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A BARGAINING MODEL WITH PLAYERS' PERCEPTIONS ON THE RETRACTABILITY OF OFFERS

ABSTRACT. This paper studies a generalization of Rubinstein's bargaining model with retractable offers. The model incorporates and parameterizes the bargainers' perceptions on the retractability of offers. Our key result characterizes the limiting set of perfect equilibria as the time interval between two consecutive offers tends to zero. In this limit, for any possible players' perceptions on the retractability of offers such that at least one of the players perceives that there is at least a small chance that offers may be retractable, the bargaining game possesses a continuum of perfect equilibria.

Keywords: Bargaining, Rubinstein's model, retractable offers, perfect equilibrium.

1. INTRODUCTION

The theory of strategic bargaining invented by Ariel Rubinstein (cf. Rubinstein, 1982) has commanded much interest and influence mainly because, for a wide class of the players' time preferences (including the often used time preferences with constant discount factors), Rubinstein's bargaining model (henceforth RBM) possesses a *unique* subgame perfect equilibrium (henceforth SPE). In Muthoo (1990) we observed that the alternating-offers procedure which underlies RBM embodies a particular *commitment* assumption, namely, that the proposer is committed to his offer for one bargaining period. More precisely, RBM contains the feature that the proposer cannot withdraw his offer after the responder accepts it; the proposer has to implement any accepted offer. In that paper we then analyzed RBM without this commitment assumption (i.e., we allowed the proposer the option of retracting any accepted offer). The objective was to investigate whether the unique SPE in RBM is robust to this change in the extensive form. It turns out that the commitment assumption is very much responsible for Rubinstein's uniqueness result. Specifically, in Muthoo (1990), we established that for any (common) discount

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factor $\delta \geqslant 1/\sqrt{2}$ ($\cong 0.7$) and any partition of the unit-size pie there exists a SPE in the modified version of RBM such that in equilibrium agreement is reached immediately on that partition. Since, for familiar reasons (see, for example, Binmore, 1987, pp. 93–94), interest in Rubinstein-type bargaining models centers on the limiting set of SPEa, as the time interval between two consecutive offers tends to zero (or, equivalently, as the common discount factor tends to one), the result contained in Muthoo (1990) is rather disturbing.

The current paper investigates further this issue of the retractability of offers within RBM. The main aim is to explore the extent to which it is possible to vindicate Rubinstein's unique bargaining equilibrium and thus, at the same time, undermine the scope of the 'folk' theorem contained in Muthoo (1990). One approach in trying to achieve such an objective is to construct a plausible refinement of the SPE concept, which would select Rubinstein's bargaining equilibrium from the multiple SPEa constructed in Muthoo (1990). In the current paper we do not follow this approach, although in the light of the results contained in this paper it will be evident that such an approach may indeed be required. In the current paper the SPE will remain as our solution concept.

The motivation for the approach adopted in the current paper derives from the basic issue of whether retractable offers should be allowed in the game form at all. We argue that this depends on the interpretation of the game form. If we were to adopt the classical interpretation (cf. Rubinstein, 1991), then it is plain and self-evident that retractable offers should indeed be allowed in the game form (since, in most bargaining situations, it is physically feasible for a proposer to retract an accepted offer). In particular, we note that the noncooperative approach (to game theory) would require a convincing story as to why and how the proposer is committed not to retract his offer, in order to vindicate the use of RBM (without retractable offers). However, we tend to agree with the comments and arguments put forward by Ariel Rubinstein in Rubinstein (1991) in favour of the perceptive interpretation of the game form, and against the classical interpretation. The point is that the 'rules of the game' as modelled by game theorists should be those 'rules' as are perceived by the players. It is such a perceived game form that is relevant in trying to understand and study the player's behaviour.

If it were the case that both players perceive that offers are retractable, then there is no escaping the 'folk' theorem contained in Muthoo (1990). On the other hand, if were the case that both players perceive that offers are not retractable, then Rubinstein's unique bargaining equilibrium is vindicated. However, it is also possible that the players' perceptions lie in between these two extreme and polar cases, in that each player perceives that offers *may* be retractable. We do not know what the players' perceptions (on the retractability of offers) are in any given bargaining situation, nor, indeed, whether the perceptions are the same for different players across all conceivable bargaining situations. Hence, in this paper, what we do is this. We study a generalization of RBM with retractable offers, which incorporates and parameterizes the players' perceptions on the retractability of offers. This allows us to address some key issues. In particular, we can examine the robustness of Rubinstein's unique bargaining equilibrium to arbitrarily tiny perturbations in the players' perceptions.

