Seniority and Incumbency in Legislatures

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Abstract

In this paper we elaborate on a strategic view of institutional features. Our focus is on seniority, though we note that this general approach may also be deployed to understand other aspects of institutional arrangements. We have taken the initial game-theoretic model of seniority of McKelvey and Riezman (1992), simplified it in order to characterize its fundamental implications, generalized these results in several ways, and extended the model by deriving additional implications.

The broad messages of our paper, articulated by McKelvey and Riezman as well, are two. First, the endogenous choice of institutional features like seniority by self-governing groups is strategic. While the fine-grained ways of doing things in an institutional context surely serve internal functional objectives, these are not the only objectives. Agents making choices on how to govern themselves have private motivations — in the case of elected politicians they often revolve around reelection.

This leads to our second broad message. The institutions through which self-governing groups conduct their business do not exist in a vacuum. They are embedded in a broader context. Those offering functional explanations for various institutional features overlook this. Particular institutional arrangements have effects outside the governance institution itself. These effects, in principle, could be accidental by-products. Our strategic approach, however, argues that they may well be the primary reasons for a practice being instituted.

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1 Introduction

Among the features that structure the organization of group affairs, seniority in one form or another is ubiquitous. In some settings the most senior enjoys all there is to enjoy. For example, the eldest male descendant inherits all property in systems of law governed by strict primogeniture. In other settings the effect is more muted: the views of seniors are respected, sometimes politely deferred to, but rarely have more impact than those of others in the group. The eldest elders in tribal societies, distinguished emeritus professors, and other eminences grise come to mind. In most settings seniority effects fall somewhere in between, connected to influence but not absolutely decisive. Even the most powerful senior leaders in the late nineteenth and early twentieth century U.S. House of Representatives — Speaker Thomas Brackett (“Czar”) Reed and Joseph Gurney (“Boss”) Cannon — had to keep their minions happy; these speakers may have had the privilege of taking the first and perhaps even the largest bite from the apple, but they could not consume the apple in its entirety and, in particular, they could not ignore the claims of senior colleagues. In short, seniority is ubiquitous, its effects and consequences varying from the extraordinary to the benign.

Seniority, moreover, is not one-size-fits-all, but actually comes in many forms. Consider a simple legislature with \( N \) agents. One form of seniority rank-orders agents according to terms of continuous service — seniority rank of one is assigned to the member having the most terms of continuous service and \( N \) for the least senior legislator. Arguably, however, seniority could be based on total terms of service with no penalty exacted for service interruptions. More common is ordinal ranking embedded inside substructures of the full legislature — within parties, committees, or party delegations on committees. Seniority in these cases is not attached to the full legislature but rather is associated with some legislative subunit.

Just as there is variety in the domain over which seniority applies, there is also variety in the amount of information impounded by a seniority designation. The most typical seniority systems, like those described in the previous paragraph, reveal nearly all of the ordinal information available. When agent \( i \) has higher seniority rank than agent \( j \), this implies he has served a greater number of continuous terms, for example. And when agents \( i \) and \( j \) have similar ranks, this implies he and she have served the same number of continuous terms. But there are other seniority systems that do not reveal all ordinal information. In the work of McKelvey and Riezman (1992), seniors are defined as any legislators who served in the immediately preceding legislative term; juniors are those newly elected. Seniority is categorical, not ordinal. In this formulation the full history of a legislator’s service is not preserved in his or her seniority designation. We will generalize this by using more of the ordinal information available. We call this generalization of McKelvey-Riezman cut-off seniority because it is governed by a cut-off rule, namely if a legislator has served \( s^* \) or more previous continuous terms then he or she is senior, whereas those having served fewer than \( s^* \) previous terms are junior.

\(^1\)Kellermann and Shepsle (2009) report a natural experiment in the U.S. House of Representatives in which randomization is used to break seniority ties when a strict rank ordering is required.
(For McKelvey and Riezman, $s^* = 1$.) Seniority is still categorical in our analysis, but it is based on more of the ordinal information about legislative service.

This paper makes two specific contributions. First, in the next section we present a simplified version of the McKelvey-Riezman model. The simplifying assumptions allow for the delivery of a version that “cuts to the bone” of their model structure. This, in turn, enables us to establish their main result and key insight in a transparent and intuitive manner. Second, in section 3 we generalize, amend and extend their model, and establish several new results pertaining to the co-existence, in equilibrium, of a (cut-off) seniority system and incumbency advantage in legislatures. Section 4 concludes with some suggestions for future development.

