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A Bargaining Model Based on the Commitment Tactic

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This paper studies a one-shot, simultaneous-move bargaining game. Each bargainer makes a partial commitment (a share of the unit size cake that she would like to get), which can later be revoked at some cost to the player. The payoffs are defined, in part, by the Nash bargaining solution, where the feasible utility set is affected by the players' partial commitments. Under certain assumptions on the two cost-of-revoking functions, we establish that the model has a unique Nash equilibrium. It is shown that a player's equilibrium share of the cake is strictly increasing in her marginal cost of revoking a partial commitment. An application of our model selects a unique equilibrium in Nash's demand game. *Journal of Economic Literature* Classification Number: C78.

1. INTRODUCTION

In many bargaining situations (e.g., international trade negotiations and firm–union wage bargaining) the bargainers often take actions prior to, and/or during, the negotiation (offer–counteroffer) process that *partially* commit them to some strategically chosen bargaining positions. Such commitments are partial in that they are revocable, but revoking a partial commitment can be costly for a bargainer. In his brilliant essay on bargaining (Schelling [6]), Thomas Schelling informally examined the role, feasibility, and pervasiveness of this type of tactic in bargaining situations. The following two extracts from Schelling [6] illustrate this "commitment" tactic:

> ... it has not been uncommon for union officials to stir up excitement and determination on the part of the membership during or prior to a wage negotiation. If the union is going to insist on $2 and expects the management to counter with $1.60, an effort is made to persuade the membership not

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> only that the management could pay $2 but even perhaps that the negotiators themselves are incompetent if they fail to obtain close to $2. The purpose ... is to make clear to the management that the negotiators could not accept less than $2 *even if they wished to* because they no longer control the members or because they would lose their own positions if they tried. In other words, the negotiators reduce the scope of their own authority and confront the management with the threat of a strike that the union itself cannot avert, even though it was the union's own action that eliminated its power to prevent the strike ... [p. 27]
> When national representatives go to international negotiations knowing that there is a wide range of potential agreement within which the outcome will depend on bargaining, they seem often to create a bargaining position by public statements, statements calculated to arouse a public opinion that permit no concessions to be made. [p. 28]

This paper aims to provide a formal game-theoretic analysis of the strategic effect of the commitment tactic on the bargaining outcome. To achieve this objective, we construct a simple bargaining game that captures the *essence* of the strategic role of the commitment tactic. The analysis of this game then allows us to formally explore the impact of this type of tactic on the bargaining outcome.

With the exception of the insightful work by Vincent Crawford (see Crawford [4]) there exist hardly any formal game-theoretic bargaining models that explore the effect of the various tactics employed by bargainers on the bargaining outcome. Yet these often quite visible tactics lie at the center stage of many bargaining processes. It is indeed important to understand and study the influence of such tactics on the efficiency of the bargaining outcome and on the players' equilibrium shares of the "cake."

Our model abstracts from the (potentially rich) processes and mechanisms by means of which the bargainers are able to partially commit themselves to some strategically chosen bargaining positions. For example, pledging one's "reputation" could be a potent means of partial commitment. The two extracts from Schelling (stated above) illustrate various other mechanisms by means of which partial commitments can be made. The model studied in this paper should be interpreted as a *reduced form* of more detailed extensive-form models in which the sources of the costs of revoking are explicitly modelled.

Formally, our bargaining model is a one-shot, simultaneous-move game. Each bargainer makes a partial commitment (a share of the cake that she would like to get), which can later be revoked at some cost to the player. The players' payoffs (when the partial commitments are incompatible) are defined by the Nash bargaining solution, where a player's partial commitment affects the shape of the feasible utility set to her advantage. We assume that the cost of revoking a partial commitment depends on the accepted share and on the partial commitment. For example, the cost could depend on the difference between the partial commitment and the accepted share.

Under certain assumptions on the cost-of-revoking functions, we establish that our model possesses a unique Nash equilibrium. The uniqueness of the equilibrium implies that we have a *theory* of strategic bargaining, in that the tactical approach to bargaining adopted in the current paper does indeed resolve the fundamental indeterminacy of the bargaining situation. The properties of this equilibrium bargaining outcome, which we now highlight, inform us as to the nature of the influence and impact of the "commitment" tactic on the equilibrium partition of the unit size cake.

The equilibrium is efficient in that neither player revokes her equilibrium partial commitment: each player's equilibrium share of the cake is exactly equal to her equilibrium partial commitment. A rather remarkable feature of the equilibrium partition of the unit size cake is that it depends only upon the relative magnitudes of the *marginal* costs to the players of revoking their respective equilibrium partial commitments; it is independent of other features of the cost-of-revoking functions. Moreover, it is shown that the equilibrium partition can be described by an asymmetric Nash bargaining solution, in which a player's "bargaining power" is a strictly increasing function of her marginal cost of revoking her partial commitment. Indeed, rather surprisingly, the *higher* a player's marginal cost of revoking her partial commitment, the *bigger* is the share of the cake received by the player.

An application of our model establishes a new, and rather compelling, equilibrium selection procedure in the Nash [5] demand game. The procedure is based on perturbing the *commitment* structure in the demand game, as against the *informational* structure–which is the basis of the equilibrium selection procedures due to Nash [5] and Carlsson [3]. Besides selecting a unique equilibrium in the Nash demand game, our theory provides a neat interpretation of the indeterminacy (multiplicity of equilibria) in the Nash demand game.

This paper has been inspired by Schelling's essay on bargaining. In addition, we have greatly benefitted from reading Crawford [4], who also studies a model of bargaining based on the commitment tactic. However, Crawford's model and analysis are significantly different from ours.

The paper is organized as follows. Section 2 provides a description of the model. Then, in Section 3 we derive the unique Nash equilibrium with the assumption that the players' costs of revoking are linear. All of the intuition for the results in the main model, which are stated in Section 4, can be obtained here. Moreover, as the arguments are geometric, it is then easier to follow the formal analysis of the general model, which is relegated to the Appendix. We defer discussion of our results to Section 5, where we shall also discuss the role and applicability of the various assumptions that we make on the cost-of-revoking functions, thus providing some informal defence for interpreting our model as a *reduced form*. We conclude in Section 6 with some final remarks.

2. The Model

Two players, A and B, bargain over the partition of a unit size cake according to the following one-shot (static) game. They simultaneously and independently choose numbers from the closed interval $[0, 1]$. Let z_i denote the number chosen by player i ($i = A, B$). The interpretation is that player i takes "actions" which *partially* commit her not to accept a share strictly less than z_i. A partial commitment can later be revoked at some cost to the player. We now turn to a description of the players' payoffs, where we denote player i's payoff from a strategy pair $z = (z_A, z_B)$ by $P_i(z)$.

