Competition Policy and Economic Growth in an Economy with Heterogeneous Industries

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Preliminary Draft

Abstract

This paper aims at studying the relationship between competition policy and economic growth in an economy with heterogeneous industries. In particular, the analysis distinguishes different types of industries – high-technology and low-technology – as well as different forms of competition policy – in the market and for the market. Then the objective is to examine the impact of various competition policies in each of these contexts.

The model predicts that a policy aimed at increasing competition for the market, through the reduction of barriers to entry, always produces a positive impact on innovation and growth, in each type of industry. On the opposite, a policy designed to improve competition in the market, by imposing the sharing of the technology invented by the leader, may generate a negative effect in high-technology industries: in fact such policy, by eliminating the expected reward due to the innovator, reduces the incentives of firms to invest in R&D and then decreases technological progress in the future.

This dynamic efficiency perspective introduces some elements of discussion about the design and the implementation of competition policy, with particular attention to the cases of abuse of dominance in high-technology industries, which involve an interaction between antitrust law and intellectual property protection.

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1. Introduction

The present work aims at analyzing the relationship between competition policy and economic growth from a theoretical point of view, in order to propose some indications about the optimal design of competition policy in a dynamic efficiency perspective, as well as to contribute to the current debate on the appropriate economic policies for encouraging long-run growth, particularly in industrialized countries. In fact, the policy recommendations usually proposed by international institutions and economic consultants for promoting sustainable growth suggest to increase the degree of competition in our economies: this outcome should be achieved by liberalizing markets such to favour the entry of new competitors and by implementing a severe antitrust policy in order to correct eventual distortions in market functioning.

In particular, this objective has been strongly emphasized in the economic policies of the European Union, through the creation of the Single Market and through the implementation of the Antitrust Policy. Consistently with this perspective, in the recent years the European Commission has adopted very important antitrust decisions against cartels and dominant firms and in some cases[1] it has shown an attitude even stricter than the one followed by the US Antitrust Authorities. Moreover, in the Lisbon Agenda competition policy is presented as one of the main tools in order to achieve the target of making the European Union "the most dynamic and competitive knowledge-based economy in the world capable of sustainable economic growth"[2]. Indeed, according to this policy perspective, a really competitive market should induce more innovation and then enhance productivity growth, so resolving the issue of the productivity slowdown observed in Europe in the last two decades.

Notwithstanding this dominant idea in the policy environment, economists have not yet given a definitive answer about the effect of competition on growth. The questions that lead such discussion are the following ones. How can competition policy affect the relevant factors for long-run growth? Does it always have a positive impact on productivity growth? Or can it also produce a negative effect?

The existing literature on endogenous growth theory has not given a clear and definitive reply about the sign of this relationship. The models based on horizontal innovation, like Romer (1990), show a positive effect of competition on growth, while the models based on vertical innovation, such as Aghion and Howitt (1992), present a negative impact of competition on growth. In fact, according to one view, also supported by empirical evidence, competition can generate strong incentives for innovation, because firms can succeed in a really competitive environment

[1] Just to make an example, we could recall the decisions adopted in the Microsoft Europe case as well as in the General Electric-Honeywell case, to make clear the somehow different attitude of the EU and US Antitrust Policies.

[2] This is the strategic goal presented in the Presidency Conclusions of the Lisbon European Council held on 23-24 March 2000. In the original intentions, this target should be achieved by 2010.

only if they are able to introduce significant improvements in the quality of the products and in the efficiency of the production processes. But, on the contrary, in the analysis of Schumpeterian models of endogenous growth, competition policies which reduce the monopoly rents gained by successful innovators can also lower the incentives for the investments of firms in R&D, and then compromise the future perspectives for technological progress.

Some explanations have been proposed to reconcile these different views and to understand which of these aspects prevails, and under which conditions. In particular, some new Schumpeterian models have provided a more articulated solution to this problem: Aghion, Bloom, Blundell, Griffith and Howitt (2005) describes a U-inverted relationship between competition and innovation, while Acemoglu, Aghion and Zilibotti (2006) identifies a negative effect of competition policy on growth for the countries far from the technological frontier and a positive one for the economies close to the frontier.

In the recent years, some new contributions have been provided in theoretical and empirical literature, in order to better analyze the complexity of the issue. In particular, it seems reasonable that the effect of competition may not be linear and so may depend on some other circumstances (the initial level of product market competition, the distance from the technological frontier, the existence of imperfections in other markets). For this reason, the recent analyses have focused the attention on such interactions and have examined in which cases and under which conditions these competitive policies may produce a beneficial or a detrimental effect on growth. So the need to distinguish different cases and conditions is a crucial point for current research on the discussed topic and this is relevant not only for theoretical purposes, but especially for policy perspectives. In fact, if the effect of competition may change depending on the features of the economic environment, the governments or the public authorities have to implement different policies for each single situation, so they cannot adopt pro-competitive policies always assuming to generate a positive effect on innovation and growth.

In particular, our theoretical analysis aims at studying this issue by distinguishing both different types of industries – high-technology and low-technology – and different forms of competition policies – for the market and in the market.

The first idea, developed in this work, is to examine the innovation activity and the growth process in an economy with heterogeneity of final goods, such that it is possible to distinguish between high-technology industries (such as software, pharmaceuticals) and low-technology industries (like food or steel), since the same policy can produce different effects depending on the characteristics of each specific industry. Indeed, the level of technology required to a firm, in order to operate in a given industry, is important for determining the incentives for innovation and then to understand how competition policy may influence such incentives: in particular, in order to avoid a negative impact on long-run growth, it should encourage these incentives or, at least, should not distort them.

Concerning the second point, a policy aimed at increasing the degree of product market competition may pursue different strategies: it can induce a higher rate of entry in a given market by reducing barriers to entry (competition policy for the market) or it can impose the same competitive conditions for all the firms in a given industry by removing all the differences among the incumbent competitors in that market (competition policy in the market). In fact these various policies can differently affect the incentives for innovation, which are again the key element for explaining the working of endogenous technological progress: so it is expected that different competition policies can produce diverse results on innovation and growth.

2. The General Framework of the Model

Given the current state of the literature, the relationship between competition policy and economic growth can have a more exhaustive explanation, by considering the differences across various types of industries, as well as the distinction across various forms of competition. Then the objective of this analysis is to clarify the different effects on growth induced by competition in the market and competition for the market both in high-technology industries and in low-technology industries. For this purpose, the analysis is divided in two parts, for each type of industry.

In particular, high-technology industries are often characterized by vertical integration between research and production activities[3], such that research costs are included in the profit function of the firms which are involved in the innovation activity. The high entry costs explain the elevated concentration of this market, characterized by a monopoly or by an oligopoly with a technological leadership. In this industry framework, the incentive for innovation is given by the monopolistic rents due to the exploitation of patents, and so it is fundamental to preserve the existence of some innovation rents for promoting research. Then, more competition in the market would imply that innovation rents are shared among all the existing firms and that innovator loses monopoly profits. Given that a long period is required for compensating innovation costs, this type of policy would eliminate any incentive for innovation and consequently more competition in the market might discourage investments in R&D and dampen technological progress. In the same context, different effects would be produced by a liberalization process designed to reduce entry barriers: in fact, provided that technological progress also depends positively on the number of

[3] Vertical integration between production and research activity is a key point for distinguishing high-technology industries from low-technology industries in the structure of our model. For this reason in the following paragraph, describing the main assumptions of the model for high-technology industries, we will provide some economic intuition as well as some justifying evidence for such organizational structure.

firms operating in the industry and potentially involved in the research activity, a policy aimed at developing competition for the market would induce more firms to invest in R&D, then increasing the innovation rate of the economy.

On the contrary, in low-technology industries, vertical integration between production and research is less frequent since firms can operate in the market without a specific research activity: as a consequence, this generally implies low entry costs for a new firm. In this industry framework, firms invest in innovation in order to reduce production costs and increase profits, and particularly to escape competition and extend market shares. So the gain from innovating in time t is equal to the additional profit obtained in time t+1 thanks to the exploitation of a new technology; however, in such industries, the higher profit obtained just for one period can be sufficient for compensating the innovation costs paid in time t. As usual, more competition in the market implies that all the other firms will have access to the new technology and share the same level of profits: but, differently from the previous case, the leader still has incentive for innovation, having already achieved in time t+1 a remuneration for the innovation effort. So, given that the escape competition effect is the determinant reason for investing in R&D, more competition in the market still produces a positive impact on innovation and then spurs economic growth. On the other hand, competition for the market increases the number of firms in the industry: so, provided that the gain from innovation is anyway an increasing function of the number of competitors, initially sharing the same pre-innovation profits, this reduction of entry barriers determines also here a positive effect on technological progress.

In conclusion, while a competition policy aimed at increasing entry always shows a positive impact whatever the type of industry, a competition policy levelling all the already existing firms in the market can produce opposite effects on innovation and growth depending on the type of industry. Then it is worth to analyze the issue by considering and contrasting the two opposite effects for each type of industry.

3. High-Technology Industries

Let start to analyze the theoretical framework by examining the innovation activity and the growth process in high-technology industries, that we denote by the subscript j. There the production process requires the adoption of an advanced technology, which is developed thanks to the investments of firms in research and development. Research activity requires both capital and labour and then implies higher costs for the firms interested in improving their production technologies. In fact, a specific feature of high-technology industries in our model is the vertical integration between research and production activities.

This assumption about vertical integration is generally supported by real-world evidence: in fact, the firms involved in high-technology industries, such as the ones supplying softwares or pharmaceuticals, are directly involved also in the research work which is propedeutic to the production process. Just to make an example, it is difficult to imagine an important pharmaceutical firm, which manages in outsourcing the research activity aimed at studying the active ingredient for a new medicine, when it has to be released to the market for the first time; similarly, it is unlikely to see a successful firm in the software field, which delegates to an external firm the research work needed to elaborate the source codes for a PC operating system, that is expected to be installed in most of the new personal computers. Clearly, there are specific rationales for vertical integration in each of these cases: for the pharmaceutical firm, some motivations of medical safety, since the firm has to sure about the quality and the effectiveness of the medicine; for the firm producing softwares, some reasons of industrial secrecy, since in a market with a limited patentability of the new inventions it is safer to manage directly all the operations related to the software development in order to avoid the diffusion of essential and easily reproducible information to the competitors. But the underlying idea, which generally justifies this choice, is the following one: in a given industry, where innovation plays a fundamental role and can determine the success or the failure of an entrepreneurial project, each firm is naturally interested in directly carrying out such activity, because it cannot rely on the other firms for such an important task. Moreover, since the share of the turnover allocated to research is usually very high in these industries, each firm prefers to directly run this activity also in order to better control the amount of costs as well as to obtain some economies from vertical integration.

So, research activity is generally integrated with production activity within the organizational structure of high-technology industries, but in any case not all the firms existing in these industries are initially involved in research activity. In fact we distinguish a leader firm (active in research and production) and some follower firms (involved only in production). As a consequence of this process innovation, the leader employs a production technology A_{jLt}, which is more advanced than the technology A_{jFt} available to the followers, by a technological step x_{jt}.

So, provided that $x_{jt} = A_{jLt} - A_{jFt}$, the size of the technological advantage x_{jt} is determinant in our framework in order to explain the market structure of such industries: in fact, the industry has a monopolistic structure if the leader has a technology level much higher than the follower, while it presents an oligopolistic structure (even with the presence of a leader) if the technological gap is quite low. At the beginning, each high-technology industry is an oligopoly: only when the innovation activity of the leader sensibly increases its technological advantage, production activity becomes much more costly for the followers and then it may induce them to exit the market. In any case, the market structure of an industry is dynamic: even a monopolized industry can become an oligopolistic one if new firms enter the market using an appropriate technology, such to compete - at least potentially - with the leader, or if a pro-competitive policy implemented by an antitrust authority imposes the leader to share - partially or totally – its technology level with the followers, then reducing or eliminating the existing technological advantage.

In high-technology industries, innovation is the main determinant of the performance of each firm, then it requires an appropriate protection by the law system. For this reason, the innovation corresponding to x_{jt} is object of a patent, so intellectual property law allows only the leader to use this new technology for the production process. Once the leader obtains the exclusive right to exploit such invention (x_{jt}), the previous innovation (x_{jt-1}) becomes object of public knowledge and then it is available for the exploitation by other firms. As a consequence of that, if technology diffusion occurred without any barrier, also the technology level of the follower should increase by an equivalent measure, because of the availability of this previously protected technology. Then, it should be $A_{jFt} = A_{jFt-1} + x_{jt-1}$.

Nevertheless, some barriers to technology diffusion, due to the technical aspects (such as the need of specialized human capital for technology implementation) or to the conduct of the leader (like exclusionary practices) may prevent the follower, totally or partially, from the adoption of the existing and available technology. For this reason, we will consider two different measures of the follower's technology level: A_{jFt}, that is the technology level available to the follower (and relevant for the maximization of the firm's profit function), which evolves as a consequence of the public availability of existing technologies; $\overline{A_{jF}}$, that is the technology level effectively determined by the barriers to technology diffusion (and relevant for the computation of the aggregate production function of industry j), which is assumed to be constant over time.

In particular, we will also assume that the follower doesn't have perfect information about the barriers to technology diffusion and then it cannot correctly forecast the impediments that it can encounter in the attempt to adopt an existing available technology: this is the reason why, even if $\overline{A_{jF}}$ is its effective technology level, the follower considers A_{jFt} as its technology level benchmark and then formulates its optimal production plans on such basis. We can justify this assumption in a various ways depending on the specific nature of the barrier to technology diffusion. In fact, when the barrier is due to technical reasons, the follower firm, which has not directly developed such innovation, but is interested in adopting the available technology, doesn't have a priori the adequate expertise for the implementation and it doesn't know the required type of technical competence. For this reason, it cannot organize a detailed plan for technology adoption, and even after it can encounter difficulties in procuring the human resources or in training the human capital. Of course, this lack of experience implies a high possibility of failure, but the follower firm is not able to quantify such probability at the beginning: in any case, this uncertainty about the final outcome of the project may discourage this activity of technological adoption. Moreover, when the barrier to technology diffusion is due to an anti-competitive conduct of the leader, it is even more difficult to foresee the future problems in technology adoption: indeed, when the leader wants to limit the diffusion of its previous technology to the followers, given that the abuse of dominance is not legal, it adopts some anti-competitive practices where the exclusionary intent is not immediately evident. So, in these cases, the follower firm can expect that the leader will adopt some exclusionary strategies but it is not able to forecast the type of conduct and especially it is not sure whether he will manage the prove the anti-competitive intent of the practice beside an antitrust authority.

3.1 The production functions of the leader and of the follower

Let define firstly the production function of the leader. It exploits the technology A_{jL} and uses specialized capital and labour both for production and for research. Depending on their utilization, we can distinguish research capital K_{jR} and production capital K_{jP}, as well as research labour L_{jR} and production labour L_{jP}. Nevertheless, from the viewpoint of the quality, capital and labour have to be considered as homogenous types of inputs, independently from the specific purposes of their usage (production or research). In fact, we can argue that high-technology industries are intensive in innovative capital and skilled labour and that the firms operating in these markets can only employ high-quality inputs in order to run both the research activity and the production process.

As a consequence of that, the same unit of innovative capital or skilled labour can be allocated both to production and to research: the only difference is that, once a given input is utilized for production rather than for research, it contributes differently to total output. This aspect is captured in the production function by the different values of the parameters for each type of input and for each final usage of that factor.

In time t, the leader produces the output Y_{jLt} according to the following function:

$$Y_{jLt} = A_{jLt} \, K_{jRt}^{\alpha} \, K_{jPt}^{\beta} \, L_{jRt}^{\gamma} \, L_{jPt}^{\delta} \quad (1)$$

where $0<\alpha<1$, $0<\beta<1$, $0<\gamma<1$, $0<\delta<1$ and $\alpha+\beta+\gamma+\delta=1$. The parameters α, β, γ and δ indicate the share of each factor in total output and are constant over time. Let assume that the factors employed in research contribute to total output quantitatively more than the factors used for production, because the first ones improve the efficiency of the production process and then present higher productivity: consequently, an increase of the amount of research capital or research labour by a multiplicative factor λ augments total output more than a corresponding rise in the quantity of, respectively, production capital or production labour. Then:

$$\alpha > \beta \quad \text{(A.1)} \quad \text{and} \quad \gamma > \delta \quad \text{(A.2)}$$

Moreover, the technology level of the leader A_{jLt} in time t is equal to:

$$A_{jLt} = A_{jFt} + x_{jt} \quad (2)$$

At each time t, the leader can improve its technology level A_{jLt} with respect to the one available to the follower A_{jFt}. Provided that the technology level, and then also the technological gap x_{jt} , can be measured as a discrete variable, we assume that:

$$1 \leq x_{jt} \leq A_{jFt} \quad \text{(A.3)}$$

Then, for a given time t, the leader can introduce a technological innovation x_{jt}, which is equal or lower than A_{jFt}. This means that in a one-period interval the leader can at most double the technology level available to the follower. Clearly this implies that, if the follower always keeps the same technology level $\overline{A_{jF}}$, while the leader increases its technological advantage during the interval from 0 to t, after this time the technological gap may be higher than the initial technology level of the follower, then it can be: $x_{jt} \geq \overline{A_{jF}}$. In particular, as it will be explained successively, this is the necessary condition for the leader to profitably invest in research and development.

