EQUILIBRIUM PARTNER SWITCHING IN A BARGAINING MODEL WITH ASYMMETRIC INFORMATION*

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We study a model in which the seller of an indivisible object faces two potential buyers and makes an offer to either of them in each period. We find that the seller's ability to extract surplus from them depends crucially on the value of the cost of switching from one buyer to the next. If the seller is pessimistic about the buyers' valuations and there is a switching cost, however small, then the market is a natural bilateral monopoly; the second buyer is never called on. If the switching cost is zero, or if the seller is optimistic, then switching, and possibly recall of the original buyer, may occur.

1.   INTRODUCTION

Recent years have witnessed an upsurge in the interest in models of decentralized markets. This interest stems from the unsatisfactory manner in which the standard Walrasian model addresses the issue of price formation, together with the obvious practical importance of the analysis of situations where the interlinked assumptions of frictionless markets and of a large number of traders on both sides of the market fail to hold.

As the excellent account by Osborne and Rubinstein (1990) highlights, the literature has drawn attention to the importance of the trading *procedure*. An important distinction among the models proposed is between complete and incomplete information. In the latter type, more realistically, (some of) the traders are privy to some relevant information; on the other hand, to keep the analysis at a manageable level, attention has focused on models of one-sided offers, where the informed agents have little opportunity to issue signals. The model we present in this article is positioned in this latter genus: We study the behavior of three traders, one of whom owns an indivisible object, which is more valuable to the other two traders than to herself. The owner of the object is aware of this, but she does not know exactly just how valuable the object is to the two potential buyers. She may make price offers to one of the two potential buyers at a time, and if an offer is turned down, then she may go back to a buyer to whom she had previously made an offer. Trade takes place only when a price offer is accepted by a buyer.

Our understanding of this situation is important: On the one hand, this is a common way of selling a large, indivisible object or, symmetrically, of awarding valuable contracts for the supply of specified commodities; typically, the owner contacts separately a limited number of potential purchasers and may well go back to some of them if the need arises (firms, houses, and patents are often sold in this way).

On the theoretical side, our model occupies an intermediate position between two related models studied in the literature: Fudenberg and Tirole (1983), Sobel and Takahashi (1983), Fudenberg, Levine, and Tirole (FLT, hereafter) (1985), Gul, Sonnenschein, and Wilson (1986), and Hart (1989) study the one-seller, one-buyer case, while FLT (1987) study the one-seller, infinitely-many-buyers case.[2] With respect to the one-seller, one-buyer case, our model endogenizes the outside option; some bargaining models assume that (one of) the parties can leave the negotiating table and take up a given payoff option elsewhere. By modeling this outside option explicitly, we provide a framework for the analysis of the role of this option.

A fundamental difference between our model and FLT (1987) is in our idea of *recall*. Given the infinite number of buyers in FLT, it makes sense to assume that the seller is unable to recall a buyer that she has met in a previous period, since meeting the same buyer is a zero-probability event. With a finite number of buyers, this anonymity is unrealistic, and we therefore assume that the seller can recall a buyer whom she met in a previous period. This is an important feature of real-life trading processes, in which only a few individuals are potentially interested in buying.

A main message of this article is that if there is a switching cost, no matter how small, then (in the limit as the discount factor approaches one) the presence of a second potential buyer adds absolutely nothing to the seller's expected payoff. In particular, the Coase conjecture continues to hold. This conclusion does not hold if there are an infinite number of potential buyers and the seller cannot recall a buyer she has previously passed over (see Proposition 3 and Section IV in FLT, 1987), and therefore, it constitutes a specific feature of the case of a finite number of buyers. We also identify situations in which the Coase conjecture does not hold. For example, when the switching cost is zero, it does not hold in the limit as the discount factor approaches one. For nonlimiting values of the discount factor, the Coase conjecture does not hold if the switching cost is sufficiently small and the seller is sufficiently optimistic about the probability of encountering a high-valued buyer.

This article is organised as follows. After presenting a description of the model in the next section, we describe the unique perfect Bayesian equilibrium of the one-seller, one-buyer case in Subsection 2.1, and we give an overview of our results in Subsection 2.2. In Section 3 we present some preliminary results and derive the conditions that must be verified for an equilibrium with no switching to exist. Section 4 studies the existence of equilibria with switching in the three cases of high and low

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[2] The one-seller, few-buyers case, which we study, therefore stands in a similar relation to these models as the industrial organization oligopoly model relates to the monopoly and perfect competition models: potentially richer and more complex, as well as more realistic. Samuelson (1992) studies a model of bargaining in markets with an infinite number of buyers and an infinite number of sellers and obtains results similar to FLT (1987).

switching cost (Subsections 4.1 and 4.3) and a boundary case between high and low switching cost (Subsection 4.2). We conclude in Section 5.

2. THE MODEL

The market considered in the model is decentralized and operates at discrete points in time, namely, at $t = 0, 1, 2, \ldots$. It opens at time 0 with a single seller, who owns an indivisible object, and two buyers, A and B. There is no further entry into the market. The seller's valuation of the object is common knowledge and is normalised at zero. Each buyer's valuation of this object is his private information. It is common knowledge that the buyers' valuations are independently and identically distributed. Buyer i's valuation ($i = A, B$) is either high (H) or low (L), where $H > L > 0$. The probability that buyer i has high valuation is α, where $0 < \alpha < 1$.[3] The market closes at time t ($t = 0, 1, 2, \ldots$) if and only if at time t the seller and one of the two buyers agree on a price.

The trading procedure underlying the market is as follows. At each time t the following events occur instantaneously but sequentially. The seller selects a buyer i, where $i = A, B$. Then the seller makes a price offer $p \geq 0$ to buyer i. If buyer i accepts this price offer, then trade occurs between the seller and buyer i. If, on the other hand, buyer i rejects the price offer, then the process is repeated one period later, at time $t + 1$. Without loss of generality, we assume that at time 0 the seller selects buyer A.

In describing the payoffs, we allow for the possibility that *switching* from buyer i to buyer j can be costly for the seller. Formally, a switch occurs at time t if the buyer to whom the seller made the offer at time $t - 1$ is different from the buyer selected at time t.

If the seller reaches an agreement with buyer i ($i = A, B$) at time t ($t = 0, 1, 2, \ldots$) on the price p ($p \geq 0$), then the payoffs to the three players are as follows. The seller obtains a payoff of $p\delta^t - \sum_{s=1}^{t} \delta^s \phi(s)$, where δ ($0 < \delta < 1$) is the players' common discount factor, $\phi(s) = c$ if the seller switched at time s ($s = 1, 2, \ldots, t$) and $\phi(s) = 0$ otherwise—where c ($c \geq 0$) is the cost per switch, which is incurred by the seller at the time a switch occurs. Buyer i obtains a payoff of $(v_i - p)\delta^t$ and buyer j ($j \neq i$) obtains a payoff of zero, where $v_i \in \{L, H\}$ denotes buyer i's valuation. Finally, if the players stay in the market forever, then each buyer obtains a payoff of zero, while the seller's payoff equals $-\sum_{s=1}^{\infty} \delta^s \phi(s)$.

We assume that the above-described *game form*, preferences, and information structure are common knowledge among the three players. This completes the description of the model, which is a dynamic game with asymmetric information. The perfect Bayesian equilibrium concept (PBE, for short) will be employed to analyze this dynamic game.

For notational convenience, let Ω denote the set of feasible values of the parameters H, L, δ, and α. Formally, $\Omega = \{(H, L, \delta, \alpha) : L \in (0, \infty), H \in (0, \infty), L < H, \delta \in (0, 1), \text{ and } \alpha \in (0, 1)\}$.

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[3] This assumption—that the seller's initial beliefs about the valuations of the two buyers are the same—is without loss of generality.

2.1. *A Benchmark Result*: *The Single-Buyer Equilibrium.* Consider our model with only a single buyer. Such a model corresponds to the standard infinite horizon bilateral bargaining game with asymmetric information, in which the uninformed seller makes repeated offers to the informed buyer.[4] The following proposition characterizes the unique PBE in this bilateral monopoly model.

