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## RATIONALITY IN THE FACE OF UNCERTAINTY

by

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# 1 Introduction

In psychology the notion of rationality tends to be viewed in a similar manner to lay thinking: a choice is rational if it is based on reasoning; impulsive choices might not seem to come into this category, nor those “leap in the dark” acts based on gut feelings (“animal spirits” as Keynes called them). Thus, psychologists tend to focus attention on the *procedure* of choosing an act — and not so much on the consequences generated by the act. This approach to rationality allows one to explore whether or not a particular procedure of choice generates dysfunctional consequences.

In economics, by contrast, the idea is tied to normative notions about what kinds of choice in general will enhance a person’s welfare. Consider the following decision problem. A consumer has to choose between two actions, namely,  $A$  and  $B$ . She obtains £10 from action  $A$  and £5 from action  $B$ . On the assumption that the consumer prefers more money to less money, she is said to be rational if she chooses action  $A$ ; a consumer who chooses action  $B$  is said to be irrational. In the context of this simple decision problem, the concept of rationality is trivial — partly because there is no uncertainty in this decision problem (i.e., in particular, the set of feasible actions and the payoff from each action are known with certainty). However, in the context of decision problems with uncertainty, the concept of rationality is far from trivial. For example, consider the following modification to the decision problem stated above. If the consumer chooses action  $B$ , then a fair coin is tossed; if heads turns up then the consumer obtains £15, and if tails turns up then she obtains £5. In this much more complex decision problem, it is far from clear as to whether it is still rational for the consumer to choose action  $A$ .

Most economic theories of rational choice (both with and without uncertainty) are based on a number of assumptions (or, axioms). One key assumption is that a rational person’s preferences should satisfy the transitivity axiom: if the person prefers  $a$  to  $b$ , and  $b$  to  $c$ , then she must prefer  $a$  to  $c$ . If the person would instead prefer  $c$  to  $a$ , she is said to violate the transitivity axiom, and in turn, said to be irrational.

Most real-life decision situations involve some uncertainty. The uncertainty can be either of the exogenous variety, in that it is resolved randomly by Mother Nature, or of the endogenous variety, in that it is resolved by the actions chosen by some other decision makers. This distinction is important. The nature of the uncertainty involved will, and ought to, affect the way the decision maker perceives and analyses the decision situation. Indeed, economists do classify decision situations under uncertainty into two

categories, namely, single-person decision problems and interactive decision problems. Decision theory, which is concerned with problems in the former category, is about rational decision-making when the uncertainty is of the exogenous variety. On the other hand, game theory, which is concerned with problems in the latter category, is about rational decision-making when the uncertainty is of the endogenous variety.

## 2 Exogenous Uncertainty

Consider the following general decision situation. A “player” (who can, for example, be either a consumer, or a firm, or a country) has to choose a “strategy” (a plan of action) from some set  $S$  of alternative strategies. The “payoff”— which we assume here to be in units of money — from any strategy is uncertain in an exogenous sense, which can be modelled as follows. Suppose there is a finite set of possible “states of nature”,  $\Omega$ : a state of nature is a complete description of the relevant features of the world. When deciding on her strategy, the player does not know the true state of nature, but she knows that the state of nature is determined in a *purely random manner* (by Mother Nature). Her payoff from any strategy may be influenced by the state of nature. For example, the payoff from walking to office with an umbrella is higher if it rains than if it does not rain. For each strategy  $s \in S$  and each state of nature  $\omega \in \Omega$ , we denote her payoff by  $m(s, \omega)$ .

Table 1 describes an example of a decision situation with exogenous uncertainty — there are two strategies,  $s_1$  and  $s_2$ , and two states of nature,  $\omega_1$  and  $\omega_2$ . The numbers in the four cells are the payoffs. If there were no uncertainty about the state of nature, then what constitutes the rational choice is trivial: if the true state of nature is  $\omega_1$  then it is rational for her to choose strategy  $s_1$ , and if the true state of nature is  $\omega_2$  then it is rational for her to choose strategy  $s_2$ . This means that when the player does not know the true state of nature, it is far from clear as to what constitutes the rational choice. It depends on the player’s beliefs about which state of nature is the true state and on her attitude to risk.

