**NB: This is an early version of a paper which subsequently evolved into "Modeling Choice and Valuation in Decision Experiments", due to appear in the Psychological Review in 2010 or 2011. In the course of redrafting, the model was simplified and much of the notation changed, as did the scope of the paper. It is made available for the record, and the content should NOT be quoted without first consulting the author, whose current e-mail address is g.loomes@warwick.ac.uk**

# The Improbability of a General, Rational and Descriptively Adequate Theory of Decision Under Risk

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## Abstract

This paper develops a parsimonious descriptive model which can account for a great many of the known systematic departures from standard theory. The paper argues that if this model is correct, there may be no theory of decision making which can accommodate observed behaviour while also being both general and rational in the sense in which decision theorists normally use those terms. From that perspective, the paper considers where various classes of alternative theories might be located in relation to the present model, why such theories fail in the ways they do, and why there may be no way of rescuing them to serve as a general, rational and descriptively adequate theory.

## 1. Introduction

There is now a very substantial body of evidence from individual decision experiments showing a variety of robust and seemingly systematic departures from standard expected utility (EU) theory. In response to this evidence, the past thirty years has seen the development of an array of 'alternative' theories which try in different ways to account for these data: see Starmer (2000) for a review of "the hunt for a descriptive theory of choice under risk"; and Rieskamp et al. (in press) for a review from a more psychological perspective.

However, no single theory has so far been able to organise more than a subset of the evidence. This is something of a puzzle, because all of the regularities in question are generated by the same kinds of people. In fact, in some experiments, the very same group of individuals exhibit many of them one after the other in the same session. So it would seem that there really ought to be a single model of individual decision making under risk that is able to account for *most if not all* of them. Moreover, since the individuals taking part in these experiments are generally not highly experienced and sophisticated decision specialists spending a lot of time and effort on each task[1], the model ought to be a reasonably parsimonious one. This paper sets out to develop such a model, and then to examine its implications for various of the alternative models and for the development and testing of decision theories more generally.

The plan of the paper is as follows. First, it will develop a descriptive model based on some simple propositions about perception and judgment. It will be shown

[1] Many participants in experiments are recruited from university student/faculty populations – although sometimes other sampling frames may be used (for example, people visiting campuses to attend conferences, or members of the local community who have volunteered to take part in university research projects). These are often people with above-average educational qualifications, but for the most part they will have no particular expertise in the kinds of tasks presented to them. Moreover, most experimental sessions will last no more than an hour or so, in the course of which participants may be asked to make dozens of decisions.

that if participants in experiments behave according to this model, a typical sample will be liable to exhibit *all* of the following 'regularities': the common ratio effect; the common consequence effect; mixed fanning; violations of betweenness; betweenness cycles; 'similarity' cycles; 'regret' cycles; the reflection effect and mixed risk attitudes; and preference reversals with money and with probability equivalents[2].

Although it will be argued that this model is a plausible source of many of the best-known experimental regularities, it is by no means obvious that the model will generalise to larger choice sets or to more complex prospects. Moreover, even if some way were found to generalise the model, it is hard to see how it could be regarded as normatively acceptable.

One important consequence of this descriptive-normative conflict is as follows. Suppose that the proposed model (or something like it) is indeed a primary generator of the observed patterns of behaviour. But suppose also that theorists are influenced by particular normative criteria and build features into their models to ensure that they meet those criteria. If those features are at odds with the actual data-generating processes, three things are liable to happen: first, there will be certain regularities which any particular model will not be able to explain; second, any such model will be mis-specified, and the behaviour that is disallowed by the normatively-imposed constraint(s) may find expression in the form of unstable or inconsistent configurations of other parts of the model; and third, attempts to deal with such problems will require further supplementary assumptions and/or extra degrees of freedom, although such 'rescue strategies' will themselves eventually fail. The later sections of the paper will consider various classes of 'alternative' model in this context, and discuss some possible implications for practice and policy.

[2] Actually, this is not an exhaustive list – for example, in earlier drafts it was shown how the model entailed certain violations of the 'reduction of compound lotteries' axiom and explained ambiguity aversion; but this list suffices to demonstrate the scope of the model.

## 2. Background

The starting point here is the same as Rubinstein's (1988) starting point for similarity theory: namely, the form of Allais paradox that has come to be referred to as the 'common ratio effect' – see Allais (1953) and Kahneman and Tversky (1979). Consider the following pair of choices between lotteries of the form (x, p; 0, 1-p) which offer payoff x with probability p and zero with probability 1-p:

Choice #1: $L_1$ = (30, 1)   vs   $L_2$ = (40, 0.80; 0, 0.20)

Choice #2: $L_3$ = (30, 0.25; 0, 0.75)   vs   $L_4$ = (40, 0.20; 0, 0.80)

The two lotteries in choice #2 can be regarded as scaled-down versions of their choice #1 counterparts, in that the probabilities of positive payoffs in choice #2 are reduced to a fraction – in this example, a quarter – of their choice #1 values, maintaining the ratio between them at 0.25 (hence the term 'common ratio'). According to the independence axiom of EU, there are only two permissible patterns of choice: if an individual prefers $L_1$ in choice #1, he should also prefer the scaled-down version in the form of $L_3$ in choice #2, and vice-versa; alternatively, if he prefers $L_2$ in the first choice, he should also prefer $L_4$ in the second. However, the evidence from numerous experiments involving variants on these parameters has shown a robust tendency for many respondents to violate independence by choosing the safer option $L_1$ in choice #1 but pick the riskier alternative $L_4$ in choice #2. The opposite combination of $L_2$ and $L_3$ is relatively rarely observed.

Similarity theory offers the following explanation for this pattern of behaviour. In choice #1, the lotteries differ substantially both on the payoff dimension and on the probability dimension; and although the expected value of $L_2$ – 32 – is somewhat higher than the certainty of 30 offered by $L_1$, the majority of respondents are liable to choose $L_1$, a result which Rubinstein ascribed to risk aversion. But in choice #2, the effect of scaling down the probabilities of the positive payoffs is to cause many respondents to consider those probabilities to be *similar* (or "approximately the same", or "inconsequential")[3], and therefore to give decisive weight to the dissimilar payoff dimension, which favours $L_4$ over $L_3$.

While this idea has some intuitive appeal and can be deployed to explain a number of other 'regularities' besides the common ratio effect (see, for example, Leland (1994), (1998)), it also has its limitations. In particular, a given pair of probabilities may be deemed to be inconsequential in one set of circumstances but that same pair may be decisive in another. To see this, consider the case where the choice is between $L_4$ = (40, 0.20; 0, 0.80) and $L_{3*}$ = (40, 0.25; 0, 0.75). These two lotteries involve exactly the same probabilities that appear in choice #2 and which may be deemed 'inconsequential' in that context. But in the comparison between $L_{3*}$ and $L_4$, the payoffs are the same, so that $L_{3*}$ stochastically dominates $L_4$.

Evidence from experiments involving choices between lotteries where one transparently dominates the other suggests that in such cases dominance is very rarely violated, even when the differences are quite small – see, for example, Loomes and Sugden (1998)[4]. So in cases where there is no difference between payoffs, *any* perceptible difference between probabilities is liable to be decisive[5]. This poses the

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[3] Rubinstein (1988, p.148) acknowledged and appeared to agree with a suggestion by Margalit that the phrase 'is approximately the same as' might be better than 'is similar to'; and Leland (1998) has been inclined to refer to the 'inconsequentiality' of the difference.

[4] Each of 92 participants was presented with five questions involving dominance scattered among forty other questions not involving dominance, with the whole set being answered twice. So there were 920 observations of choices where one alternative dominated the other – for example, a 0.2 chance of £20 and a 0.8 chance of 0 versus a 0.15 chance of £20 and a 0.85 chance of 0. Out of 920 choices, just 13 (1.4%) violated dominance.

[5] The point is not restricted to cases involving dominance. Even if the payoffs were set so that one lottery did not dominate the other – for example, if $L_{3*}$ offered 3990 rather than 4000 – we should still expect almost everyone to choose the latter, because it is easy to judge that the payoff difference is

question of how the basic intuition behind similarity theory can be modelled in a way that preserves its distinctive features while at the same time making it possible for pairs of probabilities that appear inconsequential in some contexts to be considered decisive in others. That will be the main objective of the next two sections. But before pursuing it, several general points should be clarified.

First, the model is primarily intended to be descriptive. The aim is to find a single reasonably parsimonious model that can encompass much of the experimental evidence about individual decisions under risk. So it needs to be a model that taps into what is behaviourally plausible for the kinds of people who take part in experiments and respond to the sorts of tasks they encounter there. At this stage, the normative status of the model is not a principal concern: it will be discussed in Section 6.

Second, the model will be presented in deterministic form. This is a simplification, because the experimental evidence clearly shows that actual decision behaviour has a stochastic element[6]. This has been interpreted and modelled in various ways, but one interpretation – favoured here – is that people do not have very precise preferences corresponding with some tightly specified functional form, but instead make somewhat impressionistic judgments about the relative merits of the different alternatives and respond accordingly.

However, rather than attempt at this stage to model the stochastic component of decision behaviour in detail, the focus will initially be upon modelling the central tendencies of behaviour. This is not meant to suggest that the stochastic component is necessarily neutral or unimportant[7]. But the principal aim of the next two sections is

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insignificant relative to the difference on the probability dimension, even though that probability difference is judged to be small.

[6] See, for example, Camerer (1989), Starmer and Sugden (1989) and Hey and Orme (1994).

[7] Loomes (2005) reviews various approaches to modelling the stochastic component in decision behaviour and shows that different ways of specifying this component may have very different implications.

to model perceptual/judgmental *central tendencies*, explore the way they might interact, identify the broad patterns of behaviour that are consistent with them and see how far they may account for the kinds of regularities observed in experimental data.

Third, because the model is simple, while human beings are not, it does not claim to cover *all* of the phenomena observed in decision experiments. There may be additional influences at work. The claim is not that this is a comprehensive theory of *all* decision behaviour under risk, but that by comparison with other models in the literature it has a small number of components doing a great deal of work[8].