PROPOSITION 1. (*Equilibrium in the single-buyer model*). *For any* $(H, L, \delta, \alpha) \in \Omega$ *there exists a unique PBE in the single-buyer model. Let* $V : \Omega \to \Re$ *denote the function that describes the seller's equilibrium expected payoff for each* $(H, L, \delta, \alpha) \in \Omega$. *Then*

    (*i*) *for any* $(H, L, \delta, \alpha) \in \Omega$, $V(H, L, \delta, \alpha) = L$ *if* $\alpha \leq L/H$ *and* $V(H, L, \delta, \alpha) > L$ *if* $\alpha > L/H$;

    (*ii*) *for any feasible values of* H, L, *and* δ, V *is nondecreasing in* α; *and*

    (*iii*) *as* $\delta \to 1$, $V(H, L, \delta, \alpha) \to L$ *for any feasible values of* H, L *and* α.

PROOF.   See Fudenberg and Tirole (1991:409–410).   □

Following the terminology introduced in Fudenberg and Tirole (1983), we distinguish between a *soft* seller and a *tough* seller: The seller is soft if $\alpha \leq L/H$, and the seller is tough if $\alpha > L/H$. If the seller is soft, then in the unique PBE of the single-buyer model, agreement is reached immediately at the price L.[5] In contrast, if the seller is tough, then in the unique PBE of the single-buyer model, the seller first attempts to skim the high-valuation buyer; there exists a t^* such that along the equilibrium path the price offers are decreasing and strictly greater than L, and then at time t^* the seller offers the price L.

For each $(H, L, \delta, \alpha) \in \Omega$, define $c^* = V(H, L, \delta, \alpha) - L$. It will be seen that this number, c^*, constitutes a "critical" switching cost. Notice that $c^* = 0$ if the seller is soft and that $c^* > 0$ if the seller is tough.

2.2. *An Overview of the Results.* In the following Theorem we collect the main results of our article, which are proven formally in the next two sections. The Theorem illustrates how the features of the equilibrium vary as the switching cost varies.

THEOREM 1 (THE MAIN RESULTS).   (*i*)   *If* $c > c^*$, *then there exists a unique PBE. In this equilibrium the seller never switches from buyer* A *to buyer* B, *and she obtains an expected payoff of* $V(H, L, \delta, \alpha)$.[6]

    (*ii*)   *If* $c = c^*$, *then for values of* δ *sufficiently high there exists a continuum of PBE. Specifically, for any* $n = 0, 1, 2, 3, \ldots$ *there exists a* $\delta^* < 1$ *such that for any* $\delta \in (\delta^*, 1)$ *there exist a continuum of PBE in which with strictly positive probability*

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[4] This model has been studied in FLT (1985), Gul, Sonnenschein, and Wilson (1986) and Hart (1989).

[5] Since the seller makes all the price offers, he or she will never offer a price below L.

[6] It may be noted that if we extend our model by allowing the seller to *choose* the buyer to whom she makes an offer at time 0, then there exists two such PBEs: The seller selects buyer i ($i = A, B$), and then he or she never switches from buyer i to buyer j ($j \neq i$). In either PBE, her equilibrium expected payoff is $V(H, L, \delta, \alpha)$.

exactly n switches occur. In the limit, as $\delta \to 1$, *the set of PBE payoffs to the seller is equal to the interval* $[L, \alpha H + (1 - \alpha)L]$.

(iii)   *If* $c < c^*$, *then there exists a PBE in which the seller offers buyer* A *the price* H, *which is accepted by the high type with probability one, and then at time* 1 *the seller switches to buyer* B *and does not recall buyer* A. *The seller's equilibrium expected payoff is* $\alpha H + (1 - \alpha)\delta[V(H, L, \delta, \alpha) - c]$. *Furthermore, in any other PBE the seller switches with strictly positive probability from buyer* A *to buyer* B. *In the limit, as* $\delta \to 1$, *the seller's payoff in any PBE lies in the interval* $[L, \alpha H + (1 - \alpha)L]$.

Proposition 1 and part (i) of the Theorem imply that if the seller is soft, and if there is a switching cost, no matter how small, then the equilibrium outcome in the single-buyer market is immune from the competition created by the presence of an alternative buyer.[7] This result therefore could be interpreted as endogenizing the emergence of bilateral monopoly. Models of bilateral monopoly normally assume that one potential trader is present on both sides of the market but often leave unexplained why other potential traders are not sought. We have therefore identified the conditions under which the market is a *natural* bilateral monopoly.

If the seller is soft and there is no switching cost, or if the seller is tough and the switching cost is equal to the critical value c^* (which converges to zero as δ converges to one), then there exists a continuum of equilibria—which, following Ausubel and Deneckere (1989), can be interpreted as "reputational equilibria."[8] Since in many of these equilibria the seller's payoff is strictly above $V(H, L, \delta, \alpha)$, it follows that if the seller is soft and there is no switching cost, then competition from a second buyer can increase the seller's expected payoff relative to her equilibrium payoff in the single-buyer model.[9]

Lemma 2 shows that for any parameter values and in any PBE, trade occurs in finite time with probability one. However, trade need not be efficient—either because it occurs after some costly delay or because the object is sold to a buyer with low valuation when the other buyer has high valuation. The latter type of inefficiency—allocative inefficiency—may occur only if in equilibrium the seller never switches from buyer A to buyer B. It is shown in Proposition 2 that such a *no-switching* equilibrium exists if and only if $c \geq c^*$. In particular, note from part (i) of the preceding Theorem that if $c > c^*$, then the *unique* PBE is a no-switching equilibrium. Hence, if the switching cost exceeds this critical value, then with probability $\alpha(1 - \alpha)$, trade is inefficient

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[7] This result carries through immediately to the case of more than two buyers. But it does not hold if there exists an infinite number of buyers and recall by the seller is disallowed (see Proposition 3 and Section IV in FLT, 1987). Note, therefore, the importance of the finite number of buyers and recall assumptions that underlie our model.

[8] In any one of the continuum of equilibria, a price path is supported in a PBE by the credible threat of reverting to the *worst* PBE for the seller were she to deviate from the path of play—where this worst PBE satisfies the Coase conjecture. It is *as if* the seller has a "reputation" to set prices according to the given price path, where that reputation is lost the moment she deviates from that price path—hence the term *reputational equilibrium*.

[9] Some of these equilibria may exhibit nonmonotonic price dynamics. For example, the seller makes a decreasing sequence of price offers to buyer A; then, at some point, he or she switches to buyer B, offering him a higher price than the last price offered to A, at first, and then reducing it progressively. She may then switch back to A, and so on.

in the (allocative) sense that the object is sold to a low-value buyer even though the other buyer is high-value. This conclusion illustrates the importance of the switching cost: In the limit as $\delta \to 1$, even an infinitesimally small switching cost implies that, with positive probability, the unique PBE is allocatively inefficient.[10]

In any equilibrium, trade is inefficient in the other sense that with positive probability it occurs after some *costly* delay, where the costs of delayed trade arise due to discounting and the switching cost. Note that in any limiting (as $\delta \to 1$) equilibrium, trade occurs after some costly delay only if in the limiting equilibrium the seller switches at least once and the switching cost is strictly positive. However, it follows from the preceding theorem that (since $c^* \to 0$ as $\delta \to 1$) this is not possible.

Therefore, in any limiting (as $\delta \to 1$) equilibrium, trade is efficient in the sense that it occurs with probability one without any costly delay but, with just an infinitesimal switching cost, is inefficient in the allocative sense: With positive probability, not all the gains to trade are exploited.

The results stated in the Theorem are illustrated in Figure 1, which describes the nature of the equilibria as a function of c and α for fixed values of L, H, and δ. The parameter space is divided in two regions by the curve RE. In the region CC, the switching cost is high relative to the prior α. There is a unique PBE that does not involve switching. It satisfies the Coase conjecture, and in the limit, as $\delta \to 1$, the seller's expected payoff is L. Along the boundary between the regions of high and low switching costs there exists a continuum of PBE, which may be interpreted as "reputational equilibria" (RE); in the limit, as $\delta \to 1$, the set of PBE payoffs to the seller is equal to the interval $[L, \alpha H + (1 - \alpha)L]$. In the final region, SE, there exists a (nonstationary) PBE in which switching occurs exactly once; in the limit, as $\delta \to 1$, the seller's expected payoff in this equilibrium is $\alpha H + (1 - \alpha)L$. Note that in Figure 1 the curve RE tends to the horizontal axis as $\delta \to 1$; the area SE disappears in the limit as the discount factor tends to one.

