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IMPERFECT COMPETITION AND EFFICIENCY IN LEMONS MARKETS*

Abhinay Muthoo and Suresh Mutuswami

This article studies the impact of competition on the degree of inefficiency in lemons markets. More precisely, we characterise the second-best mechanism (i.e. the optimal mechanism with private information) in a stylised lemons market with finite numbers of buyers and sellers. We then study the relationship between the degree of efficiency of the second-best mechanism and market competitiveness. A general message of our results is that increasing competition may not help lemons markets in some circumstances. Moreover, increasing competition beyond a certain degree increases the distance between the first-best and second-best levels of efficiency.

> Most cars traded will be the 'lemons', and good cars may not be traded at all. The 'bad' cars tend to drive out the good (in much the same way that bad money drives out the good).
>
> Akerlof (1970, p. 489)

It is conventional wisdom that competition is a good thing. The more the better. By fostering appropriate individual incentives, competition can help to promote aggregate (or social) welfare. Economics textbooks are replete with models in which aggregate welfare increases with the degree of competition. One classic example of this key insight is provided by Cournot's model of imperfect competition: in this model, the difference between the Nash equilibrium market price and the constant marginal cost of production is strictly decreasing (and aggregate welfare is strictly increasing) in the number of competing firms.

Does competition have a similar beneficial impact in markets with asymmetric information? Although it is well established that such markets tend in general to be inefficient (except perhaps in the restrictive, limiting scenario when they contain an arbitrarily large number of traders), much less is known about how the *degree* of inefficiency varies with the *degree* of competition. The overall aim of this article is to answer this question for markets with quality uncertainty, which, following Akerlof (1970), are termed lemons markets.

A general message that emerges from our results is that increasing competition may not help lemons markets in some circumstances. Specifically, we show that second-best efficiency (i.e. the efficiency of the second-best mechanism) is increasing in the number of competing sellers *provided* that the number of sellers in the market is below a certain critical number. When the number of competing sellers reaches the critical number, second-best efficiency remains constant thereafter, unaffected by having

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additional sellers. Indeed, our results imply that given demand, it is not optimal to increase the supply side beyond a certain number of firms.

First-best efficiency, however, is always increasing in the number of competing sellers. As such, it follows immediately that the distance between second-best and first-best levels of efficiency is increasing in the number of competing sellers after that number reaches the critical point. In consequence, the optimal number of competing firms in lemons markets is a finite number, which can be small or large depending on various specific market conditions.

It goes without saying that a better understanding of the relationship between competition and efficiency in lemons markets is useful not only from a theoretical perspective but also from a practical (market-design and policy) perspective. Such understanding should provide insight into the role played by competition on how well lemons markets function and perform.

A distinguishing characteristic of a lemons market is that when contemplating the possibility of bilateral trade, one of the traders has relatively more information about something (e.g., quality of the object for sale) that affects both traders' payoffs from trade. Such markets are ubiquitous, and have been intensely studied over the past four decades, both by economic theorists and by applied economists in the context of specific markets, such as credit and labour markets. In his seminal paper, George Akerlof was the first to argue that lemons markets will typically be inefficient: sellers owning high quality objects may fail to trade, although there are buyers who would wish to trade with them. The basic intuition for this fundamental observation stems from the incentives of the sellers owning low-quality objects: each such seller has an incentive to pretend to own a high-quality object to command a high price for her, actually, low-quality object. The consequence of such incentives is that buyers may not be prepared to buy at a price that is high enough for trade to be profitable for sellers owning high-quality objects.

We consider a stylised lemons market with finite numbers of buyers and sellers, in which each seller has private information about the quality of the object that she owns. A main objective is to characterise the maximal possible degree of efficiency that such a market can attain in any equilibrium. As such we do not study this market with any given, specific set of trading rules. This is because doing so would leave open the possibility that with a different set of trading rules it might be the case that a higher degree of efficiency is attained in equilibrium. Instead, we characterise the *second-best mechanism*, which defines the maximal achievable level of efficiency. Our approach thus involves considering all outcomes that can be achieved as a Bayesian Nash equilibrium of some game (induced by some set of trading rules). To conduct such an exercise, we use the mechanism design methodology, in which details of the trading rules are irrelevant. Of course, this exercise is made possible by appealing to the *Revelation Principle*, which allows us to confine attention to direct mechanisms in which agents truthfully report their private information.

A central contribution of this article, therefore, is the characterisation of the *second-best mechanism* (i.e. the optimal mechanism with private information) in a stylised lemons market with finite numbers of buyers and sellers. That is, we characterise the mechanism which maximises expected social surplus subject to satisfying appropriate incentive compatibility (IC) and individual rationality (IR) constraints, and being budget balanced (BB). This analysis extends Samuelson (1984) who characterised the second-best mechanism for *bilateral* lemons markets. Samuelson, like us, uses the mechanism design methods introduced by Myerson and Satterthwaite (1983) who characterised the second-best mechanism for bilateral markets with *private* values. Gresik and Satterthwaite (1989) extend Myerson and Satterthwaite by characterising the second-best mechanism for finite markets with private values.

