Taylor Rules Cause Fiscal Policy Ineffectiveness*

Guido Ascari

Dipartimento di Economia Politica
e Metodi Quantitativi
Universita degli Studi di Pavia
Via S. Felice 5
27100 Pavia
Italy

Neil Rankin

Department of Economics
University of Warwick
Coventry
CV4 7AL
UK

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Abstract

With the aim of constructing a dynamic general equilibrium model where fiscal policy can operate as a demand management tool, we develop a framework which combines staggered prices and overlapping generations based on uncertain lifetimes. Price stickiness plus lack of Ricardian Equivalence could be expected to make tax cuts, financed by increasing government debt, effective in raising short-run output. Surprisingly, in our baseline model this fails to occur. We trace the cause to the assumption that monetary policy is governed by a Taylor Rule. If monetary policy is instead governed by a money supply rule, fiscal policy effectiveness is restored.

Keywords

staggered prices, overlapping generations, fiscal policy effectiveness, Taylor Rules

JEL Classification

E62, E63

1. Introduction

The canonical ‘New Neoclassical Synthesis’ model for monetary policy analysis has been recognised by several authors recently as providing limited scope for the analysis of fiscal policy. This is because it embodies ‘Ricardian Equivalence’, so that changes in government debt, when accompanied by changes in lump-sum taxation, have no real effects on the economy. To give government debt - and thus also government budget deficits financed by borrowing - a more interesting and realistic role, these authors have begun to incorporate alternative microeconomic assumptions which break Ricardian Equivalence. An important class of models which introduce ‘non-Ricardian’ behaviour is overlapping generations models. In this paper, we examine the consequences for the effectiveness of fiscal policy of combining overlapping generations with staggered price setting.

A plausible hypothesis about the effect of fiscal policy in such an environment might be as follows. A one-period tax cut financed by an increase in government debt which is then held permanently at its new higher level would stimulate consumption demand. This is for the standard reason that, although agents would rationally anticipate higher future taxes to service the increased debt interest, a proportion of the taxes would fall on agents not yet born, so that currently-alive agents - the recipients of the tax cut - would perceive their lifetime wealth to have risen. In the presence of temporary nominal rigidities, caused by prices being re-set only intermittently and in an asynchronised way, the increase in aggregate demand would then raise output in a typical Keynesian fashion.

Below, we test this hypothesis by constructing a careful dynamic general equilibrium (DGE) model with the aforementioned features. We then use it to study the basic fiscal policy experiment just described. Surprisingly, we do not find that an expansionary short-run effect on output is bound to occur. Indeed, in the baseline version of our analysis, a fiscal expansion has no short-run effect on output different from its (relatively insignificant) long-run effect. In other words, it causes neither boom nor slump. Fiscal policy in this case is completely ineffective as a tool of macroeconomic management. This is despite setting it in a DGE model of the macroeconomy which deliberately incorporates features which might be expected to ensure its effectiveness.

What is the explanation for fiscal ineffectiveness? We show that the critical factor is the monetary policy regime. In our baseline case, we assume a Taylor Rule for monetary policy, i.e. a feedback rule making the nominal interest rate a function of current inflation. (For simplicity we omit output from the Taylor Rule, but its inclusion would not alter the result.) In recent years this has become the standard way to represent monetary policy, for reasons which have been extensively discussed in the literature. However in an overlapping-generations environment, our paper shows that the Taylor Rule has a disabling effect on fiscal policy. By way of comparison we also consider what happens if monetary policy is conducted by fixing the money supply, which was the standard assumption until it was displaced by the Taylor Rule. Here we find the expected Keynesian result: a debt-financed tax cut causes a short-run boom in output. Over time the boom dies away as prices are gradually adjusted. Why should the responses be so different? We trace the cause to the endogeneity of the money supply which applies under a Taylor Rule. We find that a debt-financed tax cut induces a sudden reduction in the money supply. This offsets the expansionary effect of the increase in government debt itself. When, instead, the money supply is held constant, such an offsetting effect is prevented.

Although other authors have also recently begun to study fiscal policy, and its interaction with monetary policy, in 'non-Ricardian' DGE models, we are not aware that the drastic effect of a Taylor Rule on the effectiveness of fiscal policy has been noted before. Bénassy (2005)[1] shows that the Taylor Principle for the determinacy of equilibrium is radically altered when overlapping generations are introduced in the manner of Weil (1987, 1991), under both flexible and staggered prices. Piergallini (2006) uses overlapping generations in the manner of Blanchard (1985) to study the determinacy question. Leith and Wren-Lewis (2006) construct a two-country model with overlapping generations and staggered prices to study the stability of a monetary union under feedback rules for monetary and fiscal policy. Galí et al. (2007) break Ricardian Equivalence by using 'rule-of-thumb'

[1] See also Bénassy (2007), Ch. 4

consumers and show that this can explain the econometric evidence of a positive effect of government spending on consumption. Finally, Chadha and Nolan (2007) look at optimal simple monetary and fiscal policy rules in a Blanchard-type framework.

The structure of the paper is as follows: the microeconomic elements of the model are presented in Section 2, while Section 3 examines the effectiveness of fiscal policy when monetary policy is governed by a Taylor Rule. Section 4 does the same when monetary policy is governed by a money-supply rule, and Section 5 concludes.

2. Structure of the Model

The model brings together staggered price setting in the manner of Calvo (1983) and overlapping generations in the manner of Blanchard (1985). We are mainly interested in the qualitative features of such an economy rather than in quantitative matching of the data. Hence we construct the model as sparingly as possible, abstracting from elements which would complicate the dynamics unnecessarily and increase the difficulty of understanding the mechanisms at work. A DGE model with overlapping generations and overlapping price setting already contains numerous intrinsic sources of dynamics. Amongst the elements omitted is capital accumulation. Although this is very commonly studied in conjunction with overlapping generations, our focus here is on short- to medium-run time spans during which changes in capital can reasonably be ignored.

(i) Household behaviour

We use a discrete-time version of Blanchard’s (1985) ‘perpetual youth’ overlapping generations model, in which agents have an exogenous probability, q ($0 < q \leq 1$) of surviving to the next period. This well-known framework conveniently permits the average length of life to be parameterised and includes infinite lives as the special case $q = 1$. In order to allow for money and for labour as an input to production we include real money balances and labour as arguments of the utility function. However the latter raises a potential difficulty, namely that, if leisure is a ‘normal’ good, a fraction of households will have a negative labour supply. To avoid this unsatisfactory implication, we assume a particular utility function which makes labour supply wealth-independent.[2]

Specifically, the household's optimisation problem may be stated as:

maximise $$\Sigma_{t=n}^{\infty}(\beta q)^{t-n}\ln\left(C_{s,t}^{1-\delta}[M_{s,t}/P_t]^{\delta} - (\eta/\varepsilon)L_{s,t}^{\varepsilon}\right) \tag{1}$$

subject to $$P_t C_{s,t} + M_{s,t} + B_{s,t}^N = (1/q)[M_{s,t-1} + (1+i_{t-1})B_{s,t-1}^N] + W_t L_{s,t} + \Pi_t - T_t, \tag{2}$$

for $t = n,...,\infty$.

Here, n is the current period and s ($\leq n$) is the household's birth-period. $C_{s,t}$ denotes the composite consumption in period t (defined below) of a household born in period s; and likewise for money holdings $M_{s,t}$, bond holdings $B_{s,t}^N$ and labour supply $L_{s,t}$. P_t, W_t, i_t indicate the price index, wage, and nominal interest rate, respectively; while Π_t, T_t denote profit receipts from firms and a lump-sum tax, which are assumed age-independent. The parameters satisfy $0 < \beta,\delta < 1$, $\varepsilon > 1$, $\eta > 0$. Note also that, as in Blanchard (1985), the household receives an 'annuity' at the gross rate $1/q$ on his total financial wealth if he survives, this wealth passing to the insurance company if he dies. This is an actuarially fair scheme which nets out across the population so that in equilibrium the profits of insurance companies are zero.

The utility function (1) is a modified version of one originating with Greenwood, Hercowitz and Huffman (1988) ('GHH'). The modification consists in introducing real money balances. Its implications for behaviour can be seen by deriving the first-order conditions for the above problem, which are as follows:

$$C_{s,t+1} - (P_{t+1}C_{s,t+1}/M_{s,t+1})^{\delta}(\eta/\varepsilon)L_{s,t+1}^{\varepsilon} = \beta(1+r_t)\left[C_{s,t} - (P_t C_{s,t}/M_{s,t})^{\delta}(\eta/\varepsilon)L_{s,t}^{\varepsilon}\right], \tag{3}$$

$$M_{s,t}/P_t C_{s,t} = (1-\delta)^{-1}\delta(1+i_t)/i_t, \tag{4}$$

$$W_t/P_t = (1-\delta)^{-1}(P_t C_{s,t}/M_{s,t})^{\delta}\eta L_{s,t}^{\varepsilon-1}. \tag{5}$$

[2] This issue is discussed in more detail in Ascari and Rankin (2007). The utility function used here is first proposed there.

