GAMES AND ECONOMIC BEHAVIOR 2, 291–297 (1990)

Bargaining without Commitment*

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Received September 26, 1989

A fundamental assumption in all sequential bargaining models is that when an offer is accepted the bargaining terminates with the implementation of that proposal. The proposer cannot change his mind; he is, in effect, committed to his proposal. In this short paper we explore the consequence of relaxing this (rather strong) *commitment* assumption. Our main result indicates that this commitment assumption is a fundamental reason for the *uniqueness* of the subgame perfect equilibrium partition in the Rubinstein bargaining model. This main result states that if the (common) discount factor of each bargainer is greater than $1/\sqrt{2}$ (≈ 0.7), then *any* partition of the unit size cake can be supported as a subgame perfect equilibrium of the *modified* Rubinstein bargaining model ("modified" in that the commitment assumption is dropped). The multiplicity of equilibria is generated by constructing supergame-type punishment strategies. *Journal of Economic Literature* Classification Number: 026 © 1990 Academic Press, Inc.

1. INTRODUCTION

A central feature common to all sequential bargaining models (for example, Ariel Rubinstein's classic model (Rubinstein, 1982)) is that when an offer is accepted the bargaining terminates with the implementation of that offer. The proposer is committed to his proposal. If the responder accepts the proposer's offer, then the proposer has to implement the offer; the proposer cannot change his mind. There is no sacred rule (or law) which says that if a proposal is accepted, then it must be imple-

* My deepest thanks go to Ariel Rubinstein for his generous advice and comments. I am grateful to David Canning for his valuable comments on the first draft of this paper. The advice of an associate editor of this journal has greatly improved the exposition at several places.

0899-8256/90 $3.00

mented. If one can commit oneself to one's proposal, then that should be modeled explicitly as part of the structure of the bargaining game.

In many real life bargaining situations offers are made verbally and there is no reason to assume that a proposer cannot change his mind, and do so with negligible cost to himself. Even if a proposer, each time he makes an offer, writes up the proposal as a legal document (in an attempt to commit himself to his proposal) he can still change his mind, but would incur some cost (if his opponent sued him). However, if all of this is possible, then it should be explicitly modeled as part of the game structure.

In this short paper we study the consequence of allowing the proposer to change his mind in the context of Rubinstein's alternating–offers bargaining model. We show that if the (common) discount factor of a bargainer is greater than $1/\sqrt{2}$ (≈ 0.7), then *any* partition of the unit size cake can be supported as a subgame perfect equilibrium of the *modified* Rubinstein (1982) bargaining model ("modified" in that a proposer can costlessly change his mind). In any such equilibrium, the equilibrium price is implemented immediately, with no delay. Moreover, in equilibrium, a proposer does not change his mind.

This result shows that although in any equilibrium a proposer will not change his mind (which may be consistent with observations) the possibility of changing one's mind costlessly has a profound effect on the set of equilibria.

The noncooperative bargaining theory literature (e.g., Binmore, 1987; Binmore *et al.*, 1986; Dasgupta and Maskin, 1988; Rubinstein, 1982; Shaked and Sutton, 1984) has commanded much interest and influence primarily for the reason that, in many cases of interest, the bargaining games analyzed possess a *unique* equilibrium. A conclusion of this paper is that this result depends critically on the (implicit) *commitment* assumption embodied in this literature; that is, a proposer is committed to his proposal and thus he cannot change his mind; the proposer has to implement the proposal accepted by the responder.

We show, in this paper, that multiplicity of equilibria prevail when this commitment assumption is dropped in the context of Rubinstein's alternating–offers bargaining model. A basic reason behind this multiplicity of equilibria can be understood with reference to the classical Folk Theorem of Repeated–Game theory (cf. Aumann and Shapley, 1976; Rubinstein, 1979). The classical Folk Theorem states that any feasible and individually rational outcome can be supported as a subgame perfect equilibrium of the repeated game. The equilibrium strategies involve a hierarchy of punishments: punishment to a deviator, punishment to those who do not punish a deviator, punishment to those who do not punish those who do not punish a deviator, and so on. In the Rubinstein bargaining game, *without* the commitment assumption, the strategies which support as an equilibrium any price also involve a similar hierarchy of punishments.

In the standard literature, *with* the commitment assumption, a responder *cannot* be punished for accepting the *wrong* offer. It seems that this is a fundamental reason for the *uniqueness* property of the standard bargaining models. In the analysis that follows, we construct equilibrium strategies (which support any particular price, denote it as the norm price) based on the following basic idea. If a proposer *deviates,* then the responder will reject that *deviation* and punish the proposer, because if the responder does not do that, then the proposer will punish the responder for not carrying out the punishment. Thus, the modified Rubinstein bargaining game possesses a multiplicity of equilibria because one can construct supergame–type punishment strategies.

