Firms’ Strategies for Reducing the Effectiveness of Consumer Price Search*

by

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Abstract

A simple model of competition with imperfect consumer information has firms setting prices using mixed strategies, and equilibrium average prices become higher as price comparisons by consumers become more difficult. For example, buyers may be comparing prices originating from the same supplier: either one firm setting multiple prices or a group of colluding firms. The resulting greater number of captive consumers implies added monopoly power. Results are given showing the shift in welfare from consumers to producers, both with exogenous and endogenous consumer behaviour. However consumers might search more or less with multiple prices. The implications of the results for competition policy and recent judgements are considered.

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I. Introduction

This paper argues that one strategy that firms can adopt to increase their profits is to prevent their customers from making price comparisons and hence from making the market competitive.¹ To proceed we have to construct a very simple model of consumer search and identify equilibrium price distributions set by suppliers. How far competition succeeds in driving prices down towards costs clearly depends on how far consumers are willing to search out and compare prices, and on how reliable and effective is such search. Our principal focus will be on how firms can reduce this effectiveness by setting more than one apparently independent price.

A number of classic papers have emphasised search costs and the breakdown of the "law of one price". A common feature of models used in Pratt, Wise and Zeckhauser (1979) and Salop and Stiglitz (1977) (and many others ²) is that buyers have different search costs. Then sellers can specialise in selling to those with low search costs who have become informed buyers or to high search cost, uninformed, buyers, with a “volume against margin” trade-off. Our model here is most closely related to that of Wilde and Schwartz (1979) and Burdett and Judd (1983). In Wilde and Schwartz (1979), the consumer has decided how many points of sale to visit and hence how many prices to observe. Some consumers only visit one store; others visit more than one. Thus the “informed” buyers are replaced by a class of buyers who

¹ Other processes such as adopting non-linear pricing, product bundling, or more general product design to differentiate from competitors are too well known to warrant additional treatment here. Our emphasis here is on strategies which lead consumers to believe that price comparisons have taken place when these are in fact illusory.

² See for example Gabszewicz and Garella (1982) in terms of product differentiation, and Arnold (2000) who finds price dispersion because the cheapest sources may suffer stock-outs and hence waste buyers’ valuable time. The latter paper does not require differences between buyers or sellers to obtain price dispersion. Ireland (1993) uses a similar price comparison model to that applied in the current paper but adds a first stage where suppliers send price information to consumers as advertising messages. Firms have incentives to restrict such information flows.
search out prices to compare. The result is a mixed strategy price equilibrium as the mixed strategy Nash equilibrium (MSNE) in the suppliers’ game. In Burdett and Judd (1983) the consumer decision of the quantity of search is also modelled. Here the buyers are all homogeneous and equilibria are proposed where buyers are indifferent between different quantities of search, and where a particular division of buyers into different behaviours can define an equilibrium. Alternatively, Burdett and Judd (1983) also consider a case where search productivity is noisy, so that one search may yield information on one or more prices. Thus in either case mixed behaviour (a random choice of action from a probability distribution) can occur on both sides of the market even when all sellers and all buyers are homogeneous.

There is robust evidence of large price dispersion, even from very homogeneous products. Pratt, Wise and Zeckhauser (1979) include convincing data to the effect that there exist wide disparities in prices quoted for virtually identical products by retail suppliers. The arrival of the internet and seemingly lower search costs has not removed price dispersion: see various papers on internet prices (mostly of books), for example Smith and Brynjolfsson (2001). A further example is given in Table 1. This example has been chosen specifically to motivate our analysis. It has been obtained by using Pricerunner.com, to find prices on 13th February 2002 of all UK internet sites selling one or more of the 10 most popular digital cameras (as indicated by search engine enquiries). Thus 51 internet suppliers were selling at least one of the models on 13th February, and there were 218 observations over all 10 cameras. Delivery costs (ranging from zero to about £8) are ignored since they reflect different qualities of delivery, ranging from next day by courier to the slowest parcel post. The websites are listed

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3 Acemoglu and Shimer (2000) extend this approach to a labour market setting. Burdett and Coles (1997) consider the case where noisy search exists in a dynamic context so that sellers have the option to build up their customer base by selling at lower prices.
in terms of the average rank of the prices for those of the 10 camera models shown as for
sale. The exercise was repeated a week later and differences noted. Briefly the data show:

**Wide coverage of models:** between 16 and 30 prices for each camera model

**Dispersion of prices** for each model measured by \((\text{Max} - \text{min}) / \text{min price}\) of between 23 and 64 per cent

**No clear pattern in "best buys"** - so that a consumer cannot model where to go for the lowest
prices. For example, there is almost zero correlation (-.209) between "number of prices" from
a website, and average rank of prices of cameras advertised.

**Reporting fluctuations of sites:** 2 sites were not reported the following week; 1 new site
appeared

**Price changes:** 23 prices changed between 13\(^{th}\) and 19\(^{th}\) February 2002. Also, 6 new prices
appeared from existing stores, and 9 disappeared.

The selection of digital cameras for this example was prompted by (i) the low ratio of
delivery cost to price (up to only 2%); (ii) the wide availability of the leading brands (for
comparison, 391 digital camera models in total were advertised on these websites but over
220 of these were each advertised by only 1 site: limiting ourselves to leading brands meant
that we obtained a reasonably large number of prices for each model). Also digital camera
models are well-identified by the manufacturer's model code. The picture that arises is of
both a high degree of price dispersion and of prices changing frequently (over 10 % in one
week). Hence there is considerable gain to be had from shopping around, and comparing
prices. On the other hand, there must be some sales made by high-priced sellers since
otherwise there is no explanation as to why they would advertise such prices at all.

(Table 1 about here)
Faced with such evidence of the failure of the "law of one price", the strategies that firms can undertake to make price comparisons less effective become of real interest. This topic has considerable importance for monopoly policy. Wilde and Schwartz (1979, p551, italics in original) suggest that “the state should reduce the costs to consumers of comparing purchase alternatives”. The policy issue that we wish to address is whether the state should indeed consider this objective more explicitly in relation to competition policy. However, little active stance on the issue appears to have been taken, although recent work suggests that this is changing.4

This paper is presented in terms of a number of sellers (“firms”) offering a homogeneous good to the market. Hence it is simplest to think of a particular brand being sold by retailer firms. However, it could also represent a number of producers each making and selling very similar products, or indeed incorporate product differentiation, if all consumers could adjust prices they observe for the different product characteristics. The key firm strategy that we consider is the sending of multiple price signals to the market. For example the firm could operate multiple retail outlets. If these outlets have the same name then consumers can avoid making price comparisons from the same firm; if the outlets have different names consumers are in danger of making spurious comparisons. Similarly a firm can market the same product under different brand names and again consumers can be making comparisons between products that provide profit for the same firm. Within the UK, there is no legislation or restrictions on groups of firms owned by a parent company trading in similar areas of activity under more than one name. From the perspective of consumer search, the presence of firms supplying through multiple trading names will be found to be very similar to the existence of

4 NERA (2001) discuss many of the difficulties of defining markets for monopoly and market dominance investigations. Hunter et al (2001) discuss the measurement of consumer surplus loss from price dispersion. Salop (2000) stresses that market shares and dominance are only means to an end, and that first principles of competition analysis should be given priority.
price fixing agreements among a subset of suppliers, a case where legislation clearly does exist. Complementary strategies can affect the set of prices available to consumers to find. For example, vertical constraints can prevent high-profile stores from stocking a product and exclusive dealing contracts and own brands can inhibit easy price comparisons across independent stores. Within the USA, there is no presumption that vertical restraints or multiple trading names is anti-competitive: rather, vertical constraints other than on price (such as retail price maintenance), are judged by a “rule of reason (which) requires an investigation into the challenged conduct” (Carlton and Perloff, 2000, p645).