Having characterised the second-best mechanism, we then study how the efficiency associated with the second-best mechanism ('second-best efficiency' for short) varies with the number of competing sellers. This comparative-static exercise allows us to address the main question of interest, namely, how the degree of inefficiency varies with the degree of competition. Some of our main results and key messages have already been summarised before. It may also be noted here that we show that lemons markets will in general not attain the first-best outcome even in the limit as the number of competing traders becomes arbitrarily large. This specific result is perhaps not that unexpected, although it should be contrasted with the positive (limiting) result that has been established for markets with other kinds of asymmetric information such as with private values (Rustichini *et al.* 1994).

The remainder of the article is organised as follows. Section 1 describes our market environment and formulates the mechanism design problem. Section 2 solves this problem, characterising the second-best mechanism and then derives the relationship between second-best efficiency and market competitiveness. We also compare the second-best to the first-best. Section 3 contains concluding remarks including a brief discussion of the related literature.

1. The Model

For reasons we shortly state, in this article we restrict attention to a market environment with a single buyer, relegating the general case to our working paper, Muthoo and Mutuswami (2010). So, the market in our model consists of a single buyer and a finite number N of sellers, where $N \geq 1$. Each seller owns one unit of an indivisible object whose quality q is her private information. It is either low quality (a 'lemon'), $q = L$, or high quality (a 'peach'), $q = H$. The probability that it is a lemon is α and the probability that it is a peach is $1 - \alpha$, where $0 < \alpha < 1$. The parameter α may alternatively be interpreted as the expected, fixed proportion of the objects that are lemons.

The buyer is interested in acquiring one and only one unit of the object. He values a lemon at v_L and a peach at v_H, where $v_H > v_L$. A seller's reservation values for a lemon and a peach are respectively c_L and c_H, where $c_H > c_L$. If the buyer acquires an object of quality q at price p, then his net payoff is $v_q - p$ (where $q = L, H$ and $p \geq 0$); and if a seller owning an object of quality q sells it at price p, then her net payoff equals $p - c_q$.[1] If an agent does not trade, then his or her net payoff is zero. All agents are risk-neutral and maximise expected utility.

[1] While the buyer may ascertain the quality of an object after acquiring it, the terms of trade cannot be made contingent on quality since we assume that is non-verifiable by 'third' parties (such as the courts).

The surpluses from trading a lemon and a peach are, respectively $s_L \equiv v_L - c_L$ and $s_H \equiv v_H - c_H$, where $s_L > 0$ and $s_H > 0$. In the model in this article, we assume that $s_H > s_L$ – that is, trading a peach generates a higher surplus than trading a lemon.

To summarise, then, in this article we restrict attention to the case with a single buyer and in which trading a peach generates a higher surplus than trading a lemon. Our working paper, Muthoo and Mutuswami (2010), extends our analysis to the general case with arbitrary but finite numbers of buyers and sellers, and deals also with the alternative assumption that trading a lemon generates a relatively higher surplus. We have relegated these cases to our working paper version since the case we are studying in this article (one buyer, N sellers and $s_H > s_L$) gets at most of the main ideas and results without too much fuss, in a transparent and straightforward manner.

We now turn to the analysis using the mechanism design methodology. By a standard appeal to the *Revelation Principle*, it suffices to focus on direct mechanisms where each seller announces her type and the mechanism selects an outcome conditional on the announcements of all sellers. Furthermore, without loss of generality, we restrict attention to symmetric mechanisms where the outcome does not depend on the names of the sellers.[2] Following the Revelation Principle, the mechanism needs to satisfy the IC constraints. We also require it to satisfy the participation (or IR) constraints and be BB.[3]

Given a direct, symmetric mechanism, let $\hat{p}^q$ be the probability that a seller sells her object if she reports that she is a q-type seller ($q = L,H$). Correspondingly, let $\hat{t}^q$ be the expected revenue that she obtains by doing so. A mechanism is *incentive-compatible* (IC) if and only if no type of a seller benefits strictly by misrepresenting her type.[4]

$$\hat{t}^H - \hat{p}^H c_H \geq \hat{t}^L - \hat{p}^L c_H \quad \text{and} \tag{1}$$

$$\hat{t}^L - \hat{p}^L c_L \geq \hat{t}^H - \hat{p}^H c_L. \tag{2}$$

Individual Rationality for an arbitrary seller of type q ($q = H,L$) requires that the expected payoff from the mechanism be at least the payoff from the outside option, which is normalised to zero.

$$\hat{t}^H - \hat{p}^H c_H \geq 0 \quad \text{and} \tag{3}$$

$$\hat{t}^L - \hat{p}^L c_L \geq 0. \tag{4}$$

Individual rationality for the buyer follows the same principle. Since the buyer does not know the type of object owned by any seller, IR requires that his expected payoff from transacting with any individual seller must be non-negative. This yields (using the requirement that the mechanism be BB)

[2] The argument for why the restriction to symmetric mechanisms is without loss of generality is based on the following two observations: an asymmetric mechanism is equivalent to a random mechanism (one that picks a symmetric mechanism randomly) and a random mechanism can do no better than a symmetric mechanism (by definition).

[3] The former requirement is motivated by the notion that trade is voluntary, whereas the latter by the notion that there are no 'third' parties (such as governments) who subsidise market trade.