The observation of the leader’s production function suggests two main considerations about the properties of that function. Firstly, we can note that the production function has constant returns to scale with respect to all the inputs, both production capital and labour, and research capital and labour. Secondly, the technology level A_{jLt} shifts the production function: then, multiplying A_{jLt} by a factor λ, the total output of the leader is also multiplied by λ. For this reason, we can divide the production function described in equation (1) by the technology level A_{jLt} and then obtain the leader’s production function per unit of technology level, that is:

$$y_{jLt} = \frac{Y_{jLt}}{A_{jLt}} = f\left(K_{jRt}, K_{jPt}, L_{jRt}, L_{jPt}\right) = K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} \qquad (3)$$

This is the amount of output that a firm involved in research and production is able to produce using the basic technology A_{jt}=1. When the leader innovates the production process and then obtains a technological advantage equal to x_{jt} compared to the follower, even using the same quantity of inputs, its total output increases by an amount equal to the product x_{jt} y_{jLt}.

Now we can consider the production function of the follower. It exploits the technology level A_{jFt} and uses capital and labour just for production purposes. It produces a total output Y_{jFt} according to the following function:

$$Y_{jFt} = A_{jFt}\ K_{jPt}^{\alpha+\beta}\ L_{jPt}^{\gamma+\delta} \qquad (4)$$

where $0<(\alpha+\beta)<1$, $0<(\gamma+\delta)<1$ and $(\alpha+\beta)+(\gamma+\delta)=1$. The parameters of this production function are defined in such a way that they correspond to the same ones used in the leader's production function: $\alpha+\beta$ is the factor share of capital (only used for production), while $\gamma+\delta$ is the factor share of labour (only employed for production). This specification of both production functions will imply important corollaries for the assumption about homogeneity of capital and labour across production and research sector.

As in the previous case, we can observe two important properties of the production function. In fact, it has constant returns to scale with respect to capital K_{jPt} and labour L_{jPt}. Moreover, the technology level A_{jLt} shifts the production function: then, dividing it by A_{jFt}, we obtain the follower's production function per unit of technology level:

$$y_{jFt} = \frac{Y_{jFt}}{A_{jFt}} = f\left(K_{jPt}, L_{jPt}\right) = K_{jPt}^{\alpha+\beta}\ L_{jPt}^{\gamma+\delta} \qquad (5)$$

This is the amount of output that a firm involved only in production is able to produce using the basic technology A_{jt}=1. When the follower adopts an existing advanced technology or when it is allowed to share a new technology developed by the leader, it innovates the production process and increases its technological level by ΔA_{jFt}: then, even using the same quantity of inputs, its total output increases by an amount equal to the product $\Delta A_{jFt}\ y_{jFt}$.

3.2 The implications of the homogeneity assumption for production and profit functions

As explained in the previous paragraph, the assumption about homogeneity of capital and labour across production and research sectors is justified by some economic considerations: the most innovative industries need high-quality capital and high-skilled workers and could not use low-quality inputs either for production or for research. Moreover, this assumption is also useful when we have to compare the production functions as well as the profit functions, in order to draw conclusions about the amount of inputs employed by different firms in high-technology industries. In fact, as a consequence of such homogeneity, we can write the production functions per unit of technology without the subscript:

$$y_{jLt} = K_{jRt}^{\alpha}\, K_{jPt}^{\beta}\, L_{jRt}^{\gamma}\, L_{jPt}^{\delta} = \left[(1+\mu)K_{jt}\right]^{\alpha}\ \left[(1-\mu)K_{jt}\right]^{\beta}\ \left[(1+\nu)L_{jt}\right]^{\gamma}\ \left[(1-\nu)L_{jt}\right]^{\delta} \qquad (6)$$

$$y_{jFt} = K_{jPt}^{\alpha+\beta}\ L_{jPt}^{\gamma+\delta} = K_{jt}^{\alpha+\beta}\ L_{jt}^{\gamma+\delta} \qquad (7)$$

where $0 \le \mu \le 1$ and $0 \le \nu \le 1$. In equation (6), indicating the leader's production function per unit of technology level, the parameters μ and ν are used to define the allocation of capital and labour among production and research activities. In fact, even if we introduce the homogeneity assumption, we don't know a priori how the leader chooses to allocate those inputs.

Let consider firstly a simplifying case. In particular, for $\mu=0$ and $\nu=0$, $K_{jPt}=K_{jRt}$ and $L_{jPt}=L_{jRt}$, then the amount of capital and labour available to the leader is equally divided among production and research. In this specific case, we can rewrite the leader's production function per unit of technology as follows:

$$y_{jLt} = K_{jt}^{\alpha} K_{jt}^{\beta} L_{jt}^{\gamma} L_{jt}^{\delta} = K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta}$$

So we can conclude that the output obtained from the production technologies y_{jL} and y_{jF} is equal. Then:

$$y_{jLt} = K_{jRt}^{\alpha} K_{jLt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} = K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta} = y_{jFt} \qquad (8)$$

In order to fulfil this equality, we must have:

$$K_{jRt}(L)=K_{jPt}(L)=K_{jPt}(F)=K_{jt} \qquad \text{and} \qquad L_{jRt}(L)=L_{jPt}(L)=L_{jPt}(F)=L_{jt}$$

Then it follows that:

$$K_{jLt} = K_{jRt} + K_{jPt} = 2K_{jt} \quad \& \quad K_{jFt} = K_{jt} \quad \rightarrow \quad K_{jLt} = 2K_{jFt}$$

$$L_{jLt} = L_{jRt} + L_{jPt} = 2L_{jt} \quad \& \quad L_{jFt} = L_{jt} \quad \rightarrow \quad L_{jLt} = 2L_{jFt}$$

This means that, if the homogeneity assumption holds and if the leader allocates the same quantity of capital and labour across production and research, the leader employs an amount of capital (and labour) as double as the follower. Such result, corresponding to a particular case, can also hold under more general conditions, i.e. the assumption that $\mu=0$ and $\nu=0$ is not a necessary condition for such equilibrium.

Let analyze the case where $0<\mu<1$ and $0<\nu<1$, which is more relevant for the development of this model. Then we have for the leader the following production function per unit of technology:

$$y_{jLt} = (1+\mu)^{\alpha}(1-\mu)^{\beta} K_{jt}^{\alpha+\beta} (1+\nu)^{\gamma}(1-\nu)^{\delta} L_{jt}^{\gamma+\delta}$$

In particular, we will consider the existence of an equilibrium with μ^* and ν^*, different from 0. For given values of α and β, there exists an equilibrium value μ^*, with $0<\mu^*<1$, such that:

$$(1+\mu^*)^{\alpha}(1-\mu^*)^{\beta} = 1 \qquad (9)$$

For given values of γ and δ, there exists an equilibrium value ν^*, with $0<\nu^*<1$, such that:

$$(1+\nu^*)^{\gamma}(1-\nu^*)^{\delta} = 1 \qquad (10)$$

Then, for the equilibrium values μ^* and ν^*, the leader's production function y_{jLt} becomes:

$$y_{jLt} = K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta}$$

Also in this case, we can conclude that the output obtained from the production technologies y_{jF} and y_{jL} is equal. Then:

$$y_{jLt} = K_{jRt}^{\alpha} K_{jLt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} = K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta} = y_{jFt} \quad (8)$$

In order to fulfil this equality, we must have:

$$K_{jRt}(L) = (1+\mu^*)K_{jt} \qquad K_{jPt}(L) = (1-\mu^*)K_{jt} \qquad K_{jPt}(F) = K_{jt}$$

$$L_{jRt}(L) = (1+\mu^*)L_{jt} \qquad L_{jPt}(L) = (1-\mu^*)L_{jt} \qquad L_{jPt}(F) = L_{jt}$$

Then it follows that

$$K_{jLt} = K_{jRt}^* + K_{jPt}^* = (1+\mu^*)K_{jt} + (1-\mu^*)K_{jt} = 2K_{jt} \quad \& \quad K_{jFt} = K_{jt} \quad \rightarrow \quad K_{jLt} = 2K_{jFt} \quad (11.a)$$

$$L_{jLt} = L_{jRt}^* + L_{jPt}^* = (1+\mu^*)L_{jt} + (1-\mu^*)L_{jt} = 2L_{jt} \quad \& \quad L_{jFt} = L_{jt} \quad \rightarrow \quad L_{jLt} = 2L_{jFt} \quad (11.b)$$

This implies that, if the homogeneity assumption holds, the leader employs an amount of capital (labour) as double as the follower. But the leader uses more capital (labour) for research than for production; then the production capital (labour) used by the follower is higher than the leader's production capital (labour) but lower than the leader's research capital (labour). Indeed we have:

$$K_{jRt}^*(L) > K_{jPt}^*(F) > K_{jPt}^*(L)$$

$$L_{jRt}^*(L) > L_{jPt}^*(F) > L_{jPt}^*(L)$$

A corollary of the homogeneity of capital and labour across sectors is that interest rates and wages are equal among production and research sector. In fact, if the input is the same, it requires the same remuneration.

$$w_t = w_{Rt} = w_{Pt} \qquad \text{and} \qquad r_t = r_{Rt} = r_{Pt}$$

The homogeneity of wages and interest rates across sectors has important implications for the computation of profit. For this reason, we are interested in comparing the profit functions for the leader and for the follower.

$$\pi_{jLt} = A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} - w_{Rt} L_{jRt} - w_{Pt} L_{jPt} - r_{Rt} K_{jRt} - r_{Pt} K_{jPt}$$

$$\pi_{jFt} = A_{jFt} K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta} - w_{Pt} L_{jPt} - r_{Pt} K_{jPt}$$

Let introduce the homogeneity assumption for production and research inputs as well as for wages and interest rates. Then we obtain:

$$\pi_{jLt} = A_{jLt}[(1+\mu)K_{jt}]^{\alpha} [(1-\mu)K_{jt}]^{\beta} [(1+\nu)L_{jt}]^{\gamma} [(1-\nu)L_{jt}]^{\delta} - w_t[(1+\mu)L_{jt}] - w_t[(1-\mu)L_{jt}] - r_t[(1+\nu)K_{jt}] - r_t[(1-\nu)K_{jt}]$$

$$\pi_{jFt} = A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}$$

We know that in equilibrium $\mu = \{0, \mu^*\}$ and $\nu = \{0, \nu^*\}$, then conditions (9) and (10) hold. So we can write the profit function for the leader and the follower, provided that $\pi_{jFt} \geq 0$ and $\pi_{jLt} \geq 0$:

$$\pi_{jLt} = A_{jLt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - 2(w_t L_{jt} + r_t K_{jt})$$

$$\pi_{jFt} = A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - \left(w_t L_{jt} + r_t K_{jt}\right)$$

A comparison between the profit functions of the leader and of the follower displays that, as shown in (11.a) and (11.b), a leader firm uses a quantity of inputs twice as big as the amount of factors employed by the follower. Then we can evaluate the profitability of the decision to invest in R&D by comparing the profit of the leader and the profit of a follower of the same size (that is employing the same amount of inputs) or – equivalently because of the constant returns to scale – twice the profit of a standard follower firm.

Let suppose that $x_{jt}>0$, so the leader has some technological advantage. Depending on the size of the technological gap, we have to consider three cases.

Firstly, if $x_{jt} = A_{jFt}$, and then $A_{jLt} = A_{jFt} + x_{jt} = 2A_{jFt}$, we can observe that:

$$\pi_{jLt} = 2\pi_{jFt} \quad (12)$$

In fact,

$$\pi_{jLt} = 2A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - 2\left(wL_{jt} + rK_{jt}\right) = 2\left[A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - \left(wL_{jt} + rK_{jt}\right)\right] = 2\pi_{jFt}$$

Then, for $x_{jt} = A_{jFt}$, we can conclude that if $\pi_{jFt} = 0$ also $\pi_{jLt} = 0$, while if $\pi_{jFt} > 0$ then $\pi_{jLt} = 2\pi_{jFt} > 0$.

Secondly, if $x_{jt} < A_{jFt}$, and then $A_{jLt} = A_{jFt} + x_{jt} < 2A_{jFt}$, we can notice that:

$$\pi_{jLt} < 2\pi_{jFt} \quad (13)$$

In fact,

$$\pi_{jLt} = \left(A_{jFt} + x_{jt}\right) K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - 2\left(wL_{jt} + rK_{jt}\right) < 2\left[A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - \left(wL_{jt} + rK_{jt}\right)\right] = 2\pi_{jFt}$$

So, in a situation where $x_{jt} < A_{jFt}$, the leader wouldn't have any incentive to invest in research and development, because a follower firm of the same size (that is using the same quantity of factors) would get a higher profit.

Finally, in the case that $x_{jt} > A_{jFt}$, and then $A_{jLt} = A_{jFt} + x_{jt} > 2A_{jFt}$, we can see that:

$$\pi_{jLt} > 2\pi_{jFt} \quad (14)$$

This is because:

$$\pi_{jLt} = \left(A_{jFt} + x_{jt}\right) K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - 2\left(wL_{jt} + rK_{jt}\right) > 2\left[A_{jFt} K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta} - \left(wL_{jt} + rK_{jt}\right)\right] = 2\pi_{jFt}$$

In conclusion, for $x_{jt} > A_{jFt}$, the profit of the leader is higher than the profit of a follower firm of the same size.

3.3 The maximization problem for the leader and for the follower

Let consider the maximization problem for the leader:

$$\max_{K_{jRt}, K_{jPt}, L_{jRt}, L_{jPt}} \pi_{jLt} = A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} - w_R L_{jRt} - w_P L_{jPt} - r_R K_{jRt} - r_P K_{jPt} \qquad (15)$$

The FOCs of such problem are the following ones:

$$\frac{\partial \pi_{jLt}}{\partial K_{jRt}} = 0 \Leftrightarrow \alpha A_{jLt} K_{jRt}^{\alpha-1} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} = r_R \quad (16)$$

$$\frac{\partial \pi_{jLt}}{\partial K_{jPt}} = 0 \Leftrightarrow \beta A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta-1} L_{jRt}^{\gamma} L_{jPt}^{\delta} = r_P \quad (17)$$

$$\frac{\partial \pi_{jLt}}{\partial L_{jRt}} = 0 \Leftrightarrow \gamma A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma-1} L_{jPt}^{\delta} = w_R \quad (18)$$

$$\frac{\partial \pi_{jLt}}{\partial L_{jPt}} = 0 \Leftrightarrow \delta A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta-1} = w_P \quad (19)$$

Given the homogeneity of wages and interest rates across production and research, we can use the above results of the profit maximization problem in order to quantify the amount of capital and labour employed by the leader in research or in production.

Since $r_t = r_{Rt} = r_{Pt}$, we can use conditions (16) and (17) and then we can compare the quantities of K_{jPt} and of K_{jRt} used by the leader.

$$r_{Rt} = r_{Pt}$$

$$\alpha A_{jLt} K_{jRt}^{\alpha-1} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta} = \beta A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta-1} L_{jRt}^{\gamma} L_{jPt}^{\delta}$$

Simplifying the equation and recalling that $\alpha > \beta$ from (A.1), we can show that:

$$K_{jRt} > K_{jPt}$$

This means that the leader optimally allocates the existing amount of capital in such a way to have more research capital than production capital.

Given that $w_t = w_{Rt} = w_{Pt}$, we can use conditions (18) and (19) and then we can compare the quantities of L_{jPt} and L_{jRt} used by the leader.

$$w_{Rt} = w_{Pt}$$

$$\gamma A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma-1} L_{jPt}^{\delta} = \delta A_{jLt} K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta-1}$$

Simplifying the equation and recalling that $\gamma > \delta$ from (A.2), we can show that:

$$L_{jRt} > L_{jPt}$$

It means that the leader optimally allocates the existing amount of labour in such a way to have more research labour than production labour.

