Corporate Control and Multiple Large Shareholders*

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Abstract

Recent evidence on financial structures within firms suggests that concentrated ownership in the form of several large shareholders, rather than dispersed ownership, is the norm. This is puzzling given that often the stakes are too big for optimal diversification and too small to guarantee control. This paper attempts to provide an explanation. We consider a setting where multiple shareholders have endogenous conflicts of interest depending on the size of their stake. Such conflicts arise because larger shareholders tend to be less well diversified and would therefore prefer the firm to pursue more conservative investment policies, while dispersed shareholders prefer high risk, high return policies. A second blockholder (or more) can mitigate the conflict by shifting the voting outcome more towards the dispersed shareholders' preferred investment policy and this raises the share price. Having a larger stake increases the ability to change decisions: however, it also changes the preferences of the shareholders, reducing the incentive to buy larger shares. The main contribution of the paper is to show the conditions under which blockholder equilibria exist when there are *endogenous* conflicts of interest. The model shows how different ownership structures affect firm value and IPO's underpricing.

1 Introduction

Finance theory has long recognized the important role of the size of shareholders in corporate governance. In order to overcome free riding problems from dispersed ownership (Grossman and Hart, 1980) a large shareholder can act as a monitor and discipline the manager (Stiglitz (1985), Shleifer and Vishny (1986), Holmstrom and Tirole (1993), Admati, Pfleiderer, and Zechner (1994), Burkart, Gromb, and Panunzi (1997)). From a positive perspective however, optimal diversification implies that no investor should want to invest too high a proportion of his wealth in a single firm. Indeed, what is observed empirically is neither dispersed ownership in the firm nor one large shareholder: the evidence is consistent with *intermediate* sizes of stakes. Indeed, there are investors who hold about 5% to 10% of the firm's shares with commensurate voting power.[1]

Of course, holding 5% to 10% of the firm's shares does not give control over the firm's decisions! In this sense, it remains a puzzle why investors hold these "intermediate" stakes – too small to control the firm, yet not small enough to be well diversified. Our paper investigates this apparent paradox, shows the mechanism by which control is exercised and provides conditions (or firm characteristics) under which we might observe such ownership patterns.

Formally, suppose there is an initial owner or founder of a firm who needs to raise capital to finance a project. The initial owner has a large stake in the firm because of an inability to commit to monitoring without having large enough stakes in the firm. He raises capital through issuing shares and/or debt. There is a set of other investors who are ex-ante identical in their preferences, whose role is to buy shares in the firm after the price is announced by the initial owner. Both the owner as well as other investors are risk averse, but their preferences on risk/return depend only on the size of their stakes in the firm: the larger the stakes the lower the risk/return they prefer.

Once the ownership structure is established, shareholders vote on the riskiness of the investment projects that the firm subsequently undertakes. At the time of buying shares therefore, investors face a trade off between holding a diversified portfolio and having little influence on the firm's decisions, or buying more shares, maybe holding a suboptimal portfolio, but influencing the firm's decisions. This trade off matters because the initial owner prefers less risky and lower return projects because of his large initial share while outside investors would ideally want the highest

[1] E.g. in eight out of nine largest stock markets in the European Union, the median size of the second largest voting block in large publicly listed companies exceeds five percent (data from the European Corporate Governance Network). The voting power of the second blockholder varies depending on the firm characteristics, but usually a second blockholder is almost always present in every firm across country, sector and type. La Porta, Lopez-De-Silanes, and Shleifer (1999) find that 25% of the firms in various countries have at least two blockholders while Laeven and Levine (2008) find that 34% (12%) of listed Western European firms have more than one (two) large owners where large owners are considered shareholders with more than a 10% stake. Even US firms which are often cited as examples of dispersed ownership have blockholders: 90% of all S&P500 firms have shareholders with more than 5% participation and more generally in US firms, 40% of the equity is owned by blockholders (Holderness, 2009).

risk/return, creating an endogenous conflict of interest.[2]

Some investors then buy bigger blocks, so as to guarantee (via higher voting power) that the risk/return profile of the firm is higher. Paradoxically, of course, when they do buy a larger fraction of shares, their preferences move closer to those of the initial large shareholder! This is the main innovation of our paper: *since the conflicts of interest are endogenous, it is not trivial to show that having a larger size will be beneficial to outside investors*, since the large size itself reduces the conflict of interest between the initial owner and large outside investors.

Our main result is that for intermediate values of the initial owner's monitoring costs, *any* subgame perfect equilibrium features multiple blockholders. When monitoring costs are very high the initial owner is in control while when they are very low there is a dispersed ownership structure. In our model, monitoring costs proxy for the frictions that are responsible for the large stake of the founder of the firm. Hence what we show is that when conflicts of interest are endogenous to the size of the stakes, blockholder equilibria exist, but only partially mitigate the conflicts of interest.

The broader message of our paper is that intermediate sized blockholders emerge as a response to the way the firm aggregates preferences of shareholders and contributes to the ongoing discussion on the objective of the firm (see for example Tirole (2001) and Becht, Bolton, and Röell (2003)). Changing the voting rules, or the exact nature of the conflict of interest may offer different results on the size and number of blockholders. But it does not change the main message of the paper, which is essentially a positive one.

In the finance literature (a seminal contribution being Shleifer and Vishny (1986)), the only reason to hold a large block is to have incentives for monitoring the manager. In contrast, we isolate another reason for an initial blockholder to hold large blocks – control of the firm. In contrast to much of the literature (see Section (2)) on the existence of multiple blockholder ownership structures, the initial owner in our model only sets the price of shares: he cannot choose the ownership structure *directly*: this is important given the existence of resale markets in shares. Addressing both the issue of monitoring and control our model shows that an initial owner might prefer to retain more shares than the monitoring incentive requires in reaction to an anticipated blockholder ownership structure.

We get a number of comparative statics results from our model. In our model the fraction of liquidity shareholders who vote can be considered a proxy for minority protection. Depending on

[2]The effects on the risk choices when a controlling shareholder is less diversified can be seen in the choices of the Swedish bank Skandinaviska Enskilda Banken (SEB). The controlling shareholder, the Wallenberg family, has a big part of its wealth invested in the bank. The bank's approach is to be prudent as "sometimes life can turn sour". For this reason it faced the financial crises with a lot of cash which helped it to perform better than its' peers and in general than the stock market (Economist, 2009). For a more systematic study see Faccio, Marchica, and Mura (2009)

country's regulation or the firm's bylaws, investors are able and/or willing to vote. This might be due to a minimum stake required to vote or because of the costly information acquisition. Our model suggests that multiple blockholder ownership structures are more likely to be seen in economic systems where the small shareholders' voting participation is higher, i.e. where investors have more power in firm's decision.

We also predict a relationship between size of the firm and the ownership structure: while the size of firms which have a single large block is usually smaller, there is less of a correlation between ownership structure and size when there is more than one block. This is confirmed by the findings of Laeven and Levine (2008) on size and ownership structure. In small firms we see one main blockholder; while in large corporations complex ownership structures are more likely to be observed.

On the question of how ownership structure affects other variables, our model predicts that firms with multiple blockholders should be characterized by higher risk/return (of course assuming that the salient conflict of interests between shareholders is on the risk profile of the firm). Carlin and Mayer (2000; 2003) and Teodora (2009) confirm this prediction. Firms with more dispersed ownership tend to invest in higher risk projects, like R&D and skill intensive activities. Laeven and Levine (2008) find that firms with several blockholders have a higher Tobin's Q than firms with only one big shareholder. Value increasing role of multiple blockholders have been found in general have been found in many studies (Lehmann and Weigand (2000), Volpin (2002), (Faccio, Lang, and Young, 2001), Maury and Pajuste (2005), Gutierrez and Tribo (2004)).

Intriguingly, our model also predicts that ownership structure affects the IPO's underpricing. In particular to guarantee the subscription of the blockholders who are undiversified, the initial owners sets a low price. This allows the diversified investors to extract some rent.[3] This is confirmed by some empirical studies(Brennan and Franks (1997), Fernando, Krishnamurthy, and Spindt (2004), Goergen and Renneboog (2002) and Nagata and Rhee (2009)).

Finally, the paper contributes to the literature on voting. Typically, the models on voting do not endogenize the individual preferences, the price of votes and hence the voting power of an agent (Dhillon, 2005). When applying voting theories to corporate governance issues, the firm value (and hence share prices) and shareholders decision are closely related. Furthermore, an investor can decide how many shares to buy and their voting decision changes depending on the block he chooses to buy. The price, being set by the initial owner, becomes an endogenous variable that affects and is affected by the existence of a second blockholder and the voting outcome.

[3]This result is similar to Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006) who show that IPO underpricing can serve to ensure the participation of large investors who can monitor and hence be value enhancing. However, in their papers control considerations are absent and the role of multiple large investors is not analyzed.

The paper is organized as follows. Section 2 discusses the related literature, Section 3 outlines the model. In section 4 we describe the equilibrium concept. Section 5 solves the model and find the possible equilibria. In section 6 we derive the empirical implications of the model. Finally, section 7 concludes. All the proofs are in the Appendix.

2 Related Literature

There is a considerable body of related theoretical literature that focuses on the role of the largest blockholder as a tool to discipline the manager or to take value enhancing actions. (See Holmstrom and Tirole (1993), Admati, Pfleiderer, and Zechner (1994) Burkart, Gromb, and Panunzi (1997), Pagano and Röell (1998), Bolton and Von Thadden (1998), Maug (1998)). Noe (2002) and Edmans and Manso (2008) focus on the role of blockholders as a tool to discipline the manager through the threat of exit which would confer information to outside investors.

A second strand of this literature focuses on the sharing of private benefits of control: Zwiebel (1995) suggests that multiple blockholders emerge as the optimal response to a situation where there are divisible private benefits of control, Bennedsen and Wolfenzon (2000) show that the initial owner in a privately held firm has incentives to choose an ownership structure that commits him to more efficient decisions. Gomes and Novaes (2001) focus on a firm's decision when blockholders have veto power. Bloch and Hege (2001) argue that the existence of two blockholders reduces the appropriation of private benefits of control.

All these approaches essentially view the emergence of multiple blockholders either (i) as a direct choice of ownership structure by the initial owner whereby he can commit himself to higher firm value, (ii) focus on exogenous private benefits of control, and (iii) allow only equity as the method of financing.[4]

In contrast, our paper lets the initial owner choose the ownership structure only *indirectly* through pricing of shares when raising capital: this is equivalent to allowing the initial owner to choose the ownership structure directly and then allowing a resale market for shares. Hence, conditional on having chosen to raise capital at least partly through shares' issue as the method of financing the firm, the owner does not *choose* the ownership structure: it is not a commitment device chosen by the owner as in much of the literature cited above. Second, we explicitly allow strategic interactions between blockholders. This is an important consideration as free-riding between blockholders can be very important when we move from single to multiple blockholders.

[4] Demichelis and Ritzberger (2008) relax some of these assumptions. In their paper the conflicts of interest among investors arise because they are not only investors but also consumers. Hence, when they hold a small fraction of shares they prefer to make consumer favorable choices while blockholders are interested in maximizing firm value.

Thirdly, we endogenize the degree of private benefits as the risk/return choice. This allows us both to highlight how private benefits can differ depending on share participation and to avoid assumptions over the divisibility of private benefits of control. Finally our results are robust to allowing the initial owner to use debt instead of equity as a way of raising funds.

Our paper is also related to Admati, Pfleiderer, and Zechner (1994) and DeMarzo and Urosevic (2006) who analyze the trade off between risk sharing and monitoring. Our paper is not about this trade off, but rather on the aggregation problems inherent when there are conflicts of interest. These conflicts happen to be on risk sharing objectives of the firm.

3 The Model

The initial sole owner of a firm needs a minimum amount of capital, K, to finance a project. He has an endowment of 1 and chooses to invest a fraction w_E of his wealth in the project. w_E is unconstrained as the initial owner can borrow at the risk free rate. The remaining amount $K - w_E$ needs to be raised by issuing equity. We allow the initial owner to raise more capital than needed for the project, i.e. $I \geq K$ and use it to diversify risk, where I is the total amount of capital raised, including the owner's contribution w_E. In this case the extra capital is invested in the risk free asset, which is the only other asset in the economy.[5] Initially we assume that $I = K$; later in section 5.2 we show that this assumption is without loss of generality.

In exchange for capital $K - w_E$, the initial owner offers an aggregate fraction $1 - \alpha_E$ of future profits to the new investors, keeping α_E for himself. Moreover we assume 'one share-one vote' so that a higher α_E implies more control rights for the initial owner. Observe that separating the capital invested and the fraction of shares retained allows the initial owner to take two different decisions, the expected return on the investment and the control rights, so our conclusions would be suitably altered. When $\alpha_E = 0$, the initial owner does not hold any shares in the firm, i.e. he exits from the firm.

The set of (potential) outside investors is denoted M and the number of such investors is also (to save on notation) denoted M. We assume that the set M is partitioned into two types of investors – those who are *active* shareholders, denoted M_A, and those who are *passive* shareholders, such that $\frac{M_A}{M} = \lambda$. Active shareholders are assumed to vote anticipating that their vote is going to have an impact on the decisions of the firm.[6] Passive shareholders act competitively and take the firm's

[5] We use the term 'diversify' loosely to capture the notion that an agent may benefit from reducing his exposure to the project risk.

[6] Institutional investors such as hedge funds can be intepreted as active investors. It is well known that the value creation effect of hedge funds is highly significant both in the short and in the long run. For more evidence on the emergence of active investors see Smith (1996), Brav, Jiang, Partnoy, and Thomas (2008) and Becht, Franks, Mayer,

decisions as given, ignoring their own potential influence. Thus they do not vote.

Investors have initial wealth of 1 and they decide how to divide their wealth between the firm's shares and a risk free asset.[7] Each investor j chooses the fraction of his wealth, w_j, to invest in the firm. There are no financial constraints so both the initial owner and the investors can borrow money at the risk free rate or go short in the firm shares, i.e. w_j is not bounded. For the initial owner this implies that the decision is not only on the ownership structure but also the composition of his portfolio between debt and equity. The higher the debt, the higher is the risk exposure but the higher the control he has. The gross return of the risk free asset is normalized to 1.

Investors and initial owner have identical preferences represented by the following utility function:

$$u_j = -\frac{1}{\gamma}e^{-\gamma Y_j} \tag{1}$$

where $j = \{i, E\}$, i refers to the generic outside investor, E refers to the initial owner, γ is the parameter of risk aversion and $Y_j = f(w_j)$ is the final wealth when a fraction w_j of the wealth is invested in the project.

The timing of the game is as follows: in period 0 the initial owner decides w_E and the fraction of the shares to retain, α_E. This is equivalent to announcing the fraction of shares retained together with the share price, which is given by the capital raised over the cash flow rights tendered, $\frac{K-w_E}{1-\alpha_E}$.

In period 1 investors decide the fraction of shares of the firm, α_i to buy, having observed the share price. A passive investor chooses α_i which optimizes her portfolio taking the voting decision as given, i.e., disregarding the effect her own votes can have on the voting outcome. An active investor, on the other hand, anticipates the effect that her vote has on the voting outcome. Hence her demand for shares will internalize the potential effect of ownership structure on the operating decisions of the firm in period 2.

We assume that there are sufficiently many investors in the market so that there is never a problem of excess supply of shares. If there is under-subscription, then the project cannot go ahead. If there is oversubscription, we assume that this is a stable situation only when no investor who gets shares is willing to sell them at a price lower than the maximum price that an excluded investor is willing to pay.[8]

In period 2 shareholders – the initial owner and/or the investors who bought the shares – have

and Rossi (2009).

[7]Since the motivation of this paper is to show that blockholders are sub-optimally diversified, we are interested in the *fraction* of their wealth that is invested in the firm. Introducing heterogeneity in wealth levels would not change the results unless attitudes to risk depend on wealth. In this case the identity of blockholders would be easier to predict: more wealthy investors who are relatively less risk averse are more likely to be blockholders.

[8]We do not explicitly model the secondary market in shares but we capture some of the spirit of the secondary market by imposing this particular refinement of the Nash equilibrium concept.

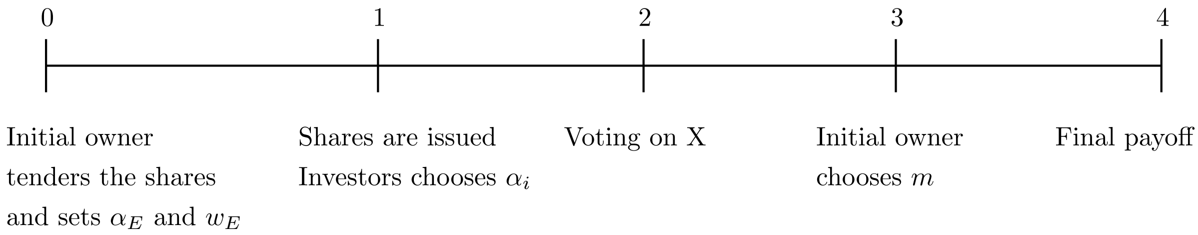

Figure 1: The Time Structure

to take a decision by voting. This decision is about the risk profile of the firm. For example, we may think of this as a decision about projects' type. The projects differ in their risk- return profile.[9] The project cash flows are affected by a variable X as follows: they are normally distributed with mean $\bar{R}X + f(m)$ and and standard deviation σX, where $f(m)$ is the extra expected cash flow from monitoring (which is chosen in period 3) and $\bar{R} > 0$ and $\sigma > 0$ are generic parameters. Shareholders thus choose a project profile $X \in [0, \bar{X}]$: the higher is X the higher is the risk and return to the firm. Hence, conflicts of interest between shareholders can be captured in a simple way through the uni-dimensional linear efficiency frontier of possible projects. We assume that $\bar{X} > \frac{\bar{R}}{\gamma\sigma^2}$. If $\bar{X}$ is too small, then we may be artificially removing any potential conflict of interest and the main driving force of our model.