[4] The first inequality is an arbitrary high-type seller's IC condition, whereas the second is an arbitrary low-type seller's IC condition.

$$(1-\alpha)(\hat{p}^H v_H - \hat{t}^H) + \alpha(\hat{p}^L v_L - \hat{t}^L) \geq 0. \quad (5)$$

The symmetry of the mechanism entails restrictions on $\hat{p}^H$ and $\hat{p}^L$. From the point of view of an individual seller of type H, the probability that there are k other sellers of type H is given by $\binom{N-1}{k}(1-\alpha)^k\alpha^{N-1-k}$; and in this case, symmetry implies that she sells her product with a probability at most $1/(k+1)$. Taking expectations across all possible realisations of k, it follows that

$$\hat{p}^H \leq \frac{1-\alpha^N}{N(1-\alpha)}. \quad (6)$$

Using a similar argument, it also follows that

$$\hat{p}^L \leq \frac{1-(1-\alpha)^N}{N\alpha}. \quad (7)$$

The probability of sale between an arbitrarily chosen seller and the buyer is $(1-\alpha)\hat{p}^H + \alpha\hat{p}^L$. Since the total probability with which trade occurs must be less than or equal to one, the mechanism must also satisfy

$$N[(1-\alpha)\hat{p}^H + \alpha\hat{p}^L] \leq 1. \quad (8)$$

The expected surplus realised from a symmetric, direct mechanism is $N[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L]$. Observe that the expression in the square brackets is the expected surplus from the transaction between an arbitrarily chosen seller and the buyer. The mechanism design problem is to choose a symmetric, direct mechanism amongst those that satisfy the two IC constraints, three IR constraints (within which has been factored the requirement of BB) and three admissibility constraints that generates the maximal expected surplus. Formally, this problem is:

$$E \equiv \max_{\hat{p}^H,\hat{p}^L,\hat{t}^H,\hat{t}^L} \quad N\left[(1-\alpha)\hat{p}^H s_H + \alpha\hat{p}^L s_L\right] \quad (9)$$
$$\text{subject to (1)} - \text{(8)}.$$

The solution to this maximisation problem defines the second-best mechanism (i.e. the optimal mechanism with private information), to which we now turn.

2. The Second-best Mechanism

2.1. *A Reduced-form Problem*

We solve (9) by defining and solving a reduced-form problem that involves the following change of variables:

$$\tilde{p}^H = N(1-\alpha)\hat{p}^H \quad \text{and} \quad \tilde{p}^L = N\alpha\hat{p}^L,$$

where $\tilde{p}^H$ and $\tilde{p}^L$ are, respectively, the total probabilities with which trade occurs between the buyer and sellers owning peaches and lemons. Given this change of variables, the constraints (6)–(8) become

$$\tilde{p}^H \leq 1 - \alpha^N, \tag{10}$$

$$\tilde{p}^L \leq 1 - (1-\alpha)^N \quad \text{and} \tag{11}$$

$$\tilde{p}^L + \tilde{p}^H \leq 1, \tag{12}$$

and the maximand in (9) becomes $\tilde{p}^H s_H + \tilde{p}^L s_L$. Note that $1 - \alpha^N$ $(1 - (1-\alpha)^N)$ is the probability that there exists at least one seller amongst the N sellers who owns a peach (a lemon). The two IC conditions are satisfied only if the following inequality holds:[5]

$$\hat{p}^H \leq \hat{p}^L,$$

which, using the change of variables defined before, becomes:

$$\tilde{p}^H \leq \left(\frac{1-\alpha}{\alpha}\right)\tilde{p}^L. \tag{13}$$

This implication of the sellers' IC conditions – namely, that the probability $\hat{p}^H$ with which trade occurs with a high-type seller is no greater than the probability $\hat{p}^L$ with which trade occurs with a low-type seller – turns out to be the binding constraint for some scenarios. In the other scenarios, it is a buyer's induced IR constraint (defined next) which will be the binding constraint. The following lemma contains several other preliminary results:

Lemma 1. *At a solution to the mechanism design problem* (9)*, the low-type seller's IC constraint,* (2)*, binds as does the high-type seller's IR constraint,* (3)*. That is,*

$$\hat{t}^H = \hat{p}^H c_H \quad \textit{and} \quad \hat{t}^L = \hat{p}^H c_H + (\hat{p}^L - \hat{p}^H)c_L.$$

Proof. See Appendix.

Using Lemma 1, substitute for $\hat{t}^H$ and $\hat{t}^L$ in the IR constraint of the buyer, (5), and it becomes (after using the change of variables defined before and some simplification):

$$\tilde{p}^H\left[s_H - \alpha(v_H - c_L)\right] + (1-\alpha)\tilde{p}^L s_L \geq 0. \tag{14}$$

It should perhaps be emphasised that inequality (14) is not the buyer's IR constraint, but the buyer's 'induced' IR constraint as it is derived after the transfers implied by two of the constraints are plugged into the buyer's IR constraint.

Now define the following *reduced-form* problem:

$$E^* \equiv \max_{\tilde{p}^H, \tilde{p}^L} \quad \tilde{p}^H s_H + \tilde{p}^L s_L \tag{15}$$
$$\text{subject to } (10) - (14).$$

The following lemma establishes the connection between the two maximisation problems:

[5] This follows by rewriting (1) and (2) as $(\hat{p}^H - \hat{p}^L)c_H \leq \hat{t}^H - \hat{t}^L \leq (\hat{p}^H - \hat{p}^L)c_L$ and then applying the assumption that $c_H > c_L$.