Here, $1+r_t$ denotes $(1+i_t)P_t / P_{t+1}$, the real interest rate. It is also helpful to define money demand per unit of consumption, $M_{s,t} / P_t C_{s,t}$, as $Z_{s,t}$. Then (4) shows that $Z_{s,t}$ is the same for all agents, irrespective of their birth date *s*, and is a simple decreasing function of the nominal interest rate. From (5) we then observe that an agent's labour supply, $L_{s,t}$, does not depend on his consumption (except through $Z_{s,t}$), and that, in consequence of this and of the fact that $Z_{s,t}$ is common for all agents, labour supply is also independent of *s*. This is our reason for using GHH preferences: it eliminates the income effect on labour supply which would otherwise be manifested by the presence of $C_{s,t}$ in (5), $C_{s,t}$ being a variable which is generally increasing with an agent's age, *t-s*. This enables us to avoid the problem of old agents having negative labour supply.[3]

Incorporating wealth-independent labour supply does have a cost, however, which is that the utility function is not additively separable. One consequence is that there is a direct effect of real balances on labour supply, as can be seen from (5). This is the 'Brock effect' (Brock (1974)). Intuitively, higher holdings of real balances (or, to be precise, higher $Z_{s,t}$) give the household an incentive to supply more labour since they complement consumption, raising the marginal utility of the latter. We would not expect this effect to be empirically important but since it is present in our theoretical model it is necessary to take account of it. Non-separability also introduces direct effects of labour supply and of real balances on consumption. This can be seen from the presence of $L_{s,t}$ and $Z_{s,t}$ in (3), which is a version of the Euler equation for consumption. The composite term $Z_{s,t}^{-\delta}(\eta / \varepsilon)L_{s,t}^{\varepsilon}$, which is subtracted from both sides of (3), acts like a 'subsistence' level of consumption. In our model the Euler equation can be viewed as determining the growth rate of 'adjusted' consumption, where the latter is defined as actual consumption minus its subsistence level. For the reasons given above, an agent's subsistence consumption level is independent of his age.

Although households of different ages choose the same labour supply and money demand per unit of consumption, in general they will have different lifetime wealth levels and choose different consumption levels. Other things being equal, households who have the good fortune to live longer will have higher wealth, and there will be a distribution of

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[3] See again Ascari and Rankin (2007) for a more extensive analysis.

consumption and wealth across the population in any period. For aggregate consumption to be a function only of aggregate wealth (and of relative prices), and thus for it to be independent of the shape of the wealth distribution, it is necessary that an individual household's consumption be linear in their total lifetime wealth. For the utility function (1), we can confirm that this is the case. We thus preserve the feature that made easy aggregation possible, and macroeconomic analysis straightforward, in the original Blanchard (1985) paper.

Given the above, we can derive a counterpart of the individual Euler equation, (5), in which individual is replaced by aggregate consumption. This is as follows:

$$C_{t+1} - Z_{t+1}^{-\delta}(\eta/\varepsilon)L_{t+1}^{\varepsilon} \;=\; \beta(1+r_t)\left[C_t - Z_t^{-\delta}(\eta/\varepsilon)L_t^{\varepsilon}\right] - (1-\delta)(1-\beta q)(1/q-1)V_t. \qquad (6)$$

Absence of an 's' subscript indicates an aggregate value (or, equivalently, an average value, since the population size is one). The relationship of a generic aggregate variable, X_t, to its constituent individual variables distinguished by generation, $X_{s,t}$, is $X_t = \Sigma_{s=-\infty}^{t}(1-q)q^{t-s}X_{s,t}$. In the cases of Z and L we have already seen that individual and aggregate values are the same, but this is not generally true in the case of consumption. Nor is it true in the case of financial wealth, V_t, which, for an individual, is defined as the sum of his real balances and bond holdings:

$$V_{s,t} \;\equiv\; (1/q)[M_{s,t-1} + (1+i_{t-1})B_{s,t-1}]/P_t.^{4} \qquad (7)$$

The 'aggregate Euler equation', (6), says that the growth rate of aggregate adjusted consumption depends positively on the real interest rate (as in the case of individual adjusted consumption), and (to the extent that $q < 1$) negatively on aggregate financial wealth. A similar relationship is found in Blanchard (1985) and other applications of the 'perpetual youth' model. The negative influence of financial wealth arises from the 'generational

[4] The relationship of aggregate to individual financial wealth is slightly different from the general one just given, being, rather, $(1/q)V_t = \Sigma_{s=-\infty}^{t}(1-q)q^{t-s}V_{s,t}$. This is because we have included the annuity payout in our definition of $V_{s,t}$, but such a payout does not apply to the aggregate variable since it is a redistribution from those who die to the survivors.

turnover effect'.[5] Such an effect occurs because some old agents are replaced by newborn agents between t and t+1, and in general the newborn, since they have no financial wealth, have lower consumption than old agents, who have had time to accumulate it over their lifetimes.

It remains to define composite consumption. We assume a continuum of types of good, indexed by $i \in [0,1]$. The household has CES utility over good types, given by:

$$C_{s,t} \;=\; \left[\int_0^1 C_{i,s,t}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}, \qquad \theta > 1. \tag{8}$$

The subsidiary part of its optimisation problem is to allocate spending amongst good types to maximise (8) subject to a budget constraint $\int_0^1 P_{i,t} C_{i,s,t} di = I_{s,t}$, where $I_{s,t}$ is its income available to spend on goods. This leads to the familiar constant-elasticity demand function for good type i:

$$C_{i,s,t} \;=\; (P_{i,t} / P_t)^{-\theta} (I_{s,t} / P_t) \qquad \text{where } P_t \equiv \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{1/(1-\theta)}. \tag{9}$$

Moreover, at an optimum, $C_{s,t} = I_{s,t} / P_t$.

(ii) Firm behaviour

Firms are monopolistic competitors who produce differentiated goods. As an input they use labour hired in a competitive market. Price staggering is introduced through Calvo's (1983) mechanism, in which a firm is allowed to adjust its nominal price with probability 1-α in any period, while it has to keep it fixed with probability α. The optimisation problem of a firm, i, which receives the opportunity to adjust its price in period n, can thus be stated as:

maximise
$$E_n \left(\Sigma_{t=n}^{\infty} \Delta_{n,t} \Pi_{i,t} / P_t\right) \tag{10}$$

where
$$\Pi_{i,t} \;=\; P_{i,t} Y_{i,t} - W_t L_{i,t},$$

$$\Delta_{n,t} \;\equiv\; (1+r_n)^{-1} (1+r_{n+1})^{-1} ... (1+r_{t-1})^{-1} \qquad (\text{with } \Delta_{n,n} \equiv 1),$$

subject to
$$Y_{i,t} \;=\; L_{i,t}^{\sigma} \qquad 0 < \sigma \leq 1, \tag{11}$$

[5] This effect is so named by Heijdra and Ligthart (2000).

$$Y_{i,t} \;=\; (P_{i,t}\,/\,P_t)^{-\theta}\,Y_t\,, \tag{12}$$

$$P_{i,t} \;=\; P_{i,t-1} \qquad \text{with probability } \alpha, \tag{13}$$

for $t = n,\ldots,\infty$.

Here, $Y_{i,t}\,, P_{i,t}\,, L_{i,t}$ are the output, price and labour input of firm i. W_t is the wage and $\Delta_{n,t}$ is the discount factor. The demand for good i is given by (12), which is the aggregation across all households of their individual demands, (9), plus the demand from the government (see below). Being infinitesimal relative to the economy as a whole, the firm treats the macro variables which shift its demand function, Y_t and P_t, as given. It also treats W_t as given.

This is a standard set-up in New Neoclassical Synthesis models. The nominal rigidity combined with monopolistic competition generates the Keynesian feature that output is demand-determined. This is because firms will always prefer to satisfy any unexpected increase in demand, given that price will have been set above marginal cost as a result of the firm's monopoly power.

Solving the optimisation problem yields the following expression for firm i's 'new' or 'reset' price:

$$X_n \;=\; \left[\frac{\theta}{\theta-1}\,\frac{\Sigma_{t=n}^{\infty}\alpha^{t-n}\Delta_{n,t}Y_t^{1/\sigma}W_tP_t^{\theta/\sigma-1}}{\Sigma_{t=n}^{\infty}\alpha^{t-n}\Delta_{n,t}Y_tP_t^{\theta-1}}\right]^{1/(1-\theta+\theta/\sigma)}. \tag{14}$$

'X_n' denotes the new price set in period n. Symmetry amongst firms means that all firms able to change their prices in period n will choose the same new price, so that no 'i' subscript is needed. (14) is a forward-looking price-setting rule typical of models with Calvo-style price staggering. It says that the new price depends on current and expected future values of output, the price level and the wage level.

The general formula for the price index was given in (9). Combining this with the Calvo pricing assumption, we obtain an expression for the price index as a function of current and lagged values of X_t:

$$P_t \;=\; \left[(1-\alpha)\Sigma_{j=0}^{\infty}\alpha^j X_{t-j}^{1-\theta}\right]^{1/(1-\theta)}. \tag{15}$$

This arises from the fact that, of all the prices in force in period t, the fraction which were last reset exactly j periods ago is $(1-\alpha)\alpha^j$.

