Reassessing the Equity Premium Puzzle Using Micro Data

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Abstract

I investigate empirically the ability of financial market incompleteness to help explaining the equity premium puzzle. I estimate the non-diversifiable component of the cross-sectional volatility of income and examine its cyclical properties. Equipped with these estimates, I compute the implied equilibrium Sharpe-ratio of excess returns and evaluate the ability of idiosyncratic risk to improve the performance of the Consumption Capital Asset Pricing model (CCAPM). Individual income is used as a proxy for consumption in estimating the aggregate pricing kernel. The results are supportive of the incomplete markets/limited participation model as a solution for the equity premium puzzle.

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1 Introduction

Representative agent models do not take into account the cross-sectional distribution of consumption abstracting from individual heterogeneity. The representative-agent paradigm relies on the assumption of complete markets, which postulates that individuals equate state by state their inter-temporal marginal rates of substitution. However there exists overwhelming evidence against the complete markets assumption (e.g. Cochrane [1991], Hayashi et al. [1996]) and hence, studying the interaction between cross-sectional volatility and aggregate volatility is of fundamental importance to understand the way we should model macroeconomic aggregates such as aggregate consumption, asset prices and business cycle fluctuations.

Mehra and Prescott (1985) uncovered a fundamental empirical flaw in the complete markets/representative agent paradigm. They showed that, given the covariance structure between security market prices and aggregate consumption growth, a reasonably parameterized representative-agent exchange economy cannot produce a mean annual premium of equity return over the risk-free rate which matches the historical premium of 6 percent, observed in U.S. data. Moreover, as stressed in Weil (1989), the standard representative agent model implies an equilibrium annual risk-free rate which is too high, about 4 percent, when compared to the 1 percent historical mean of the riskless rate of return.

Mankiw (1986) examines an economy in which aggregate shocks do not affect the population equally. Instead, ex-post, these shocks are concentrated heterogeneously across ex-ante identical individuals. He makes the point that if information asymmetries prevent individuals from achieving the full information Pareto allocation of risk, the price of risk (the equity premium) will depend on the cross-sectional distribution of aggregate shocks. Thus, inference about the degree of risk aversion from aggregate data alone will not be possible. In Mankiw's setting, whether idiosyncratic uncertainty can help solving the equity premium puzzle is shown to depend on the covariance between the cross-sectional variance of consumption and aggregate consumption. Constantinides and Duffie (1996) further explore this argument by constructing an incomplete markets exchange economy exhibiting a no-trade equilibrium consistent with the historical security and bond prices data, given a judicious specification of the individual income processes.

In this paper, I explore empirically the mechanism proposed by Mankiw. I show that, if individual log consumption admits an approximate factor structure representation and the individual factor loadings are private information (thus preventing a Pareto allocation of risk), the aggregate pricing kernel can be represented as a function of the changes in the cross-sectional variance of log consumption. Moreover, the market excess return is shown to depend on the cyclical properties of the non-diversifiable component of cross-sectional volatility (NDCV). I show that the innovations of the NDCV have to be countercyclical to help explain the equity premium puzzle. Following Balduzzi and Yao (2007), I exploit the aggregate asset pricing equation arising from the pricing kernel implied by the cross-sectional aggregation of the marginal utility functions, instead of the more commonly used pricing kernel resulting from the aggregation of the inter-temporal marginal rates of substitution.

Unfortunately consumption micro data at the household level is scarce and of poor quality and in particular the time series dimension of the existing longitudinal datasets on individual consumption is very short. Therefore, following Lettau (2002), I use data on household individual income instead of household consumption to estimate the aggregate pricing kernel and I investigate whether the implied Sharpe-ratio matches the historical market excess return[1]. Using income instead of consumption to estimate the aggregate pricing kernel is equivalent to assuming a no-trade equilibrium, in which agents simply consume their endowment. If mod-

[1]This approach is dual to the one proposed by Hansen and Jagannathan (1991), who use observable security market prices to trace admissible stochastic discount factors in terms of their means and standard deviations.

els with idiosyncratic risk are not able to generate a sufficiently high price of risk using income data, there is little hope that the same models would perform better using consumption data. Contrary to Lettau, I find that the proposed incomplete market model, estimated using income data, is able to generate a sufficiently high equity premium for reasonable parameterizations. The income data used is obtained from the Panel Study of Income Dynamics (PSID), a longitudinal survey of a representative sample of U.S. individuals.

I will therefore model and estimate the correlation structure of idiosyncratic and aggregate shocks. Thus, I exploit empirically the potential of idiosyncratic risk to help explain the equity premium puzzle, by explicitly modeling the cross-sectional variance of individual income and identifying and estimating the two components of cross-sectional volatility. One component which is non-diversifiable and is produced by the heterogeneous concentration of aggregate shocks, and a component which is produced by the idiosyncratic component of income, and which is uncorrelated with aggregate shocks and therefore, will be shown not to affect the equity premium.

Importantly, I explicitly take into account limited asset market participation, by presenting empirical results both for a general sample of households and for a tightened subsample of households who reported ownership of stocks in 1984 (one of the few years where this information was available in the data set used). The results are striking. While the aggregate pricing kernel obtained using the full sample is almost uncorrelated to the market excess returns, and is therefore unable to explain the equity premium, the pricing kernel obtained when I restrict the sample to include only stockholders, has the desirable feature of being negatively correlated to the market excess returns. Moreover, the estimated Sharpe-ratio under incomplete markets, when the sample only includes households owning stocks, equals the historically observed market excess-returns Sharpe-ratio, for low levels of risk aversion (coefficient of relative risk aversion between 2 and 4).

These results are in line with Mankiw and Zeldes (1991) and Vissing-Jorgensen (2002) that find differences between the consumption of stockholders and non-stockholders, also suggesting that limited participation matters. However in both these papers, the focus is on the co-movement between growth rates of consumption of sub-groups of the population and the market excess returns, while my focus is directed toward assessing the importance of market incompleteness for asset pricing and hence, on the cyclical properties of the NDCV of stockholders income processes. The current paper's findings are also consistent, and closer in spirit, to the work by Brav, Constantinides and Geczy (2002), who using consumption data from the consumer expenditure survey (CEX) also find support for the incomplete market/limited participation hypothesis.

The econometric methodology applied to identify and estimate the common non-diversifiable component and the idiosyncratic component of the cross-sectional variances of individual income relies on Connor, Korajczyk and Linton (2006) asymptotic principal components (APC) method to estimate an approximate factor model for endowment shocks, allowing for heteroskedasticity in both the common and idiosyncratic components. I apply this method to an unbalanced panel of observations on household income, using the approach proposed by Connor and Korajczyk (1987). An important aspect of the empirical work is that consistency of the estimator relies on large N (cross-section size) asymptotic. This is reassuring because the times series dimension of the available data set is much shorter than the cross-sectional dimension. Notice for example that, in an influential paper, Heaton and Lucas (1996) suggest that the small time series dimension might be to blame for the relatively small estimated impact of aggregate variables on cross-sectional volatility. Confidence intervals for the estimated Sharpe-ratios are produced through parametric bootstrapping methods.

The reminder of the paper is organized as follows. Section 2 describes the aggregation method used to compute the aggregate pricing kernel, as well as the implications for asset pricing of the assumption made about the stochastic process of individual consumption. The following section, section 3, describes the econometric methodology used to identify and estimate the model. Section 4 describes the data used in the empirical application. Section 5 describes the paper findings and finally section 6 concludes.