Before proceeding, it is perhaps worth noting that an underlying motivation of ours is to resurrect the McKelvey and Riezman (1992) model. As we discuss in section 2.3, their model lays down a powerful and flexible framework to study the micro-foundations of the organizational structure of legislative bodies. While theirs is a well-cited paper, it has, surprisingly, hardly been developed or applied over the past twenty years.

2 The McKelvey-Riezman Model

In “Seniority in Legislatures,” Richard McKelvey and Raymond Riezman (1992) — MR92 henceforth — launch a consideration of seniority practices in legislatures. (For an approximation to this approach, anticipating some of its arguments but in a less formal manner, see Holcombe, 1989.) They propose a dynamic, game-theoretic model of a legislative body that operates over an infinite number of periods. In each period, a three-stage game is played. First, $N$ legislators (each representing a legislative district) vote on whether or not to institute a seniority system. If so, legislators are partitioned into seniors and juniors.

Second, they engage in divide-the-dollar bargaining according to the Baron-Ferejohn (1989) random-recognition format. Initial recognition probabilities depend on whether or not a seniority system has been instituted.

Third, voters in each of the $N$ districts decide whether to reelect their incumbent legislator or not.

MR92 show that in their model there exists a subgame perfect equilibrium such that along the equilibrium path, in each period: (i) a seniority system is instituted (and thus all legislators who are reelected incumbents are senior and all legislators elected for the first time are junior); and (ii) voters (anticipating this) reelect their respective incumbent legislators (so as not to disadvantage their agents in the subsequent legislative session’s divide-the-dollar game). Thus, MR92 offers an equilibrium explanation for an incumbency advantage in the electoral arena.

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2Here, by exogenous convention, a legislator is said to be senior if he or she served in the previous legislative session and was reelected.

3If a seniority system has been instituted, then a senior legislator has a higher initial recognition probability than a junior legislator. (The only difference between juniors and seniors is in their initial recognition probabilities.) If no seniority system is in place, then each legislator has the same recognition probability. (Equal recognition probability also arises in the eventuality that a seniority system is instituted and all legislators are seniors or all are juniors.)
that is traceable to an organizational feature in the legislature, namely, a seniority system. Electoral features and legislative arrangements hang together.

2.1 The Formal Structure

Time is divided into an infinite number of periods. There is a polity which is partitioned into an odd number $N$ of districts $(N \geq 3)$. Voters in each district elect a politician to represent them in the polity’s legislature. The job of the legislature in each period is to divide a dollar amongst the $N$ districts.

In each period, a three-stage game is played. The state variable at the beginning of each period defines the “status” of each of the $N$ legislators: a legislator has either been reelected at the end of the previous period, or is newly minted.\footnote{For McKelvey and Riezman, legislators are infinitely-lived. Technically, they cannot be removed from office. Defeating a legislator for them means that the legislator does not get a salary in the period in question and, more importantly, will be a junior legislator in the next period if a seniority system is instituted. See MR92 (p. 953 and footnote 5). We do not follow this MR92 convention, without affecting their main results.} For each district $i$ (where $i = 1, 2, 3, \ldots, N$), let $q_i$ denote the status of its legislator, with $q_i = 1$ if the legislator in question has been reelected and $q_i = 0$ if newly minted.

Fix an arbitrary period, and an arbitrary state variable $q \equiv (q_1, q_2, \ldots, q_N)$. It should be noted that all the payoff-relevant bits of history at the beginning of any period are captured by this state variable. The following three-stage game ensues in this arbitrary period.

**Stage 1: Institute Seniority System?** The $N$ legislators simultaneously cast a vote between instituting a seniority system or not doing so. The seniority system is instituted for this period if and only if at least a simple majority of the legislators vote to do so.