At this stage, the ownership structure is common knowledge. The decision X is taken through majority voting. We show later that once the ownership structure is fixed, preferences on X are single peaked (Lemma 1) so that the Condorcet winner[10] in the space $[0, \bar{X}]$ is the median shareholders' choice. Voting is costless. However since only a subset of investors are assumed to vote the Condorcet winner must be chosen from among the ideal points X_j of the voting subset only.

In period 3 the initial owner decides whether to monitor or not. His choice variable is $m \in \{0, 1\}$. In particular we assume, as in Maug (1998), that if the initial owner monitors ($m = 1$), the expected firm value increases by $f(1) = K$ at a monetary cost of $c(1) = \bar{m}K$, with $\bar{m} \leq 1$.[11] If the initial owner does not monitor, $f(0) = 0$ and $c(0) = 0$. We assume that the cost of monitoring is not divisible among shareholders.[12]

Finally at period 4 the payoffs are realized. The time structure of the game is sketched in Fig.1.

[9] It could be thought of as a choice on managers who come with different reputations for risk taking.

[10] The Condorcet winner is the project that wins against every other project in pair-wise majority voting.

[11] If $\bar{m} > 1$ the initial owner would never choose to monitor.

[12] This assumption strengthens rather than weakens our results, as the main driving force as to why blockholders emerge in our model is not the sharing of monitoring costs, but rather the conflict of interest between the initial owner and other potential investors. If in addition, investors can share in monitoring there may be an additional reason for blockholders to emerge.

Hence, in our model there is both an issue of incentives and of control. Ideally, the initial owner and the investors would like to have as few shares as possible and choose maximum return, even though it comes with high risk. However, the incentive problems associated with monitoring imply that the initial owner faces a trade off between increasing the value of the firm through monitoring and diversification. In the event that the initial owner resolves this trade off by holding a less diversified portfolio, his choice of X conditional on his shareholdings is likely to be biased towards a lower X. The first best choice for the investors, on the other hand, is to hold few shares, diversify optimally, and have a high return (high X). Hence there is a conflict of interest. Notice that outside investors are identical and hence have the same preferences: the only *ex-ante* conflict on risk- return is between the initial owner on the one hand and other investors on the other. *Ex-post* conflicts depend on equilibrium ownership structure: it is in this sense that conflicts of interest are endogenous in our model. The choice of how many shares to hold changes the preferences of shareholders and therefore also the conflict of interest.

As a result, active investors too face a trade-off: holding a sufficiently big block means higher control on the voting decisions, but means that diversification is less than optimal. When holding a block an investor incurs a cost that is increasing in the price and in the risk exposure.

Assume first that X is fixed. We first derive the utility function of the initial owner and outside investors. The certainty equivalent representation of the utility function (1) is:

$$U_j = E\left[Y(w_j)\right] - \frac{\gamma}{2} w_j^2 V\left[Y(w_j)\right] \tag{2}$$

where $E\left[Y(w_j)\right]$ and $V\left[Y(w_j)\right]$ are respectively the expected value and the variance of the final wealth.

Investors get a share of the firm value that is proportional to their contribution w_i, i.e. $\alpha_i = \frac{w_i}{K-w_E}(1-\alpha_E)$ (assuming full subscription). Hence the investors' certainty equivalent is:

$$\frac{w_i}{K-w_E}(1-\alpha_E)\left(\bar{R}X + f(m)\right) - \frac{\gamma}{2}\sigma^2 X^2 \left(\frac{w_i}{K-w_E}(1-\alpha_E)\right)^2 + (1-w_i)\cdot 1 \tag{3}$$

The last part of this expression represents the share of wealth invested in the risk free asset, i.e., the opportunity cost of investing in the firm. The first part represents the expected wealth from investing in the firm and the second part represents the dis-utility from the risk of investing in the firm.

Equation (3) can be re-written as:

$$U_i = \alpha_i \left(\bar{R}X + f(m)\right) - \frac{K - w_E}{1 - \alpha_E}\alpha_i - \frac{\gamma}{2}\sigma^2 X^2 \alpha_i^2 + 1 \tag{4}$$

The first part is the expected wealth from investing in the firm, the second part is the price paid and the third is the dis-utility from investing in a risky asset. Investor i will maximize (4) by choice of α_i, given $\alpha_E, K - w_E$ and the beliefs on $X, f(m)$.

Similarly, the initial owner's exponential utility function can be re-written in terms of certainty equivalent as:

$$U_E = \alpha_E(\bar{R}X + f(m)) - \frac{\gamma}{2}\sigma^2 X^2 \alpha_E^2 + 1 - w_E - c(m) \tag{5}$$

The initial owner chooses α_E, w_E in period 1 and m in period 3 to maximize (5) subject to the constraint that he needs to raise the capital, i.e. $K - w_E \leq \sum_i w_i$, or equivalently $1 - \alpha_E \leq \sum_{i=1}^{N} \alpha_i$ where $\sum_i \alpha_i$ is the sum of the total shares demanded. In addition he needs to satisfy $\alpha_E + \sum_i \alpha_i = 1$. Note that we allow $w_E < 0$ which means that the initial owner can sell shares in return for cash from the other shareholders in order to invest in the risk free asset. $w_i < 0$ on the other hand means that investors are going short in shares, although we show later that this never occurs in equilibrium. Finally when $w_j > 1$ with $j \in \{E, i\}$ it means that the investors borrow money (at the risk free rate) in order to invest in the firm.

We can now derive the ideal point $X_j(\alpha_j) \in [0, \bar{X}]$ for any investor j. We can then determine the payoff functions of players given the ownership structure defined as the vector of shares owned by investors: $\vec{\alpha} = (\alpha_E, \alpha_1, ..., \alpha_k)$ where k is the number of active investors who hold shares in the firm.

Lemma 1 *The preferred choice of X given α_j, for any shareholder $j \in \{i, E\}$, denoted X_j is uniquely defined by:*

$$X_j = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_j}, \bar{X}\right] \tag{6}$$

The choice of X depends only on the investor's shareholdings α_j and not on the decision to monitor. Observe that $\frac{\bar{R}}{\gamma\sigma^2\alpha_j}$ is a one-to one function of α_j. Hence, we can define $\bar{\alpha} \equiv \frac{\bar{R}}{\gamma\sigma^2\bar{X}}$ as the fraction of shares $\bar{\alpha}$ such that $X_j(\bar{\alpha}) = \bar{X}$.

It follows from Lemma 1 above that once $\vec{\alpha}$ is fixed, preferences of investors and the initial owner on X are single peaked. Hence the Condorcet Winner on the set $[0, \bar{X}]$ is well-defined and is given by the preferences of the median shareholder. Denote $X_{med}(\vec{\alpha})$ as the median X when the ownership structure is $\vec{\alpha}$. To save on notation, we suppress the argument $\vec{\alpha}$. For convenience we will denote the median shareholdings as α_{med}.[13]

Notice from equations (4)-(6) that α_E determines both the price paid, $\frac{K-w_E}{1-\alpha_E}$, and (potentially) X_{med} through the share ownership structure. Hence the indirect utility function for active investors depends on α_E, as well as the anticipated m and $\vec{\alpha}$ given α_E and $K-w_E$. Passive investors' indirect utility depends also on α_E, but here X is taken as given.

Pure strategies of the owner are 3-tuples $(w_E, \alpha_E, m(\alpha_E, X_{med}))$ together with a function from α_E to a voting decision over X. Pure strategies of investors are functions from $(\alpha_E, K-w_E)$ to a shareholding α_i and a voting decision over X.[14] This describes an extensive form game, Γ where the set of players are the initial owner and other active investors, the pure strategies and payoffs are as above.

4 Equilibrium definition

We define shareholders as *liquidity shareholders*, or *blockholders* depending on whether they hold an optimally diversified portfolio or not. Liquidity shareholders' shares are denoted by $\alpha_l(X_j)$,which represents the optimal portfolio when the voting outcome is assumed to be X_j. A shareholder is a *blockholder* if in equilibrium he holds a suboptimal portfolio: $\alpha_i > \alpha_l(X_j)$ where α_i is the shareholding of blockholder i.

Our notion of equilibrium is subgame perfect equilibrium of the game described in Fig. 1. Note that because in many potential equilibria more than one investor needs to buy shares for the initial owner to find it worthwhile to start the firm, there is always a No Trade equilibrium. In this equilibrium, no investor buys any shares anticipating that no other investor will buy shares. Below we provide a definition for equilibria with positive trade.

Equilibrium is a monitoring level, $m \in \{0, 1\}$, a fraction α_E^* of shares held by the initial owner, a fraction of wealth invested w_E^*, a decision X_{med} and an allocation of shares among investors, $\vec{\alpha}^*$ such that: (i) α_E^* and w_E^* maximize the utility of the initial owner given the anticipated demand, the anticipated monitoring level, m, and the anticipated ownership structure $\vec{\alpha}(K-w_E, \alpha_E)$. (ii)Each active investor chooses α_i to maximize her utility given $K-w_E^*, \alpha_E^*$, the anticipated m and the anticipated α_{-i}. (iii) Each passive investor chooses α_i to maximize her utility given $K-w_E^*, \alpha_E^*$ and the anticipated m and X_{med}. (iv) In equilibrium there must be full subscription. There can be excess demand in equilibrium as long as no investor who owns shares is willing to sell them at a price

[13]Consider the frequency distribution of shares of *initial owner and active investors only* on the set X. The median X is the unique X_j such that exactly half the shares are on either side of it. Since it is common knowledge that passive investors never vote, α_{med} is defined only on the basis of shares of initial owner and active investors.

[14]Since there is pairwise voting, voting is assumed to be sincere and we rule out strategic agenda setting issues.

lower than the maximum willingness to pay of the excluded investors. (v) The monitoring decision must be optimal for the initial owner given his stake and the vote outcome. (vi) Expectations are rational.

We will distinguish between monitoring and no monitoring equilibria as follows: (No) monitoring equilibria are those where the initial owner is assumed to choose (not) to monitor in the last stage, and anticipating that, he chooses the optimal α_E (and w_E) in period 0.

We are interested in equilibria which are characterized by a conflict of interest among shareholders: such equilibria are of two types: (1) the initial owner is the sole blockholder and is in control of the voting outcome, i.e. $X = X_E$. We call this the *Initial Owner Equilibrium.* (2) the initial owner and a subset of the active investors are blockholders. When there are $n + 1$ blockholders, including the initial owner we call it an *n-Blockholder equilibrium.*[15] In case of blockholder equilibria we focus only on *symmetric* equilibria, where each blockholder (other than the initial owner) holds exactly the same shares. However we allow asymmetry among active investors on the decision to be a blockholder. When outside investors are in control holding a diversified portfolio we will call it a *Liquidity Shareholder Equilibrium.* When there is no conflict among any shareholders we call it a *No Conflicts Equilibrium.*

As we see later, the model admits multiple blockholder equilibria. Among these we restrict attention to those equilibria where the fraction of active investors among liquidity investors is fixed and known at $\lambda = \frac{M_A}{M}$. λ therefore captures the anticipated fraction of liquidity shareholders who take part in the voting decisions of the firm. In general we expect λ to be "small" reflecting the proportion of active investors in the market for shares of the firm (this is not needed for the model however).

5 Equilibria

We solve the game by backward induction. The last stage is the monitoring decision. The initial owner can commit to monitor only when he owns enough shares (as e.g. in Shleifer and Vishny (1986)). Observe that the decision to monitor is independent of the voting outcome:

Lemma 2 *The initial owner monitors iff $\alpha_E \geq \bar{m}$.*

The second last stage is the voting game. This is trivial given the vote shares. Each voter votes for his ideal point given his shares (see Lemma 6) and the outcome is the Condorcet winner.

[15]For expositional simplicity, we refer only to investors (and not the initial owner) as blockholders although, strictly speaking, the initial owner is the "first" blockholder.

Finally we come to what we call the *ownership subgame*. In this subgame, investors buy their shares given (α_E, w_E), anticipating the effects of their share ownership on the voting outcome and the monitoring outcome. We have seen that the monitoring outcome is independent of ownership structure except through α_E. We can therefore partition the subgames at this stage into those where $\alpha_E \geq \bar{m}$ and those where it is less. There is a continuum of such subgames. We now define the *Equilibrium Ownership Structure* (EOS) as the Nash equilibrium of the subgame for each pair $(\alpha_E, w_E) \in S \equiv [0,1] \times (-\infty, +\infty)$.

Let U_1^{nBH} denote the value function of a representative blockholder who we name blockholder 1 and who owns α_1 fraction of shares, in an ownership structure which admits n blockholders. Recall by the definition of a blockholder equilibrium that $X_{med} = X_1$. $U_{l,1}$ then denotes the value function of a liquidity shareholder in such an ownership structure.

Definition 1 *An Equilibrium Ownership Structure (EOS) corresponding to a pair* (α_E, w_E) *is an equilibrium of the subgame beginning at the information set* (α_E, w_E). *In particular the following must be satisfied in equilibrium: (1)* $U_1^{nBH} \geq 1$*(if there are any blockholders in equilibrium) and* $U_{l,j} \geq 1$ *(if there are any liquidity shareholders in equilibrium). We call this the Participation Constraint, (2) No active investor wants to unilaterally increase or decrease his shares. We call this the Incentive Constraint. (3) Passive investors maximize their utility conditional on the anticipated* $X = X_{med}$. *(4) No investor who receives shares is willing to sell them at a price lower than the maximum price that any excluded investor is willing to pay.*

The equilibrium concept is standard– that of Nash between active investors, and passive investors act to maximize their utility given their beliefs on X_j, which must be the right beliefs so $X_j = X_{med}$. Refinement (4) is imposed because in case of oversubscription, we would like a rationing rule that allocated shares in a "stable" way. In other words, we want to ensure that no excluded investor can do better by deviating unilaterally. This will turn out to affect the equilibrium price. Note that in our model, the initial owner does not directly determine the ownership structure in equilibrium, he can only influence it through the choice of share price. We believe that this structure is much more plausible: given that there is a secondary market for shares, in practice, the initial owner does not really determine ownership structure directly.

The ownership structure can be of four types based on who is the median shareholder (as discussed above in Section 4): (A) The *Initial Owner EOS*, where $X_{med} = X_E < \bar{X}$ and all investors are liquidity shareholders . (B) *Liquidity Shareholder EOS* where active liquidity investors (who hold a perfectly diversified portfolio) are in control of the firm, $X_{med} = \bar{X}$. (C) *No Conflicts EOS* where $X_{med} = X_E = \bar{X}$, hence there are no conflicts of interest between the initial owner and outside investors and all investors are liquidity shareholders. (D) An *n-Blockholder EOS* , where $X_{med} = X_1$ and n active investors hold a non-perfectly diversified portfolio. More formally:

Definition 2 *A symmetric n-Blockholder ownership structure is one where there are $n > 0$ active investors (blockholders) with shares α_1 each and $N_A \geq 0$ active investors (liquidity investors) with shares $\alpha_l(X_1)$ such that $\alpha_1 > \alpha_l(X_1) > 0$, and $X_{med} = X_1$.*

Given the investors' beliefs on $X_{med} = X_j$ we can determine the demand for shares by passive investors:

Lemma 3 *Let X_j be the belief on the voting outcome and $K - w_E$ the capital demanded. Then liquidity shareholders demand:*

$$\alpha_l(X_j) = \frac{X_j \bar{R} + f(m) - \frac{K-w_E}{1-\alpha_E}}{\gamma X_j^2 \sigma^2} \tag{7}$$

Lemma 3 shows that the fraction of shares chosen by the passive investors depends on their beliefs on the voting outcome X_j. When they believe the voting outcome is going to be low risk and return, i.e. low X_j, the optimal portfolio they choose has a higher fraction of shares and vice versa. Observe that the value function of liquidity investors, $U_{l,j}$ is always bigger than 1 since there is no restriction on $\alpha_l(X_j)$: if the utility from holding the firms' shares is less than the risk free asset, then $\alpha_l(X_j)$ could be negative, i.e. investors can go short on the shares. We show later that the constraint on full subscription by the initial owner implies that in equilibrium $\alpha_l(X_j) > 0$.

The participation constraint of liquidity investors, $U_{l,j} \geq 1$, also implies that the maximum price they are willing to pay, $\frac{K-w_E}{1-\alpha_E}$, is higher, the higher is the anticipated risk/return profile of the firm, i.e. the higher is X_j.

Lemma 4 *Liquidity shareholders are better off than blockholders regardless of the monitoring decision.*

This is true by definition: Liquidity investors hold the fraction of shares that provides optimal diversification given the anticipated voting outcome, X_j. Any other shareholding gives lower utility.

The next lemma shows that if the share price is sufficiently high,then the optimal liquidity shareholding is always smaller than $\alpha_{med} = \alpha_j$.[16]

Lemma 5 *Assume $\frac{K-w_E}{1-\alpha_E} > f(m)$, then $\alpha_l(X_j) \leq \alpha_j$.*

[16] If, to the contrary, the share price is too low, so that $f(m) \geq \frac{K-w_E}{1-\alpha_E}$, investors will demand all the shares tendered. We show later that this cannot be an equilibrium as the initial owner will always maximize the share price.

We now identify how many shares an investor buys when he decides to hold a block. Recall that λ denotes the proportion of active to total investors in the market.

Lemma 6 *Assume full subscription of shares. If an n-blockholder EOS exists with* $\alpha_E > 0$ *and* $\frac{K-w_E}{1-\alpha_E} > f(m)$*, each blockholder holds*

$$\alpha_1 = \frac{\alpha_E\,(1+\lambda)-\lambda}{(1-\lambda)\,n} \tag{8}$$

This lemma shows that blockholders buy just enough shares to be pivotal, kowing α_E and anticipating the choice of the other $n-1$ blockholders as well as the choice of active liquidity investors. They do not want to hold more shares because that would expose them to higher risk without gaining anything in terms of control. If they hold any less, they do not affect the voting outcome. From Lemmas (5) and (6), it follows that blockholders hold more shares than liquidity shareholders and less than the initial owner, $\alpha_l(X_j) < \alpha_j < \alpha_E$. Hence, X_1 will lie between the preferred choice of the liquidity shareholders and the initial owner. This is the sense in which blockholders mitigate the conflicts of interest between the initial owner and the liquidity shareholders.