2.1. *Payoffs When the Partial Commitments Are Compatible*

First we describe the payoffs when the chosen partial commitments z_A and z_B are such that $z_A + z_B \leqslant 1$. In this case we assume that neither of the two players will have to revoke their respective partial commitments: the share x_i ($0 \leqslant x_i \leqslant 1$) of the cake received by player i is such that $x_i \geqslant z_i$. Thus, we assume that the share $x_i = \lambda_i(z)$, where λ_A and λ_B are *any* functions such that $\lambda_A(z) \geqslant z_A$ and $\lambda_B(z) = 1 - \lambda_A(z) \geqslant z_B$. (For example, $\lambda_A(z) = z_A + [1 - z_A - z_B]/2$.) Hence, if $z_A + z_B \leqslant 1$, then player i's payoff $P_i(z) = \Pi_i(\lambda_i(z))$, where $\Pi_i(x_i)$ is the utility from obtaining a share x_i of the cake. We shall make the following (standard) assumptions on Π_i.

(A1) For each $i (i = A, B)$, the function $\Pi_i \colon [0, 1] \to \mathbb{R}$ is twice continuously differentiable, strictly increasing, and concave. Furthermore, without loss of generality, we set $\Pi_i(0) = 0$ and $\Pi_i(1) = 1$.

2.2. *Payoffs When the Partial Commitments Are Incompatible*

The Nash bargaining solution will be applied to define the players' payoffs when $z_A + z_B > 1$. If an agreement over the partition of the unit size cake is struck, then at least one of the players must revoke her partial commitment. If player i receives a share x_i ($0 \leqslant x_i \leqslant 1$) of the unit size cake, then her utility is $\Pi_i(x_i) - C_i(x_i, z_i)$, where $C_i(x_i, z_i)$ denotes the cost of revoking her partial commitment z_i and (instead) obtaining a share x_i. Clearly, $C_i = 0$ if $x_i \geqslant z_i$, but if $x_i < z_i$ then $C_i > 0$. An example of such a cost-of-revoking function is $C_i = \max\{0, k_i(z_i - x_i)\}$, where $k_i > 0$. Our results, however, hold for any C_i that satisfies the following assumptions (the role and applicability of these assumptions are discussed in Section 5.5).

(A2) For each i $(i = A, B)$, the cost-of-revoking function C_i: $[0, 1] \times [0, 1] \to \mathbb{R}$ is continuous. Furthermore, (i) for each $0 < z_i \leqslant 1$, it is strictly decreasing, convex, and twice continuously differentiable in x_i on the interval $(0, z_i)$; (ii) for each $0 \leqslant x_i < 1$, it is strictly increasing, convex, and twice continuously differentiable in z_i on the interval $(x_i, 1)$; and (iii) for each $0 < z_i \leqslant 1$, the left-hand derivative of C_i w.r.t. x_i, evaluated at $x_i = z_i$, is strictly negative and is independent of the value of z_i (i.e., there exists a constant $k_i > 0$ such that for any $0 < z_i \leqslant 1$, $C_i^1(z_i -; z_i) = -k_i$).

The payoff pair $(P_A(z), P_B(z))$, when $z_A + z_B > 1$, will be defined as the Nash bargaining solution computed with the set $\Phi(z)$ of feasible utility pairs that can be the outcome of bargaining, which is constructed using the set X of feasible partitions of the unit size cake and the utility functions $\Pi_A - C_A$ and $\Pi_B - C_B$, where $X = \{(x_A, x_B): 0 \leqslant x_A, x_B \leqslant 1$ and $x_A + x_B = 1\}$, and with "disagreement point" (0, 0). Let us denote $\Pi_i - C_i$ by V_i. Then the set $\Phi(z)$ is the union of all pairs $(V_A(x_A, z_A), V_B(x_B, z_B))$ for $(x_A, x_B) \in X$. Indeed, for each pair $z \in [0, 1]^2$ s.t. $z_A + z_B > 1$, the set $\Phi(z)$ is the graph of the function $F(\cdot\,; z)$ defined by

$$F(u_A; z) = V_B(1 - V_A^{-1}(u_A; z_A); z_B),$$

where the domain and range of $F(\cdot\,; z)$ are the closed intervals $[-C_A(0, z_A), 1]$ and $[-C_B(0, z_B), 1]$, respectively. Given A1 and A2 it

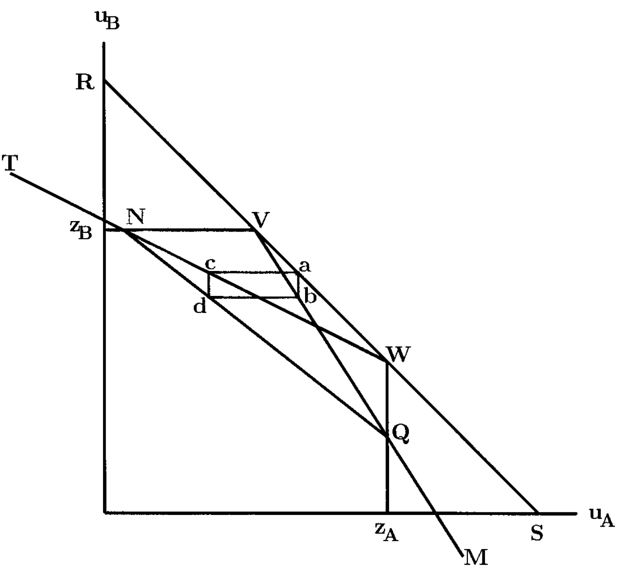

FIGURE 1

follows that, for each pair z, $F(\cdot\,; z)$ is bijective, concave, and strictly decreasing in u_A. We shall shortly illustrate the relationship between the feasible set $\Phi(z)$ and the partial commitments z_A and z_B.

It is possible (but not necessary) that there exist values of z_A and z_B such that $F(0; z) \leqslant 0$. In such a case, player i's payoff $P_i(z) = 0$ $(i = A, B)$. However, when $F(0; z) > 0$, then we define

$$(P_A(z), P_B(z)) = \arg\max[u_A \cdot u_B] \qquad \text{s.t.} \quad u_B = F(u_A; z).$$

2.3. *An Example*: *The Linear Model*

Figure 1 illustrates the relationship between the feasible set $\Phi(z)$ and the pair of partial commitments $z = (z_A, z_B)$, where $z_A + z_B > 1$, in the simplest possible case (i.e., with linear utilities): for any $0 \leqslant x_i,\ z_i \leqslant 1$, $\Pi_i(x_i) = x_i$ and $C_i(x_i, z_i) = \max\{0, k_i(z_i - x_i)\}$, where $k_i > 0$.

The line segment RS depicts the set X of feasible partitions of the unit size cake, while the graph TNQM depicts the feasible set $\Phi(z)$. If the partition lies on the line segment RV (resp., WS), then it is only player A (resp., B) who is revoking her partial commitment. But if the partition lies in the interior of the line segment VW, then both players are revoking their respective partial commitments. In this latter case, $u_A = x_A - k_A(z_A - x_A)$ and $u_B = x_B - k_B(z_B - x_B)$. Consider, for example, the partition depicted by point a. The utility to player A will be the "x-coordinate" of point c, while the utility to player B will be the "y-coordinate" of point b. Hence, this defines the utility pair (point d) associated with such a partition.