A key conclusion obtained is that in the limit, as the time interval between two consecutive offers tends to zero, Rubinstein's uniqueness result is not robust to tiny perturbations in the players' perceptions. In this limit the 'folk' theorem contained in Muthoo (1990) is re-obtained for any possible players' perceptions, provided only that at least one of the players perceives that there is at least a small chance that offers may be retractable. After presenting the model in the next section, we derive our main result (Theorem 1) in Section 3. Comments on, and implication of, this key result are also discussed in this section. Section 4 states some further results.

2. THE MODEL

Two players, A and B, are bargaining over the partition of a pie of unit size. The game form is Rubinstein's alternating-offers procedure with the following additional features. At any point in time t (where $t = 0, 1, 2, \ldots,$), immediately following the acceptance by player j (where $j = A$ or B) of an offer made by player i (where $i \neq j$ and $i = A$ or B) Mother Nature has a move. She can either terminate the bargaining game, in which case agreement is reached on the accepted offer, or She can immediately put player i on the move, who then has to decide either to retract his offer or to implement it. If player i retracts his offer, then the bargaining game continues, one time unit later, at time $t+1$, with player j now making an offer to player i. On the other hand, if player i implements his offer, then the game ends with agreement being reached on the accepted offer. It is player A who makes the first offer.

The motivation for, and indeed the interpretation of, the moves by Nature in the game form is as follows. Each player perceives that at each time t, when player i has to make an offer and when player j has to respond to a particular offer, the players at these decision nodes perceive that an accepted offer *may* be retractable. Furthermore, each player's perception on the retractability of an accepted offer is captured by a subjective probability over Mother Nature's choice. We emphasize that a player's subjective probability does not reflect that the player is uncertain, in that he has incomplete information, as to whether the 'rules of the game' allow an accepted offer to be retracted. A player's subjective probability is, in fact, determined by the manner in which he perceives the 'rules' of bargaining, which would be influenced, in part, by his information-processing ability and other mental capabilities. Indeed, we shall allow the two players to have different subjective probabilities, reflecting different perceptions on the retractability of offers. Again, we emphasize, that this does not reflect informational differences, but reflects differences in the players' perceptability of the given situation. Note, therefore, that the Common Prior Assumption does not apply in this paper.

Let p_i denote player i's (where $i = A, B$) subjective probability at time 0 that any accepted offer is retractable. We shall assume that this initial perception is exogenously given, and moreover, we assume that it can take any value in a closed interval $[0, 1]$. We now turn to the issue of whether (and if so, how) a player's perception (i.e., subjective probability) on the retractability of offers should alter during the play of the game. First, it is perhaps reasonable to assume that a responder will not alter his subjective probability after receiving an offer from the proposer. Second, it is perhaps reasonable to assume that if an offer is rejected, then neither player A nor player B will change their respective subjective probabilities. Finally, if an accepted offer is retracted, it is perhaps reasonable to assume that if a player does alter his subjective probability then it should increase (but not decrease). Our main result (Theorem 1) can be established without any further specification of the precise manner by which players' perceptions (i.e., subjective probabilities) are updated during the course of the game. However, we shall in fact assume that a player does not alter his subjective probability on the retractability of offers during the game. This assumption simplifies the formal analysis and economizes on notation. Hence, to summarize, we assume that at every move of Nature player i (where $i = A, B$) attaches a subjective probability of p_i (where $0 \leq p_i \leq 1$) that Mother Nature will give the proposer the option of retracting an accepted offer.

The payoffs to the players are as follows. If agreement is reached at time t on the partition (x_A, x_B), where $t = 0, 1, 2, \ldots$, and where x_i denotes the share of the unit-size pie received by player i (hence, for $i = A, B$, $0 \leq x_i \leq 1$ and $x_A + x_B = 1$), then the (von Neumann–Morgenstern utility) payoff to player i is $x_i \delta_i^t$, where $0 < \delta_i < 1$ denotes player i's discount factor.