## 3. Basic Framework: Perceived Relative Advantage As Applied to Probabilities

Consider again choices #1 and #2, this time set out as in Figure 1 below:

**FIGURE 1 HERE**

Let $p$ be the probability of getting a payoff of 30 under the safer (S) lottery while $q$ is the probability of 40 offered by the riskier (R) lottery. Under EU, and setting u(0) = 0, this means that in each choice the safer alternative will be preferred to, indifferent to, or less preferred than the riskier one according to whether $p \times$ u(30) is greater than, equal to or less than $q \times$ u(40). Rearranging, and denoting strict preference by $\succ$ and indifference by ~, this can be expressed as:

[8] For example, Shafir et al. (1993) proposed an advantage model which accommodated a range of departures from EU. However, that model is concerned exclusively with choices between binary lotteries and money or probability equivalences for such lotteries. Being limited to binary lotteries necessarily restricts the scope of that model: by its nature, it cannot generate predictions about the common consequence effect, mixed fanning, violations of betweenness or betweenness cycles, all of which are entailed by the present model. In addition, that model uses different parameters for gains and losses, and invokes a particular principle to allow each of those parameters to vary further according to the nature of the task, whereas the present model applies the same person-specific parameters across the board. And although other models – by Mellers and Biagini (1994) and by Gonzalez-Vallejo (2002) for example – also deploy notions of similarity and imprecise judgments to fit various patterns in the data, they too are limited in scope and/or require parameters to be allowed to vary from one class of task to another. A more detailed analysis of how these models related to the one being proposed in this paper can be obtained from the author on request.

$$S \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R \Leftrightarrow p/q \begin{matrix} > \\ = \\ < \end{matrix} u(40)/u(30) \qquad (1)$$

We might think of *p*/*q* as indicating the extent to which S is preferred to R on the probability dimension and u(40)/u(30) as indexing the extent to which R is preferable to S from the payoff perspective, with these two forces pulling the decision maker in opposite directions, and his decision depending on their relative strength. With $p/q = 1.25$ in both choices, an individual's preference depends on whether his u(40)/u(30) is less than or greater than that: if $u(40)/u(30) < 1.25$, $L_1 \succ L_2$ and $L_3 \succ L_4$; if $u(40)/u(30) > 1.25$, $L_1 \prec L_2$ and $L_3 \prec L_4$.

By contrast, similarity theory suggests, in effect, that the force acting in favour of S becomes weaker as *p* and *q* are scaled down. The intuition is that as *p* and *q* are scaled down, the *difference* between them becomes smaller, and this acts on perceptions so that although the *objective* ratio *p*/*q* is the same in both cases, the *perceived* ratio is smaller. Meanwhile, the payoffs are unaltered. This allows the *perceived p*/*q* ratio to be higher than u(40)/u(30) when the probabilities are scaled up but to fall below u(40)/u(30) when the probabilities are scaled down.

In order to model this more formally, it is helpful to rearrange *p*/*q* into the form $1 + (p-q)/q$. The element $(p-q)/q$ focuses attention on the *difference* between the two probabilities. The intuition is that it is the reduction in this difference that is causing the perceived ratio to fall, so the relationship between perceived ratio and objective ratio may be represented by some function of $(p-q)/q$ such that the weight of $(p-q)/q$ falls towards zero (and hence the weight of *p*/*q* tends towards 1) as the probabilities are scaled down.

Before proposing a particular functional form, consider what *p-q* and *q* represent. The upper half of Figure 1 shows it most clearly: $(p-q)$ is the probability that $L_1$ will give a better payoff than $L_2$, while *q* is the probability that $L_2$ will give a better payoff than $L_1$. Conceptually, then, we may regard *(p-q)* as 'the advantage of $L_1$ over $L_2$ on the probability dimension', while *q* is 'the advantage of $L_2$ over $L_1$ on the probability dimension'. On this basis, *(p-q)/q* can be regarded as 'the *relative* advantage of $L_1$ over $L_2$ on the probability dimension'.

There is a corresponding notion for the payoff dimension. The advantage that $L_2$ offers over $L_1$ is a payoff of 40 rather than a payoff of 30, which can be written in utility terms as u(40)-u(30); whereas the payoff advantage of $L_1$ over $L_2$ is u(30)-u(0). Thus u(40)/u(30) can be rearranged into 1 + [u(40)-u(30)]/[u(30)-u(0)] where [u(40)-u(30)]/[u(30)-u(0)] is 'the *relative* advantage of $L_2$ over $L_1$ on the payoff dimension'.

Under EU, expression (1) could be replaced by:

$$S \begin{matrix} \succ \\ \sim \\ \prec \end{matrix} R \Leftrightarrow (p-q)/q \begin{matrix} > \\ = \\ < \end{matrix} [u(40)-u(30)]/[u(30)-u(0)] \qquad (2)$$

However, if *actual* choices are based on how these relative advantages are judged or perceived, then what counts is how the *perceived* relative advantage of S over R on the probability dimension is weighed against the *perceived* relative advantage of R over S on the payoff dimension. It is *this* balance which is the pivotal idea underpinning the descriptive model proposed in this paper and which is therefore called the *perceived relative advantage model* (PRAM).

The bulk of experimental data are derived from decisions that can be represented in terms of pairs of alternative lotteries, each involving no more than three payoffs. This provides a basic framework, depicted in Figure 2 below, where the payoffs are $x_3 > x_2 > x_1$ and the probabilities of each payoff under the safer lottery I and under the riskier lottery J are, respectively, $p_3$, $p_2$, $p_1$ and $q_3$, $q_2$, $q_1$.

**FIGURE 2 HERE**

The advantage that J has over I is that it offers some probability of getting $x_3$ rather than $x_2$. Denote that $(q_3 - p_3)$ by $a_J$, and denote the difference between the subjective values of $x_3$ and $x_2$ by $z_J$. Likewise, the probability advantage of I over J is the probability of getting $x_2$ rather than $x_1$: that is $(q_1 - p_1)$, denoted by $a_I$; while the difference between the subjective values of $x_2$ and $x_1$ is denoted by $z_I$.

It is proposed that the *perceived relative advantage of I over J on the probability dimension* is some increasing function of $a_I/a_J$, $\phi(a_I/a_J)$; while the *perceived relative advantage of J over I on the payoff dimension* is some increasing function of $(z_J/z_I)$, $\xi(z_J/z_I)$. Thus expression (2) can be rewritten as:

$$I \begin{array}{c} \succ \\ \sim \\ \prec \end{array} J \Leftrightarrow \phi(a_I/a_J) \begin{array}{c} > \\ = \\ < \end{array} \xi(z_J/z_I) \qquad (3)$$

Thus in choice #1, $\phi(a_1/a_2)$ may outweigh $\xi(z_2/z_1)$ so that $L_1$ is chosen over $L_2$; but if $\phi(a_I/a_J)$ then falls progressively as the probabilities are scaled down, $\phi(a_3/a_4)$ is liable to fall below $\xi(z_4/z_3)$ so that $L_4$ is chosen over $L_3$ in choice #2, thereby generating the pattern typical of the common ratio effect. The question then is how to model the $\phi(.)$ and $\xi(.)$ functions in ways which are behaviourally plausible and consistent with what is known about the perception of ratios and differences[9].

When $(a_I + a_J) = 1$, as in choice #1, the probabilities are as scaled up as they can be $(p_2 = 1)$. In that case, let us suppose that the perceived ratio coincides with the objective ratio, so that $\phi(a_I/a_J) = (a_I/a_J)$. The term $(a_I + a_J)$ reflects the degree of scaling down, so that when $(a_I + a_J) = 0.25$, as in the choice between $L_3$ and $L_4$, it shows that

[9] Put simply, what that comes down to is a tendency for judgments about ratios and differences to be conflated: see, for example, Birnbaum and Sutton (1992) and Baron (2001).

the probabilities $q_3$ and $p_2$ have been scaled down to one quarter of their $\{L_1, L_2\}$ values. As $(a_I+a_J)$ falls, $\phi(a_I/a_J)$ also falls, although never below 0. One simple way of modelling this is:

$$\phi(a_I/a_J) = (a_I/a_J)^{(a_I+a_J)^{\alpha}} \quad \text{where } \alpha \leq 0 \qquad (4)$$

When $\alpha = 0$, the term $(a_I+a_J)^{\alpha} = 1$, so there is no systematic divergence between perceived and objective ratios: this is what is assumed in EU, where the decision maker always perceives ratios as they are and is not influenced by differences. So $\alpha$ may be thought of as a person-specific behavioural characteristic: someone for whom $\alpha$ is equal to 0 is someone who takes probabilities and their ratios just as they are. However, someone for whom $\alpha$ is less than 0 is inclined to have their judgment of ratios influenced by differences, so that $\phi(a_I/a_J) \leq (a_I/a_J)$; and as $\alpha$ falls, the two diverge more and more. Thus a range of values of $\alpha$ across some population reflects interpersonal differences in the extent to which $\phi(a_I/a_J)$ diverges from $(a_I/a_J)$.

However, expression (4) is only a first step. In order to see how it may be modified, consider some other choices within the Figure 2 framework. Figure 3 reproduces the $\{L_3, L_4\}$pair together with two other variants which involve the same values of $a_I$ and $a_J$, but where the 0.75 probability of 0 common to $L_3$ and $L_4$ is replaced by a 0.75 probability of 30 to give $L_5$ and $L_6$, and by a 0.75 probability of 40 to give $L_7$ and $L_8$.

**FIGURE 3 HERE**

Many authors have found it helpful to represent such choices visually by using a Marschak-Machina (M-M) triangle, as shown in Figure 4. The vertical edge of the triangle shows the probability of $x_3$ and the horizontal edge shows the probability of $x_1$. Any residual probability is the probability of $x_2$. The lotteries in the three choices in Figure 3 are depicted in Figure 4, together with $L_1$ (which is the same as $L_5$ – i.e. the certainty of 30) and $L_2$ from Figure 1, plus lottery $L_9$ = (40, 0.6; 30, 0.25; 0, 0.15), which is also included in Figure 4 for later use.

It is an implication of EU that an individual's preferences can be represented by linear and parallel indifference loci, where the gradient of those lines is a reflection of the individual's risk aversion – the more risk averse the individual, the steeper the gradient. The straight lines connecting $L_1$ to $L_2$, $L_3$ to $L_4$, $L_5$ to $L_6$ and $L_7$ to $L_8$ are all parallel, so that an EU maximiser who prefers the safer $L_1$ to the riskier $L_2$ also prefers $L_3$ to $L_4$, $L_5$ to $L_6$ and $L_7$ to $L_8$, and a less risk averse EU maximiser who prefers $L_2$ to $L_1$ also prefers $L_4$, $L_6$ and $L_8$ in their respective pairs.