It is straightforward to show that the line segments TW, VM, and NQ are defined by the respective equations

$$u_A + (1 + k_A)\, u_B = [1 + k_A(1 - z_A)] \tag{1}$$

$$u_B + (1 + k_B)\, u_A = [1 + k_B(1 - z_B)] \tag{2}$$

$$(1 + k_A)\, u_B + (1 + k_B)\, u_A = [1 + k_A(1 - z_A) + k_B(1 - z_B) + k_A k_B(1 - z_A - z_B)]. \tag{3}$$

In Section 3, the slopes of these equations will play an important role. Note that these slopes are independent of z_A and z_B.

In the next section, we derive the unique Nash equilibrium for the linear model. All of the intuition behind the results for the main model (satisfying A1 and A2) can be obtained from the study of this linear case. In particular, as most of our arguments will be geometric, it is then easier to comprehend the more formal analysis underlying the main model.

3. Nash Equilibrium in the Linear Model

This section investigates the Nash equilibria with the assumption that, for each $i = \mathrm{A}, \mathrm{B}$, $\Pi_i(x_i) = x_i$ and $C_i(x_i, z_i) = \max\{0, k_i(z_i - x_i)\}$, where $k_i > 0$.

Lemma 1. *If* $z_\mathrm{A} + z_\mathrm{B} < 1$, *then* $z = (z_\mathrm{A}, z_\mathrm{B})$ *cannot constitute a Nash equilibrium.*

Proof. Straightforward, since if $z_\mathrm{A} + z_\mathrm{B} < 1$, then there will exist an i ($i = \mathrm{A}$ or B) such that $1 - z_j > \lambda_i(z)$; for otherwise, $\lambda_\mathrm{A}(z) \geqslant 1 - z_\mathrm{B}$ and $\lambda_\mathrm{B}(z) \geqslant 1 - z_\mathrm{A}$ imply that $z_\mathrm{A} + z_\mathrm{B} \geqslant 1$. Hence, player i can benefit from a unilateral deviation to $z_i' = 1 - z_j$. ∎

Lemma 2. *If* $z_\mathrm{A} + z_\mathrm{B} > 1$, *then* $z = (z_\mathrm{A}, z_\mathrm{B})$ *cannot constitute a Nash equilibrium.*

Proof. Assume that $F(0; z) > 0$.[1] The argument, by contradiction, is as follows (allowing for the roles of A and B being reversed).[2] Suppose $P_\mathrm{B}(z) < z_\mathrm{B}$ (i.e., in Fig. 1, the payoff pair $(P_\mathrm{A}(z), P_\mathrm{B}(z))$ lies either in the interior of the line segment NQ or on the line segment QM). We shall argue that player B can benefit from a decrease in her partial commitment to $z_\mathrm{B}' = z_\mathrm{B} - \varepsilon$ for some ε such that $0 < \varepsilon < z_\mathrm{B}$. Consider Fig. 2, where the graphs TNQM and TN′Q′M′, respectively, depict the feasible sets $\Phi(z)$ and $\Phi(z')$, where $z' = (z_\mathrm{A}, z_\mathrm{B}')$. Note that the line segment N′Q′ has the same slope (namely, $-[1 + k_\mathrm{B}]/[1 + k_\mathrm{A}]$) as the line segment NQ. Hence, if the pair $(P_\mathrm{A}(z), P_\mathrm{B}(z))$ lies in the interior of the line segment NQ, such as at point D, then there will exist an $0 < \varepsilon < z_\mathrm{B}$ such that the payoff pair $(P_\mathrm{A}(z'), P_\mathrm{B}(z'))$ will be at the point D′ that lies on the ray OD.[3]

Now suppose that the payoff pair $(P_\mathrm{A}(z), P_\mathrm{B}(z))$ is at point Q, where there is a kink in the graph TNQM. If the ratio $-[P_\mathrm{B}(z)/P_\mathrm{A}(z)]$ is equal to the slope of the line segment QM (namely, $-[1 + k_\mathrm{B}]$), then (by a similar argument to that given above) it follows that player B can benefit from a decrease in her partial commitment. Now consider $-[P_\mathrm{B}(z)/P_\mathrm{A}(z)] > -[1 + k_\mathrm{B}]$. Since the slope of the line segment Q′M′ is (also) equal to $-[1 + k_\mathrm{B}]$, it follows (by continuity) that there will exist an

[1] Lemma 1A in the Appendix establishes the desired result if $F(0; z) \leqslant 0$.

[2] A central part of our argument is based on exploiting the following property of the Nash bargaining solution: that the Nash bargaining solution lies at the unique point $(u_\mathrm{A}, u_\mathrm{B})$ on the graph of the function $F(\cdot\,; z)$, where some tangent has slope $-u_\mathrm{B}/u_\mathrm{A}$.

[3] By a similar argument, if the payoff pair $(P_\mathrm{A}(z), P_\mathrm{B}(z))$ lies in the interior of the line segment QM, then player B can (also) benefit from a decrease in her partial commitment.

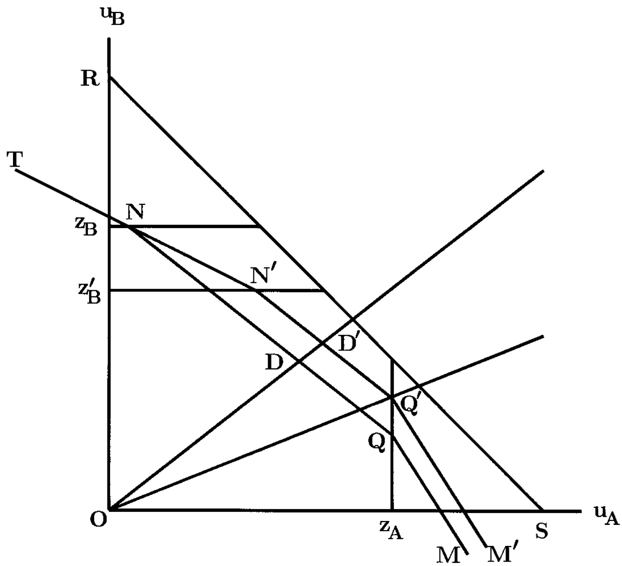

FIGURE 2

$0 < \varepsilon < z_B$ such that the slope of the ray OQ′ is (also) strictly less than $[1 + k_B]$. Consequently, the payoff pair $(P_A(z'), P_B(z'))$ must lie either at point Q′ or to the left of point Q′, i.e., $P_B(z') > P_B(z)$. ∎

LEMMA 3. *If* $z_A + z_B = 1$ *and* $z_B/z_A \neq (1 + k_B)/(1 + k_A)$, *then* (z_A, z_B) *cannot constitute a Nash equilibrium.*

Proof. By contradiction. Suppose $z_B/z_A > (1 + k_B)/(1 + k_A)$. We shall show that player A can benefit from an increase in her partial commitment to $z'_A = z_A + \varepsilon$ for some ε such that $0 < \varepsilon < 1 - z_A$. Consider Fig. 3. The graph TNQM depicts the feasible set $\Phi(z')$, where $z' = (z'_A, z_B)$. The slope of the line segment NQ is equal to $-(1 + k_B)/(1 + k_A)$. Hence, by continuity, it follows that there will exist an $0 < \varepsilon < 1 - z_A$ such that the slope of the ray OQ is (also) strictly greater than $(1 + k_B)/(1 + k_A)$. Consequently, $P_A(z') \geqslant z'_A$, i.e., $P_A(z') > z_A$.