In this paper we shall assume that it is common knowledge among the two players that the game form and the payoffs are as described above, and moreover, that it is common knowledge amongst the two players that player i (where $i = A, B$) perceives that with subjective probability p_i Mother Nature will give the proposer the option of retracting an accepted offer. Note that the Common Prior Assumption does not apply here. As we have argued above, the differences in these subjective probabilities reflect differences in the players' perceptability of the 'rules' in the given situation, and do not reflect informational differences.

We note that if $p_A = p_B = 0$ (respectively, if $p_A = p_B = 1$), then this bargaining game corresponds to RBM (respectively, to the modified version of RBM analyzed in Muthoo, 1990). In the next section we study the pure strategy SPEa of this two-player perfect information extensive form game. A player's pure strategy will tell the player what choice to make at each and every decision node belonging to him (and this includes all the decision nodes at which the player will have to decide whether to retract an accepted offer). The expected payoff to player i ($i = A, B$) of any pure strategy combination is calculated using his subjective probability p_i over Mother Nature's choice. Moreover, it follows from the complete information assumption that this fact is common knowledge amongst the two players.

3. THE MAIN RESULT

The main result of this paper is stated as Theorem 1. This result is a corollary of Proposition 2, which in turn is a straightforward consequence of Proposition 1.

3.1. Preliminary Results

Proposition 1's key import is that it constructs two extremal SPEa. In equilibrium, one of these SPEa gives player A the whole pie while the other SPE gives player B the whole pie. In both of these extremal SPEa the first offer is accepted and moreover the offer is not retracted. The equilibrium strategies are rather simple, in that they are almost stationary. Indeed, they are similar to the Abreu (1988) punishment strategies. There is an initial path of play in which player i is to receive the whole pie. If player j deviates from this path, then player i punishes player j by 're-enforcing' the initial path. Player i would prefer not to punish certain deviations made by player j. But if player i does not punish player j, then player j will in turn punish player i by 'enforcing' a new path in which player j is to receive the whole pie.

The SPEa, which are explicitly stated in Table I, are described using the language of 'states' and transition rules between 'states', a language that was introduced in Rubinstein and Wolinsky (1990) (see Osborne and Rubinstein, 1990, for an extensive use of this convenient and compact language). Basically, a player's equilibrium action at any decision node depends on the 'state' that is prevailing at that node. Furthermore, the transition rules govern the precise manner by which the 'state' is to change. As is evident, the equilibrium strategies are extremely simple, since only two 'states' are required in their description.

PROPOSITION 1. *If the parameters* $0 < \delta_A, \delta_B < 1$ *and* $0 \leqslant p_A, p_B \leqslant 1$ *are such that* $\delta_A + \delta_B \geqslant 1$, $\delta_A \geqslant 1 - p_A$ *and* $\delta_B \geqslant 1 - p_B$, *then the strategies described in Table I constitute two different SPEa. If play begins in state*

TABLE I

		state s_A	state s_B
player A	offer	$x_A = 1$	$x_B = 1$
	accept	$x_A = 1$	$0 \leqslant x_B \leqslant 1$
	reject	$0 \leqslant x_A < 1$	–
	retract	$0 \leqslant x_A < \delta_A$	–
	implement	$\delta_A \leqslant x_A \leqslant 1$	$x_B = 1$
player B	offer	$x_A = 1$	$x_B = 1$
	accept	$0 \leqslant x_A \leqslant 1$	$x_B = 1$
	reject	–	$0 \leqslant x_B < 1$
	retract	–	$0 \leqslant x_B < \delta_B$
	implement	$x_A = 1$	$\delta_B \leqslant x_B \leqslant 1$
TRANSITIONS		If an offer $x_A < 1$ is accepted by player A, then switch to state s_B	If an offer $x_B < 1$ is accepted by player B, then switch to state s_A

For any feasible parameter values such that $\delta_A + \delta_B \geqslant 1$, $\delta_A \geqslant 1 - p_A$ and $\delta_B \geqslant 1 - p_B$ this table describes two different SPEa in our bargaining game. In one SPE play begins in state s_A, while in the other SPE play begins in state s_B.

s_i (where $i = A, B$), then in equilibrium player A (who makes the first offer) offers $x_i = 1$, which is accepted by player B and subsequently would not be retracted by player A, where x_i denotes the share received by player i.