**FIGURE 4 HERE**

The three pairs of lotteries from Figure 3 are connected by lines with the same gradient and the same length. So it might be tempting to think that they are equally similar. But inspection of Figure 3 suggests otherwise. Intuitively, $L_3$ and $L_4$ appear most similar, because they entail very similar probabilities of the same zero payoff: in fact, the ratio of those probabilities is 0.75/0.8. By contrast, $L_5$ and $L_6$ are arguably the least similar of the three, with $L_5$ offering a single payoff with certainty while $L_6$ offers a mix of all three payoffs, and with the ratio of the probabilities of the common payoff being 0.75/1. $L_7$ and $L_8$ may be seen as lying somewhere between the other two pairs: like {$L_3$, $L_4$}, each lottery involves two payoffs, as contrasted with the one-versus-three pattern in the {$L_5$, $L_6$} pairing; but the ratio of the probabilities of the common payoff – 0.75/0.95 – is rather lower than that of {$L_3$, $L_4$}.

A possible objection to this is that many experiments have not displayed alternatives in the format used in Figure 3, but have simply presented them in something like the following form:

$L_3$: (30, 0.25; 0, 0.75)

$L_4$: (40, 0.20; 0, 0.80)

$L_5$: (30, 1)

$L_6$: (40, 0.20; 30, 0.75; 0, 0.05)

$L_7$: (40, 0.75; 30, 0.25)

$L_8$: (40, 0.95; 0, 0.05)

However, it could be argued that, if anything, this format goes even further towards reinforcing the idea that $\{L_3, L_4\}$ is the most similar pair, $\{L_5, L_6\}$ the least similar, with $\{L_7, L_8\}$ lying somewhere between[10].

There may be different ways of modelling this distinction, but one simple way involves scaling down the term in (4) by some multiplicative factors $f = [1-(p_1/q_1)]^\beta$, $g = [1-(q_2/p_2)]^\beta$ and $h = [1-(p_3/q_3)]^\beta$, where $\beta \geq 0$, giving expression (5) below[11]:

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[10] This is broadly in line – although for somewhat different reasons – with what was proposed by Buschena and Zilberman (1999). They suggested that when all pairs of lotteries are transformations of some base pair such as $\{L_1, L_2\}$, then the distances between alternatives in the M-M triangle would be primary indicators of similarity. However, they modified this suggestion with the conjecture that if one alternative but not the other involved certainty or quasi-certainty, this would cause the pair to be perceived as less similar, while if both alternatives had the same support, they would be regarded as more similar. That would give the same ordering, with $\{L_3, L_4\}$ as most similar of the three pairs and $\{L_5, L_6\}$ as least similar.

[11] If $\beta = 0$, the ratios of the probabilities of common consequences make no difference. As $\beta$ increases, $(fgh)^\beta$ falls for all $fgh < 1$, thereby reducing $\phi(a_I/a_J)$: which is to say, as an individual gives more and more weight to the degree of overlap between the probabilities of common consequences, he judges the two alternatives to be more and more similar on the probability dimension. Like $\alpha$, $\beta$ can be regarded as a person-specific characteristic, with some distribution of $\beta$'s across the population.

$$\phi(a_I/a_J) = (fgh)^{\beta}\left[(a_I/a_J)^{(a_I + a_J)^{\alpha}}\right] \qquad (5)$$

On this basis, it is possible to rank pairwise choices between lotteries from Figure 4 in order from the highest to the lowest perceived relative advantage for the safer alternative on the probability dimension. Table 1 shows these. It would be possible to assign arbitrary values to α and β and compute values for the various expressions, but to maintain a greater level of generality they are simply designated as 'levels', from Level 1 down to Level 6.

**TABLE 1 HERE**

It is not necessary at this stage to know exactly how the perceived relative advantage on the payoff dimension, $\xi(z_J/z_I)$, is evaluated: in all cases the set of payoffs is the same, so that however the evaluation is done, the value of $\xi(z_J/z_I)$ will be the same for a particular individual facing any of the choices: the question is then how that value compares with the relevant $\phi(a_I/a_J)$. Assuming no errors, the various implications are set out in Table 2. This table shows how a series of regularities may all be consistent with behaving according to PRAM.

**TABLE 2 HERE**

For behaviour to appear to be completely consistent with EU, it is necessary either that $\xi(z_J/z_I)$ > Level 1, so that the perceived advantage of J relative to I on the payoff dimension is such that the riskier lottery would be chosen in every case, or else that $\xi(z_J/z_I)$ < Level 6, in which case the safer lottery would always be chosen.

Any value of $\xi(z_J/z_I)$ between Level 1 and Level 6 will produce the common ratio effect. The size of this range relative to other ranges which produce different regularities may possibly help to explain why this is the violation of EU often most easily produced.

Because much of the early body of evidence focused on the common ratio effect, a number of the earlier non-EU models – Chew and MacCrimmon (1979) and Machina (1982) for example – characterised behaviour as if the individual's indifference loci were 'fanning out' from some point to the south-west of the right angle of the triangle and became flatter (less risk-averse) in the direction of the lower right-hand corner of the triangle. If this pattern were to operate consistently across the whole space of the triangle, it would entail the steepest loci being towards the top corner. However, as Camerer (1995) noted, later experiments often found some degree of fanning *in* towards that top corner (i.e. less risk aversion compared with $\{L_1, L_2\}$), which is consistent with values of $\xi(z_J/z_I)$ between Level 1 and Level 5). In response to this kind of evidence, some non-EU models – for example, Gul (1991) – were developed which have this 'mixed fanning' property.

$L_6$ is a probability mixture, or linear combination, of $L_1$ and $L_2$ (as shown by the fact that it lies on the straight line connecting them in Figure 4). According to EU, any such mixture of two 'more extreme' lotteries should not be preferred to both of them. So if $L_1$ is preferred to $L_2$, it should also be preferred to any mixture of itself and $L_2$, such as $L_6$. Equally, if $L_2$ is preferred to $L_1$, $L_2$ should also be preferred to any mixture of itself and $L_1$ – again, including $L_6$. Either way, choosing $L_6$ in preference to *both* $L_1$ and $L_2$ constitutes a violation of 'betweenness'. Yet such patterns have frequently been reported[12], and as Tables 1 and 2 show, they are consistent with the present model if $\xi(z_J/z_I)$ takes any value between Level 3 and Level 4.

[12] Camerer (1995) concluded that the balance of evidence from a number of studies that examined the issue was consistent with patterns of indifference curves in the M-M triangle which are not linear, as EU entails, but which exhibit some degree of convexity close to the bottom edge of the triangle.

Moreover, the model also entails the possibility of a *different* violation of betweenness: if $\xi(z_J/z_I)$ takes a value between Level 2 and Level 6, the mixture lottery $L_9$ will be *less* preferred than both $L_1$ and $L_2$. This region of the triangle has been less thoroughly explored experimentally, but one study that looked at mixtures like $L_6$ and $L_9$ along the $L_1$-$L_2$ chord – Bernasconi (1994) – found precisely the pattern entailed by the PRAM analysis.

The model also accommodates the other form of Allais paradox that has come to be known as the common consequence effect. This manifests itself as a switch from the safer alternative in choice #3.2 to the riskier alternative in choice #3.1, where some probability of the intermediate payoff (in this case, a 0.75 chance of 30) is substituted in both lotteries by the same probability of a zero payoff. Under EU, this should make no difference to the balance of expected utilities; but if $\xi(z_J/z_I)$ lies between Level 4 and Level 6, that switch will occur.

The common consequence effect as identified here, and the 'usual' violation of betweenness (i.e. the one where $L_6$ is preferred to both $L_1$ and $L_2$), cannot both be exhibited by the same individual at the same time, since the former requires $L_1$ to be preferred to $L_6$ while the latter requires the opposite preference. However, if there is some variability between individuals in terms of their $\alpha$'s, $\beta$'s and values of $\xi(z_J/z_I)$, we might see the same *sample* exhibit both regularities to some degree. Meanwhile, Table 2 shows that the same *individuals* can exhibit many other combinations of regularities – for example, the two 'opposite' violations of betweenness, or 'mixed fanning' alongside the common consequence effect.

Thus far, it might seem that the implications of the model developed in this paper are not very different from what might be implied by certain variants of rank dependent expected utility (RDEU) theories[13]. Some of those models can account for many of the regularities discussed above although, as Bernasconi (1994, p.69) argued, it is difficult for any particular variant to accommodate all of them via the same nonlinear transformation of probabilities into decision weights.

However, the modelling strategy in the present paper is *fundamentally* different from the strategy adopted by those theories in certain important respects, and leads to radical differences in the implications for regularities other than those discussed so far.

What is common to all of the RDEU theories is the notion that probabilities are transformed into decision weights in a manner which can accommodate violations of independence and betweenness while respecting both transitivity and stochastic dominance. To achieve that, each lottery L is evaluated separately and assigned a value index V(L), which is computed as the weighted sum of the values v(.) assigned to each payoff $x_i$ offered by that lottery, with each $v(x_i)$ multiplied by its respective decision weight, $\pi_i$. Each $\pi_i$ is determined by transforming the probability of each payoff according to that payoff's rank *within* the lottery. The preference between any two alternatives is then assumed to depend on their respective V(.) indices.

But clearly, if each lottery is evaluated separately and independently of the other, there is no direct comparison between $q_3$ and $p_3$, nor between $p_1$ and $q_1$, so that the notion of the perceived relative advantage on the probability dimension, which is pivotal to PRAM, has no status in RDEU models.

If this distinction matters, we should expect to see some significant difference between the implications of PRAM and RDEU. And Table 2 shows just such a

[13] An early form of this type of model was Quiggin's (1982) 'anticipated utility' theory. Subsequently, Starmer and Sugden (1989) proposed a form which incorporated a reference point and allowed for losses being treated differently from gains. Essentially the same basic idea is at the heart of Tversky and Kahneman's (1992) 'cumulative prospect theory'.

difference: if $\xi(z_J/z_I)$ takes a value between Level 1 and Level 3, it entails the non-transitive cycle $L_1 \succ L_2$, $L_2 \succ L_6$, $L_6 \succ L_1$; and if that value is in the range between Level 1 and Level 2, it also entails the cycle $L_1 \succ L_2$, $L_2 \succ L_9$, $L_9 \succ L_1$.

