Pre-University Summer School

Game Theory (part 3)

Strategies and Equilibrium

We started today talking about optimal strategies (so called dominant strategies).

We then learnt about *Nash equilibrium* (a method to make predictions about what is likely to happen in a game).

- ▷ Is it always optimal for me to play a Nash equilibrium strategy?
- ▷ Does every game have a Nash equilibrium?


XTL ERR
RFI
LO VOLT
HOLD
42
TARGET SPEED
PWR
PWR/VOL
LAMP TEST
MAX
TEST

A Speeding Game

Driver		Police: Monitor	Police: Don't Monitor
	Speed	-50, 20	20, -10
	Don't Speed	0, -5	0, 35

Here there is no stable Nash equilibrium if:

1. Drivers can only pick either speed or don't speed.
2. Police and only pick either monitor or don't monitor.

▷ The problem here is that both players want to be *unpredictable*.

A Speeding Game

How often should the police monitor?

- Just enough to incentivise drivers not to speed.
- But not so much that we monitor unnecessarily.

How often do we monitor to make the drivers not want to speed all the time?

$$\underbrace{p(0) + (1-p)0}_{\text{Driver payoff from Don't Speed}} \geq \underbrace{p(-50) + (1-p)20}_{\text{Driver payoff from Speed}}$$

$$0 \geq 20 - 70p$$
$$70p \geq 20$$
$$p \geq 2/7$$

Monitor no less than 2 days per week.

- Payoff for police: $\left(\frac{2}{7}\right)5 + \left(\frac{5}{7}\right)35 = 26.43$

Driver \ Police	Monitor (p)	Don't Monitor (1-p)
Speed	-50, 20	20, -10
Don't Speed	0, -5	0, 35

Monitoring Games

<u>Other Examples of Monitoring Games in Game Theory</u>

- **Employer-Employee monitoring**: Employers may monitor productivity (or effort) to prevent time wastage.
- **Environmental Agencies**: Government agencies may monitor factories to ensure compliance with pollution regulations.
- **Tax Authorities:** Random audits may be conducted to encourage accurate reporting.
- **Customs and Border Patrol**: Randomised searches may help to discourage smuggling of illegal items.

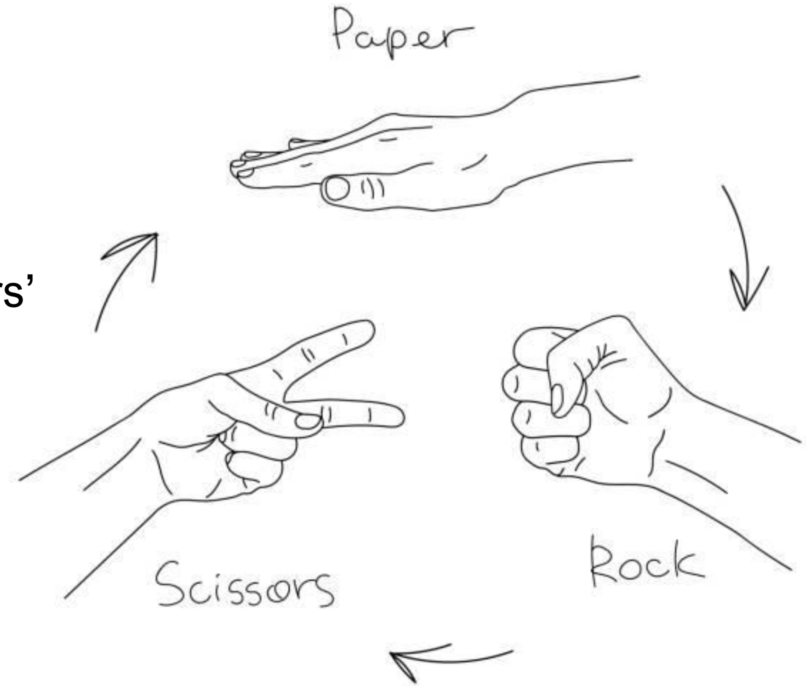
Rock Paper Scissors

<u>Rock, Paper, Scissors</u>

Rules:

You will pick either ‘Rock’, ‘Paper’ or ‘Scissors’

‘Paper’ beats ‘Rock’
‘Rock’ beats ‘Scissors’
‘Scissors’ beats ‘Paper’



Rock Paper Scissors

Suppose we each wager £1 to play this game. The winner therefore gains an extra £1 from winning and the loser will lose their £1.

	Rock	Paper	Scissors
Rock	0 , 0	-1 , 1	1 , -1
Paper	1 , -1	0 , 0	-1 , 1
Scissors	-1 , 1	1 , -1	0 , 0

Side note: This is a special type of game called a ‘zero sum’ game.

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The Optimal Strategy?

According to the now defunct 'Rock-Paper-Scissors Society', Rock is the most popular strategy over the long history of tournaments they had until 2010.