We now define the *first best* choice for investor i as the optimal α_i assuming that in the second stage agent i acts as a dictator in the choice of X.

Lemma 7 *Assume* $\frac{K-w_E}{1-\alpha_E} > f(m)$*, the first best choice of the investors is* $X = \bar{X}$ *and* $\alpha_i = \alpha_l(\bar{X})$*.*

This lemma helps to clarify the conflict of interest between investors and the initial owner. Outside investors prefer to diversify maximally, i.e. buy a small amount of shares, $\alpha_l(\bar{X})$, and choose the project with maximum risk and returns, $\bar{X}$. Absent the monitoring constraint, there is no conflict of interest, the only incentive to is maximize monopoly rents while minimizing risk, and this is achieved by selling the firm (as we show later in Proposition 1). However the monitoring constraint forces him to have a different preference ex-post (after shares are allocated) than outside investors. Given the constraint on his shareholdings ($\alpha_E \geq \bar{m}$), he prefers lower return/risk projects while outside investors prefer higher risk/return.

Finally, we solve the first stage maximization for the initial owner given what he anticipates will happen for each (α_E, w_E). This depends on the EOS that is anticipated for each pair (α_E, w_E). These will be used in the derivations of the subgame perfect equilibria: the method of proof is to solve for the Initial Owner's maximization problem, assuming a common belief about the EOS following every information set (α_E, w_E). Since the analysis of the EOS is quite technical, it is done in the Appendix (Lemmas (11)- (13) and Corollaries (3)-(5)). Before moving to the subgame perfect equilibria we need the definitions of some symbols we are going to use for the rest of the paper. Define $\epsilon_j > 0$ as a very small number such that $\epsilon_j = \frac{\gamma X_j^2 \sigma^2}{\eta_j}$ where η_j is the fraction of shares corresponding to one share, when $X_{med} = X_j$. Define:

$$\underline{w}_E^E(\alpha_E) \equiv K - \left(\frac{\bar{R}^2}{\gamma\sigma^2\alpha_E} + f(m)\right)(1-\alpha_E) + \epsilon_E \tag{9}$$

$$\underline{w}_E^n(\alpha_E) \equiv K - \left(\bar{R}X_1 + f(m) - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1\right)(1-\alpha_E) \tag{10}$$

$$\underline{w}_E^{LS}(\alpha_E) \equiv K - \left(\bar{R}\bar{X} + f(m)\right)(1-\alpha_E) + \epsilon_{LS} \tag{11}$$

Suppose α_E is fixed. Then $\underline{w}_E^E(\alpha_E)$ is the minimum w_E that the initial owner needs to invest in the firm in order to guarantee that liquidity shareholders' participation constraint is satisfied when $X_j < \bar{X}$. However at this level $w_E = \underline{w}_E^E$ the demand for shares by liquidity shareholders is zero. We need $\epsilon_E > 0$ to have a strictly positive demand by the liquidity shareholders. $\underline{w}_E^n(\alpha_E)$ on the other hand, is the analogous expression if n blockholders participate. $\underline{w}_E^{LS}(\alpha_E)$ is the analogous expression to $\underline{w}_E^E$ when $X_j = \bar{X}$: it is the minimum w_E that the initial owner needs to invest in the firm to satisfy the participation constraint of liquidity shareholders (and for them to have a strictly positive demand) when $X_j = \bar{X}$.

The first such equilibrium we analyze is the no-monitoring equilibrium: Suppose the initial owner decides not to monitor in the last stage (i.e. $\alpha_E < \bar{m}$). We show that then he always chooses $\alpha_E = 0$ and the unique ownership structure that emerges is a Liquidity Shareholder Ownership Structure: the initial owner either does not invest in the firm or sells it letting the liquidity shareholders be in control of a non-monitored firm. Our interest in the no monitoring payoff stems from the fact that in any monitoring equilibrium, the owner would have to get a higher payoff from monitoring in equilibrium than non-monitoring.

Proposition 1 *Suppose the initial owner chooses not to monitor in period 3, in period 0 he sells the firm when the firm has a positive NPV, otherwise he does not raise the capital and invests in the risk free asset (in both cases, $\alpha_E = 0 < \bar{m}$). The initial owner's value function is given by:*

$$V_E^{NM} = \max(\bar{R}\bar{X} - K + 1, 1) \tag{12}$$

If the initial owner sells the firm, he sets $w_E = \underline{w}_E^{LS}$ and the unique EOS is a Liquidity Shareholder ownership structure, $X = \bar{X}$.

This proposition illustrates the trade-offs faced by the initial owner. Since he acts as a monopolist in the pricing of shares, when he has no constraint on his shareholdings, he can extract the full value of the firm without incurring any risk, by simply selling the firm. However, investors are willing to buy shares only if the expected NPV is positive ($\alpha_E = 0$, $w_E < 0$).[17] Otherwise he prefers not to raise capital ($w_E = \alpha_E = 0$) .

Proposition 1 shows that the choice of $\alpha_E > 0$, $m = 0$ by the initial owner is dominated by the choice of either selling the firm or not raising capital. Hence, in what follows it is sufficient to show that the participation constraint and the non selling constraints are satisfied, to ensure that the initial owner prefers $\alpha_E \geq \bar{m}$, $m = 1$ to any non-monitoring equilibrium.

5.1 Monitoring Equilibria

The existence of blockholder equilibria depend on three necessary conditions (1) a conflict of interest between investors that is generated endogenously by the fact that if they have different shares in the firm, they have different preferences on the risk/return profile of the firm; (2) they are able to influence the voting decision if they strategically buy more shares than a liquidity shareholder would; (3) in equilibrium the initial owner's shareholding is large enough that active liquidity investors in the firm cannot jointly ensure that $X_{med} = \bar{X}$, without holding more than the liquidity shares. If any of these three requirements is not met, then we do not have a blockholder equilibrium.

Section 5.1.4 shows our main result, that when the monitoring costs are intermediate (so the conflict of interest is not too big and not too small and all the conditions (1)-(3) are satisfied) blockholders emerge endogenously and help to mitigate the conflict of interests between the initial owner and the liquidity shareholders. For completeness and ease of exposition however we first analyze the other equilibria.

Hence, Section 5.1.1 analyzes the case where requirement (1) is not satisfied that is where monitoring costs are so low ($\bar{m} \leq \bar{\alpha}$) that the final choice is $X_{med} = X_E = \bar{X}$. Since this is the first best point for outside investors (see Lemma 7), this implies that there is no conflict of interest: hence no blockholders in the ownership structure. Section 5.1.2, on the other hand considers the case where requirement (2) is not satisfied, i.e. where the blockholders cannot influence the voting decision because in equilibrium the initial owner holds a too large fraction of shares. Section 5.1.3 discusses when requirement (3) does not apply. In such a case there is a Liquidity Shareholder equilibrium where liquidity shareholders are in control because the initial owner holds such few shares that the active liquidity shareholders can exert control on the vote outcome without holding a block.

In all of the proofs, we do not allow the initial owner to raise more capital than the minimum needed for the firm i.e. K. So in Section (5.2) we show that this assumption is without loss of

[17] We may interpret $w_E < 0$ as rent for the initial owner for the entrepreneurial idea.

generality.

Before moving to the equilibria we show that investors are willing to receive less shares than what they proportionally contribute. Conditional on monitoring, investing in the firm increases the utility of the investors because it widens the possible portfolios they can choose among. Since the initial owner is a monopolist he can push share prices up to the point where the the participation constraint of investors is satisfied with equality.

Put another way, the initial owner contributes to capital proportionally less than what he receives in cash flow rights: $\alpha_E > \frac{w_E}{K}$ when $m = 1$. This is shown in the next Lemma:

Lemma 8 *Assume that $m = 1$. In any subgame perfect equilibrium, the price per share $\frac{K-w_E}{1-\alpha_E} > K$.*

In all monitoring equilibria, the initial owner sets the price per share as high as possible to avoid dilution of his shareholdings. If the initial owner monitors, the expected firm value is above the return on the risk-free asset. Hence, the minimum possible price that guarantees the participation of the investors is above the price of the risk-free asset.

5.1.1 No Conflicts Equilibrium

Proposition 2 *Suppose*

$$\bar{m} \in \left[0, \min\left[\bar{\alpha}, \bar{m}_{NC}^{RC}(K), \bar{m}_{NC}^{S}(K)\right]\right] \tag{13}$$

then there exists a No Conflicts (NC) equilibrium where the initial owner monitors, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_E = \bar{X}$ and $w_E = \underline{w}_E^{LS}(K)$.
(The exact values of $\bar{m}_{NC}^{RC}$ and $\bar{m}_{NC}^{S}$ as functions of the parameters are given in the proof in the appendix.)

In this equilibrium, $\bar{m}$ is so low ($\bar{m} \leq \bar{\alpha}$) that the Initial Owner's first best is the same as other investors ($X_E = \bar{X}$), so that there is no conflict of interest. The initial owner retains just enough shares to have the incentive to monitor, i.e. $\alpha_E = \bar{m}$. He does not want to retain more as this would imply a higher risk exposure without any gain in terms of monitoring: moreover he can extract all rents from liquidity shareholders using his position as a monopolist when $\alpha_E = \bar{m}$. Hence, there are no conflicts of interest, issues of control are absent and there is no incentive for any investor to hold a block.

Even when he holds a non-perfectly diversified portfolio because of the monitoring costs, the initial owner would still vote for the maximum risk/return because the "friction" in the model is low. However observe that the No Conflicts equilibrium does not imply that the initial owner has the same shareholdings as the liquidity investors. His shareholdings are determined by the monitoring costs, $\bar{m}$, and depending on the monitoring costs, the initial owner can hold more or less than the liquidity shareholders. In the extreme case when the monitoring costs are 0, he would like to sell the firm. In this way he extracts all the rent from the investment without incurring any risk. The liquidity shareholders instead would always like to hold some shares so as to take advantage of the return premium from holding some risk.

Liquidity shareholders are willing to buy the shares only if the returns are high enough to compensate for the risk, $w_E \geq \underline{w}_E^{LS}$. If the price were higher investors are better off just investing all their wealth in the risk-free asset. At the same time the initial owner sets the highest possible price in order to reduce dilution and risk exposure. Hence $w_E = \underline{w}_E^{LS}$.

The monitoring requirement sets a maximum threshold on the shares that can be distributed to the liquidity shareholders. Given this threshold there is a maximum fraction of wealth that liquidity shareholders are willing to invest. The rest of the capital (if needed) must be pledged by the initial owner ($w_E = \underline{w}_E^{LS}$). The higher the amount of capital the initial owner needs for the project, K, the higher the wealth he needs to pledge.

Note that given that there are no financial constraints, the initial owner could finance the project completely on his own through borrowing money. However, he prefers to rely on outside equity rather than issuing debt in order to limit his risk exposure.

If the value created is very high the initial owner does not need to invest *any* money; indeed, he can be compensated by the investors for the monitoring exerted and the entrepreneurial idea ($\underline{w}_E^{LS} < 0$). These characteristics of the capital invested by the initial owner are common to all equilibria we find.

Figure 2 offers a graphic representation of the No Conflicts Equilibrium. The equilibrium exists when the area in the graph that satisfies both the no selling and raising capital constraints is positive. Since $\bar{m}_{NC}^{RC}, \bar{m}_{NC}^{S}, \bar{\alpha} > 0$, we can conclude that this is the case.

Alternatively the initial owner could choose not to raise the capital or to sell the firm. To be a viable project for the initial owner, the value remaining to the initial owner after compensating the investors, must be high enough to compensate him for the money invested and the risk (i.e. $\bar{m} \leq \bar{m}_{NC}^{RC}$).

At the same time to be willing to remain a shareholder of the firm, rather than sell it outright, the value created by monitoring ($f(m) = K$) has to be high enough. The extra value due to monitoring compensates the initial owner for the direct cost of monitoring as well as the indirect costs related to holding a sub-optimal portfolio. If the extra utility created by monitoring can be achieved by a dispersed ownership structure without monitoring then he would prefer to sell the firm (if $\bar{m} \geq \bar{m}_{NC}^{S}$ and $\bar{R}\bar{X} - K \geq 0$).

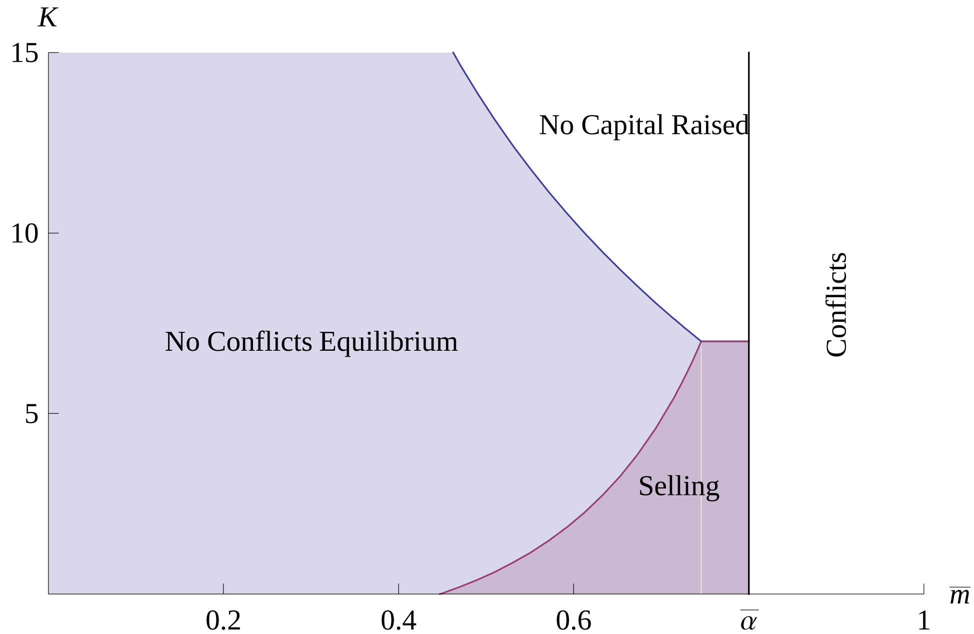

Figure 2: No Conflicts Equilibrium ($\bar{R} = 0.8$, $\gamma = 10$, $\sigma = 0.1$, $\bar{X} = 10$).

5.1.2 Initial Owner Equilibrium

The second equilibrium we consider is the one where requirement (2) is partly relaxed, i.e. outside investors *cannot* or *do not want* to influence the voting decision and there will be no blockholders apart from the initial owner. We obtain the Initial Owner equilibrium in two cases. First, when the initial owner has more than 50% of the shares and hence no outside investor can influence the decision: in this case the Initial Owner equilibrium is unique. The second case occurs when the initial owner holds less than 50% but α_E is high enough such that 1 blockholder would not have a unilateral incentive to deviate from holding liquidity shares.

Proposition 3 *Suppose that*

$$\bar{m} \in \left(\max \left[\bar{\alpha}, \min \left[\frac{1}{2}, \max \left[\hat{\alpha}(1), \tilde{\alpha}(1) \right] \right] \right], \min \left[\bar{m}_E^{RC}(K), \bar{m}_E^S(K), 1 \right] \right] \tag{14}$$

then there exists an Initial Owner equilibrium where the initial owner is the only blockholder, $m = 1$*,* $\alpha_E = \bar{m}$*,* $X_{med} = X_E = \frac{\bar{R}}{\gamma\sigma^2\bar{m}}$ *and* $w_E = \underline{w}_E^E(K)$*.*

When $\bar{m} \in \left(\max \left[\bar{\alpha}, \frac{1}{2} \right], \min \left[\bar{m}_E^{RC}, \bar{m}_E^S, 1 \right] \right]$ *the Initial Owner equilibrium is unique.*

(The exact values of $\bar{m}_E^{RC}$ *and* $\bar{m}_E^S$ *as functions of the parameters are given in the proof in the appendix.)*

If there is a wide range of projects such that there is a conflict of interests between investors and initial owner (i.e. $\bar{X}$ is sufficiently high) and the monitoring technology is sufficiently productive,

$\bar{m}$ small relative to $f(m)$, such an equilibrium always exists, i.e interval (14) is not empty.

Consider first the case, when the monitoring costs are very high ($\bar{m} \geq \frac{1}{2}$), such that the initial owner is willing to monitor only if he holds more than 50% of the shares. This implies that he is highly exposed to firm risk and because he is in control of the vote outcome he chooses a low risk/return project. Hence there is a conflict of interest between the initial owner and outside investors on the choice of X. No other blockholders emerge because under these conditions control is concentrated in the hands of the initial owner in all circumstances and there is no reason to hold a suboptimally diversified portfolio. So this equilibrium exists if $\bar{m} \geq \frac{1}{2}$ and participation constraints for liquidity shareholders are satisfied ($w_E \geq \underline{w}_E^E$) and as long as it is worthwhile to monitor rather than not (i.e. $\bar{m} \leq \min[\bar{m}_E^{RC}, \bar{m}_E^S]$).

The condition $\bar{m} > \hat{\alpha}(1)$ ensures that then an investor is not willing to unilaterally hold more shares in order to influence the voting decision, given $w_E = \underline{w}_E^E$. If the condition $\bar{m} > \tilde{\alpha}(1)$ is not satisfied then an active investor would be willing to pay a higher price than the equilibrium price given by $(\bar{m}, \underline{w}_E^E)$. Hence a passive liquidity investor, who never votes, would be willing to sell the shares to an active investor who would still hold a perfectly diversified portfolio but could become pivotal simply by going to vote, therefore destroying the Initial Owner equilibrium (if there are sufficiently many such passive investors). Finally, in order to guarantee conflicts of interests between investors and initial owner, we set $\bar{m} > \bar{\alpha}$.

