LECTURE NOTES ON THE PRINCIPAL AGENT MODEL
SIMPLIFIED MODEL
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4. Output is realized, wage is paid to worker, and the game ends.
Given effort, $e$: with probability $\eta e$, output is high and revenue associated with that is $v$. But with probability $1 - \eta e$, no output is produced and zero revenue obtained. The former is a case of the "project" on which worker works being a success, while the latter a failure.

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Expected profit to firm is: $E\pi = (\eta e)v - w$, where $w$ is wage.

Expect Utility to worker is: $EU = w - \frac{ce^2}{3}$, where $c > 0$. 
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Hence, **first-best effort level is**:

$$e^* = \frac{3\eta v}{2c}.$$
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This implies that the subgame perfect equilibrium (SPE) effort level is:

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\hat{e} = \frac{3\eta(w_S - w_F)}{2c}.
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If $w_S - w_F = v$, then $\hat{e} = e^*$. That is, in that case, SPE effort equals first-best effort level.
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Thus:

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\max_{w_S, w_F} E\pi = \eta e(v - w_S) + (1 - \eta e)(0 - w_F),
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subject to \( e = \hat{e} \).
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Substituting $w_F = 0$ into the maximand above, and also for $e = \hat{e}$, using equation 1, we need to now solve:

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Solving for \( w_S \), we get \( w_S = \frac{v}{2} \).

Conclusion: In the unique SPE, the wage contract offered is \( (w_S, w_F) = \left( \frac{v}{2}, 0 \right) \) and effort is \( \hat{e} = \frac{3\eta v}{4c} \).
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NOTE, the SPE effort is less than first-best effort.