Regulated Limits in Mixed Strategy Oligopoly Equilibria

by

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Abstract

In a simple mixed strategy equilibrium of price offers by sellers faced with possible competition, a price floor set by a cartel reduces the expected price offered, and leaves unchanged both the expected price transacted and the expected profit of oligopolists. An indirect effect is via changing incentives for buyers to implement price search and hence competition. The simple model is extended to strategies of a cartel, or of a regulator, involving both setting limits, floors or ceilings, to price and to product quality. The effects on consumers and suppliers of limiting the extreme outcomes by constraining price/quality combinations, or just price or quality (and leaving the other unconstrained), are examined. The outcomes are sensitive to whether price and quality are positively or negatively correlated within the mixed strategy equilibrium.

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1. Introduction

The main objective of this paper is to examine the impact of a one-sided constraint on the price and/or quality that individual suppliers to a market may select. The constraint may be imposed by a cartel of the suppliers (for example a minimum price) or by a regulator (for example a maximum price). The fundamental non-competitive aspect of the market arises from the tourist / native type of split of customers[1]. Thus some buyers (tourists) simply buy at a randomly selected supplier, subject only to participation constraints. We will term these buyers one-timers since they visit only one supplier. Others (natives) will conduct comparison shopping, comparing what is on offer from a limited number of suppliers and choosing the best deal. We will consider only behaviour where these buyers visit two suppliers, and will term these buyers "two-timers". All customers are assumed to have similar preferences.

We first consider the impact of a minimum price in a market for a homogeneous good where the equilibrium without any minimum price is one of a distribution of prices yielding equal expected profits for each supplier. The source of the variations in price from one supplier to another (the absence of the "law of one price" (LOOP)) is the imperfect information held by consumers. We make the point that a minimum price supposedly enforced to increase the average price set by suppliers may not work. However, it may also reduce the buyers' incentive to seek out low prices, and this may have (indirectly) the effect of raising average prices actually paid. The paradox comes from a common source of unexpected and counter-intuitive results: the equilibrium that we consider will be a mixed strategy Nash equilibrium (MSNE).[2] Although the general argument can be applied to many economic processes, our

[1] Using the terminology of Salop and Stiglitz (1977). The absence of the LOOP is discussed in Carlton and Perloff, (2005). The earliest evidence on the absence of LOOP is provided by Pratt, et al (1979).
[2] For a discussion of problems associated with MSNE, see Holler (1990) and Amaldoss (2002). The latter paper emphasises the role, of the distribution for mixing, in setting the opponent's expected profit to a constant value

model below borrows heavily on notions of equilibria from the classic papers of Wilde and Schwartz (1979), and Burdett and Judd (1983)[3]. Essentially, each firm has some monopoly power arising from the fact that one-timer consumers only observe one firm's price. Hence a high price can be charged to these consumers, or alternatively both these and some other (two-timer) consumers can be supplied if the firm charges lower prices than some other firms. Each firm has the choice of whether to set high prices and sell little, or set low prices and sell more. A Nash equilibrium with mixed strategies is easily established in these circumstances.

An important extension of the model, and one that has not been previously attempted, is to a price / quality "deal" setting strategy. The simple price-setting, homogeneous product model does not reflect the standard case of branded goods. Also, vertical product differentiation is a parallel strategy to price differences. A product can be more expensive but delivered faster (thus a "better" product) and be as good a proposition overall. Indeed, quality differences of branded products are important both in terms of the intrinsic product and in terms of the sales process and after sales service. We will assume that a visit to a supplier (which could be a physical visit, a virtual visit or assimilation of appropriate information) will inform the customer of the price and the quality of the product. The good is thus a search good, rather than an experience good. The need for a visit is enhanced by the need to check quality as well as price. For example, price lists of internet suppliers often omit costs of delivery, the after sales service description is often buried in small print and whether the product has to be sourced from another country or even another continent is sometimes less than obvious. The

within an asymmetric game. Our game will be symmetric but will have the property that a player's expected profit is unaffected by a binding constraint on the range of actions that can be selected.

[3] See also Burdett and Coles (1997).

visit to elucidate these factors may be time-consuming even though internet access is fast and cheap.[4]

This extension enables two real advances. First the contract curve between the profits that can be extracted from a customer and the customer's satisfaction (consumer surplus) from the deal comes into sharp focus. Constraining the good deals for consumers (preventing the lowest prices in a homogeneous good model) is again found not to raise expected profits unless customers change behaviour. The effects on consumers will be shown to relate to the form of the contract curve. At the other end of the contract curve, a welfare regulator can lower profits by removing the worst deals for consumers, but only by imposing a schedule of minimum value for money across all the qualities provided. The regulator's information would have to be very good for this choice of schedule not to be off the contract curve and invite inefficient price-quality combinations by suppliers. Second, the cartel or the regulator might not be able to set schedules like this, but rather set a minimum price or maximum quality (the cartel) or a maximum price or minimum quality (the regulator). We present MSNE for different consumer preferences and suppliers' cost functions. Examples are given where the best deals for the consumer are characterised by the lowest price and lowest quality; by the highest price and highest quality; and by the lowest price and highest quality. That the relative price / quality combination should vary with the attractiveness of the deal to consumers is not surprising. The lessons for the cartel or the regulator in terms of what kinds of constraint should be enforced depend on the detail of the case. The model has the added advantage of depicting the equilibrium supply of different quality products in the absence of second degree price discrimination (since all customers are assumed similar)[5].

[4] For an example of particular quality issues, see Arnold (2000) for an analysis of "stock-outs".
[5] Models demonstrating why different qualities would be supplied in oligopolies where consumers can be dividied by targeting different qualities are based on Jaskold Gabszewicz and Thisse (1979) and Shaked and Sutton (1983).

In the next section we present the basic model of a single homogeneous product where suppliers each set their own price. We then consider the imposition of a minimum price and how this affects both one-timer and two-timer consumers as well as profits. We then extend the analysis to the case where each supplier chooses both a quality of product and a price for the product. Again we consider who gains and who loses from the imposition of a maximum surplus for consumers (a limit on the best deals). Although our main focus is to look at limiting the best deals for consumers, we also contrast this with a regulator's action in limiting the worst deals for consumers. Finally, in section 4, the issue of policies which limit only one of price and quality, while leaving the other unconstrained, is considered. The argument here uses parallel properties to those in the earlier sections and can be applied to both cartel and regulator strategy. Specific illustrations of all our results are given. Conclusions are presented in a final section.

2. A base model of price-setting suppliers

We adopt the following base model for the market with homogeneous products. Many of the more limiting characteristics are removed in the later analysis.

i. Sellers offer a price for a single unit of a good or service to buyers. There are two types of buyer, and the seller cannot discriminate between them. One type (the one-timer) only visits the one seller. She accepts the price offer provided it is no more than the buyer's reservation price of 1. The other type (the two-timer) visits two firms and observes two price offers and chooses the seller offering the lower price, again provided this is no more than 1. A proportion 1-q of buyers are two-timers; q are one-timers.

ii. The seller chooses price to maximise expected profit and there are no costs of supply. The uncertainty faced by the seller only concerns whether the buyers are also seeking a further price offer: that is the type of the buyer.

iii. If the buyer obtains two price offers, and these are the same, the buyer spins a fair coin to choose between the firms.

iv. There are N buyers and n sellers. Each buyer chooses firms to visit in a random way such that any firm is equally likely to be chosen. Given the discontinuities of the payoffs, only a mixed strategy Nash equilibrium can exist: any seller will want to (just) undercut its potential competitor, and no equilibrium can exist with all sellers just undercutting each other. Thus no pure strategy equilibrium can exist.