**Stage 2: Divide-The-Dollar.** The $N$ legislators bargain over the partition of one dollar according to Baron and Ferejohn’s (1989) closed-rule bargaining game.\footnote{A legislator is randomly recognized to make a proposal on how to divide the dollar among the $N$ districts. If a majority votes to approve the proposal, then stage 2 concludes and the proposal is implemented; if not, then a new stage 2 round commences with the random recognition of a new proposer.} The initial recognition probabilities depend on the outcome of stage 1 and the state variable $q$ (in a manner described below). Recognition probabilities for all subsequent bargaining rounds in Stage 2 after the initial round (if needed) are equal across legislators (i.e., $1/N$).\footnote{The assumption of proposal parity between juniors and seniors after rejection of a proposal eliminates the advantage that juniors would otherwise enjoy ex ante, namely smaller continuation values (which would enhance their prospect of being included in proposals ex ante). We take this issue up in section 3.4.}

If a seniority system is instituted, and the state variable is such that there exists at least one senior legislator and at least one junior legislator, then the initial recognition probability of a senior legislator is $1/S$ and that of a junior legislator is zero, with $S$ denoting the number of senior legislators (where $1 \leq S \leq N - 1$). If, however, either a seniority system is not instituted, or it is but all legislators are seniors or all are juniors, then the initial recognition probabilities are the same for each legislator (i.e., $1/N$).

If $x_i$ is the share of the dollar secured by the legislator from district $i$, then he gets to...
keep a fixed positive fraction $\theta$ of it (where $0 < \theta < 1$), sending the complementary fraction to his constituency. All players (legislators and voters) are risk-neutral and share the aim of maximizing the (expected) discounted present value of the share of the dollar, where $\delta < 1$ denotes the common discount factor.\footnote{\(\theta \in (0, 1)\) insures that voter-principals have no moral hazard problem with their respective legislator-agents; their preferences are aligned.}

**Stage 3: Elections.** Elections are held simultaneously across all of the $N$ districts. Voters in each district cast a vote, either reelecting their incumbent legislator or electing a newly minted legislator.\footnote{The actions taken at this stage determine the state variable for the next period.}

This completes the description of our simplified version of the MR92 model. It is a dynamic game in which the same three-stage game is played each period.\footnote{As should be clear, this is not an infinitely-repeated game but a dynamic game because some payoff relevant aspects of the three-stage game played in any period potentially depend on history as captured by the state variable.}

We use the Markov subgame perfect equilibrium (MSPE, for short) solution concept to analyze this dynamic game. In such a subgame perfect equilibrium, players use Markov strategies. In general, in a Markov strategy a player’s action at any decision node can only be conditioned on payoff-relevant bits of history. For the dynamic game described above, a pure Markov strategy for a legislator consists of three elements: (i) a function that specifies for each possible value of the state variable $q$, the legislator’s vote for or against instituting seniority, (ii) a proposal in the $(N - 1)$-dimensional unit simplex the legislator makes whenever she is called upon to make one, and (iii) a function that specifies, for each possible proposal, the legislator’s vote to accept or to reject it.\footnote{Note that neither the state variable nor the stage 1 actions have any payoff relevance on the stage 2 bargaining game, other than in determining the initial recognition probabilities (but these are “realized” before the very first stage 2 decision node is reached).} For a voter at the election stage, a pure Markov strategy is a vote for or against her incumbent legislator.\footnote{We note that in the MSPE we characterize below in Proposition 1, a legislator’s Markov strategy is a behavioural strategy, and not a pure strategy. In equilibrium, she randomly selects the legislators who form part of her minimum winning coalition. As such, formally speaking, she randomizes over the choice of proposal when she is called upon to make one.}

### 2.2 Seniority and Incumbency in Equilibrium

The main result of MR92 is stated below in Proposition 1. We establish it in the context of our simplified version of their model as described above. The result establishes the existence of a Markov subgame perfect equilibrium (MSPE) with the desired properties. Besides providing a rationale for the existence of such an equilibrium, the proof below contains elements that will subsequently facilitate our discussion of the existence of other MSPE.

\footnote{Note that the state variable, and the actions taken in stages 1 and 2, have no payoff relevance on the stage 3 electoral game. Indeed, voters are prospective, not retrospective.}
Proposition 1 (McKelvey-Riezman 1992). There exists a Markov subgame perfect equilibrium (MSPE) in which, along the equilibrium path, in each period the seniority system is instituted and incumbents are reelected.