Strategy	Frequency of Play (data from RPS tournaments)
Rock	35.4%
Paper	29.6%
Scissors	35.0%

	Rock (35.4%)	Paper (29.6%)	Scissors (35.0%)	Total
Rock				
Paper				
Scissors				

Expected winnings

- If people play this way on average, then our best strategy is Rock.
- But if everyone else thinks this way, then our best strategy is Paper.
- And if everyone else thinks *this* way, then our best strategy is Scissors.
- …

The Optimal Strategy?

Is there a strategy which guarantees that we will always win money?

▷ No. If one existed, then our opponent could copy it and we would always draw!

Is there a strategy which guarantees that we will never *lose* money?

A ‘mixed’ strategy for RPS

Randomise between Rock, Paper and Scissors. Play each with 1/3 probability.

Expected Winnings: *If they play Rock = (1/3) 0 + (1/3) 1 + (1/3) -1 = 0*

If they play Paper = (1/3) -1 + (1/3) 0 + (1/3) 1 = 0

If they play Scissors = (1/3) 1 + (1/3) -1 + (1/3) 0 = 0

Key feature: We are unexploitable if the opponent’s choice is as difficult as possible.

Preventing them from easily picking a winning strategy will prevent us from losing!


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A Penalty Shootout

- Kicker must shoot either Left or Right
- Goalkeeper must dive either Left or Right
- If the Kicker scores they get payoff 1, Goalkeeper gets payoff -1
- If the Goalkeeper saves they get payoff 1, Kicker gets payoff -1

Kicker \ Goalkeeper	Left	Right
Left	-1, 1	1, -1
Right	1, -1	-1, 1

<u>Side note</u>: This is another example of a 'zero sum' game.

A Penalty Shootout

		Goalkeeper	
		Left	Right
Kicker	Left	-1, 1	1, -1
	Right	1, -1	-1, 1

- The kicker wants to be unpredictable.
- They do not want to give the goalkeeper a clear choice of what to do.

- What shooting strategy makes goalkeeper's decision as difficult as possible?

- What saving strategy makes the kicker's decision as difficult as possible?

- If they play like this then 50% of penalties are saved and 50% are scored.
  - (is this realistic?)

A Penalty Shootout (modified payoffs)

Now suppose the kicker has a strong right foot, so a shot to the left goes in 50% of the time even when the goalkeeper guesses correctly.

The payoffs of the game change to:

Kicker \ Goalkeeper	Left	Right
Left	0, 0	1, -1
Right	1, -1	-1, 1

Payoff for Kicker when (Left,Left) happens is

$$(1/2)(1) + (1/2)(-1) = 0$$

Payoff for Goalkeeper when (Left,Left) happens is

$$(1/2)(-1) + (1/2)(1) = 0$$

A Penalty Shootout (modified payoffs)

Kicker \ Goalkeeper	Left	Right
Left	0, 0	1, -1
Right	1, -1	-1, 1

- Is it still optimal for the kicker/goalkeeper to randomise 50-50?
- 50-50 is no longer a stable equilibrium, in fact, both players wish to adjust their strategy.

New equilibrium is actually:

- Shoot Left 2/3rds of the time and Right 1/3rd of the time.
- Dive Left 2/3rds of the time and Right 1/3rd of the time.

Now the kicker is guaranteed payoff of $1/3$ (*exercise*: check this!)

Colonel Blotto

Colonel Blotto is a game of *strategic mismatch*.

- 2 Players have T ‘troops’ each.
- There are N ‘fronts’ which must have a number of troops allocated.
- Whoever has more troops on a given front wins that front (payoff +1)
- Whoever has less troops on a given front loses that front (payoff -1)

Applications:

- ‘Troops’ = Advertising expenditure / ‘Fronts’ = Different product markets
- ‘Troops’ = R&D expenditure / ‘Fronts’ = Different product characteristics
- ‘Troops’ = Police vs. criminals / ‘Fronts’ = Areas of a city
- ‘Troops’ = Campaign spending / ‘Fronts’ = States in an election

Colonel Blotto Example

Suppose T=12 and N=3.

Player	Front 1	Front 2	Front 3
P1	4	4	4
P2	5	5	2
P1’	6	0	6
P2’	8	2	2
P1’’	4	4	4
…	…	…	…

- Any deterministic allocation of resources (troops) can be beaten by another!

Colonel Blotto

Main insights of the Colonel Blotto game:

- There is no deterministic strategy which cannot be exploited.
- Playing an unpredictable strategy is beneficial.
- You do not always need all your troops to win.

If the game is not symmetric (e.g. one player has more troops) then it may be possible to guarantee a win.

- Weaker players can try to get around this by opening more ‘fronts’

Game Theory (Part 3) - Summary

- To help make predictions in game theory we focus on stable outcomes.
- A Nash equilibrium is an outcome where each player picks their best strategy, given the strategy of the opponent.
- Sometimes these equilibrium strategies can involve randomisation.
- In a zero-sum game (if I win, you lose) we are best off picking a strategy which makes our opponent's decision as difficult as possible.