(iii) Government behaviour

The government's budget constraint in nominal terms is:

$$P_t(G_t - T_t) \;+\; i_{t-1}B_{t-1}^N \;=\; (B_t^N - B_{t-1}^N) + (M_t - M_{t-1})\,, \tag{16}$$

where G_t is purchases of firms' outputs, measured in terms of the composite good. We assume government spending on good i, G_{it}, is determined by a demand function analogous to a household's demand function, (9), and with the same price elasticity. Defining the real value of government bonds as $B_t \equiv B_t^N / P_t$, we can rewrite the budget constraint in real terms, giving:

$$G_t - T_t \;=\; B_t - (1+r_{t-1})B_{t-1} + (M_t - M_{t-1})/P_t\,. \tag{17}$$

Clearly, only three of the four policy instruments, (G_t,T_t,B_t,M_t), can be chosen independently in any period t. One of them has to be determined as a residual, to satisfy (17). Below, we always take G_t to be fixed at some exogenous, time-invariant, value, G. Furthermore we take the second independent fiscal instrument to be real government debt, inclusive of interest, which we denote as $B_t' \equiv (1+r_t)B_t$.[6] With either M_t or i_t being determined by the monetary policy rule (see below), this leaves the lump-sum tax, T_t, to be determined by (17) as the residual instrument of policy. Such a fiscal regime allows us easily to study the effect of a once-and-for-all change in the level of government debt. It is not our aim here to study an empirically realistic fiscal regime: for example, one that incorporates 'automatic fiscal stabilisers', making G_t and T_t functions of output; or one that involves rules limiting the government deficit or debt levels to some percentage of GDP. Other authors have studied such regimes using models similar to the present one, but they involve several policy instruments changing simultaneously, so that numerous effects become tangled up together.

[6] Government debt should therefore be thought of as 'indexed' debt. More precisely still, B_t' is the number of 'real treasury bills' issued, i.e. it is a promise to deliver B_t' units of the composite consumption good to the holders of the bonds at the start of period t+1.

Here our main objective is to conduct a clean and simple fiscal experiment, to elucidate the mechanics of how the macroeconomy is affected. Amongst other things, by choosing the time path of B_t' exogenously, we remove the endogenous evolution of government debt as an additional source of dynamics.

(iv) Market-clearing conditions

Equilibrium in the goods market requires that:

$$Y_t = C_t + G_t . \tag{18}$$

Here the equation is written for the composite good, but a counterpart of (18) also holds for every good, *i*.

To write down the condition for equilibrium in the labour market, we first need the aggregate demand for labour. The derivation of this is given in Appendix A. Equating aggregate labour demand to aggregate labour supply, where the latter is given by the inverse of (5) (dropping the '*s*' subscript, for reasons explained), we have:

$$Y_t^{1/\sigma}(P_t / \tilde{P}_t)^{\theta/\sigma} = \left[\eta^{-1}(1-\delta)Z_t^{\delta}W_t / P_t\right]^{1/(\varepsilon-1)} . \tag{19}$$

Here, $\tilde{P}_t$ is a price index very similar to P_t - see again Appendix A. Below, in order to study the macroeconomic implications of the model, we shall take a loglinear approximation to its equations. When we do this it turns out that, locally, $\tilde{P}_t = P_t$, in which case $\tilde{P}_t$ drops out of (19). In fact, the aggregate labour demand function on the LHS of (19) is then simply the inverse production function applied to aggregate output, as can easily be seen.

Equilibrium in the bond market requires that the exogenous, government-determined stock of bonds should equal the aggregate demand for them by households. In view of the role of aggregate financial wealth - bonds plus money - in affecting aggregate consumption, as was observed in (6), a helpful relationship to note is the following:

$$V_t = M_t / P_{t+1} + B_t' . \tag{20}$$

One component of aggregate financial wealth, B'_t, is thus exogenous, under our assumed fiscal policy regime. The other component, the real money stock, is endogenous, and in money-market equilibrium this must equal money demand as given by the aggregate version of (4).

(v) Steady-state general equilibrium

In order to study the dynamics of the general equilibrium, below we will loglinearise the model. First, however, it is useful to comment on some features of the steady-state equilibrium in which all aggregate real variables are constant over time. We will confine attention to steady states with zero inflation since, later, the monetary policy regime will be constructed so as to ensure long-run price stability.

One notable steady-state relationship is the following:

$$Y = \left[\frac{\theta\eta}{(\theta-1)\sigma(1-\delta)} \right]^{\sigma/(\sigma-\varepsilon)} Z^{\delta\sigma/(\varepsilon-\sigma)}. \tag{21}$$

A steady-state value is denoted by absence of a time subscript. (21) shows that, in the long run, the value of output is positively related to that of money demand per unit of consumption. The reason for this is the 'Brock effect' mentioned earlier: higher equilibrium real balances raise the marginal utility of consumption, due to non-separability of the utility function, which in turn provides a stimulus to households to increase labour supply. Note that in the steady state prices have had time to adjust and so are effectively perfectly flexible, whence steady-state output is determined by the 'supply side' of the model. The Brock effect will be weak to the extent that δ is rather small. Indeed, empirically, we would expect δ to be small and this effect to be of minor significance. (21) also demonstrates the absence of a direct effect of government debt, B', on steady-state output. This is despite the fact that overlapping generations, i.e. $q < 1$, imply that government bonds are 'net wealth' for the household sector, i.e. despite the fact that Ricardian Equivalence does not hold (as will be seen in another context below). The explanation is that the 'modified GHH' utility function has eliminated the wealth effect on labour supply. Were such an effect present, an increase in

B' might be expected to directly reduce output by discouraging work effort. In fact, an increase in B' *does* reduce steady-state output, but through the indirect channel of reducing Z, as will be shown later.

A second relationship of interest is the steady-state version of (6), the 'aggregate Euler equation':

$$1+r \;=\; 1/\beta \;+\; (1-\delta)(1/\beta-q)(1/q-1)V/A\,. \tag{22}$$

'A' in this denotes 'adjusted consumption', as defined above. (22) makes clear that, with infinitely-lived agents (q =1), the long-run real interest rate is simply determined by the time preference rate, i.e. by β. However, with overlapping generations (q < 1), the real interest rate exceeds this value, and the amount of the excess depends on the ratio of financial assets to adjusted consumption. Intuitively, (22) is the (inverse) 'demand function for financial assets'. A high value of r causes households to choose a positive 'tilt' to their lifetime consumption profiles, and since - in a steady state - they have constant labour incomes, in order to achieve this they need to accumulate financial assets during their lifetimes. In the aggregate, such behaviour generates a positive demand for financial assets as a store of value. From (22) we can also see how Ricardian Equivalence fails when q < 1, since an increase in the stock of government debt, B', adds to V (for the moment consider the other component of V, the stock of real balances, M/P, as given) and thus raises r. This means that government debt does affect real variables, and so is 'non-neutral' when q < 1. However, it remains to be seen exactly what form this non-neutrality takes in the complete general equilibrium.

In a zero-inflation steady state where government debt, B', is set to zero, and where G is also set to zero, we can derive the following completely reduced-form solution for r:

$$r \;=\; \tfrac{1}{2}\left\{1/\beta-1+\chi \;\pm\; [(1/\beta-1+\chi)^2+4\chi]^{1/2}\right\} \tag{23}$$

$$\text{where } \chi \;\equiv\; (1/\beta-q)(1/q-1)\delta[1-(1-\delta)\sigma(\theta-1)/\theta\varepsilon]^{-1}\,.$$

(23) is the solution of a quadratic equation in r. The larger of the two solutions for r encompassed in (23) is the relevant one. This is because the smaller solution is clearly negative, and since r also equals i in a zero-inflation steady state, it would imply a negative

value for the nominal interest rate, which is meaningless. We can easily see from (23) that when $\chi = 0$, $r = 1/\beta$-1; while when $\chi > 0$, $r > 1/\beta$ -1. One special case in which $\chi = 0$ is clearly when $q = 1$, which yields the result noted earlier that infinite lives leads to the real interest rate being equal to the pure rate of time preference. However another special case in which $\chi = 0$ is in the limit as $\delta \to 0$. It is useful to pause here to understand why it occurs. First recall that δ is the exponent on real money balances in the utility function. As $\delta \to 0$, the demand for real balances tends to zero, as is apparent from (4). The equilibrium level of real balances is essentially determined by the demand for them, since even if the nominal money stock M is exogenous, what matters is the money stock in relation to the price level, and the latter is endogenous. Hence as δ shrinks to zero, so does the equilibrium stock of real balances. Since we have assumed that $B' = 0$ in deriving (23), it is now clear that by letting δ tend to zero, we reduce the total stock of financial assets, V, to zero. The earlier equation (22) tells us that in this case, even if $q < 1$, the real interest rate will still simply equal the time preference rate. The intuitive explanation is that if the supply of financial assets is zero, for the market to clear the demand for them must also be zero, and to achieve this the real interest rate must be driven down to the time preference rate.

(vi) A loglinearised and partially-reduced form of the model

In order to investigate the macroeconomic properties of the model we now loglinearise its equations. The ‘reference’ steady state about which we take the approximation is the steady state with zero inflation, zero government debt and zero government spending. The value of the real interest rate in this steady state is given by the larger of the two solutions (23). This interest rate appears frequently as part of the coefficients of the loglinearised equations, and we henceforth denote it as r_R.