3. PRELIMINARY RESULTS

3.1. *Basic Properties of Equilibrium.*   The following lemma is straightforward:

LEMMA 1.   *For any* $(H, L, \delta, \alpha) \in \Omega$ *and* $c \geq 0$, *in any PBE the seller will never*, *for any history*, *offer a price strictly less than* L.

PROOF.   Similar to the argument on page 409 in Fudenberg and Tirole (1991). $\square$

At each point in time $t \geq 1$, the first decision node, which is the "switching" decision node, belongs to the seller: she has to choose the buyer to whom an offer will be made. We restrict attention to those PBEs in which the seller does not randomize at any switching decision node. For each n, where $n = 0, 1, 2, \ldots \infty$, we define an n-switch equilibrium to be a PBE such that in equilibrium, with strictly positive probability the seller switches exactly n times between the two buyers and with

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[10] Note that $c^* = 0$ if the seller is soft and that $c^* \to 0$ as $\delta \to 1$ if the seller is tough.
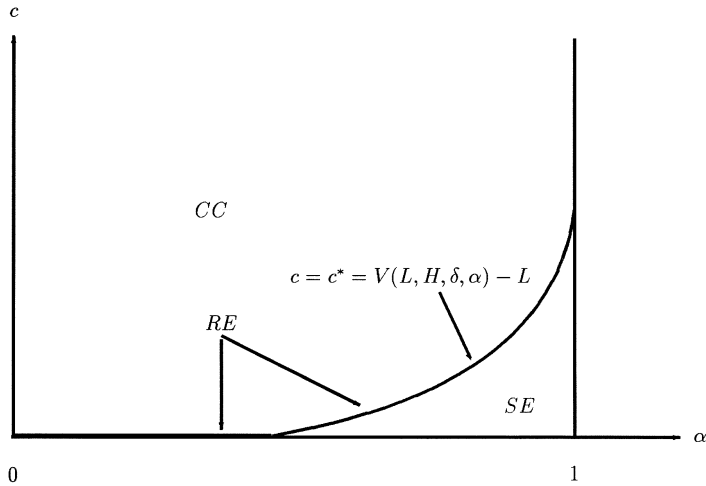
FIGURE 1

THE PARTITION OF THE (α, c) PARAMETER SPACE ACCORDING TO THE NATURE OF THE EQUI-
LIBRIA. THE THREE REGIONS ARE LABELED CC FOR COASE CONJECTURE, RE FOR REPUTATIONAL
EQUILIBRIA, AND SE FOR SWITCHING EQUILIBRIA

zero probability she switches more than n times.[11] Lemma 2(i) below establishes the
nonexistence of ∞-switch equilibria. The intuition for this result is straightforward:
Along the equilibrium path of a potential ∞-switch equilibrium, the seller would at
some finite point in time become sufficiently pessimistic[12] that, since $L > 0$, she could
profitably deviate by instead offering the price L. Lemma 2(ii) follows from a similar
line of argument.

LEMMA 2.   (i) *For any $(H, L, \delta, \alpha) \in \Omega$ and $c \geq 0$, there does not exist an ∞-switch
PBE, and (ii) for any $(H, L, \delta, \alpha) \in \Omega$ and $c \geq 0$, in any PBE trade occurs in finite
time with probability one.*

PROOF.   See the Appendix.   □

The final result of this subsection is straightforward:

LEMMA 3.   *For any $(H, L, \delta, \alpha) \in \Omega$ and $c \geq 0$, in any PBE the seller's expected
payoff is at least as large as $V(H, L, \delta, \alpha)$, the seller's equilibrium expected payoff in the
single-buyer model.*

PROOF.   The proof is trivial and hence omitted.   □

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[11] It is evident that any PBE is, for some n, an n-switch equilibrium.
[12] That is, for both $i = A$ and $i = B$, her posterior probabilistic belief that buyer i has high
valuation would become sufficiently small.

3.2. *No-Switching Equilibria.* This subsection investigates the possible existence of 0-switch equilibria; i.e., PBE in which no switching ever occurs. The following lemma characterizes the seller's expected payoff in any no-switching equilibrium.

LEMMA 4. *For any* $(H, L, \delta, \alpha) \in \Omega$ *and* $c \geq 0$, *in any 0-switch PBE the seller's expected payoff equals* $V(H, L, \delta, \alpha)$, *the seller's equilibrium expected payoff in the single-buyer model.*

PROOF. As in the single-buyer model, the equilibrium price path in a 0-switch equilibrium is given by (i) the high-type buyer A being indifferent between accepting in any given period and accepting in the following period and (ii) the fact that, by Lemmas 1 and 2, at some finite point in time the seller will offer the price L.[13] By a standard argument, it can be established that the equations that define the equilibrium path cutoff beliefs in a 0-switch equilibrium and in the single-buyer equilibrium are identical. Hence the lemma follows. ☐

The main result of this subsection is

PROPOSITION 2. *Fix* $(H, L, \delta, \alpha) \in \Omega$, *and define* $c^* = V(H, L, \delta, \alpha) - L$.

  (i) *If* $c^* > 0$ *and* $0 \leq c < c^*$, *then there does not exist a 0-switch PBE, and conversely,*
  (ii) *if* $0 \leq c^* \leq c$, *then there exists a 0-switch PBE.*

PROOF. We first establish part (i) by contradiction. Thus assume that $c^* > 0$ and $c \in [0, c^*)$, and suppose that there exists a 0-switch PBE. It follows from Lemmas 1 and 2 that with strictly positive probability along the equilibrium path at some finite time T the seller will offer buyer A the price L. Hence the seller's equilibrium expected payoff at time T is L. Suppose the seller deviates at time T from this equilibrium path by switching to buyer B. It follows from Lemma 3 that in any equilibrium beginning with the seller's offer to buyer B the seller's equilibrium expected payoff must be at least as large as $V(H, L, \delta, \alpha)$. Hence the seller's expected payoff (net of the switching cost) at time T from such a deviation is at least as large as $V(H, L, \delta, \alpha) - c$. Since $c^* > 0$ and $c \in [0, c^*)$, it follows that $V(H, L, \delta, \alpha) - c > L$, and hence the deviation is profitable.

The preceding argument fails if $c \geq c^*$. Indeed, for such values of c we now establish the existence, by construction, of a 0-switch PBE. Consider the following strategies and beliefs: The seller never, for any history, switches from buyer i to buyer j ($i, j = A, B$ and $i \neq j$). Furthermore, in any subgame beginning with the seller's offer to buyer i, play proceeds according to the (unique) PBE of the single-buyer model. It follows from the "one-shot deviation" property that this constitutes a perfect Bayesian equilibrium provided that it is optimal for the seller

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[13] It should be noted that although in our model the seller has the option to switch from buyer A to buyer B, the equilibrium price path in a 0-switch PBE is unaffected by this out-of-equilibrium option.

never to switch from buyer i to buyer j. This will be the case if for any pair of posterior beliefs μ^i and μ^j, $V(H, L, \delta, \mu^i) \geq V(H, L, \delta, \mu^j) - c$, where μ^i (respectively, μ^j) denotes the seller's posterior probability that buyer i (respectively, buyer j) has high valuation. Thus we have to establish that for any[14] $\mu^i, \mu^j \in [0, \alpha]$, $c \geq V(H, L, \delta, \mu^j) - V(H, L, \delta, \mu^i)$. This follows trivially since (by hypothesis) $c \geq c^*$ and (by Proposition 1) $V(H, L, \delta, \mu^i) \geq L$ and $V(H, L, \delta, \alpha) \geq V(H, L, \delta, \mu^j)$. □

From Proposition 1 it follows that if the seller is soft, then $c^* = 0$, and if she is tough, then $c^* > 0$. Consequently, if the seller is soft, then for any $c \geq 0$ there exists a 0-switch PBE. However, if the seller is tough, then there exists a 0-switch PBE if and only if the switching cost c is sufficiently high, namely, $c \geq c^*$. It is straightforward to verify that Proposition 2 extends immediately to the case of an arbitrary, but finite, number of buyers. Hence

COROLLARY 1. *Consider the variation of our model in which there is an arbitrary, but finite, number of buyers. Proposition 2 applies verbatim.*

4. SWITCHING EQUILIBRIA

This section investigates the possible existence of PBE in which at least one switch occurs in equilibrium.