2.1. Model Setup

The expected profit of the typical seller out of n sellers bidding to supply its share of N buyers and offering the price $p \leq 1$ is

$$p\,(p) = N[\frac{q}{n} + \frac{1-q}{n} 2(1-F\,(p))]p \qquad (1)$$

The explanation of (1) is that the seller's expected share of the N customers is N/n. There are θN/n potential buyers[6] who only consider this particular seller and 2(1-θ)N/n potential buyers who are visiting this seller and some one other seller. (Since each of (1-θ)N two-timers makes two visits there are 2(1-θ)N/n visits by two-timers to the seller.) The chance of selling the unit to the potential buyer who is a "one-timer" and is not looking elsewhere is 1 providing the price set is no higher than 1. The probability of selling to a two timer who is comparing the seller's price with another seller is 1- *F*(*p*), where *F*(*p*) is the probability that the other seller asked makes a bid lower than *p*. It is the distribution function of the other seller's price offer. The typical seller maximises (1) given the *F*(*p*) function of the other

[6] θN/n is actually the expected number of one-timer buyers, but we will drop this distinction: the risk neutrality of suppliers means that nothing is added by treating the number of visiting buyers as a random variable.

bidders. In a MSNE the value of $p(p)$ must be the same for any p played with positive density. The unique (and symmetric) MSNE can be found to be represented by[7]:

$$F(p)=\frac{(2-q)p-q}{2(1-q)p} \quad for\ \frac{q}{2-q}\leq p\leq 1 \qquad (2)$$

and at any price in θ/(2-q) ≤p≤1 we have $p(p)$=θN/n. Since expected profit is the same for all choices of price in this interval (substitute (2) into (1) to check), any firm is indifferent among these choices and cannot do better than make a random selection using the distribution function (2). Not surprisingly, the expected profit of the seller nears N/n when there is little chance of competition (a monopoly equilibrium when$q\approx 1$) and nears 0 when there is almost certain competition (a Bertrand equilibrium when$q\approx 0$). Prices less than q /(2-q) will always beat the competition but will make less profit and are thus never played. Prices above 1 make zero sales and are thus also never played. The expected price set by a seller is

$$E(p)=\int_{q/(2-q)}^{1} F'(p)\,p\,dp \quad =-\frac{q}{2(1-q)}\left[\ln(q)-\ln(2-q)\right] \qquad (3)$$

The distribution function (2) is shown in figure 1a for q =2/3. In this case the expected price in (3) is 0.693, and a firm's expected profit is 2N/3n.

2.2. Implementing a minimum price

Now assume a law is passed, or a rule is made, preventing any price offer less than p^m. For this to have any effect we assume that p^m>q / (2-q), so that some choices in the mixed strategy in section 2.1 are disallowed. The obvious first effect of such a move is to remove these choices from the strategies over which sellers mix. Intuitively we could consider that the weight of density would shift to the minimum price p^m. A spike in the price offer distribution at p^m is possible since (by law) such a price cannot be undercut. We denote the

[7] The distribution of price choice has decreasing density, that is low prices have higher densities tha high prices. This is a common feature of such mixing functions and is replicated in many models, eg Varian (1980) in the duopoly case. In Ireland(1993) the use of advertising messages to determine market size in a comparison pricing model has similar outcomes.

mass at this spike at p^m as s. We then realise that prices just above p^m will not be played since the small extra profit per sale cannot make up for the fact that at p^m the seller also has a half chance of being chosen by a two-timer even if the competitor also plays p^m. The new symmetric MSNE is defined by the size of the spike s, the lowest price greater than p^m that is played with positive density – which we will denote p*- and the distribution function for prices between p* and 1. These are found from the following conditions of equal expected profit for all choices played with positive density:

$$
\begin{aligned}
&p\,(p) = N[\frac{q}{n} + \frac{(1-q)}{n} 2(1-F(p))]\,p \quad \textit{for} \quad p^* \le p \le 1 \\
&= p\,(p^m) = N[\frac{q}{n} + \frac{1-q}{n} 2\left\{(1-s) + \frac{q}{2}s\right\})]\,p^m \quad \textit{for}\ p = p^m
\end{aligned} \qquad (4)
$$

In (4), the expected profit from any price between p* and 1 is as in (1). The expected profit from choosing p^m is given by applying the expected sales: an n^{th} share of one-timers and a $2/n^{th}$ share of two-timers conditional on the competitor not also choosing p^m, or the competitor choosing p^m but then losing the 50-50 chance of getting the two-timer's sale when both sellers play p^m. Playing a price between p^m and p* yields the same expected sales as p* but at a lower price: hence no firm plays prices in this interval.

Proposition 1. A MSNE exists and has the form

$$
\begin{aligned}
&F(p) = \frac{(2-q)\,p - q}{2(1-q)\,p}, \qquad \textit{for}\ p^* \le p \le 1 \\
&p^* = \frac{q\,p^m}{2q - (2-q)p^m} \\
&F(p^*) = s = \frac{(2-q)\,p^m - q}{(1-q)\,p^m}
\end{aligned} \qquad (5)
$$

where s is the probability spike of playing p^m, provided that $\frac{q}{2-q} < p^m < q$.

Proof. Use (4) to obtain p(p)=p(1)=Nq /n (since F(1)=1), then p(1)=Nq /n = p(p*) = (N/n)[q +(1-q)2(1-s)]p* (since F(p*)=s, the spike at p^m), and combine these with the equation for $p(p^m)$. We have three equations in $F(p)$, s and p*. If p^m is larger than θ then s=1 and only p^m is played; if p^m is smaller than the lower bound the minimum price has no effect, since we could set p^m at $\frac{q}{2-q}$, and then $p^* = \frac{q}{2-q}$.

The distribution of prices for the case q = 2/3 and p^m = 0.6 is shown in Figure 1b. Here s=2/3 and p* = 0.75 from (5). The expected price set by an individual seller is given by

$$E(p) = \int_{p^*}^{1} F'(p)\,p\,dp + sp^m = \frac{-q}{2(1-q)}\ln(p^*) + sp^m \qquad (6)$$

If p^m=q / (2-q) then (3) and (6) are of course the same since p*=p^m and s=0. If q = 2/3 and p^m = 0.6, then (6) takes the value 0.688: for these values, raising the minimum price from the unrestricted lower bound of 0.5 to 0.6 has the effect of reducing the average price set from 0.693 to 0.688. To demonstrate that this is no numerical accident, we can find how (6) changes when p^m increases to state the following:

Proposition 2. For all p^m in the interval $\frac{q}{2-q} \le p^m \le q$, we have (a) the expected price posted by a seller is decreasing in p^m.; (b) Expected profit (and equivalently expected price paid or transacted) is unchanged if p^m changes.

Proof. Differentiate the expected price set (the average price per firm) given by (6):

$$
\begin{aligned}
&\frac{dE(p)}{dp^m} = -F'(p^*)\,p^*\frac{dp^*}{dp^m} + s + p^m\frac{ds}{dp^m} = \\
&-\frac{q}{2(1-q)p^*}\frac{2q^2}{(2q-(2-q)p^m)^2} + s + p^m\frac{q}{(1-q)p^{m^2}} \\
&= -\frac{q}{2(1-q)p^*}\frac{2p^{*2}}{p^{m^2}} + \frac{q\,p^m}{(1-q)p^{m^2}} + s \\
&= \frac{1}{(1-q)p^m}\left(q - q\,p^*/p^m + (2-q)p^m - q\right) \\
&= \frac{1}{(1-q)p^m}\left(\frac{q(q-(2-q)p^m)}{2q-(2-q)p^m} + (2-q)p^m - q\right) \\
&= \frac{(2-q)p^m - q}{(1-q)p^m}\left(1 - \frac{q}{2q-(2-q)p^m}\right) = -\frac{((2-q)p^m - q)^2}{(1-q)p^m(2q-(2-q)p^m)} < 0
\end{aligned}
\qquad (7)
$$

We can also see that expected profit for each seller is still the same for any $p^m<q$: it is $q\,N/n$. Thus average prices paid by all buyers must be the **same** since each buyer buys just one unit.