Proof. Assume that $\delta_A + \delta_B \geqslant 1$, $\delta_A \geqslant 1 - p_A$ and $\delta_B \geqslant 1 - p_B$. We shall only establish the optimality of player i's ($i = A, B$) behaviour, in state s_i, to reject any offer x_i such that $\delta_i < x_i < 1$.

Suppose player i receives an offer x_i such that $\delta_i < x_i < 1$ in state s_i. If he rejects this offer (as he is supposed to, see Table I), then the state does not change (see the transition rule) and hence, his discounted expected payoff is δ_i.

Suppose he deviates from the proposed behaviour and instead accepts such an offer. This would immediately switch the state from s_i to s_j, where $j \neq i$ and $j = A, B$ (see the transition rule). Thus, there is now a possibility that player j will have to decide whether to retract the offer. From Table I we see that player j will indeed retract the offer, because $x_j = 1 - x_i < \delta_j$, which follows since $x_i > \delta_i$ implies that $x_j = 1 -$

$x_i < 1 - \delta_i$, which in turn implies that $x_j < \delta_j$ since by hypothesis $1 - \delta_i \leqslant \delta_j$. Hence, from player i's perspective, it follows that with subjective probability p_i player i will obtain a payoff of zero. And moreover, with subjective probability $1 - p_i$ Mother Nature will terminate the game, thus giving player i the share of x_i. Therefore, if player i accepts a (deviant) offer x_i such that $\delta_i < x_i < 1$, then, from player i's perspective, his subjective expected payoff is $(1 - p_i)x_i$.

Since by hypothesis $\delta_i \geqslant 1 - p_i$, it follows that $\delta_i > (1 - p_i)x_i$. Hence, it is indeed optimal for player i in state s_i to reject any offer x_i such that $\delta_i < x_i < 1$. We shall leave it for the reader to check the optimality of the other equilibrium actions. ■

PROPOSITION 2. *If the parameters* $0 < \delta_A, \delta_B < 1$ *and* $0 \leqslant p_A, p_B \leqslant 1$ *are such that* $\delta_A + \delta_B \geqslant 1$, $\delta_A \geqslant 1 - p_A$ *and* $\delta_B \geqslant 1 - p_B$, *then for any partition of the pie* (x_A, x_B) *(where* $0 \leqslant x_A, x_B \leqslant 1$ *and* $x_A + x_B = 1$*) and any time* n *(where* $n = 0, 1, 2, \ldots,$*) there exists a SPE such that along-the-equilibrium-path the first* n *offers are rejected and then the* $(n+1)$*th offer, which is the offer of the partition* (x_A, x_B), *is accepted and moreover this offer will not be retracted by the proposer. Hence, in equilibrium, the partition* (x_A, x_B) *is implemented with certainty at time* n.

This proposition is a straightforward consequence of Proposition 1. Basically, in the proof below, the equilibrium path described in Proposition 2 is supported by the threats of reverting to the extremal equilibria described in Table I.

Proof. Assume that $\delta_A + \delta_B \geqslant 1$, $\delta_A \geqslant 1 - p_A$ and $\delta_B \geqslant 1 - p_B$. This allows us to use Proposition 1. Let an arbitrary partition (x_A, x_B) (where $0 \leqslant x_A, x_B \leqslant 1$ and $x_A + x_B = 1$) and an arbitrary time n (where $n = 0, 1, 2, \ldots,$) be given. Consider the following path of play. At each time $t \leqslant n - 1$ the proposer demands to receive the whole pie and the responder rejects the demand. Then, at time n, the proposer offers the partition (x_A, x_B), which the responder accepts, and moreover, the offer is not subsequently retracted by the proposer. Hence, in this path of play, the partition (x_A, x_B) is implemented with certainty at time n.

It is easy to verify that this path of play can be supported as a SPE path by the following off-the-equilibrium-path behaviour. If a proposer, say player i (where $i = A$ or B), deviates from the proposed path of play, then immediately play begins according to the extremal SPE described in Table I that begins in state s_j (where $j \neq i$ and $j = A$ or B). It is clear that a responder, say player j (where $j = A$ or B), would not deviate from the proposed path of play at any time $t < n$. Moreover, if the responder (player j) deviates at time n, then immediately play begins according to the extremal SPE described in Table I that begins in state s_i (where $i \neq j$ and $i = A$ or B). ■

3.2. The Main Result

Our main result, which is stated below as Theorem 1, is a corollary of Proposition 2. Note that Proposition 2 holds provided $p_A > 0$ and $p_B > 0$.