2 The Aggregate Pricing Kernel

2.1 Aggregation Methodology

Following Balduzzi and Yao (2007), I exploit the aggregate asset pricing equation arising from the pricing kernel[2] implied by the cross-sectional aggregation of the marginal utility functions instead of the pricing kernel resulting from the aggregation of the inter-temporal marginal rates of substitution (IMRS). Hence, assuming that individuals have standard time and state separable von Neumann-Morgenstern utility functions, the Euler equation for individual i, which determines an optimal consumption-portfolio decision is

$$u_i'(c_{it}) = \beta E_t \left[u_i'(c_{it+1}) R_{t+1} \mid \Im_{it}\right] \tag{2.1}$$

where u_i is the flow utility function, c_{it} is period t consumption and R_{t+1} is the gross rate of return on a given asset from period t to $t+1$, given by

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \tag{2.2}$$

where D_{t+1} are the period $t+1$ dividends and P_t denotes the period t ex-dividend price. Finally, β is the subjective discount factor and $\Im_{it}$ is the information set available to individual i in period t which is allowed to differ across households although, for simplicity, I will drop the individual subscript i on the conditional expectations and simply write $\Im_t$.

[2]The pricing kernel is often called the Stochastic Discount Factor

The economy consists of N individuals which participate in the capital markets, with N large enough for the law of large numbers to apply. Taking cross-sectional averages of both sides of (2.1) one obtains

$$1 = \beta E\left[\frac{\sum_{i=1}^{N} u_i'\left(c_{it+1}\right)}{\sum_{i=1}^{N} u_i'\left(c_{it}\right)} R_{t+1} \mid \Im_t\right] \tag{2.3}$$

It follows that the ratio of the equally weighted sum of the individuals marginal utilities multiplied by the subjective discount factor is a valid pricing kernel

$$M_{t+1} = \beta \frac{\sum_{i=1}^{N} u_i'\left(c_{it+1}\right)}{\sum_{i=1}^{N} u_i'\left(c_{it}\right)} \tag{2.4}$$

And the Lucas (1978) asset pricing equation corresponds to

$$1 = E\left[M_{t+1} R_{t+1} \mid \Im_t\right] \tag{2.5}$$

Finally, by defining $\Pi_{t+1} = \left(R_{t+1}^m - R_{t+1}^f\right)$ as the excess market return, that is, the difference between the rate of return of the risky market portfolio and the risk free rate, and using the law of iterated expectations, the unconditional Euler equation on the excess market return corresponds to

$$0 = E\left[M_{t+1} \Pi_{t+1}\right] \tag{2.6}$$

Given a specification of individuals' preferences and some assumptions on the cross-sectional distribution of consumption, equation (2.6) provides testable restrictions on the joint behavior of aggregate consumption and asset prices, which depend on the second order moments of the cross-sectional distribution of consumption.

2.2 The Cross-Sectional Distribution of Consumption

I assume that the stochastic process for individual log consumption satisfies the approximate factor structure representation given by

$$\log\left(c_{it}\right) = \log(c_t) + \left(\lambda_i f_t - \frac{\Psi_t}{2}\right) + \left(\epsilon_{it} - \frac{\sigma^2_{\epsilon,t}}{2}\right) \tag{2.7}$$

where ϵ_{it} is orthogonal to c_t, f_t and λ_i, and follows a normal distribution with mean zero and variance $\sigma^2_{\epsilon,t}$. Next, f_t and λ_i are vectors, where f_t is a $k\times 1$ random column vector corresponding to the common factors and λ_i is a $1 \times k$ vector of individual i factor loadings. I will assume that $\lambda_i f_t$ is cross-sectionally normal distributed with mean zero and variance Ψ_t. It follows that c_t corresponds to period t aggregate consumption. Taking the exponential of each side of equation (2.7) reveals that individual consumption must satisfy the equation

$$c_{it} = c_t \exp\left(\lambda_i f_t - \frac{\Psi_t}{2}\right) \exp\left(\epsilon_{it} - \frac{\sigma^2_{\epsilon,t}}{2}\right) \tag{2.8}$$

And because I made the assumption that the economy is populated by a number of individuals large enough for the law of large numbers to apply, the sum of individual consumption over all individuals satisfies

$$c_t = (1/N) \sum_{i=1}^{N} c_{it} \tag{2.9}$$

Thus, individual consumption is assumed to be cross-sectionally log-normal distributed with mean c_t and variance $\Psi_t + \sigma^2_{\epsilon,t}$. The cross-sectional variance of consumption can therefore be decomposed into a non-diversifiable component (NDCV) Ψ_t and a component explained by the cross-sectional variance of idiosyncratic shocks $\sigma^2_{\epsilon,t}$. I will show later that whether cross-sectional heterogeneity can help explaining the equity premium puzzle will depend on the properties of the non-diversifiable component of cross-sectional volatility.

2.3 Individual Preferences

Individual preferences are characterized by a flow utility function which is of the constant relative risk aversion (CRRA) class, with coefficient of relative risk aversion given by $\gamma > 0$

$$u(c) = \begin{cases} (1-\gamma)^{-1} c^{1-\gamma} & if \;\; \gamma \neq 1 \\ \log(c) & if \;\; \gamma = 1 \end{cases}$$

Therefore, applying the law of large numbers to solve equation (2.4) the aggregate pricing kernel is given by

$$M_{t+1}^{IM} = \exp\left(\frac{\gamma^2+\gamma}{2}\left(\Delta\Psi_{t+1} + \Delta\sigma_{\epsilon,t+1}^2\right) - \gamma\Delta\log c_{t+1} - \delta\right) \tag{2.10}$$

where $\delta \equiv -\log(\beta)$ and the superscript IM stands for incomplete markets.

The complete markets assumption implies that individuals are able to equalize state by state their marginal rate of substitution. Thus, the equilibrium of a heterogeneous-agent, full information economy is observationally equivalent in its pricing implications to the equilibrium of a representative agent, full-information economy (Constantinides[1982]). This implies that a valid complete markets pricing kernel can be found by setting $\Psi_t = \sigma_{\epsilon,t}^2 = 0 \; \forall \; t$, which leads to the standard representative agent pricing kernel, which depends on the growth rate of aggregate consumption

$$M_{t+1}^{CM} = \exp\left(-\gamma\Delta\log c_{t+1} - \delta\right) \tag{2.11}$$

The asset pricing implications of the complete markets economy have been strongly rejected by the data. One embodiment of this rejection, is obtained by constructing a frontier which relates the variance of the pricing kernel to its mean, as implied by the realized security market prices (this frontier is known as the Hansen-Jagannahathan volatility bounds) and observing that the aggregate pricing kernel corresponding to the representative agent model is not volatile enough to be within the implied volatility bounds. Whether market incompleteness might improve or not the performance of the consumption asset pricing model will be shown to depend on the cyclical behavior of Ψ_t, the common component of cross-sectional volatility. In particular, I will show next that $\Delta\Psi_{t+1}$ must be negatively correlated with excess returns to help explain the equity premium puzzle.

2.4 Hansen-Jagannahathan Bounds and the Sharpe-ratio

Following Lettau and Uhlig (2002), I will construct the empirical HJ volatility bounds by estimating the Sharpe ratio implied by the joint process of stock market excess returns and the pricing kernel as defined in equation (2.10). The fundamental insight to understand this exercise, is obtained by observing that, applying the Cauchy-Schwarz inequality, the asset pricing equation (2.5) implies the following upper bound for the excess return Sharpe-ratio

$$\frac{E\left[\Pi_{t+1} \mid \Im_t\right]}{\sqrt{Var_t\left(\Pi_{t+1}\right)}} = -Corr_t\left(M_{t+1}, \Pi_{t+1}\right)\frac{\sqrt{Var_t\left(M_{t+1}\right)}}{E\left[M_{t+1} \mid \Im_t\right]} \leq \frac{\sqrt{Var_t\left(M_{t+1}\right)}}{E\left[M_{t+1} \mid \Im_t\right]} \quad (2.12)$$

Thus, by recognizing that the ratio $\frac{\sqrt{Var_t(M_{t+1})}}{E[M_{t+1}|\Im_t]}$ represents the market price of risk, Hansen and Jagannahathan suggest using financial market data jointly with equation (2.12) to estimate a lower bound for the market price of risk. The equity premium puzzle translates into an estimate of the lower bound for the price of risk so large that it implies, under conventional representative agent models, an amount of risk aversion unreasonably high. Given that large excess returns on equity imply large Sharpe ratios, an alternative way to view the equity premium puzzle is that the IMRS of the representative agent exhibits insufficient variability.