These results are consequential to the assumptions on the parameters of the production function: if an input employed for research increases total output more than the same input used for production, the solution of the profit maximization problem clearly implies an allocation of K_{jLt} such that $K_{jRt} > K_{jPt}$ and of L_{jLt} such that $L_{jRt} > L_{jPt}$.

This outcome has an important implication also for the allocation of capital and labour among leader and follower, as discussed in par. 2.2. Indeed, the equilibrium values of $K_{jRt}(L)$, $K_{jPt}(L)$ and $K_{jPt}(F)$, as well as of $L_{jRt}(L)$, $L_{jPt}(L)$ and $L_{jPt}(F)=L_{jt}$ are determined by the parameters μ and ν: so, among the two considered equilibria, the above solution rules out the equilibrium with μ=0 and ν=0 and only admits the equilibrium with μ=μ* and ν=ν* (where 0<μ*<1 and 0<ν*<1). But now, in order to draw clear conclusions about the input allocation for leader and followers, we also need the results of the profit maximization problem for the follower.

Let consider the maximization problem for the followers:

$$\max_{K_{jPt},L_{jPt}} \pi_{jFt} = A_{jFt} K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta} - w_P L_{jPt} - r_P K_{jPt} \qquad (20)$$

The FOCs of such problem are the following ones:

$$\frac{\partial \pi_{jFt}}{\partial K_{jPt}} = 0 \Leftrightarrow (\alpha+\beta) A_{jFt} K_{jPt}^{\alpha+\beta-1} L_{jPt}^{\gamma+\delta} = r_P \qquad (21)$$

$$\frac{\partial \pi_{jFt}}{\partial L_{jPt}} = 0 \Leftrightarrow (\gamma+\delta) A_{jFt} K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta-1} = w_P \qquad (22)$$

The solutions of the profit maximization problem can be used in order to compare the amount of production capital and labour respectively employed by the leader and the follower. Taking the FOCs from the profit maximization problem for the interest rate on production capital, that is (17) and (21), we obtain:

$$r_P(L)=r_P(F)$$

$$\beta\left(A_{jFt} + x_{jt}\right) K_{jRt}^{\alpha} K_{jPt}^{\beta-1} L_{jRt}^{\gamma} L_{jPt}^{\delta} = (\alpha+\beta) A_{jFt} K_{jPt}^{\alpha+\beta-1} L_{jPt}^{\gamma+\delta} \qquad (23)$$

Recalling the assumptions (A.1) and (A.3), we notice that:

$$\beta\left(A_{jFt} + x_{jt}\right) < (\alpha+\beta) A_{jFt}$$

Then, from equation (23), we can write the following inequality:

$$\underbrace{\left(K_{jRt}^{\alpha} K_{jPt}^{\beta} L_{jRt}^{\gamma} L_{jPt}^{\delta}\right)}_{=y_{jLt}} \left[K_{jPt}(L)\right]^{-1} > \underbrace{\left(K_{jPt}^{\alpha+\beta} L_{jPt}^{\gamma+\delta}\right)}_{=y_{jFt}} \left[K_{jPt}(F)\right]^{-1}$$

Given that in equilibrium $y_{jLt}=y_{jFt}$, as presented in equation (8), we can rewrite the inequality as:

$$K_{jPt}^{*}(L) < K_{jPt}^{*}(F) \qquad (24)$$

So this means that in equilibrium, where each firm in the industry maximizes its profit, the follower employs a greater amount of production capital than the leader. This is essentially a consequence of the technological gap: since wages for production labour have to be equal in equilibrium across the various firms in the same industry, in order to keep the equality of the marginal product of production labour for follower and leader, the follower must have a higher capital-labour ratio. And then, since the follower has to use more production capital, it has to pay higher costs for such input.

Using the FOCs from the profit maximization problem for the wage of production labour, that is (19) and (22), we obtain:

$$w_P(L)=w_P(F)$$

$$\delta\left(A_{jFt}+x_{jt}\right)K_{jRt}^{\alpha}K_{jPt}^{\beta}L_{jRt}^{\gamma}L_{jPt}^{\delta-1}=\left(\gamma+\delta\right)A_{jFt}K_{jPt}^{\alpha+\beta}L_{jPt}^{\gamma+\delta-1} \qquad (25)$$

Recalling the assumptions (A.2) and (A.3), we notice that:

$$\delta\left(A_{jFt}+x_{jt}\right)<\left(\gamma+\delta\right)A_{jFt}$$

Then, from equation (25), we can write the following inequality:

$$\underbrace{\left(K_{jRt}^{\alpha}K_{jPt}^{\beta}L_{jRt}^{\gamma}L_{jPt}^{\delta}\right)}_{=y_{jLt}}\left[L_{jPt}\left(L\right)\right]^{-1}>\underbrace{\left(K_{jPt}^{\alpha+\beta}L_{jPt}^{\gamma+\delta}\right)}_{=y_{jFt}}\left[L_{jPt}\left(F\right)\right]^{-1}$$

Given that in equilibrium $y_{jLt}=y_{jFt}$, as presented in equation (8), we can rewrite the inequality as:

$$L_{jPt}^{*}\left(L\right)<L_{jPt}^{*}(F)$$

This result means that in equilibrium the follower needs a higher amount of production labour than the leader. As already explained for production capital, this is mainly an effect of the technological gap between the follower and the leader: since the interest rates on production capital have to be equal for the various firms in the same industry, in order to balance the lower technological level, the follower must have a higher labour-capital ratio. In this way, also the equality of the marginal product of production capital for the leader and for the follower is kept. Given that the follower has to use more production labour, it has to pay a higher cost for such input.

3.4 The aggregate production function for each industry

Aggregating the product across all the firms in a given industry, the total output of industry j is given by:

$$Y_{Jt}=\sum_{1}^{M}Y_{jmt}=Y_{jLt}+\sum_{1}^{M-1}Y_{jmt} \qquad (26)$$

where $\forall m\neq L$, $Y_{jmt}=Y_{jFt}$. Given that only the leader has a higher technological level, all the other firms are followers and then each of them produces the same output, that is Y_{jFt}. Then, substituting the production functions for the leader and for the follower, we can write the aggregate production function of industry j as follows:

$$Y_{Jt} = \left(A_{jFt} + x_{jt}\right)K_{jRt}^{\alpha}K_{jPt}^{\beta}L_{jRt}^{\gamma}L_{jPt}^{\delta} + \left(M-1\right)A_{jFt}K_{jPt}^{\alpha+\beta}L_{jPt}^{\gamma+\delta}$$

Let define $\overline{A_{jF}}$ the technology level of the follower at *t=0*. Given that the follower doesn't invest in research, it keeps the same technology level $\overline{A_{jF}}$ also in time t, unless it manages to implement some of the available technologies: then, in order to determine the effective aggregate production function, we will indicate $\overline{A_{jF}}$ as the technology level of the follower. Moreover, because of the homogeneity of capital and labour across production and research sector, we can write:

$$Y_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right)\left[(1+\mu)K_{jt}\right]^{\alpha}\left[(1-\mu)K_{jt}\right]^{\beta}\left[(1+\nu)L_{jt}\right]^{\gamma}\left[(1-\nu)L_{jt}\right]^{\delta} + \left(M-1\right)\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}$$

We know that in equilibrium $\mu = \mu^{*}$ and $\nu = \nu^{*}$ (such that $0 < \mu^{*} < 1$ and $0 < \nu^{*} < 1$), then conditions (9) and (10) hold. So the aggregate production function of industry j is:

$$Y_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right)K_{jt}^{\alpha}\,K_{jt}^{\beta}\,L_{jt}^{\gamma}\,L_{jt}^{\delta} + \left(M-1\right)\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}$$

Then, rearranging, we have a general expression for Y_{Jt}:

$$Y_{Jt} = \left(M\overline{A_{jF}} + x_{jt}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} \qquad (27)$$

Depending on the size of the technological advantage of the leader, we can have three different cases. Then we define the aggregate production function of the industry for each of them.

If $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate production function of industry j is:

$$Y_{Jt} = M\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} \qquad (28)$$

In this case, the technological advantage that the leader can achieve is too small for R&D investment to be profitable. In fact, as shown in equation (13), if $x_{jt} < \overline{A_{jF}}$, the firm active in research might not recover the costs related to research activity and then could have a negative profit, or even if it obtained a positive profit, this would be lower than the profit gained by an equivalent firm active only in production. In such situation, no firm is willing to invest additional resources in research and development, then all the firms are involved in production. In conclusion, the aggregate production function is simply a M-multiple of the follower's production function.

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate production function of industry j is:

$$Y_{Jt} = \left(M+1\right)\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} \qquad (29)$$

Finally, if $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2,\infty)$, the aggregate production function of industry j is:

$$Y_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right)K_{jRt}^{\alpha}K_{jPt}^{\beta}L_{jRt}^{\gamma}L_{jPt}^{\delta} \qquad (30)$$

Differently from the previous cases, this result can occur only for $t \in [2,\infty)$. In fact, by assumption (A.3), we know that $1 \le x_{jt} \le A_{jFt}$; then, assuming that the follower doesn't improve its technology level and then maintains the same level $\overline{A_{jF}}$, the leader cannot reach in t=1 a technological advantage higher than $\overline{A_{jF}}$, but it can attain it after a time interval of at least 2 periods. In this case, since the technological advantage of the leader is quite relevant, it is much more costly for the follower to compete in the same market, then it is forced to exit because otherwise it would have negative profits.

3.5 The Dynamics of Production in High-Technology Industries

Given the expressions for the aggregate production function of industry j, let compute the growth rates of output per industry.

If $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate production function is defined as in equation (28). Then, taking logs and deriving with respect to time, the growth rate of output is given by:

$$\frac{\dot{Y}_{Jt}}{Y_{Jt}} = \frac{\dot{M}_t}{M_t} + (\alpha+\beta)\frac{\dot{K}_{jt}}{K_{jt}} + (\gamma+\delta)\frac{\dot{L}_{jt}}{L_{jt}} \qquad (31)$$

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate production function of industry j is indicated in equation (29). The growth rate of output is equal to:

$$\frac{\dot{Y}_{Jt}}{Y_{Jt}} = \frac{\dot{M}_t}{M_t + 1} + (\alpha+\beta)\frac{\dot{K}_{jt}}{K_{jt}} + (\gamma+\delta)\frac{\dot{L}_{jt}}{L_{jt}} \qquad (32)$$

Finally, if $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2,\infty)$, the aggregate production function of industry j is expressed in equation (30). Because of the homogeneity of capital and labour across production and research, we can write:

$$Y_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right) K_{jt}^{\alpha} K_{jt}^{\beta} L_{jt}^{\gamma} L_{jt}^{\delta} = \left(\overline{A}_{jF} + x_{jt}\right) K_{jt}^{\alpha+\beta} L_{jt}^{\gamma+\delta}$$

Then, taking logs and deriving with respect to time, the growth rate of the industry is described by:

$$\frac{\dot{Y}_{Jt}}{Y_{Jt}} = \frac{\dot{x}_{jt}}{\overline{A}_{jF} + x_{jt}} + (\alpha+\beta)\frac{\dot{K}_{jt}}{K_{jt}} + (\gamma+\delta)\frac{\dot{L}_{jt}}{L_{jt}} \qquad (33)$$

Given the results for the growth rates of output, we can compute the rate of technological progress for industry j as the Solow residual, in such a way to exclude the variation in capital and labour.

If $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, from equation (31) we have:

$$a_{Jt} = \frac{\dot{Y}_{Jt}}{Y_{Jt}} - (\alpha + \beta)\frac{\dot{K}_{jt}}{K_{jt}} - (\gamma + \delta)\frac{\dot{L}_{jt}}{L_{jt}} = \frac{\dot{M}_t}{M_t} \quad (34)$$

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1, \infty)$, from equation (32) we have:

$$a_{Jt} = \frac{\dot{Y}_{Jt}}{Y_{Jt}} - (\alpha + \beta)\frac{\dot{K}_{jt}}{K_{jt}} - (\gamma + \delta)\frac{\dot{L}_{jt}}{L_{jt}} = \frac{\dot{M}_t}{M_t + 1} \quad (35)$$

If $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2, \infty)$, from equation (33) we have:

$$a_{Jt} = \frac{\dot{Y}_{Jt}}{Y_{Jt}} - (\alpha + \beta)\frac{\dot{K}_{jt}}{K_{jt}} - (\gamma + \delta)\frac{\dot{L}_{jt}}{L_{jt}} = \frac{\dot{x}_{jt}}{\overline{A_{jF}} + x_{jt}} = \frac{\dot{x}_{jt}}{A_{jLt}} \quad (36)$$

From the above results, we can notice that, when $x_{jt} \leq \overline{A_{jF}}$, that is when the technological advantage of the leader is lower than or equal to the technology level of the follower, the key determinant of technological progress is the rate of entry in the industry, that is the growth rate of the number of firms. In fact, an increase of potential competition stimulates innovation, also among the firms initially active only in production, since the reduction of the profit margin due to the higher number of competitors may induce firms to invest capital and labour in research activity. On the opposite, when $x_{jt} > \overline{A_{jF}}$, that is when the technological step of the leader is higher than the technology level of the follower, the main determinant of technological progress is the variation of the leader's technological advantage: if the leader introduces further innovations and then increases its technological level, so enlarging the advantage with respect to the follower, this implies a positive rate of technological progress for industry j. For these reasons, we are now interested in studying the dynamics of these two important variables, that is the number of firms in the industry (M_t) and the technological advantage of the leader (x_{jt}).

3.6 The Dynamics of Market Structure in High-Technology Industries

The dynamics of M_t has to take into account both the firms which enter the industry and the firms which exit the market. So the variation in time t of M_t is equal to the difference between the new firms active in the market and the old firms now out of the market. Provided that the entrant can be both a firm active in another market as well as a new entrepreneur, previously employed as a worker in another firm, the entry decision is determined by various factors: the expectation about future profits, the type of entry barriers in the market, the availability of new technologies for an entrant firm (for example for foreign firms implementing direct investments). The exit decision can be caused by firm-specific factors, such as its financial condition, as well as by economy-wide factors, like the quality of bankruptcy law, the existence of imperfections in credit market. Then we can write the law of motion of firms in industry j as follows:

$$\dot{M}_t = E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t \qquad (37)$$

where $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$is the expected profit ratio (that is the ratio between the expected profit of the entrant and a reference measure of profit for a follower firm in such industry); φ is the parameter of a Poisson distribution indicating the hazard rate of entry for new firms; η is a variable indicating the type of barriers to entry, such that 0< η<1, where a value η=1 defines a completely free-entry situation while η=0 means no entry possibility in the industry; L_{Jt} is the total amount of workers, such that each of them, availing of a new technology or a new idea, can become an entrepreneur and start a new firm; χ is a parameter of a Poisson distribution denoting the hazard rate of exit for the existing firm. If the number of entrants is higher than the number of exiting firms, $\dot{M}_t$ is positive and then the total number of firms in the industry increases.

The expected profit ratio is a variable which defines the profitability of an entry decision. In particular, we compute the expected profit from entry as the ratio between the aggregate profit of industry j and the number of existing firms increased by one unit. Given that we don't know the aggregate profit of the industry in time t+1, we use as an approximation the aggregate profit in time t: then we assume that aggregate profit remains the same and that, because of such entry, it has to be divided among the existing firms plus the entrant firm. Then we compare this expected profit with a reference measure of profit, that is the profit obtained by a follower active in that industry:

$$\tilde{\pi}_{jt} = A_{jFt}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}$$

The ratio between these two variables can be equal to, lower or higher than 1. If $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$>1, the entry is profitable and then this induces more firms to enter the market, while, if $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$<1, the entry is not profitable and then firms are discouraged from entering such industry.