LEMMA 2. *Using the change of variables defined above and the expected transfer payments stated in Lemma* 1, *any solution of* (15) *defines a solution of* (9) *and vice-versa. Moreover,* $E = E^*$.

Proof. See Appendix.

Given Lemma 2, we are now ready to solve for the second-best mechanism by solving the reduced-form maximisation problem (15).

2.2. *Second-best Expected Surplus*

Before we proceed to characterise the second-best expected surplus, some preliminary observations are in order to facilitate the development and understanding of our main results.

Define $\alpha^* = s_H/(v_H - v_L)$. Note that $\alpha^* > 0$ since (by assumption) $v_H > v_L$ and $v_H > c_H$. Furthermore, $\alpha^* < 1$ if and only if $v_L < c_H$. As this may be helpful to know in developing intuition for our main results, note that

$$\alpha \lesseqqgtr \alpha* \Longleftrightarrow c_H \lesseqqgtr v^e,$$

where $v^e \equiv \alpha v_L + (1 - \alpha) v_H$, which is the buyer's expected valuation of the object held by any one of the sellers. Following the literature on bilateral bargaining with asymmetric information, we shall say that the buyer is, respectively, 'soft' and 'tough' when α is sufficiently small and sufficiently large. More precisely, in the context of our analysis and results, we say that the buyer is soft when $\alpha \leq \alpha^*$, and tough when $\alpha > \alpha^*$.

There are two, distinct kinds of inefficiencies that can arise in lemons markets: 'trading' inefficiency and 'allocative' inefficiency. The former means that trade does not occur with probability one (when it should since, by assumption, trade with either of the two types generates positive surplus) and the latter means that trade occurs with the low type seller when a high type is present in the market (which is inefficient since, by assumption, the surplus from trading with a lemon owner is strictly less than the surplus from trading with a peach owner).

In the soft buyer case, the buyer's expected valuation of the object, v^e, is greater than or equal to c_H, and this means that there exists a price (e.g., price equal to c_H) at which trade would take place with probability one, although with positive probability trade may occur between the buyer and a low-type seller. Thus, in the soft buyer case, allocative inefficiency will in general be harder to remove while trading inefficiency should be easy to remove. In contrast, in the tough buyer case, when $c_H > v^e$, one should expect both types of inefficiencies to be present.

We are now ready to present our main results. Propositions 1 and 2 respectively, state the expected surplus E associated with the second-best mechanism when $\alpha \leq \alpha^*$ and $\alpha > \alpha^*$. We sketch the main elements of the argument in the text below, but relegate the detailed calculations to Appendix (where the solution that underpins the second-best expected surplus is also derived). As will become clear, some of the analysis and most of the results differ according to whether $\alpha \leq \alpha^*$ or $\alpha > \alpha^*$. This arises because in the former (soft buyer) case, the buyer's induced IR constraint does not bind (it is the sellers' IC constraints that do), whereas in the latter (tough buyer) case the buyer's induced IR constraint is the binding constraint.

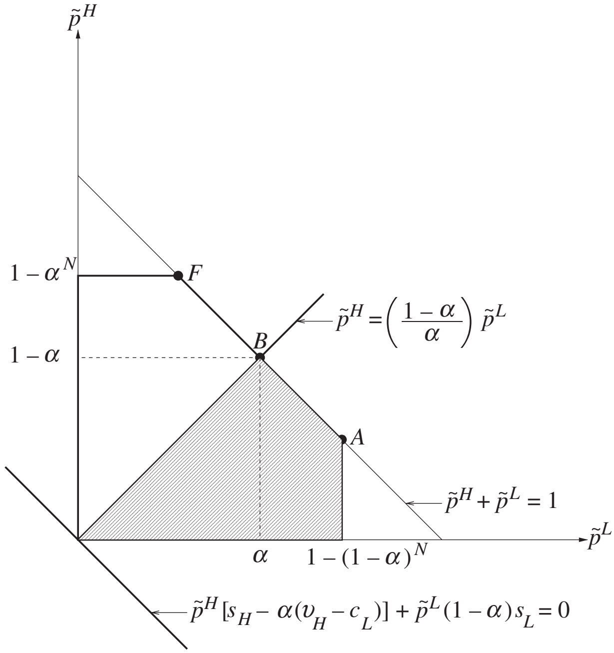

Fig. 1. *Soft Buyer: The Feasible Set When* $\alpha \leq \alpha^*$

We begin by noting that the set of all pairs $(\tilde{p}^L, \tilde{p}^H)$ that satisfy (10)–(13) comprises the shaded region in Figure 1. In the absence of (14), therefore, the solution of (15) lies at point *B*. If $\alpha \leq \alpha^*$, then all points in the shaded region shown in Figure 1 satisfy (14). Hence:

PROPOSITION 1. ('SOFT' BUYER). If $\alpha \leq \alpha^*$, *then the second-best expected surplus* $E = (1-\alpha)s_H + \alpha s_L$.

Proof. See Appendix.

