

On the Common Claim that Happiness Equations Demonstrate Diminishing Marginal Utility of Income

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This work began while I was a 2005 Jacob Wertheim Fellow at Harvard University. I have had valuable conversations on the topic with Richard Layard of the London School of Economics, Martin Seligman of the University of Pennsylvania, Dan Gilbert of Harvard University, Robert MacCulloch of Imperial College London, and Rafael Di Tella of Harvard Business School. The conclusions are mine alone, however.

Statistical research into the determinants of happiness has grown remarkably in the last few decades. This work has drawn together psychologists, economists, epidemiologists, and others. Overviews of the recent literature include, for instance, the books by Frey and Stutzer (2002) and Layard (2005), and the articles by Diener et al (1999) and by Diener and Seligman (2004).

However, one conclusion that is now very commonly found in the literature, including in emerging textbooks, appears to be incorrect. It is that modern happiness research has established that there is diminishing marginal utility of income. Happiness rises -- goes the argument -- as people get richer, but it does so in a curved way. Each dollar buys a bit less happiness than the last dollar. To put this in more mathematical language, it is asserted in the literature that wellbeing has been shown to be a concave and increasing function of income.

Cross-national scatter diagrams supportive of this claim, where each dot is a separate country, can be found in various places in the literature, including those mentioned above. Many investigators who have estimated subjective wellbeing regression equations on individual data, moreover, have found that allowing for a concave form -- something like a logarithm or some appropriate simple polynomial -- in income will fit wellbeing data better than a linear income term.

By using cohort data, Richard Easterlin (2005) has raised one set of empirical objections to the argument that there is diminishing marginal utility of income. While I agree with much of what he concludes there, the point of this note is different. It is theoretical rather than empirical.

My purpose here is to suggest that we have not, as a body of researchers, established that happiness is curved in income. It is natural, arguably, to believe in such curvature. Future research may even find a way of proving diminishing utility of income. Yet currently what we have done is to show that reported happiness is a curved function of

income. The key point is that we do not know the shape of the function relating 'reported happiness' to actual happiness. This is a serious problem when researchers try to make statements about the curvature of relationships -- though not as serious when we talk, as most of the happiness literature does, about the direction of relationships.

To put this in a different way, happiness survey answers tell us which way is up or down. They do not persuasively tell us the speed of the rise or fall.

It seems reasonable -- given only mild assumptions -- to argue that we have established, say, that greater income buys greater happiness, *ceteris paribus*. But in my judgment we have not done sufficiently more than this to allow us to be confident about rates of change.

A Sketch of a Proof

Define R as reported happiness. Let y be income. Let h be actual happiness. Let $r(h)$ be the function that maps actual happiness into reported happiness.

Let $h(y)$ be happiness as a function of income. This is akin to the utility function in conventional economics textbooks.

Assume that both $r(\cdot)$ and $h(\cdot)$ are strictly increasing functions.

To fix ideas, assume there is a single person. Assume that all variables are measured accurately (that is, without error in the statistical sense).

Here, R is what the subject in a questionnaire survey reports to an interviewer, whereas h is 'true' happiness. For the sake of simplicity, assume that all variables are measured on the real line. Reported happiness answers are usually coded into a small number of discrete boxes, but that does not affect the central argument here.

It has been found, as stated, that the reported happiness function $R(y)$ is increasing and strictly concave in income. Yet this does not prove that happiness itself exhibits diminishing marginal utility of income.

Proposition

Strictly concave $R(y)$ does not establish that $h(y)$ is strictly concave.

Assume for simplicity that all the functions are differentiable. Then reported happiness as a function of income is

$$R(y) = r(h(y)) \quad (1)$$

and so,

differentiating this once,

$$R'(y) = r'(h)h'(y) \quad (2)$$

whereupon, as long as the reported happiness function $r(h)$ is monotonically increasing in actual happiness, h , which seems a reasonably mild assumption, the finding that reported happiness rises with income means that actual happiness rises with income.

It is for the second derivative that the difficulty arises.

Differentiating equation (1) twice,

$$R''(y) = r'(h)h''(y) + r''(h)h'(y)^2 \quad (3)$$

and, thus, concavity of reported happiness in income (as the empirical literature has shown) does not prove concavity of actual happiness in income. Signing the second derivative of $R(y)$ does not unambiguously sign the second derivative of $h(y)$, and vice versa. If the reporting function $r(h)$ is strictly concave, for example, then $R(y)$ can be strictly concave even if $h(y)$ is linear.

If the reporting function $r(h)$ is linear, which could be viewed as a kind of cardinality, then if the second derivative of $R(y)$ is negative the second derivative of $h(y)$ will be negative. But such linearity seems likely to hold only in restrictive cases.

An Intuitive Statement of the Difficulty

Psychological wellbeing is not measured in objective units. It is necessary, instead, to listen to what people say. Under the (reasonably mild) assumption that happier people tend to report themselves as happier, we can learn about the direction of influences upon subjective wellbeing. The literature has done this.

However, it requires more stringent assumptions about the nature of people's answers before we can draw conclusions about curvature. We know little about the shape of the reporting function that human beings use.

Imagine, for example, that there is constant marginal utility of income, but that people, as they get happier, mark themselves happier on a questionnaire scale but do so in a way in which they are intrinsically reluctant to approach the upper possible level on the questionnaire form (the 5 on a 1-5 scale, say). Then the reporting function itself is curved, and we will have the illusion that true diminishing marginal utility of income has been shown.

In conclusion, despite what many articles and textbooks have begun to say, the literature has not established that happiness is curved in income.

References

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