The negative sign of (7) shows that any increase in the minimum price constraint, within the bounds $q/(2-q) < p^m < q$, will **reduce** the expected price set by any particular seller. This means that those buyers who rely on a single offer price will expect to pay **less**. Since buyers who rely on a single price expect to pay less, it follows that buyers who consider two price offers expect to pay **more**, the higher is the minimum price. Only in this way will the supplier's expected profit and the average transacted price remain unchanged. Thus we can state Proposition 3.

Proposition 3. The impact of the minimum price for $q/(2-q) < p^m < q$ has been to leave profits and average transaction prices the same ($q\,N/n$ and q respectively) but to have reduced the expected price difference paid between two-timers and one-timers.

Proof. Expected profit is still $q\,N/n$ as suppliers still select a price of 1 with positive density. Multiplying by n yields aggregate profit and dividing by N yields profit (equivalently revenue) per customer and hence q is the expected price transacted. The expected price for one-timers decreases with p^m (from Proposition 2, noting this is just the average price per firm since one-timers purchase at the one firm they visit), and so the expected price paid by two-timers must increase.

Our analysis has held the proportions of one-timers and two-timers as fixed. What we have shown is that the impact of the minimum price has been to leave average prices transacted unchanged but to reduce the consumer's incentive to consider two prices rather than one. We might suppose that q would increase to $q' > q$ as a result of this lower incentive. This would then have the effect of raising average transacted prices to q' (Proposition 3). Thus we have shown that although there is no direct effect on profits from introducing a minimum price on the level of prices paid, there may be an indirect effect if consumers respond by lower search activity. The lower gains from acting to compare price offers means that effective competition is reduced.

Of course, a cartel may be able to implement a minimum price above the range that Propositions 1-3 relate to. Note however that the result then would be a uniform price (p^m) set by all suppliers. This may arouse suspicion of the anti-trust authorities in a way that a MSNE does not. If this is the case, then our analysis has shown that there may be little scope for cartel action: either the minimum price is less than q and there is no direct gain in terms of suppliers' profits, or the minimum price is adopted by all cartel members and is then likely to be conspicuous and investigated. There remains however the possibility that a cartel might implement some measure of minimum price so as to reduce the tendency for consumers to shop around. Since the proportion of consumers who are one-timers is the source of all profit in the model, increasing q is a valuable outcome for the cartel members.

We can contrast this picture with the rather different situation where a regulator puts an upper limit u<1 on prices. In this case of a price ceiling, expected profit is just *uqN/n*, and the distribution of prices is shifted downwards. The lowest price played with positive density is $\frac{qu}{2-q}$ and the distribution function now becomes

$$F(p) = \frac{(2-q)p - qu}{2(1-q)p} \quad for\ \frac{qu}{2-q} \le p \le u \tag{2u}$$

and expected price set is

$$E(p) = \int_{qu/(2-q)}^{u} \frac{qu}{2(1-q)p^2} p\,dp \quad = \int_{q/(2-q)}^{1} \frac{qu}{2(1-q)x}\,dx \tag{3u}$$

by using a transformation of variables: p/u = x, dp/dx = u. Equation (3u) is just u times equation (3). Also expected profit is just u times the amount when the maximum price was 1. Hence we have the following simple proposition.

Proposition 4. An upper limit on prices of u<1 implies a MSNE with expected profit, expected price paid by one-timers, and expected price paid by two-timers all u times the results in the unlimited case.

Proof. Expected profit is *quN/n* rather than *qN/n* . (3u) gives the result for one-timers directly. If profit changes by a factor of 1-u and expected prices paid by one-timers change by the same factor, then expected prices paid by two-timers must also change by the factor 1-u.

Corollary. If the expected prices paid by one-timers and two-timers change by the same proportion then the absolute difference in the expected prices must decline. Hence there may be a reduction in the number of two-timers due to the imposition of a maximum price, and this will redress some of the effect of the maximum price u imposed by the regulator.

This corollary is the only similarity between the effects of imposing a minimum as compared to a maximum price. It reflects the reduction of extreme outcomes lessening the pay-off from search. Otherwise, the constraint at the upper end of the price distribution has an affect on the whole distribution of prices while the constraint at the lower end only leads to a local concentration around the minimum price.

3. Generalisation: sales levels, product quality and consumer value

In the model of the last section, sellers and buyers were exchanging money for a single unit of a good. Each consumer bought one unit, and so no equilibrium had a better overall welfare outcome than any other: any higher profits due to higher prices were matched by an equal loss in consumers' surplus. In this section we will extend the model to encompass a much richer and more general economic context. This will include varying quantities of sales to buyers (dependent on price and product quality), and hence buyers facing higher prices or lower quality will buy fewer units and this will result in lower aggregate welfare due to allocative inefficiency. Also the profit of sellers will have a natural upper bound, rather than one imposed by an arbitrary maximum reservation price. These extensions will come from identifying how consumer surplus depends on the per customer profit that a seller extracts. The results will build on the simple model of the last section, and will confirm that model as a special but central case.

3.1 Market operation

Consider a two-stage process of the following kind. In the first stage sellers simultaneously set quality and price of the product, and homogeneous buyers randomly select sellers to visit (one or two, in the same way as in section 2). The buyers then observe prices and product quality offered by the sellers, and two-timers choose the seller that provides them with a better deal. In the second stage each buyer chooses the quantity q she wishes to purchase, given the price and quality she faces. We define $R(p,v,\ q(p,v))$ as the profit that a seller extracts from a customer who buys from that seller. The quantity decision comes from the buyer maximising $V(v,q)$-pq with respect to q, where V is the consumer's valuation or reserve price of q units of quality v. One can also view V equivalently as the cost of obtaining the utility from the quality and quantity (v, q) if the market does not exist and only outside products can be consumed. Then $V(v,q)$-pq is the compensating variation from not being able to participate in the market. In the first stage, sellers solve the programme

$$\max_{p,v} C = V(v,q(v,p)) - pq(v,p) \text{ subject to } R(p,v,\ q(p,v)) \geq R. \tag{8}$$

The achieved value of consumer surplus is negatively dependent on the constraining value R: C=$c(R)$, with $c'(R) < 0$. We thus have the following sequence. A firm sets p and v. A buyer arrives at the seller and, if a two-timer, compares the achievable surplus there with its other observation, and goes to the seller that offers a higher c. All buyers remaining at the firm choose the optimal quantity to buy. The firm obtains a per customer profit of R and the buyer a surplus of $c(R)$. When setting p and v, the seller has to take into account the need to offer a competitively high c to buyers in order to keep some two-timers, while extracting per customer profits (R) from one-timers and from those two-timers who remain. The ranking of c offered across sellers is inverse to the ranking of R. (In section 2, R was just price p, while c was just $-p$.) In this model we can think of R^H as the maximum of R over all price, quality combinations, with just the individual buyer's demand function $q(p,v)$ acting as a constraint. Playing R^H would mean that no two-timers would be served, since other firms would play lower R values with probability one. We define $F(R)$ as the distribution function of the values of R implied by the price and quality decisions made by sellers. The expected profit of a seller is (cf equation (1))

$$p\,(R) = N[\frac{q}{n} + \frac{1-q}{n}2(1-F(R))]R \qquad R^L \leq R \leq R^H \tag{1a}$$

We want to consider the impact of a rule enforcing a minimum R, denoted R^m. If this was below R^L it would have no effect. However, for $R^m > R^L$ it would stop sellers from offering too good a deal: for any given quality, it would require a minimum price, and for any given price it would require a maximum quality.