A symmetric argument establishes that if $z_B/z_A < (1 + k_B)/(1 + k_A)$, then player B can benefit from an increase in her partial commitment. ∎

We are now ready to derive the unique Nash equilibrium for the linear model.

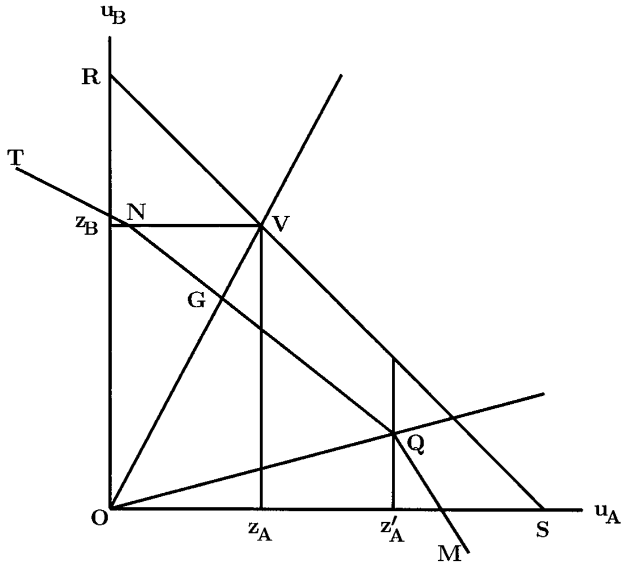

FIGURE 3

PROPOSITION 1 (Linear Model). *If, for each i* ($i = \mathrm{A}, \mathrm{B}$), $\Pi_i(x_i) = x_i$ *and* $C_i(x_i, z_i) = \max\{0, k_i(z_i - x_i)\}$, *where* $k_i > 0$, *then the bargaining model has a unique Nash equilibrium*:

$$z_{\mathrm{A}}^* = (1 + k_{\mathrm{A}})/(2 + k_{\mathrm{A}} + k_{\mathrm{B}}) \qquad \textit{and} \qquad z_{\mathrm{B}}^* = (1 + k_{\mathrm{B}})/(2 + k_{\mathrm{A}} + k_{\mathrm{B}}).$$

Proof. Lemmas 1, 2 and 3 imply that if a pair $z = (z_{\mathrm{A}}, z_{\mathrm{B}})$ is in a Nash equilibrium, then $z_{\mathrm{A}} + z_{\mathrm{B}} = 1$ *and* $(z_{\mathrm{B}}/z_{\mathrm{A}}) = (1 + k_{\mathrm{B}})/(1 + k_{\mathrm{A}})$, i.e., the pair $(z_{\mathrm{A}}, z_{\mathrm{B}})$ is as defined in the proposition. We shall now establish that, in fact, this pair is in a Nash equilibrium. First, suppose player A unilaterally deviates to $z'_{\mathrm{A}} = z_{\mathrm{A}} + \varepsilon$, where $0 < \varepsilon \leqslant 1 - z_{\mathrm{A}}$. The graph TNQM, in Fig. 3, depicts the feasible set $\Phi(z')$, where $z' = (z'_{\mathrm{A}}, z_{\mathrm{B}})$. Since the slope of the line segment NQ is equal to $-(1 + k_{\mathrm{B}})/(1 + k_{\mathrm{A}})$, it follows that the payoff pair $(P_{\mathrm{A}}(z'), P_{\mathrm{B}}(z'))$ will be at the point G that lies on the ray OV. Hence, the deviation is not profitable. Second, it is straightforward to show that player A cannot benefit from a unilateral deviation to any $0 \leqslant z'_{\mathrm{A}} < z_{\mathrm{A}}$. Finally, by a symmetric argument it follows that player B cannot benefit from a unilateral deviation to a partial commitment $z'_{\mathrm{B}} \neq z_{\mathrm{B}}$. ▌

We shall defer the discussion of this result until after we establish, formally, a similar result in the main model. In particular, further intuition and interpretation of this result is provided in Section 5.4, where we shall use this result to understand the cause of the indeterminacy in the Nash demand game. However, observe that player i's equilibrium share of the unit size cake is strictly increasing in her marginal cost k_i of revoking a partial commitment, and strictly decreasing in k_j.

4. Nash Equilibrium in the General Model

This section shows how our results generalise when Π_i and C_i satisfy A1 and A2, respectively. The proofs can be found in the Appendix. The first result here, Proposition 2, establishes the uniqueness of the Nash equilibrium, and provides a characterisation of this equilibrium.

Proposition 2 (Uniqueness and Characterisation). *If, for each* i $(i=\mathrm{A}, \mathrm{B})$, Π_i *and* C_i *satisfy* A1 *and* A2, *respectively, then the bargaining model has at most a unique Nash equilibrium. In this equilibrium, the players' partial commitments* z_{A}^* *and* z_{B}^* *constitute the unique solution to* $z_{\mathrm{A}}+z_{\mathrm{B}}=1$ and

$$\frac{\Pi_{\mathrm{B}}(z_{\mathrm{B}})}{\Pi_{\mathrm{A}}(z_{\mathrm{A}})}=\frac{\Pi'_{\mathrm{B}}(z_{\mathrm{B}})+k_{\mathrm{B}}}{\Pi'_{\mathrm{A}}(z_{\mathrm{A}})+k_{\mathrm{A}}} \tag{4}$$

This unique equilibrium is very similar to that derived in the linear model. In fact, if Π_i is linear and C_i satisfies A2, then the equilibrium is exactly the same. Indeed, a key (and rather interesting) feature of the equilibrium partial commitments is that they depend only on the players' *marginal* costs, k_{A} and k_{B} (cf. A2(iii)), of revoking their respective partial commitments.

We now turn to the issue of the *existence* of this unique Nash equilibrium. It turns out that the equilibrium exists provided the degree of convexity of the cost-of-revoking function C_i $(i=\mathrm{A}, \mathrm{B})$ is sufficiently small. (In the Appendix, we construct a simple example to show that this unique equilibrium does not exist if the degree of convexity of C_i is sufficiently large.) In order to establish our existence result, Proposition 3 below, we need to employ two further assumptions on C_i (in addition to A2), which are stated below in A3. In words, A3 states that, for any pair (x_i, z_i) such that $x_i<z_i$, (i) the increase in the cost $C_i(x_i, z_i)$ as x_i is decreased at the margin is equal to the increase in this cost as z_i is increased at the margin, and (ii) the increase in the marginal cost $-C_i^1(x_i, z_i)$ as x_i is decreased at the margin is greater than or equal to the increase in this marginal cost as z_i is increased at the margin.