Section II below presents the base model. It presents an equilibrium for a market with firms each posting an independent price. Section III considers two cases where firms set prices jointly. In the first, one “joint” firm makes two price signals while all other firms set a single price. In the second case firms each post two price signals. In this section, search behaviour is assumed exogenous. The equilibria are compared and the effect of the industry structure changing and moving the equilibrium from one form to the other is studied. In section IV, search behaviour is endogenised and we consider the effect of a change in the form of price equilibrium on the private return from a consumer’s search. We also offer a simple framework for the determination of the long-run number of firms or prices by means of entry costs or fixed costs. All our comparisons of equilibria permit welfare statements relating to consumers’ utility and to firms’ and industry profit. Finally, section V considers policy implications, particularly for competition policy, and draws conclusions. It also briefly considers the impact of other strategies that firms might take, such as own brands and exclusive dealerships, within the framework of the model.
II. The Model

II(i) Market Assumptions

Consider a market for a homogeneous product, let demand by each consumer be either one unit or no unit, and let the measure of consumers be 1 (e.g., 1 million consumers). Each consumer will choose to buy from the firm which offers to supply at the lowest price from the set of firms (suppliers) that is observed by that consumer, provided this price is no more than her reservation price of 1. Buyers are initially ignorant of the actual prices charged by different firms. They need to incur some cost to find out a randomly selected price. We can think of a consumer obtaining a list of suppliers and randomly selecting suppliers to visit and find their prices. Since all buyers are assumed to find at least one price, it is only the cost of finding further prices that is relevant for this study.\(^5\)

We assume that consumers adopt one of only two possible strategies for search. Some (for example those with relatively high search costs) simply choose a supplier at random and buy provided the price is no more than 1. These captive consumers are termed "one-timers" and constitute a proportion (and number) \(\theta\) of consumers. All other consumers (of proportion or number \(1-\theta\)) find two prices and then buy at the lower price (again assuming this is no more than 1). These are termed "two-timer" consumers.\(^6\) The number of one-timers expected to choose a particular price is thus \(\theta/n\) since each consumer chooses one of \(n\) prices at random. Similarly the number of two timers is \(2(1-\theta)/n\) since there are 2 chances in \(n\) that a particular price will be selected. A particular supplier is used by two-timers only if it sets the lower price. There are no variable costs of production, but the population of prices is fixed at \(n \geq 2\).

\(^5\) See Janssen and Moraga-Gonzalez (2004) for an analysis which incorporates the cost of finding the “first price”.

\(^6\) Note that Burdett and Judd (1983) find an equilibrium with homogeneous consumers where some consumers search once and others twice but none more than twice.
The determination of \( n \) is considered briefly in section IV(iv) when fixed costs for firms are included.

II(ii) The \( n \)-Firm Symmetric MSNE

In this sub-section each firm posts just one price and so there is a one-to-one equivalence between firms and their prices. Let each firm \( i \) (for \( i = 2, \ldots, n \); that is all firms other than firm 1) choose its offer price \( p_i \) according to some pure or mixed strategy. If firm \( i \) adopts a pure strategy then other firms would set their prices just below rather than just above firm \( i \)'s so as to win custom from two-timers. Then \( p_i \) is no longer a best response for firm \( i \). Thus any equilibrium must involve mixed strategies. Let the function \( F(p) \), with \( F(L) = 0 \) and \( F(1) = 1 \), \( F'(p) = f(p) > 0 \), \( L \leq p \leq 1 \) be the distribution function of a randomly chosen price from firms other than firm 1. We solve for this distribution function so as to ensure that firm 1 is indifferent to choosing any price in \([L, 1]\) and then we use a symmetry argument to define this as the mixing function for all firms including firm 1. We begin by defining firm 1’s expected profit, gross of any fixed costs (ignored until section IV), if it chooses price \( p_1 \) as:

\[
V_1(p_1) = p_1 \left( \frac{\theta}{n} + \frac{2(1-\theta)}{n} \left( 1 - F(p_1) \right) \right) \text{ for } p_1 \leq 1
\]  

(1)

The first and second terms in large brackets in (1) are the expected numbers, of one-timers and two-timers respectively, who buy at price \( p_1 \). Now \( V_1(1) \) is \( \theta n \), because \( \theta/n \) sales can be expected if the firm sets a higher price than other firms and thus only sells to one-timers.

Thus the distribution function \( F \) must satisfy\(^7\):

\[
V_1(p_1) = p_1 \left( \frac{\theta}{n} + \frac{2(1-\theta)}{n} \left( 1 - F(p_1) \right) \right) = \frac{\theta}{n} \text{ for } L \leq p_1 \leq 1
\]  

(2)

Equ (2) implies that firm 1 is indifferent to any choice of \( p_1 \) in \( L \leq p_1 \leq 1 \), and so is willing to mix the price randomly. Then, setting \( p_1 = L \) and \( F(L) = 0 \) in (2), we have that \( L \) has the value...
\[ L = \frac{\theta}{n} \frac{n}{2-\theta} = \frac{\theta}{2-\theta} \]  

(3)

and the distribution function itself is found from (2) to be

\[ F(p) = \frac{2-\theta-\theta}{2(1-\theta)} \]

(4)

All firms will face (4), in their competition for two-timers, if each firm plays the strategy of choosing its price from (4). No firm can then do better. This result can be summarised as

**Lemma 1**: With \( n \) independent firms each setting one price in a MSNE, a symmetric equilibrium exists with \( L \) given by (3) and each price has distribution

function \( F(p_i) = \frac{2-\theta-\theta}{2(1-\theta)} \)

for \( L \leq p_i \leq 1 \) and all \( i = 1, \ldots, n \).

This MSNE is of interest in itself. Since at most two firms are visited by any particular customer, so the *price behaviour of any firm is independent of the number of firms*. Each sale is the result of either a monopoly or a duopoly relationship. As \( \theta \to 1 \) and all consumers are one-timers, so \( L \to 1 \) (from (3)), and the outcome is virtual monopoly pricing. As \( \theta \to 0 \) and all consumers are two-timers, so \( L \to 0 \) (from (3)), and \( F(L) \to 1 \), and the outcome of zero prices is that of Bertrand competition. That the equilibrium distribution of prices in Lemma 1 is independent of the number of firms will be a key simplification in our results below.

\[ \text{The distribution must include a positive density at 1 since otherwise at least one firm would play a price which was certain to be undercut and would do better by shifting that density to the buyer’s reservation price of 1. Holes or spikes in the interior of the distribution are ruled out from similar arguments.} \]
III. Reducing the Effectiveness of Search

We will contrast the MSNE in Lemma 1 with that resulting from there being more price signals than firms. We will consider two comparisons. We first suppose that two firms combine to set their prices to maximise their joint profits, while other firms continue setting their prices independently. We then consider the (symmetric) case where each pair of firms act jointly.

III (i) Preliminaries

Interpretations of what we could mean by a “joint firm” are wide. They include a formal joint ownership of two separately-trading firms, explicit or implicit collusion of these firms in price setting, or that there is just one firm sending out two price signals. If just one pair of firms acted jointly, there would be \( n - 1 \) independent firms and \( n \) prices in the market. If the source of the price signals or the ownership structure of firms was known to buyers then two-timers may be able to avoid choosing non-independent firms' prices as their two searches. For instance, a list of suppliers with a co-ownership indicated would permit two-timers to search out prices from two independent suppliers. In this case, the analysis leading to Lemma 1 would hold with consumers choosing among \( n-1 \) independent suppliers’ prices, rather than \( n \). However, Lemma 1 does not depend on the number of firms, and so the same Lemma, and the same distribution of prices, \( F(p) \), would hold when the firm-price link was known by consumers. Clearly, the case of interest for us is rather where consumers are not aware of which firms have the same owner or where the price signals originate. For example, a supplier setting two prices, or two firms acting jointly, can use different names so that a list of suppliers does not permit the consumer to identify common sources. We will consider this case from now on. We will refer to a firm controlling two prices as a “joint firm”.

We assume consumers' behaviour is as before: they are faced with \( n \) prices, and search for
one (one-timers) or two (two-timers) prices to observe. We will first show in Lemma 2 that the joint firm will never select distinct prices. Suppose the joint firm’s prices are \( p \) and \( p' \), and let \( V = V(p, p') \) denote the expected profit of the joint firm. Let the distribution of a randomly chosen price from all those prices offered by all other firms be given by the distribution function \( H(p) \). The joint firm’s potential customers will fall into 3 classes with the following expected numbers:

- One-timers, with expected number of sales \( \frac{\theta}{n} \) to each price;
- two-timers who observe one of \( p \) or \( p' \) and also observe another price, with expected number for each price of \( \frac{2(1-\theta)(n-2)}{n(n-1)} \);
- two-timers who observe both \( p \) and \( p' \) but no other price, with expected number \( \frac{2(1-\theta)}{n(n-1)} \).