The fact that quality and price are both selected by the seller, even if under a constraint, is an important ingredient in justifying the search nature of acquiring information. It is no use for consumers to find out a seller's price without also finding out the seller's product's quality.[8] To extend the model to different qualities also means that we encompass differentiated product markets with branded goods. At an extreme we could consider both p and v as vectors, and the seller (a supermarket or a restaurant) providing a number of products. A two-timer visits two sellers and makes comparisons; a one-timer relies on others to ensure the package is competitive. A supermarket cartel might illegally restrict the bargains available as part of a market-sharing agreement; a local jurisdiction might only license expensive restaurants; drug companies might enforce minimum treatment costs. If R^m is the rule for the minimum per customer profit then the same argument as in section 2 yields the market equilibrium for three ranges of R^m.

(a) If $R^m < qR^H/(2\text{-}q)$ then the rule has no effect:

$$F(R) = \frac{(2-q)R - qR^H}{2(1-q)R} \quad for \ \frac{q}{2-q}R^H \le R \le qR^H \qquad \text{(2a)}$$

$$E(R) = \int_{qR^H/(2-q)}^{R^H} F'(R)RdR \quad = \frac{R^H q}{2(1-q)}\left[-\ln(q) + \ln(2-q)\right] \qquad \text{(3a)}$$

(b) If $qR^H/(2\text{-}q)<R^m<qR^H$ then the "minimum per customer profit" rule changes the mixed strategy equilibrium:

$$F(R)=\frac{(2-q)R-qR^H}{2(1-q)R} \quad \textit{for } R^* \le R \le R^H$$

$$F(R^*)=s \qquad \text{(2b)}$$
$$\Pr(R=R^m)=s$$

$$R^*=\frac{qR^mR^H}{2qR^H-(2-q)R^m}$$
$$s=\frac{(2-q)R^m-qR^H}{(1-q)R^m}$$

(c) If $R^m \ge qR^H$ then the rule yields a pure strategy equilibrium:

$\Pr(R=R^m)=1$

These results are no different to the outcomes in section 2, except that R replaces p and R^H is determined as the per customer profit maximum rather than consumers' reservation price. However, the issue of who gains and loses from an increase in R^m in region (b) above is more complex since it is no longer a zero sum game between sellers and buyers. We will show below that the complexity arises from any non-linearity in the trade off between the seller's profit from a consumer (R) and the maximum consumer surplus this permits: $c(R)$, with $c'(R)<0$.

3.2. Identifying winners and losers

Let expected profit, and expected consumer surplus of a one-timer consumer and a two-timer consumer, be given by π, EC_1 and EC_2 respectively. We have $\pi = qR^HN/n$ for all prices played with positive density, and

[8] If buyers started using easily-obtained price data as evidence of both price and quality it would be in the

$$E(C_1) = \int_{R^*}^{R^H} F'(R)\, c(R)\, dR + s\, c(R^m)$$
$$E(C_2) = \int_{R^*}^{R^H} F'(R) 2(1-F(R))\, c(R)\, dR + s(2-s) c(R^m) \tag{9}$$

The probability of getting the highest available surplus $c(R^m)$ for a one timer is just s while for a two-timer it is $2s(1\text{-}s) + s^2 = s(2\text{-}s)$ (the probability that at least one of the two sellers visited plays R^m). The expression $2F'(R)$ $(1\text{-}F(R))$ in the integrand in $E(C_2)$ is the density of the lower of two random variables each with distribution function $F(R)$. Clearly p is independent of R^m in region (a) and we will assume that region (c) is unattainable. We have two propositions extending propositions 2 and 3 to the more general setting.

Proposition 5. If R^m is increased in region (b), one-timers definitely gain if $c(R)$ is concave and two-timers definitely lose if $c(R)$ is convex.

Proof. See Appendix

Proposition 6. The average or aggregate consumer surplus across all consumers, that is:

$$\begin{aligned} E(AC) &= \int_{R^*}^{R_H} F'(R)\,[q + (1-q)2(1-F(R))]\, c(R)\, dR + s[q + (1-q)(2-s)]\, c(R^m) \\ &= q\, E(C_1) + (1-q) E(C_2) \end{aligned} \tag{10}$$

is increasing in R^m if $c(R)$ is strictly concave and decreasing in R^m if $c(R)$ is strictly convex. If $c(R)$ is linear then average consumer surplus is unaffected (the case in section 2). Note aggregate consumer surplus is simply N times the average consumer surplus.

Proof. See Appendix.

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interests of suppliers to drop their quality and we would have another model. We do not consider these possibilities further here.

The intuition of these results comes from the fact that the minimum profit per customer draws density from both below and above R^m. A concave function is higher at this middle value than the average of points above and below. Thus consumers do better overall when the variation of c is less if $c(R)$ is concave. Similarly they do worse if $c(R)$ is convex. Add to this the fact that the value from two-timing is reduced due to the more concentrated distribution, and both propositions become clear.

Finally, we can extend our results about the difference in incentives for two-timing to the more general case. In the Appendix, equations (9) are found to be

$$\frac{dE(C_1)}{dR^m} = 2[F(R^m)\left(\frac{R^*}{R^m} - 1\right)(c(R^m) - c(R^*)) - F(R^m)SOT]/(R^* - R^m)$$
$$\frac{dE(C_2)}{dR^m} = \frac{q}{(1-q)(R^* - R^m)} F(R^*)[\left(1 - \frac{R^*}{R^m}\right)(c(R^m) - c(R^*)) - (2 - F(R^*))SOT] \tag{9′}$$

where SOT is the second-order term of the expansion of $c(R)$. First suppose that SOT=0. Then subtraction of the first equation in (9′) from the second gives a clear negative value since $R^* > R^m$. This replicates the earlier result of section 2: a higher minimum profit per customer reduces the surplus of two-timers more than one-timers. However, the terms in SOT do not have a clear sign and so the effect of any non-linearity of the $c(R)$ function on incentives becomes unclear. The sign ambiguity reflects the changes in aggregate consumer surplus and these influence the trade-off between one-timers and two-timers. We thus have the weaker result:

Proposition 7. If the function $c(R)$ is linear then increasing the minimum R^m reduces $E(C_2)$ and increases $E(C_1)$ and thus reduces the difference between them, and hence the incentive to two-time. If the second-order term is non-zero, then the coefficient of SOT in $\frac{d(E(C_2) - E(C_1))}{dR^m}$ is certainly negative if q>0.5, and certainly positive if q<1/3. Thus the incentive to two-time is certainly reduced if $c(R)$ is convex and q>0.5, or if $c(R)$ is concave and q<1/3.

Proof. Subtract the first equation of (9′) from the second. Note that $s=F(R^*)$ is bounded between 0 and 1.

In the next sub-section we give simple examples to demonstrate the existence of both concave and convex $c(R)$ functions. We will use the illustrations again in section 4 when we consider how price and quality depend on the "deal" for the consumer and how restricting one of price or quality by imposing a floor or ceiling will affect the market equilibrium.