There are two basic theories of rational choice in decision situations with exogenous uncertainty: one is due to von Neumann and Morgenstern (1947), and the other is due to Savage (1954) — for an excellent exposition of both theories, see Kreps (1990, chapter 3) or Marschak and Radner (1972, chapter 1). Both of these theories generate an *expected utility theorem* (EUT). Before we briefly explain the key difference between these two theories, we describe this celebrated theorem, which lies at the heart of

	$\omega_1$	$\omega_2$
$s_1$	$\pounds 3$	$\pounds 1$
$s_2$	$\pounds 1$	$\pounds 2$

**Table 1:** A decision situation with exogenous uncertainty; there are two strategies,  $s_1$  and  $s_2$ , and two states of nature,  $\omega_1$  and  $\omega_2$ .

economic theory.

Two new concepts are required to define this theorem: a probability distribution over the states of nature, and a von Neumann-Morgenstern (vN-M) utility function. The former concept is relevant in order to define the rational choice, because it is necessary to assess the relative likelihoods of the states of nature. Thus, we denote by  $\pi(\omega)$  the probability that the true state is  $\omega$ . Since there is uncertainty in the decision situation, the rational choice depends upon the player's attitude towards risk. The vN-M utility function captures this aspect: we denote by  $u(m)$  the utility that the player obtains from  $\pounds m$ . The function  $u$  is the player's vN-M utility function. If it is concave then the player is averse to risk, while if it is convex then she is a risk lover. Furthermore, if it is linear then she is neutral to risk.

In the context of the general decision situation with exogenous uncertainty stated above, the expected utility theorem states that strategy  $s^* \in S$  is the rational choice if it maximises the player's expected utility; the expected utility of any strategy  $s \in S$  is the expected value of the player's vN-M utility, where the expectation is taken with respect to the given probability distribution. For example, if in the decision situation described in Table 1 the states of nature are equally likely (i.e.,  $\pi(\omega_1) = \pi(\omega_2) = 0.5$ ) and the player is risk-neutral (i.e.,  $u(m) = m$ ), then the expected utility of strategy  $s_1$  equals 2, and the expected utility of strategy  $s_2$  equals 1.5. Hence, given these probabilities and this utility function, according to the EUT it is rational to choose strategy  $s_1$ . Of course, there exist alternative probabilities and/or utility functions such that the EUT would imply that it is rational to choose  $s_2$ . Hence, rationality depends critically upon a player's beliefs and her attitude towards risk.

But where do these beliefs come from? Surely the beliefs should be rational? A main difference between vN-M's theory and Savage's theory is to do with this issue. The former theory assumes that the probabilities with which the states occur are known to the player, and hence the player does not have to form beliefs about these states — the probabilities are objective. In contrast, in Savage's theory the probabilities are subjective, in that they are the player's personal beliefs about the relative likelihoods of the states of nature. While vN-M's theory has limited applicability, because it is rare to be given objective probabilities, Savage's theory leaves unanswered the question of where the player's beliefs come from.

A fundamental weakness of these theories is that neither of them actually *describe* what constitutes rational choice in decisions situations with exogenous uncertainty. A key assumption in both theories — the “complete ordering over acts” assumption — implies that the rational choice is (implicitly) known to the player. The theories simply generate a “representation” theorem — that is, they generate (with the assistance of further assumptions) a convenient and useful *representation* of rational choice — the EUT.

The expected utility representations of preferences (as derived in the vN-M and Savage theories) have been shown to be inconsistent with various experiments, such as the well known *Allais Paradox*. For a penetrating critique of these expected utility theories, see Machina (1987), who also describes the Allais Paradox and discusses various alternative, non-expected, utility theories.

### 3 Endogenous Uncertainty

A crucial factor in defining rationality in decision situations with exogenous uncertainty are the beliefs of the player about the relative likelihoods of the states of nature, where the true state is determined in a purely random manner by Mother Nature. A main new element in decision situations with endogenous uncertainty is that the state of nature is now determined “strategically” by some other player, because the state is actually now a strategy choice of some other player.