The aggregate profit of industry j varies depending on the number of existing firms and on their technological level: then, in order to compute it, we have to distinguish three different cases, as for the determination of the aggregate output. In fact, we define the aggregate profit as follows:

$$\Pi_{Jt} = \sum_{1}^{M}\pi_{jmt} = \pi_{jLt} + \sum_{1}^{M-1}\pi_{jmt}$$

Then, substituting the profit functions for the leader and for the follower, we obtain:

$$\Pi_{Jt} = \left(A_{jFt} + x_{jt}\right)K_{jRt}^{\alpha}K_{jPt}^{\beta}L_{jRt}^{\gamma}L_{jPt}^{\delta} - w_{Rt}L_{jRt} - w_{Pt}L_{jPt} - r_{Rt}K_{jRt} - r_{Pt}K_{jPt} + \left(M-1\right)\left[A_{jFt}K_{jPt}^{\alpha+\beta}L_{jPt}^{\gamma+\delta} - w_{Pt}L_{jPt} - r_{Pt}K_{jPt}\right]$$

Applying the homogeneity assumption for production and research inputs as well as for wages and interest rates and defining $\overline{A_{jF}}$ as the technology level of the follower, we have:

$$\Pi_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right)\left[(1+\mu)K_{jt}\right]^{\alpha}\left[(1-\mu)K_{jt}\right]^{\beta}\left[(1+\nu)L_{jt}\right]^{\gamma}\left[(1-\nu)L_{jt}\right]^{\delta} - w_t\left[(1+\mu)L_{jt}\right] + \\ - w_t\left[(1-\mu)L_{jt}\right] - r_t\left[(1+\nu)K_{jt}\right] - r_t\left[(1-\nu)K_{jt}\right] + (M-1)\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]$$

We know that in equilibrium $\mu = \mu^*$ and $\nu = \nu^*$ (such that $0 < \mu^* < 1$ and $0 < \nu^* < 1$), then conditions (9) and (10) hold. So, rearranging terms, the aggregate profit function of industry j is:

$$\Pi_{Jt} = \left(M_t \overline{A_{jF}} + x_{jt}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - (M_t + 1)\left(w_t L_{jt} + r_t K_{jt}\right)$$

Then we can compute the expected profit for the three different cases.

If $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate profit function of industry j becomes:

$$\Pi_{Jt} = M_t\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]$$

In order to compute the expected profit, let divide the aggregate profit by the number of firms in the industry increased by one unit:

$$E_t\left[\pi_{jEt+1}\right] = \pi_{jEt} = \frac{\Pi_{Jt}}{M_t} = \frac{M_t}{M_t + 1}\underbrace{\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]}_{=\tilde{\pi}_{jt}}$$

Then the expected profit ratio is given by:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \frac{M_t}{M_t + 1} < 1 \qquad (38)$$

This implies that, when the technological advantage of the leader is so small to discourage innovation activity, the expected profit from entry is even lower than the current follower's profit and then entry is not profitable.

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the aggregate profit function of industry j is:

$$\Pi_{Jt} = (M_t + 1)\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]$$

The expected profit is equal to:

$$E_t\left[\pi_{jEt+1}\right] = \pi_{jEt} = \frac{\Pi_{Jt}}{M_t + 1} = \frac{M_t + 1}{M_t + 1}\underbrace{\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]}_{=\tilde{\pi}_{jt}}$$

So the expected profit ratio is given by:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = 1 \qquad (39)$$

Finally, if $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2,\infty)$, the aggregate profit function is:

$$\Pi_{Jt} = \left(\overline{A_{jF}} + x_{jt}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - 2\left(w_t L_{jt} + r_t K_{jt}\right)$$

The expected profit is equal to:

$$E_t\left[\pi_{jEt+1}\right] = \pi_{jEt} = \frac{1}{2}\left\{2\underbrace{\left[\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}\right]}_{=\tilde{\pi}_{jt}}\right\} + \frac{1}{2}\left(x_{jt} - \overline{A_{jF}}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}$$

Then the expected profit ratio is given by:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = 1 + \frac{1}{2}\frac{\left(x_{jt} - \overline{A_{jF}}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A_{jF}}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}} > 1 \qquad (40)$$

As we have seen in equations (38), (39) and (40), the expected profit ratio may assume three different values, depending on the size of the technological advantage of the leader x_{jt}. In particular, if $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, the expected profit ratio is lower than l, while if $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1,\infty)$ or $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2,\infty)$, the expected profit ratio is equal to or higher than 1. So we can conclude that the expected profit ratio is higher if the technological advantage of the leader is larger.

The analysis of market structure in high-technology industries allows us to identify the variables which better define the intervention and the impact of competition policy. As anticipated in the introduction, we want to distinguish the various effects of competition in the market and competition for the market.

The policies designed to favour competition in the market follow the purpose to guarantee the same competitive conditions for all the firms operating in a given industry through the sharing of the same technology or of the same product design. For the perspective of the analysis, compulsory licensing is a clear example of a policy promoting competition in the market. A typical situation which can eventually require this type of intervention by a competition authority is known in antitrust policy as refusal to deal. Let consider an innovative firm which has obtained a near-monopolistic position thanks to the exploitation of its own invention, protected by a patent. Another firm is interested in entering the same market or an adjacent market but, in order to supply a given product, needs to know the idea which is object of intellectual property protection. The leader doesn’t have any incentive to provide the entrant with its own idea, because by revealing the details of the patent it would share such innovation with other firms, which at this point would be able to reproduce it and to compete with the innovator. In the practice of competition authorities, such refusal to deal may be considered as an anti-competitive behaviour under some conditions[4]: if the requested intellectual property is indispensable to compete; if the refusal to deal causes the complete foreclosure of the market; and if the refusal prevents the emergence of markets for new products for which there is substantial demand. In these cases, compulsory licensing can be adopted

[4] These are the three conditions usually required in the legal practice by the European Commission and by the European Court of Justice in order to define the anti-competitive nature of the innovator’s conduct and in order to argue the pro-competitive effects of compulsory licensing.

as a remedy against the innovator. But this decision, which improves competition in the market, can be very detrimental for the incentive to innovate, especially if the product to be developed by the licensor can be in direct competition with the one of the intellectual property holder, and even more if the licensor exploits the innovative idea also to supply the same product of the patent holder.

As a result, this competition policy in the market eliminates the technological advantage of the leader and it also reduces or removes the profit of the firm active in research. But especially, this policy can sensibly modify the market structure of the concerned industry. In particular, if such policy determines a competitive situation, all the firms sharing the same technology will have zero profit; while, if this policy leaves an oligopolistic market with collusion among the existing firms, they will share a positive aggregate profit. In any case, the most relevant effect of this policy has to be observed after one period, given that the expected profit of a firm in industry j tends to decrease in the following period, because of such time-inconsistency in research policy. In fact, a firm has no incentive to invest in R&D if it knows that, notwithstanding the protection of intellectual property, it can be obliged to share the same technology with the followers: as a consequence of that, no firm will be finally active in research.

In our theoretical framework, the disincentive to innovate can be explained with reference to the variation of the expected profit ratio, as induced by this competitive policy. In fact, the implementation of the competition policy in the market, by eliminating the technological advantage of the leader, determines a reduction of the expected profit ratio $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$ to the minimum level. Then we can argue that a stronger competition in the market, or alternatively a weaker protection of intellectual property of innovations, reduces the technological advantage of the leader and then implies a lower value of the expected profit ratio. In a corresponding way, we can also state that a weaker competition in the market, and then a higher protection of intellectual property, allows for a larger technological advantage of the leader and so implies a higher expected profit ratio.

The policies aimed at improving competition for the market pursue the objective to increase the number of firms supplying a given product, by allowing more firms to enter that market. They can operate through various instruments, such as the reduction or the abolition of regulatory entry barriers or the introduction of R&D tax credits for new firms. In particular, for the scope of the analysis, a liberalization process aimed at reducing regulatory barriers is a typical example of a policy designed to enhance competition for the market. In this model, the level of barriers to entry is measured by the variable η, that we could also define as a free-entry variable, since it measures the freedom of entry in the industry: then a competition policy for the market augments the value η and, through the reduction of barriers to entry, increases the number of entrants. So we can infer that a low level of η implies less competition for the market and more entry barriers.

3.7 The Dynamics of the Technological Gap between the Leader and the Follower

The technological advantage of the leader is an outcome of R&D activity, then it is an increasing function of the amount of inputs devoted to research, as well as of the number of firms involved in the considered high-technology industry. In particular, we define the technological gap as follows:

$$x_{jt} = g(K_{jRt}, L_{jRt}, M_t) = (K_{jRt} \quad L_{jRt})^{M_t} \qquad (41)$$

So research capital and labour, which we have already seen as inputs of the production function, are relevant in this case as determinants of the technological advantage of the leader, because they are used in the innovation process for improving its production technology. Moreover, the number of firms operating in the same market positively affects the productivity of this research activity. In fact, for a given amount of research capital and labour, an increase of the number of potential competitors produces an exponential rise in the technological advantage of the leader. This is because the leader is induced to better exploit the research activity in order to obtain substantial improvements in its technology level: so the threat of entry has a clearly positive effect on the outcome of the innovative activity of the leader.

In order to analyze the dynamics of the technological gap between the leader and the follower, we have to compute the growth rate of x_{jt}. The technological advantage of the leader is defined in equation (41). Then, taking logs and deriving with respect to time, we obtain:

$$\frac{\dot{x}_{jt}}{x_{jt}} = M_t\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right) + \dot{M}_t(\ln K_{jRt} + \ln L_{jRt}) \qquad (42)$$

The law of motion of M_t is described in equation (37). Let assume that in equilibrium, for a given L_{Jt}, we have a value M_t^* such that the number of firms in industry j is constant, then $\dot{M}_t = 0$.

For $\dot{M}_t = 0$ $\qquad E_t\left[\frac{\pi_{jE\,t+1}}{\overline{\overline{\pi}}_{jt}}\right]\varphi\eta L_{Jt} = \chi M_t^* \Leftrightarrow M_t^* = E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\frac{\varphi}{\chi}\eta L_{Jt} \qquad (43)$

Then, substituting $\dot{M}_t = 0$ as well as the equilibrium value M_t^* in equation (42), we obtain the following expression for $\dot{x}_{jt}$, indicating the dynamics of the technological gap in equilibrium.

$$\dot{x}_{jt} = E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\frac{\varphi}{\chi}\eta L_{Jt}\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)x_{jt} \qquad (44)$$

3.8 The Rate of Technological Progress in High-Technology Industries

After studying the dynamics of the number of firms in the industry (M_t) and of the technological advantage of the leader (x_{jt}), we can determine the rate of technological progress in industry j and analyze its determinants, with particular attention to the variables referring to competition in the market and for the market. So let consider the results obtained from equations (34), (35) and (36) and let substitute the expressions for $\dot{M}_t$ and $\dot{x}_{jt}$.

If $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1, \infty)$, from equation (34) we have:

$$a_{Jt}\left(x_{jt} < \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t} = \frac{E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t}{M_t} \qquad (45)$$

where the expected profit ratio can assume only the minimum value, that is :

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \frac{M_t}{M_t + 1} < 1$$

Substituting this value for the expected profit ratio in equation (45), we obtain:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} < \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t} = \frac{\varphi\eta L_{Jt}}{M_t + 1} - \chi \qquad (46)$$

In this case, the technological structure of the industry is such that no investment in research and development can be profitable and then no firm is interested in acquiring a technological leadership. For this reason, we can say that the equilibrium with $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1, \infty)$ is a sclerotic equilibrium, in the sense that it is expected to persist because of the unwillingness of the existing firms to promote research and development. This explains why public authorities, and in particular competition authorities, should avoid to lead the economy to such equilibrium, given that the economy, once it has reached this equilibrium, cannot move away from it. Nevertheless, since the rate of technological progress can be however positive, some variables may influence such rate.

As we can infer from equation (46), a_{Jt} is an increasing function of η and of L_{Jt}. The first observation implies that a competition policy for the market, aimed at reducing barriers to entry, promotes technological progress because it augments the number of firms potentially engaged in the innovation activity. In fact:

$$\frac{\partial a_{Jt}}{\partial \eta} = \frac{\varphi L_{Jt}}{M_t + 1} > 0$$

The second consideration presents a scale effect related to the number of workers in industry j: since potentially each worker could become an entrepreneur, a higher amount of labour force determines a positive effect on technological progress because, given a hazard rate of entry φ, new entrepreneurs can enter the market.

$$\frac{\partial a_{Jt}}{\partial L_{Jt}} = \frac{\varphi\eta}{M_t + 1} > 0$$

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1, \infty)$, from equation (35) we have:

$$a_{Jt}\left(x_{Jt} = \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t + 1} = \frac{E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t}{M_t + 1} \qquad (47)$$

where the expected profit ratio may assume several values. In fact:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \left\{\frac{M_t}{M_t + 1}; 1; 1 + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\right\}$$

Even before substituting the various possible values for the expected profit ratio, we can observe that also in this case the rate of technological progress a_{Jt} is an increasing function of the free-entry variable η and of the number of workers in the industry L_{Jt}. Moreover, we can also see that a_{Jt} is a positive function of the expected profit ratio $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$:

$$\frac{\partial a_{Jt}}{\partial E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]} = \frac{\varphi\eta L_{Jt}}{M_t + 1} > 0$$

Indeed, if the expected profit ratio increases, not only more firms are induced to enter the market, but also the existing firms are induced to invest in R&D because in this way they can get a profit from the technological leadership and this can be higher than current profit.

In particular, if $x_{jt} = \overline{A_{jF}}$ but $x_{jt+1} < \overline{A_{jF}} \quad \forall t \in [1, \infty)$, we have:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t + 1} = \frac{M_t}{M_t + 1}\left(\frac{\varphi\eta L_{Jt}}{M_t + 1} - \chi\right) \qquad (48)$$

Moreover, if $x_{jt} = \overline{A_{jF}}$ and $x_{jt+1} = \overline{A_{jF}} \quad \forall t \in [1, \infty)$, we have:

$$a_{Jt}\left(x_{jt+1} = \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t + 1} = \frac{\varphi\eta L_{Jt}}{M_t + 1} - \chi\frac{M_t}{M_t + 1} \qquad (49)$$

Finally, if $x_{jt} = \overline{A_{jF}}$ and $x_{jt+1} > \overline{A_{jF}} \quad \forall t \in [1, \infty)$, we have:

$$a_{Jt}\left(x_{jt+1} > \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t + 1} = \frac{\varphi\eta L_{Jt}}{M_t + 1} + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\frac{\varphi\eta L_{Jt}}{M_t + 1} - \chi\frac{M_t}{M_t + 1} \qquad (50)$$

So, given that the rate of technological progress a_{Jt} is a positive function of the expected profit ratio $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$ and since the latter is increasing in the technological advantage of the leader x_{jt+1}, we can notice that a_{Jt} is an increasing function of x_{jt+1}. In fact, if we compare the previous expressions for a_{Jt}, we can observe that:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) < a_{Jt}\left(x_{jt+1} = \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) < a_{Jt}\left(x_{jt+1} > \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right) \quad (51)$$

This implies that, provided that $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1, \infty)$, the rate of technological progress augments if the technological advantage of the leader in the following period is higher. This depends not only on the innovation effort of the firms active in research, but also on the perspective of future profits that the leader is able to collect thanks to the protection of intellectual property.

In fact, if the antitrust authority imposes the leader to share its technology level with the followers, because it considers such technology as an essential facility for conducting a given economic activity, in the following period it will be $x_{jt+1} < \overline{A_{jF}}$, then the leader won't be able to get any profit from its previous effort in innovation. The consequence is that the firm active in research, after losing its technological leadership because of the compulsory licensing, won't have any other incentive to invest in R&D because the commitment of the government to protect the intellectual property on the new ideas won't be considered anymore as credible. And this lack of credibility in patent protection will affect not only the previous leader, but also the other firms, such that no firm will be interested in innovating its technology without any guarantee about the appropriate reward for research effort.

The negative impact of this time-inconsistency can be observed by comparing the rates of technological progress for $x_{jt} < \overline{A_{jF}}$ and $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1, \infty)$, as defined in equations (46) and (48), (49) and (50): usually a_{Jt} for $x_{jt} < \overline{A_{jF}}$ is lower than a_{Jt} for $x_{jt} = \overline{A_{jF}}$, because a higher technological distance also implies a higher rate of technological progress, unless in one case. This exception occurs when we compare $a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} < \overline{A_{jF}}\right)$ (46) and $a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right)$ (48): in the first situation the technological distance is lower than $\overline{A_{jF}}$ both in time t and in time t+1, while in the second case the technological advantage of the leader is equal to $\overline{A_{jF}}$ in time t but after decreases because of a competitive policy. From such comparison we can observe that:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} < \overline{A_{jF}}\right) > a_{Jt}\left(x_{jt+1} < \overline{A_{jF}} \middle| x_{jt} = \overline{A_{jF}}\right)$$

Then the rate of technological progress is higher for $x_{jt} < \overline{A_{jF}}$ than for $x_{jt} = \overline{A_{jF}}$. This reflects the negative effect that this competitive policy can produce on the expectations for future profits and then on the incentives to invest in innovation.