3.3 Illustrations

i. The fixed budget case

Let each consumer have preferences such that they spend a budget m on the product being offered by the firms in the market we are considering or on some "outside" product. The preferences are described by a utility function and are such that consumers either spend all of m or none of m on the outside good:

$$U = f(vq + x)$$
$$m = pq + x$$

where f'>0; x is the quantity of an outside good with price 1, and v is the quality of the good. This quality sets the slope of the linear indifference curves in (q, x) space. Demand for the good we are considering is

$$q = \frac{m}{p} \qquad \text{if } \frac{v}{p} \geq 1$$

$$q = 0 \qquad \text{if } \frac{v}{p} < 1$$

The monopoly behaviour in our market is thus to set $v/p=1$. Setting lower price p or higher quality v gives the consumer a surplus which we measure as the compensating variation from enforcing consumption of x alone:

$$C(p,v)=v\frac{m}{p}-m$$

The firm's profit (R) from a single consumer who chooses to buy from the firm is assumed to be revenue pq minus a cost function dependent on the quantity and quality of the good:

$$R=pq-(v+q)^{a}, \qquad a>0$$
$$R=m-(v+\frac{m}{p})^{a} \qquad \frac{v}{p}\geq 1$$

Substituting price in terms of R into $C(p,v)$ leads to

$$C(p(R,v),v)=v(m-R)^{\frac{1}{a}}-v^{2}-m$$

Maximising consumer surplus with respect to v then gives

$$\begin{aligned} v&=\tfrac{1}{2}(m-R)^{\frac{1}{a}} \\ p&=2m(m-R)^{-\frac{1}{a}} \end{aligned} \qquad (11i)$$

and then

$$c(R)=\frac{(m-R)^{\frac{2}{a}}}{4}-m \qquad (12i)$$

and $c''(R)$ is positive or negative or zero depending on whether the value of a is less than or greater than or equal to 2:

$$c''(R)=(\frac{2}{a})(\frac{2-a}{a})\frac{(m-R)^{\frac{2-2a}{a}}}{4} \qquad (13i)$$

From Propositions 5 and 6, if a<2 then SOT>0 and two-timers and aggregate consumers lose with an increase in R^{m}. If a>2 then one-timers and aggregate consumers gain. If a=2 then one-timers gain, two-timers lose and there is no change in aggregate.

We can also note, from (11i), that price increases with per customer profit, while quality decreases with per customer profit. Thus the best deals for consumers have low prices and high quality, and the worst deals have high prices and low quality. We will return to the implications of this for restrictions on just one of v or p in the next section.

ii. Where consumers' gain from quality is independent of quantity consumed.

In this illustration, quality v relates to lump sum gains for the consumer. For example, there may be an easy way to pay for or order the required number of units of the good. Also there may be a warranty which allows the transaction to be nullified (eg the goods all returned), or access to future information about other products (eg the consumer could be put on mailing lists for future catalogues or special offers). We assume quasi-linear preferences of the form

$$U = v + f(q) + x = v + f(q) + m - pq$$

and let $q = g(p)$ be the optimal demand for the consumer. The consumer's surplus from the market is then taken to be

$$C(p,v) = v + f(g(p)) - pg(p)$$

The firm's profit per customer is assumed to take the form

$$R = g(p)(p - bv) - tv$$

so that quality costs are linear, partly a fixed cost per consumer and partly a cost per unit supplied. Since both C and R are linear in v we can this time substitute v from the profit equation into the consumer surplus equation to obtain

$$v = \frac{(g(p)p - R)}{g(p)b + t} \qquad (11ii)$$

$$C(p, v(R,p)) = \frac{(g(p)p - R)}{g(p)b + t} + f(g(p)) - g(p)p \qquad (12ii)$$

Assume that $p^*(R)$ maximises $C(p,v(R,p))$ for given R. Then the result is $C(p^*(R),v(R,p^*(R)))$ and we can write this as a function $C^*(p^*(R), R)$. Write C^*_1 as the derivative of this function with respect to the first argument, etc. and we know that C^*_1=0, C^*_{11}<0 and dp^*/dR = -C^*_{12}/C^*_{11}. Total differentiation of $C^*(.,.)$ with respect to R yields

$c'(R) = C^*_1 \, dp^*/dR + C^*_2$

$c''(R) = C^*_{11} \, (dp^*/dR)^2 + 2C^*_{12} \, dp^*/dR + C^*_{22}$ when $C^*_1=0$.

Using $dp^*/dR = -C^*_{12}/C^*_{11}$ yields

$$c''(R) = -C^{*2}_{12}/C^*_{11} + C^*_{22} \qquad (13ii)$$

Since C^*_{22}=0 in this case, we have that $c''(R) = -C^{*2}_{12}/C^*_{11} > 0$ since C^*_{11}<0 as a second order condition for p^*. Thus $c(R)$ is always convex in this example. Consumers lose on average from a limit on good deals.

Again assuming that p^* maximises c for given R, elementary comparative statics and C^*_{12} < 0 shows that p^* decreases as profit per customer increases. This implies that v decreases faster than the value to the consumer of lower prices as R increases. Thus the better "deals" for the consumer are when price and quality are both highest, and the worst deals when they are lowest. (This is the case where the two-minute haircut for a dollar means you never take your hat off). Again we return to the implications of this in the next section.

iii. Where quality reduces the value of the alternative good

Consider that all individual customers have preferences over whether to buy one unit or zero units of the good (assume that other quantities are all inferior), where

$$U = \max\left\{\frac{m}{b}, v + \frac{m-p}{b+v}\right\}$$

The interpretation is as follows. The first option is to buy only the alternative product, that is m units (since the alternative product is priced at 1) and obtain m/b units of utility. The second option is to buy one unit from the market we are investigating, at quality v and price

p. The utility from consuming m-p units of the alternative good is reduced the higher the quality of this unit. The two arguments coincide in value when v=p=0. The consumer surplus can be written as the extra income needed to obtain the same utility if the market did not exist:

$$C = b[v + \frac{m-p}{b+v}] - m$$

Let the firm's profit per customer from supplying one unit per customer be

$$R = p - av^2$$

Then

$$C = b[v + \frac{m-R-av^2}{b+v}] - m \qquad (12iii)$$

A maximum of with respect to v of (12iii) exists provided a>0. Similar to the last example, we have $c(R) \equiv C^*(v(R),R)$ where $v(R)$ is the maximising value of v for given R, as having a positive second derivative: $c''(R)>0$. Thus two-timers certainly do worse with the implementation of a minimum R^m. However, the relation of quality and price to where the best and worst deals for the consumer can be found is inverted. Since $C^*_{12}>0$, we know that $v'(R)>0$. Then from the firm's profit per customer function we have that $dp/dR>0$. Thus the best deals for the consumer here are when price and quality are low, and the worst are when price and quality are high. (The worst case here is the first-class ticket that costs several times as much as tourist class, but tourist class is full.)

4. Other Policies

4.1Restricting maximum profit per customer

An obvious policy for a regulator may be to restrict the maximum R, thus reducing R^H. Suppose that at any allowable R, suppliers choose the best price and quality in terms of maximising consumer surplus (which they would want to do to retain more two-timers).

Then $c(R)$ would be the same over unconstrained values of R. Also, the expected profit of all suppliers would be reduced, so that the MSNE would reflect a lower R^H and a higher distribution of consumer surplus. Consumers benefit without doubt and the analysis is no different to Proposition 4. However, there must be some question of the regulator's ability to design an instrument, to limit the highest profit outcomes, that does not also introduce inefficiency into the contract curve between buyers and sellers by distorting the price / quality mix. We consider this and related issues below.