Because agents are likely to have information superior to the information set available to the econometrician, it is preferable to express (2.12) in terms of unconditional expectations

$$\frac{E\left[\Pi_{t+1}\right]}{\sqrt{Var\left(\Pi_{t+1}\right)}} = -Corr\left(M_{t+1},\Pi_{t+1}\right)\frac{\sqrt{Var\left(M_{t+1}\right)}}{E\left[M_{t+1}\right]} \leq \frac{\sqrt{Var\left(M_{t+1}\right)}}{E\left[M_{t+1}\right]} \quad (2.13)$$

Equation (2.13) reveals, why market incompleteness can help explaining the high equity premium. As long as the pricing kernel is negatively correlated with returns, the size of risk aversion needed to generate a Sharpe-ratio which matches the historical data, will be smaller because the pricing kernel under incomplete markets is likely to be substantially more volatile than the IMRS of the representative agent.

I will next consider what is the impact of cross-sectional heterogeneity on the equilibrium excess return Sharpe-ratio. Notice that the incomplete market pricing kernel can be expressed in terms of the complete markets pricing kernel and the change in cross-sectional dispersion as follows

$$M_{t+1}^{IM} = \exp\left(\frac{\gamma^2+\gamma}{2}\left(\Delta\Psi_{t+1} + \Delta\sigma_{\epsilon,t+1}^2\right)\right)M_{t+1}^{CM} \quad (2.14)$$

By construction, $\Delta\sigma_{\epsilon,t+1}^2$ is orthogonal to the growth rate of aggregate consumption and the market excess return. It follows that the equity premium under incomplete markets will only differ from its complete market counterpart if the non-diversifiable component of cross-sectional volatility is correlated with the relevant aggregate variables, that is, the growth rate of aggregate consumption and excess returns.

Proposition 1 *If changes in the NDCV are orthogonal to both aggregate consumption growth and excess returns, the equilibrium equity premium under incomplete markets will be the same as the equilibrium equity premium under complete markets.*

Proof: Under incomplete markets, equation (2.12) can be re-written as

$$\begin{aligned} E\left[\Pi_{t+1} \mid \Im_t\right] &= -\frac{Cov_t\left(M_{t+1}^{IM},\Pi_{t+1}\right)}{E\left[M_{t+1}^{IM} \mid \Im_t\right]} \\ &= -\frac{E_t\left[\vartheta_{t+1}M_{t+1}^{CM}\Pi_{t+1}\right] - E_t\left[\vartheta_{t+1}M_{t+1}^{CM}\right]E_t\left[\Pi_{t+1}\right]}{E_t\left[\vartheta_{t+1}M_{t+1}^{CM}\right]} \quad (2.15) \end{aligned}$$

where $E_t[\,.\,] \equiv E[\,.\,\mid \Im_t]$ and $\vartheta_{t+1} \equiv \frac{\gamma^2+\gamma}{2}\left(\Delta\Psi_{t+1} + \Delta\sigma^2_{\epsilon,t+1}\right)$.

Assuming $\Delta\Psi_{t+1}$ is orthogonal to both $\Delta\log(c_{t+1})$ and Π_{t+1}, the aggregate variables of interest, equation (2.15) can be re-written as

$$
\begin{aligned}
E[\Pi_{t+1} \mid \Im_t] &= -\frac{E_t[\vartheta_{t+1}]\,E_t\left[M^{CM}_{t+1}\Pi_{t+1}\right] - E_t[\vartheta_{t+1}]\,E_t\left[M^{CM}_{t+1}\right]E_t[\Pi_{t+1}]}{E_t[\vartheta_{t+1}]\,E_t\left[M^{CM}_{t+1}\right]} \\
&= -\frac{Cov_t\left(M^{CM}_{t+1},\Pi_{t+1}\right)}{E\left[M^{CM}_{t+1} \mid \Im_t\right]} \qquad (2.16)
\end{aligned}
$$

end of proof

The idiosyncratic component of income has a distribution which is independent from aggregate consumption growth and returns. Therefore, as a corollary of proposition (1) the equity premium under incomplete markets is orthogonal to $\Delta\sigma^2_{\epsilon,t+1}$, the change in the idiosyncratic component of cross-sectional volatility.

Corollary 1 *The unconditional expectation of the equity premium under incomplete markets corresponds to*

$$
E[\Pi_{t+1}] = -\frac{Cov\left(\exp\left(\frac{\gamma^2+\gamma}{2}\,\Delta\Psi_{t+1}\right)M^{CM}_{t+1},\Pi_{t+1}\right)}{E\left[\exp\left(\frac{\gamma^2+\gamma}{2}\,\Delta\Psi_{t+1}\right)M^{CM}_{t+1}\right]} \qquad (2.17)
$$

and is independent of $\Delta\sigma^2_{\epsilon,t+1}$, the change in the idiosyncratic component of cross-sectional volatility. The proof follows from the proof of proposition (1) and applying the law of iterated expectations.

In the empirical section of the paper, which follows, I will use data on individual income from the PSID which I will use to proxy consumption, in an approach similar to the one proposed by Lettau (2002) and I will ask whether the pricing kernel implied by the data satisfies the Hansen-Jagannahathan volatility bounds. Under the assumption that individual consumption is less volatile than individual income, an affirmative answer to the question above is a necessary condition for the presence of uninsurable idiosyncratic risk to offer an explanation for the equity premium puzzle.

Concretely, the exercise performed in the empirical section is to examine whether a model where the pricing kernel corresponds to the incomplete market pricing kernel defined in equation (2.10) produces an estimate for the excess return Sharpe-ratio, which because of corollary (1) is given by

$$\hat{SR}(\gamma) = -Corr\left(e^{\eta\,\Delta\hat{\Psi}} M^{CM}, \Pi\right) \frac{\sqrt{Var\left(e^{\eta\,\Delta\hat{\Psi}} M^{CM}\right)}}{E\left[e^{\eta\,\Delta\hat{\Psi}} M^{CM}(\gamma)\right]} \tag{2.18}$$

where $\eta \equiv \frac{\gamma^2+\gamma}{2}$, that is sufficiently high to match the realized equity premium, given a reasonably calibrated model (i.e.: given a low coefficient of relative risk aversion). The econometric methodology applied to estimate Ψ, the NDCV component, relies on Connor, Korajczyk and Linton (CLK) asymptotic principal components (APC) method to estimate an approximate factor model for income shocks, allowing for heteroskedasticity in both the common and idiosyncratic components.