THEOREM 1. *Let arbitrary values of* p_A *and* p_B *such that* $0 < p_A, p_B \leqslant 1$ *and arbitrary values of* δ_A *and* δ_B *such that* $0 < \delta_A, \delta_B < 1$ *be given.*

Let an arbitrary partition (x_A, x_B) *(where* $0 \leqslant x_A, x_B \leqslant 1$ *and* $x_A + x_B = 1$*) and an arbitrary time n (where* $n = 0, 1, 2, \ldots,$*) be given. Then in the limit, as the time interval between two consecutive offers tends to zero, there exists a SPE such that along-the-equilibrium-path the first n offers are rejected and then the* $(n+1)$*th offer, which is the offer of the partition* (x_A, x_B)*, is accepted and moreover this offer will not be retracted by the proposer. Hence, in equilibrium, the partition* (x_A, x_B) *is implemented with certainty at time n.*

The limiting SPEa characterized in this theorem are of central interest, since for familiar reasons, the main interest in Rubinstein-type bargaining models lies precisely in the limit as the time interval between two consecutive offers tends to zero. In view of this fundamental point, a main message of this paper (contained in Theorem 1) is that Rubinstein's uniqueness result is not robust to tiny perturbations in the players' perceptions on the retractability of offers. If $p_A = p_B = 0$ (i.e., if both players perceive that offers are certainly not retractable), then the bargaining game corresponds to RBM, and

TABLE II

		state s
player A	offer	$x_A = x_A^*$
	accept	$\delta_A x_A^* \leq x_A \leq 1$
	reject	$0 \leq x_A < \delta_A x_A^*$
	retract	$0 \leq x_A < \delta_A^2 x_A^*$
	implement	$\delta_A^2 x_A^* \leq x_A \leq 1$
player B	offer	$x_B = x_B^*$
	accept	$\delta_B x_B^* \leq x_B \leq 1$
	reject	$0 \leq x_B < \delta_B x_B^*$
	retract	$0 \leq x_B < \delta_B^2 x_B^*$
	implement	$\delta_B^2 x_B^* \leq x_B \leq 1$
TRANSITIONS		Absorbing

For any feasible parameter values, the (stationary) strategies described above constitute a SPE in our bargaining game, where for $i, j = A, B$ and $i \neq j$, $x_i^* = (1 - \delta_j)/(1 - \delta_i\delta_j)$.

indeed, Theorem 1 does not hold. In fact, if $p_A = p_B = 0$, then the bargaining game would possess a unique SPE, which is described in Table II, and (as expected) corresponds to Rubinstein's bargaining equilibrium. However, for any $p_A > 0$ and $p_B > 0$, Theorem 1 establishes that the uniqueness of the SPE is lost, and moreover, a continuum of SPEa is obtained. Note that there also exists SPEa that generate delayed agreements.

3.3. *Comments*

(a) The heart of the argument generating Theorem 1 lies in the construction of the extremal SPEa described in Table I. These (equilibrium) strategies are very much like supergame-type 'punishment' strategies. In fact, they are akin to the Abreu (1988) strategies, in that the 'punishment' does not fit the 'crime': an epsilon deviation is sufficient for play to move from one extremal SPE path to the other extremal SPE path. Is this type of behaviour plausible, especially so in the context of the bargaining situation? We do not know. It may be possible to formulate (as a precise criterion) some acceptable notion of plausible bargaining behaviour, which would then, not only eliminate the extremal SPEa described in Table I (thus undermining Theorem

1), but also obtain the Rubinstein bargaining equilibrium (which is described in Table II) as the unique plausible SPE in our bargaining game. We defer such an investigation for future research efforts.

(b) From the proof of Proposition 1 it is easy to verify that Proposition 1 (and hence Theorem 1) is also valid if the players update their subjective probabilities (i.e., perceptions) on the retractability of offers during the play of the game in any manner, provided only that they do not decrease their subjective probabilities (see our discussion on this issue in Section 2).