Now we can write the expected profit of the joint firm setting prices \( p \) and \( p' \) as

\[
V(p, p') = p \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)} (1 - H(p)) \right) + p' \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)} (1 - H(p')) \right) + \frac{2(1-\theta)}{n(n-1)} \min(p, p')
\]  

(5)

The explanation for (5) is that each selected price will sell to all one-timers who choose it, and to two-timers who compare it to another firm’s price drawn from the distribution \( H(.) \) and find it lower. Also the joint firms will surely sell to those two-timers who select both \( p \) and \( p' \), but the consumer will choose to buy at the lower price – hence the last term in (5).

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8 Sales to two-timers are found as follows. There is a \( \frac{1}{n} \) chance of a particular one of the joint firm’s prices being chosen for the first price and \( \frac{(n-2)}{(n-1)} \) chance of a different firm’s price (not the joint firm’s other price) for the second price. This has two permutations and the proportion of two-timers is \( 1/2 \). Hence the number: \( \frac{2(1-\theta)(n-2)}{n(n-1)} \). Also, there is a \( \frac{1}{n} \) chance of \( p \) or \( p' \) being chosen for the first price and \( \frac{1}{(n-1)} \) chance of the other joint firm’s price being chosen for the second price. Again, there are two permutations and so the number of this kind of buyer is \( \frac{2(1-\theta)}{n(n-1)} \).
To understand the implications of setting two prices, first consider that the joint firm faces the same price distribution from other firms as in Lemma 1. One might think of this as the case where other firms make no response to the collusive price setting. Replacing $H(p)$ in (5) by $F(p)$ from (4) yields

$$V^j (p, p') = p \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)} (1 - F(p)) \right) + p' \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)} (1 - F(p')) \right) + \frac{2(1-\theta)}{n(n-1)} \min(p, p')$$

$$= p \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2) (1 - \frac{1}{p})}{2n(n-1)(1-\theta)} \right) + p' \left( \frac{\theta}{n} + \frac{2(1-\theta)(n-2) (1 - \frac{1}{p'})}{2n(n-1)(1-\theta)} \right) + \frac{2(1-\theta)}{n(n-1)} \min(p, p')$$

$$= \frac{\theta(p + p') + 2\theta(n-2) + 2(1-\theta) \min(p, p')}{n(n-1)} \quad (5')$$

Clearly (5’) is strictly increasing in both $p$ and $p'$. Hence the joint firm will choose monopoly prices (consumers’ reservation prices): $p=p'=1$. An extra slice of monopoly power, from those two-timers unfortunate enough to choose both the joint firm’s prices, means that setting full monopoly prices is a strictly dominant strategy for the joint firm. Of course, if this were to occur, then the other firms would no longer be in equilibrium and would change their distribution functions. Then the joint firm would also change its strategy and would not always choose monopoly prices. However there is a particular feature of the best response of the joint firm to any distribution of other firms’ prices. This is shown in Lemma 2.

**Lemma 2** For any distribution function $H(p)$ of other firms’ prices, any best response of the joint firm will be to set $p=p'$.

**Proof.** Consider a response where $p \neq p'$ and compare this to responses of $p$ for both prices or $p'$ for both prices. From (5) we have that

$$V^j (p, p') = V^j (p', p) = \sigma(p) + \sigma(p') + \frac{2(1-\theta)}{n(n-1)} \min(p, p')$$
where
\[
\sigma(p) \equiv p\left(\frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)}(1-H(p))\right) \quad \text{and} \quad \sigma(p') \equiv p'\left(\frac{\theta}{n} + \frac{2(1-\theta)(n-2)}{n(n-1)}(1-H(p'))\right).
\]

Now
\[
V^j(p, p) + V^j(p', p') - V^j(p, p') - V^j(p', p) = \frac{2(1-\theta)}{n(n-1)}(p + p' - 2 \min\{p, p'\}) > 0
\]
so that
\[
V^j(p, p) + V^j(p', p') > V^j(p, p') + V^j(p', p) = 2V^j(p, p')
\]
and a necessary condition for this is that
\[
\max\{V^j(p, p), V^j(p', p')\} > V^j(p, p')
\]
Thus a better expected joint firm profit can be obtained by selecting one of \( p \) and \( p' \) for both prices than by selecting distinct prices. Further, if \( V^j(p, p) = V^j(p', p') \) then any random selection between the price pairs \( \{p, p\} \) and \( \{p', p'\} \) would be better than selecting either \( \{p, p'\} \) or \( \{p', p\} \). \( \text{QED} \)

The importance of Lemma 2 is that we can confine our attention to strategies of the joint firm which involve setting common rather than distinct prices. In both Theorem 1 and 2 below we use Lemma 2 to reduce the scale of the search for equilibrium strategies. We know that no joint firm will adopt a strategy with distinct prices. The intuition is that the extra monopoly power from combining two prices is gained only from those two-timers who pick both the joint firm’s prices. For these consumers a discount on either price loses money for the joint firm. It should be clear that the result extends to cases where more than two price signals are
made, or where more than two firms are jointly owned. No assumption on the competitors’ price distribution is needed and variations on how two-timers sample could be incorporated.\textsuperscript{9}

III(ii) Two Firms Setting Prices Jointly

The first case we examine here has the interpretation of one firm trading under two names (for simplicity let one joint company trade under the names of both firms 1 and 2 and set prices 1 and 2) while all other firms (all firms indexed \(i > 2\)) each trades under a single name and sets a single price. This leads to an asymmetric game and here we present a candidate equilibrium where each of \(n\) firms gains in expected profit relative to the outcome in Lemma 1. We take the MSNE in Lemma 1 with \(n\) prices and firms, and consider the event of two firms becoming jointly-owned. The expected profit of the joint firm (5) given \(p_1=p_2=p_j\) is

\[
V^J(p_j,p_j) = p_j \left( \frac{2\theta}{n} + \frac{4(1-\theta)(n-2)}{n(n-1)} (1-F^u(p_j)) + \frac{2(1-\theta)}{n(n-1)} \right)
\]  

(5a)

where \(F^u(p)\) is the distribution function of other firms’ prices. The expected profit of each of these other firms is \textsuperscript{10}

\[
V(p) = p \left( \frac{\theta}{n} + (1-\theta)\left( \frac{4}{n(n-1)} (1-G^u(p)) + \frac{2(n-3)}{n(n-1)} (1-F^u(p)) \right) \right)
\]  

(6)

Where \(G^u(p)\) is the distribution function of the joint firm’s common price \(p_j\). We solve for a mixed strategy equilibrium with the properties given in Lemma 3.

\textsuperscript{9} For example, if consumers had some but imperfect information as to which firms were co-owned, the proportion of captured two-timers would be smaller but still positive: some additional captive consumers would still drive the equal prices result in Lemma 2.

\textsuperscript{10} The chance of a two-timer visiting firm \(i\) and finding one price from the joint firm is \(2(2/n) \left(1/(n-1)\right)\) and the chance of visiting firm \(i\) and another single firm is \(2(1/n) \left((n-3)/(n-1)\right)\).
Lemma 3: An equilibrium to the asymmetric game exists with the following properties.