3 The Econometric Model

3.1 The Income Process Conditional Mean

Let's postulate following for example, Storesletten, Telmer and Yaron (2004), that the household log real income process $\{m_t^i\}_{t=0}^{\infty}$ can be decomposed into a deterministic component plus a stochastic component:

$$m_t^i = G(d_t, z_{it}) + y_t^i \tag{3.1}$$

The function $G(d_t, z_{it})$ is a linear function of aggregate time dummies and individual observable characteristics, such as education, household size, and a polynomial in the age of the head, which is necessary to capture the life-cycle component of earnings. Consistent with equation (2.7) and the assumption that agents consume their endowment, I assume that the stochastic component of income has an approximate factor structure representation and I allow for time-varying conditional second order moments

$$y_t^i = \lambda_i f_t + \epsilon_t^i \tag{3.2}$$

As before, f_t and λ_i are vectors, where f_t is a $k \times 1$ random column vector corresponding to the common factors and λ_i is a $1 \times k$ vector of individual i factor loadings. And defining $y_t = (y_t^1 \ldots y_t^N)'$, $\Lambda = (\lambda_1' \ldots \lambda_N')'$, and $\xi_t = (\epsilon_t^1 \ldots \epsilon_t^N)'$, I represent the model as:

$$y_t = \Lambda f_t + \xi_t \tag{3.3}$$

Following CLK, I allow for factor related as well as individual specific heteroskedasticity[3]. In order for the model to be estimated consistently, the following assumptions need to be made:

$$E\left[\xi_t | f_t\right] = 0 \; \forall \; t, \tau. \tag{3.4}$$

$$\lim_{N \to \infty} \frac{1}{N} \Lambda' \Lambda = M \text{ a nonsingular matrix.} \tag{3.5}$$

$$\lim_{N \to \infty} \left\| E\left[\xi_t \xi_t'\right] \right\| = c < \infty. \tag{3.6}$$

Where $\|X\|$ is the operator norm, and is equal to the maximum eigenvalue if X is a symmetric positive definite matrix. The factors f and the loadings Λ cannot be separately identified, however they are identifiable up to a rotation matrix, and this is enough to estimate the NDCV component of volatility, which is what I require to estimate the aggregate pricing kernel.

[3]CLK allow for heteroskedasticity arising from three components: a factor related component, a component related to the idiosyncratic part of income but that is common to each individual, and an individual specific component.

3.2 The Income Process Second Order Moments

To estimate the factors we apply CKL asymptotic principal components method, which relies on large N asymptotic applied to $N^{-1}\xi_t'\xi_\tau$, under the assumption that the idiosyncratic shocks are not autocorrelated[4]

$$p\lim_{N\to\infty} N^{-1}\xi_t'\xi_\tau = \begin{cases} 0 & \text{for } t \neq \tau \\ \sigma^2_{\epsilon,t} & \text{for } t = \tau \end{cases} \tag{3.7}$$

Now, defining $Y = (y_1 \ldots y_T)$, $F = (f_1 \ldots f_T)$, $\hat{\Omega} = \frac{Y'Y}{N}$ a $T \times T$ matrix of income shocks cross products[5], and Φ a $T \times T$ diagonal matrix with diagonal elements $(\sigma^2_{\epsilon,1} \ldots \sigma^2_{\epsilon,T})$, assuming that the law of large numbers applies, letting N go to infinity, and using equations (3.3) to (3.7) we obtain:

$$p\lim_{N\to\infty} \hat{\Omega} = \Omega = F'MF + \Phi \tag{3.8}$$

And given (3.8) we obtain the following (where l is a given matrix rotation):

$$p\lim_{N\to\infty} \; eigvec_s\left(\Phi^{-1/2}\hat{\Omega}\Phi^{-1/2}\right)\Phi^{1/2} = \; eigvec_s\left(\Phi^{-1/2}(F'MF)\Phi^{-1/2}\right)\Phi^{1/2} = lF$$

The model is estimated recursively using the following iterative procedure:

1. set initial conditions $\hat{\Phi}_0 = I_T$
2. compute for each iteration r, $l\hat{F}_r = \; eigvec_s\left(\hat{\Phi}_r^{-1/2}\hat{\Omega}\hat{\Phi}_r^{-1/2}\right)\hat{\Phi}_r^{1/2}$
3. estimate $\hat{\sigma}^2_{\epsilon,tr} = \frac{1}{N_t}\sum_{i=1}^{N_t} \hat{\xi}^i_t\hat{\xi}^i_t$

After experimenting with different models specifications I have chosen to estimate the model allowing for 5 factors. Further details on the estimation algorithm, as well as on the econometric model specification are provided in the appendix.

[4] This assumption needs of course to be tested against the alternative of persistent idiosyncratic income shocks. In the absence of the assumption, consistent estimation of the factors rotation lF would relie on N and T asymptotic results. Notice that l is taken to be a given matrix rotation

[5] Since we are working with an unbalanced panel we define, following Connor and Korajcyk (1987), $\hat{\Omega}_{t\tau} = \frac{1}{N_{t\tau}}\sum_{i=1}^{N_{t\tau}} y^i_t y^i_\tau$, where $N_{t\tau}$ is the number of households that are present in the panel in both t and τ

Table 1: Summary statistics

Variable	Mean	Std. Dev.	Min.	Max.
Full sample				
Age of household head	43.34	15.99	17	99
Size of household	2.97	1.58	1	14
Household total income (1984 thousand $)	31.51	28.97	0.001	1376.84
Stock market wealth (1984 thousand $)	505.32	2176.79	0	9999.99
Stockholders sub-sample				
Age of household head	45.37	15.20	17	99
Size of household	2.96	1.44	1	10
Household total income (1984 thousand $)	46.94	41.43	0.001	1376.84
Stock market wealth (1984 thousand $)	1970.62	3948.60	0.002	9999.99

Note: Summary statistics for the PSID full sample and the stockholders subsample.

Finally, confidence intervals for the implied equilibrium Sharpe-ratios estimates where produced through parametric bootstrapping methods, by generating draws from the common and idiosyncratic components, applying our estimator to each sample replication and constructing the confidence interval based on the bootstrap distribution.

4 The Data

4.1 The Household Level Data

To estimate the aggregate pricing kernel, I use household level data from the Panel Study of Income Dynamics (PSID). The PSID is a longitudinal study of nearly 8000 US households, following the same families and individuals since 1968. The original PSID sample consisted of two subsamples, a representative cross-section of 3000 U.S. families and a subsample of 2000 low-income families sampled from the Survey of Economic Opportunity (SEO). I have dropped the SEO subsample, to work with a representative sample of the U.S. population. Thereafter, both the original households and their split offs have been interviewed each year. The survey includes a variety of socioeconomic variables, including age, education, family structure and earnings. The data set was constructed by selecting the households which responded at least 10 years to the survey over the period 1968 up to 1993. An important aspect of the PSID data is that the earnings questions are retrospective. The interviews are conducted in March, and the questions refer to earnings in the previous year. I date the observations according to the year corresponding to the earnings, instead of the year of the interview. I am therefore able to construct a panel with observations of household income for the period 1967 to 1992. This in turn, allows me to estimate the pricing kernel under incomplete markets, corresponding to equation (2.10), for the period 1967-1991. I apply the econometric method to an unbalanced panel of observations on a household level measure of income which includes total taxable income of all household members plus transfers. Working with an unbalanced panel allows to keep the cross-sectional age distribution over each year fairly constant, and therefore the results will not be affected by life cycle aspects due to sampling cohort effects. Further filters which have been applied to select the final sample of households are described in detail in the appendix of the paper.

The 1984 PSID wave has information on households portfolio composition. In particular, it is possible to identify which households owned stock in positive amounts in 1984. This information is used to further tighten the sample, to include only stockholders. This approach is desirable because in equilibrium, only the stockholders are expected to satisfy with equality an asset pricing Euler equation such as equation 2.1. Table (1) shows a few sample descriptive statistics, for both the full sample and the tight sample formed by the household that where stock owners

Table 2: Correlation coeficients

	$\Delta\log(c)$ NIPA	$\Delta\log(\bar{m}^{all})$	$\Delta\log(\bar{m}^{stock})$	Π
$\Delta\log(c)$ NIPA	1	0.764	0.572	0.090
$\Delta\log(\bar{m})$ (full sample)		1	0.772	0.228
$\Delta\log(\bar{m})$ (stockholders)			1	0.121
Π				1

Note: Correlation between the growth rate of the NIPA aggregate consumption, the growth rate of the PSID average income for the full sample and for the stockholders sub-sample, and the market excess return on equity, for the sample period 1967-1991.

in 1984. Most notably, the stockholders are on average older than non-stockholders, earn higher income and the income dispersion is greater among stockholders than it is among households that do not own stocks.

