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A note on repeated-offers bargaining with one-sided incomplete information*

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Summary. This note analyzes a modified version of the standard repeated-offers bargaining game with one-sided incomplete information studied in Fudenberg, Levine and Tirole (1985), Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989). The modification, which is introduced in the extensive form, is that the (uninformed) seller can choose to withdraw her offer immediately after the (informed) buyer accepts it. This modification is important because it removes the (implicit) commitment assumption built into the standard model that the seller is committed not to withdraw her price offer. A main result obtained is, that whether or not there is a gap between the seller's valuation and the lowest possible buyer's valuation, any seller payoff between zero and the static monopoly profit can be supported by sequential equilibria. Thus, even in the "gap" case there exist equilibria that completely reverse the Coase conjecture.

1. Introduction

The standard repeated-offers bargaining game with one-sided incomplete information has been studied in Fudenberg, Levine and Tirole (1985), Gul, Sonnenschein and Wilson (1986) and Ausubel and Deneckere (1989). Henceforth, FLT (1985), GSW (1986) and AD (1989). A basic result proved in FLT (1985) and GSW (1986) is, that if there is a gap between the seller's valuation and the lowest possible buyer's valuation, then the *unique* sequential equilibrium satisfies the Coase conjecture. For familiar reasons, this is a disappointing result. More recently, AD (1989) proved, that if there is no gap between these valuations, then there exists a multiplicity of sequential equilibria, which support any seller payoff between zero and the static monopoly profit. Thus, only in the "no gap" case there exist equilibria that completely reverse the Coase conjecture.

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In this note we study a modified version of their model. The modification, which is introduced in the extensive form, is that the (uninformed) seller can choose to withdraw her offer immediately after the (informed) buyer accepts it. This modification is important because it removes the (implicit) commitment assumption built into the standard model that the seller is committed not to withdraw her price offer. The standard model needs to explain how and why the seller's commitment not to withdraw her offer is made *credible*. Indeed, the noncooperative approach demands that the process by which such commitment is achieved be explicitly modelled as part of the game structure. Hence, there is a need to investigate the consequences of relaxing this commitment assumption. In the concluding section (section 4) we provide some further comments on the choice of extensive form.

We show in this note that our modified version of the standard model generates very different results. Specifically, we prove, that whether or not there is a gap between the seller's valuation and the lowest possible buyer's valuation, any seller payoff between zero and the static monopoly profit can be supported by sequential equilibria. Thus, even in the "gap" case there exist equilibria that completely reverse the Coase conjecture. We emphasize that the sequential equilibria have the property that on-the-equilibrium-path the seller does not withdraw her offer. However, and this is the crucial point, the fact that she could do so has a profound impact on the set of equilibrium outcomes.

We prove our results by first establishing the existence of sequential equilibria that support the price *equal* to the seller's valuation. Such equilibria cannot exist in the standard model (see Lemma 2 in FLT (1985)). The folk theorems in the players' payoffs then follow trivially since, given this result, it is easy to construct what Ausubel and Deneckere in AD (1989) have called "reputational equilibria". Although the model we study is a minor modification of that studied in FLT (1985) *et al*, for the sake of completeness and in order to introduce the notation, we shall sketch our model in the next section. In this section we also characterize the set of subgame perfect equilibrium outcomes of our model with perfect information. Then in section 3 we prove the folk theorems for our model with imperfect information.

2. The model and a basic result

2.1 The model

A single seller owns an indivisible object and her valuation, which is common knowledge, is normalized at zero. There is a single buyer whose valuation v for the object is his private information, where $v \in [l, h]$ with $h > l \geq 0$. The seller's commonly known *prior* belief is described by a continuous cumulative-distribution function F with full support.

The two players negotiate over the price at which the object is sold to the buyer according to the standard repeated-offers bargaining procedure in which the seller makes all the price offers, *but* with the added feature that the seller can choose whether or not to withdraw her offer immediately after the buyer has accepted it.

If the players reach agreement at time t (where $t = 0, 1, 2, 3, \ldots,$) on the price p (where $p \geq 0$), then the payoff to the seller is $p\delta_s^t$ and the payoff to the buyer is

$(v-p)\delta_b^t$, where $0 < \delta_s, \delta_b < 1$ denote the discount factors of the seller and the buyer, respectively. The sequential equilibrium concept (SE) will be used to analyze this model.

2.2 *Perfect information*

Proposition 1 below characterizes the set of subgame perfect equilibrium (SPE) outcomes of our model with perfect information (where $v > 0$ denotes the commonly known valuation of the buyer). The proof of part (a) can be obtained from the author upon request, while the proof of part (b) will follow after we establish Lemma 1. We note that in Muthoo (1990 and 1991) we have proved that similar results also hold for the modified version of Rubinstein's (1982) bargaining model.