(i) Each firm \( i > 2 \) sets price according to \( F^a(p) \), with \( F^a(1) = 1 \) and \( F^a(L^a) = 0 \) where

\[
L^a = \left( \frac{\theta + 1/(n-2)}{2 - \theta + 1/(n-2)} \right) \quad \text{and} \quad 1 - F^a(p) = \left( \frac{(n-2)\theta + 1}{2(1-\theta)(n-2)} \right) \left( \frac{1}{p} - 1 \right).
\]

(ii) The jointly-owned firms 1 and 2 set price \( p_j \) according to \( G^a(p_j) \) where \( G^a(L^a) = 0 \), \( G^a(1) = 1-m \) and \( m \) is a mass point at \( p_j = 1 \) with value

\[
m = \frac{n-1}{2((n-2)(2-\theta)+1)},
\]

and

\[
1 - G^a(p_j) = \left( 2\theta - \frac{n-3}{n-2} \right) \frac{1}{4(1-\theta)} + \frac{m}{p_j}.
\]

(iii) The expected profit of the joint firm and the independent firms are given by

\[
V^j(p_j, p_j) = \frac{2 \theta}{n} + \frac{2(1-\theta)}{n(n-1)} \quad \text{and} \quad V(p_i) = \frac{\theta}{n} + \frac{4(1-\theta)}{n(n-1)} m \quad i > 2
\]

Proof. Given the strategies defined by (i) and (ii), the value of \( V(p_j, p_j) \) in (5a) is \( \frac{2 \theta}{n} + \frac{2(1-\theta)}{n(n-1)} \) for all \( p_j \) in \([L^a, 1]\) and \( V(p_i) \) in (6) is \( \frac{\theta}{n} + \frac{4(1-\theta)}{n(n-1)} m \) for all \( p_i \) in \([L^a, 1]\). By

Lemma 2 the joint firm would not wish to deviate to set distinct prices. Further, any price below \( L^a \) yields lower value for any firm than a price of \( L^a \) since all price comparisons would be won in either case but a price of \( L^a \) yields more profit per sale. Finally, no price above 1 makes any sales, and a price of 1 for an independent firm is (at least weakly) dominated by the limiting price \( p \to 1 \) under the joint firm’s mass point at 1.\(^{11}\) QED

\(^{11}\) This holds for most rules determining the allocation of sales if both the joint firm and firm \( i (i \geq 3) \) choose price equal to 1.
Theorem 1: Using Lemma 3 and comparing the equilibrium in the asymmetric game to that in Lemma 1, we can state the following results for any $n$:

(i) The lower support for the price distributions is higher in the asymmetric game.

(ii) All firms play a price distribution in the asymmetric game that stochastically dominates the price distribution of Lemma 1. Hence both one-timers and two-timers expect to pay more and have lower consumer surplus.

(iii) The joint firm sets prices from a distribution $G^u$ that stochastically dominates the distribution $F^u$ used by the independent firms.

(iv) In Lemma 1, gross profit is $\theta/n$ for each firm, Thus from Lemma 3 (iii) all firms make higher profit in the asymmetric game.

(v) The $n$-2 independent firms make more profit (gross of any fixed cost) per price than the joint firm.

Proof: Direct comparisons of Lemmas 1 and 3. Note that first order stochastic dominance of $G^u(p)$ and $F^u(p)$ over $F(p)$ means a higher mean price (that paid by one-timers), as well as a higher expected value of the lower of 2 observations (that paid by two-timers) in the equilibrium of Lemma 3.\(^{12}\) Hence both one-timers and two-timers pay more. Finally, (v) holds since $m > 1/4$.

QED

From (iv) and (v), no firm $i$ ($i>2$) will object to firms 1 and 2 being jointly owned. The explanation is simple: the joint firm has an added reason to play the monopoly price, since consumers who find both its constituent prices are captive, while the other firms can undercut the monopoly price (played with probability $m$). Such a result has obvious parallels in cases

\(^{12}\) This is a property of order statistics, see for instance Appendix C in Krishna (2002).
where partial cartels enforce high prices but are undercut by non-cartel members, and indeed in Cournot competition where horizontal merger benefits other firms more.

Prices paid by consumers are on average higher in the equilibrium reported in Lemma 3 than in Lemma 1. The model still has all consumers purchasing and so reducing the ability to make price comparisons has no overall welfare loss since it creates no distortion. However, the shift from consumers’ surplus to producers’ surplus is of obvious relevance to competition policy, and it may imply a response in the number of firms (due to entry) and hence the social costs of production, as well as the amount of consumer search. We will return to welfare issues in section IV.

**III(iii) Pair-wise ownership of firms**

The second comparative exercise we will undertake is when each pair of firms set joint prices. We will distinguish here between the number of prices and firms by using $n'$ as the number of prices and $n'/2$ as the number of independent firms. Again two interpretations are of interest. The first we will term the pair-wise ownership of the firms. Here each firm again sets just one price but each distinct pair of firms (firms 1 and 2, firms 3 and 4, etc.) are controlled by the same owner. Alternatively consider that the number of firms is $n'/2$ but that each firm sends two price signals making $n'$ prices in total.

We will use Lemma 2 and let the price distribution of all price pairs, other than the pair $p_1$ and $p_2$, be denoted $G(p_i)$, $i = 3, 5, ..., n'-1$, given that $p_4 = p_3$, $p_6 = p_5$, etc. Now expected profit of the joint firm setting $p_1$ and $p_2 = p_1$ is

$$V^j(p_1, p_1) = p_1 \left[ \frac{2\theta}{n'} + \frac{4(1-\theta)(n'-2)}{n'(n'-1)} \left(1 - G(p_1) + \frac{2(1-\theta)}{n'(n'-1)} \right) \right]$$

(5b)
where the first term in the square brackets are captive one-timers, the second is non-captive
two-timers whose alternative price is higher than \( p_1 \) and the third is captive two-timers. We
can state and prove

**Lemma 4:** (i) If each pair of \( n' \) prices has a common owner, then the unique symmetric MSNE
is that each pair of prices is a single random draw from the distribution function

\[
G(p) = \frac{2n' - 3 - (n' - 2)\theta - ((n' - 2)\theta + 1)}{(2n' - 4)(1 - \theta)} p \quad \text{for} \quad L^j \leq p \leq 1
\]

where the lower support is

\[
L^j = \frac{(n' - 2)\theta + 1}{2n' - 3 - (n' - 2)\theta}
\]

Then \( p_1, p_3, p_5, \ldots \) are independent random draws from \( G(p) \) and \( p_1 = p_2, p_3 = p_4, p_5 = p_6, \ldots \) The
lower support \( L^j \) is such that \( L^j > L \).

(ii) Expected profit for each pair of firms is

\[
V^j (p, p) = \frac{2}{n'} \left( \theta + \frac{1 - \theta}{n' - 1} \right)
\]

**Proof.** Playing \( p_1 = p_2 = 1 \) with positive density must form part of a symmetric equilibrium
since then, if the upper support was lower than 1, each joint firm would prefer to shift density
from that support to 1, because in both cases other firms will undercut them with certainty,
but a price of 1 yields more profit from captive customers. When \( p_1 = p_2 = 1 \), we have \( G(p_1) =
G(p_2) = 1 \), and expected profit is \( V^j (1, 1) = \frac{2}{n'} \left( \theta + \frac{1 - \theta}{n' - 1} \right) \) using (5b). This proves part (ii).

The minimum price \( L^j \) is defined such that expected profit \( V^j (L^j, L^j) \) is equal to this same
expected profit \( V^j (1, 1) \) when \( G(L^j) = 0 \), so that, putting \( p_1 = L^j \) and \( G(L^j) = 0 \) in (5b), and
setting this to \( V^j (1, 1) \) gives \( L^j \). Again, by setting (5b) equal to its value at \( p_1 = 1 \), we can use
(5b) to solve for the unique distribution function \( G(p) \) as (7). Thus Lemma 4 describes a unique symmetric equilibrium, given pair-wise pricing, when both prices in each pair are the same. Lemma 2 states that no joint firm will wish to deviate by setting distinct prices, and so Lemma 4 describes a unique symmetric MSNE even when joint firms can set distinct prices.

QED

**Theorem 2**: the comparison of the symmetric MSNE in Lemmas 1 and 4 yield the following results for any \( n \) prices in Lemma 1 and any \( n' \) prices in Lemma 4:

(i) \( G(p) < F(p) \) for all \( p<1 \), that is \( G(p) \) (first-order) stochastically dominates \( F(p) \);

(ii) The minimum price possible under pair-wise ownership or multiple signals is higher, \((L_j > L)\);

(iii) The mean price, as well as the lower of two price observations, observed under pair-wise ownership or multiple signals is higher; consumer surplus is thus lower.

(iv) Expected industry profits (gross of fixed costs) are higher under pair-wise ownership or multiple signals; sales are the same (one per consumer);

(v) \( G(p) \) is increasing in \( n' \) for a given \( p \), and decreasing in \( \theta \) for a given \( p \).