Proof. A key ingredient to the argument establishing Proposition 1 concerns the nature of the unique MSPE outcome in the appropriate version of the Baron-Ferejohn bargaining game. We first dispense with this matter. As is well-known, in the Baron-Ferejohn divide-the-dollar bargaining game (with a closed rule), if all of the legislators always have the same recognition probability (i.e., \(1/N\)), then in the unique MSPE outcome, a legislator’s expected share of the dollar is \(1/N\). This implies that the equilibrium continuation value of a legislator — junior or senior — following a failed (initial) proposal is \(\delta/N\).

Fix any state variable \(q\) such that there are \(S\) seniors and \((N - S)\) juniors (where \(1 \leq S \leq N - 1\)). If at stage 1 a seniority system is instituted, then it immediately follows that the unique MSPE expected shares of the dollar received by a senior and a junior are respectively

\[
x_S = \frac{1}{S} \left[ 1 - \left( \frac{N - 1}{2} \right) \frac{\delta}{N} \right] + \left[ \frac{S - 1}{S} \right] \left( \frac{1}{2} \right) \frac{\delta}{N} = \frac{\delta}{2N} + \frac{1}{S} \left[ 1 - \frac{\delta}{2} \right] \tag{1}
\]

and

\[
x_J = \left[ \frac{1}{2} \right] \frac{\delta}{N} = \frac{\delta}{2N}. \tag{2}
\]

Notice that:

\[
x_S > 1/N > x_J. \tag{3}
\]

If, on the other hand, either a seniority system is not instituted, or the state variable is such that either \(S = 0\) (all legislators are juniors) or \(S = N\) (all legislators are seniors), then the unique MSPE expected share of the dollar received by every legislator is \(1/N\).

Given the results just established, we can now proceed to show: (i) that a legislator’s equilibrium voting strategy at the institutional stage is to cast her vote for the establishment

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13The crux of the argument for this result runs as follows. Let \(V\) denote the expected share of the dollar received by an arbitrary legislator. Then, in a MSPE, \(V\) must satisfy the following recursive equation:

\[V = \left( \frac{1}{N} \right) \left[ 1 - \frac{1}{2(N - 1)} \delta V \right] + \frac{1}{N} \frac{N - 1}{2} \delta V.\]

The rationale behind the two sets of terms on the RHS of this equation is as follows. With probability \(1/N\) the legislator in question is randomly recognized to make a proposal. In that eventuality he or she selects any \((N - 1)/2\) of the other legislators (to then form a minimum winning coalition), and offers to each a share \(\delta V\) of the dollar; the proposer keeps the residual for herself. But with probability \((N - 1)/N\), one of the other legislators is randomly recognized. Since this other legislator is indifferent as to which of the \(N - 1\) legislators to select, the probability that any particular legislator is part of the minimum winning coalition is \(1/2\) and such a legislator receives a share \(\delta V\) of the dollar. Solving the recursive equation for \(V\) implies that \(V\) has a unique solution, namely, \(V = 1/N\), as desired.

14With probability \(1/S\) a senior legislator is randomly recognized to make a proposal. In that eventuality he or she selects any \((N - 1)/2\) of the other legislators (to then form a minimum winning coalition), offers to each a share \(\delta(1/N)\) of the dollar, and retains the residual. But with probability \((S - 1)/S\), one of the other \(S - 1\) senior legislators is randomly recognized. Since this other senior legislator is indifferent as to which of the \(N - 1\) legislators to select, the probability that any particular legislator is part of the minimum winning coalition is \(1/2\) and such a legislator receives a share \(\delta(1/N)\) of the dollar. It may be noticed that, as expected, \(Sx_S + (N - S)x_J = 1\).
of the seniority system if and only if she is senior, and (ii) that a voter’s equilibrium Markov strategy at the election stage is to cast her vote for her incumbent. Proposition 1 then follows immediately (since, with these equilibrium strategies, incumbents from all $N$ districts are reelected, and thus a seniority system is instituted, as all legislators are seniors).

We first consider a legislator’s voting decision. If at stage 1 either all incumbent legislators are reelected or none of them are, then each legislator is indifferent between voting for or against having a seniority system, since in either case each legislator’s unique MSPE expected share of the dollar is $1/N$. Now suppose that the state variable is such that there is at least one senior and at least one junior. In this case, inequality (3) implies that a senior legislator strictly prefers to have the seniority system instituted while the opposite is the case for a junior legislator. This means that if a legislator is pivotal then he will vote for the seniority system if and only if he is a senior. However, if he is not pivotal then he is indifferent between voting for or against the seniority system. It thus follows that a legislator cannot profit from a unilateral, one-shot deviation from the proposed equilibrium voting strategy at the institutional stage of casting her vote for the establishment of the seniority system if and only if she is senior.