4.2. Restricting price or quality but not both

In section 3 we saw how a cartel might limit the competition for consumers by restricting the lowest R (highest c). One can imagine the cartel gathering its members together, assessing the qualities of products that could be offered for sale and coming up with a minimum price for each such product. We saw that this had no direct effect on expected profit, but did have effects on consumers' benefit from the market, and these effects were different for those consumers who bought at the first supplier they met rather than compared provision across two suppliers. Also, a regulator who imposes a restriction on the maximum R that suppliers could extract would achieve a lower distribution of R values and a higher distribution of c values for all consumers: simply reduce R^H in the analysis. Regulators' ability to do this would depend on their ability to assess qualities, profits and consumer value. In this section we consider the model as a testing ground for partial regulator and cartel policy. We ask questions about the effect of restricting price or quality (but not both) from above or below. The key point is whether the supplier who does worst or best for the consumer (extracts highest or lowest profit per customer) sets the highest or lowest quality and the highest or lowest price. We can clearly discard the possibility that the supplier who does worst for the consumer sets the highest quality and the lowest price! We are then left with three cases, and these have been illustrated in section 3.3. In both cases i and ii, the lowest quality is provided by a supplier playing R^H, the highest profit per customer, and lowest consumer surplus. In case i, that supplier also sets the highest price, while in case ii that supplier sets the lowest price. In case iii, the highest quality and highest price is provided at the highest profit per customer and the lowest consumer surplus. Figure 2 provides the basis for the analysis.

Consider the four diagrams in figure 2i relating to case i. In (a), we see that the maximum R is achieved when $v=p$, and lower R, permitting positive consumer benefit, occurs when $v>p$. In (b) and (c) we see the effect of a regulator constraining v and p respectively. In (b), a minimum quality level is set. Then the previous R^H is no longer feasible for suppliers since they are not allowed to adopt the same quality / price combination. Thus the maximum profit per customer is $R^{H'} < R^H$ and so the lowest profit per customer $R^{L'} = q\ R^{H'} < R^L$. The distribution has thus shifted to the left. The outcome for suppliers is that their expected profit $Nq\,R^{H'}/n$ has declined, while those customers fortunate not to choose the highest R suppliers have gained or been unaffected. The customers choosing the highest R suppliers have an ambiguous outcome: R values are lower but the mix of quality and price is inefficient and this reduces benefits. A very similar story exists for the regulator setting a maximum price: the only consumers who could suffer are those most affected by the inefficient mix of quality and price.

In (d) we see the result of the supplier's cartel setting a minimum price when higher quality permits the same distribution of profit per consumer as before. Thus the supports of the distribution are unchanged but when the minimum price is binding there is an inefficiency due to the enforced higher price and quality compared to the no-constraint case. The c value that can be supported at any R affected by the constraint is less than without the constraint.

Thus suppliers have not gained from the minimum price due to competition over quality. The inefficiently high quality and price has led to consumer losses.

In case (ii) we see from Figure 2ii (a) that both quality and price decline with profit per customer: thus price declines but quality declines even faster, leading to higher profits for the suppliers. In (b) we see that setting a minimum quality has the same impact as in case i. The only difference is that, for the region where quality is constrained not to fall further, price has to increase to improve per customer profits. Thus the same price can be linked to two different qualities in equilibrium.

When price is constrained from above (presumably by a regulator) or below (presumably by the cartel) than very different outcome are seen compared to case (i). Restriction by setting a maximum price in case (ii) is qualitatively similar to setting a minimum price in case (i): good deals for the consumer (low R values for the suppliers) are less good since the mix of price and quality is inefficient. On the other hand, setting a minimum price will affect bad deals for the consumer (high R for the suppliers). Since maximum R is reduced by the constraint, the distribution is shifted down, benefiting consumers generally. One can see that this case is one where the regulator should not set a maximum price and the cartel should not set a minimum price. Indeed it is in the interests of the regulator to set a minimum price. The argument is that profits per customer are highest for those suppliers who offer very low qualities and fairly low prices. Only the one-timers buy from these suppliers, but expected profit for all suppliers is conditioned by the return from the low-quality strategy. A minimum price implies that the "monopoly" quality level is higher (to persuade customers to buy more at the higher price) and this reduces the profit per customer which drives the competition for two-timer customers. The outcome is fairly similar to a minimum quality constraint.

In case iii, the interpretation is just the inverse of case ii and so no figure is provided. Here the regulator may do well to limit either high prices or high quality since either reduces the monopoly profit per customer, albeit by also introducing an inefficiency. A minimum price or minimum quality level would have the effect of introducing an inefficiency with no effect on firms' profits, and would not be a robust policy choice for a cartel.

5. Discussion and Conclusion

We have investigated exogenous floors and ceilings within a MSNE. The floors and ceilings have been limited in their extent: we have not been concerned with the case where a cartel could implement monopoly prices for all members with impunity, nor where a regulator could implement competitive pricing. Our exogenous control has left a revised MSNE, not uniform behaviour. We have seen stark differences in the efficacy of controlling the monopoly end of the distribution relative to the competitive end. A simple price-setting, homogeneous product model showed that a regulator could essentially shift the price distribution down by limiting the top values, while a cartel could only effect a concentration from above and below around the minimum price it set. The regulator could shift profit into consumer surplus but the cartel could not do the reverse. Furthermore there is a more adverse effect on two-timers than one-timers by any attenuation of the distribution of prices, so that either a minimum price or a maximum price might lead to fewer two-timers and thus a stronger monopoly position for suppliers.

The extension to a more general model where profits per customer are traded for the generosity of the deal offered to consumers was found to be very straightforward. We considered just two dimensions of the "deal": price and quality of the product. We could have extended this to any number of relevant dimensions, or have disaggregated the notion of quality into a number of constituent parts. In the extended model there is a non-zero effect of the choice of profit/deal mix on the level of aggregate welfare. Thus average or aggregate consumer surplus will be affected by a minimum level of deal either positively or negatively according to whether the profit/ deal trade-off is concave or convex. The intuition here is that the minimum constraint will reduce the range of profit per customer and thus improve the average deal for consumers if customers' surplus is a concave function of profit – in the same way that a risk averse individual will benefit from a lower variance of outcomes. One of the most interesting outcomes from the extended model is the inefficiency introduced if only one part of the "deal" is constrained. Substitution of lower quality for a disallowed higher price, for example, is a welfare cost to put against benefits arising from restricting full monopoly outcomes. In different cases the best deals can occur at the lowest prices and highest qualities; lowest prices and lowest qualities; and highest prices and highest qualities. Identifying where the most and least abuse of monopoly power takes place is a precursor to either restricting the best deals or restricting the worst deals for consumers. The illustrations we have considered in section 3ii and used in section 4 are all monotonic in the relation of price and quality to profit per customer. Clearly preferences could change over the range of product price and type so that monotonicity could be lost. We have not considered extensions to such cases here.

The application of the analysis can be put into different settings with no added complexity. A different interpretation is obtained by thinking of suppliers making a random price / quality offer for a single customer contract (for example to paint a house). A supplier does not know if the potential customer is to obtain a further quotation or not (thus making the customer a two-timer or one-timer). In this case each customer is a potential contract. A minimum price for such a contract may be imposed in order to protect factor incomes: either the price is the wage for the job, or it is a price of a product which then determines the wage or capital return that is paid in its production. The assumption that profits are simply the sum of profits from each customer is particularly appropriate here. One application would be to the labour market: in bidding for employment, discounts on the monopoly wage give a greater chance of success of obtaining the job and the model of section 2 has direct relevance.[9]

Our application to minimum prices or best "deals" represents an example of a more general process. Consider any symmetric MSNE, with one support defined endogenously (in section 2 the reservation price of buyers was 1 and this defined the upper support; in section 3 it was found from monopoly behaviour). Then imposing a change on the lower support has no effect on the expected payoff for a player since the upper support will still always win, and its value is unchanged. The treatment of the supports of the mixed strategy distributions could be reversed. For example, consider bidding for a dollar prize in a symmetric, complete information, all-pay auction (Baye et al (1996)). The lower support of the distribution of bids has to be zero since any other lower support would always lose the auction but would lose more than playing zero. Putting on a **maximum** limit for bids to replace the upper support has no effect on the payoff from playing zero, and hence no effect on the payoffs. Since the prize is always a dollar, this means that the expected bid is also unchanged.

