

*Strategic Foundations of General Equilibrium: Dynamic Matching and Bargaining Games.* by Douglas Gale. Cambridge University Press, 2000. xi+219 pp. £14.95 (paperback).

Adam Smith's classic treatise *An Inquiry into the Nature and Causes of the Wealth of Nations* effectively laid down the foundations of the economics discipline. Implicit in his analyses lies a theory of perfect competition, which was subsequently developed by Alfred Marshall and Leon Walras. The theory of perfectly competitive equilibrium (or competitive equilibrium, for short), which was built on the key insights of these great economic thinkers, was developed by Kenneth Arrow, Gerard Debreu and Lionel McKenzie. There is no doubt that this theory of competitive equilibrium occupied until very recently a central place in economic theory and analysis. It is a theory of resource allocation achieved *via* the market mechanism. This theory, it can be argued, formally and rigorously established an important fundamental insight provided by Adam Smith that individuals acting in their own self-interest, will through the invisible hand of the market mechanism, achieve an efficient allocation of resources. This is an amazingly deep insight, and even today fills one with much awe.

As is well-known amongst most economists, the theory of competitive equilibrium however has some fundamental weaknesses and shortcomings. One major weakness is that it fails to provide a convincing theory of price *formation*. This may sound strange since a theory that aims to solve the resource allocation problem via the market mechanism should (by definition, I suppose) be based on prices. Indeed, prices do lie at the very core of the theory of competitive equilibrium. They comprise its central element. However — as is clear from, for example, Gerard Debreu's classic *The Theory of Value*, which provides an elegant, rigorous, complete and beautiful exposition of it — the theory of competitive equilibrium is silent on the issue of *who* sets the prices. The theory has nothing significant to say on that important issue, and this is partly because the theory's approach, perspective and contribution lie elsewhere.

Another important weakness of the theory of competitive equilibrium is that it does not provide a rationale (or explanation) for itself; that is, the circumstances and conditions under which it is applicable. Let me give but just two examples to illustrate this weakness. It is often informally argued that there should be a large number of agents in the economy in order for this theory to be applicable. But, the unanswered

question is “how large”? It is also informally argued that agents should have perfect information in order for this theory to be applicable. Literally or approximately? Since not all agents in the real world will literally have perfect information it should be applicable to economies where some agents are slightly imperfectly informed; for otherwise the theory is inapplicable to any real economy — and hence, useless. But then, what exactly are the informational requirements for the theory to be applicable?

Douglas Gale’s book is part of the large and growing literature that studies models of economies in which the *process* (or *procedure*) through which prices are formed is central to these models and analyses, unlike in the theory of competitive equilibrium. While a significant part of the motivation and focus of this literature is to study the role of *frictions* on equilibrium outcomes, an important part of the analyses is to address some of the weaknesses and shortcomings of the theory of competitive equilibrium. In particular, a key aim is to explore the conditions under which the (game-theoretic) equilibrium outcome in such models is close to the competitive equilibrium outcome. For example, to see whether or not this is the case when the frictions become arbitrarily small.

The literature to which I have referred can be called the literature on *dynamic matching and bargaining games* (or DMBGs, for short). Like the main model studied in Gale’s book, a central characteristic of such DMBGs is that agents in the model economy are matched in pairs through some matching process, and then each matched pair of agents bargain over the terms of trade. The DMBGs differ not only with respect to the details of the adopted matching and bargaining processes, but also with respect to the underlying economic environment.

Douglas Gale begins his book in chapter 1 with a most stimulating, clear and insightful discussion of the *raison d’etre* for these DMBGs. In particular, he provides a clear and coherent account of the need to study what he calls the *strategic* foundations for competitive equilibrium. This chapter should be compulsory reading for all students of economics (even advanced undergraduates), and all academic economists who think deeply about economic theory and analyses.