(vi) \( G(p) \rightarrow F(p) \) as \( n' \rightarrow \infty \);

**Proof**: (i) is easily checked by comparing (4) and (7). Thus \( G(p) \) stochastically dominates \( F(p) \) for all finite \( n' \). (ii) is shown by comparing (3) and (8). Hence (iii) and (iv) hold: the mean price is higher, and a higher price is paid both by one-timers and by two-timers. Next, (v) can be shown to hold by differentiating (7) with respect to \( n' \) and \( \theta \) to obtain

\[
\frac{\partial G(p)}{\partial n'} = \frac{1}{2(n' - 2)^2(1-\theta)}\left(\frac{1}{p} - 1\right) > 0
\]

\[
\frac{\partial G(p)}{\partial \theta} = -\frac{n' - 1}{2(n' - 2)^2(1-\theta)^2}\left(\frac{1}{p} - 1\right) < 0
\]

\[(10)\]
Finally (vi) holds by letting \( n' \to \infty \) in (7), noting that \( F(p) \) in Lemma 1 is independent of \( n \).

QED

**Theorem 2 Corollary:** (i) The results in Theorem 2 all still hold if the number of firms (equals number of prices) in the MSNE of Lemma 1 changes. (ii) Thus the equilibrium price distribution, with joint-pricing firms and \( n' \to \infty \), is equivalent to the equilibrium price distribution in the MSNE in Lemma 1, with any number of firms \( n \).

**Proof.** (i) is immediate, because the MSNE in Lemma 1 is independent of the number of firms/prices. (ii) follows from Theorem 2 (vi).

QED

The Corollary to Theorem 2 has two important implications. One is that, in comparing the two MSNE, there is no difference in applying the theorem between a treatment of \( n \) firms each moving to setting two price signals so that \( n' = 2n \) price signals exist rather than \( n \), and \( n \) firms effectively becoming \( n/2 \) firms by moving to joint ownership and continuing to provide \( n' = n \) price signals. This is because the price distribution found in Lemma 1 is independent of the number of firms/prices. Hence any \( G(p) \) (which increases with \( n' \) at any \( p \)) can be compared with the \( n \)-independent \( F(p) \). Secondly, we also note that price distributions with and without joint pricing can be compared by using the price distribution under joint pricing at different numbers \( n' \), with \( n' = \infty \) replicating \( F(p) \). We will use this result in comparative statics exercises in the next section.

Clearly the price paid both by the one-timer consumers and the two-timer consumers is higher on average with the extra monopoly power which comes with each pair of firms being
jointly owned. This effect has nothing to do with the smaller residual demand (as would occur in a Cournot model) but rather to do with the buyers’ lower ability to check prices. Indeed, prices are on average higher if there are, say, 8 joint firms setting 16 prices than if there are 2 independent firms, setting only 2! The effect is reduced as $n^j$ increases and the likelihood of two-timers choosing both prices from the same joint firm declines, but only disappears completely when there are an infinite number of firms (then $L^j$, $G(p)$ collapse to $L$, $F(p)$ respectively as $n^j \to \infty$). ¹³

IV. Incentives and Endogenous Consumer Behaviour

In the analysis so far we have treated consumer behaviour as exogenous. In this section we make endogenous the consumer’s decision to be a one-timer or a two-timer, and then consider whether a change to pair-wise ownership still reduces consumer utility and still increases aggregate profit. We first model the incentives and cost of search, then discuss some of the issues of endogenous search in terms of graphs and a table, and finally undertake a comparative static exercise to show that our previous conclusions about utility and profits will be unchanged.

IV(i) The incentive to search

We will compare the independent pricing case of Lemma 1 with the full pair-wise pricing case of Lemma 4. If the incentive to visit two prices rather than one is smaller in the pair-wise case, then $\theta$ would be higher as fewer consumers feel that the additional search is worth

¹³ This result has an application in terms of price-fixing cartels. In a perfect information homogeneous-product world, where all prices are known, it only takes two independent firms for the Bertrand equilibrium to be the competitive price. In our framework of imperfect consumer information, moving, from any number of firms with one firm / one price, to the case where one pair (in Theorem 1) or each pair (in Theorem 2) of firms has formed a price-fixing cartel, will increase average prices.
the time or money. The expected difference in product price paid for changing behaviour from a one-timer to a two-timer can be shown to be \(^1\)

\[
\Gamma^*(\cdot, \theta) = \int \frac{1}{L} F(1 - F) \, dp \tag{11}
\]

for the independent firm case with any number of firms and prices \((n \geq 2)\) depicted in Lemma 1, and

\[
\Gamma(n^j; \theta) = \frac{n^j - 2}{n^j - 1} \int \frac{1}{L} G(1 - G) \, dp \tag{12}
\]

for the pair-wise-owned structure with \(n^j\) prices \((n^j \geq 4)\), and \(n^j/2\) price pairs depicted in Lemma 4. The difference in incentive is

\[
\Delta = \Gamma(n^j; \theta) - \Gamma^*(\cdot; \theta)
\]

The calculation of \(\Delta\) and the determination of its sign is not immediate. Part of the difficulty of coming to an analytical conclusion is because \(\Gamma(n^j; \theta)\) is not monotonic in \(\theta\). Thus at large \(\theta\) (or small \(\theta\)) there is little to be gained from a further search since prices are already closely packed near the monopoly price of 1 (or competitive price of 0). At middle values of \(\theta\) there is more variation and thus more to gain. For example with \(n^j = 8, 16\) or 100 in Table 2 the relation is \(\cap\)-shaped: reading along one of these rows gives highest values of \(\Gamma\) for middle levels of \(\theta\). Figure 1 sketches a typical result for \(\Gamma(8; \theta)\) as a function of \(\theta\).

(Table 2 about here)

(Figure 1 about here)

\(^1\) With all integrals from \(L\) to 1, the expected price paid by a one-timer is \(\int p \, dF(p)\), and the expected price paid by a two-timer is \(\int p \, d\{(1-F(p))^2\}\). Subtracting the second expression from the first and integrating by parts yields \(1 - \int F(p) \, dp \cdot L + \int (1-F(p))^2 \, dp = \int (1-F(p)) \cdot (1-F(p))^2 \, dp = \int (1-F(p))^2 F(p) \, dp\). (12) is derived in a similar way, except that the gain is 0 with probability \(1/(n^j-1)\) (since the consumer might choose the same joint firm for the second price and gain no benefit) and \(\int (1-G(p))G(p) \, dp\) with probability \((n^j-2)/(n^j-1)\). This gives (12). Note that (12) tends to (11) as \(n^j\) becomes large.
A simple model of the consumer’s choice would have individuals indexed by \( \theta \) and with costs of observing a second price as \( C(\theta) > 0 \), where \( C'(\theta) < 0 \): thus the higher the index, the lower the cost to that individual in changing to being a two-timer. \( C(\theta) \) as a constant would be an extreme case where consumers were homogeneous, and is considered in Burdett and Judd (1983). The issue is now whether there is a unique solution to the determination of \( \theta \) and \( \Gamma \) for a given \( n' \). Also, whether this solution implies a higher value of \( \theta \) (and thus less consumer search and more monopoly power) than the value that would exist if \( \Gamma = \Gamma^*(\cdot; \theta) \), (no pair-wise structure). In figure 1, we attempt the first part of just such an analysis. The additional cost of two-timing is shown as the schedule \( C(\theta) \). The additional gain is shown as \( \Gamma(8; \theta) \) for the pair-wise structure with 4 joint firms setting 8 prices in all. The system is in equilibrium when all types are choosing the best (for them) strategy for searching. As drawn, there are three equilibria. One is the full monopoly case. Here no-one searches twice (and \( \theta = 1 \)). Costs are greater than benefits (equal to \( \Gamma(8; 1) \)), even for the individual (type or index equal to 1) who has the least cost of two-timing. Thus this no-search position is an equilibrium where no prospect of two-timer customers leads to monopoly pricing and there is no gain for any individual from becoming the only two-timer. However, it may be an unlikely equilibrium since it is based on monopoly pricing being pervasive. In this situation, firms may compete in advertising messages, and more importantly the anti-trust authorities might seek to cap prices or improve consumers' price information. There are two other equilibria, shown as \( \theta_1 \) and \( \theta_2 \) in figure 1, where the schedule of gains crosses that of the costs. At these interior equilibria, we have for \( \theta \) to be an equilibrium:

\[
\Gamma(n'; \theta) = C(\theta) \quad (13)
\]
At $\theta_1$, the equilibrium is stable since small departures from $\theta_1$ will lead to self-correcting behaviour. Those of types less than $\theta_1$ will have an incentive to move to one-timer behaviour and those with more than $\theta_1$ will have an incentive to move to two-timer behaviour.