Analogously to the No Conflicts equilibrium, liquidity shareholders are willing to buy shares if the price is low enough to compensate them for the risk they bear. This implies that the amount of capital that they contribute (the demand for shares) depends on their belief on the voting outcome and hence in the risk profile of the firm. This amount of capital can be greater or smaller than the capital needed. The initial owner anticipates this given $X = X_E$ and chooses w_E to maximize his utility. Therefore he may choose $\underline{w}_E^E > 0$, or $\underline{w}_E^E < 0$, and cash in the rest as compensation for the entrepreneurial idea.

Monitoring not only implies a direct cost of $\bar{m}$, but it also has an indirect cost for the initial owner in terms of holding a sub-optimally diversified portfolio. Hence the initial owner is willing to monitor and hold a large fraction of shares only if he is able to increase the firm value sufficiently to compensate both for the monitoring cost and the risk of holding a suboptimal portfolio. When the marginal returns on firm value from monitoring are not very high the initial owner prefers not to be involved in the project. In this case as Proposition 1 stated, there are two possible outcomes. If the extra value created by monitoring is not very high and the firm is very valuable even without monitoring (high $\bar{X}$), i.e. $\bar{m} > \bar{m}_E^S$, the initial owner sells the firm to the investors. In this case the firm will show dispersed ownership (a Liquidity Shareholder equilibrium). Instead, when the value

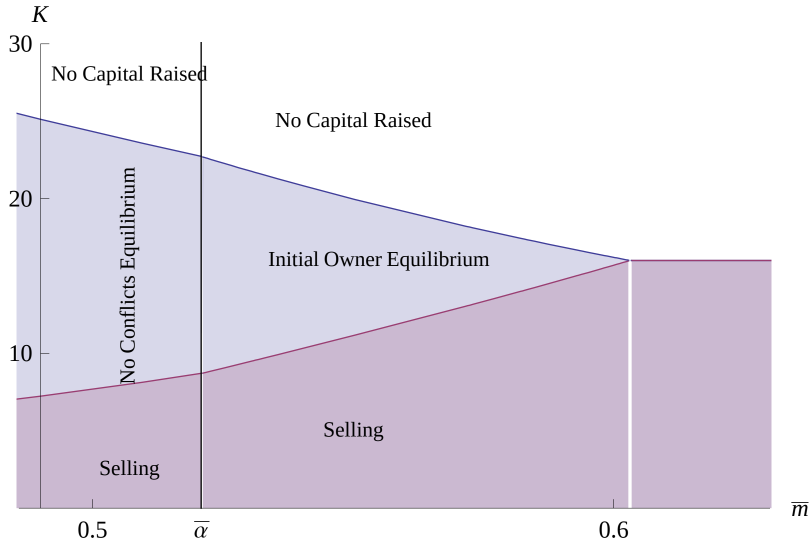

Figure 3: Initial Owner Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

created by the project is not high enough (with or without monitoring) to compensate the initial owner for the capital invested $\bar{m} > \bar{m}_E^{RC}$, the initial owner does not raise the capital and invests all his wealth in the risk free asset. The possible outcomes are shown in Figure 3.

A less obvious result is that an Initial Owner equilibrium also exists if the monitoring costs are *smaller* than $\frac{1}{2}$, although in this case it is not unique. There may be blockholder equilibria as well, but we cannot rule out an Initial Owner equilibrium. To ensure existence of an Initial Owner equilibrium when $\bar{m} < \frac{1}{2}$, we need conditions such that no single investor has a unilateral incentive to deviate to become a blockholder.[18] This case is discussed in Section (5.1.4)–see Corollary (1).

5.1.3 Liquidity Shareholders equilibrium

We now consider equilibria where the liquidity shareholders are in control and there are no blockholders. Intuitively this happens when there are sufficiently many active liquidity shareholders and $\bar{m}$ is not too high. Recall that N_A is the number of active investors in the equilibrium that satisfy $\lambda(1 - \alpha_E) = N_A \bar{\alpha}_l$.

Proposition 4 *Suppose*

$$\bar{m} \in \left(\bar{\alpha}, \min\left[\frac{1}{2}, \tilde{\alpha}(n), \bar{m}_1^{NC}(K), \bar{m}_2^{NC}(K)\right]\right] \tag{15}$$

then there exists a Liquidity Shareholders equilibrium with $N_A + n$ *active investors where* $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = \bar{X}$ *and* $w_E = \underline{w}_E^{LS})(K)$.

[18]However it is possible to have blockholder equilibria if $n \geq 2$.

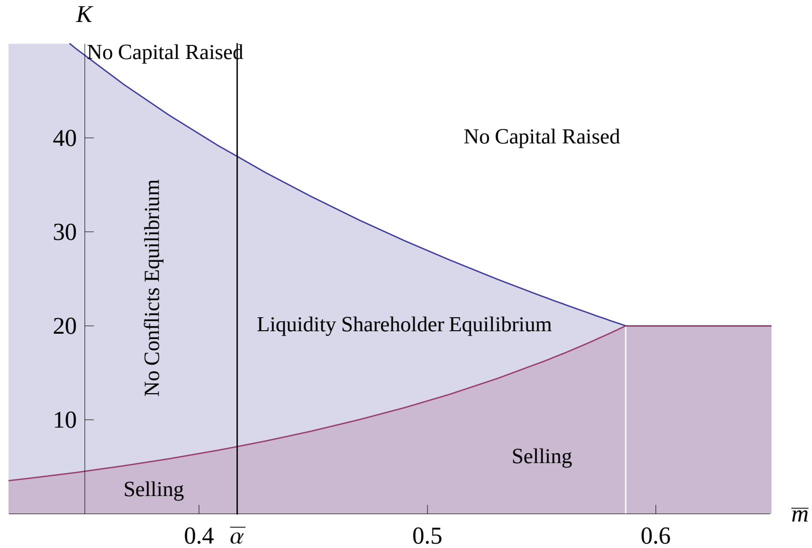

Figure 4: Liquidity Shareholder Equilibrium ($\bar{R} = 2$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 10$, $\lambda = 0.05$).

(The exact values of $m_1^{NC}(K), \bar{m}_2^{NC}(K)$ as functions of the parameters are given in the proof in the appendix.)

A Liquidity Shareholder equilibrium exists when the monitoring costs are relatively low, but not low enough that there are no conflicts (i.e. $\bar{m} > \bar{\alpha}$). In this equilibrium, the initial owner finds it optimal to choose $\alpha_E = \bar{m}$, sufficiently small so that there are enough active liquidity shareholders to be pivotal. In particular we can have two cases: one where the fraction λ of the liquidity shareholders is sufficient to change the vote outcome (i.e. $n = 0$) and the other where there are n active investors in addition to the fraction λ (i.e. N_A) active liquidity investors who vote and ensure that the outcome is $\bar{X}$. If monitoring costs are higher ($\bar{m} > \tilde{\alpha}(n)$), then there is an n blockholder EOS and if $\bar{m} > \frac{1}{2}$ then there is an Initial Owner EOS. Finally, as in the case of the No Conflicts and the Initial Owner equilibrium, when the monitoring costs are higher $\bar{m}_1^{NC}$ than the initial owner prefers to raise no capital and if monitoring costs are higher than $\bar{m}_2^{NC}$, he prefers to sell the firm.

As in the other equilibria, the participation constraint of liquidity shareholders is satisfied (when $X = \bar{X}$), at $w_E = \underline{w}_E^{LS}(K)$.

Notice, that the initial owner could always choose to retain a strictly higher fraction of shares ($\alpha_E > \bar{m}$) to induce a blockholder EOS or even to hold α_E bigger than half, to induce an Initial Owner EOS. In either of these cases, the vote outcome is closer to his own preferred point. However the higher control comes at the expense of both a lower price paid by the investors, more dilution and a less diversified portfolio for the initial owner.

5.1.4 Blockholder equilibria

This section shows our main result: the existence of blockholder equilibria for intermediate monitoring costs. In the following, we discuss two different cases of blockholder equilibria. For one range of parameters, it is possible to have both Initial Owner and n-Blockholder equilibria (Proposition 5). However, we have a stronger result: for some parameter values of monitoring costs, there exist *only* blockholder equilibria. So, even though there are multiple equilibria in terms of the number of blockholders, we know that there will be no non-blockholder equilibria (Corollary 2).

Proposition 5 *Suppose:*

$$\bar{m} \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n), \bar{m}_1^E(n,\lambda), \bar{m}_{1,n}^{RC}(n,K), \bar{m}_{1,n}^{S}(n,K)\right], \min\left[\frac{1}{2}, \hat{\alpha}(n), \bar{m}_2^E(n,\lambda), \bar{m}_{2,n}^{RC}(n,K), \bar{m}_{2,n}^{S}(n,K)\right]\right] \tag{16}$$

then there exists an n Blockholder equilibrium ($n \in [1, M_A]$) where the initial owner monitors, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_1$ and $w_E = \underline{w}_E^n$.
($\bar{m}_{1,n}^{RC}(n), \bar{m}_{2,n}^{RC}(n), \bar{m}_{1,n}^{S}(n), \bar{m}_{2,n}^{S}(n), \bar{m}_1^E(n), \bar{m}_2^E(n)$ are given in the proof in the appendix.)

Proposition 5 demonstrates our main result: i.e there exist blockholder equilibria where n investors prefer to hold a large block of shares (and a sub-optimally diversified portfolio) in order to shift the decision to a higher level of risk/return. This mitigates the conflicts of interests between the initial owner and the investors ($\bar{X} > X_1 > X_E$). There can be multiple EOS, involving different n depending on the beliefs on the EOS.

Now we explain where we get the conditions (16) from. The initial owner could choose to set an $\alpha_E > \frac{1}{2}$ to guarantee himself control and end up in a Initial Owner EOS. Setting $\bar{m} \in [\bar{m}_{1,n}^E, \bar{m}_{2,n}^E]$ guarantees that the initial owner prefers to be in a n-Blockholder equilibrium rather than in an Initial Owner one (he gets a higher price though he gives up control). If the initial owner were to choose a different α_E he may end with a different EOS. Thus, e.g. if $\alpha_E \geq \frac{1}{2} > \bar{m}$ then a dispersed ownership structure is the unique EOS (assuming appropriate prices to guarantee participation). Any α_E between $\bar{m}$ and $\frac{1}{2}$ does not give him control and only dilutes the shares (since we can assume that the EOS following such a choice has at least n blockholders). Hence it is never chosen.

Following from Propositions (2) and (4), if $\bar{m} < \bar{\alpha}$ there are no conflicts and no blockholders; if $\bar{m} \leq \tilde{\alpha}(n)$ then there would be an incentive for passive shareholders to sell shares to active liquidity investors, possibly leading to a dispersed ownership structure with $X_{med} = \bar{X}$ (this condition affects the equilibrium price).

To guarantee that the initial owner prefers to monitor rather than not with this EOS we have the condition $\bar{m} \in [max\left(\bar{m}_{1,n}^{RC}(n), \bar{m}_{1,n}^{S}(n)\right), min\left(\bar{m}_{2,n}^{RC}(n), \bar{m}_{2,n}^{S}(n)\right)]$ (See Fig. 5). If the monitoring costs are too high the residual value for the initial owner after compensating the blockholder for the extra risk they bear due to the undiversified portfolio, is not high enough to compensate for holding an undiversified portfolio and the monitoring costs. When the monitoring costs are too low, there are blockholder equilibria with small blocks or liquidity shareholder equilibria where there are enough active investors to determine X. In such a case the risk/return outcome is very high compared to the initial owner's preferred point. Then the risk the initial owner bears is so high and the gain from monitoring is so low that he prefers to sell the firm or not to raise capital.

What if the initial owner were to choose a different w_E? The condition $\alpha_E < \hat{\alpha}(n)$ guarantees that whenever the participation constraint of liquidity shareholders is satisfied, so is that of blockholders. Hence the initial owner cannot prevent the entry of blockholders through setting a lower price – he only loses rent. Hence he sets the highest $w_E = \underline{w}_E^n$ that guarantees participation by investors. Increasing the price, (below $\underline{w}_E^n$) drives out participation (see Lemma 10) and so the initial owner would not be able to raise capital. Finally, $\bar{m}$ must be be smaller than $\hat{\alpha}(n)$ so that no blockholder in a Nash equilibrium with $n-1$ other blockholders has a unilateral incentive to deviate: recall that in a blockholder equilibrium, each blockholder is pivotal. Therefore if a single blockholder reduces his shareholdings, the voting outcome in the next period would be X_E. The incentive compatibility condition that the utility of an investor is higher when he is a blockholder with decision X_1 than a liquidity shareholder with decision $X_E < X_1$, holds if $\alpha_E \leq \hat{\alpha}(n)$.

The reader may find it puzzling that we start with ex-ante identical outside (active) investors, yet only some of them decide to become blockholders.[19] This is because there are multiple Nash equilibria for every n, and the identity of the blockholders could be different in each of these equilibria and the rest of the investors (liquidity shareholders) free ride on these. Like in a (discrete) public goods provision problem, the blockholders contribute to the public good provision (i.e. moving the decision on the project closer to the most preferred point of all outside investors) because given the other shareholders contributions, it is a Nash equilibrium for them to contribute as long as the value of the public good to them is sufficiently high.[20] This translates into the condition that α_E is sufficiently low: as α_E decreases, the incentives to hold larger blocks increases. This is because, in the first place, as α_E decreases, fewer shares are needed in order to gain control over X and hence the cost of holding a block is lower. Second, because of the convexity of X_j

[19]It may be the case that some large institutional investors are *less* risk averse relative to other investors. This destroys the ex-ante symmetry between outside investors, but this actually helps in our model, in the sense that there are some investors who are believed to be natural blockholders giving us a nice focal point equilibrium.

[20]This is also the case in the Nash demand game or the Divide the Dollar game : each of two players announces the share he demands of an amount of money that may be split between them. If the demands can be satisfied, they are. Otherwise neither player receives any money. This game has multiple pure strategy Nash equilibria: any demands that add upto the feasible amount are Nash equilibria.

with respect to α_j (equation (6)), it follows that the smaller α_E is, the larger is the shift in X (for the same $\alpha_E - \alpha_1$), i.e. $X_E - X_1$ is greater. This implies a higher increase in the expected return of becoming a blockholder. Hence there exists a threshold, $\hat{\alpha}(n)$, such that when $\alpha_E \leq \hat{\alpha}(n)$ and $w_E = \underline{w}_E^n$, the utility of being a blockholder is higher than being a liquidity shareholder with the initial owner in control.

Unlike the usual public goods contribution game, however, when blockholders buy a larger block of shares, their preferences over X are closer to those of the initial owner. This is why the presence of blockholders mitigates, but does not remove, the conflict of interest between the initial owner and the outside investors.

Lemma (4) implies that in the blockholder equilibrium, liquidity shareholders free ride on blockholders as they hold the optimal portfolio. The price which satisfies the participation constraints of the blockholders is lower than the maximum price that liquidity shareholders are willing to pay. Therefore the existence of blockholders allows the liquidity shareholders to extract some of the rent, that in all other equilibria goes entirely to the initial owner. In all the other equilibria the initial owner sets the price low enough to satisfy the participation constraint of the liquidity shareholders with equality, and hence he extracts all the rent. As the initial owner is not able to extract all the rent, when switching from an Initial Owner to an n Blockholder equilibrium there is a jump in the level of utility of the initial owner. This discontinuity can be seen in Fig. 5 at $\hat{\alpha}$ which represents the switching point between the Initial Owner to 1 Blockholder equilibrium as the monitoring costs decrease. Because the initial owner's utility decreases at $\hat{\alpha}$, the possibility to sell or not raise money becomes more attractive.

The next corollary demonstrates the existence of an Initial Owner equilibrium where the initial owner retains full control ($\alpha_E = \frac{1}{2} + \eta_E$) in order to avoid an n-Blockholder equilibrium.

Corollary 1 *Suppose*

$$\bar{m} \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n), \right], \min\left[\bar{m}_E^{RC}(K), \bar{m}_E^{S}(K)\right]\right) \tag{17}$$

and either $\bar{m} \leq \bar{m}_{1,n}^E(n, \lambda)$*, or* $\bar{m} > \bar{m}_{2,n}^E(n, \lambda)$ *then an Initial Owner Equilibrium exists and is unique where* $m = 1$*,* $\alpha_E = \frac{1}{2} + \eta_E$*,* $X_{med} = X_E$ *and* $w_E = \underline{w}_E^E$*.*

The initial owner is willing to hold $\alpha_E > \frac{1}{2} > \bar{m}$ when the monitoring costs are either very high, $\bar{m} > \bar{m}_{2,n}^E$ or very low $\bar{m} < \bar{m}_{1,n}^E$. The intuition behind this result is that the initial owner faces a trade off between two types of costs: the cost of holding a very risky asset (this cost increases the bigger the gap between α_E and α_1) if he does not control the vote outcome, and the cost from being

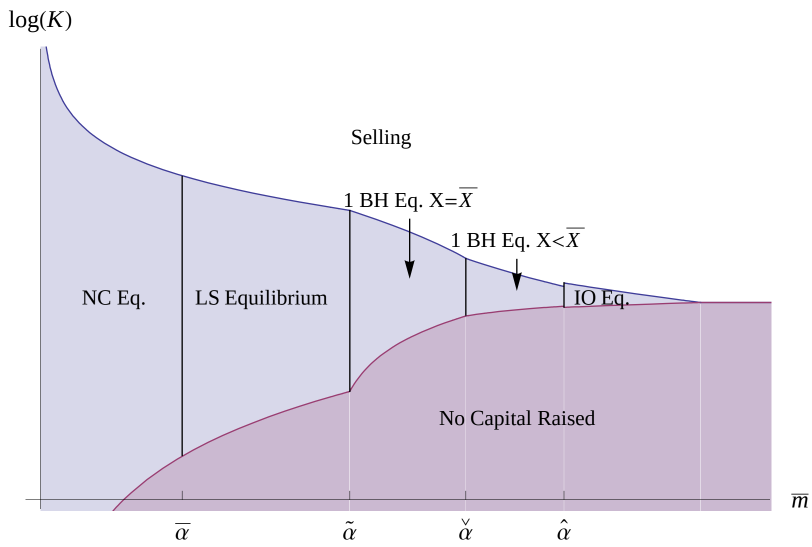

Figure 5: 1 Blockholder equilibrium ($\bar{R} = 1$, $\gamma = 12$, $\sigma = 0.2$, $\bar{X} = 50$, $\lambda = 0.1$). Note that in the graph there is not the boundary $\bar{\alpha}$ and $\bar{m}_2^{1E}$ as they are not binding and far out from the plot range considered.

relatively undiversified (this cost increases with α_E). When the monitoring costs are very high, $\bar{m} > \bar{m}^E_{2,n}$, the cost of increasing α_E a little bit and reducing diversification is very small relative to the gain from controlling the vote outcome and choosing his preferred risk/return combination. Hence he chooses to retain more shares than the monitoring incentive requires and retains control. When the monitoring costs are very low, on the other hand, blockholders face a very low cost in terms of diversification to be in control as now the vote of the liquidity shareholders has relatively more weight. Hence, the initial owner faces a large cost arising from the conflict of interest on the risk/return vote outcome. For this reason he prefers to increase α_E to a point where he can control the vote outcome, $\alpha_E > \frac{1}{2}$. The other conditions have the same rationale as Proposition 5 so we do not repeat them here.