Appendix B provides a complete list of the underlying loglinearised equations. In the New Neoclassical Synthesis model to which ours is closely related, the standard way of combining these equations is in the form of a ‘New Keynesian Phillips Curve’ (NKPC) equation and an ‘IS’ equation. A third equation is also needed which depends on the monetary policy rule: if, for example, an interest-rate feedback rule such as the Taylor Rule is used, this provides the third equation; while if the money supply is the instrument of monetary policy, then an ‘LM’ equation completes the model. We adopt the same general approach to summarising our model’s structure here. This yields the following equations (derived in Appendix B):

$$\pi_t \;=\; (1+r_R)^{-1}\pi_{t+1} \;+\; \kappa[y_t - (\varepsilon/\sigma - 1)^{-1}\delta z_t\,], \tag{24}$$

$$\begin{aligned}
&[1-(1-\delta)(1-1/\theta)]y_{t+1} \;+\; (1-\zeta^{-1})\delta z_{t+1} \\
&\quad = \beta(1+r_R)\Big\{[1-(1-\delta)(1-1/\theta)]y_t \;+\; (1-\zeta^{-1})\delta z_t\Big\} \\
&\qquad + \beta(1+r_R)\zeta^{-1}(\hat{i}_t - \pi_{t+1}) \;-\; \beta\chi(1+r_R^{-1})\zeta^{-1}v_t\,,
\end{aligned} \tag{25}$$

$$z_t \;\;(=\; m_t - p_t - y_t) \;=\; -r_R^{-1}\hat{i}_t\,, \tag{26}$$

$$v_t \;=\; m_t - p_{t+1} + b_t'\,. \tag{27}$$

Unless otherwise stated, lower-case variables are the log-deviations of their upper-case counterparts; e.g. $y_t \equiv \ln Y_t - \ln Y_R$ (where ‘R’ denotes the value in the reference steady state). π_t is defined as $p_t - p_{t-1}$, the rate of inflation. $\hat{i}_t \equiv \ln(1+i_t) - \ln(1+i_R)$, and similarly for $\hat{r}_t$ which appears below. Since $B_R' \equiv 0$, the log-deviation of B_t' is not well-defined; hence we use $b_t' \equiv (B_t' - B_R')/V_R$. In the above and henceforth we have also set government spending, G_t, permanently to zero; hence y_t and c_t are the same variable. Two new composite parameters which appear here are $\kappa \equiv (1-\alpha)[\alpha^{-1} - (1+r_R)^{-1}](1-\theta+\theta/\sigma)^{-1}(\varepsilon/\sigma - 1)$ and $\zeta \equiv [1-(1-\delta)(1-1/\theta)\sigma/\varepsilon]^{-1}$. These are both positive.

(24) is the NKPC equation for our model, giving inflation as an increasing function of current output and of expected future inflation. It essentially derives from the price-setting and price index equations, (14) and (15), together with the condition for labour market clearing, (19). Compared to the standard model, its novel feature is the inclusion of z_t, which is the result of the ‘Brock effect’ of real balances in stimulating labour supply, as discussed earlier. (25) is the IS equation which, as in the related literature, derives from the Euler equation for consumption. One of its distinctive features here is the inclusion of financial wealth, v_t. This is the result of the ‘generational turnover’ effect on aggregate consumption,

as explained previously. Another of its distinctive features is the presence of z_t and z_{t+1}. These are a further consequence of non-additively-separable utility, which causes Z_t to be a component of 'adjusted consumption', as seen above. Having seen that z_t enters the NKPC and IS equations, it is clear that our model cannot be solved (unlike the standard model) without also using the LM equation, given by (26). With the LM equation we can eliminate z_t, introducing further instances of $\hat{i}_t$.

The system consisting of (24)-(27) still does not provide a complete description of the economy's dynamics because it remains to add a monetary policy rule. In the next section we do this using a Taylor Rule for the interest rate, and we study the macroeconomic behaviour which results. In the following section we carry out a similar analysis by instead assuming a simple rule for the money supply.

3. Fiscal Policy when Monetary Policy is Governed by a Taylor Rule

The standard way of representing monetary policy in recent years has been to assume that the nominal interest rate is set as a function of the inflation rate and of output. The best known example of such a rule is that of Taylor (1993). In our log-deviation notation, such a rule could be expressed as:

$$\hat{i}_t = \bar{i}_t + \phi_\pi \pi_t + \phi_y y_t . \tag{28}$$

In this, ϕ_π and ϕ_y are positive feedback parameters and $\bar{i}_t$ is the 'intercept term', changes in which represent the purely discretionary aspect of policy. In fact, in what follows, we shall work with a simplified version of the rule in which $\phi_y = 0$. This helps to limit the amount of algebra while not affecting the main conclusion.

The steady-state rate of inflation which results from this policy will depend, inter alia, on the steady-state value chosen for $\bar{i}_t$ (which we may call $\bar{i}$). It is usual to parameterise the Taylor Rule in such a way that it delivers an exogenously-chosen 'target' level of long-run inflation. To achieve this, $\bar{i}$ must be chosen appropriately. In the present paper, our interest is mainly in fiscal policy, so we take the value of the target steady-state inflation rate to be zero. In the standard, infinite-lives model, where the steady-state real interest rate $\hat{r}$ ($\equiv \hat{i} - \pi$) is simply $1/\beta$-1, zero steady-state inflation is obtained by setting $\bar{i}$ = $1/\beta$-1. (This can easily be seen from (28), with ϕ_y = 0.) However in the overlapping-generations model $\hat{r}$ has a more complex set of determinants, one of which is the level of government debt. The level at which $\bar{i}$ needs to be set in order to ensure zero steady-state inflation is therefore endogenous and remains to be calculated below: it will simply be whatever is the associated level of the steady-state real interest rate.

It is simplest to start by examining the effects of fiscal policy on the steady state, turning to the dynamics subsequently. The Taylor Rule itself plays no role in determining the steady state, other than via our assumption that steady-state inflation is zero. Hence for now we set aside (28), but we will return to it shortly.

The system (24)-(27) may be used to solve for the steady state. In doing so, first note that the real-balances component of v_t, m_t - p_{t+1}, may be substituted out as z_t + y_t - π_{t+1}. Setting variables to time-invariant values, and π to zero, we then have four equations with ($y,\hat{i}$,z,v) as the unknowns. From this we can derive the following expressions for the steady-state values (y, $\hat{i}$) as functions of the government debt level, b' (see Appendix C):

$$y = \left\{\beta(1+r_R)-1-\psi + \left[[\beta(1+r_R)-1]\rho-\psi r_R^{-1}-\nu\right]\xi^{-1}\kappa\right\}^{-1}\psi b', \tag{29}$$

$$\hat{i} = -\xi^{-1}\kappa\left\{\beta(1+r_R)-1-\psi + \left[[\beta(1+r_R)-1]\rho-\psi r_R^{-1}-\nu\right]\xi^{-1}\kappa\right\}^{-1}\psi b'. \tag{30}$$

Four new composite parameters are introduced in these expressions, namely:

$$\rho \equiv [1-(1-\delta)(1-1/\theta)]^{-1}(1-\zeta^{-1})\delta r_R^{-1}, \qquad \nu \equiv [1-(1-\delta)(1-1/\theta)]^{-1}\zeta^{-1}\beta(1+r_R),$$

$$\psi \equiv [1-(1-\delta)(1-1/\theta)]^{-1}\zeta^{-1}[\beta(1+r_R)-1], \qquad \xi \equiv \kappa(\varepsilon/\sigma-1)^{-1}\delta r_R^{-1}.$$

ρ, ν and ξ are unambiguously positive. ψ is positive if q < 1 or zero if q = 1, since then 1+r_B > $1/\beta$ or = $1/\beta$, respectively, as noted above. The signs of the coefficients on b' in (29) and (30) can be seen to hinge on the sign of the bracketed term {.}, which is at first glance ambiguous. However the sign of {.} can in fact be resolved to be negative (see again

Appendix C). This reveals that, when $q < 1$, the effect of permanently higher government debt is to increase the steady-state interest rate (real as well as nominal, since inflation is zero) and to reduce steady-state output. We also see clearly that when $q = 1$ these effects are zero (since ψ is then zero), which is the manifestation of Ricardian Equivalence.

The finding that higher government debt raises the interest rate is to be expected, since it is a standard result in other overlapping-generations models (e.g. Diamond (1965), Blanchard (1985)). The intuitive explanation for it here is that, as seen, overlapping generations gives rise to a demand for financial assets as a store of value which is increasing in the interest rate. Hence when the supply of such assets is increased by increasing b', the interest rate has to rise to clear the asset market. The finding that output falls is perhaps less expected, especially since there is no capital in our model. The only variable input is labour, so the mechanism must involve a fall in labour supply. One might at first think that the mechanism is that the increased bond stock, being perceived as ‘net wealth’ by households, increases the demand for leisure and so reduces work effort. However, this is not correct, since our use of GHH preferences has removed the usual wealth effect on labour supply. Instead, the mechanism is the Brock effect: the increased nominal interest rate reduces the equilibrium stock of real money balances, and the non-separability of the latter in utility then acts as an alternative disincentive to provide labour. We would not expect this effect to be very significant empirically. In particular, the effect will be weak to the extent that δ, the weight on real balances in the utility function, is small.