Proposition 1. (*a*) *If $\delta_b + \delta_s < 1$, then, with perfect information, there exists a unique SPE. In equilibrium trade occurs immediately on the price equal to the buyer's valuation.* (*b*) *If $\delta_b + \delta_s \geq 1$, then, with perfect information, any outcome can be supported by SPEa, where an outcome is either some price agreement at some finite time or perpetual disagreement.*

(*Note*: The proof of the next lemma is based on the equilibrium strategies described in Table 1. We note here that the description, of the equilibrium, uses the language of states and transition rules between the states. A player's equilibrium action at any point in the game depends on the state that is prevailing at that point. Moreover, a transition rule dictates when and if the state changes. The game begins in the state s_1. For further illustration of this compact and convenient representation of equilibria see Osborne and Rubinstein, 1990).

Lemma 1. *If $\delta_b + \delta_s \geq 1$, then, with perfect information, the strategies described in Table 1 constitute a SPE. In equilibrium agreement is reached immediately on the price of zero.*

The idea behind the proof of this lemma is that one can construct equilibrium strategies with the property that if the seller offers any price p such that $v(1-\delta_b) > p > 0$, then the buyer will reject it. This is because by accepting such a

Table 1. A SPE of our model with perfect information, where $v > 0$ denotes the commonly known valuation of the buyer.

		state s_1	state s_2
Seller	offer	$p = 0$	$p = v$
	implement	$p = 0$	$p \geq \delta_s v$
	withdraw	–	$p < \delta_s v$
Buyer	accept	$p = 0$	$p \leq v$
	reject	$p > 0$	$p > v$
Transitions		Go to state s_2 if price $p > 0$ is accepted	Absorbing

price offer the buyer does *not* obtain a payoff of $v - p$ "today", but instead he obtains a payoff of zero "tomorrow", since the seller (in equilibrium) chooses to withdraw such a price offer.

Proof of Lemma 1. Quite straightforward. We only show why $\delta_b + \delta_s \geq 1$. In state s_1 if the buyer deviates by accepting a price $0 < p < v(1 - \delta_b)$, then he obtains a payoff of zero. This is because such a deviation moves play into state s_2 where, since $1 - \delta_b \leq \delta_s$, the price $p < \delta_s v$. Hence, such a deviation is not profitable. QED

The result contained in Lemma 1 is quite remarkable; it overturns the long-held belief that a bargainer who cannot make offers has no bargaining power. Given this result it is now easy to construct SPEa that support any outcome of our model with perfect information. For example, the following path of play leading to an outcome (p, t) can be supported as a SPE path. At any point in time before t the seller offers the price equal to the buyer's valuation and the buyer rejects this offer, and then at time t the seller offers the price p which is implemented immediately. If the seller deviates from this path, then immediately play proceeds according to the zero price SPE described in Table 1. The buyer obviously has no incentive to deviate from this path at any point before time t, and if he deviates at time t, then immediately play proceeds according to the SPE that supports the equilibrium price equal to the buyer's valuation. The following corollary is implied by Proposition 1.

Corollary 1. *If $\delta_b + \delta_s \geq 1$, then any seller payoff and any buyer payoff between zero and the buyer's valuation can be supported by SPEa of our model with perfect information.*

The objective of the next section is to show that the folk theorems in the players' payoffs obtained above for the perfect information case can be "extended" (for any $h > l \geq 0$) to the imperfect information case. Indeed, a main task is to construct strategies in the imperfect information model, similar to those described in Table 1, that would form part of a SE, thus proving the existence of a zero price SE outcome.

Table 2. A SE of our model with imperfect information. The cumulative-distribution function D_h is the following degenerate distribution function: for any $v \in [l, h)$, $D_h(v) = 0$ and $D_h(h) = 1$

		state s_1	state s_2
Seller	offer	$p = 0$	$p = h$
	implement	$p = 0$	$p \geq \delta_s h$
	withdraw	–	$p < \delta_s h$
	belief	F	D_h
Buyer with valuation v where $v \in [l, h]$	accept	$p = 0$	$p \leq v$
	reject	$p > 0$	$p > v$
Transitions		Go to state s_2 if price $p > 0$ is accepted	Absorbing

3. Imperfect information

Lemma 2. *If $\delta_b + \delta_s \geq 1$, then the strategies and beliefs described in Table 2 constitute a SE of our model with imperfect information. In equilibrium trade occurs immediately with probability one on the price of zero.*

The proof of this lemma is straightforward. As is evident, this SE is an "extension" of the SPE described in Table 1. We note that if a price $p > 0$ is accepted in state s_1, then the seller conjectures that the deviation was made by the buyer with the highest valuation (the game is now in state s_2). This *optimistic* conjecture is not unreasonable, because the "type" who has the strongest incentive to deviate from this SE is the buyer with the highest valuation. Thus, many refinements of the SE concept (including, for example, those proposed and used in Rubinstein (1985) and Grossman and Perry (1986)) would not eliminate this SE. Having said this, in Table 3 we describe another SE, which also supports the price of zero, but has the feature that the seller makes only reasonable *pessimistic* conjectures. We therefore suspect that the zero price SE outcome should be robust to any refinement of the SE concept that only acts to constrain the seller's off-the-equilibrium-path beliefs.