Neither of these implications can be accommodated by any RDEU model because, as noted earlier, RDEU models entail transitivity[14]. So the possibility of systematic patterns of cyclical choice is a key feature of this model which distinguishes it from the RDEU family. Moreover, as discussed shortly, the implication of non-transitive choice is by no means limited to 'betweenness cycles' of the kind considered above.

The experimental evidence relating to betweenness cycles is limited: few studies seem to have set out to look for such cycles. One exception is reported by Buschena and Zilberman (1999), who examined choices between mixtures on two chords within the M-M triangle: they reported a significant asymmetric pattern of cycles along one chord, although no significant non-transitive asymmetries were observed along the other chord. More recently, Bateman et al. (2005) reported more such asymmetries: again, these were statistically significant in one area of the triangle; and in the expected direction, although not significantly so, in another area[15].

Reviewing the discussion so far relating to Tables 1 and 2, it is clear that PRAM entails some readily testable implications. Although a more detailed discussion about the configuration of $\xi(.)$ is left until Section 4, it is easy to see that if $x_2$ and $x_1$ were to be held constant (so that $z_I$ is fixed), it would be possible to start with a value of $x_3$ marginally greater than $x_2$ (i.e. start with $z_J$ small) and then steadily

[14] Indeed, one motivation for the development of these models was to 'fix' features of Kahneman and Tversky's (1979) prospect theory that allowed certain violations of transitivity and stochastic dominance to 'slip through'.

[15] Given the relative narrowness of the range within which $\xi(.)$ must lie in order to produce such cycles – see Tables 1 and 2 – their occurrence in some areas but not others within the same triangle may not be surprising.

increase $x_3$ so that $z_J$ and hence $\xi(z_J/z_I)$ progressively rise. Thus one could take some set of lotteries such as those in Figure 4 and steadily raise $\xi(z_J/z_I)$, thereby checking the implications set out in Table 2[16].

However, the implications of PRAM are not confined to the M-M triangle. In particular, the possibility of non-transitive choice is not limited to 'betweenness' cycles. Consider a triple of lotteries consisting of $L_3$ and $L_4$ as above, plus another lottery $L_{10}$ = (55, 0.15; 0, 0.85), and consider the three pairwise choices $L_3$ vs $L_4$, $L_4$ vs $L_{10}$ and $L_3$ vs $L_{10}$. The three pairwise comparisons are as shown in Table 3:

**TABLE 3 HERE**

Suppose, purely for illustrative purposes, that we take the perceived relative advantage of J over I on the payoff dimension simply to be the ratio of the subjective value differences, with those subjective values being drawn from a conventional v(.) function[17] with v(0) = 0: that is, $\xi(z_J/z_I) = [v(x_3)-v(x_2)]/v(x_2)$, which can be rewritten as $[v(x_3)/v(x_2)] - 1$.

An individual will be indifferent between I and J when $\xi(z_J/z_I) = \phi(a_I/a_J)$. Applying this (after minor rearrangement) gives:

$$L_3 \sim L_4 \Leftrightarrow \frac{v(40)}{v(30)} = 1 + \phi(a_3/a_4) \qquad (6)$$

$$L_4 \sim L_{10} \Leftrightarrow \frac{v(55)}{v(40)} = 1 + \phi(a_4/a_{10}) \qquad (7)$$

$$L_3 \sim L_{10} \Leftrightarrow \frac{v(55)}{v(30)} = 1 + \phi(a_3/a_{10}) \qquad (8)$$

With a v(.) function with the standard properties it must be the case that

16 As will be discussed later, this set of implications holds for cases such as the one depicted in the Figures so far where both $a_I/a_J$ and $z_J/z_I$ are less than 1. In due course it will become apparent that other, somewhat different, sets of implications can be derived for other cases.

17 The 'most' conventional such function would be the vN-M utility function; however, a wider class of value functions, including the forms of v(.) function used in RDEU models, will also suffice.

$$\frac{v(55)}{v(30)} = \frac{v(55)}{v(40)} \times \frac{v(40)}{v(30)} \qquad (9)$$

so that transitivity would require $1 + \phi(a_3/a_{10}) = [1 + \phi(a_3/a_4)] \times [1 + \phi(a_4/a_{10})]$. However, inspection of Table 3 shows that this will be true only if α and β are both equal to zero. If either $\alpha < 0$ or $\beta > 0$, $1 + \phi(a_3/a_{10}) > [1 + \phi(a_3/a_4)] \times [1 + \phi(a_4/a_{10})]$ so that $L_3 \succ L_{10}$. In other words, PRAM here allows the possibility of the choice cycle $L_{10} \succ L_4$, $L_4 \succ L_3$, $L_3 \succ L_{10}$, but not its opposite. This cycle will be denoted by RRS to signify that in each of the two more similar pairs the riskier (R) alternative is chosen, while in the least similar pair the safer (S) option is chosen.

Such cycles are, of course, consistent with similarity theory. However, there appear to have been few studies reporting them. One notable exception is a paper by Tversky (1969), which both Rubinstein (1988) and Leland (2002) acknowledge as influential, where *some* evidence was reported; but the data relating to lotteries were generated by just eight respondents, who had themselves been selected from an initial sample of eighteen on the basis of a 'screening' procedure which established a predisposition to violate transitivity in that particular direction. Recently, however, Bateman et al. (2005) reported that such cycles had been observed in two separate experiments, each involving around 150 participants. The main purpose of those experiments had been to explore the common ratio effect, and the data concerning cycles were to some extent a by-product. Nevertheless, in all four cases they observed, RRS cycles outnumbered SSR cycles to a highly significant extent.

On the other hand, that asymmetry is in quite the *opposite direction* to the patterns reported in a number of papers testing the implications of regret theory and/or examining the causes of the preference reversal phenomenon. The preference reversal phenomenon – see Lichtenstein and Slovic (1971), and Seidl (2002) for a review – occurs when individuals place a higher certainty equivalent value on one item in a pair, but pick the other item in a straight pairwise choice.

In the context of lotteries, this phenomenon has typically involved a lottery offering a fairly high chance of a moderate prize (a 'P-bet'), and an alternative offering a considerably lower chance of a rather bigger prize (a '$-bet'). If the certainty equivalents of the two bets are denoted by CE(P) and CE($), the preference reversal phenomenon takes the form that many individuals state CE($) > CE(P) while exhibiting P $\succ$ $ in a straight choice between the two. The opposite reversal – CE(P) > CE($) with $ $\succ$ P – is relatively rarely observed.

For some sure sum of money M such that CE($) > M > CE(P), the common form of preference reversal translates into the choice cycle $ $\succ$ M, M $\succ$ P, P $\succ$ $. A number of studies have tested for the existence of such cycles, and their predominance over cycles in the opposite direction has been reported in several papers – see, for example, Tversky, Slovic and Kahneman (1990) and Loomes, Starmer and Sugden (1991). However, note that such a cycle involves choosing the safer alternative in each of the two more similar pairs {M, P} and {P, $} while choosing the riskier alternative from the least similar pair {M, $} – that is, it involves the cycle SSR, which is in exactly the opposite direction to the one which is consistent with Tversky (1969) and with the model in this paper, as developed so far. Does this constitute a refutation of the model? Or can such seemingly contradictory results from different studies be reconciled? That is the issue addressed in the next section.

## 4. Extending The Framework: Perceived Relative Advantage As Applied to Payoffs

Up to this point, the main focus has been upon the degree of similarity between *probabilities* and little has been said about payoffs and the specification of $\xi(z_J/z_I)$. However, if interactions between ratios and differences are a general perceptual phenomenon, there is no reason to suppose that they operate on the probability dimension but not on the payoff dimension. In this section, the model will be extended accordingly.

In decision theory, it is usual to suppose that what individuals are concerned with on the payoff dimension is not the payoffs themselves but the utility of those payoffs: indeed, when the payoffs take some non-monetary form such as 'a weekend in Paris' or 'the amputation of a leg', there is no obvious alternative except to use some such index. To avoid any possible confusion with the notation from EU or RDEU, the primitives on the payoff dimension in this and subsequent sections will either be the payoffs themselves or some function of them expressed in terms of what might be thought of as 'basic' or 'choiceless' subjective value indices[18]. It will be shown that such a function, denoted by c(x), may be smooth and everywhere weakly concave and yet be compatible with patterns that are sometimes taken to signify that v(.) is convex in some regions and/or kinked at certain points. In PRAM the working assumption is that c(.) is everywhere a weakly concave function of x (or of W + x, where W represents status quo wealth[19]).

[18] The notion here is much the same as that proposed in Loomes and Sugden (1982): something that may be thought of as a cardinal utility in the Bernouillian tradition – that is, the anticipated utility of any given payoff or consequence (which might, in many applications, be non-monetary) as it will be experienced in consumption.

[19] This latter way of conceiving of c(.) emphasises the point that there is no kink in the function at current wealth; but to simplify the exposition, we can work with a transformation such that c(x) = 0 when x = 0.

Let $c(x_i)$ be denoted by $c_i$ for all i. Thus the payoff advantage of J over I, $z_J$, is given by $c_3–c_2$, and the payoff advantage of I over J is $z_I = c_2–c_1$. These terms are thus analogous to the probability differences $a_I$ and $a_J$. On that basis, the most direct analogue to expression (4) would then be:

$$\xi(z_J/z_I) = (z_J/z_I)^\gamma \qquad (10)$$

where $\gamma \geq 1$ and plays a role which corresponds to $(a_I+a_J)^\alpha$ on the probability dimension[20].

To illustrate how this operates, apply expression (10) to pairwise choices between the three lotteries $L_3$, $L_4$ and $L_{10}$. Letting $c(x) = x$, we get $\xi(z_4/z_3) = (10/30)^\gamma$, $\xi(z_{10}/z_4) = (15/40)^\gamma$, and $\xi(z_{10}/z_3) = (25/30)^\gamma$. If preferences were linear in probabilities – so that $1 + \phi(a_{10}/a_3) = [1 + \phi(a_{10}/a_4)] \times [1 + \phi(a_4/a_3)]$ – transitivity would require $1 + \xi(z_{10}/z_3) = [1 + \xi(z_{10}/z_4)] \times [1 + \xi(z_4/z_3)]$. But this will only be the case when $\gamma = 1$; when the perceived relative advantage on the payoff dimension is specified as above, $1 + \xi(z_{10}/z_3) > [1 + \xi(z_{10}/z_4)] \times [1 + \xi(z_4/z_3)]$ for all $\gamma > 1$. Thus if preferences *were* linear in probabilities, this would allow the cycle $L_3 \succ L_4$, $L_4 \succ L_{10}$, $L_{10} \succ L_3$, – that is, an SSR cycle[21].