In chapter 1 Gale also lays out the main motivation for the analyses in this book. In particular, it is noted that many of the main models studied in the literature such as in Rubinstein and Wolinsky’s 1995 *Econometrica* paper and Gale’s 1996 *Econometrica* papers assume that there exists a non-atomic continuum of traders. It is important to relax this *continuum assumption* and study models with a finite number of traders. A main objective is to show that there exist conditions under which in the limit as the number of traders becomes arbitrarily large, the equilibrium outcome in a DMBG converges to the competitive equilibrium outcome. That is, Gale’s main objective is to derive a *limit theorem*, rather than a *theorem in the limit* (where the latter establishes

such an “equivalence” in a model economy with a continuum of traders). Since the continuum assumption is not literally speaking plausible, such a limit theorem is important in order to provide a rationale for the applicability of the theory of competitive equilibrium.

In chapter 2, Gale constructs an interesting and somewhat novel DMBG of an economy characterised by a *finite* number of traders. The description is rigorous and clear. Any student interested in this literature would learn a great deal about the modelling assumptions that characterise it by studying the specific model described here. The bulk of chapter 2 contains a thorough, careful and rigorous analysis of the equilibria of this dynamic game. In particular, the focus is on deriving conditions under which a limit theorem can be established. One learns a lot from Gale’s arguments and analyses, some of which should prove useful when studying models with alternative assumptions about the matching and bargaining processes, and about the economic environment.

In order to establish his limit theorem, Gale argues that he needs to adopt three restrictions on the equilibrium strategies of the traders. One is the familiar restriction that is built into the *Markov* perfect equilibrium concept, namely, that a trader’s equilibrium behaviour at any point in time may be conditioned only on the payoff-relevant parts of the history of the game. The other two restrictions are not standard in game theory, but are restrictions that formalise two key properties that are associated with competitive markets, namely, that each trader is small and that such markets are anonymous mechanisms (in the sense that traders’ identities are irrelevant). Gale formulates a continuity assumption and an anonymity assumption about the equilibrium strategies of agents in his model, which respectively capture the two properties mentioned above, and shows in chapter 2 that under these assumptions a limit theorem is obtainable. In chapter 2, he formalises these assumptions, and carefully elucidates the formal role they play in the argument. This formal and rigorous exposition is preceded in chapter 1 by an informal and intuitive motivation and discussion of these assumptions, which enhances depth of understanding obtained about his limit theorem, and in particular of the importance of the Markov strategies, continuity and anonymity.

In chapter 3, Gale provides an interesting and persuasive rationale for these three restrictions, which is based on a model in which players are *boundedly rational*. Specifically, his analyses here is built on the literature on repeated game models in which players have *imperfect memory*. Much valuable insight is obtained here, providing further understanding about the limit theorem, which lies at the heart of the important contribution made in this book.

Chapter 4 presents and analyses a model of an economy in which traders do not optimise, but follow simple rule-of-thumb, adaptive behavior. The model and analyses are based on recent developments in evolutionary game theory that explicitly allow

for imitation and experimentation. A main purpose of this analyses is to show that the induced dynamics leads play in the long-run to the competitive outcome. The analyses in this chapter is novel and significant. Furthermore, it provides ideas for future research along the lines of the chapter.

Gale concludes at the end of chapter 4 and in chapter 5 by raising and briefly discussing some thought-provoking and challenging issues (which should be addressed in future research). These relate to the fundamental and perceptive observation that both the theory of competitive equilibrium and DMBGs are really *partial* equilibrium models, and not *general* equilibrium models. A genuine model of general equilibrium should, for example, allow for the possibilities that markets are distinct (albeit inter-related) and that not all agents participate in all markets at a given time.

To sum up, this book makes important and significant contributions on several fronts. In particular, it enhances our understanding of the strategic foundations for the theory of competitive equilibrium, and provides some novel ways of studying economies with boundedly rational players. Although an understanding of the main body of this book does require some technical proficiency, there are many significant and important ideas and arguments here which should be accessible to the interested, non-technically oriented reader; especially the material in chapter 1, end of chapter 4 and in chapter 5, which should be compulsory reading for us all.

*University of Essex*

ABHINAY MUTHOO