If $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2, \infty)$, from equation (36) we have:

$$a_{Jt}\left(x_{jt} > \overline{A_{jF}}\right) = \frac{\dot{x}_{jt}}{A_{jLt}} = M_t\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)\frac{x_{jt}}{A_{jLt}} + \underbrace{\left\{E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t\right\}}_{=\dot{M}_t}\left(ln\,K_{jRt} + ln\,L_{jRt}\right)\frac{x_{jt}}{A_{jLt}} \quad (52)$$

where the expected profit ratio may assume several values. In fact:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \left\{\frac{M_t}{M_t+1}; 1; 1 + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\right\}$$

Also in this case, we notice from equation (52) that the rate of technological progress is an increasing function of the free-entry variable η, of the number of workers in the industry L_{Jt} and of the expected profit ratio $E_t\left\lfloor \pi_{jEt+1}/\tilde{\pi}_{jt}\right\rfloor$. Then, substituting the different values of the expected profit ratio, we obtain the specific expressions for a_{Jt}.

In particular, if $x_{jt} > \overline{A_{jF}}$ but $x_{jt+1} < \overline{A_{jF}} \quad \forall t \in [2,\infty)$, we have:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) = \frac{\dot{x}_{jt}}{A_{jLt}} = M_t\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)\frac{x_{jt}}{A_{jLt}} + \underbrace{\left\{\frac{M_t}{M_t+1}\varphi\eta L_{Jt} - \chi M_t\right\}}_{=\dot{M}_t}\left(\ln K_{jRt} + \ln L_{jRt}\right)\frac{x_{jt}}{A_{jLt}} \quad (53)$$

Moreover, if $x_{jt} > \overline{A_{jF}}$ and $x_{jt+1} = \overline{A_{jF}} \quad \forall t \in [2,\infty)$, we have:

$$a_{Jt}\left(x_{jt+1} = \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) = \frac{\dot{x}_{jt}}{A_{jLt}} = M_t\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)\frac{x_{jt}}{A_{jLt}} + \underbrace{\left\{\varphi\eta L_{Jt} - \chi M_t\right\}}_{=\dot{M}_t}\left(\ln K_{jRt} + \ln L_{jRt}\right)\frac{x_{jt}}{A_{jLt}} \quad (54)$$

Finally, if $x_{jt} > \overline{A_{jF}}$ and $x_{jt+1} > \overline{A_{jF}} \quad \forall t \in [2,\infty)$, we have:

$$a_{Jt}\left(x_{jt+1} > \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) = \frac{\dot{x}_{jt}}{A_{jLt}} = M_t\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)\frac{x_{jt}}{A_{jLt}} + \underbrace{\left\{\left[1 + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t\right\}}_{=\dot{M}_t}\left(ln\,K_{jRt} + ln\,L_{jRt}\right)\frac{x_{jt}}{A_{jLt}} \quad (55)$$

The comparison among equations (53), (54) and (55) allow us to draw some conclusions about the impact of the technological advantage of the leader on the rate of technological progress. In fact, we observe that:

$$a_{Jt}\left(x_{jt+1} < \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) < a_{Jt}\left(x_{jt+1} = \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) < a_{Jt}\left(x_{jt+1} > \overline{A_{jF}}\middle| x_{jt} > \overline{A_{jF}}\right) \quad (56)$$

As discussed for the inequalities presented in (51), technological progress is higher when the distance between the leader and the follower is expected to be larger in the following period. This is because the leader is induced to invest more in R&D when it expects that it will get the exclusive right to exploit the new technology and that it will obtain the appropriate reward for innovation. A policy aimed at improving competition in the market, by imposing the sharing of an innovation, can reduce the technological distance between the leader and the follower in the following period. For this reason x_{jt+1} can be used as a measure of competition policy in the market: a low value of x_{jt+1}

(such as $x_{jt+1} < \overline{A_{jF}}$) is caused by a full implementation of such policy, while a high value of x_{jt+1} (such as $x_{jt+1} > \overline{A_{jF}}$) is the result of a limited application of this policy. In conclusion, competition in the market decreases the technological advantage of the leader in the future period but, at the same time, it also reduces the rate of technological progress by lowering the incentives for innovation.

3.9 The Rate of Technological Progress with Equilibrium Dynamics in Market Structure

In the previous paragraph, we have computed the rate of technological progress for various measures of the distance from the technological frontier, but always considering a positive or negative dynamics of market structure in industry j. In other words, we have analyzed the situation where $\dot{M}_t \neq 0$. Now we want to study the case where, in equilibrium, $\dot{M}_t = 0$, such that the total number of firms in industry j remains constant, and then to analyze the determinants of technological progress under this equilibrium dynamics for market structure. As before, we distinguish the three different cases for the value of x_{jt}.

In the case that $x_{jt} < \overline{A_{jF}} \quad \forall t \in [1,\infty)$, if $\dot{M}_t = 0$, from equation (45) we obtain:

$$a_{Jt}\left(x_{jt} < \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t} = \frac{E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t^*}{M_t^*} = 0 \quad (57)$$

where the expected profit ratio can assume only the minimum value, that is :

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \frac{M_t}{M_t + 1} < 1$$

If $x_{jt} = \overline{A_{jF}} \quad \forall t \in [1,\infty)$, provided that $\dot{M}_t = 0$ in equilibrium, from equation (47) we have:

$$a_{Jt}\left(x_{Jt} = \overline{A_{jF}}\right) = \frac{\dot{M}_t}{M_t + 1} = \frac{E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right]\varphi\eta L_{Jt} - \chi M_t}{M_t + 1} = 0 \qquad (58)$$

where the expected profit ratio may assume the following values:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \left\{\frac{M_t}{M_t + 1}; 1; 1 + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\right\}$$

Then, for $x_{jt} \leq \overline{A_{jF}}$, if $\dot{M}_t = 0$, the rate of technological progress is 0. As we have discussed in par. 2.5, for such values of the technological distance, the key determinant of technological progress is the rate of entry in the industry, that is the growth rate of the number of firms. So if M_t is constant, because the amount of entrant firms is equal to the number of exiting firms in a given time t, the only source of technological progress is missing. As a consequence of that, the growth rate of output in industry j, which is measured by equations (31) and (32), is only determined by the variation in capital and labour inputs.

Such equilibrium with constant number of firms in industry j and with zero technological progress is expected to persist until an exogenous shock modifies one of the concerned variables. For example, if the government implements a competition policy designed to reduce barriers to entry, the variable η increases and then the number of entrants augments: as a consequence of that, $\dot{M}_t$ becomes positive. Then a new equilibrium with a constant number of firms will arise for a higher value of M_t^*. During the transition to the new equilibrium, technological progress can assume the values described in the previous paragraph depending on the distance between the leader and the follower.

Finally, if $x_{jt} > \overline{A_{jF}} \quad \forall t \in [2, \infty)$, provided that $\dot{M}_t = 0$, we can determine the rate of technological progress by substituting $\dot{x}_{jt}$ from equation (44) in equation (36) and then we have:

$$a_{Jt}\left(x_{jt} > \overline{A_{jF}}\right) = \frac{\dot{x}_{jt}}{A_{jLt}} = E_t\left[\frac{\pi_{jE\,t+1}}{\overline{\overline{\pi}}_{jt}}\right]\frac{\varphi}{\chi}\eta L_{Jt}\left(\frac{\dot{K}_{jRt}}{K_{jRt}} + \frac{\dot{L}_{jRt}}{L_{jRt}}\right)\frac{x_{jt}}{A_{jLt}} \qquad (59)$$

where the expected profit ratio may assume the following values:

$$E_t\left[\frac{\pi_{jE\,t+1}}{\tilde{\pi}_{jt}}\right] = \left\{\frac{M_t}{M_t + 1}; 1; 1 + \frac{1}{2}\frac{\left(x_{jt+1} - \overline{A}_{jF}\right)K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta}}{\overline{A}_{jF}K_{jt}^{\alpha+\beta}L_{jt}^{\gamma+\delta} - w_t L_{jt} - r_t K_{jt}}\right\}$$

Provided that the sum of the growth rates of research capital and research labour is positive, the rate of technological rate is positive and it is an increasing function of the free-entry variable η, of the number of workers in the industry L_{Jt} and of the expected profit ratio $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$. Also in this case, competition for the market (that is higher value of η because of the reduction of entry barriers) determines an increase of technological progress, while competition in the market (that is lower value of the expected profit ratio $E_t\left[\pi_{jEt+1}/\tilde{\pi}_{jt}\right]$ because of lower technological distance between the leader and the follower) implies a reduction of technological progress.

3.10 Final Remarks on the Effects of Competition Policy in High-Technology Industries

The study of the theoretical framework for high-technology industries has shown that competition policy may produce different effects depending on the pursued objectives and of the employed instruments. In fact, competition for the market generally produces a positive effect on technological progress because it increases the number of firms in a given market and then stimulates the investments in research and development. On the opposite, competition in the market may have a negative impact on technological progress, because it reduces the technological advantage of the leader and then eliminates the incentive to invest in innovation.

These results, firstly obtained for a given dynamics (positive or negative) of market structure, have been confirmed for an equilibrium dynamics of the industry, which requires a constant number of firms in the industry.

In this analysis, disaggregated by industry, the target variable is the rate of technological progress, rather than the growth rate of output. In fact, we are interested in observing the endogenous determinants of technological progress and, for this purpose, the variations in capital or labour input are less relevant, while the incentives explaining the investments in research and development are much more important.

4. Low-Technology Industries

After analyzing the theoretical framework for high-technology industries, we are now interested in studying the growth process in low-technology industries. There the production process doesn't require necessarily a specific innovation activity because the existing firms can produce their goods or services also by using a long-standing technology: nevertheless, they can also improve their productivity by investing in new production technologies. In particular, they can spend a given amount c_{it} in order to modernize their production processes and then this increases their technology level A_{it} by $z(c_{it})$, where z_{it} is an increasing and concave function of c_{it}. The firms operating in low-technology industries don't need to manage directly a research activity because they can buy the license on a new technology from the developers.

In this model, we will not consider the functioning of the research sector, which elaborates the new ideas to be implemented in process innovations: we will simply assume that this is a perfectly competitive sector and that at given time t it supplies the producers with a technology advancement z_{it}, which can be acquired through the payment of an amount c_{it}. Not all the firms are interested in purchasing the new available technology and in fact some of them keep their previous technology. The firms which decide to buy such license obtain a technological advantage in the industry and then can produce a higher amount of output: in particular, more than one firm can buy the new technology at the same cost and so many firms get such technological leadership [5].

[5] Clearly, in low-technology industries we cannot use the idea of technological leadership in the same way as for high-technology industries, where it implies a monopoly in the exploitation of the process innovation. In that case the exclusivity in the usage of the new technology is due to the need to reward the research effort of the leader firm and to compensate the high costs related to the innovation activity. In fact, the monopoly rent gained by the innovator, which provides the most important incentive for conducting research activity, has to be extracted for an adequate period of time in order to justify a relevant investment in R&D as profitable.

In any case, the technological step between the innovators and the other firms in time t cannot be higher than the difference z_{it}-z_{it-1}: indeed, when the innovators improve their technology by z_{it}, the other firms adopt by imitation the technology corresponding to z_{it-1}. It means that in this case barriers to technology diffusion are sensibly lower than in high-technology industries: in fact, since technology is less relevant in determining the success or the survival of a given firm, the innovator is not interested in protecting the licensed innovation for more than one period, by adopting exclusionary practices towards the competitors. Moreover, since the limitation in the usage of a given technology would not produce as a result the exit of any firm from the market, the exclusionary purpose would not be achieved.

4.1 The Production Function in Low-Technology Industries

In low-technology industries, which we denote by subscript *i*, each firm *n* produces by using capital K_{int} and labour L_{int} and by exploiting a technology level A_{int} . So the production function of a firm *n* in industry *i* at time *t* has the following Cobb-Douglas form:

$$Y_{int} = A_{int} K_{int}^{\zeta} L_{int}^{1-\zeta} \qquad (60)$$

where $0 < \zeta < 1$ and $\zeta + (1-\zeta) = 1$.

The technology level of a producer is defined as:

$$A_{int} = A_{i0} + z_{int} \qquad (61)$$

where $z_{int} = z(c_{int}) = c_{int}^{\vartheta}$ and $0 < \vartheta < 1$, such that $z'(c_{int}) = \vartheta c_{int}^{\vartheta-1} > 0$ and $z''(c_{int}) = (\vartheta - 1)\vartheta c_{int}^{\vartheta-2} < 0$. The value of the parameter ϑ indicates the measure by which the expenditure for investments effectively determines a technological improvement for the producer. Just to make an example, let consider the extreme values, which are not included in the interval for ϑ: in particular, if $\vartheta = 0$, investment expenditure doesn't imply any progress in the technology level (in fact, whatever is the positive value of c_{it}, z_{int} is always equal to 1); on the contrary, if $\vartheta = 1$, investment expenditure determines without any waste of resources a corresponding advancement in the technology level. In this model, we define for all firms a fixed value of ϑ, which is higher than 0 but lower than 1: this means that in any case there is some waste in the implementation of the new technologies, due to some features of the industry (such as the lack of adequate human capital for adopting such process innovations). In general, there could also be some determinants of ϑ, which are specific for each firm: but for simplicity, we will assume that ϑ has the same value for all the firms belonging to a given industry.

Each firm solves the following profit maximization problem:

$$\max_{K_{int},L_{int},c_{int}} \pi_{int} = p_i\left\{\underbrace{\left[A_{i0}+z(c_{int})\right]}_{A_{int}}K_{int}^{\zeta}L_{int}^{1-\zeta}\right\}-\left(w_{it}L_{int}+r_{it}K_{int}+c_{int}\right) \qquad (62)$$

where p_i is the price of the final good produced by the firms in industry i, which is assumed to be constant over time.

The FOCs for the profit maximization problem are the following ones:

$$\frac{\partial \pi_{int}}{\partial K_{int}} = 0 \Leftrightarrow p_i\zeta A_{int}K_{int}^{\zeta-1}L_{int}^{1-\zeta} = r_{it} \Leftrightarrow p_i\zeta A_{int}k_{int}^{\zeta-1} = r_{it} \qquad (63)$$

$$\frac{\partial \pi_{int}}{\partial L_{int}} = 0 \Leftrightarrow p_i(1-\zeta)A_{int}K_{int}^{\zeta}L_{int}^{-\zeta} = w_{it} \Leftrightarrow p_i(1-\zeta)A_{int}k_{int}^{\zeta} = w_{it} \qquad (64)$$

$$\frac{\partial \pi_{int}}{\partial c_{int}} = 0 \Leftrightarrow p_iK_{int}^{\zeta}L_{int}^{1-\zeta} = \frac{1}{z'(c_{int})} \Leftrightarrow p_ik_{int}^{\zeta}L_{int} = \frac{1}{z'(c_{int})} \qquad (65)$$

where k_{int} is the capital-labour ratio, that is the amount of capital per worker.

Using the above FOCs for capital and labour as well as for innovation expenditure, we can draw some conclusions about the impact of new technologies on the optimal amount of production inputs.

Combining (63) and (65), we obtain that:

$$\frac{1}{\zeta}\underbrace{\frac{r_{it}}{A_{int}}}_{\bar{r}_i}K_{int} = \frac{1}{z'(c_{int})} \qquad (66)$$

where $\bar{r}_i$ is the technology-adjusted interest rate, that is the ratio between interest rate r_{it} and the technology level A_{int}, which is assumed to be constant over time. In particular:

$$\bar{r}_i = \frac{r_{it}}{A_{int}} = p_i\,\zeta\,k_{int}^{\zeta-1}$$

where p_i and ζ are constant by definition. Then this implies that also capital per worker k_{int} is constant over time: so, even if capital and labour increase, the ratio is unchanged.

From equation (66), rearranging terms, we can compute the optimal quantity of capital:

$$K_{int} = \zeta\frac{A_{int}}{r_{it}}\frac{1}{z'(c_{int})} \qquad (67)$$

We can notice that, if the expenditure for investment in new technologies c_{int} increases, assumed that the technology-adjusted interest rate is constant, the optimal amount of capital K_{int} is expected to augment. In fact:

$$c_{int}\uparrow \;\Rightarrow z'(c_{int})\downarrow \;\Rightarrow \frac{1}{z'(c_{int})}\uparrow \;\Rightarrow K_{int}\uparrow$$

Combining equations (64) and (65), we observe that:

$$\frac{1}{1-\zeta}\underbrace{\frac{w_{it}}{A_{int}}}_{\overline{w_i}}L_{int} = \frac{1}{z'(c_{int})} \qquad (68)$$

where $\overline{w_i}$ is the technology-adjusted wage rate, that is the ratio between wage rate w_{it} and the technology level A_{int}, which is assumed to be constant over time. In particular:

$$\overline{w_i} = \frac{w_{it}}{A_{int}} = p_i(1-\zeta)k_{int}^{\zeta}$$

where p_i and ζ are constant by definition. Then this implies that also capital per worker k_{int} is constant over time: so, even if capital and labour increase, the ratio is unchanged.