We now consider an arbitrary voter’s problem, from some arbitrary district $i$. When she is not pivotal, she is indifferent between casting her vote for or against her incumbent legislator. Now consider the case when she is pivotal (e.g., she is the median voter in district $i$). In the proposed MSPE, voters in each of the other districts reelect their respective incumbents. Given that, suppose the pivotal voter from district $i$ considers undertaking a one-shot, unilateral deviation to elect a newly minted politician to represent district $i$ in the legislature in the next period. We now show that this deviation is unprofitable. This is because in that eventuality, at the beginning of the next period there will be $N - 1$ legislators who are seniors while the legislator from district $i$ will be a junior. A seniority system will be instituted in equilibrium, and thus the unique MSPE expected share of the dollar secured by the legislator from district $i$ will be $x_J$ as given in equation 2. This is strictly less than $1/N$, which is the unique MSPE expected share of the dollar secured when the pivotal voter from district $i$ does not undertake this unilateral deviation. It thus follows that a voter cannot profit from a unilateral, one-shot deviation from the proposed equilibrium voting strategy at the election stage of casting her vote for her incumbent.

We now offer some comments on the result stated in Proposition 1, and on the matter of the existence of other MSPE outcomes. Notice that along the equilibrium path of the MSPE stated in Proposition 1, all legislators are seniors. This means that although a seniority system is instituted, each legislator’s MSPE expected share of the dollar is the same (namely, $1/N$). Indeed, there exists another MSPE which differs from the one stated in Proposition 1 in the following (minor) aspect: along the equilibrium path, all legislators vote against the institution of a seniority system (even though all legislators are senior since voters reelect their respective incumbent legislators). This MSPE is payoff-equivalent to the MSPE stated in Proposition 1; and along both equilibrium paths, an incumbency advantage exists. They differ from each
other on whether or not, along the equilibrium path, a seniority system exists. The reason why voters reelect their incumbent legislators in this other MSPE, where no seniority system is instituted along the equilibrium path, runs as follows (and this highlights the importance of off-the-equilibrium path behavior, which is the same in both MSPE).

Voters from each district reelect their respective incumbent legislators, even if along the equilibrium path the seniority system is not instituted, because if voters from some district \( i \) were unilaterally to deviate and not reelect their incumbent legislator, then, in equilibrium, there would be \( N - 1 \) seniors and one junior. In that (off-the-equilibrium-path) eventuality, the seniority system would, for sure, be instituted. This, then, makes such a unilateral deviation unprofitable for the voters from district \( i \). So, it is the credible threat that the seniority system will be instituted (when there are \( N - 1 \) seniors and one junior) which ensures that, along the equilibrium path, voters from each district reelect their incumbent legislator (irrespective of whether or not, along the equilibrium path, a seniority system is instituted).\(^{15}\)

This feature is revealing: it drives home the insight that it is the equilibrium anticipation of a seniority system that provides sharp incentives to voters to reelect their respective incumbent legislators (so as not to disadvantage them in the subsequent legislative session’s divide-the-dollar game).

One may then ask whether there exists a MSPE with the property that, along the equilibrium path, neither a seniority system nor an incumbency advantage exist. It is straightforward to verify, by using the arguments presented in the proof of Proposition 1, that such a MSPE does exist. But it is equally straightforward to verify by the same arguments that such a MSPE will involve legislators in the stage 1 voting game and voters in the stage 3 electoral game using weakly dominated actions. Indeed, using the solution concept introduced in Baron and Kalai (1993), the "stage-undominated" MSPE concept, it is easy to verify that in any stage-undominated MSPE, voters will always reelect their respective incumbent legislators. But, for reasons just discussed above, there will be two such stage-undominated MSPE paths of play: one in which a seniority system is instituted and one in which it is not.

2.3 Discussion

The MR92 model offers a powerful and compelling insight, namely, that there is an intimate link between seniority and incumbency advantage in legislative bodies. The incumbency effect is driven by the equilibrium anticipation by voters of the institution of a seniority system. The latter, in turn, is driven by the advantage it confers on incumbents. To see this, suppose to the contrary that voters in all the districts were not to reelect their respective incumbent legislators. In that case all legislators in any period would be juniors, and thus the seniority system would not be instituted.

