Pre-University Summer School

Game Theory (part 1)

Game Theory

Outline for today

Part 1 (10am-11am)
- What is ‘game theory’?
- How should I play a ‘game’?

Part 2 (11am-12pm)
- How will other people play?
- What is likely to happen?

Part 3 (12pm-1pm)
- Being unpredictable.
- Zero sum games.

Split or Steal

Consider the following situation:

You are on a game show and have the chance to win up to £1m.

You and one other person must choose to either:

Split the money, or,
Steal the money

- If you both *split* then both players £500k (50%).
- If both *steal* then both players get £0.
- If one player *steals* and the other *splits* then the stealer gets £1m and the splitter gets £0.

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Winnings for Split or Steal

Let's try to analyse the winnings and strategies in this game.

<u>Start with Player A:</u>

- For Player A they get zero whenever Player B picks 'steal'.
- But if B picks 'split' it is best to 'steal'.
- So, 'steal' is strictly better in one case and no worse in the other case.

<u>For Player B:</u>

- They are in an identical position to A.
- Again, 'steal' is at least as good as 'split' not matter what A does.

	Player B	
A's Winnings (£1000s)	Split	Steal
Player A: Split		
Player A: Steal		

Combining the Tables

Player A \ A's Winnings (£1000s)	Player B: Split	Player B: Steal
Split	500	0
Steal	1000	0

Player A \ B's Winnings (£1000s)	Player B: Split	Player B: Steal
Split	500	1000
Steal	0	0

Player A \ Player B	Split	Steal
Split	500, 500	0, 1000
Steal	1000, 0	0, 0

From this analysis, it would seem that 'Steal', 'Steal' is the most likely outcome.

- Is that what we would see in reality?
- (if not then why not?)

Game Theory

- *Game theory* is the toolkit used to make predictions in *strategic* situations.
- "What is the likely outcome?"

- We model these strategic situations as 'games' with 'players'.
  - 'Strategic' means my action affects your payoff and vice versa.
  - Strategic situation: Poker
  - Non-strategic situation: Roulette

- Does game theory only apply to game shows, board games, card games and alike?

Applications of Game Theory

- Firm behaviour (industrial economics)
- Voting (political economy)
- Climate change (environmental economics)
- Trade agreements (international economics)
- Auctions (mechanism design)

- Armed conflict
- Traffic management
- Biology
- Computer science
- Linguistics

A Famous Game

The Prisoner's Dilemma

Two people stand jointly accused of a serious crime. The police only have evidence of a lesser crime and therefore need a confession.

The prisoners are locked in different cells and are given the opportunity to either 'confess' to the more serious crime, or 'stay quiet'.

If one prisoner confesses and the other does not, then they will get a pardon and will be free to go. The other prisoner gets a sentence of 10 years.

If both confess they each get a sentence of 5 years.

If both stay quiet they each get a sentence of 1 year for the lesser crime.

The Payoff Matrix

Prisoner A \ Prisoner B	Confess	Stay Quiet
Confess	-5, -5	0, -10
Stay Quiet	-10, 0	-1, -1

Let's apply our previous technique:

- If Prisoner B picks 'Confess' what should A do?
- If Prisoner B picks 'Stay Quiet' what should A do?
- 'Confess' is the *best response* for A, no matter what B does.
- In game theory we call such an 'always best' strategy a *dominant strategy*.

Dominant and Dominated Strategies

Not all games have dominant strategies.

- But if one exists, this gives a very strong prediction.

Dominant strategies are stable choices…it doesn't matter what the other player chooses.

Picking a dominant strategy is a great idea.

- We cannot do any better!

As well as giving advice on what to pick, we can also highlight what not to pick:

Prisoner A \ Prisoner B	Confess	Stay Quiet
Confess	-5, -5	0, -10
Stay Quiet	-10, 0	-1, -1

In the Prisoner's dilemma game, the strategy 'Stay Quiet' is *dominated*.

Dominated means that there exists an alternative strategy which is always better.

Applications of the Prisoner's Dilemma

The prisoner's dilemma represents a common problem in economics:

- Conflict between the interests of the individual and the interests of the group.