The result is that the initial owner holds more shares than what the monitoring incentives require. An initial owner wants to hold a block in order to retain control and be sure to obtain his preferred choice on the project chosen. Hence the model offers another rationale for why one investor would choose to hold the biggest block in a firm quite apart from the monitoring incentive proposed so far in the literature. (Shleifer and Vishny, 1986)

A possible objection to our main result is that there are multiple equilibria for this configuration of monitoring costs. In particular it can be that for the same monitoring costs there exist both an n Blockholder and an Initial Owner equilibrium. Indeed this is true for n Blockholder equilibria where $n > 1$. This result depends on the multiplicity of EOS for a given (α_E, w_E) combination. However, Corollary 2 shows that for some values of the monitoring costs only n Blockholder equilibria are possible. The intuition behind this result is that when monitoring costs are low enough then the cost of losing diversification is low enough for a single blockholder to deviate and demand a block, thus ruling out an Initial Owner equilibrium. This is because he needs a smaller block to both win the vote (using the vote of the active liquidity investors) and to have a large shift in X. If monitoring costs are too low of course, there may be no conflict between the initial owner and outside investors and hence no desire to hold blocks.

Corollary 2 *Suppose:*

$$\bar{m} \in \left(\max[\bar{\alpha}, \tilde{\alpha}(1), \bar{m}_1^E(1\lambda), \bar{m}_1^{RC}(1)(K), \bar{m}_1^S(1,K)], \min\left[\frac{1}{2}, \hat{\alpha}(1), \bar{m}_2^E(1,\lambda), \bar{m}_2^{RC}(1,K), \bar{m}_2^S(1,K)\right]\right] \tag{18}$$

then there exist n Blockholder equilibria with $n \in [1, M_A]$ where the initial owner is monitoring, $m = 1$, $\alpha_E = \bar{m}$, $X_{med} = X_1 < \bar{X}$, $w_E = \underline{w}_E^1$. No other types of equilibria with positive trade exist for these parameter values.

This corollary is a special case of Proposition 5, so the conditions are the same except that n (blockholders) is replaced with 1 blockholder.

Among the possible blockholder equilibria we might want to consider the one which is most preferred by the initial owner, since he can always "induce" this EOS by appropriate choice of (α_E, w_E).

Proposition 6 *The optimal number of blockholders for the initial owner is:*

$$n^* = \left[\frac{(1+\bar{m})(\bar{m} - \lambda + \bar{m}\lambda)}{2\bar{m}^2(1-\lambda)}\right] > 0 \tag{19}$$

The optimal number of blockholders for the initial owner arises from the trade-off from having few blockholders and a vote outcome closer to his optimal outcome or having many blockholders and being able to sell the shares at a high price, i.e. low w_E. The preferred number of blockholders is a function of the monitoring costs (and hence, in equilibrium, of the amount of shares he retains in a monitoring equilibrium) and the proportion of active liquidity investors.

When the monitoring costs are high a further increase of the monitoring costs would induce the initial owner to prefer less blockholders. In such a case blockholders are already holding a highly undiversified portfolio and the initial owner needs to set a very low price in order to induce them to buy the shares. Decreasing the number of blockholders further does not reduce the share price by much but he gains in terms of risk/return outcome that is it decreases the costs of holding a suboptimal portfolio.

On the other hand, when the monitoring costs are very low an increase in the monitoring costs induces the initial owner to prefer more blockholders. In this case the discrepancy between the preferences of the blockholders and the initial owner is not that high so that increasing the number of blockholders allows the initial owner to raise the price at which the shares are tendered. Hence the initial owner would prefer to have more blockholders.

Finally the effect of the proportion of active investors among the liquidity shareholders, λ, has a negative effect on the optimal number of blockholders preferred by the initial owner. When more liquidity investors vote it becomes cheaper for blockholders to hold sufficiently large blocks so that they are jointly pivotal in the voting. This is very costly for the initial owner in terms of risk exposure. Hence when the λ is higher the initial owner would prefer to set a lower price, but to have the vote outcome closer to his preferred point. Hence the higher the participation of the liquidity shareholders to the vote the lower the number of blockholders the initial owner would like to have.

5.2 Optimal amount of capital raised

In this section we relax the assumption that the initial owner raises just enough capital needed to implement the project, K. We allow the initial owner to invest more capital $I \geq K$ and use the difference, $I - K$ to buy the risk free asset. This would offer him the possibility to achieve the preferred degree of diversification through the firm's investment in the risk free asset: hence the conflict of interest between the initial owner and outside investors may disappear. We show in the proposition below that our results are robust to relaxing this assumption.

Proposition 7 *The initial owner always strictly prefers to raise the minimum amount of capital, i.e.* $I = K$.

Observe that the initial owner acts as a monopolist when setting the share price. Hence, if he increases I, this lowers the price per share and it lowers the risk of the project for the same X. The decrease in price decreases the initial owner's utility in such a way that it more than offsets the increase in utility due to a lower risk of the project. The compensation demanded by the investors in terms of lower price is higher than the gain that the initial owner obtains. Hence the initial owner prefers to hold a suboptimally diversified portfolio rather than issue the shares at a lower price.

6 Comparative Statics and Empirical Implications

In this section we discuss some of the important predictions of our model and discuss how they relate to the empirical literature on ownership structure.

6.1 What does Ownership Structure depend on?

Our first prediction is that the *number* and *size* of the blockholders will depend on the degree of co-ordination required for voting decisions captured by the size of the first blockholder (hence e.g. on the monitoring technology or the degree of complementarity between the firms production and the input of the initial owner). We expect firms to have blockholders only in cases where they hold less than 50% of the stock. No blockholders should be observed in firms that keep the voting control when they release the rest.

Secondly, we find that the degree of concentration in the ownership structure is decreasing in the monitoring costs, $\bar{m}$, or in the size of the first block. For very small monitoring costs (or small size of the initial block) there is dispersed ownership, for intermediate levels of monitoring costs there are multiple large blocks and for high monitoring costs there is a single large block. For example, high monitoring costs are more common in firms dedicated to innovation or R&D where moral hazard issues are much more pervasive. In such firms we should see an initial owner who has control.[21]

Third, we can capture the effect of riskier sectors through $\bar{X}$. High $\bar{X}$ corresponds to industries where risk can be potentially very high. Then our model predicts that blockholder ownership structures emerge in more risky industries. Thus, in more mature industries the projects among which the shareholders can choose is limited to the low risk ones, that is in our models, i.e. low $\bar{X}$. Ceteris paribus, in more mature sectors having a choice of low risk/low return projects, dispersed ownership structures with more than one blockholder are less likely. Anecdotal evidence suggests that in more mature sectors it is more common to see families in control of firms. In very innovative industries, on the other hand, we should see blockholders which ease the conflicts of interests between investors and initial investor and induce more risky decisions. These blockholders are usually represented by institutional investors, e.g. venture capitalists, who professionally look for firms with a high risk/return profile.

There is also an inverse relationship between dispersion in the ownership structure and the size clustering of the projects, K. When K is small, the initial owner may prefer not to raise the capital

[21] Although in our model the monitoring technology can be considered more broadly as a friction which determines the first block, this does not exclude these empirical predictions applying in firms where the largest blockholder exists because of monitoring.

as the monitoring costs are too high to compensate for the risk incurred. On the other hand, when the project size K is very big the extra value added by the monitoring is such a small fraction of the total cost that it becomes more attractive to sell the idea and have a totally dispersed ownership structure.Hence, concentrated ownership should invest in similar size projects, while more dispersed ownership should be common in any type of project size. Even more interesting it is the feature of the ownership structure for intermediate levels of monitoring costs. As Fig. 5 shows dispersed ownership is common in all firms whatever is the siwe (given low monitoring costs). Concentrated ownership instead arises only for projects of specific intermediate size.

Fourth, we analyze the effects of λ on the ownership structure. The effect of higher λ in our model is ambiguous because we do not impose any capital constraints on investors: on the one hand it helps to reduce the costs of participation for blockholders implying, ceteris paribus, a more diversified structure. On the other hand, it becomes more costly for the initial owner to give up control since the vote outcome will be further from his first best. The balance between these two forces determines the equilibrium outcome. Voting participation can be influenced by country regulation or firm bylaws. For example until a few years ago in Germany only shareholders with more than 5% participation could vote. Alternatively, shareholders' vote participation can be related to information disclosure and hence the ability to make informed decisions. Hence, we may interpret higher λ as equivalent to an increasing minority shareholder protection. In financial markets with high minority shareholder protection and where investors are not capital constrained we should expect to see either an initial owner with a big block and many small investors or situations where there are few blocks – the initial owner and some others who need to hold relatively few shares to be able to gain control over the firm's decision.

Finally notice that because the utility of the initial owner is decreasing in λ, the threshold of monitoring costs below which he is willing to invest in the project is lower. Hence minority shareholder protection has a positive and a negative effect on investments. On one side minority shareholder protection leads to higher value projects indirectly through its effect on the ownership structure. However it also reduces the willingness of the initial owner to invest money or to exert monitoring. Hence minority shareholder protection is detrimental to induce the investment, but on the other side if an investment is taken minority shareholder protection is a tool to undertake more value enhancing projects.

6.2 The implications of the Ownership Structure

Let us now look at the predictions we make on how ownership structure influences firm choices.

In our model, the risk/return decision depends not only on whether there is concentration of ownership but rather on the number and size of blockholders: firm value decreases with the size of the first block. This is because for low values of monitoring costs the initial owner prefers a high return as well so both agree on the choice of projects. Ceteris paribus, the smaller the size of the median block the higher is the predicted firm value. These results are consistent with the empirical literature which shows that the effect of the first blockholder on the value of the firm is usually negative, (Barclay and Holderness (1989) and Kirchmaier and Grant (2005) and Lehmann and Weigand (2000)). This is in keeping with our results on the Initial Owner equilibrium. Some of the papers distinguish between the role of the first and subsequent blockholders: The empirical consensus seems to be (again in keeping with our results) that a second blockholder increases firm value. This phenomenon is present across countries and across publicly listed or private firms (Volpin (2002), Maury and Pajuste (2005), Faccio, Lang, and Young (2001), Gutierrez and Tribo (2004), Isakov and Weisskopf (2009) and Laeven and Levine (2008)). Roosenboom and Schramade (2006) studying French IPOs find that when the owner is powerful, the firm is less valued; when the initial owner shares control with other blockholders the value increases. Helwege, Pirinsky, and Stulz (2007) find that firms where insider ownership gets reduced over time or firms with widely held ownership are more highly valued.

Our paper also offers an extra prediction to differentiate our theoretical results from other theories explaining why blockholders emerge: in our model the risk profile of a firm also depends on how widely dispersed is the ownership. There are few empirical papers looking at this relationship. Carlin and Mayer (2000; 2003) and Teodora (2009) find that multiple blockholders are present in high risk firms, while a single blockholder is common in low risk firms.

Our paper offers an explanation for the underpricing observed in IPOs. Empirically, it has been found that IPOs are usually associated with a first day positive return (i.e. the underpricing).[22] In the literature there is no agreement on the reasons why this phenomenon arises (see Jenkinson and Ljungqvist (2001) and Welch and Ritter (2002) for reviews). Brennan and Franks (1997) among others (Boulton, Smart, and Zutter (forthcoming), Nagata and Rhee (2009) and Yeh and Shu (2004)) found that ownership structure contributes to the degree of underpricing. In particular they argue that underpricing can be more severe when the initial owner wants to avoid blocks. However, they note that this is not a stable outcome and over time blocks are formed anyway. The findings of Brennan and Franks (1997) are in line with our predictions. If the initial owner could decide the share allocation and retain control, he would be willing to do so even though this implies a lower price. However, if share trade is allowed this outcome cannot be stable. The findings of Brennan and Franks (1997) contrast instead with the predictions of Stoughton and

[22]The average underpricing is between 15% and 17% though it varies a lot over time

Zechner (1998) and DeMarzo and Urosevic (2006) where the higher the ownership concentration the higher the underpricing. When the initial owner has no discretion on share allocation and trade is allowed, blockholder equilibria cannot be avoided and underpricing occurs in the sense that liquidity shareholders would be willing to pay more than the equilibrium price to buy shares. Our theory predicts therefore that underpricing occurs when the size of the initial block (initial owner) in the shares is not too large (in particular less than the relevant voting threshold) and not too small. In such a case the predictions of our model are similar to those of Stoughton and Zechner (1998) and DeMarzo and Urosevic (2006): underpricing occurs when blockholders are present and it is higher the higher the degree of concentration.

7 Conclusions

This paper attempts to reconcile two well documented empirical regularities in the corporate governance literature: the presence of multiple large shareholders, some of them without a controlling interest on the one hand and increased firm value when such large shareholders are present on the other hand (e.g. Carlin and Mayer (2000; 2003) and Laeven and Levine (2008) among others).

Our model relies on an endogenous conflict of interest over the choice of risk between the initial owner of the firm and outside investors. This problem is similar to a public good contribution game: if there are multiple blockholders which are intermediate in size between the entrepreneur and small shareholders, then the decision on the project is more favorable to all investors, but it comes at a cost to the large shareholders as they hold suboptimal portfolios. Hence blockholders provide a public good to other investors. We show that at very low levels and at very high levels of monitoring costs, we get equilibria where either the initial owner has full control when monitoring costs are high or liquidity shareholders having full control when they are low. For intermediate values we get equilibria with multiple blockholders. The corresponding choice of projects goes from low risk/return (the entrepreneur's first best) when monitoring costs are low, to intermediate levels of risk/return in the blockholder equilibria (depending on how many blockholders there are), to very high risk/return projects when the monitoring costs are very high.

The main contribution of our paper is to generate a multiple blockholder equilibrium with a very simple and standard model of corporate control. Unlike many of the papers that explain this phenomenon, we show that blockholders arise in the absence of any coordination between investors and without a direct choice of share allocation on the part of the initial owner – indeed the initial owner only chooses the ownership structure indirectly through the pricing of shares. This simple model is able to explain a variety of stylized facts about the links between ownership size and firm characteristics.

Although our paper assumes that all outside investors are identical, the paper could be easily extended to the case of heterogenous agents. Indeed this would help to reduce the problem of multiple equilibria. In this case less risk averse investors would be the natural blockholders and we would expect that if this occurs, the risk/return choice is even higher. This is in line with empirical papers which find that the presence of institutions as blockholders enhances firm performance (Ben Dor (2003), Hartzell, Kallberg, and Liu (2008), Barber (2007) and Chen, Harford, and Li (forthcoming)).

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A Appendices

A.1 The Model

A.1.1 Lemma 1:

Proof. The proof is obvious. We just maximize the objective functions (over X) of the outside investors, equation (4), and of the initial owner, equation (5), given the fraction of shares held, α_j. Concavity ensures uniqueness of the solution. ■

A.2 The equilibria.

A.2.1 Lemma 2:

Proof. At date 3, the ownership structure, $\vec{\alpha}$, and thus X_{med} are already fixed. Given the initial owner's objective function (5), he monitors iff the utility from monitoring is greater than from not monitoring, that is when:

$$\alpha_E \left(X_{med}\bar{R} + K \right) - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 - \bar{m}K \geq \alpha_E X_{med}\bar{R} - \frac{\gamma}{2}\alpha_E^2 X_{med}^2 \sigma^2 \tag{20}$$

Rearranging, we get the condition $\alpha_E \geq \bar{m}$. ■

A.2.2 Lemma 3

Proof. Investor l chooses $\alpha_l(X_j)$ to maximize equation (4) where $X_j = X_{med}$. The first order condition implies equation (7). The second order condition is satisfied as long as $X_j > 0$, so this is a maximum. ■

A.2.3 Lemma 4

Proof. As the liquidity shareholding $\alpha_l(X_j)$ maximizes the utility of an investor given a vote outcome, any shareholding $\alpha_1 \neq \alpha_l(X_j)$ gives less utility. ■

A.2.4 Lemma 5

Proof. Consider first the case where $\alpha_j \geq \bar{\alpha}$. Hence by Lemma 1 $X_j = \frac{\bar{R}}{\gamma\sigma^2\alpha_j}$ and by Lemma 3 $\alpha_l(X_j) = \alpha_j + \frac{f(m)-\frac{K-w_E}{1-\alpha_E}}{\frac{\bar{R}^2}{\gamma\sigma^2\alpha_j^2}}$. By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ and thus $\alpha_l(X_j) < \alpha_j$.