We continue with our argument (which turns to the characterisation of the second-best expected surplus when $\alpha > \alpha^*$) after the following remark that provides some intuition behind Proposition 1 (a fuller discussion of this proposition is deferred to Sections 2.3 and 2.4).

Remark 1. Point *B* in Figure 1 depicts the second-best solution in the soft buyer case, which entails setting $\tilde{p}^L = \alpha$ and $\tilde{p}^H = 1 - \alpha$ or equivalently, $\hat{p}^H = \hat{p}^L = 1/N$. The intuition for this solution runs as follows. Since $s_H > s_L$, maximising expected surplus entails making $\tilde{p}^H$ as large as possible. This means (given admissibility) getting it to be as close to $1 - \alpha^N$ as possible. But the implication of the sellers' IC constraints, that is, $\hat{p}^H \leq \hat{p}^L$, bites, and hence $\alpha\tilde{p}^H = (1-\alpha)\tilde{p}^L$ (which means that $\hat{p}^H = \hat{p}^L = \hat{p}$). The desired conclusion then follows immediately because of the admissibility requirement that the total probability with which trade occurs cannot exceed one (i.e. $N\hat{p} \leq 1$), and optimality then entails setting $\hat{p} = 1/N$. The following observations shed further light on the second-best mechanism in the case under consideration. Using Lemma 1, we obtain that in this case $\hat{t}^H = \hat{t}^L = c_H/N$. An indirect mechanism that implements the second-best is the following fixed-price mechanism: The buyer makes a 'take-it-or-leave-it' fixed-price offer, and then he chooses amongst those sellers who accept to trade at the announced price. It is easy to verify that there exists a perfect Bayesian equilibrium of this indirect mechanism in which the buyer announces that he is willing to trade at price equal to c_H, each seller of either type accepts to trade at this price, and the buyer then selects to trade with each seller with equal probability (which is $1/N$). The expected surplus generated in this equilibrium is

$$N\left\{\frac{1}{N}\left[(1-\alpha)s_H + \alpha s_L\right]\right\},$$

which equals $(1-\alpha)s_H + \alpha s_L$, the second-best expected surplus. The buyer's expected payoff equals $N[(v^e - c_H)/N] = v^e - c_H$, which is greater than or equal to zero if and only if $\alpha \leq \alpha^*$ (which is, of course, the hypothesis of Proposition 1). The expected payoffs to a high-type seller and a low-type seller are, respectively, zero and $(c_H - c_L)/N$.

Now let us return to the main argument and assume that $\alpha > \alpha^*$. In this case not all points in the shaded region shown in Figure 1 satisfy (14). Figures 2 and 3 show how (14) affects the feasible set depending on whether or not point A remains a feasible point. Point A remains a feasible point if and only if the following inequality holds:

$$s_L \geq \frac{(1-\alpha)^{N-1}}{1-(1-\alpha)^N}\left[\alpha(v_H - c_L) - s_H\right]. \tag{16}$$

Since the right-hand side of (16) is strictly decreasing in N, converges to zero in the limit as $N \to \infty$ and is strictly greater than s_L when $N = 1$, there exists an N^*, where $N^* \geq 2$, such that (16) holds if and only if $N \geq N^*$.

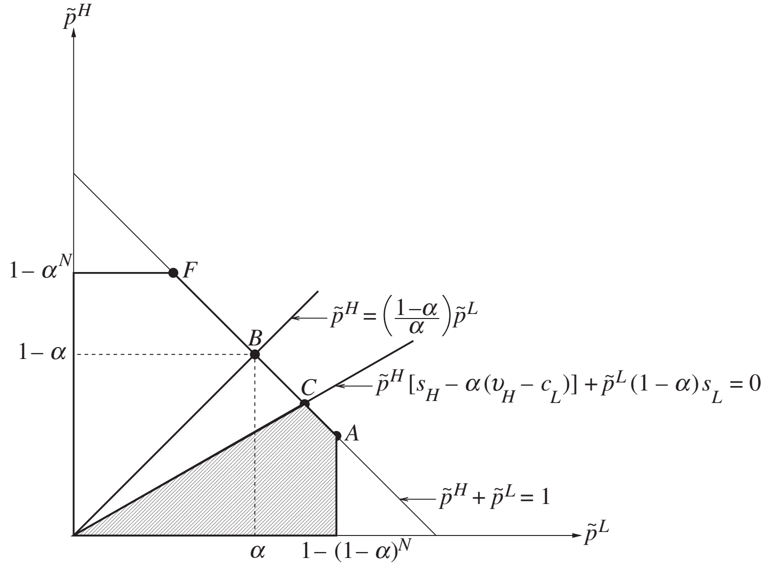

Fig. 2. *Tough Buyer and Large Market: The Feasible Set When* $\alpha > \alpha^*$, *and Inequality* (16) *Holds* (*i.e.* $N \geq N^*$)