Of course, the point of PRAM applied to the probability dimension is to model preferences as *not* being linear in probabilities. But when probabilities are scaled *up* to

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[20] It is possible to configure $\gamma$ more elaborately, with a 'scale' component analogous to $(a_I+a_J)$ and a separate 'perception' element corresponding with $\alpha$. However, none of the results in this paper depend on such a separation.

[21] Notice the resemblance between the inequality $1 + \xi(z_{10}/z_3) > [1 + \xi(z_{10}/z_4)] \times [1 + \xi(z_4/z_3)]$ and the condition that characterises regret theory – especially as specified in Loomes and Sugden (1987). In regret theory, the *net advantage* of one payoff over another is represented by the $\psi(. , .)$ function, which is assumed to be strictly convex, so that for all $x_3 > x_2 > x_1$, $\psi(x_3, x_1) > \psi(x_3, x_2) + \psi(x_2, x_1)$. Regret theory assumes preferences to be linear in probabilities, so for binary lotteries such as $L_3$, $L_4$ and $L_{10}$ convexity of the $\psi(. , .)$ function allows SSR but not RRS cycles.

the point where $(a_I+a_J) = 1$, the perceived ratio and the objective ratio are assumed to coincide. So for two of the three choices in the cycles that are the choice analogue of preference reversals – that is, for {M, $} and for {M, P} – there is no divergence between $\phi(a_I/a_J)$ and $(a_I/a_J)$, so that the only 'distortion' occurs in the perception of probabilities in the {P, $} choice: but since the P-bet is usually set with a probability quite close to 1, the extent of the divergence from linearity may be relatively small.

Thus the inequality $1 + \xi(z_{10}/z_3) > [1 + \xi(z_{10}/z_4)] \times [1 + \xi(z_4/z_3)]$ on the payoff dimension may outweigh the inequality in the opposite direction on the probability dimension when probabilities are scaled up and most dissimilar, producing significantly more SSR cycles than RRS cycles; but when those probabilities are scaled down, the inequality $1 + \phi(a_3/a_{10}) > [1 + \phi(a_3/a_4)] \times [1 + \phi(a_4/a_{10})]$ on the probability dimension may weigh more heavily, overturning the effect of the payoff inequality and producing significantly more RRS than SSR cycles.

That rather striking implication of the model turns out to have some empirical support. Following the first two experiments reported in Bateman et al. (2005), a third experiment was conducted in which every pairwise combination of four scaled-up lotteries, together with every pairwise combination of four scaled-down lotteries, were presented in conjunction with two different sets of payoffs. All these choices were put to the same sample in the same sessions under the same experimental conditions. The results are reported in Day and Loomes (2005). As PRAM suggests, there was a tendency for SSR cycles to outnumber RRS cycles when the lotteries were scaled up, while the opposite asymmetry was observed among the scaled-down lotteries.

In short, if perceived relative advantage as modelled in this paper operates on *both* dimensions – and there seems no reason why, if it operates at all, it should operate only on one dimension and not the other – then it sets up interesting tensions and offers an account of all of the regularities considered above, and more besides, as discussed in the next section.

## 5. Some Further Implications

5.1 Mixed Risk Attitudes

All of the examples discussed so far have been ones where the numerators of $\phi(a_I/a_J)$ and $\xi(z_J/z_I)$ have been smaller than the denominators, but there are many potentially interesting cases where this will not be so. What does the model entail when the numerators become equal to, or larger than, the denominators?

First, on the probability dimension. Consider the choice between the certainty of $x_2$ and a lottery offering $x_3$ and $x_1$, both with probability 0.5. Here $\phi(a_I/a_J) = 1$, irrespective of the individual's values of $\alpha$ and $\beta$. Thus an individual will be indifferent between I and J when $\xi(z_J/z_I) = 1$. Examining such cases and observing the relationship between $x_3$–$x_2$ and $x_2$–$x_1$ would give insights into the extent to which it can be assumed that $c(x_i) = x_i$ for all i, or whether concavity of c(.) needs to be assumed.

People's aversion to actuarially fair 50-50 gambles in the domain of gains – and even more so, when $x_2$ is the *status quo* while $x_3$ is a gain relative to current wealth and $x_1$ is a loss of the same magnitude relative to current wealth – has been taken as a sign of risk aversion, compounded perhaps by loss aversion[22]. However, the PRAM analysis does not operate in terms of either risk aversion or loss aversion as individual characteristics. Under PRAM, the individual characteristics are represented

[22] Although there is at least some evidence casting some doubt on this 'stylised fact': Battalio et al. (1990, Table 4) found their sample roughly equally split between opting for the status quo and choosing a 50-50 chance of a gain and a loss of equal magnitude.

by $\alpha$, $\beta$ and $\gamma$ operating on the probabilities and choiceless subjective value indices given by c(x).

On this basis, suppose that some set of payoffs $x_3 > x_2 > x_1 \geq 0$ have been identified such that the individual is indifferent between the certainty of $x_2$ and a 50-50 lottery paying $x_3$ or else $x_1$. Holding those payoffs constant but scaling down the probabilities of $x_3$ and $x_2$ will, according to expression (5), result in a change in $\phi(a_I/a_J)$ attributable solely to the 'beta component' $(fgh)^\beta$ of that expression, since the 'alpha component' from expression (4) is held equal to 1 under these conditions; and this allows an exploration of the sign and magnitude of $\beta$.

Keeping I as the certainty of $x_2$ but reducing the probability of $x_3$ below 0.5 (and correspondingly increasing the probability of $x_1$) results in $a_I > a_J$, so that $(a_I/a_J) > 1$. For simplicity, let $x_1 = 0$, and suppose that values of $x_2$ and $x_3$ have been identified such that I ~ J when $(a_I+a_J) = 1$. Holding those payoffs constant and scaling down the probabilities $q_3$ and $p_2$ now has two effects on expression (5): if $\beta > 0$, the beta component tends to reduce the value of $\phi(a_I/a_J)$; but if $\alpha < 0$, the alpha component increases when $(a_I/a_J) > 1$ and $(a_I+a_J)$ falls. Depending on the relative strength of these two influences, we might observe switching in either direction or, if they more or less cancel out, no particular common ratio effect pattern at all.

Thus when it is the zero payoff whose probability is increased, any pattern of the standard common ratio effect kind is, at best, likely to be much weaker here, and the opposite asymmetry is at least possible, whereby switches from R in the scaled-up pair to S in the scaled-down pair may outweigh switches in the 'standard' direction. Something analogous may occur when the common consequence whose probability increases is either $x_2$ or else $x_3$: if the alpha component pushes in the direction of switching from R to S when $(a_I/a_J) > 1$ and $(a_I+a_J)$ falls, and if this outweighs the influence of the beta component, we could see what looks like fanning *out* in the top corner of the M-M triangle, and violations of betweenness that look like quasiconvexity of preferences near the right-angle of the triangle as opposed to the quasiconcavity attributed to cases where $L_6$ is preferred to both $L_1$ and $L_2$ in Figure 4.

There is evidence that the degree, and even the direction, of departures from EU may be (highly) sensitive to the parameters used. Both Battalio et al. (1990) and Prelec (1990) were able to construct problems that generated more 'non-standard' than 'standard' violations. For example, Battalio et al.'s Set 2 in their Table 7 involved $x_3–x_2$ = \$14 and $x_2–x_1$ = \$6 while $a_I/a_J = 2.33$. Scaling down by a factor of 0.2 resulted in 16 departures from EU (out of a sample of 33), with 10 of those 16 involving switching from R in the scaled-up pair to S in the scaled-down pair, while only 6 exhibited the pattern consistent with fanning out in the bottom right-hand corner of the M-M triangle. Prelec (1990, p.255) also found variable degrees of fanning, and violations of betweenness in both directions, and concluded that "the relationship between the local attitude to risk and gamble value is, in general, nonmonotonic".

For another perspective on the way that the PRAM framework can accommodate what appears to be a manifestation of within-person mixed attitudes to risk, consider the implied pattern of certainty equivalents for binary lotteries with the same means but different degrees of skewness. To illustrate, consider lotteries of the form ($x_3$, $q_3$; 0, 1-$q_3$) with the means held constant at some $x_2$, so that $x_2 = x_3 \times q_3$. Since we are dealing with certainty equivalents, $(a_I+a_J) = 1$, and $\phi(a_I/a_J)$ is therefore simply $(1–q_3)/q_3$. To keep things as simple as possible, let $c(x_i) = x_i$ for all i. Then the ratio $(z_J/z_I) = (1–q_3)/q_3 = \phi(a_I/a_J)$, so that $\xi(z_J/z_I) = [\phi(a_I/a_J)]^\gamma$. If $\gamma > 1$, then whenever $\phi(a_I/a_J)$ is less than 1, $\xi(z_J/z_I)$ is even smaller, so that the certainty is preferred to the lottery, so that the certainty equivalent of the lottery must be less than its expected value – an observation that is conventionally taken to signify risk aversion; but whenever $\phi(a_I/a_J) > 1$, $\xi(z_J/z_I)$ is bigger, so that the lottery is preferred, and the certainty equivalent of the lottery will be greater than its expected value – conventionally interpreted as risk seeking.

Notice that these seemingly mixed risk attitudes are obtained even when c(.) is assumed to be linear. In other words, the patterns are not – as they would conventionally be interpreted – due to curvature in the utility/value function, but rather the result of the way that relative advantages on the payoff dimension are perceived: when $z_J$ is small relative to $z_I$, the effect of $\gamma > 1$ is to reduce the weight on the riskier lottery, whereas when $z_J$ is large relative to $z_I$, perceptual influences enhance the appeal of the riskier alternative. If c(.) were concave, the result would be modified somewhat: when $x_2/x_3 = 0.5$, $(c_3-c_2)/c_2 < 0.5$, so the certainty would be strictly preferred for $q_3 = 0.5$ – and for some range of values below 0.5, depending on the curvature of c(.) and the value of $\gamma$. Nevertheless, it could easily happen that below some value of $q_3$ there is a range of probabilities for which the certainty equivalent of the lottery would be greater than its expected value.