Now let $\alpha_j < \bar{\alpha}$. By definition, j is the median shareholder, hence $X_{med} = X_j = \bar{X}$. As $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$ a liquidity investor always chooses $\alpha_l(\bar{X}) < \bar{\alpha}$ and hence it is sufficient to show that no active investors hold $\alpha_j < \alpha_l(\bar{X})$. Assume to the contrary investors j holds $\alpha_j < \alpha_l(\bar{X})$. (Active) Investor j can improve his utility by choosing $\alpha_l(\bar{X})$ and voting for $X = \bar{X}$ without changing X_{med}. Contradiction to the equilibrium definition where shareholders are maximizing their utility when X is fixed. This proves that in equilibrium $\alpha_j \geq \alpha_l(X_j)$. ■

A.2.5 Lemma 6:

Proof. Before moving to the proof of the Lemma we have an intermediary step given by Lemma 9 which describes the necessary conditions for an n Blockholder EOS to exist.

Denote the size of the shareholdings of other *active* investors excluding investor 1 as α_{-1}. So $\alpha_{-1} = (n-1)\alpha_1 + N_A\alpha_l(X_1)$.

Lemma 9 *Suppose $\alpha_E > 0$ and $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$. If a n-Blockholder EOS arises then (1) $\alpha_{-1} > 0$; (2) $\alpha_1 + \alpha_{-1} \geq \alpha_E > \alpha_1$; (3) $\alpha_E \geq N_A\alpha_l(X_1)$. In any such equilibrium $X_1 > X_E$ and $N_A\alpha_l(X_1) + n\alpha_1 \geq \alpha_E$.*

Proof.

We now prove the lemma first computing the utility of the median shareholder (a) when $X_{med} = X_1 < \bar{X}$ and (b) when $X_{med} = X_1 = \bar{X}$ and then showing the lemma by contradiction.

(a) Since $X_{med} = X_1 < \bar{X}$, in Definition 2, shareholder 1 is the median shareholder and $\alpha_1 > \bar{\alpha}$. Using equation (4), the utility of the median shareholder is:

$$U_1^{nBH} = 1 + \alpha_1\left(X_1\bar{R} + f(m) - \frac{K-w_E}{1-\alpha_E}\right) - \frac{\gamma}{2}\alpha_1^2X_1^2\sigma^2$$

By Lemma 1 and the fact that $X_{med} = X_1 < \bar{X}$, this is equivalent to

$$\alpha_1(f(m) - \frac{K-w_E}{1-\alpha_E}) + 1 + \frac{\bar{R}^2}{2\gamma\sigma^2} \tag{21}$$

By assumption, $f(m) - \frac{K-w_E}{1-\alpha_E} < 0$, hence U_1^{nBH} is decreasing in α_1.

(b) Suppose instead that $X_{med} = X_1 = \bar{X}$. By definition $\alpha_l(\bar{X}) < \alpha_1$ in an n-Blockholder EOS and $\alpha_l(\bar{X}) = \text{argmax}\, U_i(\alpha_i)|_{X=\bar{X}}$. Hence, U_1^{nBH} is decreasing in α_1.

Now we are ready to prove the Lemma by contradiction:

(1) Suppose to the contrary that $\alpha_{-1} = 0$ in an n-Blockholder EOS. In order to have $X_{med} = X_1$ we must have $\alpha_1 \geq \alpha_E$, since investor 1 is the only outside investor who votes. From part (a), we know that U_1^{nBH} above is decreasing in α_1 and hence he is better off setting $\alpha_1 = \alpha_E$, otherwise the initial owner becomes the median shareholder. But in this case, $X_{med} = X_1 = X_E$ and *de facto* investor 1 does not affect the vote outcome. Hence he

prefers to be a liquidity shareholder and his optimal shareholding is given by $\alpha_l(X_E)$. Contradiction to the fact that this is an EOS (since investor 1 wants to deviate unilaterally). Hence, $\alpha_{-1} > 0$ in any n-Blockholder EOS.

(2) $\alpha_E > \alpha_1$ follows from the proof of part (1) above. We need to prove that $\alpha_1 + \alpha_{-1} \geq \alpha_E$. Suppose to the contrary that $\alpha_1 + \alpha_{-1} < \alpha_E$. Then $\alpha_E = \alpha_{med}$ and so $X_{med} = X_E$, so this is not an n-Blockholder EOS by Definition 2. Contradiction.

(3) $\alpha_E \geq N_A \alpha_l(X_1)$. Given (2) above, this holds iff $\alpha_1 = 0$ which contradicts Definition 2. This implies that $N_A \alpha_l(X_1) + n\alpha_1 > \alpha_E$ (since $\alpha_1 > 0$ and $\alpha_E \geq N_A \alpha_l(X_1)$). ■

Lemma 9 shows that U_1^{nBH} is decreasing in $\alpha_1 (\geq \alpha_l(\bar{X}))$. So, $n\alpha_1 = \alpha_E - N_A \alpha_l(X_1)$. Recall that the total shares of the firm (assuming full subscription) must add up to 1 and that we consider equilibria where the proportion of active liquidity investors among the liquidity shareholders is given by λ. Hence N_A must satisfy $\lambda(1 - \alpha_E - n\alpha_1) = N_A \alpha_l(X_1)$. Hence $n\alpha_1 + \lambda(1 - n\alpha_1 - \alpha_E) = \alpha_E$. Solving for α_1 we get equation (8). ■

A.2.6 Lemma 7:

Proof. By Lemma 1, given the shareholding α_i of investor i the first best X for the investor is given $X_i = \min\left[\frac{\bar{R}}{\gamma\sigma^2\alpha_i}, \bar{X}\right]$. By the proof of Lemma 9, we know that the investors' utility function is decreasing in α_i when $\alpha_i \geq \alpha_l(\bar{X})$. Hence, the first best choice of $\alpha_i = \alpha_l(\bar{X})$. ■

A.2.7 Proposition 1:

Proof. We solve the problem using backward induction. The initial owner's objective function is different depending on the anticipated ownership structure.

The initial owner maximizes the following objective function:

$$\max_{\alpha_E, w_E} U\left(m=0\right) = \bar{R}X_{med}(\alpha_E)\alpha_E - \frac{\gamma}{2}X_{med}(\alpha_E)^2\sigma^2\alpha_E^2 + 1 - w_E \tag{22}$$

Given the objective function, regardless of the ownership structure, he chooses the lowest w_E, that is $w_E = w_E^j(\alpha_E)$ where $w_E^j(\alpha_E) = \{\underline{w}_E^E, \underline{w}_E^n, \underline{w}_E^{LS}\}$.

Substituting for w_E^j in the objective function, it can be checked that $U\left(m=0\right)$ is decreasing in α_E for all w_E^j, for $X_{med} \leq \bar{X}$. Therefore $\alpha_E = 0$ is the optimal choice of the initial owner for *any* ownership structure.

When $\alpha_E = 0$, and there is at least one active investor ($\lambda > 0$), all active investors vote for $\bar{X}$ and hence $X_{med} = \bar{X}$. The participation constraint of outside investors is satisfied if $w_E \geq \underline{w}_E^{LS}$. This is the first best for outside investors hence there is no incentive to become a blockholder. Hence, the Liquidity Shareholder EOS is the unique EOS. Also it is trivial to see that no investor is willing to sell his shares at a price lower than the maximum that an excluded investor is willing to pay.

Furthermore the initial owner sets $w_E = \underline{w}_E^{LS}$ as the lowest w_E^j. Hence, the initial owner's utility is given by:

$$\bar{R}\bar{X} - K + 1 \tag{23}$$

Finally we check if the participation constraint of the initial owner is satisfied: i.e if he invests in the riskfree asset his utility is 1. Hence when the project has a positive NPV he sells the firm, otherwise he does not raise capital. The initial owner's value function is then given by equation (12). ■

A.3 Monitoring Equilibria

A.3.1 Lemma 8:

Proof. Before moving to the proof of the Lemma we show which are the various EOS for different pair of $(\alpha_E, w_E) \in S \equiv [0,1] \times (-\infty, \infty)$.

First, observe that for certain combinations of (α_E, w_E) there always exists a no trade equilibrium where no investors participate in the share issue. Define $\underline{w}_E^{NT} = \begin{cases} \underline{w}_E^E & \text{if } \alpha_E > \max\left[\frac{1}{2}, \bar{\alpha}\right] \\ \underline{w}_E^1 & \text{if } \bar{\alpha} \leq \alpha_E \leq \frac{1}{2} \\ \underline{w}_E^{LS} & \text{if } \alpha_E \leq \bar{\alpha} \end{cases}$.

This is the minimum amount the initial owner needs to invest in order to guarantee that at least one investor is willing to buy shares. The next lemma shows the conditions under which the No Trade equilibrium exists.

Lemma 10 *There always exists a No Trade EOS if* $w_E < \underline{w}_E^{NT}$.

Proof. Suppose to the contrary that there exists a No Trade EOS and $w_E \geq \underline{w}_E^{NT}$. Then a single shareholder can buy $1-\alpha_E$ shares and ensure full subscription since $w_E \geq \underline{w}_E^{NT}$, contradiction. ■

Define:

$$\underline{\underline{w}}_E(\alpha_E) \equiv K - f(m)(1-\alpha_E) \tag{24}$$

$$\hat{\alpha}(n) \equiv \max[\hat{\alpha}_1(n), \hat{\alpha}_2(n)] \tag{25}$$

$$\hat{\alpha}_1(n) \equiv \frac{2\lambda}{2(1+\lambda) - n(1-\lambda)} \tag{26}$$

$$\hat{\alpha}_2(n) \equiv \frac{2\bar{\alpha}(1-\lambda)n + \lambda - \sqrt{\lambda^2 + 4\bar{\alpha}(1-\lambda)n\left(\bar{\alpha}(1-\lambda)n - 1 - 2\bar{\alpha}(1+\lambda)\right)}}{2\left(1+\lambda\right)} \tag{27}$$

$$\check{\alpha}(n) \equiv \frac{n(1-\lambda)\bar{\alpha} + \lambda}{1+\lambda} \tag{28}$$

$$\tilde{\alpha}(n) \equiv \frac{\lambda}{1+\lambda} - \frac{n}{\eta_j} \tag{29}$$

Suppose α_E is fixed, then $\underline{\underline{w}}_E(\alpha_E)$ is the minimum w_E that guarantees that $f(m) > \frac{K-w_E}{1-\alpha_E}$. Note that when $m=1$ this condition implies that the price of the risky asset is lower than that of the risk free one. Later we show this is always the case in equilibrium. $\hat{\alpha}(n)$ is the value of α_E such that if $\alpha_E \leq \hat{\alpha}(n)$ then $\underline{w}_E^E \leq \underline{w}_E^n$. In particular if $\alpha_E \leq \hat{\alpha}_1(n)$, then $\underline{w}_E^E \leq \underline{w}_E^n$ where $X_1 < \bar{X}$, and if $\alpha_E \leq \hat{\alpha}_2(n)$, then $\underline{w}_E^E \leq \underline{w}_E^n$ where $X_1 = \bar{X}$. $\check{\alpha}(n)$ is the value of α_E such that when $\alpha_E \leq \check{\alpha}(n)$, $\alpha_1 = \bar{\alpha}$. Finally $\tilde{\alpha}(n)$ is the value of α_E such that when $\alpha_E \leq \tilde{\alpha}(n)$ n extra active liquidity shareholders can become pivotal in the voting decision and change it from X_E to $\bar{X}$.

Lemma 11 *There exists an Initial Owner EOS, with* $X_{med} = X_E < \bar{X}$ *for any pair* (α_E, w_E)*, satisfying the following conditions:*

$$\alpha_E \in \left(\max\left(\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \tilde{\alpha}(1)\right]\right]\right), 1\right] \tag{30}$$

$$w_E \in \left[\underline{w}_E^E(\alpha_E), \min\left[\underline{w}_E^1(\alpha_E), \underline{\underline{w}}_E(\alpha_E)\right]\right) \tag{31}$$

Proof. We use Definition 1: An Initial Owner EOS exists for any combination of (α_E, w_E) iff (a) the participation constraint of investors is satisfied; (b) the incentive constraint of active investors is satisfied; (c) $X_{med} = X_E < \bar{X}$ in the EOS and (d) no shareholder is willing to sell his participation at a price lower than the price at which the excluded investors are willing to buy.

(a) The participation constraint for liquidity shareholders is satisfied iff $U_{l,E} \geq 1$. A sufficient condition for $U_{l,E} \geq 1$ is that $\alpha_l(X_E) \geq 0$, i.e. iff $w_E \geq \underline{w}_E^E$. This is the first part of condition (31).

(b) We check that no liquidity investor has an incentive to switch to becoming a blockholder. Notice that $\underline{w}_E^E < \underline{w}_1^E$ whenever $\alpha_E > \hat{\alpha}(1)$. This implies that, given the conditions of the lemma, if a liquidity investor switches to becoming a blockholder, his utility is negative while if he stays as a liquidity investor his utility is strictly positive since $w_E \geq \underline{w}_E^E$. Hence condition (31) implies that $U_{l,E} \geq U_1^{1BH}$ (assuming that $\frac{K-w_E}{1-\alpha_E} > f(m)$ – this is the case since $w_E \leq \underline{\underline{w}}_E$).

(c) $\alpha_E > \bar{\alpha}$ ensures that $X_E < \bar{X}$.

(d) Recall that there are a fraction λ of active investors among the liquidity shareholders. For the EOS we also need to guarantee that none of the investors who hold shares want to sell them given the maximum price that excluded investors are willing to pay. Any active investor who is excluded can do better by buying liquidity shares from a *passive* investor (who does not vote) at a price slightly higher than the initial owner's price, if by voting he is able to become pivotal and change the outcome. This occurs when

$$\lambda(1 - \alpha_E - \bar{\alpha}_l) + \alpha_l(X_E) > \alpha_E$$

which corresponds to $\alpha_E \leq \tilde{\alpha}(1)$. In such a case the participation constraint of the investors is also satisfied since it was satisfied for the investors who originally held the shares.

To rule this out we set $\alpha_E > \max[\hat{\alpha}(1), \tilde{\alpha}(1)]$. However these conditions become irrelevant when the initial owner has the majority of the shares since then, no active investors can change the vote outcome. This implies that $\alpha_E > \min\left[\frac{1}{2}, \max[\hat{\alpha}(1), \tilde{\alpha}(1)]\right]$.

Putting together the constraints on α_E from point (c) and (d), the lower bound on α_E is given by (30).

Observe that the interval in condition (31) is non empty whenever $m = 1$ i.e. $f(m) = K$. This is because then $\underline{w}_E^E < \underline{\underline{w}}_E$, and we already showed that $\underline{w}_E^E < \underline{w}_1^E$ whenever $\alpha_E < \hat{\alpha}(1)$. ■

Lemma 12 *There exists an n-Blockholder EOS, with $X_{med} = X_1 < \bar{X}$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left[\breve{\alpha}(n)\right], \min\left[\hat{\alpha}_1(n), \frac{1}{2}\right]\right] \tag{32}$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{33}$$

where $1 \leq n \leq M_A$.[23]

Proof. An n-Blockholder EOS with $X_1 < \bar{X}$ exists iff the conditions of Definition 1 are satisfied: (a) $U_1^{nBH} \geq 1$, $U_{l,1} \geq 1$; (b) No investor wants to unilaterally increase or decrease his shares; (c) No investor wants to sell his shares at a price lower than the maximum willingness to pay of excluded investors.

Observe that $X_1 < \bar{X}$ iff $\alpha_1 > \bar{\alpha}$ (Lemma 1). From Corollary 6, this condition is equivalent to $\alpha_E > \breve{\alpha}(n)$, the lower boundary of condition (32) .

(a) The participation constraint of blockholders is satisfied iff

$$U_1^{nBH} = \left(\bar{R}X_1 + f(m)\right)\alpha_1 - \frac{K - w_E}{1 - \alpha_E}\alpha_1 - \frac{\gamma}{2}X_1^2\sigma^2\alpha_1^2 + 1 \geq 1 \tag{34}$$

[23]Observe that $\hat{\alpha}(n) > \frac{\lambda}{1+\lambda} > 0$ for all $n \geq 1$. Moreover when $n \geq 2$, $\hat{\alpha}(n) \geq \frac{1}{2}$. So $\frac{1}{2} = \min\left(\hat{\alpha}(n), \frac{1}{2}\right)$ for $n \geq 2$. Finally the interval of α_E is always positive when $n \geq \frac{2((1+\lambda)\bar{\alpha}-\lambda)}{(1-\lambda)\bar{\alpha}}$ and in such a case $\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right] = \bar{\alpha}$.

Rearranging, this is equivalent to $w_E \geq \underline{w}_E^n(\alpha_E)$, the lower boundary of condition (33). By Lemma 4, liquidity shareholders participation constraint is always satisfied whenever the blockholders' is, hence $U_{l,1} \geq 1$.

(b) Claims 1 and 2 show that given conditions (32) and (33), no blockholder wants to change his shares unilaterally from α_1. Claim 3 shows that no liquidity investor wants to increase or decrease unilaterally his shareholding.

Claim 1: Suppose conditions (32) and (33) are satisfied. No blockholder wants to decrease his shares unilaterally from α_1.

Proof. Observe that $w_E \leq \underline{\underline{w}}_E$ by condition (33). By Lemma 6, a blockholder chooses α_1 as given in equation (8). If any blockholder reduces his shares, given α_{-i}, X_{med} shifts to $X_E < X_1 < \bar{X}$ as $\alpha_E > \alpha_1$. In such a case the highest possible utility a blockholder investor can achieve, is given by being a liquidity shareholder. Hence, it is sufficient to consider the deviation to liquidity shareholding only. Hence, the incentive compatibility constraint is $U_1^{nBH} \geq U_{l,E}$, where $U_{l,E}$ denotes the utility of a liquidity shareholder when $X_{med} = X_E$.