(A3) For each i $(i=\mathrm{A}, \mathrm{B})$ and for any pair (x_i, z_i) such that $x_i<z_i$: (i) $-C_i^1(x_i, z_i)=C_i^2(x_i, z_i)$ and (ii) $C_i^{11}(x_i, z_i)\geqslant -C_i^{12}(x_i, z_i)$.

These two assumptions embody the notion that x_i and z_i are to be treated symmetrically in the cost function C_i. For example, if the cost of revoking z_i and obtaining $x_i < z_i$ is a (possibly non-linear) function of the difference $z_i - x_i$, then A3 will be satisfied. A simple example of a convex cost function satisfying A2 and A3 is $C_i = 0$ if $x_i \geqslant z_i$, and $C_i = k_i(z_i - x_i) + (\delta_i/2)(z_i - x_i)^2$ if $x_i < z_i$, where $k_i > 0$ and $\delta_i \geqslant 0$.

Proposition 3 (Existence). *If, for each i* ($i = \mathrm{A, B}$), Π_i *satisfies* A1, C_i *satisfies* A2 *and* A3, *and, for any pair* (x_i, z_i) *such that* $x_i < z_i$, $C_i^{11}(x_i, z_i) < (k_i)^2/\Pi_i(z_i^*)$, *then the unique Nash equilibrium described in Proposition* 2 *does exist.*

Therefore, if the "degree of convexity," C_i^{11}, of the cost function C_i ($i = \mathrm{A, B}$) is strictly less than $(k_i)^2/\Pi_i(z_i^*)$, then the unique equilibrium does exist. Consider, for example, the following function: $C_i = 0$ if $x_i \geqslant z_i$, and if $x_i < z_i$, then $C_i = k_i(z_i - x_i) + (\delta_i/2)(z_i - x_i)^2$, where $k_i > 0$ and $\delta_i \geqslant 0$. And assume that $\Pi_i(x_i) = x_i$. Thus, if $\delta_i < [(k_i)^2\,(2 + k_i + k_j)]/(1 + k_i)$ (where $i \neq j$), then the unique equilibrium does exist. Notice that this bound on δ_i is strictly increasing in k_i (and in k_j), and it converges to plus infinity as either k_i or k_j or both k_i and k_j converge to plus infinity. Clearly, therefore, for a large class of convex cost functions satisfying A2 and A3 the unique Nash equilibrium, described in Proposition 2, does exist.

5. Discussion

Sections 5.1, 5.2, and 5.3 state, and comment upon, some of the main properties of the unique Nash equilibrium, which is described in Proposition 2. Then, in Section 5.4, we show that our model can be applied to select a unique equilibrium in Nash's demand game. Section 5.5 contains a brief discussion of A2, the assumptions made on the cost-of-revoking function C_i. In Section 5.6 we argue that our results would *not* change if player A were to choose her partial commitment *after observing* player B's partial commitment. Finally, in Section 5.7 we point out that our results are (also) robust to a modification in the payoff rule.

5.1. *Efficiency*

In equilibrium neither player revokes her partial commitment. Naturally, the complete information assumption must be crucial in establishing this feature of the equilibrium. It would be interesting to analyze our model with the assumption that a player's marginal cost k_i of revoking a partial commitment is her private information. We suspect that there may then exist an equilibrium in which some player does revoke her partial commitment.

5.2. *Comparative Statics*

The equilibrium share of the cake to a player depends only upon k_{A} and k_{B}, the *marginal* costs to the players of revoking their respective partial commitments. It is independent of other features of the cost-of-revoking functions. For example, it is independent of the degrees of convexity in these functions.

Player i's equilibrium share is strictly increasing in k_i, and strictly decreasing in k_j. Thus, if a high k_i is interpreted as a weak bargaining position for player i (as indeed one's intuition *a priori* would suggest), then our model supports the viewpoint that, in bargaining, weakness can often be a source of strength (cf., e.g., Schelling [6]).

5.3. *Nash Program*

If and only if $\Pi_i'(z_i^*)=1$ (for both $i=\mathrm{A}$ and B), the equilibrium partition of the unit size cake $(x_{\mathrm{A}}, x_{\mathrm{B}})=(z_{\mathrm{A}}^*, z_{\mathrm{B}}^*)$ can be described by an asymmetric Nash bargaining solution computed with the feasible utility set $\{(\Pi_{\mathrm{A}}(x_{\mathrm{A}}), \Pi_{\mathrm{B}}(x_{\mathrm{B}})): 0 \leqslant x_{\mathrm{A}}, x_{\mathrm{B}} \leqslant 1 \text{ and } x_{\mathrm{A}}+x_{\mathrm{B}}=1\}$, "disagreement point" $(0, 0)$, and "bargaining powers" $\tau_{\mathrm{A}}=1+k_{\mathrm{A}}$ and $\tau_{\mathrm{B}}=1+k_{\mathrm{B}}$.

5.4. *Equilibrium Selection in the Nash Demand Game*

The Nash demand game (see [5]) possesses a continuum of Nash equilibria. By perturbing the *informational* structure of his demand game, John Nash was able to select a unique equilibrium (see Binmore [1 and 2] and van Damme [7] for a thorough discussion of Nash's equilibrium selection procedure). For an alternative, but related, equilibrium selection procedure see Carlsson [3].

At the heart of the Nash demand game lies the assumption that the bargainers can make *irrevocable* commitments to their respective demands. Our model may be interpreted as a perturbation to the *commitment* structure of the Nash demand game, where this perturbation is made arbitrarily small in the limit as both k_{A} and k_{B} diverge to plus infinity.

It follows from Proposition 2 that in the limit, as $k_{\mathrm{A}} \to +\infty$ and $k_{\mathrm{B}} \to +\infty$, keeping the ratio $k_{\mathrm{A}}/k_{\mathrm{B}}$ constant at some value $\gamma>0$, the unique equilibrium partial commitments (or demands) will depend on the value of γ. For example, if $\Pi_i(x_i)=x_i$, then, in this limit, $z_{\mathrm{A}}^* \to \gamma/(1+\gamma)$ and $z_{\mathrm{B}}^* \to 1/(1+\gamma)$. This result, besides selecting a unique equilibrium in the Nash demand game, provides an interpretation of the indeterminacy (multiplicity of equilibria) in the demand game: the ratio $k_{\mathrm{A}}/k_{\mathrm{B}}$ is an important determinant of the equilibrium demands, but is undetermined in Nash's demand game (since, in the demand game, $k_{\mathrm{A}}=+\infty$ and $k_{\mathrm{B}}=+\infty$).