5.2 The Reflection Effect and 'Loss Aversion'

Since Kahneman and Tversky (1979), a number of studies have shown that when the sign in front of all payoffs is changed from positive to negative, the preference between two alternatives is liable to be reversed. This has been interpreted as evidence that the value function is concave for gains relative to the status quo but convex for losses over some range in the vicinity of the status quo reference point (usually also supposing the gradient of the function to be steeper for losses than for gains of the same magnitude, and with a kink at the status quo).

The PRAM framework also implies the reflection effect, but without any convexity of the c(.) function in the domain of losses, nor any kink at x = 0 or anywhere else. Indeed, as shown below, it produces the effect even when c(x) = x.

Figure 5 reproduces the two choices from Figure 1, but with all positive payoffs replaced by losses of the corresponding magnitudes. The requirement for $L_1$ to be preferred to $L_2$ as depicted in Figure 1 is that $\phi(a_I/a_J) > \xi(z_J/z_I)$. Since $\phi(a_I/a_J) = 0.25$ in this case, $L_1 \succ L_2$ requires $(10/30)^\gamma < 0.25$, which will hold iff $\gamma > 1.26186$.

**FIGURE 5 HERE**

In the choice between $L_{11}$ and $L_{12}$, the effect of reversing the signs on the payoffs is to invert both $a_I/a_J$ and $z_J/z_I$, so that preference depends on the relative magnitudes of $\phi(0.8/0.2)$ and $\xi(30/10)$. If $\gamma > 1.26186$, as was required for $L_1 \succ L_2$, then $(30/10)^\gamma > 4$, so that $L_{12} \succ L_{11}$. Thus PRAM implies the reflection effect.

What is happening here is essentially the same as was being described at the end of the previous subsection. In the comparison between $L_1$ and $L_2$, the lottery is a 'small-gain-large-stake' gamble, with $z_J < z_I$, so that the effect of $\gamma$ is to favour $L_1$ and generate risk averse choices. However, reversing the signs on the payoffs has the effect of making $L_{12}$ a 'large-gain-small-stake' gamble *relative to* $L_{11}$, with $\gamma$ enhancing its appeal to produce what manifests itself as risk seeking behaviour.

It is also easy to see that if the probabilities of losses are scaled down so as to produce $L_{13}$ and $L_{14}$, $\xi(z_J/z_I)$ will continue to take whatever value it had for the $\{L_{11}, L_{12}\}$ choice, but $\phi(a_I/a_J)$ will be increased as the ratio of 4 from $a_I/a_J$ is raised to the power $(0.25)^\alpha$. If this raises $\phi(a_I/a_J)$ to the extent that $\phi(a_I/a_J) > \xi(z_J/z_I)$, the safer alternative $L_{13}$ will be chosen, implying a switch from choosing R in the scaled-up loss pair to choosing S in the scaled-down pair – a reflection of the standard common ratio effect observed in the domain of gains and another regularity reported in the literature (again, see Kahneman and Tversky, 1979, or Battalio et al., 1990, among others).

What if c(.) were everywhere concave? For lotteries such as $L_2$, where $q_3 > 0.5$, the effect would be to reinforce the appearance of risk aversion in the domain of gains. For lotteries involving a 50-50 chance of a gain and a loss, individuals would also behave in a risk averse manner, which in (Cumulative) Prospect Theory terms would be represented by the value function being steeper for losses than for gains of the same absolute magnitude. For choices between prospects such as $L_{12}$ and $L_{13}$, the outcome would depend on the interaction between the curvature of c(.) and the value of $\gamma$. For modest losses of the sort examined in incentive-compatible experiments, it could well be the case that the impact of $\gamma$ relative to any curvature of c(.) could produce risk seeking behaviour. Thus it is at least arguable that behaviour which has been interpreted in terms of a value function that is concave for gains but convex for losses and steeper for losses than for gains of the same magnitude, might actually be generated by the processes modelled by PRAM with c(.) everywhere concave.

## 5.3 Preference Reversals: Money Equivalents and Probability Equivalents

Earlier it was shown how the PRAM analysis allowed the choice cycle analogue of the preference reversal phenomenon, namely $\$ \succ M$, $M \succ P$, $P \succ \$$. It follows that the sure money equivalent of the \$-bet, $M_\$$, is strictly greater than the sure money equivalent of the P-bet, $M_P$. While this may not be the only influence on the certainty equivalent values that individuals state in experiments, it is nevertheless consistent with the preference reversal phenomenon, which could therefore be expected to occur even if those other influences were not at work. And because it is entails the reflection effect, the PRAM analysis also accommodates the opposite asymmetry when losses are involved, as reported in Loomes & Taylor (1992).

Moreover, there is another form of reversal which the model can explain. In addition to a P-bet and a $-bet, consider some lottery T which offers an even higher payoff than the $-bet but with a lower probability. Since the probability of winning offered by the $-bet is usually (a lot) less than 0.5, $ and T may both be regarded as scaled-down. So if we can identify some {P, $} pair such that an individual has a *slight* preference for the $-bet, the balance of perceived relative advantages would allow an RRS cycle that would translate into T $\succ$ $, $ $\succ$ P, P $\succ$ T.

If the individual is then asked to give a *probability equivalent* by adjusting the probability of winning the payoff of T until she is indifferent between P and T, she will adjust it *upwards* from its initial value. Let this probability equivalent of P be denoted by PE(P). Asked to undertake the corresponding task to establish the probability equivalent of the $-bet, the initial preference T $\succ$ $ requires that she adjust the original probability of the T payoff *downwards*. Thus PE($) < PE(P) at the same time as $ $\succ$ P. So the PRAM analysis is consistent not only with the predominance of the classic money preference reversal, but also with the opposite asymmetry when value is elicited in probability equivalence form.

Butler and Loomes (2005) reported an experiment which elicited both certainty equivalents and probability equivalents for a particular {P, $} pair via an iterative choice procedure. They observed the standard asymmetry whereby reversals of the form P $\succ$ $ but CE($) > CE(P) outnumbered the opposite money reversals to a highly significant extent; at the same time, they observed probability reversals of the form $ $\succ$ P but PE(P) > PE($) significantly outnumbering those in the opposite direction. Since the elicitation of certainty equivalents and probability equivalents play a significant role in informing various important areas of public policy[23], the fact that both are prone to disparities between choice and valuation, and moreover prone to disparities in opposite directions, may be a cause of some practical concern.

However, there is an even more distinctive implication of PRAM, as follows. The \$-bet offers some payoff $x_\$$ with probability q, while the P-bet offers $x_P$ with probability p. Typically, q is small and p is several times larger, so that (p-q) often exceeds q to a considerable extent. Supposing (p-q) > q and c(x) = x, and denoting the certainty equivalent of the \$-bet by $M_\$$, we can express the point at which an individual is indifferent between the \$-bet and the P-bet as:

$$\mathrm{P} \sim \$ \;\Leftrightarrow\; [(p-q)/q]^{p^{\alpha}} = [(x_\$ - x_P)/x_P]^{\gamma} \qquad (13)$$

The certainty equivalent of the \$-bet, $M_\$$, can be expressed as:

$$\$ \sim \mathrm{M}_\$ \;\Leftrightarrow\; [(1-q)/q] = [(x_\$ - M_\$)/M_\$]^{\gamma} \qquad (14)$$

It can be seen immediately that so long as (p-q)/q is greater than 1, there will exist a value of $p^{\alpha}$ such that $M_\$ = x_P$: that is to say, an individual could be indifferent between P and \$ and also state a certainty equivalent for the \$-bet which is equal to the positive payoff offered by the P-bet. Higher values of $p^{\alpha}$ are therefore compatible with $\mathrm{P} \succ \$$ and at the same time $M_\$ > x_P$. This latter is what Fishburn (1988) called a

[23] For example, certainty equivalents are elicited to estimate the value of reducing the risks of death and injury to guide public safety policy, while probability equivalents (often referred to as ‘standard gambles’) have been widely used to elicit values of health states for cost utility purposes in health care provision.

‘strong reversal’: that is, a case where the P-bet is chosen even though the certainty equivalent of the $-bet is strictly greater than the positive payoff offered by the P-bet.

Even models such as regret theory, which can accommodate preference reversals up to a point by relaxing transitivity, cannot accommodate strong reversals. Nor can strong PE reversals[24] be reconciled with any existing formal theories (at least, not in their deterministic forms). Yet strong reversals *are* a feature of the evidence: indeed, in the Butler and Loomes (2005) data, more than 40% of both CE and PE reversals were ‘strong’; and PRAM can accommodate them.

5.4 Other Considerations

As stated in Section 2 that the model offered here is descriptive and intended to explain a large number of systematic departures from standard theory observed in numerous experimental studies. However, it is not claimed that the behaviour modelled above is the *only* source of these effects, nor that it can explain *all* of the regularities that have been reported.

For example, there may be various ‘framing’ effects at work which cannot be accommodated by this model as it stands. As set out above, the model entails respect for dominance in straight choices between two single-stage lotteries; but as Tversky and Kahneman (1986) were able to show, it may be possible to present two such lotteries in a way which masks dominance so that the majority of participants chose the dominated alternative. Other influences, such as strategic biases in buying and selling responses, or ‘anchoring and adjustment’ effects, may also play a role in some cases. The claim in this paper is simply that such additional effects are not *necessary* to produce the various regularities, even though they may reinforce them.

[24] Strong PE reversals occur when $ $\succ$ P in the straight choice while the probability equivalent of the P-bet is strictly greater than the chance offered by the $-bet of receiving its positive payoff. A parallel analysis to the one for certainty equivalents shows that strong PE Reversals are also allowed by PRAM.

## 6. Generality and Rationality

It has been argued above that PRAM (or something very much like it) represents the way that many participants in experiments make pairwise choices and judge equivalences in cases where there are no more than three payoffs. However, such problems are a small subset of the kinds of risky decisions which are of interest to economists and psychologists. Yet if those same participants were asked to make decisions involving more complex lotteries and/or more alternatives, we should expect them to be able to do so. So is there some more general form of PRAM that characterises behaviour in those richer tasks as well as in the simpler cases that have been the focus of the bulk of individual decision experiments to date?