One of the simplifications of our analysis has been to treat all customers the same at the service point they select. The possibility of discriminating between them has not been considered but one can envisage ways in which this might be achieved. In some situations two-timers may be "new" customers, and then introductory bonuses may enable more

[9] The analysis of a minimum wage has largely considered its effect on a matching equilibrium and hence its

competitive bids for such new customers while denying these to “old” customers. Either from the point of view of the single supplier, or from any cartel representing suppliers as a group, this leads to issues in the design of such bonuses, and how these are affected by attenuation of the distribution of consumer deals. These issues will be explored in further research but would relate to the optimal design of prizes in contests (Moldovanu and Sela, 2001).

effect on the single equilibrium wage for some sub-market. The key issue is usually the effect on the amount of transactions (employment). See for example Masters (1999).

Appendix: Proofs of Propositions 5 and 6.

Proposition 5. If R^m is increased in region b, one-timers definitely gain if $c(R)$ is concave and and two-timers definitely lose if $c(R)$ is convex.

Proof.

Before proceeding note the following useful relationships which can be derived immediately from (2b):

$L1$: $s = F(R^*) = 2F(R^m)$

$$L2: \quad \frac{dR^*}{dR^m} = \frac{2R^{*2}}{R^{m^2}}$$

$$L3: \quad F'(R^*)\frac{R^{*2}}{R^{m^2}} = F'(R^m)$$

$$L4: \quad F'(R^m)(R^* - R^m)R^m / R^* = F(R^m)$$

We can also use the following exact Taylor's expansion of $c(R^*)$ around $c(R^m)$:

$c(R^*) = c(R^m) + c'(R^m)\,(R^*\text{-}R^m) + \text{SOT}$

where the second order term (SOT) is evaluated at some R in the open interval (R^m, R^*) and is positive when the function is convex and negative when it is concave. We then obtain

$L5$: $c'(R^m) = \{[c(R^*) - c(R^m)] - \text{SOT}\}/(R^*\text{-}R^m)$

Now write

$$\frac{dE(C_1)}{dR^m} = [-F'(R^*)c(R^*) + F'(R^*)c(R^m)]\frac{dR^*}{dR^m} + sc'(R^m)$$
$$= 2[-F'(R^m)c(R^*) + F'(R^m)c(R^m)] + 2F(R^m)c'(R^m)$$

By using $L1$, $L2$ and $L3$. Then apply $L5$ and $L4$ to obtain

$$\frac{dE(C_1)}{dR^m} = 2[F(R^m)\left(\frac{R^*}{R^m} - 1\right)(c(R^m) - c(R^*)) - F(R^m)SOT]/(R^* - R^m) \qquad (9?a)$$

A sufficient condition for (9?a) to be positive and for the one-timer consumer to gain from the higher minimum per customer profit is that SOT $\leq$ 0. This is assured if $c(R)$ is concave.

Now consider the effect on two-timers. We have

$$\frac{dE(C_2)}{dR^m} = [-F'(R^*)2(1-F(R^*))c(R^*) + F'(R^*)2(1-F(R^*))c(R^m)]\frac{dR^*}{dR^m} + (2-s)sc'(R^m)$$
$$= 2[-F'(R^m)c(R^*) + F'(R^m)c(R^m)](1-F(R^*)) + 2F(R^m)(2-F(R^*))c'(R^m)$$

(using $L1$, $L2$ and $L3$)

$$= \{[2F'(R^m)(1-F(R^*))(R^*-R^m) - 2F(R^m)(2-F(R^*))](c(R^m)-c(R^*)) - 2F(R^m)(2-F(R^*))SOT\}/(R^*-$$
$$= \frac{q}{(1-q)(R^*-R^m)} F(R^*)\{(1-R^*/R^m)(c(R^m)-c(R^*)) - (2-F(R^*))SOT\}$$

(using $L4$ and $L5$), which completes the derivation of (9?)

Thus expected consumer surplus for a two-timer decreases with R^m if SOT ≥ 0, as 1 - R^*/R^m is negative, and all other terms are positive. If $c(R)$ is convex then this is a sufficient condition for two-timers to become worse off as the minimum per customer profit increases. We have that one-timers definitely gain if $c(R)$ is concave and two-timers definitely lose if $c(R)$ is convex.

Proposition 6. The average or aggregate consumer surplus across all consumers, that is:

$$E(AC) = \int_{R^*}^{R_H} F'(R)[q + (1-q)2(1-F(R))]\,c(R)\,dR + s[q + (1-q)(2-s)]\,c(R^m)$$
$$= q\,E(C_1) + (1-q)E(C_2)$$

is increasing in R^m, if $c(R)$ is strictly concave and decreasing in R^m if $c(R)$ is strictly convex. If $c(R)$ is linear then average consumer surplus is unaffected (the case in section 2). Note aggregate consumer surplus is simply N times the average consumer surplus.

Proof. We have found that

$$\frac{dE(C_1)}{dR^m} = 2[F(R^m)\left(\frac{R*}{R^m}-1\right)(c(R^m)-c(R*))-F(R^m)SOT]/(R*-R^m)$$

(9?)

$$\frac{dE(C_2)}{dR^m} = \frac{q}{(1-q)(R*-R^m)}F(R*)\{(1-R*/R^m)(c(R^m)-c(R*))-(2-F(R*))SOT\}$$

and so

$$\frac{dE(AC)}{dR^m} = q\frac{dE(C_1)}{dR^m} + (1-q)\frac{dE(C_2)}{dR^m}$$

$$= q\left\{2[F(R^m)\left(\frac{R*}{R^m}-1\right)(c(R^m)-c(R*))-F(R^m)SOT]/(R*-R^m)\right\}+$$
$$(1-q)\left\{\frac{q}{(1-q)(R*-R^m)}F(R*)\{(1-R*/R^m)(c(R^m)-c(R*))-(2-F(R*))SOT\}\right\}$$
$$= \frac{q}{(R*-R^m)}F(R*)[-1-2(1-F(R^m))]SOT$$

Clearly it is necessary and sufficient for aggregate consumer surplus to increase (decrease) with R^m if SOT < 0 (>0). In section 2, we had the special case where SOT=0 and so no aggregate change in consumer surplus was caused, only a transfer from two-timers to one-timers. The sign of SOT depends on the form of the individual buyer's preferences and the cost function of the seller. A sufficient condition for SOT<0 (>0) is that $c(R)$ is strictly concave (convex). It is sufficient that $c(R)$ is linear for SOT=0 and then aggregate and average consumers surplus does not change with R^m.

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Figure 1: Distribution function of prices for a seller's price offers with and without a minimum price: *q*=2/3.

