

## Bargaining in a Long-Term Relationship with Endogenous Termination\*

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This paper studies a dynamic game of perfect information, in which two players start bargaining over the partition of a *new* cake each time agreement is struck over the partition of an *existing* cake. Negotiations over the partition of each cake take place according to Rubinstein's alternating-offers model. The parameters of the model are the players' rates of time preference, the time interval between two consecutive offers, and the lag time between the end of one set of negotiations and the start of the next. We characterize the unique stationary perfect equilibrium and investigate the possible existence of non-stationary equilibria. *Journal of Economic Literature* Classification Numbers: C73, C78. © 1995 Academic Press, Inc.

### 1. INTRODUCTION

Rubinstein's bargaining theory (see [3]) is concerned with the classic bilateral bargaining situation, in which two players can divide a unit-size cake if and only if they can agree on the partition of the cake. In many bilateral relationships, however, the two players may have the opportunity to cooperate over the production of more than one cake. For example, a seller and a buyer of some input can engage in mutually beneficial trade more than once; a husband and wife are involved in bargaining situations "day-in-and-day-out"; a firm and a trade union engage in wage bargaining year after year; and Japan and United States engage in trade negotiations involving more than one issue.

In this paper we study a bilateral relationship in which two players have the opportunity to produce an infinite number of cakes, but they can choose to terminate their relationship after having produced and divided

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only a finite number of such cakes. Subject to two important qualifications, our model is an infinite repetition of Rubinstein's bargaining game. The first qualification is that the players start bargaining over the partition of the  $(n + 1)$ th cake (where  $n = 1, 2, \dots$ ) if and only if they reach agreement at some finite time on the partition of the  $n$ th cake. Thus, if the players perpetually disagree over the partition of the  $n$ th cake, then their relationship is terminated. The second qualification is that the time at which the players start bargaining over the partition of the  $(n + 1)$ th cake is determined by the time at which agreement is struck over the partition of the  $n$ th cake.

Section 2 provides a detailed description of the model. It will be evident that the repeated play of Rubinstein's equilibrium does not constitute a perfect equilibrium. We therefore, in Section 3, characterize the unique stationary perfect equilibrium. It will be shown, for example, that in this equilibrium, when the players are sufficiently patient and the time interval between two consecutive offers is sufficiently small, the players split each cake equally, irrespective of their relative rates of time preference. Section 4 investigates the possible existence of non-stationary perfect equilibria. It will be shown that, for some parameter values, there exists a continuum of such equilibria. We shall argue that this result suggests that Rubinstein's unique equilibrium is not robust to small "external effects," in that the indeterminacy of the basic bargaining problem is re-obtained if the players expect to bargain, with an arbitrarily small probability, over the partition of another cake each time they reach agreement over the partition of an existing cake. We conclude, in Section 5, with a brief discussion of the results that would be obtained in some alternative models.

## 2. THE MODEL

Our model of bargaining in a long-term relationship is a dynamic game of perfect and complete information. The game begins at time 0 with the two players,  $A$  and  $B$ , beginning to play Rubinstein's bargaining game over the partition of a unit-size cake. If agreement is reached at some finite time  $t_1$ , where  $t_1 = 0, \Delta, 2\Delta, \dots$ , and where  $\Delta > 0$  denotes the time interval between two consecutive offers, then immediately and instantaneously the players consume their respective shares. Then  $\tau > 0$  units of time later, at time  $t_1 + \tau$ , the players play another Rubinstein bargaining game, bargaining over the partition of a second cake of unit size. Agreement at some finite time  $t_2$  (where  $t_2 = t_1 + \tau, t_1 + \tau + \Delta, t_1 + \tau + 2\Delta, \dots$ ) is followed immediately and instantaneously with the players consuming their respective shares, and then  $\tau > 0$  units of time later, at time  $t_2 + \tau$ , the players play another Rubinstein game, bargaining over the partition of a third cake of unit size. This process continues indefinitely, provided that the players

always reach agreement in finite time. However, if the players perpetually disagree over the partition of some cake, then there is no further bargaining over new cakes: the players have terminated their relationship. We shall assume that player  $i$  ( $i = A, B$ ) makes the first offer in the Rubinstein bargaining game over the partition of the  $(n + 1)$ th cake ( $n = 1, 2, \dots$ ) if it was player  $j$  ( $j \neq i, j = A, B$ ) whose offer over the partition of the  $n$ th cake was accepted by player  $i$ . Finally, we assume that it is player A who makes the (first) offer at time 0.