Stability for an interior equilibrium thus requires

$$\frac{\partial \Gamma(n^i; \theta)}{\partial \theta} > \frac{\partial C(\theta)}{\partial \theta}$$

that is, the schedule of the benefit from additional search cuts the cost of additional search schedule from below. At $\theta_2$ in figure 1 the stability condition does not hold. Rather, if the initial share of one-timers is slightly higher than $\theta_2$ (lower than $\theta_2$) then $\Gamma(8; \theta_1) < C(\theta_2)$ ($\Gamma(8; \theta_2) > C(\theta_2)$), and individuals will change to one-timing (change from one-timing) and a share $\theta_2$ will not be reached. Finally, note that if the cost schedule shifts upwards then the three equilibria may collapse to just a single equilibrium at 1. If the schedule shifted downwards the set of equilibria may reduce to a single point of type $\theta_1$. It is $\theta_1$ that represents the most plausible basis for endogenising the search decision since it is an interior stable equilibrium.

(figure 2 about here)

Figure 2 contrasts the graphs of $\Gamma(8; \theta)$ and $\Gamma^*$ (approximated by $\Gamma(100; \theta)$ as suggested by Theorem 2 Corollary). In this case, the equilibrium at $\theta_1$ shifts to the left when the benefit schedule shifts from $\Gamma^*$ to $\Gamma(8; \theta)$. Pair-wise ownership leads to more monopoly pricing and a greater return from searching and hence to more two-timers. However, it is relatively easy to draw $C(\theta)$ to produce a different result. For example $\hat{C}$ gives an alternative outcome, where the equilibrium $\hat{\theta}_1$ is high (few two-timers, many captive consumers and high prices predominating) and is at the declining part of both benefit functions. Now the shift is a shift downwards (due to a greater concentration of prices at monopoly pricing levels), leading to an even higher $\hat{\theta}_1$ and thus even more monopoly power of firms. The broad conclusion of our
analysis of endogenous search when moving to a pair-wise structure is that a competitive situation (already low $\theta$) would be mitigated, due to more individuals searching, while an already monopolistic situation (high $\theta$) would get worse, since fewer individuals would tend to search. It is straightforward to see this precisely within an algebraic approach, and also necessary since we will have to assess whether increased search might overturn the results of the fixed search case. The difference in the $\Gamma$ schedules in figure 2 can be viewed as the result of different $n^j$. To find the change in search due to a change in $n^j$ we need to change $n^j$ and $\theta$ such that $\Gamma(n^j;\theta)$ is still equal to $C(\theta)$. Thus

$$\frac{d\Gamma}{dn^j} = \frac{\partial \Gamma}{\partial n^j} + \frac{\partial \Gamma}{\partial \theta} \frac{d\theta}{dn^j} = C'(\theta) \frac{d\theta}{dn^j}$$

or

$$\frac{d\theta}{dn^j} = -\frac{\frac{\partial \Gamma}{\partial n^j}}{\frac{\partial \Gamma}{\partial \theta} - C'(\theta)} \quad (15)$$

Thus, assuming the stability condition (14) holds\(^\text{15}\), shifting the $\Gamma$ function by changing the number of prices increases or decreases the number of one-timers according to whether the shift is upwards or not. We have seen that Figure 2 contrasts the two situations: $\theta_1$ has

$$\frac{\partial \Gamma}{\partial n^j} < 0 \quad \text{while} \quad \hat{\theta}_1 \quad \text{has} \quad \frac{\partial \Gamma}{\partial n^j} > 0.$$ 

We can find the partial derivatives of $\Gamma(n^j;\theta)$ directly as:

$$\frac{\partial \Gamma}{\partial n^j} = \frac{n^j - 2}{n^j - 1} \int_a^b \frac{\partial G}{\partial n^j} dp + \frac{\Gamma(n^j;\theta)}{(n^j - 1)(n^j - 2)} = \int_a^b (1 - 2G) \frac{\partial G}{\partial n^j} dp$$

$$\frac{\partial \Gamma}{\partial \theta} = \frac{n^j - 2}{n^j - 1} \int_a^b (1 - 2G) \frac{\partial G}{\partial \theta} dp \quad (16)$$

Use (10) in (16) to obtain an easier form yields

\(^{15}\) We will use stability as a property to help us with comparative static analysis; that is we will apply Samuelson’s (1947) Correspondence Principle.
\[
\frac{\partial \Gamma}{\partial n'} = \frac{1}{2(n'-2)(n'-1)(1-\theta)} \int_{\nu} (1-2G)\left(\frac{1}{p} - 1\right) dp + \frac{1}{(n'-1)(n'-2)} C(\theta)
\]

\[
= \frac{I}{2(n'-2)(n'-1)(1-\theta)} + \frac{1}{(n'-1)(n'-2)} C(\theta)
\]

(17)

\[
\frac{\partial \Gamma}{\partial \theta} = -\frac{1}{2(1-\theta)^2} \int_{\nu} (1-2G)\left(\frac{1}{p} - 1\right) dp = -\frac{I}{2(1-\theta)^2}
\]

where \( I = \int_{\nu} (1-2G)\left(\frac{1}{p} - 1\right) dp \), so that

\[
\frac{\partial \Gamma}{\partial n'} = \frac{-(1-\theta)}{(n'-1)(n'-2)} \frac{\partial \Gamma}{\partial \theta} + \frac{1}{(n'-1)(n'-2)} C(\theta)
\]

(18)

Hence, \( \frac{\partial \Gamma}{\partial n'} > 0 \) if \( \Gamma \) is a negatively sloped function of \( \theta \), but is negative if \( \Gamma \) is sufficiently positively-sloped with respect to \( \theta \). Figure 2 demonstrates the two possibilities at \( \theta_l \) and \( \hat{\theta} \).

IV(ii) Comparative welfare with endogenous search

Whether changing \( n' \) leads to more or less search, a key question is whether the results comparing consumer utilities and industry profits reported in Theorem 2 would be overturned. In this subsection we investigate this using the observation in the Theorem’s Corollary and focussing on how a stable interior equilibrium such as \( \theta_l \) changes to maintain (13) as \( n' \) increases, and how these changes lead to a shift in the distribution function of prices \( G(p) \) to a stochastically dominated distribution.

\textit{Lemma 5}: In two pair-wise-pricing interior stable equilibria with endogenous search, one with \( n' \) prices and one with \( n > n' \) prices, the price distribution with \( n' \) prices first-order stochastically dominates that with \( n \) prices.

\textit{Proof}. From (10) we see that the ratio of the partial derivatives of \( G(p) \) is given by:

\[
\frac{\partial G}{\partial n'} = \frac{-(1-\theta)}{(n'-1)(n'-2)} \frac{\partial G}{\partial \theta}
\]

(19)
We have by substituting (18) and then (19) into (15)

\[
\frac{d\theta}{dn'} = -\frac{\partial \Gamma}{\partial n'} C'(\theta) = -\frac{-(1-\theta)}{(n'-1)(n'-2)} \frac{\partial \Gamma}{\partial \theta} + \frac{1}{(n'-1)(n'-2)} C(\theta)
\]

(20)

\[
\frac{\partial G}{\partial n'} - \frac{\partial G}{\partial \theta} C(\theta)
\]

(21)

Now consider the effect of changing \(n'\), including the adjustment in \(\theta\), on the distribution function \(G(p)\). We have

\[
\frac{dG(p)}{dn'} = \frac{\partial G(p)}{\partial n'} + \frac{\partial G(p)}{\partial \theta} \frac{d\theta}{dn'} = \frac{\partial G(p)}{\partial n'} \left(1 + \frac{\partial G}{\partial \theta} \frac{d\theta}{dn'}\right)
\]

(22)

Hence, using (21) in (22) we arrive at

\[
\frac{dG(p)}{dn'} = \frac{\partial G(p)}{\partial n'} \left(1 + \frac{\partial G}{\partial \theta} \frac{1-\theta}{\partial \theta} C'(\theta)\right) = \frac{\partial G(p)}{\partial n'} \frac{C(\theta)}{\partial \theta} - C'(\theta)
\]

(23)

Now \(\frac{\partial G(p)}{\partial n'} > 0\) from (10), and for stability (14) holds: \(\frac{\partial \Gamma}{\partial \theta} - C'(\theta) > 0\). Thus (22) is positive since \(C'(\theta) \leq 0\) and \(C(\theta) > 0\). This generalises to comparing the distributions at \(n'\) and \(n\). The difference between them is made up of the integral of the small changes (23). Stochastic dominance of the price distribution at the higher number of prices by that at the lower number of prices is then assured.