Next, consider the perfect-foresight transition path to the steady state following a once-and-for-all increase in government debt. To be precise, suppose that in t = -1 the economy is in a steady state with no debt, and then in t = 0 there is an unanticipated tax cut, i.e. a fall in τ_t, which lasts for one period only. In all subsequent periods, taxation is adjusted to hold debt constant at its new, higher, level. The economy’s laws of motion in this situation can be written as:

$$\pi_{t+1} = (1+r_R)[\pi_t - \kappa y_t - \xi \hat{i}_t], \tag{31}$$

$$y_{t+1} - \rho \hat{i}_{t+1} = \beta(1+r_R)(y_t - \rho \hat{i}_t) + \nu(\hat{i}_t - \pi_{t+1}) - \psi(y_t - r_R^{-1}\hat{i}_t - \pi_{t+1} + b'_t), \tag{32}$$

$$\hat{i}_t = \bar{i} + \phi_\pi \pi_t . \tag{33}$$

This system has been obtained from (24)-(27) by substituting (27) into (25) (rewriting m_t - p_{t+1} as $z_t + y_t - \pi_{t+1}$), and then (26) into (24) and (25), using the definitions of (ρ, ν, ψ, ξ) above. (31) and (32) are slightly more reduced-form expressions for the NKPC and IS equations. Relative to the standard NKPC and IS equations, the differences are the term in ψ in the IS, which is present when $q < 1$ and represents the generational turnover effect; and the terms ξ and ρ in the NKPC and IS (respectively), which arise from non-separability of the utility function.

The main result of the paper is visible from this system. It is clear that (31) and (32), with $\hat{i}_t$ being governed by the Taylor Rule, (33), constitute a pair of simultaneous first-order difference equations in (π_t, y_t). π_t and y_t are both non-predetermined variables, so for a determinate perfect-foresight equilibrium to exist, we need the two eigenvalues of the system to lie outside the unit circle. Let us assume this holds - we return to this question below. Now notice that if the economy is initially in a steady state with $b' = 0$ (and hence $\pi = y = 0$), and then in $t = 0$ b_t' is raised to some positive value b' and held there ever after, then there is no time-variation in any exogenous variable of the system over $t = 0,...,\infty$. The only exogenous variable in (31)-(32) is b_t', and by assumption it is held constant at b' for $t = 0,...,\infty$. It then follows that the economy must jump immediately to its new steady state. This means that the impact effect on output, inflation and all other endogenous variables is the same as the long-run effect. In other words, despite price stickiness and despite the lack of Ricardian Equivalence, the attempt to give a short-run Keynesian stimulus to output by a temporary tax cut fails. Output moves straight away to its new steady-state level which, as seen above, is *lower* - even if not much lower - than its initial level.

We have constructed a model with the aim of formalising, in a modern DGE framework, the positive effect of a fiscal expansion on output which is familiar from more ad-hoc models such as the textbook IS-LM model. However, we have not succeeded, so the question arises of ‘why?’ We will argue that the culprit is the monetary policy regime. To show this, in Section 4 we study the same fiscal experiment under a different rule for monetary policy. An intuitive understanding of why the Taylor Rule destroys the effectiveness of fiscal policy can best be obtained in the light of this comparison: see the end of the next section.

First, however, we return to the question of determinacy of the perfect foresight equilibrium under a Taylor Rule. In Appendix D we prove that, conditional on δ being sufficiently close to zero, a necessary and sufficient condition for both eigenvalues of the system (31)-(33) to lie outside the unit circle is that the following should hold:

$$\phi_\pi > \frac{1}{\kappa}\frac{(1-\delta)(1-1/\theta)(1-\sigma/\varepsilon)}{1-(1-\delta)(1-1/\theta)\sigma/\varepsilon}\frac{\beta(1+r_R)-1}{\beta(1+r_R)^2-1}\frac{r_R^2}{1+r_R} + \frac{r_R}{\beta(1+r_R)^2-1}. \tag{34}$$

In the standard 'New Neoclassical Synthesis' model with no feedback of the interest rate on output (ϕ_y = 0), the condition for determinacy is that $\phi_\pi > 1$ (the 'Taylor Principle'). In our model we see that if ϕ_π is greater than unity, this may or may not ensure determinacy. Notice that the critical value on the RHS of (34) is decreasing in κ. A key determinant of κ (which is the slope of the 'short-run' Phillips curve), is α, the 'price survival' probability and hence the degree of price stickiness, with κ tending to zero as α tends to one, and κ tending to infinity as α tends to zero. Sketching the inequality (34) as in Figure 1 below, we see that, for high degrees of price stickiness (low κ), a value of ϕ_π much greater than one may be needed for determinacy. Conversely, for low degrees of price stickiness (high κ), a value of ϕ_π less than one may be sufficient.[7]

[FIGURE 1 ABOUT HERE]

Bénassy (2005), on the other hand, finds that the magnitude of the feedback coefficient on inflation becomes irrelevant for determinacy when overlapping generations are introduced. It is notable that he assumes a different fiscal policy to ours, in which the total

[7] Notice that the second RH term in (34) is less than one, insofar as $1+r_R > 1/\beta$.

nominal stock of government liabilities (bonds plus money) is held constant over time. Piergallini (2006), meanwhile, finds that overlapping generations relax the normal Taylor Principle, independently of the degree of price stickiness.

4. Fiscal Policy When Monetary Policy is Governed by a Money-Supply Rule

We now replace the Taylor rule by a rule which makes the monetary growth rate the exogenous instrument of monetary policy:

$$\mu_t \equiv m_t - m_{t-1}.$$

We will presently assume $\mu_t = 0$ in all periods, i.e. that the money supply is pegged at a constant value; but to show how μ_t enters the equations we begin with the more general case.

The economy's laws of motion under this regime are still given by (31) and (32). However the Taylor Rule, (33), is now replaced by:

$$\hat{i}_{t+1} = \hat{i}_t + r_R(y_{t+1} - y_t + \pi_{t+1} - \hat{\mu}_{t+1}). \tag{35}$$

(35) is just the first-differenced version of the LM equation, (26). It can be seen that (31), (32) and (35) constitute a system of three simultaneous first-order difference equations in (π_t, y_t, $\hat{i}_t$), i.e. a third-order system. To characterise its behaviour is hence more difficult than for the second-order system which represented the economy under a Taylor Rule. Under a Taylor Rule, the state variables (π_t, y_t) were both non-predetermined; here, although the same is true of each *individual* state variable (π_t, y_t, $\hat{i}_t$), a linear combination of them is, however, predetermined. This combination is:

$$\pi_t + y_t - r_R^{-1}\hat{i}_t = m_t - p_{t-1}, \tag{36}$$

which has been obtained by subtracting m_{t-1} - p_{t-1} from both sides of the LM equation, (26). As of period *t*, the RHS of (36) is clearly exogenous (predetermined), so there are only two degrees of freedom in the way (π_t, y_t, $\hat{i}_t$) can 'jump' if an unexpected shock occurs. Our third-order system is hence equivalent to a system with one predetermined and two non-predetermined state variables. The fact that there is now a predetermined variable means that the economy will not in general jump straight to its new steady state when there is a permanent shock. This already hints that, under a money supply rule, the short-run impact of an increase in government debt is unlikely to be the same as the long-run impact. The question to be answered is what is the nature of this short-run impact: in particular, does it involve a boom in output?

We can rearrange the system (31), (32) and (35) into a matrix equation with the following general form:

$$\begin{bmatrix} \pi_{t+1} \\ y_{t+1} \\ \hat{i}_{t+1} \end{bmatrix} = A \begin{bmatrix} \pi_t \\ y_t \\ \hat{i}_t \end{bmatrix} + B \begin{bmatrix} \hat{\mu}_{t+1} \\ b_t' \end{bmatrix}. \tag{37}$$

From the foregoing we can say that determinacy of the perfect foresight equilibrium - ‘saddlepoint stability’ - requires that the coefficient matrix A possess one eigenvalue inside, and two outside, the unit circle. In fact we can show that this condition holds with no additional parameter restrictions.[8] Moreover we can show that the unique stable eigenvalue (which we denote as λ_1) lies in the interval (0,1), rather than the interval (-1,0). This implies that convergence to the new steady state following a shock is monotonic, rather than oscillatory.

We now consider the same fiscal policy shock as in Section 3, namely a once-and-for-all increase in government debt which occurs in $t = 0$, brought about by a tax cut in period 0 only. Monetary growth, $\hat{\mu}_t$, is held at zero for all t. The particular point of interest is to solve for the initial value of output, y_0. The perfect foresight solution can be written as:

$$\begin{bmatrix} \pi_t \\ y_t \\ \hat{i}_t \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} \lambda_1^t + \begin{bmatrix} \pi \\ y \\ \hat{i} \end{bmatrix} \qquad \text{for } t = 0,\ldots,\infty, \tag{38}$$

where the vector multiplying λ_1^t is the stable eigenvector of A. This vector is in general only unique up to a scalar multiple of itself. However, by making use of the initial condition (36)

[8] The calculations required are lengthy and hence are not reproduced in the paper. They are available on request.

we can determine the absolute values of its elements and not just their ratios. To see this, first note that, setting t = 0 in (38), the absolute values (c_{11}, etc.) can be interpreted as the initial values of variables measured as deviations from their new steady-state values (π_0 - π, etc.) Let us next write (36) in a more schematic way, as follows:

$$d_1\pi_0 \ + \ d_2 y_0 \ + \ d_3\hat{i}_0 \ = \ d_0 \, .$$

In view of the preceding point, this is equivalently:

$$d_1 c_{11} \ + \ d_2 c_{21} \ + \ d_3 c_{31} \ = \ d_0 \ - \ d_1\pi \ - \ d_2 y \ - \ d_3\hat{i} \quad (\equiv \bar{d}, \ \text{say}) \, . \tag{39}$$

(39) and the matrix equation which determines the stable eigenvector of A are sufficient to allow us to solve for (c_{11},c_{21},c_{31}). We can combine them in a single matrix equation, such as:

$$\begin{bmatrix} d_1 & d_2 & d_3 \\ a_{11} - \lambda_1 & a_{12} & a_{13} \\ a_{21} & a_{22} - \lambda_1 & a_{23} \end{bmatrix} \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \end{bmatrix} \ = \ \begin{bmatrix} \bar{d} \\ 0 \\ 0 \end{bmatrix} . \tag{40}$$

Here, a_{ij} is an element of the matrix A. In writing (40), we have arbitrarily discarded the third equation of the system which defines the stable eigenvector, since one of the three in that homogeneous system is redundant.