(c) It is quite straightforward to establish (by extending the arguments presented above) that the multiplicity of equilibria that underlies Theorem 1 is also obtained if the game form incorporates the following additional features. After the proposer decides not to retract an accepted offer, Mother Nature has another move. She can either terminate the game or she can immediately put the responder on the move, who has to decide whether to retract his acceptance of the offer . . . , and so forth. This process of confirming, and then reconfirming, one's offer and one's acceptance of the offer must be stopped exogenously after some finite number of such moves, for otherwise the game would not have any terminal nodes. These additional features capture the player's perceptions on the retractability of the acceptances of accepted offers . . . , and so forth.

(d) It can be established that our bargaining game has a unique stationary SPE, which corresponds to Rubinstein's bargaining equilibrium. Furthermore, it can also be established that Rubinstein's bargaining equilibrium is the unique limiting SPE, in the limit of the finite-horizon version of our bargaining game as the horizon approaches infinity. However, as is well-known, both the stationarity assumption and the finite-horizon assumption have been severely criticised, and rightly so (see Rubinstein, 1991, for a penetrating commentary on such issues).

(e) By extending the arguments contained in the proofs of Propositions 1 and 2, it can be shown that Theorem 1 would be robust to the introduction of a (relatively) small cost of retracting accepted offers (see Chaudhuri, 1992, who has established this kind of result in the model analyzed in Muthoo, 1990, i.e., in our bargaining game with $p_A = p_B = 1$).

4. SOME OTHER RESULTS

In this section we establish some further results that characterize the SPEa in our bargaining game for the other feasible parameter values (not covered by our results in Section 3). In particular, we shall show that for any $p_i > 0$ (where $i = A, B$) and $p_j = 0$ (where $j \neq i$ and $j = A, B$) in the limit, as the time interval between two consecutive offers tends to zero, there exists a continuum of SPEa. Thus, this result strengthens the main message of this paper. That is, the 'folk' theorem contained in Muthoo (1990) is re-obtained provided only that at least one of the players perceives that there is at least a small chance that offers may be retractable.

We begin by noting that for any feasible parameter values the (stationary) strategies described in Table II constitute an SPE in our bargaining game. Note that this SPE corresponds to Rubinstein's bargaining equilibrium.

(4.1) The next proposition establishes that a continuum of SPEa exists for feasible parameter values such that $\delta_i \geqslant 1 - p_i$, $\delta_j < 1 - p_j$ and $\delta_j^2 + \delta_i\delta_j \geqslant 1$ (where $i, j = A, B$ and $i \neq j$). The proof is omitted, since it is essentially based on arguments similar to those presented in Section 3.1.

PROPOSITION 3. *If the parameters* $0 < \delta_A, \delta_B < 1$ *and* $0 \leqslant p_A, p_B \leqslant 1$ *are such that* $\delta_i \geqslant 1 - p_i$, $\delta_j < 1 - p_j$ *and* $\delta_j^2 + \delta_i\delta_j \geqslant 1$ *(where* $i, j = A, B$ *and* $i \neq j$*) then for any partition of the cake* (x_A, x_B) *(where* $0 \leqslant x_A, x_B \leqslant 1$ *and* $x_A + x_B = 1$*) such that* $(1 - \delta_j)/(1 - \delta_i\delta_j) \leqslant x_i \leqslant 1$ *there exists an SPE such that along-the-equilibrium-path player A (who makes the first offer) offers the partition* (x_A, x_B)*, which is accepted by player B, and moreover, the offer will not be retracted by player A. Hence, in equilibrium, the partition* (x_A, x_B) *is implemented with certainty at time* 0.

It thus follows from Proposition 3 that for any $p_i > 0$ and $p_j = 0$ (where $j \neq i$ and $i, j = A, B$) in the limit, as the time interval between two consecutive offers tends to zero, there exists a continuum of SPEa.

(4.2) The next proposition states that for any possible players' perceptions (i.e., for any $0 \leqslant p_A, p_B \leqslant 1$) and for any feasible values of the discount factors such that $\delta_A + \delta_B < 1$ our bargaining game has a unique SPE, which is described in Table II, and corresponds to Rubinstein's bargaining equilibrium.