4.2 The Aggregate Level Data

The market risky portfolio is taken to be the Annual average Standard and Poor's Composite Stock Price Index divided by the consumption deflater and was obtained along with the annualized risk-less rate of interest from Robert Schiller's website. The aggregate consumption data corresponds to the total personal consumption expenditure NIPA series, provided by the Bureau of Economic Analysis (BEA), deflated using the US CPI price index. I have chosen to use the NIPA data on consumption independently of the sample used to estimate the incomplete market Sharpe-ratio. An alternative would have been to average over the income of the PSID households. Thus in table (2) I present correlation measures between aggregate consumption growth rates, the growth rate of PSID households average income ($\bar{m}^{all}$), the growth rate of the PSID stockholders subsample average income ($\bar{m}^{stock}$), and the market excess return on equity. Figure (1) plots the market excess return on equity, as well as the growth rate of aggregate consumption.

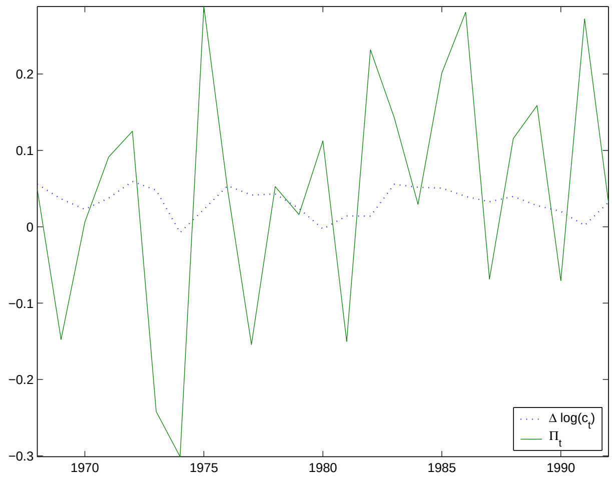

Figure 1: Market excess returns vs consumption growth

5 Empirical Results

Remember that the key quantity of interest is the excess return Sharpe-ratio implied by the estimated pricing kernel under incomplete markets, (henceforth, the equilibrium Sharpe-ratio), which is given by

$$\hat{SR}(\gamma) = -Corr\left(e^{\eta\,\Delta\hat{\Psi}}M^{CM}, \Pi\right)\frac{\sqrt{Var\left(e^{\eta\,\Delta\hat{\Psi}}M^{CM}\right)}}{E\left[e^{\eta\,\Delta\hat{\Psi}}M^{CM}(\gamma)\right]} \quad , \quad \eta \equiv \frac{\gamma^2+\gamma}{2}$$

Thus, the equilibrium Sharpe-ratio depends on the co-movement between the pricing kernel and the equity premium. In turn, the incomplete market pricing kernel (IMPK) was shown to depend on the changes in the cross-sectional variance of consumption. Figures (2) and (3) show a decomposition of the cross-sectional variance of household endowments Ω into the NDCV component which is explained by the common factors, Ψ, and the idiosyncratic component σ_ϵ^2, for both samples, the full PSID sample and the stockholders sub-sample. The findings of the paper are driven by the cyclical properties of Ψ_t, the NDCV. Therefore I will first present results concerning the estimates of Ψ_t, in particular, I am interested in how $\Delta\hat{\Psi}_t$ co-moves with the growth rate of aggregate consumption and the equity excess return, the relevant aggregate variables. Moreover, in constructing confidence intervals for the estimates of $\hat{SR}(\gamma)$, I have considered sampling error in the estimation of the pricing kernel under incomplete markets. This sampling error, translates into uncertainty concerning the precision of the estimates of Ψ_t. As explained before, the confidence intervals have been constructed through parametric bootstrapping, imposing the factor structure (3.2) as I re-sampled observations on individual consumption.

5.1 The NDCV Cyclical Properties

Figures (2) and (3) which shows the decomposition of the cross-sectional variance of household endowments into the NDCV component and the idiosyncratic component for the full PSID sample and the stockholders subsample, respectively, already allows to anticipate one of the paper's findings. Inspection of the two figures, reveals that for the stockholders subsample, the NDCV component accounts for a much greater share of the cross-sectional variance. Moreover, the stockholders NDCV exhibits more pronounced cyclical movements. These observations suggest that the IMPK estimated using the stockholders subsample, is likely to be more strongly correlated with the market excess returns.

In section 2, the IMPK, was shown to depend on the correlation between changes in Ψ, the non-diversifiable component of cross-sectional variance of log consumption and the relevant aggregate variables, $\Delta \log(c)$ and Π. Table (3) shows the correlation between the two available estimates of $\Delta\Psi$ (one obtained using the full sample, and another one obtained using only the stockholders subsample) and these aggregate variables. The first central finding is that the NDCV seems to be countercyclical and thus the mechanism conjectured by Mankiw (1986) as a possible solution to the equity premium puzzle, is likely to be operative.

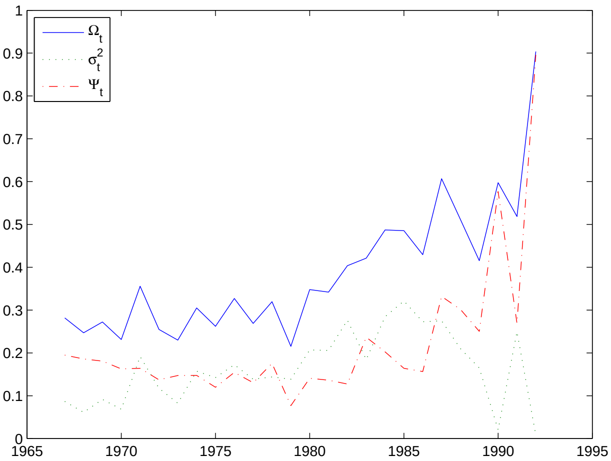

Figure 2: Cross-sectional variance decomposition (full sample)

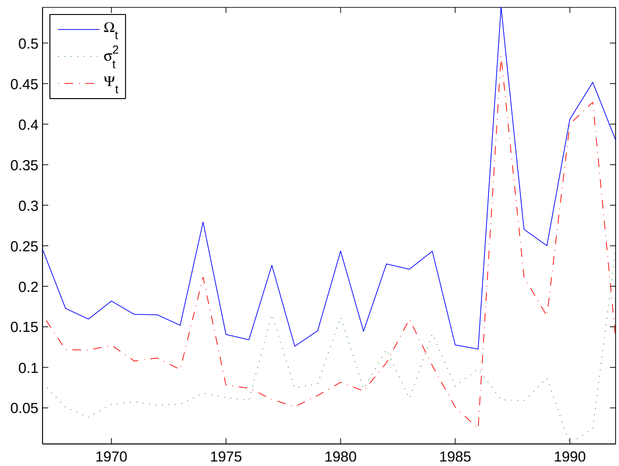

Figure 3: Cross-sectional variance decomposition (stockholders subsample)

5.2 The IMPK vs the Representative Agent Pricing Kernel

As was said before, an alternative manifestation of the equity premium puzzle is the observation that the IMRS, which roughly corresponds to the representative agent pricing kernel exhibits insufficient variability to justify the historically observed Sharpe-ratio on the equity excess return. As one would expect, the pricing kernel under incomplete markets seems to be substantially more volatile. This can be seen in table (4) which shows the standard deviation, for the sample period running between 1967 and 1991, of the pricing kernel, under complete markets and incomplete markets (for both the full sample and the stockholders sub-sample), implied by different coefficients of relative risk aversion, chosen to be within a range deemed reasonable. Table (4) also shows the ratio between the standard deviation and the mean of the pricing kernel. As can be seen by inspecting equation (2.13) this ratio corresponds to the unconditional Sharpe-ratio maximum bound. Because the incomplete markets pricing kernel is much more volatile than the one of the representative agent, the Sharpe-ratio maximum bound under incomplete markets is larger than its complete markets counterpart by several orders of magnitude.