Note that, the utility of the blockholders increases monotonically in w_E, while the liquidity shareholders' utility is convex quadratic in w_E with the minimum equal to 1 at $\underline{w}_E^E$. However, because the firms' shares are issued only if the initial owner raises the capital, the liquidity shareholders' utility is convex quadratic in w_E for $w_E \geq \underline{w}_E^E$ and it is 1 for $w_E < \underline{w}_E^E$. Hence $U_1^{nBH} \geq U_{l,E}$ iff $\underline{w}_E^E > \underline{w}_E^n$ and $w_E \leq \bar{w}_E$ where $\bar{w}_E$ the biggest solutions of $U_1^{nBH} = U_{l,E}$. Note also that when $\underline{w}_E^E > \underline{w}_E^n$, $\bar{w}_E$ is real and is smaller than $\underline{\underline{w}}_E$ (this can be seen substituting $\underline{\underline{w}}_E$ in $U_1^{nBH} - U_{l,E}$ and verifying that this is always positive). Hence the only relevant condition to ensure the incentive compatibility is $\underline{w}_E^E > \underline{w}_E^n$. This condition is always satisfied when $2\alpha_1 \leq \alpha_E$ and substituting for α_1 from expression (8), this becomes:

$$\alpha_E \leq \hat{\alpha}_1(n) \tag{35}$$

This is the first upper bound in condition (32) .

Finally because an investor wants to hold a block only if he is pivotal in the vote outcome, we need to impose that $\alpha_E \leq \frac{1}{2}$. Suppose $\alpha_E > \frac{1}{2}$ then $X_{med} = X_E$ always, so there is no Blockholder EOS. This gives the second upper bound in condition (32). ■

Claim 2: Suppose conditions (32) and (33) are satisfied. No blockholder wants to increase his shares from α_1.

Proof. Since $w_E \leq \underline{\underline{w}}_E$ by condition (33) his utility U_1^{nBH} is decreasing in α_1 so the blockholder holds just enough to be pivotal. ■

Claim 3: Liquidity Shareholders cannot gain from unilateral deviation.

Proof. No (active) liquidity shareholder has an incentive to hold shares bigger than α_1 in order to change $X_{med} < X_1$, as this would reduce their utility (which is increasing in X and decreasing in shareholdings when $\alpha_i > \alpha_l(X_j)$).

If the investors choose any $\alpha_i < \alpha_1$ they do not change the outcome, hence $\alpha_l(X_1)$ maximizes their utility.

(c) No shareholder, either liquidity or blockholder, is willing to sell his shares for a price lower than what he paid to the initial owner. Excluded investors are willing to buy the shares at a higher price from the liquidity shareholders, but not from the blockholders as this would shift the vote outcome to X_E. Hence the EOS does not change. ■ ■

Corollary 3 *There exists an n-Blockholder EOS, with $X_{med} = X_1 = \bar{X}$, for any pair (α_E, w_E), satisfying the following conditions:*

$$\alpha_E \in \left(\max\left[\bar{\alpha}, \tilde{\alpha}(n)\right], \min\left[\check{\alpha}(n), \hat{\alpha}_2(n), \frac{1}{2}\right]\right] \tag{36}$$

$$w_E \in [\underline{w}_E^n(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{37}$$

where $1 \leq n \leq M_A$.

Proof. It follows directly from Lemma 12 and taking into account that when $\tilde{\alpha}(n) < \alpha_E < \check{\alpha}_E(n)$, $\alpha_l(\bar{X}) < \alpha_1 < \bar{\alpha}$. ■

Both Lemma 12 and Corollary 3 do not guarantee uniqueness: it is possible that there is an Initial Owner EOS for the same parameters. However when $n = 1$, we can show that there is no Initial Owner EOS for these parameters: one investor is willing to deviate unilaterally from being a liquidity shareholder and hold a block. Hence in this case the Initial Owner EOS cannot be a Nash equilibrium.

We present this case of a one blockholder equilibrium as Corollary.

Corollary 4 *There exists a* 1*-Blockholder EOS, with* $X_{med} = X_1$*, for any pair* (α_E, w_E)*, satisfying the following conditions:*

$$\alpha_E \in (\max[\bar{\alpha}, \tilde{\alpha}(n)], \hat{\alpha}_1(1)] \tag{38}$$

$$w_E(\alpha_E) \in [\underline{w}_E^1(\alpha_E), \underline{\underline{w}}_E(\alpha_E)) \tag{39}$$

Moreover, there does not exist an Initial Owner EOS for any w_E *when* α_E *is in interval (38).*

Proof. The proof of the first part follows directly from Lemma 12 and Corollary 3, setting $n = 1$ and observing that $\min\left[\hat{\alpha}(1), \frac{1}{2}\right] = \hat{\alpha}(1)$. For the second part, suppose to the contrary that there is an EOS with $X_{med} = X_E$ when α_E is in the interval (38). We know from the proof of Lemma 12, that any active liquidity shareholder has an incentive to switch to becoming a single blockholder when $\alpha_E \leq \hat{\alpha}_1(1)$. Contradiction to the Definition 1 of EOS. ■

Lemma 13 *Let* $n \in [1, M_A - N_A]$*. There exists a Liquidity Shareholder EOS, with* $X_{med} = \bar{X}$ *with* $N_A + n$ *active investors for any pair* (α_E, w_E)*, if:*

$$\alpha_E \in \left(\bar{\alpha}, \min\left(\tilde{\alpha}(n), \frac{1}{2}\right)\right] \tag{40}$$

$$w_E(\alpha_E) \geq \underline{w}_E^{LS}(\alpha_E) \tag{41}$$

Proof. We divide the proof in two parts. We first consider the case $n \geq 1$ and then $n = 0$.

(A) $n \geq 1$

Let $U_{l,\bar{\alpha}}$ denote the value function of a liquidity shareholder when $X = \bar{X}$, and he holds the optimal shareholdings. A liquidity shareholder EOS exists iff the following conditions are satisfied: (1) $U_{l,\bar{\alpha}} \geq 1$; (2) No active investor wants to unilaterally increase or decrease his shares; (3) Passive investors maximize their utility conditional on $X_{med} = \bar{X}$. Moreover all active investors are liquidity shareholders. (4) No investor is willing to sell his shares at any price lower than the maximum that excluded investors are willing to pay.

1. By the proof of Lemma 3, $U_{l,\bar{\alpha}} \geq 1$ iff $w_E \geq \underline{w}_E^{LS}(\alpha_E)$.
2. No active liquidity investor wants to increase or decrease his shareholdings since this is the most preferred point (see Lemma 7), as long as $\alpha_E \leq \tilde{\alpha}_E(n)$, since this condition guarantees that $X_{med} = \bar{X}$.
3. Passive investors hold $\alpha_l(\bar{X})$ which maximizes their utility.
4. No investor is willing to sell his shares to any excluded investors as the maximum price at which excluded investors are willing to buy the shares is the minimum price at which the liquidity shareholders are willing to sell.

(B) $n = 0$

In such a case the interval (40) becomes $\alpha_E \in \left(\bar{\alpha}, \frac{\lambda}{1+\lambda}\right]$. In such a case there are sufficiently many active liquidity shareholders that even with liquidity shares, $\alpha_l(\bar{X})$ they can get their most preferred point and $X_{med} = \bar{X}$. The participation constraint of the liquidity shareholders is satisfied when $w_E \geq \underline{w}_E^{LS}(\alpha_E)$. No active liquidity shareholders find it profitable to hold a suboptimal portfolio to switch the decision to $X < \bar{X}$, as this is their first best. Note that no (passive) investors are willing to sell their shares to the excluded investors. The excluded investors cannot improve the vote outcome and hence the maximum price they are willing to buy is equal to the minimum price the shareholders are willing to sell. ■

Corollary 5 *There exists a No Conflicts EOS, with $X_{med} = X_E = \bar{X}$ iff $\alpha_E \in (0, \bar{\alpha}]$, $w_E \geq \underline{w}_E^{LS}(\alpha_E)$.*

Proof. When $\alpha_E \in (0, \bar{\alpha}]$ there are no conflicts of interests between outside investors and the initial owner, $X_{med} = X_E = \bar{X}$. Active investors hold $\alpha_l(\bar{X})$, and are at their most preferred X. Hence they have no incentive to deviate. The participation constraint of the liquidity shareholders is satisfied when $w_E \geq \underline{w}_E^{LS}$. To prove necessity note that if $\alpha_E > \bar{\alpha}$, $X_E < \bar{X}$ hence there are conflicts between investors. If $w_E < \underline{w}_E^{LS}$, no investors would buy the shares hence it is not an EOS. Finally note no investor is willing to sell shares at a price lower than the maximum that an excluded investor will pay. ■

We are now ready to show the Lemma. First note that in any EOS the utility of the initial owner is decreasing in w_E. Further, from the Lemmas above, all the possible EOS are characterized by outside investors who are either liquidity shareholders or blockholders.

Consider the case where no blockholders exist. Suppose to the contrary, that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$, $\alpha_l(X_j) > 0$ and $U_{l,j} > 1$. By assumption there are sufficiently many investors in the market, so there always exist passive shareholders who have a strictly positive demand for shares for any $0 < X_j \leq \bar{X}$. Hence, the initial owner can increase his utility by decreasing w_E, for any α_E and still ensure that there is a (smaller) positive demand by passive investors, ensuring full subscription. As the demand of the liquidity shareholders is given by equation (7), the initial owner will do this until $\frac{K-w_E}{1-\alpha_E} > K$. Contradiction.

Consider the case where blockholders and liquidity shareholders exist in equilibrium. In such a case either blockholders or liquidity shareholders will have the most binding constraint. We already showed above that when the liquidity investors participation is more binding then $\frac{K-w_E}{1-\alpha_E} > K$. So it is sufficient to show that this is true when the binding constraint is that of blockholders.

When the blockholders's participation constraint is more binding given $m = 1$ the value function for blockholders given $X_{med} = X_j$ is given by :

$$U_1^{nBH} = \bar{R}X_j\alpha_1 + \left(K - \frac{K-w_E}{1-\alpha_E}\right)\alpha_1 - \frac{\gamma}{2}X_j^2\sigma^2\alpha_1^2 + 1 \tag{42}$$

By the same logic as for the first part, suppose that there is an equilibrium with $\frac{K-w_E}{1-\alpha_E} < K$. Because $\alpha_1 \leq \max[\alpha_j, \bar{\alpha}]$ when $\frac{K-w_E}{1-\alpha_E} < K$, then the participation constraint of the blockholders is satisfied with strict inequality, i.e. $U_1^{nBH} > 1$. The initial owner can decrease w_E and still satisfy the constraints and ensure full subscription. Contradiction to the fact that it is an equilibrium. ■

A.3.2 Proposition 2:

Proof. Before we prove the next proposition, we need few lemmas which provide expressions for the value function of the initial owner under the alternative ownership structures that could be obtained in a subgame perfect equilibrium.

Lemma 14 *Suppose the conditions for the No Conflicts EOS (Corollary 5) are satisfied and the equilibrium of the game is the No Conflicts equilibrium, then the initial owner sets* $\alpha_E = \bar{m}$*,* $X_{med} = X_E = \bar{X}$*,* $w_E = \underline{w}_E^{LS}$ *and the value function of the Initial Owner is given by:*

$$V_E^{NC} = \bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\bar{m}^2\sigma^2 - \bar{m}K - \epsilon \tag{43}$$

Proof. By Lemma 2, in any monitoring equilibrium, $\alpha_E \geq \bar{m}$. By Corollary 5, the No Conflicts EOS with $X_{med} = X_E = \bar{X}$ exists if $\alpha_E \in (0, \bar{\alpha}]$ and $w_E \geq \underline{w}_E^{LS}(\alpha_E)$. Therefore the maximization problem of the Initial Owner in the No Conflicts equilibrium is:

$$\max_{\alpha_E, w_E} U_E = (\bar{R}\bar{X} + K)\alpha_E - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 + 1 - w_E - \bar{m}K \tag{44}$$

$$\text{s.t} \quad w_E \geq \underline{w}_E^{LS}(\alpha_E) \tag{45}$$

$$\alpha_E \in [\bar{m}, \bar{\alpha}] \tag{46}$$

The initial owner's utility is decreasing in the wealth invested, w_E. Hence he chooses w_E such that it satisfies the participation constraint of the liquidity investors, (45), at equality. Hence $w_E = \underline{w}_E^{LS}$ where $\underline{w}_E^{LS}$ is given by equation (11). Inserting it in the initial owner's objective function we obtain:

$$\bar{R}\bar{X} + 1 - \frac{\gamma}{2}\bar{X}^2\alpha_E^2\sigma^2 - \bar{m}K - \epsilon \tag{47}$$

This expression is decreasing in α_E. Hence the initial owner will retain just enough shares to satisfy the monitoring constraint with equality: $\alpha_E = \bar{m}$. Inserting $\alpha_E = \bar{m}$ in the initial owners utility function, we have expression (43). ■

Let $\underline{b} = \left(\max\left[\bar{\alpha}, \min\left[\frac{1}{2}, \max\left[\hat{\alpha}(1), \tilde{\alpha}\right]\right]\right]\right) + \eta_E$. Remember that η_E is the fraction correspondent to one share when the vote outcome is X_E.

Lemma 15 *Suppose the conditions of the Initial Owner EOS are satisfied (Lemma 11), and the equilibrium of the game is an Initial Owner equilibrium, then* $X_{med} = X_E < \bar{X}$*,* $w_E = \underline{w}_E^E$*,* $\alpha_E = \max[\bar{m}, \bar{b}]$*, and the value function of the Initial Owner is given by:*

$$V_E^E = \bar{R}X_E + 1 - \frac{\gamma}{2}X_E^2\max[\bar{m}, \bar{b}]^2\sigma^2 - \bar{m}K - \epsilon = \frac{\bar{R}^2}{\gamma\sigma^2}\left(\frac{1}{\max[\bar{m}, \bar{b}]} - \frac{1}{2}\right) + 1 - \bar{m}K - \epsilon \tag{48}$$

Proof. The proof follows the same steps as for the proof of Lemma 14 and it applies Lemmas 2 and 11. Detailed proof is available upon request. ■

Let $\underline{c} = \max[\bar{\alpha}, \tilde{\alpha}_E(n)] + \bar{\eta}$ where $\bar{\eta}$ is the fraction correspondent to one share when the vote outcome is $\bar{X}$.

Lemma 16 *Suppose the conditions of the n-Blockholder EOS are satisfied (Lemma 12 and Corollary 3) and the equilibrium of the game is an n Blockholder equilibrium with* $X_{med} = X_1$*, the initial owner sets* $\alpha_E = \max[\underline{c}, \bar{m}]$*,* $w_E = \underline{w}_E^n$ *and the value function of the Initial Owner is given by:*

$$V_E^n = \bar{R}X_1 + 1 - \bar{m}K - \frac{\gamma}{2}X_1^2\sigma^2(\max[\underline{c}, \bar{m}]^2 + \alpha_1 - \alpha_1\max[\underline{c}, \bar{m}]) \tag{49}$$

Proof. The proof follows the same steps as for the proof of Lemma 14 and it applies Lemmas 2 and 12. Detailed proof is available upon request. ■

Lemma 17 *Suppose the conditions for the Liquidity Shareholder EOS are satisfied and there exists a Liquidity Shareholder equilibrium with monitoring (Lemma 13). Then,* $\alpha_E = \max(\bar{\alpha}+\bar{\eta}, \bar{m})$*, there are at least* $N_A + n$ *active investors,* $w_E = \underline{w}_E^{LS}$ *and the value function of the Initial Owner is given by:*

$$V_E^{LS} \equiv \bar{R}\bar{X} + 1 - \bar{m}K - \frac{\gamma}{2}\bar{X}^2\sigma^2 \max[\bar{m}, \bar{\alpha}+\bar{\eta}]^2 - \epsilon \tag{50}$$

Proof. The proof follows the same steps as for the proof of Lemma 14 and it applies Lemmas 2 and 13. Detailed proof is available upon request. ■

Finally define $\bar{m}_{NC}^{RC}$ and $\bar{m}_{NC}^{S}$ the values such that when $\bar{m}$ is smaller the initial owner prefers monitoring rather than selling or not raising capital. Hence:

$$\bar{m}_{NC}^{RC} \equiv \frac{\sqrt{K\left(K + 2\bar{X}^2\gamma\sigma^2\right)} - K}{\bar{X}^2\gamma\sigma^2} \tag{51}$$

$$\bar{m}_{NC}^{S} \equiv \frac{\sqrt{2\bar{R}\gamma\sigma^2\bar{X}^3 + K^2} - K}{\bar{X}^2\gamma\sigma^2} \tag{52}$$

We are now ready to prove the proposition. We solve the game by backward induction. The initial owner chooses α_E and w_E, anticipating the ownership structure. His problem can be broken into the following: (1) $\alpha_E \in [\bar{m}, \bar{\alpha}]$, (2) $\alpha_E \in (\bar{\alpha}, 1]$, (3) $\alpha_E \in [0, \bar{m}]$.

We first describe the beliefs on the EOS and the corresponding value functions in each interval.

Case (1). By Corollary 5 there exists a No Conflicts EOS. Hence in this interval the beliefs of all players on the EOS are the No Conflicts EOS when $w_E \geq \underline{w}_E^{LS}$. As $w_E \leq \underline{w}_E^{LS} < \underline{w}_E^{NT}$ for any possible X there exists a No Trade EOS by Lemma 10, and we assume that the belief is on the No Trade EOS. The initial owner's value function is given by equation (43) when $w_E \geq \underline{w}_E^{LS}$ and by the no trade value function, $V_E^{NT} = 1$ in case $w_E < \underline{w}_E^{LS}$.