Indeed, a game situation is the key type of situation that exhibits endogenous uncertainty. A player is in a game when her payoff depends not only on her strategy but also on the strategy of some other player. Consider the following “Battle of Sexes” game, which involves two players, Romeo and Juliet. It is about 6pm and they are

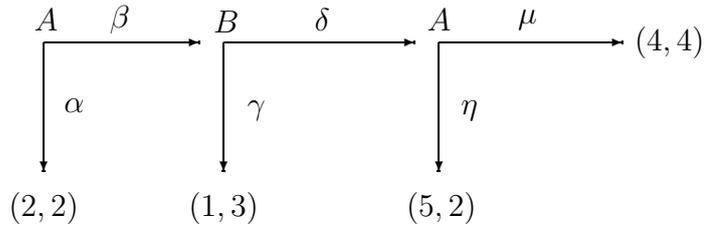
		<u>Juliet</u>	
		<i>theater</i>	<i>colosseum</i>
<u>Romeo</u>	<i>theater</i>	2,3	0,0
	<i>colosseum</i>	0,0	3,2

**Table 2:** A decision situation with endogenous uncertainty — the Battle of Sexes game.

unable to communicate with each other. However, they must decide where to spend the evening. Each must decide whether to go to the theater to see the ballet or to the colosseum to see the big fight. If they end up at different performances, then they become rather upset and hence each receives zero utility. However, if they meet at the same venue, then each receives two utils. In addition, Romeo gets another util if they meet at the colosseum, while Juliet gets an additional util if they meet at the theater. All this information is compactly summarised in Table 2.

What constitutes the rational choice for Romeo? It depends on what choice Juliet makes. If she goes to the theater, then the rational choice for Romeo is go to the theater. But if she goes to the colosseum, then the rational choice for Romeo is go to the colosseum. Hence, the rational choice for Romeo depends on his beliefs about where Juliet will go. The concept of rationality in this interactive decision situation is non-trivial, because Juliet’s choice depends on her beliefs about where Romeo will go. Unlike exogenous uncertainty, where the uncertainty is resolved in a purely random manner, endogenous uncertainty is resolved strategically, and makes the concept of rationality pretty complex.

Game theorists have suggested that *if* there exists a rational choice for Romeo and a rational choice for Juliet, then these choices must form a *Nash Equilibrium* — which means that neither player has an incentive to unilaterally deviate to an alternative strategy, given the rational choice of the other player. Attention thus focuses on studying the Nash equilibria of the game. For a brief discussion of this and related concepts of game theory, see the entry on “game theory”; and for a more detailed exposition, see Kreps (1990, chapters 11 and 12). The Nash equilibrium concept, however, fails to



**Figure 1:** The centipede game.

satisfactorily and persuasively identify what constitutes rational behaviour in decision situations with endogenous uncertainty. In fact, this is an equilibrium concept, and it simply evades addressing the problems involved in defining the concept of rationality in game situations.

Matters become more complex in the context of *dynamic* interactive decision situations, where the concept of rationality may not be well-defined — for example, some (small) degree of irrationality may be required to define and make sense of the concept of rationality. To briefly illustrate this point, consider the following dynamic game with two players,  $A$  and  $B$ , which is illustrated in Figure 1. The game begins with player  $A$  choosing between  $\alpha$  and  $\beta$ . If she chooses the former then each player obtains  $\mathcal{L}2$ ; otherwise the game continues with player  $B$  having to choose between  $\gamma$  and  $\delta$ . If she chooses the former then player  $A$  obtains  $\mathcal{L}1$  and player  $B$  obtains  $\mathcal{L}3$ ; otherwise the game continues with player  $A$  having to choose between  $\eta$  and  $\mu$ . If she chooses the former then player  $A$  obtains  $\mathcal{L}5$  and player  $B$  obtains  $\mathcal{L}2$ ; otherwise the game ends with each player obtaining  $\mathcal{L}4$ .