Moreover, table (4) also shows the point estimate correlation between the pricing kernels and the equity premium. The pricing kernel under incomplete markets is substantially more volatile. Therefore, as long as the correlation between the market excess returns and the pricing kernel under incomplete markets has the right sign, allowing for household heterogeneity can go a long way toward explaining the equity premium puzzle. As shown before, the pricing kernel under incomplete markets has to be negatively correlated with the market excess return on equity to boost the equilibrium Sharpe-ratio to the historically observed levels. As can be seen in table (4) the correlation between the pricing kernel under incomplete markets and the market excess return on equity has the right sign. Remarkably, when the full sample is used to estimate the pricing kernel, the correlation between the pricing kernel of the representative agent and the market excess return is al-

Table 3: Cyclical Dynamics NDCV

	$\Delta \log(c)$ NIPA	$\Delta\Psi$ (all)	$\Delta\Psi$ (stock)	Π
$\Delta \log(c)$ NIPA	1	-0.039	-0.214	0.090
$\Delta\Psi$ (full sample)		1	0.403	-0.207
$\Delta\Psi$ (stockholders)			1	-0.330
Π				1

Note: Correlation between the growth rate of the NIPA aggregate consumption, the changes in the non-diversifiable component of cross-sectional variance for the full sample and for the stockholders sub-sample, and the market excess return on equity.

most the same as the correlation between the incomplete markets pricing kernel and the market excess return. Instead, when the sample is tightened, to include only stockholders, the correlation obtained is much higher in absolute value. This suggests that taking notice of limited market participation is important to understand security prices. Moreover, the IMPK is substantially more volatile than the representative agent pricing kernel, which suggests that market incompleteness may explain a substantial part of the equity premium. This is well illustrated in figures (4) and (5), which plot $\Delta\Psi$, the change in the NDCV, for the entire PSID sample and for the stockholders subsample, against the market excess return on equity. The comparison of both this figures with figure 1 is striking. $\Delta\Psi$ is substantially more volatile than the representative agent pricing kernel, which roughly corresponds to the volatility of the aggregate growth rate of consumption. Moreover, it is also easy to verify that $\Delta\Psi$ is negatively correlated with the excess returns on equity. However, one also observes that the IMPK estimated using the full PSID sample is less volatile and, more importantly, less correlated with the market equity excess return. Hence, for a given model parameterization, the Sharpe-ratio obtained when the sample is restricted to include only stockholders will be higher.

Table 4: Sample moments of the pricing kernel estimates

$\gamma = 1$	*Standard Deviation*	$\frac{Std}{Average}$	$corr(M, \Pi)$
$\hat{M}^{CM}$	0.020	0.019	−0.091
$\hat{M}^{IM}$ (full sample)	0.141	0.127	−0.205
$\hat{M}^{IM}$ (stockholders)	0.157	0.144	−0.297
$\gamma = 2$			
$\hat{M}^{CM}$	0.040	0.038	−0.091
$\hat{M}^{IM}$ (full sample)	0.541	0.451	−0.167
$\hat{M}^{IM}$ (stockholders)	0.607	0.527	−0.273
$\gamma = 3$			
$\hat{M}^{CM}$	0.058	0.057	−0.092
$\hat{M}^{IM}$ (full sample)	1.931	1.210	−0.111
$\hat{M}^{IM}$ (stockholders)	2.391	1.535	−0.222
$\gamma = 5$			
$\hat{M}^{CM}$	0.091	0.096	−0.094
$\hat{M}^{IM}$ (full sample)	60.768	4.107	−0.031
$\hat{M}^{IM}$ (stockholders)	105.272	4.602	−0.155

Note: The first column shows the pricing kernel standard deviation, the second column the maximum Sharpe-ratio consistent with each pricing kernel and the last column, the correlation between the pricing kernel and excess returns.

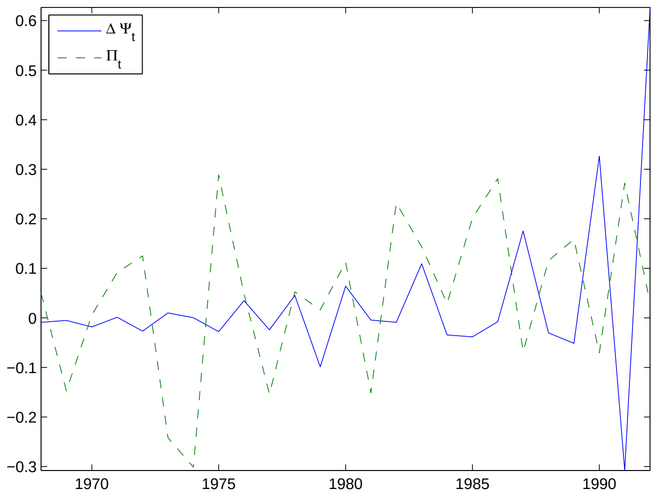

Figure 4: Market excess returns and changes in NDCV (full sample)

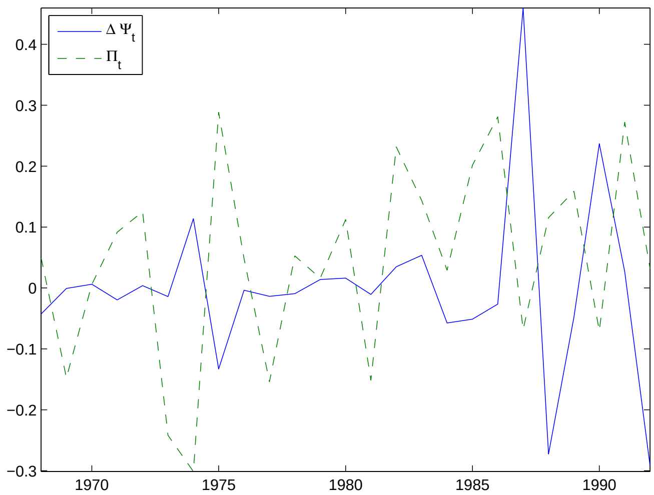

Figure 5: Market excess returns and changes in NDCV (stockholders)

Table 5: Equilibrium Sharpe-ratio in percentage

	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 5$
CM	0.171	0.346	0.527	0.527
IM full sample	3.717	10.264	14.882	9.317
(95% confidence interval)	(3.712, 3.733)	(10.240, 10.294)	(14.716, 15.092)	(9.229, 9.420)
IM stockholders	4.711	16.602	40.070	72.895
(95% confidence interval)	(4.613 , 4.764)	(16.216 , 16.866)	(39.007 , 40.999)	(72.536 , 73.164)

Note: Implied equilibrium Sharpe-ratio in percentage for selected RRA coefficients and 95% bootstrap confidence intervals.

5.3 The Implied Equilibrium Sharpe-ratio

In the post-war period 1945-2004, the observed average excess return on equity, when the S&P500 is used as a proxy for the market portfolio has been of 7.59% with a standard deviation of 16.67%. Hence, the observed excess return on equity Sharpe-ratio was 0.45. As can be seen in table (5) the standard representative agent model is unable to generate an equilibrium Sharpe-ratio anywhere near the same ballpark. For example, the representative agent implied equilibrium Sharpe-ratio when the RRA coefficient is chosen to be equal to 3 is smaller than the observed return on equity by two orders of magnitude. Table (5) also shows the estimated equilibrium Sharpe-ratio under incomplete markets, implied by different and economically reasonable coefficients of relative risk aversion (RRA). The results are encouraging. In particular, when the model is estimated using the stockholders subsample, for a RRA coefficient equal to 3, a 5% confidence interval for the Sharpe ratio is given by (0.39 , 0.41). Hence a RRA coefficient between 2 and 4 would generate an estimate for the equilibrium Sharpe-ratio which is consistent with the historical security and bond prices data. On the other hand, when the estimation is done using the full sample of PSID households, the model is not able to generate a high enough Sharpe-ratio. This is because the correlation between the incomplete market pricing kernel (estimated using the full sample) and the market excess return on equity is very week. As can be seen by inspecting figures (6) and (7) the implied equilibrium incomplete market Sharpe-ratio implied by a coefficient of RRA which is within a reasonable range is only able to match the historical data when the model is estimated on the stockholders subsample.