Case (2). By Lemmas 11, 12 and 13 the possible EOS in this interval are the Initial Owner, the n Blockholder or the Liquidity Shareholder ones if $w_E \geq \underline{w}_E^{j}$ where $j = \{IO, n, LS\}$. Again if $w_E < \underline{w}_E^{j}$ the No Trade EOS exists and $V_E^{NT} = 1$. If an Initial Owner EOS exists the initial owner sets $\alpha_E = \max[\bar{m}, \underline{b}] = \underline{b}$ as $\bar{m} \leq \bar{\alpha}$ and his value function, V_E^{E} is given by equation (48). If an n Blockholder EOS exists, Lemma 16 shows that the initial owner's utility is decreasing in α_E. Observe that for an n Blockholder equilibrium to exist $\tilde{\alpha}(n) > \bar{\alpha}$ otherwise the initial owner would choose $\alpha_E = \bar{\alpha}$. Hence $\underline{c} = \tilde{\alpha}(n)$ and the value function V_E^{n} is given by (49). If a Liquidity Shareholders EOS arises, Lemma 17 shows that $\alpha_E = \max[\bar{\alpha}+\bar{\eta}, \bar{m}] = \bar{\alpha}+\bar{\eta}$. Hence the value function V_E^{LS} is given by equation (50).

Case (3). In this interval the Liquidity Shareholders EOS exists as we showed in Proposition 1, hence we assume that the belief is that if $w_E \geq \underline{w}_E^{LS}$ the Liquidity Shareholders EOS emerges. Lemma 2 shows that the initial owner chooses not to monitor in this interval, and by Proposition 1 $\alpha_E = 0$ and the initial owner's value function is $V_E^{NM} = \max\left[1, \bar{R}\bar{X} - K + 1\right]$. If $w_E < \underline{w}_E^{LS}$ then the belief is on the No Trade EOS (Lemma 10) and the corresponding value function is $V_E^{NT} = 1$.

The initial owner will choose α_E to maximize his value function across the intervals (1)–(3) above.

First consider Case (2), ignoring V_E^{NT} for the moment: It is easy to see from (48) that $V_E^{E}|_{\alpha_E=\underline{b}} < V_E^{E}|_{\alpha_E=\bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a No Conflicts equilibrium than in an Initial Owner equilibrium. If an n-Blockholder EOS exists, it is easy to see from equation (49) that $V_E^{n}|_{\alpha_1<\bar{m}} < V_E^{n}|_{\alpha_1=\bar{m}}$. $V_E^{n}|_{\alpha_1=\bar{m}} < V_E^{NC}$ iff

$$\frac{1}{\alpha_1}\left(1 + \alpha_E - \frac{\alpha_E^2}{\alpha_1}\right) < \frac{2}{\bar{\alpha}} - \frac{\bar{m}^2}{\bar{\alpha}^2}$$

which is always true as $\bar{m} \leq \bar{\alpha} \leq \alpha_E$. Hence the initial owner is better off in a No Conflicts equilibrium than in an n-Blockholder equilibrium. If a Liquidity Shareholder EOS exists then by Lemma 17 $V_E^{LS}|_{\alpha_E > \bar{\alpha}} < V_E^E|_{\alpha_E = \bar{m}} = V_E^{NC}$. Hence the initial owner is better off in a No Conflicts equilibrium than in an Liquidity Shareholders equilibrium.

Now consider Case (3): The initial owner's value function is $V_E^{NM} = \max\left(1, \bar{R}\bar{X} - K + 1\right)$. Hence he prefers to monitor iff $V_E^{NC} \geq V_E^{NM}$, i.e. iff $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$ and $V_E^{NC} \geq 1$.

This first condition is satisfied when $\bar{m} \in [c, \bar{m}_{NC}^S]$, where $c < 0$. Hence $V_E^{NC} \geq \bar{R}\bar{X} - K + 1$, whenever $\bar{m} < \bar{m}_{NC}^S$.[24] When the above condition is not satisfied the initial owner sells out the firm. The second condition, $V_E^{NC} \geq 1$, is satisfied when $\bar{m} \in [d, \bar{m}_{NC}^{RC}]$, where $d < 0$. Hence, if $\bar{m} > \bar{m}_{NC}^{RC}$ then the initial owner does not raise capital.

Finally, consider the No Trade equilibrium. Clearly this gives the same value to the initial owner as not raising capital, so under the conditions of the proposition, the No Conflicts equilibrium is preferred by the initial owner. ■

A.3.3 Proposition 3:

Proof. The proof follows the same steps as the proof of Proposition 2. Let

$$\bar{m}_E^{RC} \equiv \frac{1}{2}\left(1 - \frac{\bar{R}\bar{X}}{K}\right) - \frac{\bar{R}^2}{4K\gamma\sigma^2} + \frac{\sqrt{16K\bar{R}^2\gamma\sigma^2 + \left(\bar{R}^2 + 2\gamma\sigma^2\left(\bar{R}\bar{X} - K\right)\right)^2}}{4K\gamma\sigma^2} \tag{53}$$

$$\bar{m}_E^S \equiv -\frac{\bar{R}^2}{4K\gamma\sigma^2} + \frac{\bar{R}\sqrt{\bar{R}^2 + 16K\gamma\sigma^2}}{4K\gamma\sigma^2} \tag{54}$$

We break up the maximization problem of the initial owner into the following cases: (1) $\alpha_E \in [\max[\underline{b} + \eta_E, \bar{m}], 1]$, (2) $\alpha_E \in [0, \bar{m}]$. We first describe the beliefs on the EOS and the corresponding value functions in each interval.

Case (1). By Lemma 2 the initial owner monitors. By Lemma 11 an Initial Owner EOS exists in this interval of α_E as long as w_E satisfies condition (31) and we assume that the anticipated EOS is the Initial Owner EOS for $w_E \geq \underline{w}_E^E$. Lemma 15 implies then that: $\alpha_E = \bar{m}$, $w_E = \underline{w}_E^E$ and the initial owner's value function, V_E^E, is given by equation (48). Otherwise when $w_E < \underline{w}_E^E$ the No Trade EOS (with value function V_E^{NT}) is anticipated.

Case (2). This case is the same as in Proposition 2, Case (3). The initial owner's value function is $V_E^{NM} = \max[\bar{R}\bar{X} - K + 1, 1]$.

Maximizing across intervals of Cases (1) and (2) the initial owner will choose $\alpha_E = \bar{m}$ as long as $V_E^E \geq \max(V_E^{NM}, V_E^{NT}) = V_E^{NM}$. This occurs when $\bar{m} \leq \min\left[\bar{m}_E^{RC}, \bar{m}_E^S, 1\right]$.

Note that when $\bar{m} > \left[\frac{1}{2}, \bar{\alpha}\right]$ this is the unique equilibrium (under the conditions of the proposition), induced by the uniqueness of the Initial Owner EOS. ■

A.3.4 Proposition 4:

Proof. Following the same steps as in the proof of Proposition 2, we break up the maximization problem into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \min\left(\tilde{\alpha}_E(n), \frac{1}{2}\right)\right)$; 2) $\alpha_E \in \left[\tilde{\alpha}_E(n), \min\left[\hat{\alpha}(n), \frac{1}{2}\right], 1\right]$; (3) $\alpha_E > \min\left[\hat{\alpha}(n), \frac{1}{2}\right]$; (4) $\alpha_E \in [0, \bar{m})$. As before, we first describe the beliefs on the EOS in each interval and the corresponding value functions.

Case (1). By Lemma 2, $m = 1$. Then all investors anticipate monitoring in the last stage. We will assume the following beliefs about the EOS in date 1: if $w_E \geq \underline{w}_E^{LS}$ then the anticipated EOS is the Liquidity Shareholder EOS which exists by Lemma 13. If $w_E < \underline{w}_E^{LS}$ the EOS is the No Trade EOS with value function V_E^{NT} (Lemma 10). By

[24]Note that as ϵ is a very small number we just consider a strict inequality.

Lemma 17, if a Liquidity Shareholder equilibrium exists the initial owner's value function, V_E^{LS}, is given by equation (50).

Case (2). If this interval is non-empty, and $w_E \geq \underline{w}_E^n$ there exists an n-Blockholder EOS by Lemma 16 and Corollary 3. The proof of Lemma 16 shows that the initial owner's minimizes α_E and the initial owner's utility function is continuous between these two intervals of α_E and is given by expression (49). Hence he prefers to minimizes α_E, i.e. $\alpha_E = \tilde{\alpha}(n)$. The value function is therefore given by V_E^n, expression (49) with $\underline{c} = \tilde{\alpha}(n) + \bar{\eta}$. If $w_E < \underline{w}_E^n$ then the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.[25]

Case (3). In this case the unique EOS is the Initial Owner EOS for $w_E \geq \underline{w}_E^E$. Using the proof of Lemma 15 we know that the initial owner minimizes α_E, i.e. $\alpha_E = \underline{d} \equiv \min[\hat{\alpha}(n), \frac{1}{2}]$ and the value function is given by V_E^E. If $w_E < \underline{w}_E^E$ the belief on the EOS is the No Trade EOS, with value function V_E^{NT}.

Case (4). This interval is the same as in Proposition 2, Case 3. By Proposition 1, the unique equilibrium is the no monitoring equilibrium and the value function is given by V_E^{NM} (expression (12)).

We now show that the initial owner chooses $\alpha_E = \bar{m}$, i.e. a Liquidity Shareholder EOS. This is true whenever $V_E^{LS} \geq \max(V_E^E, V_E^{NM}, V_E^n, V_E^{NT})$. Because the initial owner's value function is decreasing in α_E, $V_E^E|_{\alpha_E = \underline{d}} < V_E^E|_{\alpha_E = \bar{\alpha}} = V_E^{NC}$. Hence the liquidity shareholder ownership structure of Case (1) is preferred over the initial owner one. Second, as in the proof of Proposition 2, $V_E^n|_{\alpha_E = \tilde{\alpha}(n)} < V_E^n|_{\alpha_E = \bar{m}} < V_E^{NC}$. Hence the liquidity shareholder ownership structure of Case (1) is preferred over the n Blockholder ownership structure of Case (2). Moreover $V_E^{NM} \geq V_E^{NT}$. Hence we only need to check that $V_E^{LS} \geq V_E^{NM}$ and this is true iff $\bar{m} \leq \min\left(\bar{m}_{NC}^{RC}, \bar{m}_{NC}^S\right)$. ■

A.3.5 Proposition 5:

Proof. Define as $\bar{m}_{1,n}^{RC}(n)$ and $\bar{m}_{2,n}^{RC}(n)$ the first two biggest solutions of the equation $V_E^n = 1$ and $\bar{m}_{1,n}^S(n)$ and $\bar{m}_{2,n}^S(n)$ the two biggest solutions of the equation $V_E^n = \bar{R}\bar{X} + 1 - K$. Let:

$$\bar{m}_1^E(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 - \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda + 1}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{55}$$

$$\bar{m}_2^E(n) \equiv \frac{\lambda(1+\lambda) + n(1-\lambda)\left(1 + \sqrt{8(3n-1)\lambda^2 + (4-24n)\lambda + 1}\right)}{11n^2(1-\lambda)^2 + (n(1-\lambda) - (1+\lambda))^2} \tag{56}$$

$\bar{m}_1^E(n)$ and $\bar{m}_2^E(n)$ are the two monitoring costs for which the initial owner is indifferent between being in a n Blockholder equilibrium or in an Initial Owner equilibrium holding the majority of the shares, i.e. $V_E^n = V_E^E\left(\alpha_E = \frac{1}{2}\right)$. For monitoring cost values within this interval $[\bar{m}_1^E(n), \bar{m}_2^E(n)]$ the initial owner prefers to be in an n Blockholder equilibrium.

Following the steps of the proof of Proposition 2 above, we break up the maximization problem of the initial owner into the following intervals of α_E : (1) $\alpha_E \in \left[\bar{m}, \frac{1}{2}\right]$; (2) $\alpha_E \in (\frac{1}{2}, 1]$; (3) $\alpha_E \in [0, \bar{m}]$. We first describe the beliefs on the EOS and the corresponding value functions in each of these intervals.

Case (1). By Lemma 2, $m = 1$. Then all investors anticipate monitoring in the last stage. We assume the following beliefs about the EOS at date 1: if $w_E \geq \underline{w}_E^n$ then the anticipated EOS is the n Blockholder equilibrium which exists by Lemma 12. By Lemma 16 in such a case the initial owner's value function, V_E^n is given by equation (49). If $w_E < \underline{w}_E^n$ then the EOS is the No Trade EOS with corresponding value function V_E^{NT}.

Case (2). By Lemma 2, $m = 1$. In this interval whenever $w_E \geq \underline{w}_E^E$ there exists an Initial Owner EOS. By Lemma

[25] In this interval there can be also an Initial Owner EOS if $n > 1$ and $w_E \geq w_E^E$. In such a case the proof that shows that the initial owner prefers the Liquidity Shareholder EOS follow the same steps as Case (3).

15, he minimizes α_E, i.e. $\alpha_E = \frac{1}{2}$ and his value function becomes:

$$V_E^E = \frac{3}{2}\frac{\bar{R}^2}{\gamma\sigma^2} + 1 - \bar{m}K - \epsilon \tag{57}$$

Case (3). This is the same as Proposition 2, Case 3 and generates a value of V_E^{NM}.

Now we show that the conditions under which the initial owner chooses $\alpha_E = \bar{m}$, i.e. Case (1).

We first check that $V_E^n \geq V_E^E \left(\alpha_E = \frac{1}{2}\right)$. This occurs when:

$$\frac{1}{\alpha_1}\left(1 - \frac{\bar{m}^2}{\alpha_1} + \bar{m}\right) \geq 3$$

This condition is satisfied iff:

$$\frac{1 + \bar{m} - \sqrt{1 + 2\bar{m} - 11\bar{m}^2}}{6} \leq \alpha_1 \leq \frac{1 + \bar{m} + \sqrt{1 + 2\bar{m} - 11\bar{m}^2}}{6} \tag{58}$$

Substituting α_1 we obtain that $V_n^E \geq V_E^E$ iff $\bar{m} \in [\bar{m}_{1,n}^E, \bar{m}_{2,n}^E]$. Note also that in order to guarantee real values of condition (58) $\bar{m}$ has to be below 40%. This means that in order to guarantee an n-Blockholder equilibrium $\bar{m} < 40\%$.

Second we check that $V_n^E \geq V_{NM}^E$:

(i) $V_n^E \geq 1$ iff:

$$n(1-\lambda)\lambda\bar{R}^2 + \bar{m}\left(\bar{m}n^2(1-\lambda)^2\bar{R}^2 - n(1-\lambda)(\lambda\bar{m} + \bar{m} + 1)\bar{R}^2 + 2K\gamma(\lambda\bar{m} + \bar{m} - \lambda)^2\sigma^2\right) < 0 \tag{59}$$

The left hand side is a third degree inequality which goes from $-\infty$ to ∞, and it is positive at $\bar{m} = \frac{\lambda}{1+\lambda}$. Note also that when $\bar{m} = \frac{1}{2}$, the left hand side can be either positive or negative. Hence of the 3 potential roots for which the left hand side is equal to 0, we are interested for the two biggest ones which are defined as $\bar{m}_{1,n}^{RC}$ and $\bar{m}_{2,n}^{RC}$ and the negative values are between these two values, that is $\bar{m}_{1,n}^{RC} < \bar{m} < \bar{m}_{2,n}^{RC}$.

(ii) $V_n^E \geq \bar{R}\bar{X} - K + 1$ iff:

$$\begin{aligned} &2\bar{\alpha}K\gamma(1+\lambda)^2\sigma^2\bar{m}^3 + \left(\bar{R}^2\left(2(1+\lambda)^2 - \bar{\alpha}n(1-\lambda)(\lambda+1)\right) - n(1-\lambda) - 2\bar{\alpha}K\gamma\left(3\lambda^2 + 4\lambda + 1\right)\sigma^2\right)\bar{m}^2 \\ &+ \left(\bar{\alpha}\left(2K\gamma\lambda(3\lambda+2)\sigma^2 - n\bar{R}^2(1-\lambda)\right) - 4\bar{R}^2\lambda(1+\lambda)\right)\bar{m} + \lambda\left(2\lambda\bar{R}^2 + \bar{\alpha}\left(n\bar{R}^2(1-\lambda) - 2K\gamma\lambda\sigma^2\right)\right) < 0 \end{aligned} \tag{60}$$

The left hand side has the same features of the left hand side of condition (59). Hence this condition is satisfied when $\bar{m}_{1,n}^S < \bar{m} < \bar{m}_{2,n}^S$ where $\bar{m}_1^S$ and $\bar{m}_2^S$ are the biggest solutions of the left hand side set equal to zero. ■

A.3.6 Corollary 1:

Proof. It follows directly from Propositions 3 and 5. ■

A.3.7 Corollary 2:

Proof. This is special case of Proposition 5. Note that in such a case no Initial Owner equilibrium can arise as one active investor is always willing to unilaterally deviate and hold a block and become pivotal. ■

A.3.8 Proposition 6:

Proof. The initial owner's value function has a maximum for $n = n^*$. As $\bar{m} > \frac{\lambda}{1+\lambda}$, $n > 0$. ■

A.3.9 Proposition 7

Proof. When the initial owner can raise an amount of capital $I \geq K$ and invest the remaining amount in the risk free asset his objective function becomes:

$$\alpha_E(X\bar{R} + K + I - K) - \frac{\gamma}{2}\alpha_E^2 X^2 \sigma^2 - w_E - \bar{m}$$

The objective function of the investors is instead:

$$(1 - \alpha_i \frac{I - w_E}{\alpha_I}) + \alpha_i \left[X\bar{R} + K + I - K\right] - \frac{\gamma}{2}\alpha_i^2 X^2 \sigma^2$$

Repeating the same steps of Propositions 2, 3, 4, 5, we obtain the optimal w_E. Inserting it in the initial owner objective function, we obtain that the initial owner's objective function is decreasing in I. ■