Lemma 2 establishes the existence of SEa that give the seller a payoff of zero. Thus, given this result, it is easy to construct what Ausubel and Deneckere in AD (1989) have called "reputational equilibria". In the context of our model, such equilibria have the feature that *any* price path $(p_t)_{t=0}^{\infty}$ can be supported by a SE as

Table 3. For any $l > 0$ and δ_b sufficiently large this table describes a SE of our model with imperfect information. For each $v \in [l, h]$ the cumulative-distribution function D_v is the degenerate distribution that gives probability one to the buyer's valuation being equal to v. Furthermore, $\alpha = (1 - \delta_b)v + \delta_b l$ and $\beta = (1 - \delta_b)v + \delta_b \min\{v, p^*\}$.

		state s_1	state s_2	state $s(p^*)$	state s_3
Seller	offer	$p = 0$	$p = l$	$p = p^*$	$p = h$
	implement	$p = 0$	$p \geq \delta_s l$	$p \geq \delta_s p^*$	$p \geq \delta_s h$
	withdraw	–	$p < \delta_s l$	$p < \delta_s p^*$	$p < \delta_s h$
	belief	F	D_l	D_{p^*}	D_h
Buyer with valuation v where $v \in [l, h]$	accept	$p = 0$	$p \leq \alpha$	$p \leq \beta$	$p \leq v$
	reject	$p > 0$	$p > \alpha$	$p > \beta$	$p > v$
Transitions		Go to state s_2 if price $l \geq p > 0$ is accepted.	Absorbing	Absorbing	Absorbing
		Go to state $s(p^*)$ if price $h > p^* > l$ is accepted.			
		Go to state s_3 if price $p \geq h$ is accepted.			

follows. If the seller, at any point in time t, deviates from this price path or withdraws her offer of the price p_t, then immediately play proceeds according to the zero price SE. The buyer's strategy constitutes a best response to the seller's strategy. (Note that for any sequence of probabilities $(\pi_t)_{t=0}^{\infty}$ such that $\Sigma\pi_t \leq 1$ one can construct such "reputational equilibria" with the property that trade occurs at time $t \in \{0, 1, 2, \ldots,\}$ with probability π_t). Hence, the following proposition can be established by constructing such "reputational equilibria" (the details are left to the reader).

Proposition 2. *If $\delta_b + \delta_s \geq 1$, then any seller (resp. buyer) payoff between zero (resp. zero) and the static monopoly profit (resp. his valuation) can be supported by SEa of our model with imperfect information, where the static monopoly profit equals* $\max_{p \geq 0} p[1 - F(p)]$.

4. Concluding comments

In this note we have established folk theorems in the players' payoffs for a modified version of the standard repeated-offers bargaining game both with perfect and with imperfect information. In the perfect information case, our results overturn the long-held belief that a bargainer who cannot make offers has no bargaining power. In the imperfect information case, our results imply that even in the "gap" case the Coase conjecture can be completely reversed. In view of these radically different conclusions, the question of which of the two extensive forms is "correct" becomes of utmost importance. We conclude this note by offering some comments on this issue.

If we adopt the classical interpretation of the game form that it represents a full description of the events in the bargaining situation, then it seems plain that the onus is on the standard model to explain how and why the seller's commitment not to withdraw her offer is made *credible*. The noncooperative approach demands that the process by which such commitment is achieved be explicitly modelled as part of the game structure.

In a most stimulating paper (Rubinstein, 1991), Ariel Rubinstein has argued in favor of a perceptive interpretation of the game form. Although this idea that the game form should represent the rules as *perceived* by the players is indeed compelling, it appears to raise more questions and issues than the classical interpretation does. In particular, not only do we (as game theorists) have to account for our chosen "perceptions" of the players, but we also have to explain how conflicting perceptions are moulded into one unified game form. This latter point raises, among other questions, the question of how these perceptions are "updated" given that the players have different *prior* perceptions which are subsequently assumed to be common knowledge. Having noted all this, it is clear that a perceptive interpretation may allow us to rigorously justify both bargaining with and without the feature that offers can be withdrawn.

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