The payoffs to the players will depend upon the total number  $N$  (where  $N = 0, 1, 2, \dots$ ) of cakes that they partition. If  $N = 0$  (i.e., if the players perpetually disagree over the partition of the first cake), then each player's payoff is zero. And if  $1 \leq N \leq \infty$  (i.e., if the players partition a total of  $N$  cakes and (if  $N$  is finite) perpetually disagree over the partition of the  $(N + 1)$ th cake), then player  $i$ 's ( $i = A, B$ ) payoff is  $\sum_{n=1}^N [x_i^n \exp(-r_i t_n)]$ , where  $0 \leq x_i^n \leq 1$  ( $n = 1, 2, \dots, N$ ) denotes player  $i$ 's share of the  $n$ th cake,  $t_n \geq 0$  denotes the time at which agreement over the partition of the  $n$ th cake is struck, and  $r_i > 0$  denotes player  $i$ 's rate of time preference.

For notational convenience we define, for  $i = A, B$ ,  $\delta_i = \exp(-r_i A)$  and  $\alpha_i = \exp(-r_i \tau)$ . The parameters  $\delta_A$  and  $\delta_B$  capture the bargaining frictions: they represent the costs to players A and B, respectively, of haggling over the partition of a cake. In contrast, the parameters  $\alpha_A$  and  $\alpha_B$ , respectively, can be interpreted as capturing the values to the players of future bargaining situations.

It is straightforward to verify that the repeated play of Rubinstein's equilibrium does not constitute a perfect equilibrium in our model. This is not particularly surprising. In Rubinstein's model, the cost to player  $i$  of rejecting an offer is captured by  $\delta_i$ ; rejecting an offer shrinks, from player  $i$ 's perspective, the *single* available cake by a factor of  $\delta_i$ . In contrast, in our model the rejection of an offer not only shrinks the current cake, but it also shrinks *all* the future cakes, thus inducing a relatively higher cost of rejecting an offer.

### 3. STATIONARY PERFECT EQUILIBRIA

A *stationary* pure strategy for player  $i$  ( $i = A, B$ ) is defined by two objects,  $x_i^*$  and  $f_i$ , where  $x_i^*$  is a number in the closed interval  $[0, 1]$  and  $f_i$  is a function from the closed interval  $[0, 1]$  to  $\{A, R\}$ .

Whenever player  $i$  must make an offer, she demands a share  $x_i^*$  (leaving  $1 - x_i^*$  for her opponent). Moreover, whenever she must respond to an offer that gives her a share  $x_i$ , she accepts  $x_i$  if  $f_i(x_i) = A$  and she rejects  $x_i$  if  $f_i(x_i) = R$ . Proposition 1 characterizes the unique stationary perfect equilibrium in our model.

PROPOSITION 1. *In the unique stationary perfect equilibrium, agreement is reached immediately over the partition of every single cake. The equilibrium partition of the  $n$ th cake is  $(x_A^*, 1 - x_A^*)$  if  $n$  is odd (i.e.,  $n = 1, 3, 5, \dots$ ), and if  $n$  is even (i.e.,  $n = 2, 4, 6, \dots$ ) then it is  $(1 - x_B^*, x_B^*)$ , where  $x_i^* = \min\{z_i^*, 1\}$  with*

$$z_i^* = \frac{(1 - \delta_i \alpha_i)(1 - \delta_j)(1 + \alpha_j)}{[(1 - \delta_i \alpha_i)(1 - \delta_j \alpha_j) - (\delta_i - \alpha_i)(\delta_j - \alpha_j)]}$$

( $i, j = A, B$  with  $i \neq j$ ), and where  $x_i^*$  (resp.,  $1 - x_i^*$ ) denotes the share to player  $i$  (resp., to player  $j$ ). Furthermore, player  $i$  accepts an offer  $x_i$  if and only if  $x_i \geq 1 - x_j^*$ . (Note that  $z_i^* \geq 1$  if and only if  $\Delta \geq \tau$ .)