QED

The implication of lemma 5 is that all consumers are better off, since the expected price, as well as the expected lower price from two observations, are both lower with \(n > n'\). The
consumer could continue with the search strategy she had before and be better off, or change to an even better strategy. We can now state

**Theorem 3.** (i) Expected consumer surplus for all consumers is higher and (ii) aggregate expected profit is lower, in the stable interior equilibrium with any \( n \) independent prices than with any \( n' \) pair-wise prices even if consumers’ search behaviour changes endogenously.

**Proof.** (i) If \( n' \) is infinite, the distribution \( G \) is the same as the distribution \( (F) \) with all independent prices for any \( n \), (Theorem 2 Corollary). From Lemma 5, as \( n' \) falls to any finite level consumers become worse off than in the case of independent prices with any \( n \).

(ii) Aggregate profit in the pair-wise case is (by multiplying (9) by \( n'/2 \))

\[
\Pi = \theta + \frac{1-\theta}{n'-1}
\]

If \( n' \) changes then the total effect on aggregate profit, including the effect of search response, is given (using (20)) by the total derivative:

\[
\frac{d\Pi}{dn'} = -\frac{1-\theta}{(n'-1)^2} + \left(1 - \frac{1}{n'-1}\right) \frac{d\theta}{dn'} = -\frac{1-\theta}{(n'-1)^2} + \left(1 - \frac{1}{n'-1}\right) \left(\frac{1-\theta}{(n'-1)(n'-2)} \frac{\partial \Gamma}{\partial \theta} - \frac{1}{(n'-1)(n'-2)} C'(\theta)\right)
\]

This simplifies to

\[
\frac{d\Pi}{dn'} = \left(\frac{1-\theta}{(n'-1)^2}\right) \left(-1 + \frac{\partial \Gamma}{\partial \theta} \frac{1-\theta}{C'(\theta)}\right) = \frac{(1-\theta)}{(n'-1)^2} \left(\frac{C'(\theta) - \frac{1}{1-\theta} C(\theta)}{\frac{\partial \Gamma}{\partial \theta} - C'(\theta)}\right)
\]

Since \( C'(\theta) \leq 0 \) by definition and \( \frac{\partial \Gamma}{\partial \theta} - C'(\theta) > 0 \) due to the stability property (14), we have

\[
\frac{d\Pi}{dn'} < 0.
\]

Since aggregate profit is falling with the number of prices so must profit per price and per joint firm. In order to replicate the price distribution of the single price per firm
model, \( n' \) has to be infinite. Then aggregate profit is \( \theta \). For all such decreases in \( n' \) from this infinite level, aggregate profit and profit per joint firm will be increasing. Hence we can state that, at any \( n \) independent prices compared to any \( n' \) pair-wise prices, aggregate profit is higher in the pair-wise case. Also at any finite \( n = n' \), profit per price is higher in the pair-wise case.

QED

Obviously if there is fixed cost of operating firms or promoting prices then a comparison involving a different number of firms is likely to be determined in part by the relative number of firms. We come to this next.

\textit{IV(iii) Long-run zero-net-profit equilibrium with endogenous search.}

We now suppose that there is a fixed cost per price offered and denote this as \( T \). We will distinguish between the profit discussed in the previous sections and profit minus fixed cost by referring to the latter as net profit. Aggregate welfare is of limited interest in our model since the sum of consumers’ surplus and aggregate net profit is just \( 1 - C(\theta) - nT \): the value of product minus search costs and fixed costs. This is maximised when there is just one firm, and then there is no search costs (by definition). Welfare is then equal to \( 1 - T \), which is also the monopolist’s net profit. If the number of firms is endogenous then we can look for equilibria where net profit per firm is zero, and this is of more interest.

Consider a long run zero-net-profit equilibrium in the independent price case (Lemma 1). That is profit per firm just covers the fixed cost so that \( \theta \) and \( n \) now solve two conditions:

\[
\frac{\theta}{n} = T \text{ (zero net profit) and } \Gamma^*(\cdot, \theta) = C(\theta) \text{ (equilibrium search behaviour).}
\]

Let all firms convert from independent prices to pair-wise ownership with the same number of prices, so that \( n' = n \). Hold the level of search as constant at \( \theta \). The extra monopoly power increases
aggregate profit and profit per price. The positive net profit will lead to entry of further pair-wise firms. Suppose for the moment that entry is so high that the number of prices approaches infinity. Of course profits will now be negative! At this stage, the distribution $G(\theta; n') = G(\theta, \infty) = F(\theta)$ and the same $\theta$ is again an equilibrium value, with consumer surplus equal to that at the original independent firm zero-profit equilibrium. Now let $n'$ decrease from $\infty$. This will lead to a smaller loss and (a fortiori) a smaller loss per price and per firm. The process continues until a new zero-net-profit equilibrium is found. However the ultimate $n'$ at this equilibrium will be finite (assuming a finite fixed cost), and so, compared to the independent pricing equilibrium, consumers’ surplus will be lower (Lemma 5) because the price distribution will stochastically dominate that at an infinite $n'$, which would be equivalent to independent pricing (Theorem 2 Corollary).

Formally, we can define consumers’ surplus as $S(\theta, n, F(\theta))$ in the initial independent price case. Then

$$S(\theta, \infty, G(\theta, \infty)) = S(\theta, n, F(\theta))$$ (from Theorem 2 Corollary) and

$$S(\theta_1, n', G(\theta_1, n')) < S(\theta, \infty, G(\theta, \infty))$$ (from Lemma 5)

where $n'$ is the finite number of prices, and $\theta_1$ is the associated equilibrium number of one-timers, such that net profit per price in the pair-wise case is zero.

Since aggregate net profit is zero before and after the switch to pair-wise firms, and consumers’ surplus has decreased, aggregate welfare has also decreased. This will be the case whether the ultimate $n'$ is greater or less than the initial $n$. It will also be unaffected if joint firms have fixed cost different to $2T$: if fixed cost are less then more entry will occur (and more incidence of the fixed costs will be incurred) until net profits are zero.
V. Policy and Conclusions

We have shown that prices are on average higher with pair-wise ownership. Higher prices benefit firm owners at the expense of consumers and consumer search has a cost to the searchers. What we have shown additionally is that it is not the reduction in the number of independent firms that is the problem, but rather that the pair-wise ownership leads to consumers wasting their search activity. The possibility of a price being compared with another price which is in fact controlled by the same owner limits the effectiveness of search both as an anti-monopoly mechanism and as an efficient way of making individual purchasing decisions. Thus the prices are higher in the pair-wise structure than when there are half the firms (or indeed any number of firms greater than one) but each independently owned. If the joint ownership of firms was clearly labelled (by each jointly-owned firm having the same trading name for example) then the monopoly problem might be less since two-timers can adjust their search behaviour. The lesson for anti-trust authorities is thus to ensure that consumers know about ownership links of each candidate supplier by ensuring transparent trading names. If such knowledge is insufficient to permit random searches to avoid sampling from prices from the same source, then the prevention or elimination of multiple price signals or joint ownership would be the only policy instrument.

There are further points that accompany this lesson. Most obviously, the analysis has given a justification for limiting price-fixing agreements even when the share of the market affected is relatively small.16 This is in contrast to the typical result from Bertrand competition with homogeneous products which suggests that price-fixing by sub-sets of suppliers can have little impact. Another scenario is when the product offered is differentiated. Consumers would then have to seek out comparable product specifications as well as prices. If a

16 When the share of the market is large, a cartel has to exert discipline on its members. The folk theorem that underlies such analysis is affected by consumers’ information, see Ireland and Waterson (2006).
particular variety is sold by two firms, then their prices make ideal comparators. However, if the firms share the same owner this comparison is controlled by the owner and is ineffective for the consumer. Such an event is very likely if the owner makes a sole distributor agreement with a manufacturer, or indeed sells the same "own-brand" across the jointly-owned firms. A vertical agreement extends the ability of the jointly owned firm to portray its prices as fair and does not depend on market dominance of either the particular product or the jointly owned firm. In the UK, vertical agreements are normally excluded from prohibition orders, unless the parties concerned have dominant market positions.

