Y_H increases, the upper bound for δ' will increase. The downward sloping BC is drawn for an arbitrary upper bound of δ'. Points of (R, x) below BC are feasible.

The intersection of the above three restrictions forms the feasible set $G(R, x)$, represented by the shaded area in the figure. It is clear from the figure that if Y_H is large enough, δ' is sufficiently high so that the intersection is not empty. It is also clear that this set is convex and compact. Since any point in $G(R, x)$ satisfy $P(e^*)R \geq 1$, so $V(\cdot) \geq 0$. ■

Claim 7 *There exists a unique and concave value function.*

Proof.

We work with a metric space with continuously differentiable and bounded functions V (and/or W) mapping δ' onto the real line and with the metric $d_\infty = sup_{\delta'}|V(\delta') - W(\delta')|$. This metric space is complete. An operator T in this metric space is defined as

$$T[V(\delta')] = \max_{R,x}\{(P(e)R - 1)(1 - \alpha) + P(e)\beta V(\delta')\},$$

where T maps a continuously differentiable and bounded function into a continuously differentiable and bounded function $T[V(\delta')]$. We now prove that T is a contraction mapping using Blackwell's sufficient conditions.

For a given δ', suppose $V(\delta') \geq W(\delta')$ for all $\delta' \in [0, \delta_{max}]$. Note $T(V) = \max_{R,x}\{(P(x)R - 1)(1 - \alpha) + P(x)\beta V(\delta')\}$, and $T(W) = \max_{R,x}\{(P(x)R - 1)(1 - \alpha) + P(x)\beta W(\delta')\}$ for $(R, x) \in G(R, x)$. Since $\max_{R,x}\{(P(x)R - 1)(1 - \alpha) + P(x)\beta V(\delta')\} \geq \max_{R,x}\{(P(x)R - 1)(1 - \alpha) + P(x)\beta W(\delta')\}$ for $(R, x) \in G(R, x)$, so $T(V) \geq T(W)$. So T satisfies the monotonicity.

It is straightforward to show that

$$\begin{aligned} T(V+c) &= \max_x\{(P(x)R-1)(1-\alpha)+P(x)\beta[V(\delta')+c]\} \\ &= T(V)+P\beta c \leq T(V)+\beta c. \end{aligned}$$

So T satisfies discounting.

By Blackwell's sufficient condition, T is a contraction on a complete metric space and the functional equation $V(\delta) = T[V(\delta)]$ has a unique fixed point in the space of continuously differentiable and bounded continuous functions. Moreover T maps concave functions into concave functions so that the solution is concave.

■

Other properties of the value function

In what follows, we show that the value function is decreasing in the promised value, and $V(\delta) > 0$ for sufficiently large Y_H.

Construct the Lagrangean

$$L = \max_{R,\delta'}\{[P(e^*)R-1](1-\alpha)+P(e^*)\beta V(\delta')+\lambda[P(e^*)(Y_H+x)-e^*(x)-\delta]+\mu x+\phi(P(e^*)R-1)\} \tag{C.6}$$

where $\delta' = (x+R)/\beta$, λ and μ are Langrange multipliers. The first order conditions (FOCs) to (C.6) determines R and δ'. Using the envelope theorem (see (C.6)) we have: $V'(\delta) = \partial L/\partial\delta = -\lambda$, so $V(\delta)$ is decreasing.

From Figure 3, we have shown that the feasible set contain some point (R, x) strictly above the convex schedule AC. Given this feasible point, $P(e^*)R > 1$, so by the definition of the value function in (C), $V(\cdot) > 0$.

■

D Proof of Proposition 4

Proof.

In what follows, we first show the existence of a stationary equilibrium when Y_H is sufficiently large. Then we look at the properties of the stationary equilibrium. The

FOCs of the maximization problem are given by:

$$\begin{aligned}\frac{\partial L}{\partial \delta'} &= -P'(e)e'(x)(1-\alpha)R + (1-\alpha)P - \beta P'(e)e'(x)V(\delta') - \mu \\ &\quad -\lambda[P'(e)e'(x)(Y_H + x) + P - e'(x)] - \phi(P'(e)e'(x)R - P) = 0 \quad \text{(D.1)} \\ \frac{\partial L}{\partial R} &= \beta\{P'(e)e'(x)(1-\alpha)R + \beta P'(e)e'(x)V(\delta') + PV'(\delta') + \mu \\ &\quad +\lambda[P'(e)e'(x)(Y_H + x) + P - e'(x)] + \phi P'(e)e'(x)R\} = 0 \quad \text{(D.2)}\end{aligned}$$

Solving for $V'(\delta')$ yields:

$$V'(\delta') = -(1-\alpha) \tag{D.3}$$

As δ' is independent of δ, (D.3) indicates that all future contracts are stationary (if δ' exists), and the stationarity is reached in just one period.

Note that without imposing $\delta \geq 0$ and $V(\delta) \geq 0$, Proposition 3 ensures that there always exists some δ' such that (D.3) can be satisfied. However, from Proposition 3, the rationality conditions $\delta \geq 0$ and $V(\delta) \geq 0$ imply that the value function is restricted to some domain $\delta \in [0, \delta_{max}]$ where $V(\delta_{max}) = 0$. To show the existence of the stationary equilibrium δ', we only need to show that $\delta' \in [0, \delta_{max}]$.