Evidently, the share  $x_i^*$  appears to depend on the parameters in a rather complex manner. After proving the proposition we shall derive some of its rather revealing properties.

*Proof.* Consider an arbitrary stationary perfect equilibrium. Let  $V_i$  (respectively,  $W_i$ ) denote the equilibrium payoff to player  $i$  in any subgame beginning with  $i$ 's offer to  $j$  (respectively,  $j$ 's offer to  $i$ ), where  $i, j = A, B$  with  $i \neq j$ . It is straightforward to see that  $0 \leq V_i, W_i \leq [1/(1 - \alpha_i)]$ . In equilibrium, therefore,  $j$ 's payoff from rejecting any offer is  $\delta_j V_j$ , while her payoff from accepting an offer  $x_i$  is  $[(1 - x_i) + \alpha_j V_j]$ . If  $\Delta \geq \tau$ , then perfection requires that  $j$  accept any offer. Hence,  $x_i^* = 1$ .

Now assume that  $\Delta < \tau$ . In this case, perfection requires that  $j$  accept an offer  $x_i$  if and only if  $(1 - x_i) \geq (\delta_j - \alpha_j) V_j$ . (Note that  $0 \leq (\delta_j - \alpha_j) V_j < 1$ .) Hence, either  $x_i^* = [1 - (\delta_j - \alpha_j) V_j]$  or  $x_i^* > [1 - (\delta_j - \alpha_j) V_j]$ . The former (resp., the latter) is the case if  $[1 - (\delta_j - \alpha_j) V_j + \alpha_i W_i] \geq$  (resp.,  $<$ )  $\delta_i W_i$ . From Step 1 (below) it follows that in fact, for  $i, j = A, B$  with  $i \neq j$ ,

$$x_i^* = [1 - (\delta_j - \alpha_j) V_j] \tag{1}$$

Therefore, for  $i, j = A, B$  with  $i \neq j$ ,  $V_j = [x_j^* + \alpha_j(1 - x_i^*) + \alpha_j^2 V_j]$ , i.e.,

$$V_j = [x_j^* + \alpha_j(1 - x_i^*)] / [1 - \alpha_j^2]. \tag{2}$$

From (1) and (2), it follows that  $x_i^* = z_i^*$ .

STEP 1. For  $i, j = A, B$  with  $i \neq j$ ,

$$[1 - (\delta_j - \alpha_j) V_j + \alpha_i W_i] \geq \delta_i W_i. \tag{3}$$

*Proof of Step 1.* By contradiction. First, suppose that inequality (3) does not hold for either  $i = A$  and  $j = B$  or  $i = B$  and  $j = A$ . Hence, in equilibrium, neither A nor B will make acceptable offers. Thus,  $V_i = W_i = 0$  ( $i = A, B$ ), which leads to a contradiction.

Now suppose that inequality (3) does hold for  $i = A$  and  $j = B$  but *not* for  $i = B$  and  $j = A$ . Hence, in equilibrium, A (resp., B) will make an acceptable (resp., unacceptable) offer. Therefore,  $V_A = [x_A^* + \alpha_A W_A]$ ,  $V_B = \delta_B W_B$ ,  $W_A = \delta_A V_A$ , and  $W_B = [1 - x_B^* + \alpha_B V_B]$ , where  $x_A^* = [1 - (\delta_B^* - \alpha_B) V_B]$ . It then follows that  $W_B = 0$  and  $V_A = [1/(1 - \alpha_A \delta_A)]$ , which leads to a contradiction. ■

We now take a closer look at how the parameters influence the equilibrium partitions of the cakes. If  $\Delta \geq \tau$ , then  $z_A^* \geq 1$  and  $z_B^* \geq 1$ . Hence, in this case the equilibrium partitions are easy to describe: player A obtains the whole of the  $n$ th cake when  $n$  is odd, while B obtains the whole of the  $n$ th cake when  $n$  is even. The intuition for this result is, that if  $\Delta \geq \tau$ , then the cost of rejecting any offer *relative* to the cost of accepting it is infinite; consequently, it is "as if" the proposer is able to make a take-it-or-leave-it offer. We state this result as:

**COROLLARY 1.** *If  $\Delta \geq \tau$ , then in the unique stationary perfect equilibrium player A obtains the whole of the  $n$ th cake when  $n$  is odd, while player B obtains the whole of the  $n$ th cake when  $n$  is even.*

Although this result is rather provocative, it is perhaps not that interesting since  $\Delta \geq \tau$  is not reasonable. It is far more typical for  $\Delta < \tau$ : the time interval between two consecutive offers during bargaining over the partition of some cake would typically be much *smaller* than the time taken for the "arrival" of a new cake. If  $\tau > \Delta$ , then  $z_A^* < 1$  and  $z_B^* < 1$ . Hence, the equilibrium partitions depend on the parameters in a far more complex manner. However, the nature of the equilibrium partitions become far more transparent in the limit as  $\Delta \rightarrow 0$  (keeping  $\tau$ ,  $r_A$ , and  $r_B$  fixed). For well-known reasons (see Binmore [2]) this limiting case is, perhaps, the most compelling. Hence, by applying L'hôpital's rule, we obtain:

**RESULT 1.** *For  $i, j = A, B$  with  $i \neq j$ , in the limit as  $\Delta \rightarrow 0$ ,  $z_i^* \rightarrow z_i$ , where  $z_i = r_j/(r_j + \varepsilon_{ij} r_i)$  with  $\varepsilon_{ij} = [(1 + \alpha_i)(1 - \alpha_j)]/[(1 - \alpha_i)(1 + \alpha_j)]$ .*

Using Result 1, the following discussion (summarized below in Corollaries 2–4) elucidates some properties of the equilibrium partitions of the cakes in the limit as  $\Delta \rightarrow 0$ . First, we observe that if  $r_A = r_B$ , then  $z_A = z_B = \frac{1}{2}$ . Now consider  $r_A \neq r_B$ . By applying L'hôpital's rule, it follows from Result 1 that  $z_i \rightarrow \frac{1}{2}$  as  $\tau \rightarrow 0$  (keeping  $r_A$  and  $r_B$  fixed). Thus, the equilibrium partitions of the cakes in the limit as  $\Delta \rightarrow 0$  and  $\tau \rightarrow 0$ , with  $\Delta \rightarrow 0$  infinitely faster than  $\tau \rightarrow 0$ , are *independent* of the players' rates of time preference: irrespective of the absolute or relative magnitudes of  $r_A$  and  $r_B$ , each player receives one-half of each and every unit-size cake.

Furthermore, it follows from Result 1 (by applying L'hôpital's rule) that, for any  $\tau > 0$ , in the limit as  $r_A \rightarrow 0$  and  $r_B \rightarrow 0$ , keeping the ratio  $r_A/r_B$

constant at any arbitrary positive number,  $z_i \rightarrow \frac{1}{2}$ . Thus, the players split each cake equally, irrespective of their relative rates of time preference.

**COROLLARY 2 (RUBINSTEINIAN SOLUTION).** *For any  $\tau > 0$ ,  $r_A > 0$ , and  $r_B > 0$  such that  $r_A = r_B$  in the limiting unique stationary perfect equilibrium, as  $\Delta \rightarrow 0$ , each player receives one-half of each and every unit-size cake.*

**COROLLARY 3 (NON-RUBINSTEINIAN SOLUTION).** *For any  $r_A > 0$  and  $r_B > 0$  such that  $r_A \neq r_B$  in the limiting unique stationary perfect equilibrium, as  $\Delta \rightarrow 0$  and  $\tau \rightarrow 0$ , with  $\Delta \rightarrow 0$  infinitely faster than  $\tau \rightarrow 0$ , each player receives one-half of each and every unit-size cake.*

**COROLLARY 4 (NON-RUBINSTEINIAN SOLUTION).** *For any  $\tau > 0$  in the limiting unique stationary perfect equilibrium, as  $\Delta \rightarrow 0$ ,  $r_A \rightarrow 0$ , and  $r_B \rightarrow 0$ , with  $\Delta \rightarrow 0$  infinitely faster than either  $r_A \rightarrow 0$  or  $r_B \rightarrow 0$  and keeping the ratio  $r_A/r_B$  constant at an arbitrary positive number, each player receives one-half of each and every unit-size cake.*