4.1. *High Switching Cost*: $c > c^*$. In the following proposition it is established that if the switching cost c is above its critical value c^* [where $c^* = V(H, L, \delta, \alpha) - L$], then for any finite $n \geq 1$ there does not exist an n-switch PBE. Indeed, the unique PBE is a 0-switch equilibrium.

PROPOSITION 3. *For any $(H, L, \delta, \alpha) \in \Omega$ and $c > c^*$ there exists a unique PBE, which is a 0-switch equilibrium, and in which the seller's expected payoff equals $V(H, L, \delta, \alpha)$, her equilibrium expected payoff in the single-buyer model.*

PROOF.   We first establish, by contradiction, that for any finite $n \geq 1$ there does not exist an n-switch PBE. Thus suppose that there exists, for some finite $n \geq 1$, an n-switch PBE. Along the equilibrium path, immediately after the nth switch, play must proceed according to a 0-switch PBE. It follows from Lemma 4 and Proposition 1 that in such an equilibrium the seller's expected payoff must be bounded above by $V(H, L, \delta, \alpha)$. Thus, since by hypothesis $V(H, L, \delta, \alpha) - c < L$, the seller can profitably deviate from the n-switch equilibrium path by not making the nth switch but instead offering the price L. The proposition now follows immediately from Lemma 4 and Proposition 2(ii).   □

Therefore, if the seller is soft, then the existence of a switching cost, no matter how small, makes the single-buyer equilibrium outcome immune from the competition

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[14] Note that in any PBE, $\mu^i \leq \alpha$ because a low-type buyer rejects with probability one any price $p > L$, while both types will accept $p = L$. The only way that $\mu^i > \alpha$ is following the acceptance of a price $p > L$, but then the game ends.

created by a second buyer. If the seller is tough, then this conclusion equally applies provided the switching cost $c > c^*$. The following result is an immediate consequence of Proposition 3, since $c^* \to 0$ as $\delta \to 1$.

COROLLARY 2.   *Fix feasible values of H, L, and α, and assume that $c > 0$. Then in the limit, as $\delta \to 1$, the seller's expected payoff in any PBE is uniquely defined and equals L.*

Corollary 2 contains a main message of this article: If there is a switching cost, no matter how small, then (in the limit as the discount factor approaches one) the presence of a second potential buyer adds absolutely nothing to the seller's expected payoff. In particular, the Coase conjecture continues to hold. This result continues to hold in the variation of our model in which there is an arbitrary, but finite, number of buyers. This conclusion does not, however, hold if there are an infinite number of potential buyers and the seller cannot recall a buyer she has previously passed over (see Proposition 3 and Section IV in FLT, 1987), and therefore, it constitutes a specific feature of the case of a finite number of buyers.

4.2.   *Switching Cost $c = c^*$.*   We now establish that if $c = c^*$, then there exists a continuum of switching PBEs. Furthermore, we shall show that in the limit, as $\delta \to 1$, the set of PBE payoffs to the seller is *equal* to the interval $[L, \alpha H + (1 - \alpha)L]$. The main idea behind the switching equilibria is straightforward and is as follows: A path of play is supported in a PBE by the credible threat of reverting to the worst equilibrium for the seller were the seller to deviate from the path of play. This worst equilibrium is a no-switching PBE that gives the seller an expected payoff of at most $V(H, L, \delta, \alpha)$; it exists since $c = c^*$ [see Proposition 2(ii)]. Following Ausubel and Deneckere (1989), these equilibria may be interpreted as "reputational equilibria." The following result characterizes (for values of δ sufficiently high) a continuum of 1-switch (switching-without-recall) PBEs:

PROPOSITION 4.   *Fix feasible values of H, L, and α. There exists $\underline{\delta} < 1$ such that for $\delta \in (\underline{\delta}, 1)$ and any $p_0 \in [p^*, H]$, where $p^* = [V(H, L, \delta, \alpha) - (1 - \alpha)\delta L]/\alpha$, if $c = c^*$, then there exists a 1-switch PBE in which along the equilibrium path the seller offers buyer A the price p_0 at time 0, which is accepted with probability one by the high type, and then at time 1 the seller switches to buyer B, where play proceeds according to a no-switching PBE. The seller's expected payoff in this PBE is $\alpha p_0 + (1 - \alpha)\delta L$.*

PROOF.   Notice that such an equilibrium exists provided $H > p^*$. Since in the limit, as $\delta \to 1$, $p^* \to L$, and since p^* is continuous in δ, there exists a $\delta < 1$ such that for any $\delta \in [\underline{\delta}, 1]$, $H > p^*$. The remaining features of the 1-switch PBE are as follows. If the price p_0 is rejected, and if the seller were to stay with buyer A at time 1, then immediately play proceeds according to a no-switching PBE, which gives the seller an expected payoff of L. Furthermore, if at time 0 the seller were to offer a price $p_0' \neq p_0$, then immediately (before buyer A responds) play proceeds according to a no-switching PBE, which gives the seller an expected payoff of at most $V(H, L, \delta, \alpha)$. It is straightforward to verify that this constitutes a PBE.   □

We now use Proposition 4 to characterize the set of *all* PBE payoffs to the seller in the limit, as $\delta \rightarrow 1$. First, notice that in this limit the set of PBE payoffs to the seller supported by the set of 1-switch PBE constructed in Proposition 4 *equals* the interval $[L, \alpha H + (1 - \alpha)L]$. In the following lemma we establish an *upper bound* on the limiting, as $\delta \rightarrow 1$, set of *n*-switch (where $n \geq 1$) PBE payoffs to the seller:

LEMMA 5. *Fix any feasible values of the parameters H, L, α, and c. In any limiting (as $\delta \rightarrow 1$) n-switch PBE, where $n \geq 1$, the seller's expected payoff is bounded above by $\alpha H + (1 - \alpha)L$.*

PROOF. See the Appendix. □

Since the seller can guarantee a payoff of L, it thus follows that in the limit, as $\delta \rightarrow 1$, the seller's payoff in any *n*-switch PBE (where $n \geq 1$) lies in the interval $[L, \alpha H + (1 - \alpha)L]$. Hence, in conjunction with Lemma 4 and Proposition 1(iii), we have the following result[15]:

COROLLARY 3. *Fix feasible values of H and L.*

(i) *If the seller is soft (i.e., $\alpha \leq L/H$ and $c = 0$), then in the limit, as $\delta \rightarrow 1$, the set of PBE payoffs to the seller is equal to the interval $[L, \alpha H + (1 - \alpha)L]$.*

(ii) *If the seller is tough (i.e., $\alpha > L/H$), then there exists a path in the (c, δ) space such that in the limit, as $c \rightarrow 0$ and $\delta \rightarrow 1$ along this path, the set of PBE payoffs to the seller is equal to the interval $[L, \alpha H + (1 - \alpha)L]$.*

The next result shows that for any finite $n \geq 1$, provided δ is sufficiently high, there exists a continuum of *n*-switch PBEs. The result follows from a construction similar to that of Proposition 4 above: A path of play involving exactly n switches is supported by an *n*-switch "reputational equilibrium." It is evident that in an *n*-switch equilibrium, the successive price offers made to the same buyer are decreasing, both within a spell of offers to that buyer and across successive spells. However, while successive price offers made to the same buyer must be decreasing through time, the first offer made to a buyer after the seller has switched to him can be higher than the last offer made before the switch.

PROPOSITION 5. *For any feasible values of H, L, and α, and for any finite $n \geq 1$, there exists a $\delta^* < 1$ such that for any $\delta \in (\delta^*, 1)$, if $c = c^*$, then there exists a continuum of n-switch PBE.*

PROOF. See the Appendix. □

In general, the lower bound δ^* of the discount factor for which an *n*-switch equilibrium exists depends on n. This is so because the possibility of trade occuring in the *n*th period represents a costly delay for the seller, which is higher the higher is n.

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[15] It should be noted that if $\alpha \leq L/H$, then $c^* = 0$, and if $\alpha > L/H$, then $c^* \rightarrow 0$ as $\delta \rightarrow 1$.

In order for this costly delay to be preferable to a deviation to a no-switching equilibrium, the seller must be sufficiently patient (which also implies that the switching cost $c = c^*$ is sufficiently small), and hence the range of values of δ for which an n-switch equilibrium exists shrinks as n becomes large.