Figure 1a: no minimum price

Figure 1b: minimum price at 0.6

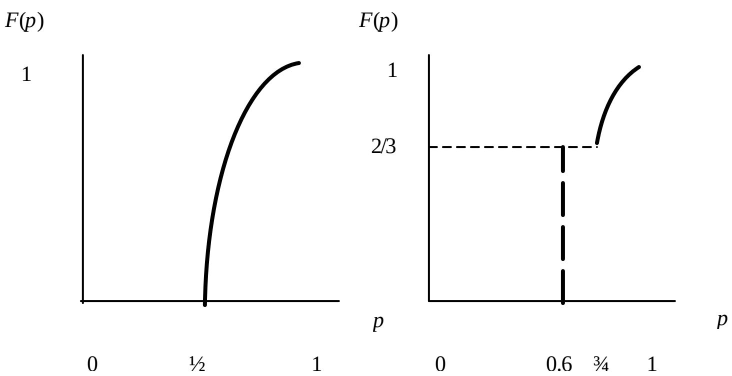
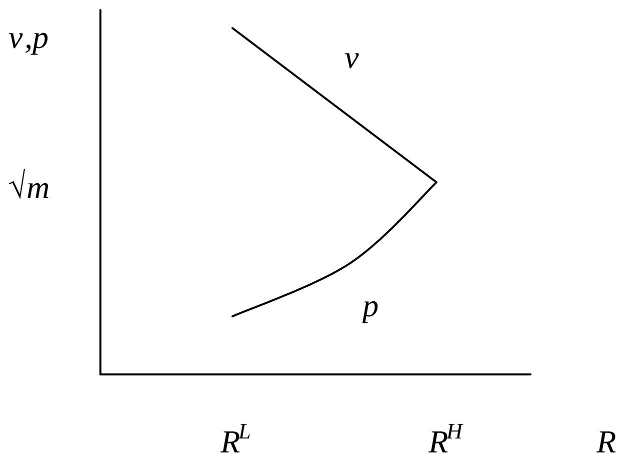


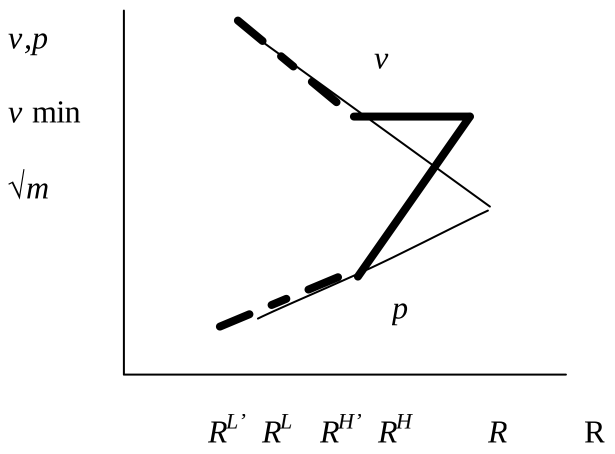
Figure 2: Quality and price for the distribution of profit per consumer

Figure 2i: case (i) (a) No constraints; (b) Minimum Quality constraint; (c) Maximum price constraint; (d) Minimum price constraint. Heavy lines are the graphs after the constraint is imposed; dashed heavy line is common to both before and after imposition.

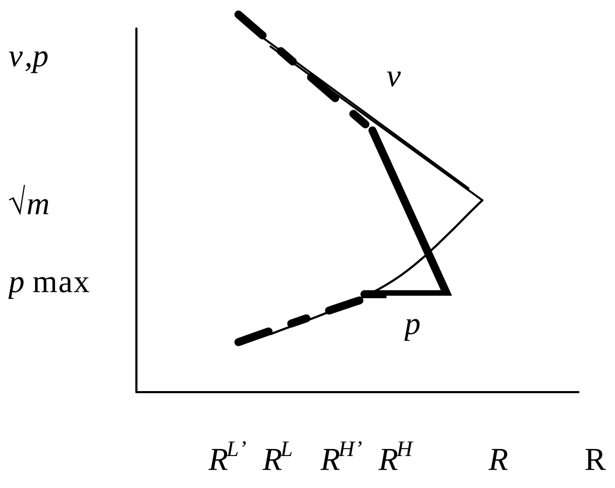
2i (a) No constraints



2i (b) Minimum quality at *v* min



2i (c) Maximum price at *p* max



2i (d) Minimum price at *p* min

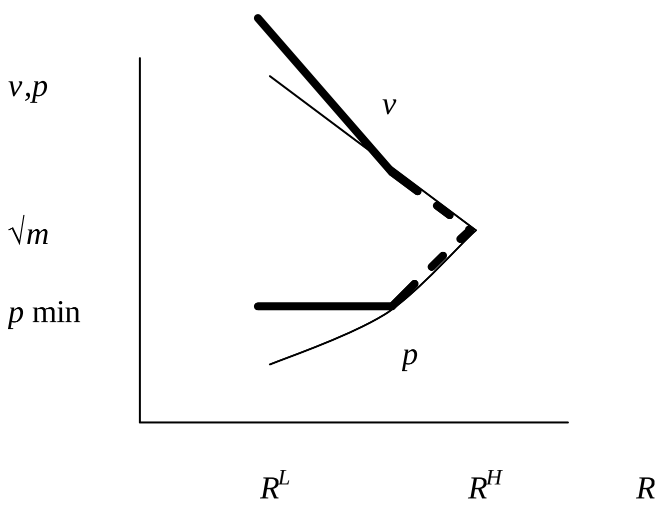
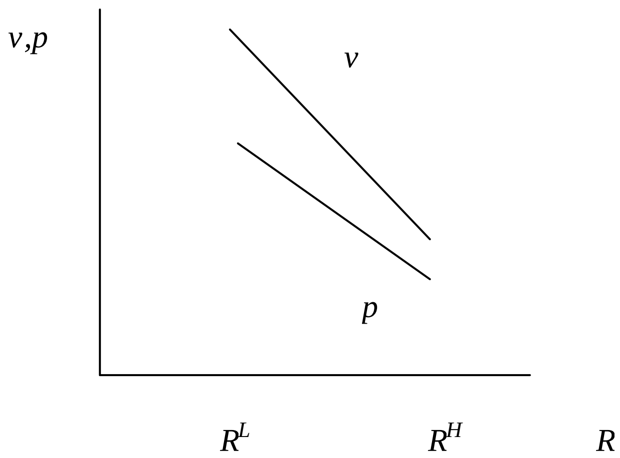
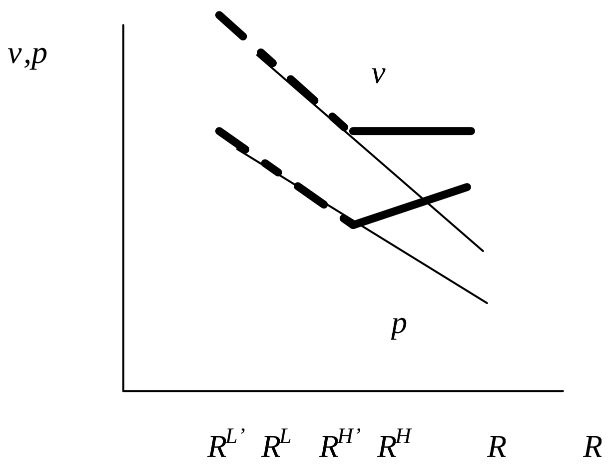


Figure 2ii: case (ii) (a) No constraints; (b) Minimum Quality constraint; (c) Maximum price constraint; (d) Minimum price constraint. Heavy lines are the graphs after the constraint is imposed; dashed heavy line is common to both before and after imposition. Measures for v and p should be considered to have separate scales.

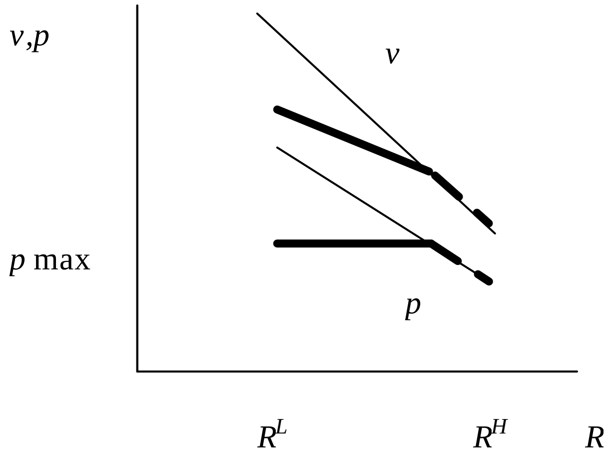
2ii (a) No constraints



2ii (b) Minimum quality at *v* min



2ii (c) Maximum price at *p* max



2i (d) Minimum price at *p* min