It seems unlikely. The key to explaining the various choice and equivalence tasks discussed above is that many individuals are making somewhat impressionistic assessments in cases where there are just two contending advantages – one for each alternative – on the payoff dimension and two corresponding advantages pulling in opposite directions on the probability dimension. If there were larger numbers of different payoffs, individuals would no doubt find some means of processing them, but there is more room for various different ways in which they might do so: for example, some might be inclined to form some rough and ready impression of the average forces pulling in different directions, while others might focus principally on extremes (maxima, or minima).

Moreover, even if each alternative only involved at most two outcomes, asking respondents to process three or more prospects at a time might well modify pairwise judgments and/or introduce additional considerations. Indeed, in certain cases, it could hardly be otherwise. For example, consider cases where PRAM allows cycles of choice in the three pairwise combinations of three lotteries. Then consider asking individuals who exhibit such cycles to rank-order the three lotteries or to choose one of them from a set consisting of all three. Clearly, in order to do so, they must reverse at least one of the orderings expressed in their pairwise choices. However, it is not obvious that different individuals would use the same means of deciding what to do: some might weigh up the perceived relative advantage for each of the three pairs, then overturn the one felt to be weakest; others might start with one pair (selected according to some personally salient feature, perhaps), make a pairwise choice and then pit the 'winner' against the third alternative. No doubt, other possibilities exist. Moreover, the desire to simplify a more complex task such as ranking may allow other influences to play some role.

So without being able to deny categorically the *possibility* of any generalisation of the model, there are reasons to suspect that such a general theory is highly improbable. Of course, this is not a criticism of PRAM: it is a descriptive model, and if it is a good description of the domain to which it applies, and if there is no more general model that describes that domain equally well, that may simply be how the world is, and the model cannot be blamed for that. On the contrary, the model's success in accommodating such a broad range of regularities across those tasks can provide insights into the ways in which decision theory has developed in the past few decades, and indicate why no general theory of rational decision making under risk has had more than limited descriptive success.

Suppose it is true that the sorts of stripped-down problems which are typical of experiments are processed by participants in the manner proposed in PRAM. The result is behaviour which exhibits a high degree of regularity and replicability, but which also departs systematically from EU. Seeing such systematic behaviour exhibited so reliably by people who are among the most educated members of the population and who appear to be good candidates for behaving rationally, it is tempting to suppose that the behaviour must be some manifestation of a less restricted, but still rational, theory[25]; and the challenge may then be seen in terms of discovering the structure of that theory. This has typically involved maintaining those assumptions which are thought to be normatively indispensable while modifying other assumptions in ways which can accommodate at least some of the departures from EU without threatening 'core' principles of rationality.

However, if the PRAM analysis is correct, this strategy will inevitably fail. It is understandable that decision theorists may wish to build certain normatively compelling properties into their theories; but if the data are actually generated by a PRAM-like process, trying to make the data fit such theories is rather like trying to shoehorn an Ugly Sister's foot into one of Cinderella's elegant glass slippers.

To illustrate this point, consider regret theory. Because it seemed compelling to have preferences linear in probabilities, there were regularities that regret theory could not easily accommodate. For example, if common ratio and common consequence problems were presented in a form that preserved the act-state juxtaposition and thereby invoked Savage's (1954) sure-thing principle, no systematic switching was predicted by regret theory. The 'solution' was to assume that the lotteries were statistically independent of each other, so that scaling down the probabilities allowed greater weight to be given to juxtapositions where regret favoured the riskier alternative. But the fact is that these effects do *not* in practice require statistical independence. By building in a normatively-driven requirement that preferences should be linear in probabilities, regret theory cannot accommodate some

[25] This is certainly the way I was thinking about things when I worked with Robert Sugden to develop both regret theory and disappointment theory.

of the effects that arise from the $\phi(.)$ component in PRAM without qualifying assumptions or 'special pleading' of one sort or another.

On the other hand, the reason why regret theory succeeded in accommodating certain violations of transitivity that many other models could not explain was that it tapped into the kind of behaviour entailed by the $\xi(.)$ component of PRAM. In contrast, by designing RDEU so as to maintain transitivity, modellers imposed restrictions that are incompatible with the behaviour generated by $\xi(.)$. In such models, the only way to accommodate this source of behaviour is to somehow configure the value and/or probability weighting functions to try to absorb it.

But that asks too much. Despite making the value function concave in gains and convex in losses and kinked at the status quo[26], models such as cumulative prospect theory are still not able to account for violations of transitivity and the preference reversal phenomenon without recourse to 'outside' considerations[27]. Moreover, the complex array of patterns observed within the framework of the M-M triangle cannot all be squared with the same pair of $v(.)$ and $\pi(.)$ functions. Indeed, if PRAM is an essentially correct model of how perceptions operate on the probability dimension, no theory – RDEU or otherwise – which entails non-intersecting indifference curves everywhere in the M-M triangle can be descriptively adequate[28].

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[26] And possibly requiring yet more points of inflection to accommodate other patterns – see the section discussing the shape of the value function in Kahneman and Tversky (1979).

[27] For example, Schmidt, Starmer and Sugden (2005) have proposed a 'third-generation' form of prospect theory which aims to explain preference reversal. Their explanation involves allowing the reference point to be different for a selling task than for a direct choice. However, while this might account for reversals in cases where values are elicited in a *selling* framework, it does not account for the analogous SSR choice cycles discussed earlier, and still less for the phenomenon when values are elicited via a buying task, such as those reported in Lichtenstein and Slovic (1971) and in Loomes, Starmer and Sugden (2005).

[28] For any three payoffs $x_3 > x_2 > x_1$, PRAM entails an individual having some value of $\xi(.)$. Taking the certainty of $x_2$ and setting $q_3$ sufficiently low, it will always be possible to construct a pair of lotteries S = $(x_2, 1)$ and R = $(x_3, q_3; x_1, 1\text{-}q_3)$ such that $\phi(a_S/a_R) > \xi(.)$, because in the limit as $q_3 \rightarrow 0$, $\phi(a_S/a_R) \rightarrow \infty$. It will then be possible to find some linear combination of S and R – call it V – such that $\phi(a_S/a_R) > \xi(.) > \phi(a_S/a_V) > \phi(a_V/a_R)$, or alternatively such that $\phi(a_S/a_R) > \xi(.) > \phi(a_V/a_R) > \phi(a_S/a_V)$. In either case,

To sum up, PRAM encapsulates some simple propositions about the ways in which people actually perceive and process the kinds of decisions under risk presented to participants in experiments. It is essentially a descriptive model, making no normative claims. However, it entails systematic patterns of behaviour that are liable to violate every normative principle except respect for transparent dominance.

It is important to appreciate that these patterns do not rely on framing effects *per se*. In the PRAM analysis, no distinction is made between choice and equivalence procedures – certainty equivalents and probability equivalents are derived on the same basis as pairwise choices are made. Nor is selling treated differently from buying, and there are no reference point or status quo effects required to generate the results. Such effects may well occur, but they are not essential to the analysis, which nevertheless entails a wide variety of patterns of response which, between them, violate independence, betweenness, transitivity. This being the case, any general decision theory that incorporates any of the conventional normative principles into its structure is liable to be confronted with at least some experimental data that contradict it systematically.

## 7. The Role of Noise and Error

In the last 10-15 years there has been a revival of interest in building some stochastic component into EU and the various alternative deterministic theories – see, for example, Sopher and Gigliotti (1993), Harless and Camerer (1994), Hey and Orme (1994), Loomes and Sugden (1995). Part of the momentum came from the desire to estimate the parameters of the competing models and compare their performances. But adding random error to a deterministic core could also be seen as a strategy for

PRAM entails a betweenness cycle, which is incompatible with a complete set of non-intersecting indifference loci, so that any theory which entails such an indifference map will fail in such cases.

‘rescuing’ theories in the face of the patterns they could not accommodate in their deterministic form.

And indeed, adding an error term *can* appear to allow some ‘core’ theories to accommodate certain regularities – at least, up to a point. For example, as shown in Loomes and Sugden (1995), adding a homoscedastic symmetrically-distributed error term to EU allows a proportion of the reported common ratio patterns to be accommodated so long as the great majority of participants choose S in the scaled-up pair and a smaller majority choose S in the scaled-down pair. What cannot be accommodated, however, are cases where scaling down the probabilities results in the *modal* choice switching from S to R (as, for example, in Kahneman and Tversky’s 1979 data); or cases, such as those reported by Bateman et al. (2005, Table 2, first three cases) where R choices constituted a clear the majority in the scaled-up pairs, and where an even bigger majority chose R in the scaled-down pairs, whereas this error specification would entail a more even split in the scaled-down choices.

Moreover, if the variance of the error term is held constant across all problems, this cannot accommodate the common consequence effect, since replacing some probability of $x_2$ by the same probability of $x_1$ leaves the *difference* between EU’s unaffected. Hence, choosing the riskier alternative in one case is exactly as likely as choosing it in the other, contrary to the evidence.

Another strategy has been to allow heteroscedasticity. Using the dataset from Hey and Orme (1994), Buschena and Zilberman (2000) let the variance of the error term depend on “exogenous variables [that] relate to behavioural similarity models … that were designed to model the same behavioural regularities that have driven the development of the GEU [Generalized Expected Utility] models” (p.68). They found that this substantially increased the fit relative to a homoscedastic specification, and concluded that "EU with heteroscedasticity performed comparably and often better than GEU models with homoscedastic error specifications" (p.83). Moreover, changing the error specification for GEU models from homoscedastic to heteroscedastic did not greatly improve the fit.

This result has an interpretation in terms of the PRAM framework, as follows. If PRAM is a good model of the actual data-generation process, it produces systematic departures from EU within the M-M triangle that are due to the effects of $\alpha$ and $\beta$. GEU models try to accommodate those effects by various modifications to the deterministic 'core' theory. By contrast, Buschena and Zilberman's heteroscedastic specifications tap into them via their error specifications – especially their (5) and (6) on p.71, which are closely related to the beta and alpha factors in the PRAM $\phi(.)$ function. Since these are two different ways of picking up essentially the same influences, it is hardly surprising that there is little to choose between their performances. Nor is it surprising that adding this kind of heteroscedastic error to a GEU core provides little improvement: if they are both tapping into what is really the impact of $\alpha$ and $\beta$ on the probability dimension, adding one to the other is unlikely to give much additional value for the extra degrees of freedom.