4.3. *Low Switching Cost*: $c < c^*$. Note that the switching cost c can be strictly less than c^* only if the seller is tough, since if the seller is soft, then $c^* = 0$. It has been established that if $c < c^*$, then there does not exist a no-switching PBE [see Proposition 2(i)]. Hence the reputational-type switching equilibria as constructed in Subsection 4.2 do not exist when $c < c^*$. However, in the next proposition we establish the existence of a switching PBE:

PROPOSITION 6.   *For any $(H, L, \delta, \alpha) \in \Omega$ such that $\alpha > L/H$ and for any $0 \le c < c^*$, there exists a 1-switch (nonstationary) PBE. Along the equilibrium path, at time 0 the seller offers buyer A the price H, which is accepted by the high-type with probability one, and then at time 1 the seller switches to buyer B and stays with him until trade occurs in finite time. The seller's equilibrium expected payoff is $\alpha H + (1 - \alpha)\delta[V(H, L, \delta, \alpha) - c]$.*

PROOF.   See the Appendix.   □

If $c < c^*$, then in any PBE the seller will switch at least once. In Proposition 6 we have established the existence (by construction) of such an equilibrium. Although we have not addressed the issue of the uniqueness of the PBE when $c < c^*$, it follows from Lemma 5 that in the limit, as $\delta \to 1$, in any PBE the seller's expected payoff will lie in the interval $[L, \alpha H + (1 - \alpha)L]$. Hence, in conjuction with Proposition 6, we have the following result:

COROLLARY 4.   *For any feasible values of H and L and for any $\alpha > L/H$, if $c = 0$, then in the limit, as $\delta \to 1$, the seller's expected payoff in any PBE lies in the interval $[L, \alpha H + (1 - \alpha)L]$. Furthermore, there exists a PBE that gives the seller an expected payoff of $\alpha H + (1 - \alpha)L$ in the limit, as $\delta \to 1$.*

5. CONCLUDING REMARKS

An important message of this article is that switching costs play a significant role in determining the nature of the market outcome. We have shown that if the seller is sufficiently pessimistic about the chance of finding a keen buyer, and if there is a switching cost, no matter how small, then the presence of a second potential buyer adds absolutely nothing to the seller's expected payoff. In particular, the Coase conjecture holds, and the outside option is useless. These conditions, therefore, could be seen as endogenously defining a bilateral monopoly situation. Once a buyer is chosen, the seller is locked in with that particular buyer, even when other buyers are potentially available. Competition *ex ante* becomes bilateral monopoly *ex post*: This situation therefore could be defined a *natural bilateral monopoly*. If the seller is more optimistic, then the presence of an alternative potential buyer can increase

the seller's expected payoff, as long as the switching cost is not too high. However, the maximum value of the switching cost that allows the seller to increase his or her expected payoff shrinks to zero as the discount factor approaches one. A loose but appealing way of interpreting this result is that in order for the seller to escape the outcome associated with the Coase conjecture, the cost of switching from one buyer to the next must be smaller than the cost of waiting for a time period to lapse.

In terms of the functioning of markets, our results are disappointing on the one hand and encouraging on the other. Given our trading mechanism, the presence of switching costs make a large finite market essentially identical to bilateral monopoly.[16] Since, in practice, markets are finite, this may cast serious doubts on the reliance on Walrasian ideas for the analysis of nonanonymous markets, however large, at least for the types of markets resembling the stylized model studied in this article. On the plus side, however, we seem to have identified in the plausible and appealing concept of the switching cost a powerful tool to reduce the multiplicity of equilibria endemic in these types of models. The existence of extra costs involved in contacting a different trade partner naturally complements the other standard friction normally introduced, namely, the cost of letting time lapse without trade being accomplished. They are both reasonable ideas, and their analytical bite is most apparent in the limit, as they become negligible, but without quite disappearing. Further analyses will clearly be needed, but we believe that the important goal of understanding the functioning of markets with a finite number of traders will hinge around the twin ideas of discounting and switching costs.[17]

Given the importance of this kind of study, alternative models of small, decentralized markets with asymmetric information ought to be studied, including models that allow for alternating offers and models that assume common values. The purpose of this research program is to explore, in a precise and rigorous manner, the impact and role of asymmetric information and competition on the equilibrium market outcomes. These, to quote Robert Wilson (1987), "are building blocks in the construction of a genuine theory of price formation" (pp. 33–34).

In our model it is implicitly assumed that the seller has located two buyers and incurs a cost when she switches between them. Consider an extension of our model in which the seller has already located one buyer and has to decide whether or not to search for a second buyer *before* negotiating with any buyer. If the seller chooses to search for a second buyer, then she incurs a search cost $s > 0$ and locates a second buyer. The game continues according to our model if the seller locates a second buyer, but otherwise it continues according to the standard bilateral monopoly model (mentioned in Subsection 2.1). Using the results derived in Sections 3 and 4, we can sketch the main properties of the limiting PBE, as $\delta \to 1$, of this simple extension of our model. It follows from Proposition 1(iii) and Corollary 2 that for any search cost $s > 0$ and any switching cost $c > 0$, in the unique limiting PBE, the seller will

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[16] This is reminiscent of the well-known Diamond (1971) result: A large market with price setting firms in which consumers have a strictly positive search cost will generate the monopoly outcome.

[17] We note that the discontinuity in the equilibrium set at the zero switching cost ($c = 0$) parallels closely the situation encountered with respect to the other source of friction, the discount factor, at $\delta = 1$. This confirms that the appropriate way to study these types of market games is by analyzing their limit behavior.

choose not to search for a second buyer. Now suppose that $c = 0$. In this case, there exists a continuum of limiting PBEs. If the search cost $s > \alpha(H - L)$, then in any such limiting equilibrium the seller will not search for a second buyer. However, if $s < \alpha(H - L)$, then there exists limiting equilibria in which the seller will search for a second buyer.[18]

<center>APPENDIX</center>

PROOF OF LEMMA 2. We shall only prove part (i), since—given Lemma 2(i)—part (ii) can be proven along very similar lines. We argue by contradiction and thus suppose that there exists an ∞-switch PBE. Let $\langle p_t \rangle_{t=0}^{\infty}$ denote the infinite sequence of price offers made along the equilibrium path. And let $\langle \mu_t^A, \mu_t^B \rangle_{t=0}^{\infty}$ denote the seller's posterior beliefs along the equilibrium path, where μ_t^i is the probability at time t assigned to high-type buyer i. For each t $(t = 0, 1, 2, \ldots)$, $p_t > L$, for otherwise a contradiction is obtained by appealing to Lemma 1.[19] Consequently, for each i $(i = A, B)$, $\mu_{t+1}^i \leq \mu_t^i$ for all $t = 0, 1, 2, \ldots$—since $p_t > L$ for all t implies that the low type of either buyer never trades.

We first note that if, at some finite point in time T, $\mu_T^A \leq \alpha^*$ and $\mu_T^B \leq \alpha^*$, where $\alpha^* = 1 - \sqrt{1 - (L/H)}$, then the seller can profitably deviate from the equilibrium path at time T by instead offering the price L. This is so because if, at time T, $\mu_T^A \leq \alpha^*$ and $\mu_T^B \leq \alpha^*$, then the seller's expected payoff along the equilibrium path at time T will be bounded above by $\alpha^* H + (1 - \alpha^*)\alpha^* \delta H$, which is strictly less than $\alpha^*(2 - \alpha^*)H$, which, in turn, is equal to L. Note that $\alpha^* > 0$, since $L > 0$.

Consequently, if $\mu_0^A = \mu_0^B = \alpha \leq \alpha^*$, then a contradiction is obtained. It follows from the Claim below that if $\mu_0^A = \mu_0^B = \alpha > \alpha^*$, then there exists a finite point in time T such that $\mu_T^A \leq \alpha^*$ and $\mu_T^B \leq \alpha^*$. Hence a contradiction is also obtained if $\alpha > \alpha^*$.

CLAIM. For each $i = A, B$, there exists $T_i \in \{0, 1, 2, 3, \ldots\}$ such that $\mu_{T_i}^i \leq \alpha^*$.