From equation (68), rearranging terms, we can determine the optimal amount of labour:

$$L_{int} = (1-\zeta)\frac{A_{int}}{w_{it}}\frac{1}{z'(c_{int})} \qquad (69)$$

We can notice that, if the amount of innovation expenditure c_{int} increases, assumed that the technology-adjusted wage rate is constant, the optimal amount of labour L_{int} is expected to augment. In fact:

$$c_{int}\uparrow \;\; \Rightarrow z'(c_{int})\downarrow \;\; \Rightarrow \frac{1}{z'(c_{int})}\uparrow \;\; \Rightarrow L_{int}\uparrow$$

Using the solution of the profit maximization problem, we have shown that the optimal amount of capital and labour employed in the production process is greater when the technology level of the firm is higher. This is relevant for the computation of the aggregate production function in a low-technology industry, when it includes some innovative firms with an advanced technology and some laggard firms with a standard technology. In fact, as demonstrated in equations (67) and (69), firms with higher technology level also employ a larger amount of capital and labour for the production of final output.

4.2 The Profitability Condition for Investments in New Technologies

A firm is willing to invest in new technologies insofar as this investment decision can guarantee a post-innovation profit higher than or at least equal to the pre-innovation profit. This means that the difference between the profit obtained in time *t+1* thanks to a technology A_{it+1} and the profit gained in time t with the technology A_{it} has to be non-negative, as indicated in the following inequality:

$$b_{int+1} = \pi(A_{int+1}) - \pi(A_{int}) \geq 0 \qquad (70)$$

This means that the rise in earnings, due to the higher output produced by the firm, has at least to compensate the increase in costs, due to the investments in new technologies. Moreover, this additional profit has to be obtained in period t+1, given that after one period the new technologies are publicly available and then some other firms may exploit the same technology without paying any innovation cost. So, in order to justify the profitability of such investment decision, the additional profit obtained in time t+1 has to be equal to or higher than 0.

Substituting the expressions for the profit functions $\pi(A_{int+1})$ and $\pi(A_{int})$ from equation (62), the profitability condition can be expressed as follows:

$$b_{it+1} = p_i\left\{\left[A_{i0}+z(c_{it+1})\right]K_{it+1}^{\zeta}L_{it+1}^{1-\zeta}\right\} - \left(w_{it+1}L_{it+1}+r_{it+1}K_{it+1}+c_{it+1}\right) - p_i\left\{\left[A_{i0}+z(c_{it})\right]K_{it}^{\zeta}L_{it}^{1-\zeta}\right\} + \left(w_{it}L_{it}+r_{it}K_{it}+c_{it}\right) \ge 0$$

Replacing the optimal values of capital and labour from equations (67) and (69) and rearranging terms, we obtain:

$$b_{it+1} = p_i\left[\left(\frac{\zeta}{\overline{r_i}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{w_i}}\right)^{1-\zeta}\right]\left\{A_{i0}\left[\frac{1}{z'(c_{it+1})}-\frac{1}{z'(c_{it})}\right]+\left[\frac{z(c_{it+1})}{z'(c_{it+1})}-\frac{z(c_{it})}{z'(c_{it})}\right]\right\}-\left[\frac{A_{i0}+z(c_{it+1})}{z'(c_{it+1})}-\frac{A_{i0}+z(c_{it})}{z'(c_{it})}\right]-\left(c_{it+1}-c_{it}\right)\ge 0$$

Then, we substitute the value $z(c_{it})=c_{it}^{\vartheta}$ and $z(c_{it+1})=c_{it+1}^{\vartheta}=x^{\vartheta}c_{it}^{\vartheta}$, such that $z'(c_{it})=\vartheta c_{it}^{\vartheta-1}>0$ $z'(c_{it+1})=\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}>0$, and we assume that the basic level of technology is $A_{i0}=1$. Moreover, recalling that $c_{it+1}=xc_{it}$ and moving the variation in investment costs to the RHS of the inequality, we can write:

$$p_i\left[\left(\frac{\zeta}{\overline{r_i}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{w_i}}\right)^{1-\zeta}\right]\left\{\left[\frac{1}{\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}}-\frac{1}{\vartheta c_{it}^{\vartheta-1}}\right]+\left[\frac{x^{\vartheta}c_{it}^{\vartheta}}{\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}}-\frac{c_{it}^{\vartheta}}{\vartheta c_{it}^{\vartheta-1}}\right]\right\}-\left[\frac{1+x^{\vartheta}c_{it}^{\vartheta}}{\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}}-\frac{1+c_{it}^{\vartheta}}{\vartheta c_{it}^{\vartheta-1}}\right]\ge c_{it}(x-1)$$

Rearranging and simplifying terms, we have:

$$\frac{p_i}{\vartheta}\left\{\left[\left(\frac{\zeta}{\overline{r_i}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{w_i}}\right)^{1-\zeta}\right]-1\right\}\left[c_{it}^{-\vartheta}\left(x^{1-\vartheta}-1\right)+(x-1)\right]\ge(x-1)$$

Given that p_i is a constant by definition, we can assume that $p_i=1$, then it figures as a numéraire. Finally the profitability condition is expressed as follows:

$$\left\{\left[\left(\frac{\zeta}{\overline{r_i}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{w_i}}\right)^{1-\zeta}\right]-1\right\}\left[c_{it}^{-\vartheta}\frac{x^{1-\vartheta}-1}{x-1}+1\right]\ge\vartheta \qquad (71)$$

In this condition, only one variable is endogenously determined by a decision of a firm, as a solution of the profit maximization problem, that is the amount of investments in new technologies c_{it}. All the other variables are given as constant: in particular, ζ and $1-\zeta$ are the parameters of the Cobb-Douglas production function, $\overline{r_i}$ is the technology-adjusted interest rate and $\overline{w_i}$ is the technology-adjusted wage rate; moreover, x is the multiplicative factor denoting the increase of investment expenditure c_{it} from t to $t+1$, while ϑ is the exponential parameter indicating the impact of investment expenditure c_{it} in terms of technological improvement z_{it}. For this reason, we are interested in defining the amount of investment in new technologies $\tilde{c}_{it}$, or equivalently the corresponding technological improvement $\tilde{z}_{it}$, which satisfies such profitability condition. In particular, rearranging terms from the inequality (71), we obtain the following result for $\tilde{z}_{it} = \tilde{c}_{it}^{\vartheta}$:

$$\tilde{z}_{it} = \tilde{c}_{it}^{\vartheta} \geq \left(\frac{x^{1-\vartheta} - 1}{x - 1} \right) \left[\frac{\left(\frac{\zeta}{\overline{\overline{r_i}}} \right)^{\zeta} \left(\frac{1-\zeta}{\overline{\overline{w_i}}} \right)^{1-\zeta} - 1}{1 + \vartheta - \left(\frac{\zeta}{\overline{\overline{r_i}}} \right)^{\zeta} \left(\frac{1-\zeta}{\overline{\overline{w_i}}} \right)^{1-\zeta}} \right] \qquad (72)$$

This means that an investment in new technologies can be profitable if the firm, allocating an amount of expenditure equal to or larger than $\tilde{c}_{it}$, obtains a technological advantage equal to or higher than $\tilde{z}_{it}$. When the distance between the leader and the follower is sufficiently relevant, the innovator can expect to obtain a post-innovation profit higher than pre-innovation profit and then to recover the costs related to the implementation of the new technologies. And given that the technological advantage depends anyway on the investment expenditure, even if it doesn't determine a proportional improvement in the technology level, a firm can foresee its future technological advantage and then evaluate the profitability of the investment also on the basis of the amount of such expenditure. In fact, on one hand a too large expenditure may imply difficulties in recovering the costs, but on the other hand a too limited expenditure can be insufficient for guaranteeing an adequate and profitable technological improvement.

4.3 Aggregate Output, Growth Rate and Market Competition in Neck-and-neck Industries

We are interested in computing the aggregate production function, by aggregating the output for all the firms in a given industry. From the technological point of view, we can distinguish two cases, depending on whether all the firms have the same technology level or some innovators present a technology level higher than the other firms. Recalling a terminology already used in the endogenous growth literature, and in particular in the paper by Aghion, Bloom, Blundell, Griffith and Howitt (2005), we can distinguish low-technology industries in two categories: neck-and-neck industries and unlevelled industries.

Firstly, we compute the aggregate production function for neck-and-neck industries:

$$Y_{It}\left(A_{int} = A_{it}\right) = \sum_{1}^{N_t} A_{int} K_{int}^{\zeta} L_{int}^{1-\zeta} \qquad (73)$$

Recalling from equation (61) that $A_{int} = A_{i0} + z_{int}$ and given that in a neck-and-neck industry the innovation level of all firms is the same, that is $z(c_{int}) = z(c_{it})$ we have:

$$Y_{It}(z_{int} = z_{it}) = A_{i0}\sum_{1}^{N_t} K_{int}^{\zeta} L_{int}^{1-\zeta} + \sum_{1}^{N_t} z(c_{it}) K_{int}^{\zeta} L_{int}^{1-\zeta}$$

Substituting the optimal values of K_{int} and L_{int} from equations (67) and (69) and rearranging terms, we obtain:

$$Y_{It}(z_{int} = z_{it}) = A_{i0}\sum_{1}^{N_t}\left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta}\frac{1}{z'(c_{it})} + \sum_{1}^{N_t} z(c_{it})\left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta}\frac{1}{z'(c_{it})}$$

We know that $A_{it} = A_{i0} + z_{it}$. Then, substituting the value $z_{it} = z(c_{it}) = c_{it}^{\vartheta}$, such that $z'(c_{it}) = \vartheta c_{it}^{\vartheta-1} > 0$, and assuming that the basic level of technology is $A_{i0} = 1$, we write

$$Y_{It}(z_{int} = z_{it}) = \sum_{1}^{N_t}\left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta}\frac{1}{\vartheta c_{it}^{\vartheta-1}} + \sum_{1}^{N_t}\left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta}\frac{c_{it}^{\vartheta}}{\vartheta c_{it}^{\vartheta-1}}$$

Given that the considered variables have the same value for each firm, we can substitute the summation operator by a multiplicative factor N_t . So rearranging terms we obtain the aggregate production function for low-technology industries when all firms have the same technology level:

$$Y_{It}(z_{int} = z_{it}) = \left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta} N_t \frac{c_{it}}{\vartheta}\left(c_{it}^{-\vartheta} + 1\right) \qquad (74)$$

Taking logs and deriving with respect to time, after some manipulations we obtain the following growth rate of output in low-technology industries:

$$\frac{\dot{Y}_{It}}{Y_{It}} = \frac{\dot{N}_t}{N_t} + \frac{\dot{c}_{it}}{c_{it}}\left[1 - \frac{\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta} + 1}\right] \qquad (75)$$

Now we want to determine the growth rate of expenditure in new technologies c_t. Let assume that in discrete time:

$$c_{t+\upsilon} = x^{\upsilon} c_t \qquad \text{where } 1 < x < 2 \qquad (76)$$

which implies, for example, that $c_{t+1} = x c_{it}$. Then the growth of investment expenditure in discrete time from t to $t+\upsilon$ can be indicated as:

$$\frac{c_{t+\upsilon} - c_t}{c_t} = \frac{(x^{\upsilon} - 1)c_{it}}{c_{it}}$$

Dividing the time interval by υ periods and taking the limit for υ in order to compute the growth rate, we obtain an undetermined form:

$$\lim_{\upsilon\to 0}\frac{\frac{c_{t+\upsilon} - c_t}{\upsilon}}{c_t} = \lim_{\upsilon\to 0}\frac{x^{\upsilon} - 1}{\upsilon} = \frac{0}{0}$$

Then, by applying the Hôpital rule, we get the following outcome:

$$\lim_{\upsilon\to 0}\frac{\frac{c_{t+\upsilon}-c_t}{\upsilon}}{c_t}=\lim_{\upsilon\to 0}\frac{\frac{\partial\left[x^{\upsilon}-1\right]}{\partial\upsilon}}{\frac{\partial\upsilon}{\partial\upsilon}}=\ln x\; x^{\upsilon} \qquad (77)$$

Then, substituting such result for the growth rate of investment expenditure, the growth rate of output for low-technology industries is equal to:

$$\frac{\dot{Y}_{It}}{Y_{It}}=\frac{\dot{N}_t}{N_t}+\ln x\cdot x^t\left[1-\frac{\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta}+1}\right] \qquad (78)$$

Now we are interested in studying the impact of new technology on the growth rate of output. Then we derive the growth rate with respect to c_{it}:

$$\frac{\partial\gamma_Y}{\partial c_{it}}=\ln x\cdot x^t\,\frac{\vartheta^2 c_{it}^{-\vartheta-1}}{\left(c_{it}^{-\vartheta}+1\right)^2}>0 \qquad (79)$$

This result[6] shows that an increase of the expenditure in new technologies produces a positive impact on the growth rate of industry i, when all the firms have the same technology level. This result is quite intuitive on the basis of the assumptions of the model: provided that the investment expenditure c_{it} determines the technological improvement z_{it} and so the technology level A_{it}, and given that in neck-and-neck industries aggregate output is simply a N-multiple of the output of the individual firm, an increase of the technology level for all the firms has a positive effect on the growth rate of output in industry i. But this outcome has also an important policy implication: in fact, when the market structure of a low-technology industry is characterized by the existence of many firms, all of them using the same production technology, a growth-enhancing policy should encourage the expenditure for investment in new technologies. In fact, when all the firms share the same technology level, the industry is quite competitive but this doesn't mean necessarily that the firms are induced to invest in innovation, because such decision depends on a profitability condition, which requires that $b_{int+1}=\pi\left(A_{int+1}\right)-\pi\left(A_{int}\right)\geq 0$, that is the variation in the profit due to the acquisition of the new technology must be non-negative. So, for example, even in a fairly competitive industry, a policy intervention aimed at subsidizing the investments in new technologies could be useful in order to improve the growth rate of the industry. In this case, if a part of this expenditure is subsidized by the government, the firm will pay an investment cost $(1-\varepsilon)c_{it}$, while the technological improvement for the production function would be in any case equal to $z(c_{it})$. In this way, the profitability condition for investments in new technologies could be more easily satisfied and this could increase technological progress in the industry.