Before we turn to our generalizations of the MR92 model, we comment on the main simplification of their model that we have built into our version – that if a seniority system is instituted, the initial recognition probability of a junior legislator is zero. While this appears

\(^{15}\)It is possible, therefore, to observe an incumbency advantage in a legislature without a seniority system.
to be a restrictive assumption, and MR92 in fact allow for juniors to have positive initial recognition probabilities, making the simplifying assumption reduces considerably the notation, analysis and algebra, and allows for the development of the core insights in a transparent manner. For example, the expressions for the equilibrium expected shares would be somewhat different and more involved than what is stated in (1) and (2) if a junior legislator has a positive initial recognition probability. But, and this is the significant point, it would nonetheless remain the case that the key property about the equilibrium shares, namely, the two inequalities stated in (3), would continue to hold.\footnote{This is of course provided that a junior’s initial recognition probability is strictly less than a senior’s initial recognition probability, which is the fundamental characteristic of a seniority system that underlies MR92.}

The main idea of MR92, linking seniority and incumbency advantage, is developed in a dynamic game that combines legislative bargaining and elections. This is novel. Even after twenty years, work on legislative bargaining (initiated by Baron and Ferejohn 1989) and dynamic principal-agent models (initiated by Barro 1973 and Ferejohn 1986, and recently surveyed in Besley 2006) have not been integrated. There are very few models that, like MR92, combine legislative and electoral interactions within a single framework.\footnote{A few exceptions include Austen-Smith and Banks (1988) and Muthoo and Shepsle (2008 and 2010).} Their argument, however, offers a number of opportunities for generalization. We note some of these here, in part a preview of coming attractions in later sections of this paper.

First, MR92 offers a restrictive notion of seniority. The seniority system is categorical — legislators are either senior or junior — and history matters in a very limited fashion. The seniority system only conditions on a legislator’s status as a successfully reelected incumbent or a new member of the legislature, i.e., only on $t - 1$ characteristics of the legislator and the $t - 1$ election outcome. In sections 3.1 and 3.2 we address this limitation and show how the McKelvey-Riezman argument extends under a relaxation of their seniority concept.

Second, in the equilibrium seniority system of MR92, there is no turnover. Their MSPE has a legislature composed entirely of reelected incumbents; on the equilibrium path no incumbent loses. A consequence of this is that there is no mix of seniors and juniors in the legislature. Both of these features are odd in light of evidence from empirical legislatures. In the generalization we report in the next section, there is a mix of juniors and seniors in equilibrium. In section 3.3 we introduce the possibility of exogenous turnover.

Third, in MR92 the recognition advantage of seniors applies only to the initial round; if a proposal fails in the initial round, then the seniority recognition advantage is suspended for subsequent rounds. Our second elaboration of MR92 focuses precisely on this feature. In section 3.4 we explore the consequences of allowing the seniority recognition advantage to persist in our generalization of MR92.\footnote{This issue is touched upon in MR92 and a sequel (McKelvey and Riezman, 1993).}

Fourth, in the McKelvey and Reizman game form the first-stage proposal to implement a seniority system is considered under a closed rule. It is a take-it-or-leave-it proposal. In section 4.1 we consider the possibility of amendments from the floor. Our motivation is to accommodate the possibility of juniors attempting to affect recognition probabilities in the
stage 2 divide-the-dollar game.

We will not tackle all of these possibilities in the present paper, but a number of them figure in our attempt to extend the framework of MR92. We begin in the next section with a more general formulation of the seniority concept.

3 Generalization of McKelvey-Riezman: Cut-off Seniority

In MR92, seniority depends only upon having been reelected at the end of the previous period. A legislator is senior in period \( t \) under a seniority system if he were reelected at the end of period \( t-1 \); otherwise he is junior. This implies, in particular, that a legislator’s service prior to period \( t-1 \) is irrelevant in determining his level of seniority in period \( t \). The main idea of the generalization developed in this section is to extend MR92 by allowing for a legislator’s entire length of tenure (the number of times he was reelected) to matter in determining his level of seniority in any period. As motivation, we mention one key implication of this extension.