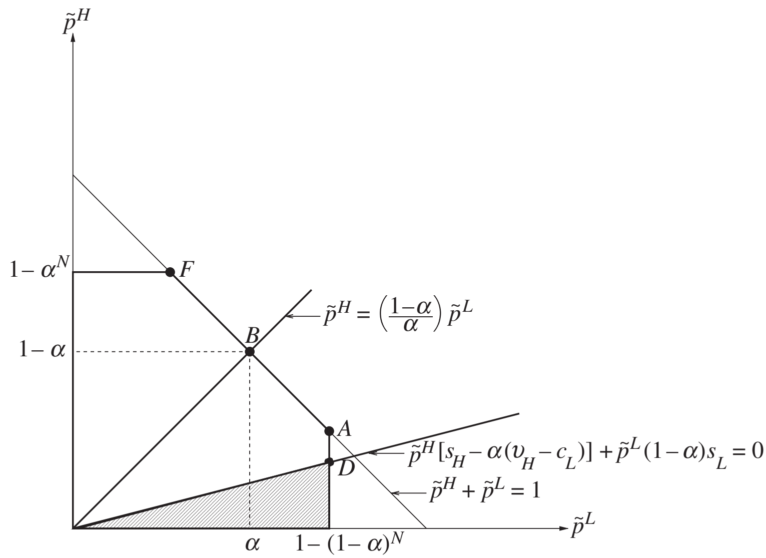

Fig. 3. *Tough Buyer and Small Market: The Feasible Set When* $\alpha > \alpha^*$, *and* (16) *Does Not Hold* (*i.e.* $1 \leq N < N^*$)

If point *A* does remain a feasible point (i.e. $N \geq N^*$), then the shaded region shown in Figure 2 comprises the feasible set; in this case, the solution of (15) lies at point *C*. If, in contrast point *A* does not remain a feasible point (i.e. $1 \leq N < N^*$), then the shaded region shown in Figure 3 comprises the feasible set; in this case the solution of (15) lies at point *D*. Hence:

Proposition 2. ('Tough' Buyer) *Assume that* $v_L < c_H$. *If* $\alpha > \alpha^*$ *and:*

(*i*) (*Large Markets*). *If* $N \geq N^*$ (i.e. *equation* (16) *holds*), *then the second-best expected surplus*

$$E = \frac{\alpha(c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)(s_H - s_L)}.$$

(*ii*) (*Small Markets*). *If* $1 \leq N < N^*$ (i.e. *equation* (16) *does not hold*), *then the second-best expected surplus*

$$E = \frac{\alpha[1-(1-\alpha)^N](c_H - c_L)s_L}{\alpha(c_H - c_L) - (1-\alpha)s_H}.$$

Proof. See Appendix.

Remark 2. Notice that the fixed-price mechanism of Remark 1 does not implement the second-best mechanism here since it would give the buyer a negative expected payoff.[6] This is consistent with the observation – obtainable from Figures 2 and 3 – that the buyer's induced IR constraint plays a decisive role in pinning down the second-best

[6] Recall that his expected payoff in this indirect mechanism is non-negative if and only if $\alpha \leq \alpha^*$.

mechanism. This constraint, it may be recalled, is the buyer's IR constraint after the transfers implied by the low-type's IC constraint and the high-type's IR constraint are factored into it. If N is sufficiently small – how small depends on the 'tightness' of the buyer's induced IR constraint (cf. Figures 2 and 3) – then the total probability with which trade occurs is strictly less than one. Only when the market has enough sellers does trade occur for sure, although with positive probability the 'wrong' object is traded (i.e. a lemon is traded instead of a peach).

2.3. *Comparison with the First-best*

The first-best lies at point F; this is because (since $s_H > s_L$), in the first-best trade occurs with a high-type seller unless all sellers are of low type. Hence, the first-best has $\tilde{p}^H = 1 - \alpha^N$ and $\tilde{p}^L = \alpha^N$. It thus follows that the first-best cannot be attained by the second-best mechanism except in the special case when there is a single seller *and* the buyer is soft (in this special case, points F and B in Figure 1 coincide). We summarise this in the following corollary.

Corollary 1. (Comparison with the First-Best). *The second-best mechanism cannot attain the first-best (except in the special case when the market contains a single seller who owns a lemon with a sufficiently small probability).*

So, the second-best mechanism does not in general implement the first-best outcome. This result shows that Akerlof's (1970) central message about the inefficiency of lemons markets – which he developed in the context of perfectly competitive markets – is robust to imperfect competition. As can be seen from Figures 1 and 2, when the market contains a sufficient number of sellers, trade occurs with probability one. The second-best mechanism is inefficient, however, because of *allocative inefficiency*: trade occurs with positive probability between the buyer and a low-type seller even when the market contains a high-type seller. To understand this better, note that when N is large, it is commonly known that a fraction α are low-type sellers and $1 - \alpha$ high-type sellers. However, this information cannot be used to identify whether a given seller is a high type or a low type. The uncertainty regarding the type of a seller remains and this explains why the allocative inefficiency persists even in the limit.

2.4. *The Role of Competition*

Using the results established before, we can now answer the following two questions of interest: What impact does an increase in the number of competing sellers have on the second-best expected surplus? and What impact does an increase in the number of competing sellers have on relative efficiency (i.e. the 'distance' between the second-best and first-best expected surpluses)?

Using Figures 1–3 or Propositions 1 and 2, it is easy to see that: if $\alpha \leq \alpha^*$, then the level of second-best expected surplus E is independent of N; and if $\alpha > \alpha^*$, then there is an integer N^* (where $N^* \geq 2$) such that E is strictly increasing in N over the set $\{1,2, \ldots, N^* - 1\}$, attains the same or a higher value at N^* as it does at $N^* - 1$, and is a constant for all $N \geq N^*$.