From (40), we can now obtain an expression for c_{21} (= y_0 - y) by using Cramer's Rule:

$$c_{21} \ = \ \bar{d}[(\lambda_1 - a_{11})a_{23} \ - \ a_{21}a_{13}]\det{}^{-1} . \tag{41}$$

Here, 'det' is the determinant of the matrix in (40). The sign of c_{21} determines whether, on impact, output jumps to a value above or below its new steady-state level. Note that the new steady state is the same under a constant money supply as it is under the Taylor Rule, since by construction $\pi = 0$ under the Taylor Rule, while a constant money supply clearly requires that inflation likewise be zero in the steady state. The steady-state values $(y, \hat{i})$ are therefore still given by (29) and (30) (recalling that $\pi = 0$ was the only feature of monetary policy used in their derivation). We hence know that the permanent increase in government debt will again cause output in the long run to be slightly lower. Given the price stickiness and the lack of Ricardian Equivalence, however, we might conjecture that on impact output would exceed its long-run level, i.e. that there would be a boom. For this, we need to show that $c_{21} > 0$.

Consider the signs of the constituent terms in (41). In the general case, the algebraic expressions for det and for the term [...] are complicated and not easy to sign. They are given in Appendix E. However, we can easily sign them if we let $\delta \to 0$, a restriction also appealed to earlier. This is the case where the weight on real balances in the utility function tends to zero. If δ is actually set to zero, the expressions simplify to (see again Appendix E):

$$\det \;=\; \nu(1-\beta^{-1}-\beta^{-1}\kappa) \;-\; (1-\lambda_1)\,, \tag{42}$$

$$(\lambda_1 - a_{11})a_{23} \;+\; a_{21}a_{13} \;=\; \nu(\lambda_1-\beta^{-1})\,. \tag{43}$$

Recalling that $\beta^{1} > 1$, these are clearly both negative.[9] By continuity, they will also be negative for δ in some sufficiently close neighbourhood of zero. $\bar{d}$, the third factor in (41), is generally defined as the RHS of (39), i.e. it is just a linear combination of the new steady-state values. Using the steady-state solutions (29)-(30) and the definitions of the '*d*' coefficients implicit in (36), we have:

$$\bar{d} \;=\; -y \;+\; r_R^{-1}\hat{i}\,. \tag{44}$$

We saw earlier that $y < 0$, $\hat{i} > 0$ when $b' > 0$. Hence $\bar{d} > 0$, implying that $c_{21} > 0$.

The above establishes that, under a constant money supply, fiscal policy *is* effective in raising output, unlike under a Taylor Rule. In fact we can prove that not only does $y_0 - y > 0$ hold, but also $y_0 > 0$: output rises on impact relative to its initial steady-state value (zero) as well as relative to its new steady state value. Since we know that convergence to the new steady state thereafter is monotonic, the complete time path of output can then be depicted - in a purely qualitative way - as in Figure 2. In Figure 2 we also sketch the associated paths of inflation, the nominal interest rate and the real interest rate. For reasons of space, the algebra which demonstrates the qualitative features of these paths (i.e. y_0, π_0, $\hat{i}_0 - \hat{i} > 0$ and $\hat{r}_0 < 0$) is not presented here, but it is available on request. We again use the assumption that δ is

[9] λ_1, although affected by δ, can be shown to remain in the interior of the interval [0,1] when $\delta = 0$

sufficiently close to zero (and, in the case of $\hat{r}_0 < 0$, the assumption that β is sufficiently close to 1).

[FIGURE 2 ABOUT HERE]

Why does the monetary policy rule make such a sharp difference to the effectiveness of fiscal policy? This can be answered by examining what happens to the money supply under a Taylor Rule. Under interest-rate control, the money supply is of course endogenous. We have that:

$$m_t \equiv z_t + y_t + p_t . \tag{45}$$

In the model of Section 3, the fiscal expansion was seen to cause the economy to jump immediately to its new steady state in which z and y are lower while π remains at zero. From (45) we then see that hiding behind the Taylor Rule is a sudden reduction in the money supply. This negates the expansionary effect on aggregate demand of the increase in the stock of government bonds. Under a money supply rule, on the other hand, the money stock is fixed, so it cannot adjust to offset the higher bond stock. In this case we obtain the ‘Keynesian’ outcome which the textbook IS-LM model would lead us to expect. The endogeneity of the money supply under a Taylor Rule also explains why the new steady state is attained immediately. We know that real money balances must be lower in the new steady state. Under a money supply rule, this can only be achieved by a rise in the price level, which takes time, since the latter is sticky. Under a Taylor Rule, it is achieved by a fall in the nominal money supply, which ‘bypasses’ the price stickiness.

5. Conclusions

We have constructed a DGE model with staggered prices and overlapping generations, and have used it to study the macroeconomic effects of a one-off debt-financed tax cut. Surprisingly, given the lack of Ricardian Equivalence, we found that this does not give a short-run stimulus to output in our baseline model, but that output moves immediately to its new steady-state value, which is in fact lower (though probably not much lower) than its initial value. We then repeated the experiment, replacing the assumption of a Taylor Rule for monetary policy by the assumption of a constant money supply. In this case the expected short-run boom in output does occur. The choice of monetary policy regime is hence crucially important when assessing the effectiveness of fiscal policy.

A debt-financed tax cut is the crucial policy experiment for assessing the effectiveness of fiscal policy as a tool of demand management because it provides the cleanest test of how much leverage the absence of Ricardian Equivalence generates. Another standard type of fiscal policy change would be a balanced-budget increase in government spending. This does not rely on a lack of Ricardian Equivalence for it to be non-neutral. We have not, up to now, formally analysed such a policy change in our model, but in similar models one usually finds that it *lowers* consumption, and thus has a multiplier of at most unity on output, even under price stickiness. We would expect this to be true here too.

The fact that Taylor Rules cause fiscal policy ineffectiveness in a framework such as this leads on to the question of what role, if any, fiscal policy has as a tool of optimal macroeconomic stabilisation policy. We hope to investigate this in future work. The present result might suggest that fiscal policy has no useful role in stabilisation, and that stabilisation should be done entirely through monetary policy. However, such a simple conclusion is unlikely to be correct, because fiscal policy still has *some* real effects - such as on the real interest rate - even under a Taylor Rule. This makes it unlikely that a zero fiscal reaction to shocks is optimal.

Appendix A The Aggregate Demand Function for Labour

By inverting firm *i*'s production function we may write its demand for labour as a function of its output: $L_{it} = Y_{it}^{1/\sigma}$. Y_{it} is demand-determined and given by (12). If firm *i* last changed its price *j* periods ago, then $P_{it} = X_{t-j}$, so that firm *i*'s demand for labour is:

$$L_{it} = (X_{t-j} / P_t)^{-\theta} Y_t \, . \tag{A1}$$

The proportion of firms who last changed their price *j* periods ago is $(1-\alpha)\alpha^j$. Summing across $j = 0,...,\infty$, we then obtain aggregate labour demand as:

$$L_t \;=\; \Sigma_{j=0}^{\infty}(1-\alpha)\alpha^j (X_{t-j} / P_t)^{-\theta/\sigma} Y_t^{1/\sigma} \, . \tag{A2}$$

If we define $\tilde{P}_t \equiv [\Sigma_{j=0}^{\infty}(1-\alpha)\alpha^j X_{t-j}^{-\theta/\sigma}]^{-\sigma/\theta}$ this can also be written in the form:

$$L_t \;=\; Y_t^{1/\sigma}(P_t / \tilde{P}_t)^{\theta/\sigma} \, , \tag{A3}$$

which yields the LHS of (19).

Appendix B The Underlying Loglinearised Equations and the Derivation of the Partially-Reduced Form of the Model

$$p_t \;\; (= \tilde{p}_t) \;=\; \alpha p_{t-1} \;+\; (1-\alpha) x_t \, , \tag{A4}$$

$$x_t \;=\; \alpha(1+r_R)^{-1} x_{t+1} \;+\; (1+r_R-\alpha)(1+r_R)^{-1}(1-\theta+\theta/\sigma)^{-1}[w_t + (1/\sigma - 1)(\theta p_t + y_t)] \, , \tag{A5}$$

$$w_t \;=\; p_t \;+\; \sigma^{-1}(\varepsilon - 1) y_t \;-\; \delta z_t \, , \tag{A6}$$

$$y_t \;=\; c_t \;+\; g_t \, , \tag{A7}$$

$$z_t \;\equiv\; m_t \;-\; p_t \;-\; c_t \, , \tag{A8}$$

$$z_t \;=\; -r_R^{-1} \hat{i}_t \, , \tag{A9}$$

$$a_{t+1} \;=\; \beta(1+r_R) a_t \;+\; \beta(1+r_R)\hat{r}_t \;-\; \beta\chi(1+1/r_R) v_t \, , \tag{A10}$$

$$\hat{r}_t = \hat{i}_t - \pi_{t+1}, \tag{A11}$$

$$\pi_t \equiv p_t - p_{t-1}, \tag{A12}$$

$$a_t = \zeta c_t - (\zeta - 1)\varepsilon l_t + \delta(\zeta - 1)z_t, \tag{A13}$$

$$l_t = \sigma^{-1} y_t, \tag{A14}$$

$$v_t = m_t - p_{t+1} + b_t', \tag{A15}$$

$$g_t - \tau_t = \delta(1-\delta)^{-1}(1+r_R^{-1})[(1+r_R)^{-1}b_t' - b_{t-1}' + m_t - m_{t-1}]. \tag{A16}$$

Government spending and taxation are zero in the reference steady state, so their log-deviations are not well defined. Hence we define $g_t \equiv G_t/Y_R$, $\tau_t \equiv T_t/Y_R$.