This raises questions about such an error-modelling strategy. It might be argued that an error term should be used to account only for those variations in the dependent variable that are due to independent random factors and/or to explanatory variables that cannot be identified and entered into the core regression. Putting explanatory variables, or proxies for them, into the error specification and then claiming that EU with this kind of (non-random) error does just as well as another model with something more like truly random error seems debatable. If what is being detected is actually a systematic departure from EU, it would be better to recognise that by building it into the core model.

Moreover, if the interpretation suggested by PRAM *is* correct, then trying to accommodate observed patterns of behaviour via an error specification which taps into the influence of $\alpha$ and/or $\beta$ in a general or proxied way will not succeed so well when an important source of the deviation from EU comes via the $\xi(.)$ term – as in the case of non-transitive behaviour, for example – or when the dataset includes dominance questions, where an error term geared to accommodating significant deviations from EU is liable to overpredict violations of transparent dominance (see Loomes, 2005, p.307).

This is not to deny that there *is* noise and imprecision in people's judgments; indeed, those are exactly the circumstances under which we might expect people to resort to somewhat impressionistic judgments about the relative merits of the different alternatives as modelled above. As stressed earlier, the deterministic form of the model proposed in this paper was simply intended to represent plausible tendencies, and the occasions when numerical examples have generated four decimal places should not be taken to suggest precision. On the contrary, we should expect variability in people's judgments, with this being seen in terms of the variability of the parameters that act on perceptions of relative advantage. That is, we might expect any one individual *not* to be characterised by a *single* set of values of $\alpha$, $\beta$ and $\gamma$, but to act as if on the basis of those parameters being to some extent stochastic.

## 8. Concluding Remarks

This paper has proposed a descriptive model of how individuals generate responses to many of the decision tasks typical of experiments. This model, based on some straightforward ideas about how perceptions of relative advantage operate on both the probability and the payoff dimensions, turns out to account for a much wider spectrum of regularities than any of the main alternatives to EU.

It has been argued that those alternative theories have proved descriptively limited because they have tried in various ways to incorporate certain features which are driven by normative considerations and/or by the desire to achieve generality, but which run counter to the actual data-generating process as modelled in this paper. Moreover, if the core assumptions are wrong, tacking on some stochastic specification may at best provide temporary relief from some of the symptoms but does not constitute a cure.

In the face of such a seemingly irreconcilable descriptive-normative conflict, what can or should be done? This paper cannot provide any simple or definitive answer to that question. That is a debate still to be had. What this paper has sought to show is that the question should not be postponed in the hope that someone may eventually devise a theory which simultaneously satisfies the requirements of generality, normative acceptability and descriptive adequacy: it is highly improbable that such a theory exists, and we might be better advised to turn our attention to the question of what to do in terms of prediction and prescription in the absence of any such theory.

**Figure 1: Example of ‘Common Ratio Effect’ Choices**

| | | |
|---|---|---|
| | 1 | |
| $L_1$ | 30 | |
| $L_2$ | 40 | 0 |
| | 0.8 | 0.2 |

| | | |
|---|---|---|
| | 0.25 | 0.75 |
| $L_3$ | 30 | 0 |
| $L_4$ | 40 | 0 |
| | 0.2 | 0.8 |

**Figure 2: The Basic Choice Framework**

<table>
<tr><td></td><td>$p_3$</td><td colspan="3">$p_2$</td><td>$p_1$</td></tr>
<tr><td>I</td><td>$x_3$</td><td colspan="3">$x_2$</td><td>$x_1$</td></tr>
<tr><td>J</td><td colspan="2">$x_3$</td><td>$x_2$</td><td colspan="2">$x_1$</td></tr>
<tr><td></td><td colspan="2">$q_3$</td><td>$q_2$</td><td colspan="2">$q_1$</td></tr>
</table>

**Figure 3: Three Different, But Equally Scaled-Down, Pairs**

Choice #3.1

| | | |
|---|---|---|
| | 0.25 | 0.75 |
| $L_3$ | 30 | 0 |
| $L_4$ | 40 | 0 |
| | 0.2 | 0.8 |

Choice #3.2

| | | | |
|---|---|---|---|
| | 1 | | |
| $L_5$ | 30 | | |
| $L_6$ | 40 | 30 | 0 |
| | 0.2 | 0.75 | 0.05 |

Choice #3.3

| | | |
|---|---|---|
| | 0.75 | 0.25 |
| $L_7$ | 40 | 30 |
| $L_8$ | 40 | 0 |
| | 0.95 | 0.05 |

**Figure 4: Lotteries $L_1$ to $L_9$ Represented Diagrammatically**

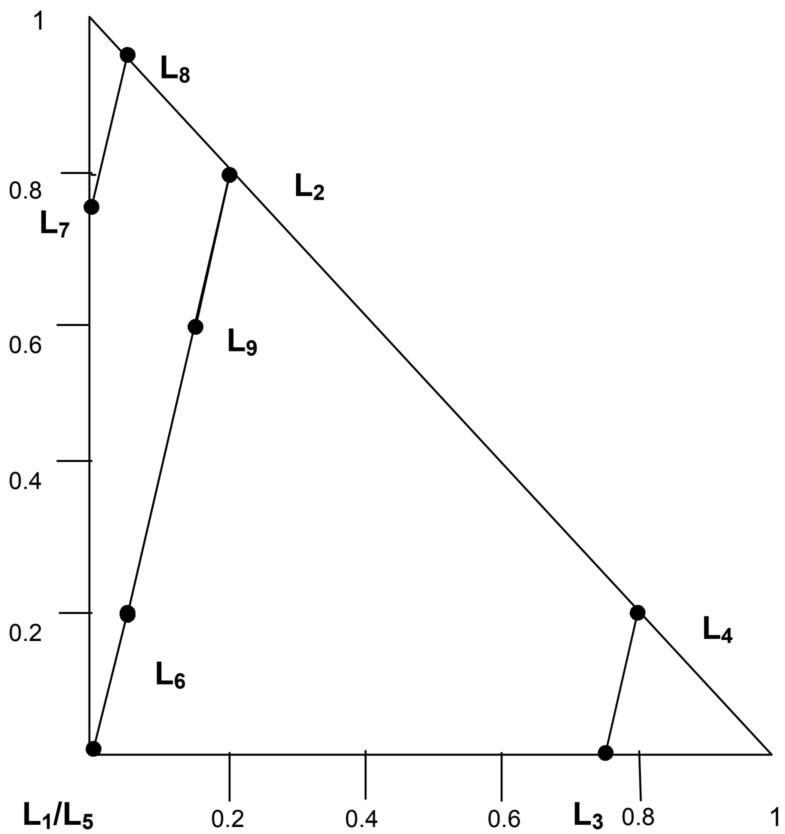

**Figure 5: The ‘Reflections’ of the Common Ratio Effect Choices**

| | | |
|---|---|---|
| | 1 | |
| $L_{11}$ | –30 | |
| $L_{12}$ | –40 | 0 |
| | 0.8 | 0.2 |

| | | |
|---|---|---|
| | 0.25 | 0.75 |
| $L_{13}$ | –30 | 0 |
| $L_{14}$ | –40 | 0 |
| | 0.2 | 0.8 |

**TABLE 1**

**Pairs Ranked by Perceived Relative Advantage on Probability Dimension**

| | | **Level** |
|---|---|---|
| $\{L_1, L_2\}$ | $\phi(a_1/a_2) = 1^{\beta}\left[0.25^{(1)^{\alpha}}\right]$ | 1 |
| $\{L_1, L_9\}$ | $\phi(a_1/a_9) = 0.75^{\beta}\left[0.25^{(0.75)^{\alpha}}\right]$ | 2 |
| $\{L_6, L_2\}$ | $\phi(a_6/a_2) = 0.5625^{\beta}\left[0.25^{(0.75)^{\alpha}}\right]$ | 3 |
| $\{L_1, L_6\}$ | $\phi(a_1/a_6) = 0.25^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$ | 4 |
| $\{L_7, L_8\}$ | $\phi(a_7/a_8) = 0.2105^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$ | 5 |
| $\{L_9, L_2\}$ | $\phi(a_9/a_2) = 0.0625^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$ | 6 |
| $\{L_3, L_4\}$ | $\phi(a_3/a_4) = 0.0625^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$ | 6 |

**TABLE 2**

**Implications of Different Levels of $\xi(z_J/z_I)$**

| Position of $\xi(z_J/z_I)$ | Nature of Regularity |
|---|---|
| Level 6 > $\xi(z_J/z_I)$ | Consistent with EU: Safer lottery always chosen |
| Level 1 > $\xi(z_J/z_I)$ > Level 6 | Common ratio effect: $L_1 \succ L_2$ but $L_3 \prec L_4$ |
| Level 1 > $\xi(z_J/z_I)$ > Level 5 | 'Fanning in' in top corner: $L_1 \succ L_2$ but $L_7 \prec L_8$ |
| Level 2 > $\xi(z_J/z_I)$ > Level 6 | Betweenness violated: $L_9$ less preferred than both $L_1$ and $L_2$ |
| Level 3 > $\xi(z_J/z_I)$ > Level 4 | Betweenness violated: $L_6$ preferred to both $L_1$ and $L_2$ |
| Level 4 > $\xi(z_J/z_I)$ > Level 6 | Common consequence effect: $L_1 \succ L_6$ but $L_3 \prec L_4$ |
| Level 1 > $\xi(z_J/z_I)$ > Level 3 | Transitivity violated: $L_1 \succ L_2$; $L_2 \succ L_6$; but $L_1 \prec L_6$ |
| Level 1 > $\xi(z_J/z_I)$ > Level 2 | Transitivity violated: $L_1 \succ L_2$; $L_2 \succ L_9$; but $L_1 \prec L_9$ |
| $\xi(z_J/z_I)$ > Level 1 | Consistent with EU: Riskier lottery always chosen |

**TABLE 3**

$$\phi(a_3/a_4) = (0.05/0.80)^{\beta}\left[0.25^{(0.25)^{\alpha}}\right]$$

$$\phi(a_4/a_{10}) = (0.05/0.85)^{\beta}\left[0.33^{(0.20)^{\alpha}}\right]$$

$$\phi(a_3/a_{10}) = (0.10/0.85)^{\beta}\left[0.67^{(0.25)^{\alpha}}\right]$$

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