PROOF OF THE CLAIM. It is helpful first of all to introduce some notation and define some concepts. For each $i = A, B$, define $\Sigma_i \subseteq \{0, 1, 2, 3, \ldots\}$ as follows: $\Sigma_i = \{t \geq 0$: at time t the seller offers the equilibrium price p_t to buyer $i\}$. Furthermore, for any $t \in \Sigma_i$, let γ_t^i denote the equilibrium probability with which the high-type buyer i accepts the equilibrium price offer p_t. It follows from Bayes' rule that for any $t \in \Sigma_i$,

$$(A.1) \qquad \gamma_t^i = \frac{\mu_t^i - \mu_{t+1}^i}{\mu_t^i (1 - \mu_{t+1}^i)}$$

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[18] Due to the underlying stationary structure of our bargaining game, these results continue to be valid if, instead, the seller is allowed to choose to search for a second buyer at any point during her negotiations with the first buyer whom she has already located.

[19] Lemma 1 implies that for all t, p_t cannot be strictly less than L. Furthermore, if for some t, $p_t = L$, then the game ends at time t with probability one—since either buyer of low or high type would accept this price offer with probability one, which contradicts the supposition that this is an ∞-switch PBE.

Define for each $i = A, B$ and $t = 0, 1, 2, \ldots$:

$$\theta_t^i = \begin{cases} \mu_t^i \gamma_t^i & \text{if } t \in \Sigma_i \\ 0 & \text{if } t \in \Sigma_j \end{cases}$$

For each $t \in \Sigma_i$, θ_t^i is the equilibrium probability that at time t the equilibrium price p_t is accepted (since $p_t > L$ implies that the low-type buyer i rejects p_t with probability one). Now define for each $i = A, B$, for any $t = 0, 1, 2, \ldots$ and for any $k \geq 1$:

$$\pi^i(t, k) = \theta_t^i + \sum_{n=0}^{k-1} \theta_{t+n+1}^i \left[\prod_{s=0}^{n} (1 - \theta_{t+s}^i) \right]$$

$\pi^i(t, k)$ is the equilibrium probability with which buyer i and the seller trade between times t and $t + k$ (inclusive), conditional on buyer j rejecting with probability one any price offered to him between times t and $t + k$ (inclusive). Hence $\pi^i(t, k)$ is an *upper bound* on the equilibrium probability that an offer is accepted by buyer i between times t and $t + k$ (inclusive). Since $1 - \theta_t^i \leq 1$ for all $t = 0, 1, 2, \ldots$, it follows that

$$\pi^i(t, k) \leq \sum_{n=0}^{k} \theta_{t+n}^i$$

If $\{t, t + 1, t + 2, \ldots, t + k\} \subseteq \Sigma_i$, then [after substituting for γ_t^i in θ_t^i using (A.1)], we obtain that

$$\sum_{n=0}^{k} \theta_{t+n}^i = \sum_{n=0}^{k} \frac{\mu_{t+n}^i - \mu_{t+n+1}^i}{1 - \mu_{t+n+1}^i} \leq \frac{\mu_t^i - \mu_{t+k+1}^i}{1 - \alpha}$$

where the latter inequality is obtained by using the fact that $\mu_t^i \leq \alpha$ for all $t \geq 0$. Hence, since $\theta_t^i = 0$ if $t \in \Sigma_j$, it follows that

(A.2) $$\pi^i(t, k) \leq \frac{\mu_t^i - \mu_{t+k+1}^i}{1 - \alpha}$$

We now establish the following preliminary result:

STEP. *Fix $i \in \{A, B\}$. If $\mu_t^i > \alpha^*$ for all $t = 0, 1, 2, \ldots$, then $\forall k \geq 1$ and $\forall \epsilon > 0$ $\exists T \geq 0$ such that $\forall t > T$, $\pi^i(t, k) < \epsilon$.*

PROOF OF THE STEP. Given inequality (A.2), it suffices to show that $\forall k \geq 1$ and $\forall \epsilon > 0$ $\exists T \geq 0$ such that $\forall t > T$, $\mu_t^i - \mu_{t+k+1}^i < \epsilon(1 - \alpha)$. Suppose, to the contrary, that $\exists k \geq 1$ and $\exists \epsilon > 0$ such that $\forall T \geq 0$ $\exists t(T) > T$ such that $\mu_{t(T)}^i - \mu_{t(T)+k+1}^i \geq \epsilon(1 - \alpha)$. Define the sequence $\langle t_n \rangle$ as follows: $t_1 = t(0)$, and $t_{n+1} = t(t_n + k + 1)$ for $n = 1, 2, 3, \ldots$. Thus, $\mu_{t_n+k+1}^i \leq \mu_{t_n}^i - \epsilon(1 - \alpha)$ for all $n = 1, 2, \ldots$. Hence, since the sequence $\langle \mu_t^i \rangle$ is nonincreasing (and since $t_{n+1} > t_n + k + 1$ for all $n = 1, 2, 3, \ldots$), it follows that $\mu_{t_n+k+1}^i \leq \mu_{t_1}^i - n\epsilon(1 - \alpha)$ for all $n = 1, 2, \ldots$. Hence, when

$$n = \left[\frac{\mu_{t_1}^i - \alpha^*}{\epsilon(1 - \alpha)} \right]_{\text{int}} + 1$$

$\mu_{t_n+k+1}^i \leq \alpha^*$, which is a contradiction. This establishes the Step.

We are now ready to establish the Claim. First suppose, to the contrary, that there exists $T_A \geq 0$ such that $\mu_{T_A}^A \leq \alpha^*$ *and* that for any $t \geq 0$, $\mu_t^B > \alpha^*$. This implies that for any $t > T_A$, the seller's equilibrium payoff $V_S(t)$ at time t satisfies

(A.3) $$V_S(t) \leq \alpha^* H + \pi^B(t, k)H + \delta^{k+1} H$$

This is because the seller's equilibrium payoff at time $t > T_A$ from trading with buyer A cannot exceed $\mu_{T_A}^A H$ (which, by supposition, is less than or equal to $\alpha^* H$), and her equilibrium payoff at time t from trading with buyer B cannot exceed $\pi^B(t, k)H + \delta^{k+1} H$—since $\pi^B(t, k)$ is an upper bound on the equilibrium probability that an offer is accepted by buyer B between times t and $t + k$ (inclusive).

Since $\alpha^*(2 - \alpha^*)H = L$, the right-hand side (RHS) of inequality (A.3) is *strictly less* than L if $\pi^B(t, k) + \delta^{k+1} < \alpha^*(1 - \alpha^*)$. Hence, for any $t > T_A$, $V_S(t) < L$ if $\pi^B(t, k) < \alpha^*(1 - \alpha^*) - \delta^{k+1}$. Since $\alpha^*(1 - \alpha^*) > 0$, there exists a $\bar{k}$ such that for any $k > \bar{k}$, $\alpha^*(1 - \alpha^*) - \delta^{k+1} > 0$. It follows from the Step above that for any $k > \bar{k}$ and $\epsilon = \alpha^*(1 - \alpha^*) - \delta^{k+1}$, there exists $T \geq 0$ such that for any $t > T$, $\pi^B(t, k) < \epsilon$. Hence there exists $t > T_A$ such that $V_S(t) < L$, which is a contradiction (since the seller can benefit by deviating at time t by instead offering the price L).

Now suppose that there exists $T_B \geq 0$ such that $\mu_{T_B}^B \leq \alpha^*$ *and* that for any $t \geq 0$, $\mu_t^A > \alpha^*$. A symmetric argument to that given above—with the roles of A and B reversed—establishes a contradiction.

Finally, suppose that for each $i = A, B$ and for any $t \geq 0$, $\mu_t^i > \alpha^*$. For any $t \geq 0$, the seller's equilibrium payoff $V_S(t)$ at time t satisfies

(A.4) $$V_S(t) \leq \pi^A(t, k)H + \pi^B(t, k)H + \delta^{k+1} H$$

Hence, for any $t \geq 0$, $V_S(t) < L$ if $\pi^A(t, k) + \pi^B(t, k) < \alpha^*(2 - \alpha^*) - \delta^{k+1}$. Since $\alpha^*(2 - \alpha^*) > 0$, there exists k' such that for any $k > k'$, $\alpha^*(2 - \alpha^*) - \delta^{k+1} > 0$. Letting $\epsilon = [\alpha^*(2 - \alpha^*) - \delta^{k+1}]/2$, it follows from the Step above that for any $k > k'$ there exists $T_i' \geq 0$ such that for any $t > T_i'$, $\pi^i(t, k) < \epsilon$. Hence there exists $t > \max\{T_A', T_B'\}$ such that $V_S(t) < L$, which is a contradiction.   □

PROOF OF LEMMA 5.   Fix any feasible values of the parameters H, L, α, and c and any limiting (as $\delta \to 1$) n-switch PBE, where $n \geq 1$. The argument to follow concerns the equilibrium *path* of this limiting PBE.