To derive the NKPC equation, (24), we first use (A6) to eliminate w_t from (A5):

$$\begin{aligned} &x_{t+1} - \alpha^{-1}(1+r_R)x_t \\ &\quad = -\alpha^{-1}(1+r_R-\alpha)[p_t + (1-\theta+\theta/\sigma)^{-1}(\varepsilon/\sigma-1)y_t - (1-\theta+\theta/\sigma)^{-1}\delta z_t]. \end{aligned} \tag{A17}$$

Next, we 'quasi-difference' the price-index equation, (A4), to the same pattern as the LHS of (A17). That is, we advance (A4) by one period, multiply through the original equation by $\alpha^{-1}(1+r_R)$ and then subtract the latter from the former. This gives:

$$p_{t+1} - \alpha^{-1}(1+r_R)p_t = \alpha p_t - (1+r_R)p_{t-1} + (1-\alpha)[x_{t+1} - \alpha^{-1}(1+r_R)x_t]. \tag{A18}$$

(A17) can now be used to eliminate the x variables from (A18). The p variables can then be grouped such that they can be replaced by π variables, which yields (24).

To derive the IS equation, (25), we substitute (A11), (A13) and (A14) into (A10). Then we use (A7), with g_t set to zero, to replace c_t by y_t.

Appendix C The Algebra of the Steady-State Solution

To solve for the steady state we use the system (31) and (32). This is a slightly more reduced form of the system (24)-(27). Its derivation is described in the main text. Setting variables to time-invariant values in (31) and (32), and also setting $\pi = 0$, we obtain a pair of equations in $(y, \hat{i})$. Solving these then yields the steady-state solutions (29) and (30).

The common denominator of (29) and (30), namely the bracketed term {.}, is at first sight of indeterminate sign. Here we show that the sign is in fact negative. Using the definitions of the composite parameters already introduced, we may re-express two of the terms which appear inside {.} as follows:

$$\beta(1+r_R) \; - \; 1 \; - \; \psi \;=\; [\beta(1+r_R)-1]\frac{(1-\delta)(1-1/\theta)(\sigma/\varepsilon-1)}{1-(1-\delta)(1-1/\theta)}, \tag{A19}$$

$$[\beta(1+r_R)-1]\rho \; - \; \psi r_R^{-1} \;=\; [\beta(1+r_R)-1]\frac{(1-\delta^2)(1-1/\theta)\sigma/\varepsilon \; - \; 1}{1-(1-\delta)(1-1/\theta)} r_R^{-1}. \tag{A20}$$

Recall that $0 < \delta, \sigma < 1$ and $\theta, \varepsilon > 1$. Moreover in the main text we saw that $1+r_R > 1/\beta$ when $q < 1$. Hence both the expressions in (A19) and (A20) are negative. It then follows that the common denominator, {.}, in (29) and (30) is also negative.

Appendix D Determinacy of Equilibrium Under a Taylor Rule

The characteristic equation of the system (31)-(33) can be computed as:

$$a\lambda^2 \; + \; b\lambda \; + \; c \;=\; 0, \tag{A21}$$

where λ denotes an eigenvalue, $a = 1$ and

$$b \;=\; -(1+r_R)\left\{1+\beta-\kappa\left[\psi-\nu+[(\varepsilon/\sigma-1)^{-1}\delta/r_R+\rho]\phi_\pi\right]\right\} \; + \; \psi,$$

$$c \;=\; (1+r_R)\left\{[1-\kappa(\varepsilon/\sigma-1)^{-1}(\delta/r_R)\phi_\pi][\beta(1+r_R)-\psi] \; + \; \kappa[\nu-\beta(1+r_R)\rho+\psi/r_R]\phi_\pi\right\}.$$

Necessary and sufficient conditions for both eigenvalues to lie outside the unit circle are:

$$\frac{a+b+c}{a-b+c} \; > \; 0, \tag{A22}$$

$$\frac{c-a}{a-b+c} \; > \; 0. \tag{A23}$$

First, consider the sign of the common denominator *a-b+c*. By manipulation of the terms for *b* and *c*, we can obtain:

$$a-b+c \;=\; (2+r_R)[1+\beta(1+r_R)-\psi]$$

$$+\;(1+r_R)\kappa\Big(\nu-\psi \;+\; \big\{\nu \;-\; [1+\beta(1+r_R)][(\varepsilon/\sigma-1)^{-1}\delta/r_R+\rho]\big\}\phi_\pi$$

$$+\;[(\varepsilon/\sigma-1)^{-1}\delta/r_R+1/r_R]\phi_\pi\psi\Big)\,. \tag{A24}$$

We now claim that a-b+c > 0 for δ sufficiently close to zero. As $\delta \to 0$, $1+r_R \to \beta^{-1}$, as was shown in the main text (see the discussion of (23)). This implies that $\psi \to 0$ (from the definition of ψ). Hence the term on the first line of (A24) $\to$ $(1+\beta^{-1})2$, which is positive. Concerning the term on the second line of (A24), note that ν-ψ is always positive (from the definitions of ν and ψ). The term {.} at first sight has an ambiguous sign. However, as $\delta \to 0$, ν remains strictly positive, while $\rho \to 0$ (see the definitions of ν and ρ). Therefore {.} is unambiguously positive for δ sufficiently close to zero. Concerning the term on the third line of (A24), we note that it is always positive. This set of observations proves our claim.

Second, consider the sign of *c-a*, the numerator of (A23). We can write *c-a* as:

$$c-a \;=\; (1+r_R)\Big(\beta(1+r_R) \;-\; (1+r_R)^{-1} \;-\; \psi$$

$$+\;\kappa\big\{\nu \;-\; \beta(1+r_R)[(\varepsilon/\sigma-1)^{-1}\delta/r_R+\rho]\big\}\phi_\pi$$

$$\kappa[(\varepsilon/\sigma-1)^{-1}\delta/r_R+1/r_R]\phi_\pi\psi\Big)\,. \tag{A25}$$

We now claim that c-a > 0 for δ sufficiently close to zero. As $\delta \to 0$, $1+r_R \to \beta^{-1}$, as was shown in the main text (see the discussion of (23)). This implies that $\psi \to 0$ (from the definition of ψ). Hence the term on the first line of (A25) $\to \beta^{-1}(1-\beta)$, which is positive. The term on the second line of (A25) at first sight has an ambiguous sign. However, as $\delta \to 0$, ν remains strictly positive, while $\rho \to 0$ (see the definitions of ν and ρ). Therefore {.} is unambiguously positive for δ sufficiently close to zero. Concerning the term on the third line of (A25), we note that it is always positive. This set of observations proves our claim.

From the foregoing it follows that, for δ sufficiently close to zero, condition (A23) is satisfied with no further parameter restrictions. It also follows that, for δ sufficiently close to zero, condition (A22) will be satisfied if and only if a+b+c > 0. Now, eliminating (ψ,ρ,ν) using their definitions, with some manipulation we can express a+b+c as:

$$a+b+c \;=\; [1-(1-\delta)(1-1/\theta)]^{-1}(1+r_R)\kappa\zeta^{-1}[\beta(1+r_R)^2-1]r_R^{-1}$$

$$\times\left(\phi_\pi \;-\; \frac{r_R}{\beta(1+r_R)^2-1} \;-\; \frac{1}{\kappa}\frac{(1-\delta)(1-1/\theta)(1-\sigma/\varepsilon)}{1-(1-\delta)(1-1/\theta)\sigma/\varepsilon}\frac{\beta(1+r_R)-1}{\beta(1+r_R)^2-1}\frac{r_R^2}{1+r_R}\right). \qquad \text{(A26)}$$

a+b+c is clearly positive if and only if the term on the second line is positive. This is the same as the condition (34) in the main text.