The following, rather persuasive, backward induction argument is meant to identify the rational outcome of this game. Consider  $A$ 's second decision node, where she has to choose between  $\eta$  and  $\mu$  — which is reached if and only if  $A$  has chosen  $\beta$  at her first decision node and  $B$  has chosen  $\delta$ . It seems rational for  $A$  to choose  $\eta$ , because she then obtains  $\mathcal{L}5$  (rather than  $\mathcal{L}4$ , which is obtained if she were to instead choose  $\mu$ ). Proceeding backwards, this implies that it is rational for  $B$  to choose  $\gamma$  and obtain  $\mathcal{L}3$  (rather than  $\delta$ , which would give her  $\mathcal{L}2$ , because we have just shown that it is rational for  $A$  to choose  $\eta$ ). Proceeding backwards, this implies that it is rational for player  $A$  to choose  $\alpha$  at her first decision node. Hence, *backward induction* rationality implies that the game will terminate immediately after  $A$  makes her first decision. It should be noted that backward induction rationality is the basic concept of rationality in dynamic games.

Backward induction rationality also implies that player  $B$  will not be called upon to

make the choice between  $\gamma$  and  $\delta$ . But this conclusion rests on the counterfactual that player  $A$  does make the irrational choice of  $\beta$ . How should we interpret this? The standard interpretation is that  $A$  has made a mistake, which is uncorrelated with any mistake that  $A$  might make at her second decision node. Hence, the standard way to get round this fundamental problem with backward induction rationality is to assume that players are prone to some (albeit tiny) mistakes/irrationality. There are, however, some serious problems with such an interpretation; for a detailed discussion of this and related issues, see Reny (1992), who uses this game to discuss the notion that it might be rational to be irrational. It should also be mentioned that most solution concepts in game theory, such as backward induction rationality, implicitly rest on the common knowledge assumption, which includes the assumption that the rules of the game are common knowledge amongst the players: both players know the rules, both players know that both players know the rules, both players know that both players know that both players know the rules of the game, and so on *ad infinitum*.

## 4 Substantive and Procedural Rationality

Herbert Simon coined the terms *substantive* rationality and *procedural* rationality. He argued that economists have focused attention on the former concept, while psychologists on the latter concept. The issue of defining/discovering *what* constitutes rationality in a decision situation is a matter of substantive rationality, while the issue of defining/discovering the rational *procedure* of choice is a matter of procedural rationality. That is, procedural rationality is about the rationality of the procedure used to reach a decision, while substantive rationality is about the rationality of the decision itself.

The following example illustrates the distinction between procedural and substantive rationality. Consider the situation in which an individual has to buy a Television (TV). Suppose that an omniscient observer knows which brand and model of TV the individual should buy to maximise her utility — say, it is Sony 234. Thus, the substantively rational choice is to buy Sony 234. Now suppose that the individual does not know (for one reason or another) what the substantively rational choice is. She would then use some procedure (which is based on her information about TVs, etc.) in order to arrive at her decision as to which TV to buy. The procedure may be considered to be rational if it generates a satisfactory choice, whether or not the choice is the same as the substantively rational choice (which here is Sony 234). The

concept of *satisficing*, which is developed by Herbert Simon and on which there is a separate entry, is intimately connected with the concept of procedural rationality.

In our discussion above, we only addressed the issue of substantive rationality: given a decision situation with exogenous/endogenous uncertainty, we discussed the question of *what* constitutes rationality. We did not address the issue of *how* the player reaches a decision — thus, issues of thought, reasoning and computation were omitted from our discussion. However, such issues are important, especially if we are to understand and develop models of *human* rationality. On this latter point, it should be noted that human rationality might differ to some lesser or greater extent from the concepts of substantive rationality developed by economists, which is linked to the thorny issue of human reasoning/thought processes. Although psychologists have done much research on this latter issue and amassed much data about it, we are far from developing general models of such complex processes.

Procedural rationality has recently been somewhat forced onto the agenda of economic theory, especially due to the recent dissatisfaction with the concepts of substantive rationality and equilibrium developed by economists. Furthermore, the concept of *bounded* rationality, which arises naturally when thinking about procedural rationality and is inextricably linked with the concept of human rationality, has been the subject of much recent investigation in economic theory. This work builds on the ideas developed by Herbert Simon, which are collected together in Simon (1982). A major difficulty involves constructing useful and tractable *formal* models of bounded rationality that take into explicit account the procedural aspects of rationality. For an exposition of some of the recently developed tools and models, and in particular of the difficult modelling issues involved, see Rubinstein (1997).

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