2. The Model

There is a single seller who owns an indivisible object which he values at zero dollars and a single buyer who wants to buy the object and values it at one dollar. They negotiate over the price of the object according to the (Rubinstein) alternating–offers bargaining procedure, *but* with the added feature that a *proposer* can change his mind costlessly. That is, if a responder accepts ("A") a proposer's offer, then the proposer has to decide whether to accept the responder's acceptance ("AA") of his proposal (and thus implement the proposal) or to reject the responder's acceptance ("RA") of his proposal (that is, change his mind) and thus continue to bargain.

Without loss of generality, we shall assume that it is the seller who makes the first offer. Thus the seller makes offers at times $0, 2, 4, \ldots, 2n, \ldots$, and the buyer makes offers at times $1, 3, 5, \ldots, 2n + 1, \ldots$, where the length of a single bargaining period is normalized at unity. Figure 1 shows the basic structure of this bargaining game; denote the game by G.

If the parties reach an agreement of price p at time n, then the utility payoffs are: for the seller, $U_s(p, n) = p\delta^n$ and for the buyer, $U_b(p, n) = (1 - p)\delta^n$, where δ is the common discount factor, $\delta \in (0, 1)$.

The game described above is a perfect information extensive–form game. Moreover, we assume that there is complete information.

3. Equilibria

Proposition 1. *For any* $\delta \in (0, 1)$ the price $1/(1 + \delta)$ *can be supported as a subgame perfect equilibrium of the bargaining game.*

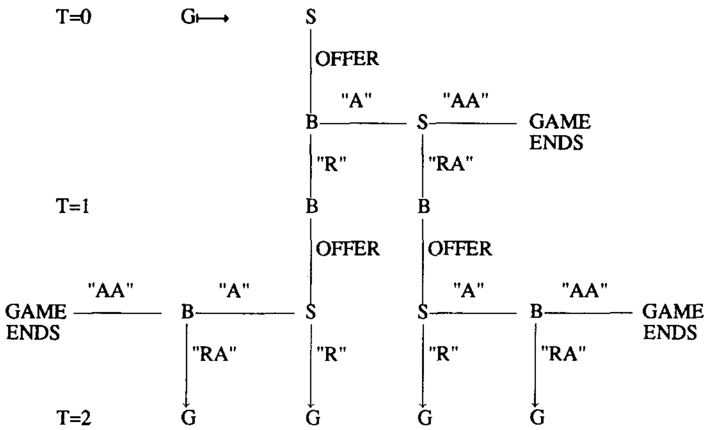

FIGURE 1

Proof. The following pair of *stationary* strategies are in subgame perfect equilibrium. The seller: offers the price $1/(1 + \delta)$, accepts those acceptances of the buyer such that the price is $p \geq \delta^2/(1 + \delta)$, and accepts any price $p \geq \delta/(1 + \delta)$.

The buyer: offers the price $\delta/(1 + \delta)$, accepts those acceptances of the seller such that the price is p where $(1 - p) \geq \delta^2/(1 + \delta)$, and accepts any price $p \leq 1/(1 + \delta)$. Q.E.D.

PROPOSITION 2. *For any* $\delta \in (1/\sqrt{2}, 1)$ *the price one can be supported as a subgame perfect equilibrium of the bargaining game.*

In the proof, below, we construct supergame–type punishment strategies which support the price one as an equilibrium of the bargaining game. For example, suppose the buyer deviates from the norm price (namely, one) and instead offers the price δ. A priori, we would assume that the seller would accept such a deviation since the seller would be indifferent between the agreement of price one tomorrow and the agreement of price δ today. However, if the seller accepts the price δ (thus, not punishing the buyer for deviating from the established norm price, namely price one), then the buyer will punish the seller for not punishing him, by obtaining the price $1/(1 + \delta)$ tomorrow. Thus, by accepting the price δ the seller does not obtain agreement at that price today, but instead the agreement is at price $1/(1 + \delta)$ tomorrow. Hence, the seller will not accept the price δ.

Proof. The equilibrium strategies are described using the language of two states and one transition rule between them. A player's equilibrium action at any point in the game will depend on the state that is prevailing at that point in the game. The state s_1 denotes the *initial* state, which prevails at the beginning of the game. The state s_2 denotes the *absorbing* state. If a certain event (to be described below) occurs during the play of the game, then both players will switch from state s_1 to state s_2.

The seller's equilibrium strategy is as follows. If state s_1 is prevailing, then the seller: offers price one, accepts only price one, and accepts those acceptances of the buyer such that the price is $p \geq \delta$. If state s_2 is prevailing, then the seller behaves as in the Proposition 1 equilibrium.

The buyer's equilibrium strategy is as follows. If state s_1 is prevailing, then the buyer: offers price one, accepts any price, and accepts those acceptances of the seller such that the price is either one or $p < \delta$. If state s_2 is prevailing, then the buyer behaves as in the Proposition 1 equilibrium.