[6] The derivation of this result is presented in the Appendix (A.1)

4.4 Aggregate Output, Growth Rate and Market Competition in Unlevelled Industries

Now we want to compute the aggregate production function for unlevelled industries:

$$Y_{It}\left(A_{iLt} > A_{iFt}\right) = \sum_{1}^{D_t} A_{iLt} K_{iLt}^{\zeta} L_{iLt}^{1-\zeta} + \sum_{D_t}^{N_t} A_{iFt} K_{iFt}^{\zeta} L_{iFt}^{1-\zeta} \qquad (80)$$

where D_t is the total number of leaders, which have a higher technological level, while N_t is the total number of firms in low-technology industries, such that N_t-D_t is the total number of followers, whose technology level is lower by one step. Recalling from equation (61) that $A_{int} = A_{i0} + z_{int}$ and given that in an unlevelled industry the innovation level of the various firms is different, such that $z(c_{iLt}) > z(c_{iFt})$ where $z(c_{iFt}) = z(c_{it}) = c_{it}^{\vartheta}$ and $z(c_{iLt}) = z(c_{it+1}) = c_{it+1}^{\vartheta}$ (with $c_{it+1} = xc_{it}$ and $1 < x < 2$), we have:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = A_{i0}\sum_{1}^{D_t} K_{iLt}^{\zeta} L_{iLt}^{1-\zeta} + \sum_{1}^{D_t} z\left(c_{it+1}\right) K_{iLt}^{\zeta} L_{iLt}^{1-\zeta} + A_{i0}\sum_{D_t}^{N_t} K_{iFt}^{\zeta} L_{iFt}^{1-\zeta} + \sum_{D_t}^{N_t} z\left(c_{it}\right) K_{iFt}^{\zeta} L_{iFt}^{1-\zeta}$$

Then we substitute the optimal values of capital and labour from equations (67) and (69). In particular, as we have shown in the discussion of these results, we must take into account that, because of the higher technology level, also the optimal amount of capital and labour is higher for the innovative firms. Indeed, for the leader firms $K_{iLt} = K\left(A_{it+1}\right) = K_{it+1}$ and $L_{iLt} = L\left(A_{it+1}\right) = L_{it+1}$, while for the followers $K_{iFt} = K\left(A_{it}\right) = K_{it}$ and $K_{iFt} = K\left(A_{it}\right) = K_{it}$. After rearranging terms, we obtain:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = A_{i0}\left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta}\left[\sum_{1}^{D_t}\frac{1}{z'\left(c_{it+1}\right)} + \sum_{D_t}^{N_t}\frac{1}{z'\left(c_{it}\right)}\right] + \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta}\left[\sum_{1}^{D_t}\frac{z\left(c_{it+1}\right)}{z'\left(c_{it+1}\right)} + \sum_{D_t}^{N_t}\frac{z\left(c_{it}\right)}{z'\left(c_{it}\right)}\right]$$

We know that $A_{it} = A_{i0} + z_{it}$. Then, substituting the value $z\left(c_{it}\right) = c_{it}^{\vartheta}$ and $z\left(c_{it+1}\right) = c_{it+1}^{\vartheta} = x^{\vartheta}c_{it}^{\vartheta}$, such that $z'\left(c_{it}\right) = \vartheta c_{it}^{\vartheta-1} > 0$ $z'\left(c_{it+1}\right) = \vartheta x^{\vartheta-1}c_{it}^{\vartheta-1} > 0$, and assuming that the basic level of technology is $A_{i0} = 1$, we can write:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta}\left[\sum_{1}^{D_t}\frac{1}{\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}} + \sum_{D_t}^{N_t}\frac{1}{\vartheta c_{it}^{\vartheta-1}}\right] + \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta}\left[\sum_{1}^{D_t}\frac{x^{\vartheta}c_{it}^{\vartheta}}{\vartheta x^{\vartheta-1}c_{it}^{\vartheta-1}} + \sum_{D_t}^{N_t}\frac{c_{it}^{\vartheta}}{\vartheta c_{it}^{\vartheta-1}}\right]$$

Given that the considered variables have the same value for each firm included among the leaders or the followers, we can substitute the summation operator respectively by a multiplicative factor D_t and $\left(N_t - D_t\right)$. So rearranging terms we have:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta}\frac{1}{\vartheta}\left[D_t x^{1-\vartheta}c_{it}^{1-\vartheta} + \left(N_t - D_t\right)c_{it}^{1-\vartheta} + D_t x c_{it} + \left(N_t - D_t\right)c_{it}\right]$$

Let assume that the number of innovative firms is a constant fraction τ of the total number of firms N_t. Then, substituting D_t with τN_t and rearranging terms, we obtain the aggregate production function for low-technology industries when the firms are distinguished among leaders and followers:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta} \left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta} N_t \frac{c_{it}}{\vartheta} \left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1\right\} \qquad (81)$$

Taking logs and deriving with respect to time, after some manipulations we obtain the following growth rate of output in low-technology industries:

$$\frac{\dot{Y}_{It}}{Y_{It}} = \frac{\dot{N}_t}{N_t} + \frac{\dot{c}_{it}}{c_{it}} \left\{1 - \frac{\vartheta c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]}{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1}\right\} \qquad (82)$$

Using the same law of motion for the expenditure in new technologies as the one presented in equation (76), we know that $\dot{c}_{it}/c_{it} = ln\, x \cdot x^t$. Then, substituting such result, we obtain:

$$\frac{\dot{Y}_{It}}{Y_{It}} = \frac{\dot{N}_t}{N_t} + ln\, x \cdot x^t \left\{1 - \frac{\vartheta c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]}{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1}\right\} \qquad (83)$$

Now we are interested in examining how a variation of τ, that is the fraction of innovative firms in the industry, can have an effect on the growth rate of output in low-technology industries. In this framework, τ is an exogenous variable, which can be affected by a policy intervention, such as a decision of a competition authority. In particular, an increase in the access to a new technology, determined by the execution of an antitrust decision, may augment the number of the innovative firms, because in this way more firms can exploit a given process innovation, and then this enhances competition among the leader firms in that market: so a policy aimed at raising this variable τ can be considered as a policy promoting competition in the market.

Clearly, the market structure of a low-technology industry is quite different from the market structure of a high-technology industry. In that case, the industry is characterized by the dominance of a monopolistic firm, given that the process innovation x_{jt} is protected by a patent lasting for a long period, or however until the invention of a new technology x_{jt+1}; then the intervention of a competition authority may impose the sharing of such technology with the other firms in the market, with the consequence of disregarding the expected protection of the intellectual property. On the opposite, in low-technology industries, more than one firm can obtain the license on the new technology and in addition the other firms are able to adopt such technology in the following period. So the policy intervention can produce the effect of extending the set of firms having access to the new technology in time t and in this way it may determine a pro-competitive result.

In order to have a better intuition about the functioning of this competition policy in the market for low-technology industries, we could refer to some concrete issues in the antitrust practice on vertically fragmented industries, such as the industries that we are now examining. Let consider a market situation where some producers need to buy a given technology from the developers of a process innovation: in many cases, the producers of the final good may conclude some exclusive contracts with the intermediate firms providing the new technology, in order to limit their possibility to deal with other competitors and then to supply their technology to other firms. In this way, exclusive dealing may have an exclusionary effect towards the other producers of final good, which are compelled to use a previous and less efficient technology. Moreover, even if a single producer is not able to impose directly an exclusive contract to the supplier of a new technology, a limitation in the supply of such technological facility can be determined as a result of the collusion among the leader firms in the downstream industry: in fact, in order to keep their technological advantage in the industry and then to preserve the existing oligopoly, they want to avoid that other firms can acquire the usage of the new technology and then they can pursue this objective also thanks to a collective agreement with the technology providers.

As we have seen, in both cases the technological advantage of some firms in the unlevelled industries is not determined by their efficiency, but by the adoption of an anti-competitive conduct. In such situations, the antitrust authority can oblige the developers of the new technology to provide a compulsory license, against the payment of a given fee, to all the producers interested in using such innovation. Cases like this are very frequent in antitrust practice: quite often competition authorities have to evaluate the exclusionary impact, on the downstream market, of exclusive contracts between one or more producers of a final good and a supplier of an intermediate product or service, such as a technological facility. In such context, a compulsory licensing decision has the effect of improving competition in the market, because it is aimed at ensuring the same competitive conditions to all the firms operating in that industry.

The presentation of these paradigmatic cases for the implementation of a competition policy in the market provides the right intuition on the real world in order to reconsider and clarify the analysis of our theoretical framework. In particular, in order to observe the impact of the competition policy in the market, we have to derive the growth rate of output for industry *i* with respect to the variable τ :

$$\frac{\partial \gamma_{Y_{It}}}{\partial \tau} = ln\, x \cdot x^{t} \cdot \vartheta c_{it}^{-\vartheta} \frac{\left(x - x^{1-\vartheta}\right)}{\left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta} - 1\right) + 1\right] + \tau\left(x - 1\right) + 1\right\}^{2}} > 0 \qquad (84)$$

The positive sign of the derivative[7] shows that in low-technology industries a policy aimed at increasing competition in the market has a beneficial impact on the growth rate of output. In fact, given that in any case more than one firm can gain access to the new technology by purchasing the license, the increase of τ doesn't change the incentive of the existing firms to acquire the new technology. Indeed, as discussed in par. 3.2, such decision is adopted if it satisfies a profitability

[7] The derivation of this result is provided in the Appendix (A.2)

condition, which simply requires that $b_{int+1} = \pi(A_{int+1}) - \pi(A_{int}) \geq 0$, that is the variation in the profit due to the acquisition of the new technology must be non-negative. So, in this case the number of firms sharing the same technology doesn't affect the profit of the individual firm, because the additional profit is determined by the usage of the new technology and not by the market power of the firm. In conclusion, a competition policy in the market may have a positive impact on the growth rate for low-technology industries because it fosters the investments in innovation by the follower firms and then promotes technological progress at the industry level, without affecting at all the incentives for innovation of the leader firms.

4.5 A Comparison of the Growth Rates in Unlevelled and Neck-And-Neck Industries

After analyzing the aggregate production function and the growth rate of output for unlevelled and neck-and-neck industries, we are interested in examining which market structure is able to ensure the highest growth rate for a low-technology industry. So let compare the results obtained under equation (78) for neck-and-neck industries and equation (82) for unlevelled industries.

$$\gamma_{Y_{It}}(A_{int} = A_{it}) = \frac{\dot{N}_t}{N_t} + ln\, x \cdot x^t \left[1 - \frac{\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta} + 1}\right] \gtrless \frac{\dot{N}_t}{N_t} + ln\, x \cdot x^t \left\{1 - \frac{\vartheta c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta} - 1\right) + 1\right]}{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta} - 1\right) + 1\right] + \tau(x-1) + 1}\right\} = \gamma_{Y_{It}}(A_{iLt} > A_{iFt})$$

Comparing the considered equations, we observe[8] that:

$$\gamma_{Y_{It}}(A_{int} = A_{it}) > \gamma_{Y_{It}}(A_{iLt} > A_{iFt}) \quad (85)$$

Then, in low-technology industries the growth rate of output is higher if the industry presents a neck-and-neck structure rather than if the industry has an unlevelled composition with some leaders and some followers. In particular, in neck-and-neck industries all the firms share a standard technology level $A_{it} = A_{i0} + z_{it} = A_{i0} + c_{it}^{\vartheta}$, while in unlevelled industries the innovative firms use an advanced technology $A_{it+1} = A_{i0} + z_{it+1} = A_{i0} + x^{\vartheta} c_{it}^{\vartheta}$ and the laggard firms employ a standard technology $A_{it} = A_{i0} + z_{it} = A_{i0} + c_{it}^{\vartheta}$. This also implies that, in the aggregation of the production functions at industry level, the aggregate output is greater in unlevelled industries than in neck-and-neck industries.

This outcome, that is greater aggregate output but lower growth rate in unlevelled industries and vice versa in neck-and-neck industries, can be explained because of decreasing marginal returns of the investments in new technologies. In the model, this is due to the value of the parameter ϑ in the function of technological improvement $z(c_{int})$: provided that $z(c_{int}) = c_{int}^{\vartheta}$, a value of ϑ lower than

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[8] The proof of such result is presented in the Appendix (A.3)

1 $(0 < \vartheta < 1)$ implies that the implementation of a new technology generates decreasing marginal returns for the production of final output. Then the adoption of a new technology augments the output of the individual firm as well as the aggregate output of the industry (according to the composition of the industry), but this marginal increase is lower for further improvements of the technology level A_{int} .

This explanation is also confirmed by the fact that, if we assume constant marginal returns from technology adoption, that is if we consider a value of $\vartheta = 1$, we notice that the result is reversed: then the growth rate of output is higher in unlevelled industries than in neck-and-neck industries. The results on aggregate output and growth rate of industry *i* in the case of constant marginal returns from investments in technology are presented in the Appendix (A.4).

4.6 The Dynamics of Market Size in Low-Technology Industries

An important determinant of the growth rate of output in low-technology industries, both in the unlevelled ones and in the neck-and-neck ones, is the growth rate of the total number of firms N_t in the industry. For this reason, we are interested in examining the dynamics of market size, in order to understand what type of policy may affect the total number of firms in industry *i* and then the growth rate of output.

As discussed in par. 2.6, the variation in time *t* of the total number of firms in industry *i* is given by the difference between the amount of entrant firms and the quantity of exiting firms. In particular, the law of motion of N_t is defined as follows:

$$\dot{N}_t = \varphi \eta L_{It} - \chi N_t \qquad (86)$$

where φ is the parameter of a Poisson distribution indicating the hazard rate of entry for new firms; η is a variable denoting the type of barriers to entry, such that $0< \eta<1$, where a value $\eta=1$ defines a completely free-entry situation while $\eta=0$ means no entry possibility in the industry; L_{It} is the total amount of workers, such that each of them, exploiting the standard technology available in the industry, can become an entrepreneur and start a new firm; χ is a parameter of a Poisson distribution representing the hazard rate of exit for the existing firm.

As we can notice from the comparison with equation (37), presenting the law of motion of M_t for high-technology industries, the determinants of entry and exit are very similar: in particular, a reduction in barriers to entry increases the value of η and then encourages more firms to enter the market; moreover, a scale effect is associated to the amount of labour force L_{It} in the industry. However, we can observe an important difference: in low-technology industries, the expected profit ratio for the entrant is not included among the variables affecting the number of the new firms. This is because the expectation of high monopolistic profits, coming from the exploitation of a patent, is essentially a specific feature of high-technology industries.

Given the law of motion for N_t, we are interested in determining the growth rate of output in low-technology industries, by substituting the expression for $\dot{N}_t$ in the corresponding equations for neck-and-neck and unlevelled industries.

From equation (78), the growth rate in neck-and-neck industries is given by:

$$\gamma_{Y_{It}}\left(A_{int} = A_{it}\right) = \frac{\dot{Y}_{It}}{Y_{It}} = \underbrace{\left(\varphi\eta L_{It} - \chi N_t\right)}_{=\dot{N}_t} + ln\, x \cdot x^t \left[1 - \frac{\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta} + 1}\right] \qquad (87)$$

We want to study the impact of a policy aimed at reducing entry barriers on the growth rate of output. Then, deriving $\gamma_{Y_{It}}\left(A_{int} = A_{it}\right)$ with respect to η, we obtain:

$$\frac{\partial \gamma_{Y_{It}}}{\partial \eta} = \varphi L_{It} > 0$$

The positive sign of the derivative shows that competition for the market has a beneficial effect on growth, because the potential competition due to the higher number of producers induces firms to invest more in the adoption of new technologies.

From equation (83), the growth rate in unlevelled industries is equal to:

$$\gamma_{Y_{It}}\left(A_{iLt} > A_{iFt}\right) = \frac{\dot{Y}_{It}}{Y_{It}} = \underbrace{\left(\varphi\eta L_{It} - \chi N_t\right)}_{=\dot{N}_t} + ln\, x \cdot x^t \left\{1 - \frac{\vartheta c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta} - 1\right) + 1\right]}{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta} - 1\right) + 1\right] + \tau\left(x - 1\right) + 1}\right\} \qquad (88)$$

Also in unlevelled industries, a competition policy for the market, designed to decrease the barriers to entry, produces a positive impact on the growth rate of output. In fact, deriving $\gamma_{Y_{It}}\left(A_{iLt} > A_{iFt}\right)$ with respect to η, we have:

$$\frac{\partial \gamma_{Y_{It}}}{\partial \eta} = \varphi L_{It} > 0$$

Given that the derivative of the growth rate of output with respect to the free-entry variable assumes the identical value in both cases, we can argue that competition for the market produces the same positive impact on growth both in a neck-and-neck industry and in an unlevelled one, provided that they present similar characteristics regarding the labour force or the hazard rate of entry. Then, regardless of the technological structure of industry i, a competition policy for the market has always the effect of enhancing the investments in new technologies.

In fact, if the industry is neck-and-neck, more competition for the market may induce an escape-competition effect, corresponding to the one presented in the paper by Aghion, Bloom, Blundell, Griffith and Howitt (2005): since the entry of new firms may reduce the pre-innovation rent of the existing firms, they are induced to improve their technology level in order to obtain the post-innovation rent, provided that the profitability condition is satisfied.

At the same time, if the industry is unlevelled, more competition for the market still produces the effect of fostering innovation because, if the profitability condition is fulfilled, both the leaders and the followers have incentive to implement the new technologies: the leaders are interested in acquiring the new process innovations in each period because in this way they can keep their technological advantage over time; the followers, once they have adopted by imitation the technology introduced by the leaders in the previous period, are induced to innovate in order to reach a technological leadership in the industry. Clearly, in such context, the existence of numerous potential competitors can simply reinforce the incentives for innovation for each firm, no matter whether it is leader or follower. So, with regard to unlevelled industries, this result is different from the outcome proposed in the article by Aghion, Bloom, Blundell, Griffith and Howitt (2005), where the leaders have no reason for further innovation, while the followers –which could be interested in adopting new technologies - have lower incentives to innovate in this case because more competition here reduces the post-innovation rent for the developers of a leading-edge technology. This is due to the use of different assumptions: in our framework, after one period, the technology diffusion implies that the followers may access to the previous leader's technology, even if the leader has not introduced any other innovation; on the contrary, in the model by Aghion, Bloom, Blundell, Griffith and Howitt (2005), the followers may adopt the previous leader's technology, only if the leader has adopted a further innovation.

4.7 Final Remarks on the Effects of Competition Policy in Low-Technology Industries

The analysis of low-technology industries has shown that competition policy, both in the market and for the market, always has a positive impact on the growth rate of the industry. In particular, we have distinguished low-technology industries in two categories, neck-and-neck and unlevelled industries, depending on the technology level of the firms.