The recent OFT decision (Office of Fair Trading (2001)) in the case of the DSG Retail Ltd’s agreements for exclusive dealer status with Compaq and Packard Bell (on some of their products) was that the exclusion should be confirmed and the agreements permitted. Obviously, many factors went into that decision. In particular, the downstream and upstream market shares were thought not to be dominant (less than 40%) and evidence was cited to the effect that customers were not attracted to particular brands: rather, “competitive price” was the main reason stated for buying from a particular retailer. Hence, the OFT concluded that buyers were not being disadvantaged by being limited in retail outlets where these particular products were available, since other brands were available elsewhere and brand loyalty or reputation was not an important characteristic. Since dominant market positions were absent, and alternative products available, DSG could not be abusing the competitive operation of the market.

The analysis of the OFT concentrated on market shares and the importance of price in determining sales. The latter would ensure that, for example, Packard Bell computers would need to be priced low if customers were to be persuaded away from other retailers selling
other brands. However, the submission of the complainant was rather different: that retailers needed to “stock prominent brands in order to compete” (Office of Fair Trading, 2001, paragraph 73). Our model of comparison shopping by two-timers indeed suggests that the testing of price levels between retailers needs similar brands to be sold across at least some competing retailers. An additional factor that did not seem to be taken into consideration by the OFT is that DSG can sell under three different retail chain labels (Dixons, Currys and PC World). Consumers who were unaware of the connections (i) may find false comfort from price comparisons between say Dixons and PC World and (ii) will find it difficult to make price comparisons between one of the Dixons group and another, independent, chain, since other chains do not sell these particular products. Of course, even if due attention were given to these arguments, it would still be possible that the advantages of vertical integration may still imply net benefits for the consumer because of reductions in double margins or scale economies of maintenance and expertise.

However, the DSG case is relatively well-known compared to other co-ownerships which, for example, may be hidden in the data of Table 1. There is no simple way of telling whether the prices in Table 1 reflect independent websites, either independent in terms of ownership or of price collusion. Just as some authors choose to write books under various noms-de-plume, so some website operators may choose to operate under more than one trade name.

The extent to which our model is relevant to a particular market may well reflect competition policy and judgements. The case of Levi Strauss and Tesco was decided in the European Court of Justice in November 2001 (European Court of Justice, 2001). The decision was that Tesco would be operating contrary to European law if it sourced Levi jeans from the USA without the trademark owner (Levi Strauss) specifically granting permission. Thus Levi
Strauss could enforce higher prices in the UK by preventing Tesco from importing from the USA at the cheaper prices which exist there. The issue was based around the legal rights conferred by owning trademarks. From the perspective of enabling price comparisons, however, the elimination of a low price / high volume chain retailer like Tesco is unfortunate. Suppose Tesco had won the case. Then the fact that there is a “Tesco” in most towns in England would mean that a benchmark was available to all consumers, and this would have had two effects. First the cost of making price comparisons would decline. This would be partly because the research necessary to find stores which stock the product would be much easier, and also because Tesco being a supermarket would be visited by many consumers on a regular basis. Second, Tesco might wish to obtain a nationwide reputation for low prices and hence would price aggressively. The model we discussed in sections II and III would be replaced by one where random selection of stores would no longer be an appropriate assumption. Rather, most customers would visit Tesco, and then some would make a further search to check on price, designs, fit, etc. at another store. The supermarket would become the price leader, and this might suggest lower prices for consumers. By preventing Tesco from selling its products, both by declining to supply direct and by stopping “grey market” supplies, Levi Strauss had removed a potential price benchmark, and thus placed the market for its products within the scope of our model.

There are a number of issues and strategies closely associated with our main thesis. One is that the retailer’s own brand products are equally difficult to test by price comparisons. This may be one reason why in many fields own brands are not promoted as such. Indeed, many own brands seem to have names which conjure up other brand names and indeed might be confused in the buyer’s mind for a product which is not an own brand. Perhaps one explanation of own brands which have such hidden identities (for example, not the name of
the store itself) is again that retailers suggest that price comparisons are possible when they are not. Secondly, own-brands and exclusive brands are excluded from price-matching or price-beating guarantees by definition. (The possibility of such exclusion, if identified by the consumer, will add to the “hassle” factor (Hviid and Shaffer, 1999)).

Although the model we have adopted is stylised and simplified, the key points should be robust to extensions. For example, if two-timers were augmented by different groups of consumers who search over 3, 4 or more, prices then there would still be some who searched only those provided by a particular (joint) firm setting 3, 4 or more prices, and hence were additional captive consumers. Expressions for expected sales would be more complex, but the principle would remain. Similarly, marginal cost could be non-zero and variable, and some kinds of product differentiation introduced, as well as heterogeneous reservation prices for consumers. All these factors would make the analysis more complex but would not stop firms being able to increase the number of their captive consumers by setting multiple prices. Also, some of our assumptions have tended to reduce the possible effect. For example, we have assumed equal chances of any price being selected; however, firms may attempt to place their promotions (for example advertise multiple trading names in the same newspaper) so that if one of their prices is selected another may become more likely to be selected as well.

The conclusion to this paper must be that price comparisons need to be transparent and readily available. In this way, consumers have an incentive to test prices, confident that this is a fair check on price competitiveness. The broader question is whether the state can or should specifically include such issues in judgements in competition policy. Competition policy is currently dominated by market share issues. The theoretical justification for the importance of market share is the Cournot framework where firms exercise monopoly power on the
residual market share they control. Within a price-setting, Bertrand framework, market share is perhaps not so relevant except as an indicator of product substitutability (just as it is not so relevant within the basic model with a homogeneous product that we have used here), but the ability to compare prices is paramount. This conflict can be seen in the response of competition regulators to the promise of both trading names continuing after two firms merge. The response is generally positive: it suggests that less dominance in terms of market share can be achieved through advertising and promotion. However, it also indicates more difficulty for consumers in making true price comparisons, and we have found that this shifts equilibrium price distributions upwards. Perhaps the policy focus should shift at least some way to issues of judging how appropriate price comparisons can be made, so that competition can properly take place.
References


Ireland, N.J. and M. Waterson, 2005, Cartels and Search, mimeo, University of Warwick.


Table 1: Digital Camera Prices on UK Internet Sites, Feb 13th 2002

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| number of prices   | 22       | 30       | 25       | 18       | 20       | 27       | 18       | 25       | 15       | 18       |
| mean               | 644.23   | 522.07   | 410.00   | 251.22   | 359.10   | 292.44   | 629.33   | 294.24   | 486.80   | 299.50   |
| standard deviation | 47.79    | 33.95    | 50.26    | 19.72    | 20.04    | 36.97    | 40.65    | 23.41    | 38.50    | 29.06    |
| (max-min)/min      | 0.40     | 0.40     | 0.53     | 0.30     | 0.23     | 0.64     | 0.28     | 0.43     | 0.39     | 0.43     |
| min price          | 559      | 439      | 359      | 230      | 320      | 230      | 588      | 248      | 400      | 263      |
| max price          | 783      | 613      | 550      | 299      | 395      | 378      | 750      | 355      | 556      | 375      |

Source: Pricerunner.com; all prices rounded to nearest £1; all values in £.
Table 2: Returns from two-timing - $\Gamma(n^j; \theta)$ varying with $n^j$ and $\theta$. In the table, $\Gamma(100; \theta)$ approximates $\Gamma^*(.; \theta)$. The sign of $\Delta$ is the sign of $\Gamma(8; \theta) - \Gamma(100; \theta)$.

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<tr>
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<td>0.072782</td>
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</tr>
</tbody>
</table>

Sign of $\Delta$: positive positive negative negative negative negative negative negative
Figure 1: Relative gains and losses, from two-timing rather than one-timing, for consumers of index $\theta$. 

\[ C(\theta) \]

\[ \Gamma(8; \theta) \]

$\theta_1 \quad \theta_2$
Figure 2: Shift in structure from $\Gamma^*(\cdot; \theta)$ to $\Gamma(8; \theta)$. This decreases $\theta_i$ (and increases the number of two-timers) when the cost schedule is $C$ and increases $\hat{\theta}_i$ (decreases the number of two-timers) when the cost schedule is $\hat{C}$. 