6 Conclusion

In this paper, I have examined empirically the ability of financial market incompleteness to help explaining the equity premium puzzle. Given that large excess returns on equity imply large Sharpe ratios, an alternative way to view the equity premium puzzle is that the IMRS of the representative agent exhibits insufficient variability. Hence, provided that the incomplete markets pricing kernel co-moves with market excess returns in an appropriate way, market incompleteness is likely to explain a substantial part of the equity premium puzzle, because the incomplete markets pricing kernel is substantially more volatile than the representative agent IMRS. The paper's first finding is that the non-diversifiable component of cross-sectional variance of income, which is the corner-stone of the incomplete market pricing kernel, co-moves with the growth rate of aggregate consumption and the market excess return on equity with the right sign. Thus, I found the changes in the NDCV to be countercyclical and in particular, negatively correlated to the market excess returns on equity.

Therefore, provided that the IMPK has enough volatility, the equilibrium market excess return Sharpe-ratio can be substantially larger, for reasonable levels of risk aversion. The main finding of the paper is therefore an encouraging result. Financial market incompleteness is a promising solution for the equity premium puzzle. However, the paper also highlights an important caveat. Limited financial market participation has to be taken seriously when constructing and calibrating equilibrium asset pricing models. This is one of the central empirical findings of this paper. All the results provided have been obtained for two samples of households. One sample which included all households, independently of there asset holdings, and a second tightened subsample, which only included households reporting to own a positive level of stock. This restriction is likely to play a very important role, because only households who participate in financial markets are expected to satisfy with equality the Euler equation (2.1). Thus, taking into account limited asset market participation is important, for estimating the equilibrium Sharpe-ratio. Three reasons justify this statement. First, the NDCV component of cross-sectional variance was shown to explain a larger share of the total cross-sectional variance when the sample was restricted to only include stockholders. Second, the IMPK estimated on the stockholders subsample was found to co-move more strongly with market excess returns. And third, the volatility of the stockholders IMPK is greater than the volatility of the IMPK estimated using the full sample of households.

The central question asked in the empirical investigation was whether a model where the pricing kernel corresponds to the incomplete market pricing kernel defined in equation (2.10) implied an estimate for the equilibrium Sharpe-ratio which is sufficiently high to match the historical Sharpe ratio for the excess returns on equity, given a reasonably calibrated model (i.e.: given a low coefficient of relative risk aversion). An affirmative answer to this question is a necessary condition for financial market incompleteness to provide a solution for the equity premium puzzle. The findings are encouraging. For the model estimated on the stockholders subsample, a coefficient of relative risk aversion between 2 and 4, produces an estimate for the equilibrium Sharpe-ratio around 0.5, matching the Sharpe-ratio of 0.45 observed in annual post-war data obtained when the S&P500 is used as a proxy for the true market portfolio. The results are, therefore, supportive of the incomplete markets/limited participation model offering a solution for the equity premium puzzle.

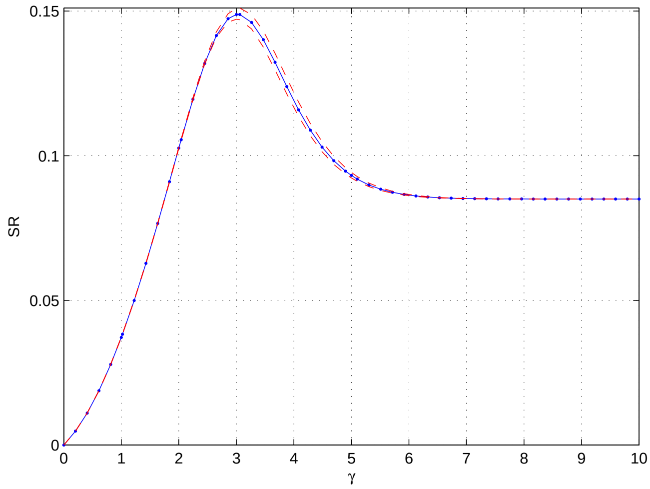

Figure 6: Equilibrium Sharpe Ratio (full sample)

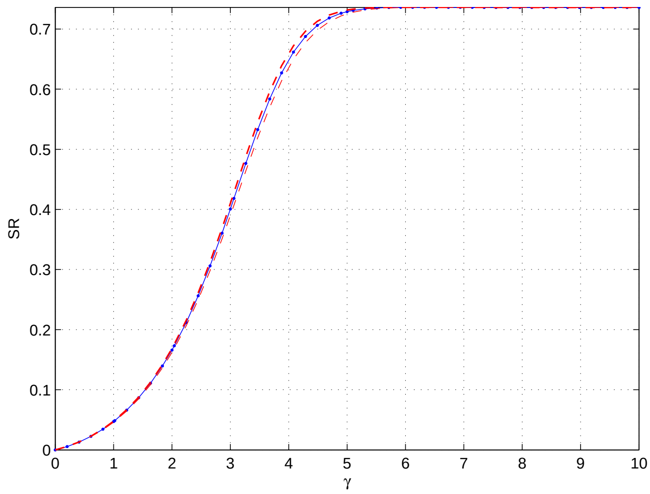

Figure 7: Equilibrium Sharpe Ratio (stockholders)

A Appendix

A.1 Sample Selection

Households where selected into the sample only if they satisfied the following eligibility criteria: they were not part of the initial SEO subsample; the average income was positive; they were present in the 1984 cross-section; they were present in the survey and eligible in at least ten sample periods. Moreover, the following observations were dropped: observations for whom any of the explanatory variables which are in $G(d_t, z_{it})$ where missing or miss-reported; observation where income was not positive; observations in which the household head changed. Tables (6) and (7) show information for both unbalanced panels, the one obtained using the full sample and the one corresponding to the stockholders subsample.

A.2 Model Estimation

The household log real income process $\{m_t^i\}_{t=0}^{\infty}$ is decomposed into a deterministic component plus a stochastic component:

$$m_t^i = G(d_t, z_{it}) + y_t^i \tag{A.1}$$

The function $G(d_t, z_{it})$ is a linear function of aggregate time dummies and individual observable characteristics, such as education, household size, and a polynomial in the age of the head, which is necessary to capture the life-cycle component of earnings. $G(d_t, z_{it})$ is estimated through pooled least squares. In table (8) the coefficient estimates are reported.

The model specified for the stochastic component of income corresponds to

$$y_t = \Lambda f_t + \xi_t \tag{A.2}$$

Where $y_t = (y_t^1 \ldots y_t^N)'$, $\Lambda = (\lambda_1' \ldots \lambda_N')'$, and $\xi_t = (\epsilon_t^1 \ldots \epsilon_t^N)'$. In order for the model to be estimated consistently, the following assumptions need to be made:

$$E\left[\xi_t | f_t\right] = 0 \; \forall \; t, \tau. \tag{A.3}$$

$$\lim_{N\to\infty} \frac{1}{N}\Lambda'\Lambda = M \text{ a nonsingular matrix.} \tag{A.4}$$

$$\lim_{N\to\infty} \left\| E\left[\xi_t \xi_t'\right] \right\| = c < \infty. \tag{A.5}$$

Where $\|X\|$ is the operator norm, and is equal to the maximum eigenvalue if X is a symmetric positive definite matrix.