Thus, second-best expected surplus does not change with an increase in the number of competing sellers once the market contains a certain critical number of them. This is a surprising result. It implies, moreover, that if there is an infinitesimal cost of getting a seller into the market, then second-best welfare (i.e. second-best expected surplus minus the total cost of having N sellers in the market) is strictly decreasing in the degree of market competition N, once it is sufficiently intense. Contrary to conventional wisdom, only a limited degree of competition is good; too much is bad. In particular, if the buyer is 'soft' then the optimal number of sellers is one, and if the buyer is 'tough' then the optimal number of sellers is $N^* \geq 2$. To put it differently, if the likelihood of sellers owning lemons is small then a bilateral monopoly is the optimal market structure but if the likelihood of sellers owning lemons is high then an oligopoly is the optimal market structure.

The first-best expected surplus G is strictly increasing in N.[7] Hence, if the buyer is soft, then the distance between first-best and second-best levels of efficiency is strictly increasing in the degree of market competition. But, if the buyer is tough, then the relationship between this distance and the degree of competition is a bit more complex: For sufficiently large markets, this distance is strictly increasing in the degree of market competition but for sufficiently small markets this distance can be increasing, decreasing or non-monotonic since both E and G are strictly increasing in N over the set $\{1,2, \ldots, N^* - 1\}$. Finally, notice that in the limit as N tends to infinity, the second-best mechanism does not attain the first-best.[8] The following corollary summarises two of our main insights regarding the role of market competition:

Corollary 2. (Role of Competition). *The second-best expected surplus does not change with any increase in the number of competing sellers once the market contains a certain critical number of them. Moreover, the second-best mechanism becomes less efficient relative to the first-best with any further increase in the degree of competition beyond a certain critical point.*

3. Concluding Remarks

In this article, we studied the question of how the degree of inefficiency varies with the degree of competition in lemons markets. The general message that emerges from our results is that increasing competition may not help to enhance efficiency in some circumstances. Furthermore, given demand, it is not optimal to increase the supply side beyond a certain number of firms.

We restricted our analysis to the case of a single buyer and many sellers, and we assumed that trading a peach generates a higher surplus than trading a lemon. As we noted earlier, we did so since this case gets at most of the main ideas and results. We have relegated various extensions to our working paper version, Muthoo and Mutuswami (2010). In particular, in that paper we extend our analysis and results to the general case of many buyers and many sellers. We also analyse the scenario when trading a lemon generates a higher surplus than trading a peach. Several other issues

[7] First-best expected surplus (when, by assumption, $s_H > s_L$) is $G = \alpha^N s_L + (1 - \alpha^N) s_H$.

[8] It may be noted that, in contrast, in markets with other kinds of asymmetric information the first-best is typically attainable in the limit as the number of traders increases without bound (Rustichini *et al.* (1994) establish such a limiting result in markets with private values.)

and extensions are also explored or discussed. For example, we argue that our main results are robust to extensions in which there are a continuum of seller types and the sellers' types are correlated.

The literature on the market for lemons is too large to be summarised here; we confine ourselves to discussing those papers which have a direct bearing on our article. The use of the mechanism design methodology can be regarded as a direct follow-up to the work of Samuelson (1984) who studied the bilateral lemons problem; our extension consists of analysing the general case of finite but arbitrary number of sellers and buyers.[9]

The inefficiency pointed to by Akerlof (1970) has prompted economists to examine ways by which this inefficiency can be overcome. For instance, Klein and Leffler (1981) and Tirole (1996) have suggested that repeated interactions may overcome the adverse selection problem; on another dimension, the works of Hendel and Lizzeri (1999), Janssen and Roy (2002) and Hendel *et al.* (2005) suggest that particular features of the durable goods market, when taken into account, can overcome partially or even fully the inefficiency associated with the lemons market.

Our previous analysis and the various extensions studied in our working paper version are in the same spirit as these papers. However, there are at least two key differences. First, the other papers mostly use models with a continuum of agents and thus their models cannot directly address the question of interest to us, which is the impact of competition on market efficiency. Secondly, in contrast to the other papers which use the dynamic element, our model is still a static one and, as such, closer to the basic Akerlof model.

It is interesting to note that in the context of standard models of oligopolistic price competition but with the novel feature that consumers engage in costly search, Stahl (1989) and Janssen and Moraga-Gonzalez (2004) have shown that, under certain conditions, increasing the number of competing firms reduces welfare and that the optimal market structure is one with a small number of firms. Furthermore, under some conditions there can be a non-monotonic relationship between welfare and the number of competing firms. These results, which are derived by bringing the standard oligopoly and search models together within a single framework, challenge the conventional wisdom that welfare is increasing in the number of firms. Although these results are derived in a world with symmetric information, they nonetheless offer an interesting parallel to the 'similar' results obtained in this article in the context of lemons markets.

Appendix

Proof of Lemma 1. Fix a pair $(\hat{p}^H, \hat{p}^L)$ that satisfies the three admissibility constraints and the constraint $\hat{p}^H \leq \hat{p}^L$. We can now regard the sellers' IC and IR constraints as specifying the set of transfers which are compatible with the given $(\hat{p}^H, \hat{p}^L)$. In general, there will be more than one transfer that are compatible with the given $(\hat{p}^H, \hat{p}^L)$.