As given in our Proposition 1 above, MR92 identifies a Markov subgame perfect equilibrium in which a seniority system is established and all legislators are reelected in every period. Hence, in equilibrium all legislators are senior. This, in turn, implies that seniority has no legislative bite.\(^ {19} \) In contrast, in our model described below, in equilibrium a seniority system is instituted but only a strict subset of the legislators is endowed with seniority (those with sufficiently long continuous service in the legislature).

3.1 The Generalized Formal Structure

There are two ingredients of our generalized model that make it different substantively from the MR92 model structure. First, since we allow the entire legislative experience of a legislator to matter in determining whether or not he or she is endowed with seniority, each of the \( N \) elements of the state variable (which now give the number of terms of service of each legislator) can take any positive integer as its value (and not just 0 or 1 as in MR92). Second, since we allow legislators to establish a seniority system that endows only those who have sufficient legislative experience with senior status, voting over whether or not to institute a seniority system will now mean voting over what the cut-off should be. We now turn to a description of our generalized model structure that makes all this clear.

Time is divided into an infinite number of periods, and there is a polity which is partitioned into an odd number \( N \) of districts \(( N \geq 3 )\). Voters in each district elect a politician to represent them in the polity’s legislature. The job of the legislature in each period is to divide a dollar amongst the \( N \) districts.

In each period, a three-stage game is played. Fix an arbitrary period \( t \). Let \( s^i_t \) denote the number of previous terms served by the legislator from district \( i \) \((i = 1, 2, 3, \ldots, N)\) at the beginning of period \( t \), where \( s^i_t \in \{0, 1, 2, 3, \ldots\} \); this is equal to the number of times the

\(^{19}\) However, it definitely has electoral bite.
legislator in question has been reelected.²⁰ Let the $N$-tuple of these tenure lengths at the beginning of period $t$ be denoted by $\underline{s}_t = (s^i_t)_{i=1}^N$, which is the state variable of our dynamic game. Given the state $\underline{s}_t$ at the beginning of period $t$, the following three-stage game ensues:

**Stage 1: Cut-Off Seniority System.** Through a voting mechanism (to be specified momentarily), a number $s^*_t \in \{0, 1, 2, \ldots \}$ is determined that has the following implication: A legislator from district $i$ is endowed with seniority if and only if $s^i_t \geq s^*_t$. Thus, the $N$ legislators are partitioned into the subset of legislators who possess seniority (the seniors) and those who do not (the juniors). Like MR92, this entails a categorical seniority system in which a legislator either has seniority or does not. But the difference from MR92 lies in the fact that the entire electoral history now matters: a legislator’s full tenure determines (in conjunction with the endogenous cut-off) whether or not he or she is endowed with seniority. Notice that MR92 is a special case of this cut-off seniority system with the choice of $s^*_t$ restricted to zero and one; the choice of $s^*_t = 0$ means all are endowed with seniority (equivalent to none with seniority), while setting $s^*_t = 1$ means only current legislators who were also elected in the immediately preceding period are senior.

We assume that Stage 1 commences under “general parliamentary procedure,” the practice of most legislatures before they have formally adopted rules for a session. Accordingly, a motion is offered by a (randomly) recognized legislator, e.g., “I move $s^*_t = x$.” If it is approved by a simple majority, it becomes the status quo against which other motions, e.g., “I move $s^*_t = y$,” are in order. This process of motion-making and voting continues until no further motions are forthcoming — because no legislator is inclined to move another alternative, or because all feasible alternatives have already been proposed and disposed of. The status quo prevailing at this point in the process is the selected cut-off.

In effect, given the state variable $\underline{s}_t$, the $N$ legislators engage in a voting game (consisting of a pairwise majority-rule contest) that determines the selected cut-off $s^*_t$ chosen from amongst the $N$ tenure lengths as defined in the state variable $\underline{s}_t$, i.e., legislators vote over the set $\{s^1_t, s^2_t, s^3_t, \ldots, s^N_t\}$.²¹ Given the state $\underline{s}_t$ and the selected cut-off $s^*_t$, we denote the number of seniors by $S_t$. The next two stages are the same as in our simplified version of MR92. **Stage 2** is a divide-the-dollar game among legislators. If seniority cut-off $s^*_t$ has been selected at Stage 1, then the $S_t \geq 1$ seniors each have recognition probability $1/S_t$; if there are no seniors then each legislator is recognized with probability $1/N$. In either case the recognition probability for each legislator is $1/N$ in subsequent rounds if a proposal is rejected and subsequent rounds are required. In

²⁰ We assume that if a legislator is not reelected in some period, then he or she may not seek election in any subsequent period. This means that, unlike in MR92, if an incumbent legislator is not reelected he or she is replaced by a newly minted legislator, i.e., a defeat ends an incumbent’s legislative career. It also means, then, that $s^i_t$ is the number of previous consecutive terms served.