It is interesting to note that these results are analogous to the results obtained in Rubinstein's alternating-offers bargaining model: if the players' rates of time preference, r_A and r_B, are such that $r_A = r_B = 0$, then the model has a continuum of subgame perfect equilibria, but if $r_A > 0$ and $r_B > 0$, then there exists a unique subgame perfect equilibrium, and moreover, in the limit as $r_A \to 0$ and $r_B \to 0$, the unique equilibrium partition depends on the ratio r_A/r_B. By studying the limiting model ($k_A \to +\infty$ and $k_B \to +\infty$), rather than the model at the limit ($k_A = k_B = +\infty$), one discovers the significant role played by the ratio k_A/k_B. In fact, it is rather intuitive that in order to generate a unique (equilibrium) partition, one needs to know how "strong" player A is *relative* to player B. Since this question cannot be addressed when $k_A = k_B = +\infty$, it is not surprising that therefore every partition can be supported as an equilibrium in the Nash demand game.

5.5. *Comments on Some of the Assumptions*

The continuity and monotonicity assumptions on C_i capture the notion that the cost to player i of revoking her partial commitment z_i and obtaining a share $x_i < z_i$ depends on the amount by which she revokes (deviates from) her partial commitment (e.g., on the difference $z_i - x_i$). Consider, for example, wage bargaining between a union leader and management. If the cost to the union leader of revoking a partial commitment depends on her expected loss of reputation (credibility) and on the probability of her being fired from the post of union leader, then it may seem reasonable to assume that the cost is continuous in the difference $z_i - x_i$ (cf., e.g., the extracts from Schelling that were stated in Section 1).

The assumption that the marginal cost $-C_i^1(z_i-; z_i)$ of revoking a partial commitment is strictly positive is quite reasonable. However, let me emphasize that this assumption plays a fundamental role in our analysis and results. (The kinks in the graphs of the feasible sets are created by this assumption.) In fact, if both players' marginal costs were equal to zero, then almost any pair of partial commitments can be supported in equilibrium.

The assumption that the marginal cost $-C_i^1(z_i-; z_i)$ of revoking a partial commitment is independent of the value of the partial commitment seems reasonable, especially since there appear to be no good (or compelling) reasons to suggest otherwise. Indeed, why would it be relatively easier (i.e., less costly) to revoke, at the margin, moderate (or extreme) partial commitments. Let me, nevertheless, briefly discuss whether and how our results would be affected by any alternative assumption describing the dependence of the marginal cost $-C_i^1(z_i-; z_i)$ on z_i. The characterization of an equilibrium is essentially unaffected: $z_A + z_B = 1$ and Eq. (4) is to be satisfied, but with k_i replaced by $-C_i^1(z_i-; z_i)$. Continuity of the marginal cost in z_i would ensure that the model possesses at most an odd number $n \geqslant 1$ of equilibria. If in addition, for example, the marginal cost $-C_i^1(z_i-; z_i)$ is non-increasing in z_i, then uniqueness is (re)obtained. The necessary and sufficient conditions for the existence of an equilibrium are unaffected (cf. Lemma 4A in the Appendix). However, whether they would be satisfied will depend on how the marginal cost varies with z_i.

Finally, note that the differentiability assumptions on C_i (and on Π_i) have been made to simplify the analysis and the exposition. Our results can be obtained without such assumptions, especially since the concavity in $\Pi_i - C_i$ does ensure differentiability almost everywhere.

5.6. *No First-Mover Advantage*

How would the results obtained in this paper change if player A were to choose her partial commitment z_A *after observing* the partial commitment z_B chosen by player B? Would player B benefit from being the "first-mover"? We now sketch an argument which will establish that the outcome is, in fact, *unaffected*: that is, the unique subgame perfect equilibrium partial commitments (in the sequential-move structure) are *identical* to the unique Nash equilibrium partial commitments (in the simultaneous-move structure). Our argument (and therefore this conclusion) is for the linear utilities case.

The argument below will exploit our main result, namely that the simultaneous-move (linear) model has a unique Nash equilibrium (z_A^*, z_B^*), as stated in Proposition 1, with the property that $z_B^*/z_A^* = K$, where $K = (1+k_B)/(1+k_A)$.

If $z_B < z_B^*$, then $z_B/(1-z_B) < K$. It can therefore be verified, using geometric arguments (similar to those contained in the proofs to Lemmas 2 and 3 and Proposition 1), that player A's unique best response is to set $z_A = 1 - z_B$. Now suppose that player B chooses $z_B > z_B^*$. This implies that $z_B/(1-z_B) > K$. It can therefore be verified, again using geometric arguments, that player A's unique best response is to set z_A equal to the unique solution of the following equation: $F(z_A; z)/z_A = K$, where $z = (z_A, z_B)$. With linear utilities, $F(z_A; z) = [(1+k_B)(1-z_A) - k_B z_B]$. Hence, player A's unique best response to a $z_B > z_B^*$ is $z_A = [1 + k_B(1-z_B)]/K(2+k_A)$. For each $z_B > z_B^*$ player B's payoff will equal $F(z_A; z)$ *with* z_A being player A's unique best response to z_B. Straightforward computation, therefore, establishes that player B's optimal choice is z_B^*.[4]

[4] The intuition for this result, that the sequential-move structure generates the same outcome as the simultaneous-move structure, comes partly from the observation that each player's indifference curves are kinked at certain key points. For example, player B's indifference curve through (z_A^*, z_B^*) is kinked at that point. This observation can be obtained, at least informally, through our geometric arguments.

The result obtained here re-enforces a main message of this paper: the equilibrium partial commitments chosen by the players depend critically (and only) on the *relative* magnitudes of the *marginal* costs to the players of revoking their respective partial commitments. Even the order in which the players make their respective partial commitments does not matter.

5.7. *On the Payoff Rule*

We have assumed that if the players' partial commitments are compatible ($z_A + z_B \leqslant 1$), then neither player will have to revoke their respective partial commitment. It may, however, be reasonable (in some contexts) not to make this assumption. In that case one can define the players' payoffs using the Nash bargaining solution, as we did if $z_A + z_B > 1$. With such a change in the rule that defines the payoffs, our results are *unaffected*: Propositions 1–3 still describe the unique Nash equilibrium.[5]

6. Concluding Remarks

This paper has explored the influence of a particular type of *tactic* on the outcome of bargaining situations. Of course, we are far from a complete understanding of this bargaining tactic. Much more research is required, and the interested reader is advised to consult Schelling [6] and Crawford [4] for further discussion of the other issues involved.

We see some scope for further development and extension of the model studied in this paper, in order to increase and improve our understanding of commitment tactics in bargaining. One line of research could involve modifying the rules that define the payoffs, (for example, by choosing a different bargaining solution, and or by changing the assumptions on the structure of the costs of revoking). Another line of research could involve enlarging the strategy spaces (for example, by allowing the players to revise their respective partial commitments, with costs being incurred if they are revised downwards and, perhaps, benefits being obtained if revised upwards).

Appendix

This appendix contains the analysis of the general model, in which Π_i and C_i satisfy A1 and A2, respectively. Specifically, we provide proofs to Propositions 2 and 3, which were stated in Section 4. In order to establish

[5] However, with this change, the arguments and proofs are lengthier; they are available from the author upon request.