#### 4. NON-STATIONARY PERFECT EQUILIBRIA

This section investigates the possible existence of non-stationary perfect equilibria. For analytical and notational convenience, we shall, *in this section*, assume that  $r_A = r_B = r > 0$ ; the results extend to the case of  $r_A = r_B$ . Define  $\delta = \exp(-r\Delta)$  and  $\alpha = \exp(-r\tau)$ . Our first result here, which applies if and only if  $\alpha + \delta \geq 1$ , establishes the existence of two "extremal" perfect equilibria. In both of these equilibria, agreement is reached immediately on the partition of each and every cake. In one of these equilibria it is player A who, for all  $n$  ( $n = 1, 2, \dots$ ), receives the whole of the  $n$ th cake, while in the other equilibrium it is player B who, for all  $n$ , receives the whole of the  $n$ th cake. These equilibria can be interpreted (following Abreu, see [1]) as constituting the optimal penal code.

**PROPOSITION 2.** *If and only if  $\alpha + \delta \geq 1$ , the strategy profile described in Table I is in a perfect equilibrium. If play begins in state  $s_i$  ( $i = A, B$ ), then in equilibrium, for all  $n$  ( $n = 1, 2, \dots$ ), agreement over the partition of the  $n$ th cake is reached immediately with player  $i$  receiving the whole of the  $n$ th cake.*

*Proof.* Using the "one-shot deviation" property, it is straightforward to verify that the pair of strategies described in Table I are in a perfect equilibrium if and only if  $\alpha + \delta \geq 1$ . Note that if and only if  $\alpha + \delta \geq 1$  is it optimal for player  $i$  ( $i = A, B$ ) in state  $s_i$  to reject any offer such that  $0 \leq x_i < 1$ . ■

TABLE I

	State $s_A$	State $s_B$
Player A		
Offer	$x_A = 1$	$x_B = 1$
Accept	$x_A = 1$	$0 \leq x_B \leq 1$
Reject	$0 \leq x_A < 1$	
Player B		
Offer	$x_A = 1$	$x_B = 1$
Accept	$0 \leq x_A \leq 1$	$x_B = 1$
Reject	...	$0 \leq x_B < 1$
Transitions	If an offer such that $0 \leq x_A < 1$ is accepted by player A, then immediately switch to state $s_B$ .	If an offer such that $0 \leq x_B < 1$ is accepted by player B, then immediately switch to state $s_A$ .

Given this result it is straightforward to construct a continuum of other non-stationary perfect equilibria. In particular, the following folk theorem can be established.

**PROPOSITION 3.** *If  $\alpha + \delta \geq 1$ , then almost any path of play can be supported by a perfect equilibrium.*

Note that this folk theorem is obtained under far weaker conditions than usual, since (for  $\delta$  plausibly close to one) we only require the discount factor  $\alpha$  between stages to be strictly positive. In the next section we shall argue that this conclusion is probably robust and carries over to a large variety of repetitions of sequential bargaining games.

Our model may be interpreted as a perturbation to Rubinstein's [3] model, where this perturbation is made arbitrarily small in the limit as  $\alpha \rightarrow 0$ . If, in addition to  $\alpha \rightarrow 0$ , we allow  $\delta \rightarrow 1$  (keeping  $\alpha + \delta \geq 1$ ), then the above result suggests that Rubinstein's *unique equilibrium* is not robust to small "external effects," in that the indeterminacy of the basic bargaining problem is re-obtained if the players expect to bargain, *with an arbitrarily small probability*, over the partition of another cake each time they reach agreement over the partition of an existing cake.

In the case of  $\alpha + \delta < 1$ , we have not been able to establish that the stationary equilibrium constitutes the unique perfect equilibrium. However, Proposition 4 below derives upper and lower bounds on the payoffs that the players can achieve in any perfect equilibrium.