Let t_k ($k = 1, 2, \ldots, n$) denote the time at which the kth switch occurs; note that $t_k \geq t_{k-1} + 1$ ($k = 1, 2, \ldots, n$), where $t_0 = 0$. Furthermore, let λ_k ($k = 0, 1, 2, 3, \ldots, n - 1$) denote the probability with which the seller trades between times t_k and $t_{k+1} - 1$ (inclusive). Let buyer i denote the buyer to whom the seller makes the nth switch; since (by assumption) at time 0 the seller makes a price offer to buyer A, it follows that $i = A$ if n is even and $i = B$ if n is odd.

Define a number N as follows: $N = n/2$ if n is even, and $N = (n + 1)/2$ if n is odd. Now define for each $m = 1, 2, \ldots, N$:

$$Z_{n-(2m-1)} = 1 - \prod_{s=1}^{m} \left(1 - \lambda_{n-(2s-1)}\right)$$

By definition, $Z_{n-(2m-1)}$ is the probability with which the seller trades with buyer j (where $j \neq i$) from time $t_{n-(2m-1)}$ onward. Since the seller does not trade with the low-type buyer j, it follows that

(A.5) $$Z_{n-(2N-1)} \leq \alpha$$

For any k ($k = 0, 1, 2, \ldots, n-1$) such that $\lambda_k > 0$, consider the sequence of prices accepted with positive probability by buyer g between times t_k and $t_{k+1} - 1$ (inclusive), where $g = A$ if k is even and $g = B$ if k is odd. Since the high-type buyer g is indifferent between accepting any one of *these* prices, it follows that (in this limiting equilibrium) these prices are identical; let p_k denote this price.

Let V_k ($k = 0, 1, 2, \ldots, n$) denote the seller's expected payoff (along the equilibrium path) at the beginning of time t_k. We have to establish that $V_0 \leq \alpha H + (1 - \alpha)L$. The Claim stated below establishes that $V_0 \leq Z_{n-(2N-1)}H + \left(1 - Z_{n-(2N-1)}\right)L$. Hence, using (A.5), it follows that $V_0 \leq \alpha H + (1 - \alpha)L$.

CLAIM. $V_0 \leq Z_{n-(2N-1)}H + [1 - Z_{n-(2N-1)}]L$.

PROOF OF THE CLAIM.   We first establish the result stated in the following Step.

STEP.   *Define a number N^* as follows: $N^* = N$ if n is even, and $N^* = N - 1$ if n is odd. Then, for each $m = 1, 2, 3, \ldots, N^*$: (i) $V_{n-(2m-1)} \leq U_{n-(2m-1)}$, and (ii) $V_{n-2m} \leq U_{n-(2m-1)}$, where $U_{n-(2m-1)} = Z_{n-(2m-1)}H + [1 - Z_{n-(2m-1)}]L$.*

PROOF OF THE STEP.   (By induction.) We first establish the Step for $m = 1$. Then we assume that the Step is true for $m = r$, where $1 \leq r < N^*$, and deduce that it is true for $m = r + 1$.

It follows from Lemma 4 and Proposition 1(iii) that $V_n = L$. Hence it follows that if $\lambda_{n-1} > 0$, then $V_{n-1} \leq \lambda_{n-1}p_{n-1} + (1 - \lambda_{n-1})L$, and if $\lambda_{n-1} = 0$, then $V_{n-1} = L$.[20] This implies that $V_{n-1} \leq U_{n-1}$. Now note that if $\lambda_{n-2} > 0$, then $V_{n-2} \leq \lambda_{n-2}p_{n-2} + (1 - \lambda_{n-2})V_{n-1}$, and if $\lambda_{n-2} = 0$, then $V_{n-2} = V_{n-1}$. Furthermore, if $\lambda_{n-2} > 0$, then $H - p_{n-2} \geq (1 - \lambda_{n-1})(H - L)$, which implies that $p_{n-2} \leq U_{n-1}$. Hence we obtain that $V_{n-2} \leq U_{n-1}$. In summary, we have thus established the Step for $m = 1$.

Now assume that the Step is true for $m = r$, where $1 \leq r < N^*$. We shall deduce that it is true for $m = r + 1$. Since part (ii) of the Step holds for $m = r$, it follows that if $\lambda_{n-(2r+1)} > 0$, then

(A.6) $$V_{n-(2r+1)} \leq \lambda_{n-(2r+1)}H + \left(1 - \lambda_{n-(2r+1)}\right)U_{n-(2r-1)}$$

and if $\lambda_{n-(2r+1)} = 0$, then $V_{n-(2r+1)} \leq U_{n-(2r-1)}$. Since the right-hand side of (A.6) is equal to $U_{n-(2r+1)}$—and since $U_{n-(2r-1)} \leq U_{n-(2r+1)}$—we have thus established part (i) of the Step for $m = r + 1$. Hence it follows that if $\lambda_{n-(2r+2)} > 0$, then

(A.7) $$V_{n-(2r+2)} \leq \lambda_{n-(2r+2)}p_{n-(2r+2)} + \left(1 - \lambda_{n-(2r+2)}\right)U_{n-(2r+1)}$$

and if $\lambda_{n-(2r+2)} = 0$, then $V_{n-(2r+2)} \leq U_{n-(2r+1)}$. Furthermore, if $\lambda_{n-(2r+2)} > 0$, then $H - p_{n-(2r+2)} \geq \left(1 - Z_{n-(2r+1)}\right)(H - L)$, which implies that $p_{n-(2r+2)} \leq U_{n-(2r+1)}$.

[20] Recall that p_k is defined above only for any k ($k = 0, 1, 2, \ldots, n-1$) such that $\lambda_k > 0$. As such, the expression $\lambda_k p_k$ is not defined for any k ($k = 0, 1, 2, \ldots, n-1$) such that $\lambda_k = 0$.

It thus follows [using (A.7)] that $V_{n-(2r+2)} \leq U_{n-(2r+1)}$. We have thus established part (ii) of the Step for $m = r + 1$. This completes the proof of the Step.

It follows immediately from the Step above that if n is even, then $V_0 \leq Z_1 H + (1 - Z_1)L$. Now suppose that n is odd. If $\lambda_0 > 0$, then $V_0 \leq \lambda_0 H + (1 - \lambda_0)V_1$, and if $\lambda_0 = 0$, then $V_0 = V_1$. It follows from the Step above that if n is odd, then $V_1 \leq U_2$. Since $\lambda_0 H + (1 - \lambda_0)U_2 = U_0$ and $U_2 \leq U_0$, it follows that $V_0 \leq U_0$, where $U_0 = Z_0 H + (1 - Z_0)L$. This completes the proof of the Claim.   □

PROOF OF PROPOSITION 5.   (By construction.) Fix $n \geq 1$, and assume (without loss of generality) that n is odd.[21] Since Proposition 4 has established the existence of a continuum of 1-switch PBEs, we assume that $n \geq 3$. First we introduce some parameters. Fix $\gamma \in [0, 1)$ and define $\hat{\mu} = \alpha(1 - \gamma)^{\hat{m}}/[1 - \alpha\gamma \sum_{s=0}^{s=\hat{m}-1}(1 - \gamma)^s]$, where $\hat{m} = (n - 1)/2$. It is easy to verify that since $\gamma < 1$, $\hat{\mu} > 0$. Furthermore, fix $\eta \in [\hat{p}, H]$, where $\hat{p} = [V(H, L, \delta, \hat{\mu}) - (1 - \hat{\mu})\delta L]/\hat{\mu}$. If the seller is tough, then fix $\gamma = 0$, but if the seller is soft, then choose any $\gamma \in [0, 1)$.