To estimate the factors we apply CKL asymptotic principal components method, which relies on large N asymptotic applied to $N^{-1}\xi_t'\xi_\tau$, under the assumption that the idiosyncratic shocks are not autocorrelated.

$$p\lim_{N\to\infty} N^{-1}\xi_t'\xi_\tau = \begin{cases} 0 & \text{for } t \neq \tau \\ \sigma^2_{\epsilon,t} & \text{for } t = \tau \end{cases} \tag{A.6}$$

Now, defining $Y = (y_1 \dots y_T)$, $F = (f_1 \dots f_T)$, $\hat{\Omega} = \frac{Y'Y}{N}$ a $T \times T$ matrix of income shocks cross products and Φ a $T \times T$ diagonal matrix with diagonal elements $(\sigma^2_{\epsilon,1} \dots \sigma^2_{\epsilon,T})$, assuming that the law of large numbers applies, letting N go to infinity, and using equations (A.2) to (A.6) we obtain:

$$p\lim_{N\to\infty} \hat{\Omega} = \Omega = F'MF + \Phi \tag{A.7}$$

And given (A.7) we obtain the following:

$$\begin{aligned} p\lim_{N\to\infty} \; eigvec_s\left(\Phi^{-1/2}\hat{\Omega}\Phi^{-1/2}\right)\Phi^{1/2} &= eigvec_s\left(\Phi^{-1/2}(F'MF)\Phi^{-1/2}\right)\Phi^{1/2} \\ &= lF \end{aligned} \tag{A.8}$$

where l is taken to be a given matrix rotation.

Given that we are working with an unbalanced panel we define, following Connor and Korajcyk (1987), $\hat{\Omega}_{t\tau} = \frac{1}{N_{t\tau}}\sum_{i=1}^{N_{t\tau}} y_t^i y_\tau^i$, where $N_{t\tau}$ is the number of households that are present in the panel in both t and τ. And given (A.8) we estimate Φ and F using the following iterative procedure:

1. set initial conditions $\hat{\Phi}_0 = I_T$
2. compute for each iteration k, $l\hat{F}_r = \; eigvec_s \left(\hat{\Phi}_r^{-1/2} \hat{\Omega} \hat{\Phi}_r^{-1/2} \right) \hat{\Phi}_r^{1/2}$
3. Regress each household income on a constant and the previous-iteration $\hat{F}$ and keep the time-series regression residuals $\hat{\xi}_t^i$. Next estimate $\hat{\sigma}^2_{\epsilon,tr} = \frac{1}{N_t} \sum_{i=1}^{N_t} \hat{\xi}^i_{tr} \hat{\xi}^i_{tr}$.

The convergence criteria was taken to be a lower bound for the minimum R^2 from the regression of each factor on all the k factors from the previous iteration. Table (10) reports the number of iterations until convergence as well as the the evolution of the convergence criteria.

Finally, because I am working with an unbalanced panel, standard tests for the number of factors are not available. After experimenting with different models specifications I have chosen to estimate the model allowing for 5 common factors. Tables (10) and (11) report selected quantiles of the cross-sectional distribution of individual Adjusted-R^2 which provide some support for the 5 factor model.

Table 6: Full sample information

Year	# Households	Mean Age	Std Age
1967	1385	42.10	13.76
1968	1352	43.04	13.69
1969	1456	42.83	13.99
1970	1646	42.12	14.62
1971	1840	41.71	14.99
1972	2052	41.31	15.45
1973	2270	41.06	15.75
1974	2551	40.57	15.94
1975	2787	40.47	15.99
1976	2972	40.39	16.11
1977	3193	40.56	16.22
1978	3440	40.63	16.19
1979	3633	40.80	16.30
1980	3859	40.97	16.33
1981	4034	41.56	16.40
1982	4192	42.05	16.52
1983	4404	42.49	16.52
1984	4312	43.19	16.29
1985	4200	43.85	16.05
1986	4109	44.56	15.84
1987	4028	45.22	15.60
1988	3911	46.09	15.46
1989	3754	46.88	15.24
1990	3662	47.79	15.20
1991	3553	48.49	14.97
1992	3377	49.57	14.79

Note: Full sample size and age statistics. The age statistic corresponds to the head of the household. The total number of observations is 81972.

Table 7: Stockholders subsample information

Year	# Households	Mean Age	Std Age
1967	466	40.52	12.76
1968	455	41.25	12.36
1969	480	41.41	12.67
1970	532	41.43	13.26
1971	570	41.71	13.60
1972	611	41.53	14.07
1973	666	41.61	14.45
1974	715	41.55	14.62
1975	758	41.85	14.80
1976	803	41.92	14.92
1977	840	42.35	15.11
1978	880	42.73	15.12
1979	918	43.08	15.27
1980	957	43.41	15.37
1981	978	44.15	15.44
1982	1004	44.80	15.54
1983	1039	45.50	15.61
1984	1013	46.41	15.62
1985	994	47.18	15.51
1986	975	48.00	15.32
1987	952	48.63	14.91
1988	923	49.42	14.74
1989	902	50.17	14.49
1990	881	51.16	14.30
1991	868	51.92	14.25
1992	840	52.79	14.00

Note: Stockholders subsample size and age statistics. The age statistic corresponds to the head of the household. The total number of observations is 21020.

Table 8: Deterministic component of income (first stage regression)

	Full Sample		**Stockholders**	
Variable	Coefficient	t-Stat	Coefficient	t-Stat
constant	9.928	55.66	9.195	29.26
age	-0.166	-10.57	-0.078	-2.84
age^2	0.741	15.26	0.529	6.28
age^3	-1.143	-18.02	-0.926	-8.46
age^4	0.006	19.06	0.005	9.51
gender	-0.762	-94.07	-0.860	-54.59
size	0.415	189.51	0.380	100.43
high school	0.432	59.81	0.179	12.86
college	0.803	84.70	0.473	31.89
black	-0.390	-50.07	-0.118	-6.53
latino	-0.112	- 4.12	-0.037	-0.81
N	81972		21020	
R^2	0.56		0.61	

Note: first stage regression. The specification includes year dummy variables which I do not report here.

Table 9: Stockholders subsample information

	Full sample	Stockholders
Iteration	R^2	R^2
1	-	-
2	0.8874	0.9468
3	0.9786	0.9839
4	0.9934	0.9959
5	0.9971	0.9969
6	0.9988	0.9987
7	0.9996	0.9986
8	0.9999	0.9992
9	1.0000	0.9996
10	-	0.9998
11	-	0.9998
12	-	0.9999
13	-	1.0000

Note: Algorithm convergence. $R^2 = 1$ is reported when the convergence criteria is reached. The convergence criteria was chosen to be $1 - 10^{-4}$

Table 10: Full sample cross-sectional individual Adjusted-R^2

# Factors	5% quantile	Median	95 %quantile
1	0.001	0.174	0.722
2	0.042	0.415	0.810
3	0.108	0.508	0.843
4	0.164	0.584	0.886
5	0.215	0.604	0.905
6	0.277	0.703	0.939

Note: Time-series regression of each individual income against all factors. Selected quantiles of the cross-sectional distribution of individual adjusted-R^2

Table 11: Stockholders cross-sectional individual Adjusted-R^2

# Factors	5% quantile	Median	95 %quantile
1	0.001	0.203	0.718
2	0.040	0.414	0.810
3	0.106	0.506	0.843
4	0.161	0.582	0.885
5	0.201	0.602	0.904
6	0.253	0.672	0.933
7	0.307	0.717	0.961

Note: Time-series regression of each individual income against all factors. Selected quantiles of the cross-sectional distribution of individual adjusted-R^2

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