Many other applications:

1. Price competition between two firms in a market.
2. Overfishing (tragedy of the commons).
3. Working hours.
4. Teamwork (free riding).
5. Performance enhancing drugs in sport.
6. Queuing.
7. …Others!

The Studying Game

		Student B		
		No Effort	Some Effort	Very High Effort
Student A	No Effort	0, 0	-100, 400	-100, 150
	Some Effort	400, -100	100, 100	0, 150
	Very High Effort	150, -100	150, 0	50, 50

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The Studying Game

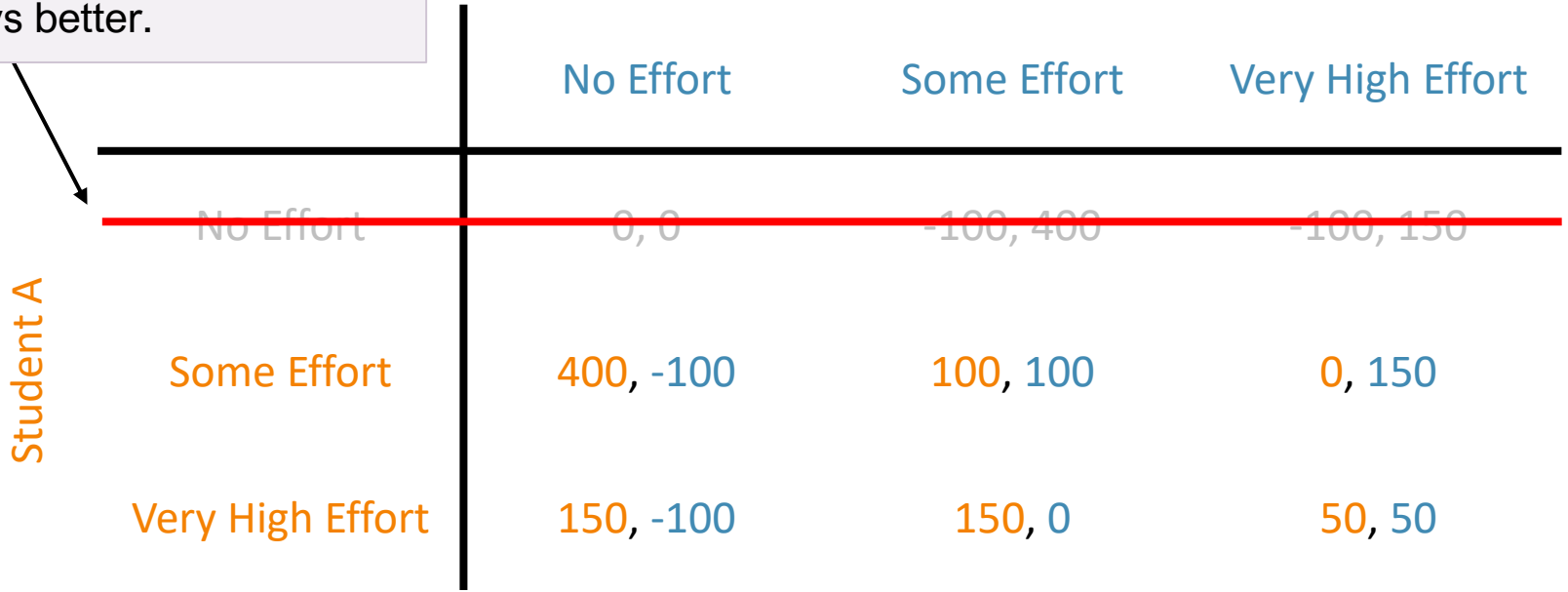
Is there a *dominant strategy*?

No effort is a *dominated strategy*, the other two are always better.