PROPOSITION 4. *If the parameters* $0 < \delta_A, \delta_B < 1$ *and* $0 \leqslant p_A, p_B \leqslant 1$ *are such that* $\delta_A + \delta_B < 1$, *then our bargaining game has a unique SPE, which is described in Table II. In equilibrium, player A (who makes the first offer) offers* $x_A = (1 - \delta_B)/(1 - \delta_A\delta_B)$, *which is accepted by player B, and moreover, the offer will not be retracted by player A. Hence, in equilibrium, agreement is reached immediately and with certainty.*

The proof of this proposition is based on the classic arguments invented in Rubinstein (1982) and Shaked and Sutton (1984) and therefore it is omitted, except that we shall briefly elucidate the role played by the constraint $\delta_A + \delta_B < 1$. Let M_i (respectively, m_i), where $i = A, B$, denote the supremum (respectively, infimum) of the set S_i of SPE expected payoffs to player i in any subgame beginning with player i's offer. In order to establish that $m_i \geqslant 1 - \delta_j M_j$ (where $i \neq j$ and $i, j = A, B$) we need to assume that $\delta_A + \delta_B < 1$. The argument is by contradiction and goes as follows. Suppose, to the contrary, that $m_i < 1 - \delta_j M_j$. Then there exists an $x_i \in S_i$ such that $m_i \leqslant x_i < 1 - \delta_j M_j$. Suppose player i (deviates and) offers $x_i^* = 1 - \delta_j M_j - \varepsilon$, where $\varepsilon > 0$ is such that $x_i^* > x_i$. Then, in any SPE, player j will certainly accept this offer provided player i would not subsequently retract it. It is easy to establish that if $\delta_i + \delta_j < 1$, then player i would not retract such an offer.

The result contained in Proposition 4 is of less interest and is relatively unimportant, since it would not hold for arbitrarily small time intervals between two consecutive offers. It establishes that Rubinstein's bargaining equilibrium is the unique SPE for any possible players' perceptions on the retractability of offers provided the time interval between two consecutive offers is sufficiently large.

(4.3) Is the SPE described in Table II, which corresponds to Rubinstein's bargaining equilibrium, the unique SPE for feasible values of the parameters such that $\delta_A + \delta_B \geqslant 1$, $\delta_A < 1 - p_A$ and $\delta_B < 1 - p_B$? We do not know. It seems clear that for such parameter values extremal paths of play giving the whole pie to either player cannot be supported as SPE paths. But, at the same time, the standard arguments that establish the uniqueness of the SPE for parameter values such that $\delta_A + \delta_B < 1$ (cf. Proposition 4) seem not to apply, at least in any obvious way. Hence, it remains an open question as to whether uniqueness is obtainable for these parameter values. However, note that if for some i ($i = A, B$) $p_i > 0$, then this issue is not important, because (if for some i $p_i > 0$) this set of parameter values would be 'inconsistent' (i.e., would not arise) with an arbitrarily small time interval between two consecutive offers.

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REFERENCES

Abreu, D.: 1988, 'On the theory of infinitely repeated games with discounting', *Econometrica* **56**, 383–396.

Binmore, K. G.: 1987, 'Perfect equilibria in bargaining models', in *The Economics of bargaining*, P. S. Dasgupta and K. G. Binmore (Eds.), Basil Blackwell, Oxford.

Chaudhuri, P. R.: 1992, 'Bargaining without commitment but with small reneging costs', mimeo, Indian Statistical Institute, New Delhi.

Muthoo, A.: 1990, 'Bargaining without commitment', *Games and Economic Behaviour* **2**, 291–297.

Osborne, M. J. and Rubinstein, A.: 1990, *Bargaining and Markets*, Academic Press, San Diego.

Rubinstein, A.: 1982, 'Perfect equilibrium in a bargaining model', *Econometrica* **50**, 97–109.

Rubinstein, A.: 1991, 'Comments on the interpretation of game theory', *Econometrica* **59**, 909–924.

Rubinstein, A. and Wolinsky A.: 1990, 'Decentralized trading, strategic behaviour and the Walrasian outcome', *Review of Economic Studies* **57**, 63–78.

Shaked, A. and Sutton J.: 1984, 'Involuntary unemployment as a perfect equilibrium in a bargaining model', *Econometrica* **52**, 1351–1364.

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