**PROPOSITION 4.** *If  $\alpha + \delta < 1$ , then for all  $i$  ( $i = A, B$ ),*

$$m^* \leq m_i \leq M_i \leq M^*,$$

where

$$m^* = \frac{1 + \alpha - \delta}{(1 + \alpha)[1 - \alpha^2 - \delta \max\{\alpha, \delta\}]} \quad \text{and} \quad M^* = \frac{1 + \alpha - \max\{\alpha, \delta\}}{1 - \alpha^2 - \delta \max\{\alpha, \delta\}},$$

and where  $M_i$  (resp.  $m_i$ ) denotes the supremum (resp., infimum) of the set of perfect equilibrium payoffs to player  $i$  in any subgame beginning with  $i$ 's offer.

*Proof.* Using fairly standard (Rubinstein-type) arguments, it can be shown that  $M_j = [1/(1 - \alpha) - m_i \max\{\alpha, \delta\}]$  and  $m_j \geq [1 - \delta M_i]/(1 - \alpha^2)$  (for  $i, j = A, B$  with  $i \neq j$ ). It is then straightforward to establish, by induction, that for all  $n$  ( $n = 0, 1, 2, \dots$ ),  $a_n \leq m_i \leq M_i \leq b_n$ , where  $b_0 = [1/(1 - \alpha)]$ ,  $a_n = [(1 - \delta b_n)/(1 - \alpha^2)]$ , and  $b_{n+1} = [1/(1 - \alpha) - a_n \max\{\alpha, \delta\}]$ . Proposition 4 then follows immediately, since the infinite sequence  $a_n$  (resp.,  $b_n$ ) is strictly increasing (resp., decreasing) and converges to  $m^*$  (resp.,  $M^*$ ). ■

Note, for example, that in the limit as  $\delta \rightarrow 0$  the bounds  $m^*$  and  $M^*$  converge to  $1/(1 - \alpha^2)$ , which equals a player's payoff in the (limiting, as  $\delta \rightarrow 0$ ) stationary perfect equilibrium.

## 5. FINAL REMARKS

We have established that the unique stationary equilibrium in our model is non-Rubinsteinian, in that the (limiting) equilibrium partition of each cake is independent of the players' relative rates of time preference. An important feature of our model that contributes to this result is that the rejection of an offer not only shrinks the current cake but it also shrinks all the future cakes. This is because, in our model, if agreement over the partition of the  $n$ th cake is delayed by  $\Delta > 0$  time units, then the arrival of the  $(n + 1)$ th cake is delayed by an additional  $\Delta$  time units (from  $\tau$  to  $\tau + \Delta$ ). Hence, in our model, the cost to a player of rejecting an offer is relatively higher than that in the Rubinstein model.

Consider the following, alternative, repeated Rubinstein game in which the rejection of an offer does *not* affect the times of arrival of future cakes. At each "date"  $n$  (where  $n = 1, 2, \dots$ ) the players bargain over the partition of the  $n$ th cake where the time interval  $\Delta$  between two consecutive offers is *equal* to zero, and  $\delta_i$  measures the factor by which the cake shrinks when player  $i$  ( $i = A, B$ ) rejects an offer. Furthermore,  $\alpha_i$  denotes player  $i$ 's discount factor between two consecutive dates. In this alternative model, the unique stationary equilibrium would constitute the repeated play of the Rubinstein equilibrium.



Finally, we shall now comment on the robustness of the folk theorem that was obtained under very weak assumptions on the discounting between the stage games. Consider any infinitely repeated sequential bargaining game, in which  $\alpha$  denotes the discount factor between the stages and  $\delta$  measures the factor of shrinkage of a cake following the rejection of an offer.

If the horizon of the underlying stage (bargaining) game is infinite, then the following argument suggests that Proposition 2 (and hence, the folk theorem, Proposition 3) can be obtained for any  $\alpha > 0$  (provided that  $\delta$  is plausibly close to one). For  $\delta$  close to one, the gain from accepting a deviant offer is close to zero. However, the loss from accepting such a deviant offer is strictly positive, provided only that  $\alpha > 0$ .

On the other hand, if the horizon of the underlying stage (bargaining) game is finite, then the gain from accepting a deviant *final* offer is strictly positive (for any value of  $\delta$ ). Hence, in this “finite horizon” case, a folk theorem can be obtained only under standard conditions, namely, when the discount factor  $\alpha$  is sufficiently large.

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