Both players will switch from state s_1 to state s_2 only at a subgame beginning with the seller's offer IF the immediately preceding price offer p (made by the buyer) is $p \in [\delta, 1)$ *and* it is accepted by the seller *and* then the buyer rejects the seller's acceptance.

This pair of strategies is in subgame perfect equilibrium. The reason for $\delta > 1/\sqrt{2}$ is the following. Suppose at any subgame beginning with the buyer's offer (such that state s_1 prevails) the buyer offers the price p, where $p \in [\delta, 1)$, and the seller accepts. The strategy of the buyer prescribes that the buyer should reject this acceptance of the seller. By doing so the buyer obtains a payoff equal to $\delta^2/(1 + \delta)$ (note that in this eventuality both players will switch to state s_2). If he deviated from his prescribed strategy and instead accepted the acceptance of the seller, then his payoff would be equal to $(1 - p)$, where $p \in [\delta, 1)$. If $\delta > 1/\sqrt{2}$, then $\delta^2/(1 + \delta) > (1 - \delta)$. Moreover, $p \in [\delta, 1)$ implies that $(1 - p) \leq (1 - \delta)$. Hence, $\delta^2/(1 + \delta) > (1 - p)$. Thus, deviation from prescribed strategy is not profitable. Q.E.D.

The three conclusions, below, follow from Proposition 2. Conclusion 1 is basically a restatement of Proposition 2. Conclusions 2 and 3 follow from the construction of Proposition 2.

CONCLUSION 1. *For any $\delta \in (1/\sqrt{2}, 1)$ the price zero can be supported as a subgame perfect equilibrium of the bargaining game.*

CONCLUSION 2. *For any $\delta \in (1/\sqrt{2}, 1)$ the price zero can be supported as a subgame perfect equilibrium of the bargaining game with the assumption that it is the buyer who makes the first offer.*

CONCLUSION 3. *For any $\delta \in (1/\sqrt{2}, 1)$ the price one can be supported as a subgame perfect equilibrium of the bargaining game with the assumption that it is the buyer who makes the first offer.*

PROPOSITION 3. *For any $\delta \in (1/\sqrt{2}, 1)$ any price $\hat{p} \in (0, 1)$ can be supported as a subgame perfect equilibrium of the bargaining game.*

Once again the proof involves the construction of supergame–type punishment strategies. A key aspect is that a player will get punished for not carrying out a punishment. At time 0, in equilibrium, the seller and the buyer will agree to the (norm) price $\hat{p}$. If either of them deviates, then at time 1 the players will behave as in the Conclusion 2 equilibrium or as in the Conclusion 3 equilibrium.

Proof. The equilibrium is as follows. At *time 0*: the seller offers the price $\hat{p}$, where $\hat{p} \in (0, 1)$; the seller accepts those acceptances of the buyer such that the price is p, where either $p \leq \hat{p}$ or $p > \max\{\delta, \hat{p}\}$; and the buyer accepts any price $p \leq \hat{p}$.

If at *time 0* price $p > \hat{p}$ is offered (by the seller), accepted by the buyer, and then the seller rejects the buyer's acceptance, *then* at *time 1* (at the subgame beginning with the buyer's offer) the seller and the buyer behave as in the Conclusion 3 equilibrium (which supports price one).

If at *time 0* price $p > \hat{p}$ is offered (by the seller) and rejected by the buyer, *then* at *time 1* (at the subgame beginning with the buyer's offer) the seller and the buyer behave as in the Conclusion 2 equilibrium (which supports price zero).

If at *time 0* price $p \leq \hat{p}$ is offered (by the seller), accepted by the buyer, and then the seller rejects the buyer's acceptance, *then* at *time 1* (at the subgame beginning with the buyer's offer) the seller and the buyer behave as in the Conclusion 2 equilibrium (which supports price zero).

If at *time 0* price $p \leq \hat{p}$ is offered (by the seller) and rejected by the buyer, *then* at *time 1* (at the subgame beginning with the buyer's offer) the seller and the buyer behave as in the Conclusion 3 equilibrium (which support the price one). Q.E.D.

4. CONCLUDING REMARKS

We have shown that any price can be supported as a subgame perfect equilibrium of the alternating–offers bargaining game (without the commitment assumption) by supergame–type punishment strategies. For example, if the seller deviates from the established norm price, then the buyer will punish the seller. Moreover, if the buyer does not carry out the punishment (thus himself deviating from established norms), then the seller will punish the buyer.

Finally, it has also been shown in Muthoo (1989) that any price can be supported as a subgame perfect equilibrium of the modified Repeated–Offers bargaining game ("modified" in that the proposer can costlessly change his mind). Indeed, the equilibria of that game are also based on supergame–type punishment strategies.

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