From Proposition 3, if Y_H is large enough, $V(\cdot) > 0$. Consider the case where $\delta = 0$. Note that the borrower's period utility $u(e^*; x) = P(e^*)(Y_H + x) - e^*(x)$ is increasing in Y_H, so for large enough Y_H, $u(e^*; x) > 0$. In this case constraint (C.3) is binding and (C.2) is not (for $\delta = 0$). Using the envelope theorem, it is clear that $\partial V(0)/\partial \delta = -\lambda = 0$.

Now, we look at the local behaviour of the value function $V(\delta)$ near δ_{max}. Since V is decreasing in δ and $V(0) > 0$, so $V(\delta_{max}) = 0$ only if $V'(\delta_{max}) < 0$. From the envelope theorem, this is the case where $\lambda > 0$, so constraint (C.2) is binding, i.e., $P(e^*)(Y_H + x) - e^*(x) = \delta_{max}$. Differentiating both sides of the constraint with respect to δ and incorporating the FOC (7) yields

$$P(e^*)\frac{\partial x^*}{\partial \delta} = 1. \tag{D.4}$$

Using the Bellman equation (C) at δ_{max},

$$V(\delta_{max}) = P(e^*)[(1-\alpha)R^* + \beta V(\delta')] - (1-\alpha). \tag{D.5}$$

one can differentiate both sides with respect to δ to obtain

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P(e^*)(1-\alpha)\frac{\partial R}{\partial \delta} + P(e^*)\beta V'(\delta')\frac{\partial \delta'}{\partial \delta}. \tag{D.6}$$

Using $x = \beta\delta' - R$ we have $\frac{\partial R}{\partial \delta} = \beta\frac{\partial \delta'}{\partial \delta} - \frac{\partial x}{\partial \delta}$. Hence equation (D.6) can be re-written as:

$$\frac{\partial V(\delta_{max})}{\partial \delta} = P(e^*)(1-\alpha)[\beta\frac{\partial \delta'}{\partial \delta} - \frac{\partial x}{\partial \delta}] + \beta\frac{\partial \delta'}{\partial \delta}V'(\delta') \tag{D.7}$$

Noting that $P(e^*)\frac{\partial x}{\partial \delta} = 1$ we get

$$\frac{\partial V(\delta_{max})}{\partial \delta} = -\beta\frac{\partial \delta'}{\partial \delta}[(1-\alpha)(1-P) - (1-\alpha)] \tag{D.8}$$

Given the stationarity of δ', $\partial\delta'/\partial\delta = 0$, then

$$\frac{\partial V(\delta_{max})}{\partial \delta} = 0 > V'(\delta') = -(1-\alpha) \tag{D.9}$$

Since the value function is continuously differentiable and concave, there must be an interior solution such that $V'(\delta') = -(1-\alpha)$ which is stationary. ■

E Proof of Proposition 5

Before proving this Proposition we need the following lemma:

Lemma 1 *Suppose there exists an optimal contract which is stationary. Then in equilibrium, the "promise keeping" constraint of borrowers (46) is always binding.*

Proof.

By Proposition 4, in a stationary equilibrium we have $V'(\delta') = -(1-\alpha)$. Moreover by the Lagrangean equation (C.6), $V'(\delta) = \lambda$. By Stationarity, $\delta' = \delta$ so $V'(\delta) \neq 0$ implies that $\lambda \neq 0$. Hence the constraint (46) is always binding in equilibrium.

■

Proof. First notice that at equilibrium we have

$$V'(\delta') = -(1-\alpha) \tag{E.1}$$

Hence, by the implicit function theorem we have $\frac{d\delta'}{d\alpha} = \frac{1}{V''(\delta')} < 0$ since $V'' < 0$.

From Lemma (1), we know that the participation constraint of the borrower, (46) is binding, so:

$$P(e^*)(Y_H + x) - e^*(x) = \delta \tag{E.2}$$

By the implicit function theorem:

$$P(e^*)\frac{\partial x}{\partial \alpha} = \frac{\partial \delta}{\partial \alpha} \tag{E.3}$$

Now, $x = \beta\delta - R$, in the stationary equilibrium, so again, using the Implicit Function theorem:

$$\frac{\partial R}{\partial \alpha} = \beta\frac{\partial \delta}{\partial \alpha} - \frac{\partial x}{\partial \alpha} \tag{E.4}$$

Using equation (E.3) above, we get:

$$\frac{\partial R}{\partial \alpha} = \beta\frac{\partial \delta}{\partial \alpha} - \frac{1}{P}\frac{\partial \delta}{\partial \alpha} = \frac{\beta P - 1}{P}\frac{\partial \delta}{\partial \alpha} > 0. \tag{E.5}$$

Consider the first order condition for the borrower equation (41): By the Implicit Function theorem, we have that

$$\frac{\partial e}{\partial x} = -\frac{1}{(Y_H + x)^2}\frac{1}{P''} > 0 \tag{E.6}$$

Also by assumption $P' > 0$ so $\frac{\partial P}{\partial \alpha} = P'\frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha} < 0$. Hence, when the constraint (46) is binding then as α increases, R increases and P decreases.

Finally, $\frac{\partial e^*}{\partial \alpha} = \frac{\partial e}{\partial x}\frac{\partial x}{\partial \alpha} < 0$.

■