We begin by describing the path of play of the (proposed) n-switch PBE. At each time t $(t = 0, 1, 2, \ldots n - 3, n - 2)$ the seller offers a price $p_t > L$, where p_t is specified below. At time 0 the seller makes her offer to buyer A. Thereafter, if t is odd, then the seller switches to buyer B, while if t is even, she switches to buyer A. At any such time $t \leq n - 2$, the high-type buyer i accepts the price offer p_t with probability γ, and the low-type buyer i rejects p_t with probability one (where $i = A$ if t is even and $i = B$ if t is odd). Then, at time $t = n - 1$, the seller switches to buyer A, and play immediately proceeds according to a 1-switch PBE: The seller offers buyer A a price $p_{n-1} = \eta$, which is accepted with probability one by the high-type buyer A (and rejected by the low-type buyer A with probability one), and then at time $t = n$, the seller switches to buyer B, where play proceeds according to a no-switching PBE. Let μ_t^i denote the seller's posterior belief (at the beginning of time t) that buyer i is high-type. Given $p_{n-1} = \eta$, the prices $p_0, p_1, p_2, \ldots, p_{n-2}$ are defined recursively as follows: For each $m = 0, 1, 2, \ldots, (n - 3)/2$,

$$H - p_{2m} = \delta^2(1 - \gamma\mu_{2m+1}^B)(H - p_{2m+2})$$

and for each $m = 0, 1, 2, \ldots, (n - 3)/2$,

$$H - p_{2m+1} = \delta^2(1 - \gamma\mu_{2m+2}^A)(H - p_{2m+3})$$

where p_n is the first price offer in the single-buyer equilibrium (where p_n will depend on the value of μ_n^B)

If the seller deviates at any point along the above-described path of play, then immediately play proceeds according to a no-switching equilibrium.

Along the path of play the seller's posterior beliefs are derived using Bayes rule. It is easy to show that for $m = 1, 2, 3, 4, \ldots, (n - 3)/2, (n - 1)/2$,

$$\mu_{2m-1}^A = \frac{\alpha(1 - \gamma)^m}{1 - \alpha\gamma \sum_{s=0}^{s=m-1}(1 - \gamma)^s} \qquad \text{and} \qquad \mu_{2m-1}^A = \mu_{2m}^A$$

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[21] A slightly modified argument establishes the proposition when n is even.

Furthermore, for $m = 0, 1, 2, 3, 4, \ldots, (n-5)/2, (n-3)/2$,

$$\mu^B_{2m+2} = \frac{\alpha(1-\gamma)^{m+1}}{1 - \alpha\gamma \sum_{s=0}^{s=m}(1-\gamma)^s} \qquad \text{and} \qquad \mu^A_{2m+1} = \mu^A_{2m}$$

Note that $\mu^B_n = \mu^B_{n-1} = \mu^A_{n-1} = \hat{\mu} > 0$.

It is straightforward to verify that the above-described path of play is in a perfect Bayesian equilibrium. The equilibrium path beliefs described above have been derived using Bayes rule. Neither high-type buyer at any point will have an incentive to deviate because the two recursive equations that define the equilibrium path prices ensure (by construction) that the high-type buyers are indifferent between accepting and rejecting equilibrium price offers.[22] Finally, provided δ is sufficiently high, the seller will not have any incentive to deviate at any point of this path of play. Let us verify that this is the case at the beginning of time $t = n$ (when the seller has to decide whether to switch from buyer A to buyer B). If she conforms with the above-described path of play, she will switch to buyer B, and her payoff equals $V(H, L, \delta, \hat{\mu}) - c^*$. However, if she deviates and stays with buyer A, then her payoff is L. If the seller is soft, $c^* = 0$, and hence the seller cannot profit from deviating [for any $\gamma \in [0, 1)$—i.e., for any $0 < \hat{\mu} \leq \alpha$]. If the seller is tough, $c^* = V(H, L, \delta, \alpha) - L > 0$, and hence the deviation is not profitable provided $V(H, L, \delta, \hat{\mu}) \geq V(H, L, \delta, \alpha)$. This is true if and only if $\hat{\mu} = \alpha$ (which follows since we have fixed $\gamma = 0$ when the seller is tough).

Let us also verify that when the seller is tough, she does not have an incentive to deviate at time 0 (provided that δ is sufficiently high). If she does deviate at time 0, then play immediately proceeds according to a no-switching PBE. Hence, her payoff from such a deviation is $V(H, L, \delta, \alpha)$. By the construction of the n-switch PBE, when the seller is tough, $\gamma = 0$ and $\hat{\mu} = \alpha$. Hence it follows that when the seller is tough, her payoff in the n-switch PBE is

$$W = -c(\delta + \delta^2 + \cdots + \delta^{n-1}) + \delta^{n-1}\Big\{\alpha\eta + (1-\alpha)\delta[V(H, L, \delta, \alpha) - c]\Big\}$$

Since $c = c^*$, it follows that

$$W = -c^*(\delta + \delta^2 + \cdots + \delta^{n-1}) + \delta^{n-1}[\alpha\eta + (1-\alpha)\delta L]$$

Since $c^* \to 0$ as $\delta \to 1$, it follows that

$$\lim_{\delta \to 1} W = \alpha\eta + (1-\alpha)L$$

Furthermore, it follows from Proposition 1 that

$$\lim_{\delta \to 1} V(H, L, \delta, \alpha) = L$$

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[22] If price p_t is rejected by buyer i, then, with probability $\gamma\mu^j_{t+1}$, the price p_{t+1} will be accepted by buyer j next period and hence, with (the complement) probability $1 - \gamma\mu^j_{t+1}$, buyer i will be recalled in period $t + 2$.

Since $\eta > L$, it thus follows that

$$\lim_{\delta \to 1} W > \lim_{\delta \to 1} V(H, L, \delta, \alpha)$$

This implies that there exists a $\hat{\delta} < 1$ such that for any $\delta \in (\hat{\delta}, 1)$,

$$W > V(H, L, \delta, \alpha)$$

as required. $\square$

PROOF OF PROPOSITION 6.   It is straightforward to verify that the strategies and beliefs described below are in a PBE. Following Osborne and Rubinstein (1990), it is convenient to describe these (nonstationary) strategies and beliefs using the finite automata language involving "states" and "transition rules." Play begins, at time 0, in state s_A.

State s_i ($i = A, B$). In this state the seller believes that each buyer, A and B, is high-type with (the prior) probability α. She will offer buyer i the price H. The high-type buyer i accepts any price $p \leq H$, whereas the low-type buyer i accepts any price $p \leq L$.

State $\gamma(\mu)$ ($\gamma = A, B$ and $0 \leq \mu < 1$). In this state the seller is with buyer γ, and she believes that buyer γ is high-type with probability μ and that buyer γ^* (where $\gamma^* \neq \gamma$, $\gamma^* = A, B$) is high-type with probability zero. The seller always selects buyer γ and will never, for any history, switch from buyer γ to buyer γ^*. The seller's and buyer γ's behavior is characterized by the unique equilibrium in the single-buyer model (with the seller's "initial" belief being that buyer γ is high-type with probability μ).

State $\gamma^*(\mu)$ ($\gamma^* = A, B$ and $0 \leq \mu < 1$). In this state the seller is with buyer γ^*, and she believes that buyer γ^* is high-type with probability zero and that buyer γ (where $\gamma \neq \gamma^*$, $\gamma = A, B$) is high-type with probability μ. She offers buyer γ^* a price L if $L \geq \delta[V(H, L, \delta, \mu) - c]$, and if $L < \delta[V(H, L, \delta, \mu) - c]$, then she will offer him some price $p > L$.

Transitions from state s_i ($i = A, B$).

  1. If a price $p > H$ is offered and rejected, then play remains in state s_i.
  2. If the seller selects buyer j ($j \neq i$, $j = A, B$), then play immediately switches to state s_j.
  3. If a price $p \leq H$ is offered and rejected, then play immediately switches to state $\gamma(\alpha)$ with $\gamma = j$, where $j \neq i$, $j = A, B$.

Transition from state $\gamma(\mu)$ ($\gamma = A, B$ and $0 \leq \mu < 1$). If the seller selects buyer γ^* (where $\gamma^* \neq \gamma$, $\gamma^* = A, B$), then play immediately switches to state $\gamma^*(\mu)$.

Transitions from state $\gamma^*(\mu)$ ($\gamma^* = A, B$ and $0 \leq \mu < 1$)

  1. If buyer γ^* rejects any offer *and* $V(H, L, \delta, \mu) - c > L$, then play immediately switches to state $\gamma(\mu)$ (where $\gamma \neq \gamma^*$, $\gamma = A, B$).
  2. If the seller selects buyer γ ($\gamma \neq \gamma^*$, $\gamma = A, B$), then play immediately switches to state $\gamma(\mu)$.

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