Proposition 3, we first derive necessary and sufficient conditions for the existence of the unique Nash equilibrium (Lemma 4A below). Finally, an example is constructed to show that the unique Nash equilibrium will not exist when the degree of convexity of the cost-of-revoking function C_A is sufficiently large.

Proof of Proposition 2. The proof of Lemma 1 establishes that if $z_A + z_B < 1$, then (z_A, z_B) cannot constitute a Nash equilibrium. Proposition 2 follows immediately from Lemmas 1A–3A (below), since (given A1) there exists a unique solution to $z_A + z_B = 1$ *and* Eq. (4). ▮

LEMMA 1A. *If* $z_A + z_B > 1$ *and* $F(0; z) \leqslant 0$, *then* (z_A, z_B) *cannot constitute a Nash equilibrium.*

Proof. First, suppose $z_A + z_B < 2$. In this case there will exist an i ($i = A$ or B) s.t. $1 - z_i > 0$. Hence, player j ($j \neq i$) can benefit by unilaterally deviating to $z'_j = 1 - z_i$.

Now suppose $z = (1, 1)$, and let $z'(\varepsilon) = (1, \varepsilon)$ where $0 \leqslant \varepsilon < 1$. Since $F(0; z'(0)) > 0$ and $F(0; z'(\varepsilon))$ is continuous in ε, there will exist an $0 < \varepsilon < 1$ s.t. $F(0; z'(\varepsilon)) > 0$. Consequently, player B can benefit by unilaterally deviating to $z'_B = \varepsilon$. ▮

LEMMA 2A. *If* $z_A + z_B > 1$ *and* $F(0; z) > 0$, *then* $z = (z_A, z_B)$ *cannot constitute a Nash equilibrium.*

Proof. The argument, by contradiction, is broken into three cases (allowing for the roles of A and B being reversed).

Case (i). Either $P_A(z) > \Pi_A(z_A)$ or $P_A(z) = \Pi_A(z_A)$ and $-[P_B(z)/P_A(z)] = F^1(\Pi_A(z_A)+; z)$, where this last term is the right-hand derivative of $F(\cdot; z)$ w.r.t. u_A evaluated at $u_A = \Pi_A(z_A)$. Consider a unilateral deviation by player A to z'_A such that $\Pi_A(z'_A) = P_A(z) + \varepsilon$, where $0 \leqslant \varepsilon < 1 - P_A(z)$. Define, for each $0 \leqslant \varepsilon < 1 - P_A(z)$, $f_1(\varepsilon) = -[F(P_A(z) + \varepsilon; z)]/[P_A(z) + \varepsilon]$ and $g_1(\varepsilon) = F^1([P_A(z) + \varepsilon]-; z'(\varepsilon))$, where $z'(\varepsilon) = (z'_A, z_B)$ and $g_1(\varepsilon)$ is the left-hand derivative of F w.r.t. u_A. f_1 is continuous on the interval $[0, 1 - P_A(z))$. Furthermore, by writing $g_1(\varepsilon)$ in terms of Π_i and C_i ($i = A$ and B), it is straightforward to verify that g_1 is also continuous on the interval $[0, 1 - P_A(z))$ (since Π_A and Π_B are continuously differentiable, C_B is continuously differentiable in x_B on $(0, z_B)$, and $C^1_A(z_A-; z_A) = -k_A$ for any $0 < z_A \leqslant 1$). Hence, since $f_1(0) < g_1(0)$, there will exist an $0 < \varepsilon < 1 - P_A(z)$ such that $f_1(\varepsilon) < g_1(\varepsilon)$. Consequently, by the concavity of $F(\cdot; z'(\varepsilon))$ in u_A, it follows that $P_A(z'(\varepsilon)) \geqslant P_A(z) + \varepsilon$, i.e., $P_A(z'(\varepsilon)) > P_A(z)$.

Case (ii). $P_A(z) = \Pi_A(z_A)$ and $-[P_B(z)/P_A(z)] > F^1(\Pi_A(z_A)+; z)$. Consider a unilateral deviation by player B to z'_B such that $\Pi_B(z'_B) =$

$\Pi_B(z_B)-\varepsilon$, where $0 \leqslant \varepsilon < \Pi_B(z_B)$. Define, for each $0 \leqslant \varepsilon < \Pi_B(z_B)$, $f_2(\varepsilon) = -[F(\Pi_A(z_A); z'(\varepsilon))]/\Pi_A(z_A)$ and $g_2(\varepsilon) = F^1(\Pi_A(z_A)+; z'(\varepsilon))$, where $z'(\varepsilon) = (z_A, z'_B)$. Both f_2 and g_2 are continuous on the interval $[0, \Pi_B(z_B))$. Hence, since $f_2(0) > g_2(0)$, there will exist an $0 < \varepsilon < \Pi_B(z_B)$ such that $f_2(\varepsilon) > g_2(\varepsilon)$. Consequently, $P_B(z'(\varepsilon)) > P_B(z)$.

Case (iii). $P_A(z) < \Pi_A(z_A)$ and $P_B(z) < \Pi_B(z_B)$. In this case either player can benefit from a unilateral decrease or from a unilateral increase in her partial commitment. Consider a unilateral deviation by player A to $z'_A = z_A - \varepsilon$, where $-\hat{\varepsilon} \leqslant \varepsilon \leqslant \hat{\varepsilon}$ for some small $\hat{\varepsilon} > 0$. Define, for each $-\hat{\varepsilon} \leqslant \varepsilon \leqslant \hat{\varepsilon}$, $f_3(\varepsilon) = -[F(P_A(z); z'(\varepsilon))]/P_A(z)$ and $g_3(\varepsilon) = F^1(P_A(z); z'(\varepsilon))$, where $z'(\varepsilon) = (z'_A, z_B)$. Given the current hypothesis, there will exist an $\hat{\varepsilon} > 0$ such that both f_3 and g_3 are differentiable on the open interval $(-\hat{\varepsilon}, \hat{\varepsilon})$. In particular, by writing the derivatives $f'_3(\varepsilon)$ and $g'_3(\varepsilon)$ in terms of Π_i, C_i and the derivatives of Π_i and C_i $(i = \text{A, B})$, one can show that $f'_3(0) \neq g'_3(0)$. Consequently, since $f_3(0) = g_3(0)$, there will exist either an $0 < \varepsilon < \hat{\varepsilon}$ or an $-\hat{\varepsilon} < \varepsilon < 0$ such that $g_3(\varepsilon) > f_3(\varepsilon)$. Hence, $P_A(z'(\varepsilon)) > P_A(z)$. ∎

Lemma 3A. *If* $z_A + z_B = 1$ *and Eq.* (4), *stated in Proposition* 2, *does not hold, then* (z_A, z_B) *cannot constitute a Nash equilibrium.*