Neck-and-neck industries, since they include many firms with the same technology level, are rather competitive: for this reason, they don't require a competition policy in the market but they could benefit from a competition policy for the market, aimed at reducing barriers to entry. In any case, the competitive structure of the industry doesn't guarantee per se an adequate investment in innovation: so, in certain cases, an industrial policy based on government subsidies could encourage the choice of some firms to adopt new technologies, allowing them to better satisfy the profitability condition.

Unlevelled industries are characterized by the existence of some leaders and some followers: there a policy designed to improve competition in the market can have a positive impact on growth by enlarging the diffusion of the existing technologies and also a competition policy for the market can encourage innovation both among the leaders and among the followers.

In this framework, the expenditure for investments in new technologies determines the technological advantage of the innovative firms, but the process of technology adoption presents decreasing marginal returns for the production of final output: this also explains why the growth rate of output is higher in neck-and-neck industries than in unlevelled industries.

5. Conclusions

The present paper analyzes the relationship between competition policy and economic growth in an economy with heterogeneous industries and with different types of competition policy. Then it aims at extending the existing literature on the topic in two directions.

Firstly, this analysis proposes a distinction between high-technology industries and low-technology industries, showing that competition policy may produce different effects depending on the type of industry. In this perspective, the separate treatment of different types of industries also requires the adoption of different assumptions regarding the organization of production and research activity, the market structure, the incentives for innovation in each type of industry. On the contrary, in the traditional models of endogenous growth, the final sector is assumed to be homogeneous and then the main hypotheses are uniformly applied to all the firms in the economy. But this can produce some misleading outcomes: in fact, insofar as such assumptions concern some key aspects of the issue, they can affect in a determinant way the results of the analysis. So, in order to avoid inappropriate conclusions, a reasonable approach to the problem is to study the relation between competition and growth by differentiating various types of industries. Indeed a theoretical analysis, disaggregated for different types of industries, is likely to offer more significant and useful conclusions than a generic investigation, focused on a unique model of industry, unable to explain the diversities existing in production and innovation decisions.

The second element of innovation proposed by this paper is the distinction between competition in the market and competition for the market. In fact, the literature on competition and growth has focused the attention on a notion of competition, which considers only the actual interaction among the existing firms in the market and then neglects the role of entry in determining potential competition. In particular, the entry threat plays a very important function in high-technology industries, where market structure is extremely dynamic, both because new firms may enter the industry thanks to a leapfrogging technology, and because the boundaries of the market are not clearly defined and are subject to a constant evolution. Moreover, the distinction among various types of competition policy is worthy of interest, because competition in the market and for the market operate in different ways, and in fact they can produce diverse effects on technological progress and economic growth. For this reason, such distinction is useful in order to capture the heterogeneity of the policy interventions and of their effects on the economy.

Using such framework, our analysis of the relation between competition policy and economic growth also provides some policy implications for the design and the implementation of antitrust policy in a growth-enhancing perspective. In particular, this study shows that a policy aimed at increasing competition for the market always produces a positive impact on innovation and growth, both in high-technology and in low technology industries. On the opposite, a policy designed to improve competition in the market may generate a positive effect in low-technology industries while a negative one in high-technology ones. The reason of this diversity is related to different incentives to innovate as well as to different periods of time needed to achieve an adequate reward for innovation. In high-technology industries, firms are induced to innovate because they are interested in obtaining the monopolistic profits due to the exploitation of a patent, but given the high costs of research activity they need to maintain such technological advantage for a relatively long period. On the opposite, in low-technology industries, firms innovate in time t if they expect that the additional profit gained in time $t+1$ is higher than the costs paid for the investments in new technologies; then, if this profitability condition is satisfied, that is if firms attain a non-negative profit for each period, they are willing to invest in innovation also in the following periods.

So the key issue is the expectation of an adequate reward for innovation, that we have modelled in high-technology industries as an expected profit ratio. A competition policy which compromises this return from innovation may discourage firms from running a research activity and then it can lower technological progress in the long-term, as it is the case for a policy aimed at improving competition in the market. Consequently, this result raises some doubts on the dynamic efficiency, in a long-run perspective, of those competition policies which forbid the abuse of dominance, and then punish the firms that take advantage of a monopolistic position, even if they have gained this monopoly power thanks to important innovations protected by a patent. In fact, if the disincentive effect due to competition is significantly strong, then the consequences of a reduced effort in innovation may be serious in an endogenous growth framework, where the technological progress depends also on the efforts of firms in the R&D sector.

For example, the implementation of antitrust policy in Europe shows that in many cases the abuse of dominance is defined and sanctioned no matter how a firm has obtained that position. Then, the key issue for the policy-maker is to judge whether a competition policy like this one can be detrimental for long-run growth and, in case, how this policy should be designed in order to avoid negative effects on economic growth. In particular, it is worth to pay specific attention to the issue of the intersection between antitrust policy and intellectual property protection: in fact, the approach adopted by the Antitrust Authorities on this point might require a revision, in the direction of introducing a specific consideration for IP protection as a criterion for evaluating (and eventually also excluding) the abuse of dominant position.

As a general rule, the protection of intellectual property has to be guaranteed for a given period of time as an appropriate recompense for innovation. But, after the expiration of the patent, the new technology has to be completely available to the other firms interested in using it. This implies that, in an innovating economy, all the barriers to technology diffusion which are not related to patent protection have to be removed. So, in this situation, antitrust policy can be useful in order to avoid that, after the expiration of the patent, the leader can exploit its advantage in order to adopt exclusionary practices against the competitors.

In any case, as the results of the model demonstrate, the best competition policy to be implemented in a high-technology industry is a policy designed to facilitate the entry of new firms in the market, through the reduction of previous entry barriers. In fact, in these industries the threat of entry by new innovating firms, since it increases potential competition, induces the incumbent firms and in particular the leader firm to invest more in research and development. Indeed, if other firms are expected to enter the market, the only way that the leader can keep its technological advantage and so its dominant position is to introduce further innovations.

In conclusion, the most important contribution of this work is that in high-technology industries competition policy can be detrimental for innovation and growth because it compromises the required remuneration for R&D investment. Of course, this doesn’t mean that antitrust authorities should completely renounce to intervene in these industries against anti-competitive practices, also because a monopoly always determines a deadweight loss and then a competition policy can anyway produce some gains in terms of static efficiency. Nevertheless, when a competition policy reduces the technological advantage of an innovator and then it affects the required compensation for an innovation effort, it could be useful, in a public policy perspective, to compare the gains in terms of static efficiency with the losses in terms of dynamic efficiency, in order to have a complete view of the effects. At the end, the trade-off between the two objectives may have different solutions depending on the features of the economy and on the preferences of the society. But, for instance, in a society with a low rate of time preference, an innovation policy aimed at increasing the rate of technological progress in the long-term could be more appropriate than a competition policy designed to achieve the highest possible static efficiency in the present.

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Appendix

A.1 Derivation of result (79)

Deriving the growth rate of output for low-technology unlevelled industries $\gamma_{Y_{It}}$ - where $Y_{It} = Y_{It}\left(z_{int} = z_{it}\right)$ - with respect to the amount of investment expenditure c_{it}, we obtain:

$$\frac{\partial \gamma_Y}{\partial c_{it}} = \ln x \cdot x^t \, \frac{\vartheta c_{it}^{-\vartheta-1}\left[-\left(1-\vartheta\right)\left(c_{it}^{-\vartheta}+1\right)+c_{it}^{-\vartheta}\left(1-\vartheta\right)+1\right]}{\left(c_{it}^{-\vartheta}+1\right)^2} > 0$$

Solving the operations in brackets, we finally have:

$$\frac{\partial \gamma_Y}{\partial c_{it}} = \ln x \cdot x^t \, \frac{\vartheta^2 c_{it}^{-\vartheta-1}}{\left(c_{it}^{-\vartheta}+1\right)^2} > 0$$

A.2 Derivation of result (84)

Deriving the growth rate of output for low-technology unlevelled industries $\gamma_{Y_{It}}$ - where $Y_{It} = Y_{It}\left(z_{iLt} > z_{iFt}\right)$ - with respect to the fraction τ of innovative firms in industry I, we obtain:

$$\frac{\partial \gamma_{Y_{It}}}{\partial \tau} = -\ln x \cdot x^t \cdot \vartheta c_{it}^{-\vartheta} \, \frac{\left(x^{1-\vartheta}-1\right)\left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1\right\}-\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]\left[c_{it}^{-\vartheta}\left(x^{1-\vartheta}-1\right)+\left(x-1\right)\right]}{\left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1\right\}^2}$$

In order to determine the sign of this derivative, we have to verify the sign of the following inequality:

$$\left(1-x^{1-\vartheta}\right)\left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1\right\} \gtrless -\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]\left[c_{it}^{-\vartheta}\left(x^{1-\vartheta}-1\right)+\left(x-1\right)\right]$$

Solving the operations in LHS and RHS, we obtain:

$$\tau x^{1-\vartheta}c_{it}^{-\vartheta} - \tau c_{it}^{-\vartheta} + c_{it}^{-\vartheta} + \tau x - \tau + 1 - \tau x^{2(1-\vartheta)}c_{it}^{-\vartheta} + \tau x^{1-\vartheta}c_{it}^{-\vartheta} - x^{1-\vartheta}c_{it}^{-\vartheta} - \tau x^{2-\vartheta} + \tau x^{1-\vartheta} - x^{1-\vartheta} \gtrless -\tau x^{2(1-\vartheta)}c_{it}^{-\vartheta} + \tau x^{1-\vartheta}c_{it}^{-\vartheta} +$$
$$- x^{1-\vartheta}c_{it}^{-\vartheta} + \tau x^{1-\vartheta}c_{it}^{-\vartheta} - \tau c_{it}^{-\vartheta} + c_{it}^{-\vartheta} - \tau x^{2-\vartheta} + \tau x - x + \tau x^{1-\vartheta} - \tau + 1$$

Simplifying terms in RHS and LHS and recalling that $1<x<2$ and $0 < \vartheta < 1$ we finally have:

$$x > x^{1-\vartheta}$$

So we can conclude that the derivative of $\gamma_{Y_{It}}$ with respect to τ is positive, that is:

$$\frac{\partial \gamma_{Y_{It}}}{\partial \tau} = \ln x \cdot x^t \cdot \vartheta c_{it}^{-\vartheta} \, \frac{\left(x - x^{1-\vartheta}\right)}{\left\{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau\left(x-1\right)+1\right\}^2} > 0$$

A.3 Proof of result (85)

We have to compare the growth rates of output for neck-and-neck and unlevelled industries. Then:

$$\gamma_{Y_{It}}\left(A_{int}=A_{it}\right)=\frac{\dot N_t}{N_t}+\ln x\cdot x^t\,\frac{c_{it}^{-\vartheta}+1-\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta}+1}\gtrless\frac{\dot N_t}{N_t}+\ln x\cdot x^t\left\{1-\vartheta\frac{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]}{c_{it}^{-\vartheta}\left[\tau\left(x^{1-\vartheta}-1\right)+1\right]+\tau(x-1)+1}\right\}=\gamma_{Y_{It}}\left(A_{iLt}>A_{iFt}\right)$$

In order to determine the sign of such inequality, we have to compare the two following expressions on LHS and RHS:

$$\frac{c_{it}^{-\vartheta}+1-\vartheta c_{it}^{-\vartheta}}{c_{it}^{-\vartheta}+1}-1\gtrless\frac{-\vartheta c_{it}^{-\vartheta}\left[\tau x^{1-\vartheta}-\tau+1\right]}{c_{it}^{-\vartheta}\left[\tau x^{1-\vartheta}-\tau+1\right]+\tau x-\tau+1}$$

Solving the operations in LHS and RHS, we obtain:

$$-\frac{1}{c_{it}^{-\vartheta}+1}\gtrless-\frac{\tau x^{1-\vartheta}-\tau+1}{\tau x^{1-\vartheta}c_{it}^{-\vartheta}-\tau c_{it}^{-\vartheta}+c_{it}^{-\vartheta}+\tau x-\tau+1}$$

Rearranging terms in LHS and RHS and solving, we finally have:

$$-\tau x^{1-\vartheta}c_{it}^{-\vartheta}+\tau c_{it}^{-\vartheta}-c_{it}^{-\vartheta}-\tau x+\tau-1\gtrless-\tau x^{1-\vartheta}c_{it}^{-\vartheta}+\tau c_{it}^{-\vartheta}-c_{it}^{-\vartheta}-\tau x^{1-\vartheta}+\tau-1$$

Simplifying terms in RHS and LHS and recalling that $1<x<2$ and $0<\vartheta<1$ we finally have:

$$-x<-x^{1-\vartheta}\Leftrightarrow x>x^{1-\vartheta}$$

So we can conclude that the growth rate of output for neck-and-neck industries is higher than for unlevelled industries:

$$\gamma_{Y_{It}}\left(A_{int}=A_{it}\right)>\gamma_{Y_{It}}\left(A_{iLt}>A_{iFt}\right)$$

A.4 Growth rate in low-technology industries with constant marginal returns from technology

Firstly, we have to compute the aggregate production function for neck-and-neck industries as in equation (70). In this case, we assume that $z_{it}=z(c_{it})=c_{it}$ and $z_{it+1}=z(c_{it+1})=c_{it+1}$, such that $z'(c_{it})=1$ and $z'(c_{it+1})=1$. Then the aggregate output is given by:

$$Y_{It}\left(A_{int}=A_{it}\right)=\left(\frac{\zeta}{\overline{\overline{r}}_i}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w}}_i}\right)^{1-\zeta}N_t\left(1+c_{it}\right)$$

The growth rate of output in neck-and-neck industries is given by:

$$\gamma_{Y_{It}}\left(A_{int}=A_{it}\right)=\frac{\dot Y_{It}}{Y_{It}}=\frac{\dot N_t}{N_t}+\frac{\dot c_{it}}{1+c_{it}}=\frac{\dot N_t}{N_t}+\ln x\cdot x^t\,\frac{c_{it}}{1+c_{it}}$$

where $\dot c_{it}=\ln x\cdot x^t\cdot c_{it}$ is the variation in time t of investment expenditure, as in equation (74).

Secondly, we have to determine the aggregate production function for unlevelled industries as in equation (80). Also in this case, we assume that $z_{it}=z(c_{it})=c_{it}$ and $z_{it+1}=z(c_{it+1})=c_{it+1}$. Moreover, we assume that the number of innovative firms is a constant fraction τ of the total number of firms N_t. Then the aggregate output is equal to:

$$Y_{It}\left(z_{iLt} > z_{iFt}\right) = \left(\frac{\zeta}{\overline{\overline{r_i}}}\right)^{\zeta}\left(\frac{1-\zeta}{\overline{\overline{w_i}}}\right)^{1-\zeta} N_t\left[1 + c_{it} + \tau c_{it}\left(x-1\right)\right]$$

The growth rate of output in unlevelled industries is equal to:

$$\gamma_{Y_{It}}\left(A_{iLt} > A_{iFt}\right) = \frac{\dot{Y}_{It}}{Y_{It}} = \frac{\dot{N}_t}{N_t} + \frac{\dot{c}_{it}\left[1+\tau\left(x-1\right)\right]}{1 + c_{it} + \tau c_{it}\left(x-1\right)} = \frac{\dot{N}_t}{N_t} + \ln x \cdot x^t \frac{c_{it}\left[1+\tau\left(x-1\right)\right]}{1 + c_{it} + \tau c_{it}\left(x-1\right)}$$

where $\dot{c}_{it} = \ln x \cdot x^t \cdot c_{it}$ is the variation in time t of investment expenditure, as in equation (74).

Now we are interested in comparing the growth rates for neck-and-neck industries $\gamma_{Y_{It}}\left(A_{int} = A_{it}\right)$ on the LHS and for unlevelled industries $\gamma_{Y_{It}}\left(A_{iLt} > A_{iFt}\right)$ on the RHS. For this purpose we have to determine the sign of the following inequality:

$$\frac{1}{1+c_{it}} \gtrless \frac{1+\tau\left(x-1\right)}{1 + c_{it} + \tau c_{it}\left(x-1\right)}$$

Rearranging and simplifying terms in LHS and RHS, we finally obtain:

$$\tau < \tau x$$

So we can conclude that the growth rate of output for unlevelled industries is higher than for neck-and-neck industries:

$$\gamma_{Y_{It}}\left(A_{int} = A_{it}\right) < \gamma_{Y_{It}}\left(A_{iLt} > A_{iFt}\right)$$