[9] See also Evans (1989), Vincent (1989), Gul and Postlewaite (1992) and Manelli and Vincent (1995).

It is straightforward to verify that the 'south-west' corner of the feasible set of transfers is represented by the point $(\hat{t}^H, \hat{t}^L)$, where $\hat{t}^H = \hat{p}^H c_H$ and $\hat{t}^L = \hat{p}^H c_H - (\hat{p}^L - \hat{p}^L)c_L$. But these transfers correspond to the case where the low-type seller's IC constraint and the high-type seller's IR constraint binds. Furthermore, it is easy to note that the significance of the 'south-west' corner of the feasible set of transfers is that it is the smallest set of transfers required to satisfy the sellers' IC and IR constraints.

Now, if the buyer's IR constraint cannot be met with this set of transfers (for the given $(\hat{p}^H, \hat{p}^L)$) then it will never be satisfied. In effect, therefore, we can regard the IC of the low type and the IR of the high type as binding. This will fix the (minimum) transfers and we can then substitute these transfers into the buyer's IR constraint and see whether it is satisfied. If yes, then we have a feasible $(\hat{p}^H, \hat{p}^L)$; otherwise not.

Proof of Lemma 2. Suppose that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ solves the mechanism design problem (9). Then, $\tilde{p}^H = N(1-\alpha)\hat{p}^H$ and $\tilde{p}^L = N\alpha\hat{p}^L$ satisfy (10)–(14), and so $E \leq E^*$. Now suppose that $(\tilde{p}^H, \tilde{p}^L)$ solves the reduced-form mechanism design problem (15). Define $\hat{p}^H = \tilde{p}^H/N(1-\alpha)$, $\hat{p}^L = \tilde{p}^L/N\alpha$, $\hat{t}^H = \hat{p}^H c_H$ and $\hat{t}^L = \hat{p}^L c_L + \hat{p}^H(c_H - c_L)$. It is straightforward to verify that $(\hat{p}^H, \hat{p}^L, \hat{t}^H, \hat{t}^L)$ satisfies (1)–(8) and, hence, $E^* \leq E$. Therefore, $E = E^*$.

Proof of Propositions 1 and 2. We conveniently break our argument into two main cases, depending on whether Z is negative or positive, where

$$Z \equiv s_H - \alpha(v_H - c_L),$$

is the coefficient of $\tilde{p}^H$ in (14). First consider the case when $Z \geq 0$ (i.e. $\alpha < s_H/(v_H - c_L)$).[10] In this case, (14) can be rewritten as

$$\tilde{p}^H \geq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L$$

and hence (since $Z \geq 0$) the feasible set of the maximisation problem (15) is the shaded region in Figure 1. It thus follows that in this case the unique solution of (15) is at point B, i.e.

$$(\tilde{p}^L, \tilde{p}^H) = (\alpha, 1-\alpha).$$

Now consider the case when $Z < 0$ (i.e. $\alpha > s_H/(v_H - c_L)$). In this case, (14) can be rewritten as

$$\tilde{p}^H \leq \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L.$$

Notice that in this case, the line

$$\tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right]\tilde{p}_L, \tag{A.1}$$

is positively sloped; whereas in the previous case when $Z \geq 0$, the line (A.1) was non-positively sloped. There are three subcases to consider here, depending on the relative position of the line (A.1).

If the slope of the line (A.1) is greater than or equal to $(1-\alpha)/\alpha$ – which is the case if and only if $\alpha \leq \alpha^*$ – then (A.1) lies above the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$ and hence the feasible set of the maximisation problem (15) in this case (when $\alpha \in (s_H/(v_H - c_L), \alpha^*)$) continues to be the shaded region in Figure 1. It thus follows that in this case, the unique solution of (15) is the same as for the case above when $\alpha < s_H/(v_H - c_L)$.

[10] It may be noted that $s_H/(v_H - c_L) < \alpha^*$ and hence this case refers to Proposition 1.

Now suppose that $\alpha > \alpha^*$ – which means that the line (A.1) lies below the line $\tilde{p}^H = [(1-\alpha)/\alpha]\tilde{p}^L$. This is shown in Figures 2 and 3, depending on whether it intersects the line $\tilde{p}^H + \tilde{p}^L = 1$ to the left of (or at) point *A* or to the right of point *A*. After some simplification, it can be shown that the former is the case if and only if inequality (16) holds; and that the latter is the case if and only if (16) does not hold – notice that Proposition 2(*i*) concerns the former case, whereas Proposition 2(*ii*) the latter.

When (16) holds, the unique solution of (15) lies, as shown in Figure 2, at point *C*, that is,

$$(\tilde{p}^L, \tilde{p}^H) = \left[\frac{-Z}{(1-\alpha)s_L - Z}, \frac{(1-\alpha)s_L}{(1-\alpha)s_L - Z}\right].$$

When (16) does not hold, then the unique solution of (15) lies, as shown in Figure 3, at point *D*, that is

$$\tilde{p}^L = 1 - (1-\alpha)^N \quad \text{and} \quad \tilde{p}^H = \left[\frac{-(1-\alpha)s_L}{Z}\right][1 - (1-\alpha)^N].$$

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