Appendix E Impact Effects of an Increase in Government Debt Under a Money Supply Rule

The general expression for the determinant of the matrix in (40) can be computed as:

$$\det \;=\; (1-\rho r_R)^{-1}\Big(-\big\{\nu - r_R^{-1}\lambda_1[\nu-\psi-\rho r_R]+r_R^{-1}[\nu-\psi-\rho r_R][\beta(1+r_R)-1]\big\}(1+r_R)\kappa$$

$$+\; \big\{(1-\rho r_R)(1-\lambda_1)+\lambda_1[\nu-\psi-\rho r_R]+\beta(1+r_R)-1-\psi\big\}(1+r_R)\xi$$

$$+\big\{\nu + r_R^{-1}(1-\rho r_R)(1-\lambda_1)+r_R^{-1}[\nu-\psi-\rho r_R][\beta(1+r_R)-1]+r_R^{-1}[\beta(1+r_R)-1-\psi]\big\}[\lambda_1-(1+r_R)]\Big)$$

(A27)

The general expression for (λ_1-a_{11})a_{23} + $a_{21}a_{13}$ is:

$$(\lambda_1-a_{11})a_{23}+a_{21}a_{13} \;=\; (1-\rho r_R)^{-1}\Big(\big\{\rho[1-\beta(1+r_R)]+\nu+\psi r_R^{-1}\big\}[\lambda_1-(1+r_R)]$$

$$+\; [\nu-\psi-\rho r_R](1+r_R)\xi\lambda_1\Big). \qquad \text{(A28)}$$

In the limit as $\delta \to 0$, 1+$r_R \to \beta^{-1}$, as was shown in the main text (see the discussion of (23)). From the definition of ψ, this implies that $\psi \to 0$. From the definitions of ρ and ξ, $\delta \to 0$ also implies that ρ, $\xi \to 0$. From the definition of ν, $\delta \to 0$ implies that:

$$\nu \;\to\; 1 \;+\; (\theta - 1)(1 - \sigma / \varepsilon)\,.$$

Thus ν remains positive (and greater than one).

Applying these special cases to the expression for *det* gives:

$$\det \;=\; \nu(\lambda_1 - \beta^{-1} - \beta^{-1}\kappa) \;-\; (\beta^{-1} - 1)^{-1}[(1-\lambda_1)(\beta^{-1} - \lambda_1) - \lambda_1 \nu \beta^{-1}\kappa]\,. \qquad \text{(A29)}$$

The first RH term is negative while the term [.] has an ambiguous sign, so the sign of *det* is still indeterminate. To proceed further, we now appeal to the characteristic equation. This is a third-order polynomial equation in λ. When $\delta = 0$ one factor of the polynomial turns out to be λ-β^{1}. Hence one eigenvalue is simply β^{1}. This is clearly one of the two unstable eigenvalues. The stable eigenvalue, λ_1, must therefore satisfy the second-order polynomial equation which remains when we cancel the factor λ-β^{1}, namely:

$$(1-\lambda)(\beta^{-1} - \lambda) \;+\; (1-\lambda)(\beta^{-1} - 1)(\nu - 1) \;-\; \lambda \nu \beta^{-1}\kappa \;=\; 0\,. \qquad \text{(A30)}$$

Now notice that, by rearranging this equation, we can re-express the term [.] in (A29) as (1-λ_1)(β^{1}-1)(1-ν). Substituting this into (A29) and simplifying, we obtain:

$$\det \;=\; \nu(1 - \beta^{-1} - \beta^{-1}\kappa) \;-\; (1 - \lambda_1)\,,$$

which is (42) in the main text.

Applying the above special cases to the expression for (λ_1-a_{11})a_{23} + $a_{21}a_{13}$ gives:

$$(\lambda_1 - a_{11})a_{23} + a_{21}a_{13} \;=\; \nu(\lambda_1 - \beta^{-1})\,,$$

which is (43) in the main text.

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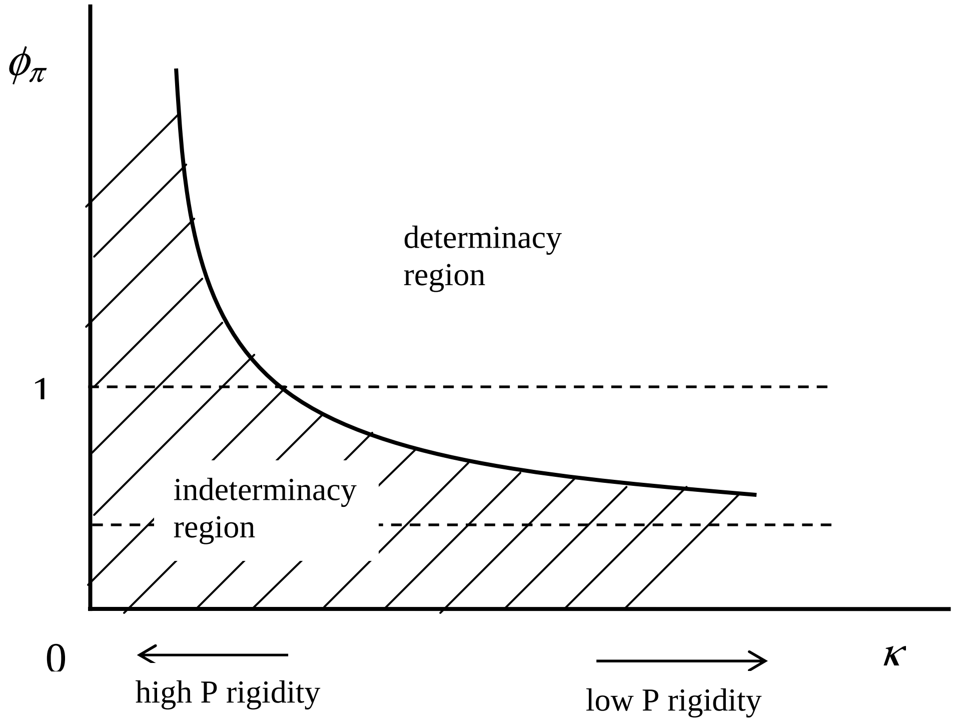

Figure 1

Parameter combinations for determinacy of equilibrium
under a Taylor Rule

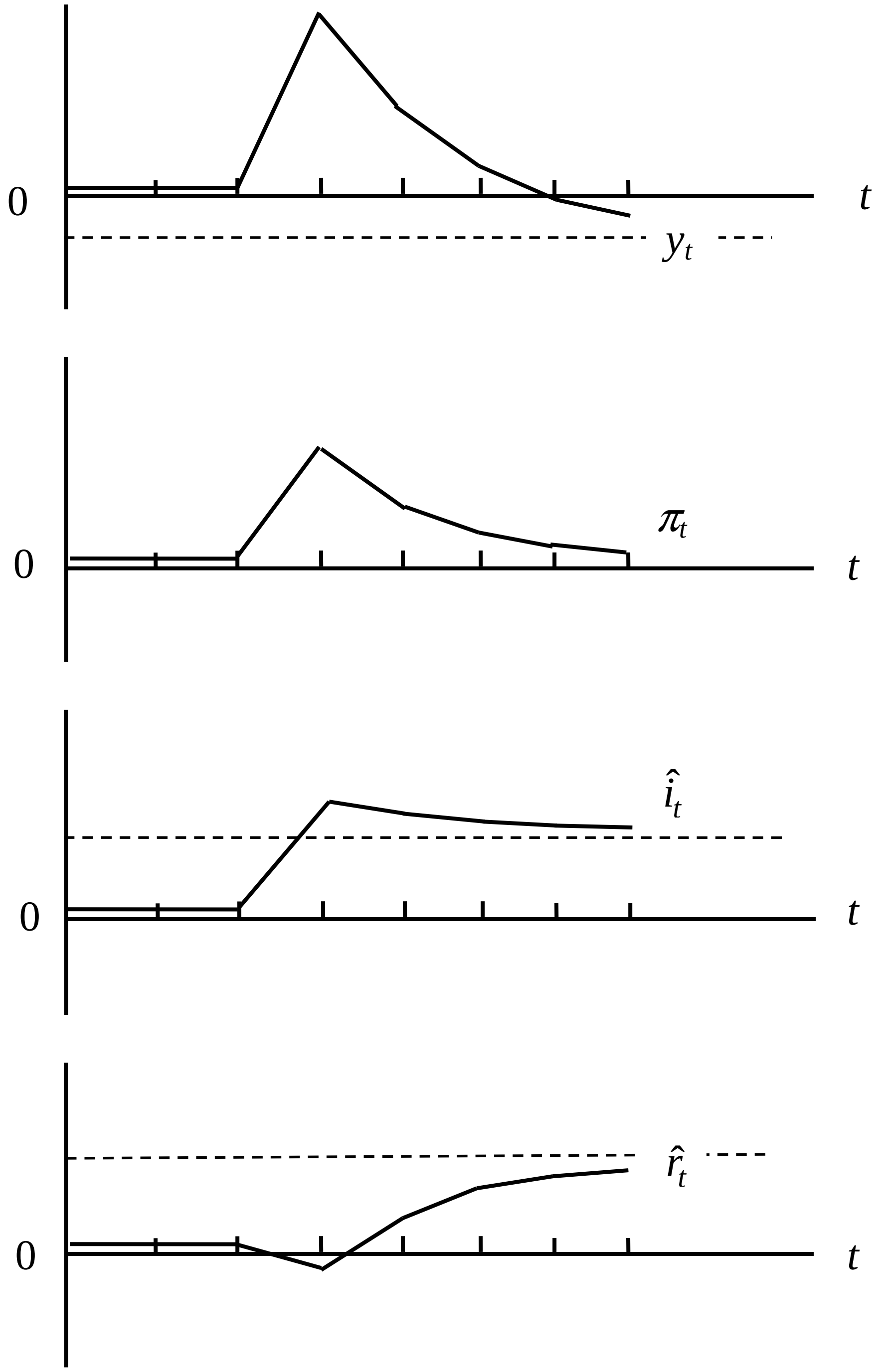

Figure 2

Time paths in response to a one-period debt-financed tax cut
when the money supply is held constant