Student A \ Student B	No Effort	Some Effort	Very High Effort
No Effort	0, 0	-100, 400	-100, 150
Some Effort	400, -100	100, 100	0, 150
Very High Effort	150, -100	150, 0	50, 50

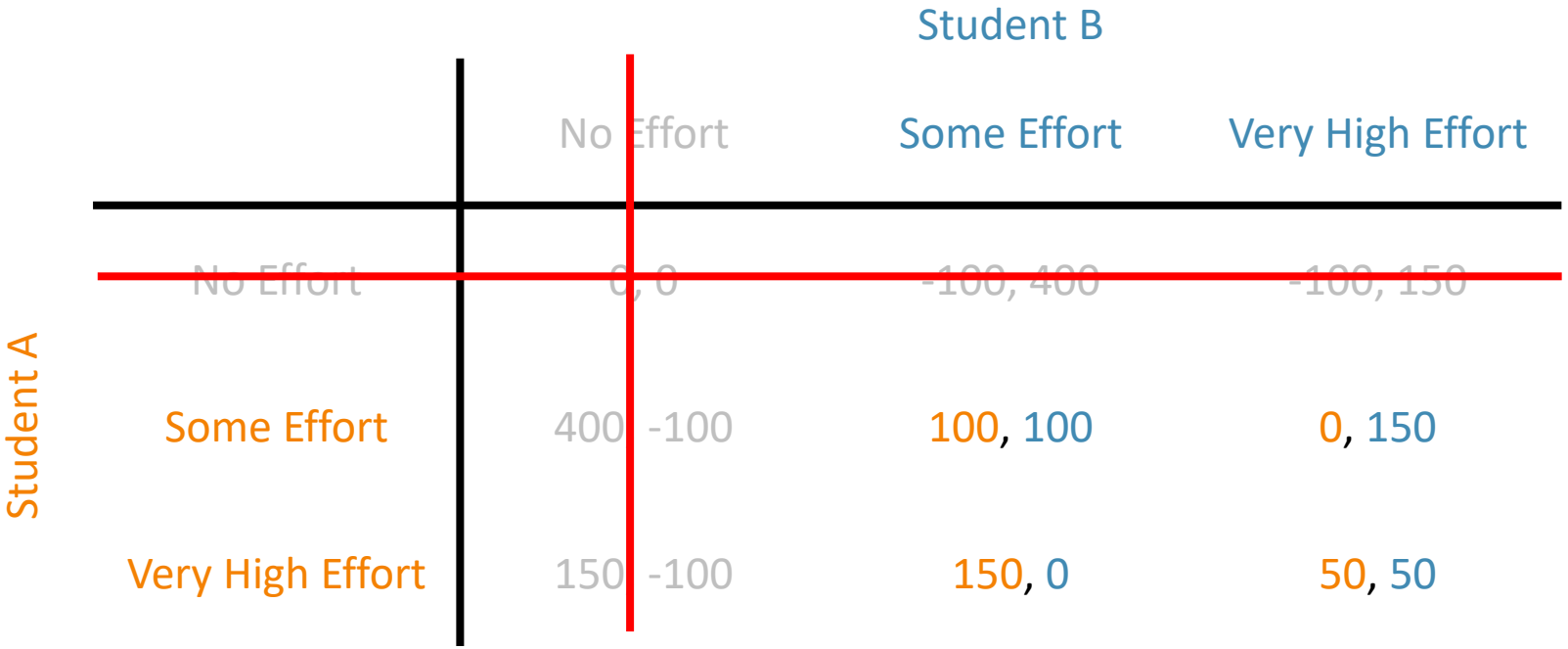
The Studying Game

No effort is a *dominated strategy*, the other two are always better.



Student A \ Student B	No Effort	Some Effort	Very High Effort
No Effort	0, 0	-100, 400	-100, 150
Some Effort	400, -100	100, 100	0, 150
Very High Effort	150, -100	150, 0	50, 50

The Studying Game



Now very high effort is dominant!

Game Theory (Part 1) - Summary

- Game theory is used to make predictions in strategic situations.
- It can also be used to give advice on how to play in some cases.
  (more on this later!)
- Dominant strategies do not exist in every game, but if they do exist then they are always optimal.
- Dominated strategies (if they exist) can be removed from the game to help us narrow down the optimal strategy to play.