Proof. By contradiction. Suppose $[\Pi_B(z_B)/\Pi_A(z_A)] > [\Pi'_B(z_B) + k_B]/[\Pi'_A(z_A) + k_A]$. Consider a unilateral deviation by player A to z'_A such that $\Pi_A(z'_A) = \Pi_A(z_A) + \varepsilon$, where $0 \leqslant \varepsilon < 1 - \Pi_A(z_A)$. Let f_1 and g_1 be defined as in Case (i) of the proof of Lemma 2A. It is straightforward to verify that g_1 is not right-continuous at $\varepsilon = 0$, but that it is continuous on the open interval $(0, 1 - \Pi_A(z_A))$, and that $g_1(\varepsilon)$ converges to $-[\Pi'_B(z_B) + k_B]/[\Pi'_A(z_A) + k_A]$ as $\varepsilon \to 0+$. Furthermore, f_1 is continuous on the closed interval $[0, 1 - \Pi_A(z_A)]$. Hence, since $f_1(0)$ equals $-[\Pi_B(z_B)/\Pi_A(z_A)]$, it follows that there exists an $0 < \varepsilon < 1 - \Pi_A(z_A)$ such that $f_1(\varepsilon) < g_1(\varepsilon)$. Consequently, $P_A(z'(\varepsilon)) > P_A(z)$, where $z'(\varepsilon) = (z'_A, z_B)$. A symmetric argument with the roles of A and B reversed completes the proof. ∎

In order to prove Proposition 3 we need the following result.

Lemma 4A. (Necessary and Sufficient conditions for existence). *The unique pair* (z^*_A, z^*_B), *defined in Proposition* 2, *is in a Nash equilibrium* if and only if *the following two* (*symmetric*) *conditions are satisfied. For each pair* $(i, j) = (\text{A, B})$ *and* $(i, j) = (\text{B, A})$, *it is the case that*, *for any* $0 < \varepsilon \leqslant 1 - z^*_i$,

$$\frac{V_j(1 - x^*_i(\varepsilon); z^*_j)}{\Pi_i(z^*_i)} \leqslant \frac{V^1_j(1 - x^*_i(\varepsilon); z^*_j)}{V^1_i(x^*_i(\varepsilon); z^*_i + \varepsilon)}, \tag{5}$$

where $x_i^*(\varepsilon) = V_i^{-1}(\Pi_i(z_i^*); z_i^* + \varepsilon)$ *and where* V_i^1 *denotes the derivative of* V_i *w.r.t.* x_i.

Proof. When $i = A$ and $j = B$, inequality (5) can be rewritten as follows. For any $0 < \varepsilon \leqslant 1 - z_A^*$,

$$\frac{-F(\Pi_A(z_A^*); z^{**}(\varepsilon))}{\Pi_A(z_A^*)} \geqslant F^1(\Pi_A(z_A^*); z^{**}(\varepsilon)), \tag{6}$$

where $z^{**}(\varepsilon) = (z_A^* + \varepsilon, z_B^*)$. It is now evident that for any $0 < \varepsilon \leqslant 1 - z_A^*$, $\Pi_A(z_A^*) \geqslant P_A(z^{**}(\varepsilon))$ *if and only if* inequality (6) is satisfied. Hence, since it is trivial to show that player A cannot benefit from a unilateral deviation to any $0 \leqslant z_A < z_A^*$ (given $z_B = z_B^*$), it follows that z_A^* is player A's best response to z_B^* *if and only if* inequality (6) is satisfied for all $0 < \varepsilon \leqslant 1 - z_A^*$. A symmetric argument establishes the same for player B. ▮

Proof of Proposition 3. Let $G_i(\varepsilon)$ and $H_i(\varepsilon)$ denote the left-hand and right-hand sides of inequality (5), respectively. Differentiating G_i and H_i w.r.t. ε, we obtain that $H_i'(\varepsilon) > G_i'(\varepsilon)$ *if*

$$\frac{x_i'(\varepsilon)}{\Pi_i(z_i^*)} > \frac{V_i^{11}[x_i'(\varepsilon)] + V_i^{12}}{(V_i^1)^2},$$

where $x_i'(\varepsilon)$ denotes the derivative of $x_i^*(\varepsilon)$, and where the derivatives of V_i are evaluated at $x_i = x_i^*(\varepsilon)$ and $z_i = z_i^* + \varepsilon$. It is straightforward to verify that, given A1–A3, this inequality is satisfied *if* $C_i^{11}(x_i, z_i) < (k_i)^2/\Pi_i(z_i^*)$ for all $x_i < z_i$. Hence, this establishes that, for any $0 < \varepsilon < 1 - z_i^*$, $H_i'(\varepsilon) > G_i'(\varepsilon)$. Furthermore, as $\varepsilon \to 0+$, $G_i(\varepsilon)$ converges to $\Pi_j(z_j^*)/\Pi_i(z_i^*)$, and $H_i(\varepsilon)$ converges to $[\Pi_j'(z_j^*) + k_j]/[\Pi_i'(z_i^*) + k_i]$. Consequently, by appealing to Eq. (4), it follows that, for any $0 < \varepsilon \leqslant 1 - z_i^*$, $H_i(\varepsilon) > G_i(\varepsilon)$. ▮

A.1. *A Counterexample*

Suppose $\Pi_i(x_i) = x_i$ (for $i = $ A and B), $C_B = \max\{0, k_B(z_B - x_B)\}$, and $C_A = 0$ if $x_A \geqslant z_A$, but if $x_A < z_A$, then $C_A = k_A(z_A - x_A) + (\delta_A/2)(z_A - x_A)^2$, where $k_A > 0$ and $k_B > 0$. We now establish that if $\delta > k_A(1 + k_A)(2 + k_A + k_B)$, then the unique equilibrium does not exist. As in the proof of Proposition 3, let $G_A(\varepsilon)$ and $H_A(\varepsilon)$ denote the left-hand and the right-hand sides of inequality (5) with $(i, j) = (\text{A}, \text{B})$. It is straightforward to verify that

$$G_A'(\varepsilon) - H_A'(\varepsilon) = -(1 + k_B)\left[\frac{x_A'(\varepsilon)}{z_A^*} + \frac{[\delta_A x_A'(\varepsilon) - \delta_A]}{(V_A^1)^2}\right],$$

where $V_A^1 = 1 + k_A + \delta_A[z_A^* + \varepsilon - x_A^*(\varepsilon)]$ and

$$x'_A(\varepsilon) = \frac{k_A + \delta_A[z_A^* + \varepsilon - x_A^*(\varepsilon)]}{1 + k_A + \delta_A[z_A^* + \varepsilon - x_A^*(\varepsilon)]}.$$

Hence, if $\delta_A > k_A(1 + k_A)(2 + k_A + k_B)$, then $[G'_A(\varepsilon) - H'_A(\varepsilon)]$ converges to a *strictly positive* number as $\varepsilon \to 0+$. Consequently, since $G_A(\varepsilon)$ and $H_A(\varepsilon)$ converge to the same value as $\varepsilon \to 0+$, it follows that there exists an $0 < \varepsilon < 1 - z_A^*$ such that $G_A(\varepsilon) > H_A(\varepsilon)$.

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