²¹ There are other extensive forms that could also be employed. It could, for example, consist of a round-robin tournament among the $N$ components of the state variable. Or it could be a fixed agenda of votes over the $N$ tenure lengths (or some subset of them) — a binary voting tree — with the loser at each node of the extensive form eliminated. For the class of circumstances we consider below, a unique $s^*_t$ prevails for any $\underline{s}_t$. 

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Stage 3 elections are held in each district where the fates of the $N$ incumbents are determined. For further specifics the reader may consult our previous rendering of these stages.

This completes the description of our generalization of our simplified version of the MR92 model. As before, we use Markov subgame perfect equilibrium to analyze this dynamic game. For this dynamic game, Markov strategies are slightly different from before, in the following respects. A legislator’s stage 1 behavior is now defined by a function that specifies for each possible value of the state variable $s_t$, the legislator’s voting behavior in stage 1 that determines the cut-off. A voter’s stage 3 behavior is now defined by a function that specifies for each possible value of the state variable $s_t$, a vote, either for or against her incumbent legislator. Unlike in the MR92 model and its simplified version studied in the previous section, the state variable $s_t$ is now payoff-relevant at stage 3, since $s_{t+1}$ depends upon whether incumbents are reelected or not.

3.2 Equilibrium Cut-Off Seniority

We now turn to explore the existence of a Markov subgame perfect equilibrium in the generalized model that has properties similar to those of MR92 equilibrium (as stated in Proposition 1). The substantive difference with respect to the MR92 equilibrium will, perhaps not surprisingly, lie in the nature of the equilibrium seniority system. Before turning to the novel aspect of the analysis, which concerns determining this equilibrium seniority system (or, more formally, the equilibrium in stage 1), we first note that the equilibrium in stage 2 is exactly as in the MR92 equilibrium. Specifically, the unique MSPE expected shares in the stage 2 bargaining game (when there is at least one senior and one junior present) are as stated in (1) and (2). The two inequalities stated in (3) continue to hold. The incentives of voters in each district to reelect their respective incumbent are similar in some but not all respects to those in the MR92 equilibrium.

Let us now turn to an analysis of the stage 1 voting game amongst the $N$ legislators to determine the equilibrium cut-off. Fix an arbitrary state $s ≡ (s^1, s^2, s^3, . . . , s^N)$. If we could appeal to the Median Voter Theorem (MVT henceforth), it would be a straightforward exercise to determine the equilibrium of the voting game. First we need to establish whether or not legislator preferences (over the cut-off, $s^*$) are single-peaked. In Figure 1 we show that they are not. As can be seen, legislator $i$’s preferences for the seniority cut-off increase monotonically from $s^i$ to $s^i$ (as implied by equation (1)), since this entails increasingly smaller senior cohorts that still include her; however, she is indifferent among all cut-offs greater than $s^i$ up to and including $s^N$ (all less preferred than $s^i$ as implied by equation (3)), since for all of these cut-offs she would be junior and her expected payoff, $\delta/2N$, is invariant to the number of seniors (as implied by equation (2)). For cut-offs beyond $s^N$, her preference function jumps upwards to $1/N$ since for all such cut-offs there are no seniors. Even if we restrict the domain for the choice of $s^*$ to be the relevant finite set of feasible choices, namely, $X ≡ \{s^1, s^2, s^3, . . . , s^N\}$,

\[Without loss of generality, we assume that for all $i = 1, 2, \ldots, N - 1$, it the case that $s^i ≤ s^{i+1}$.\]
preferences are single-peaked only in a “weak” form, an insufficient basis for the MVT. This we now show.

![Figure 1: Legislator Preferences](image1)

In the context of a three person legislature, Figure 2 depicts for each legislator his utilities for the three possible cutoffs. It may be seen that $s^2$ is preferred by a majority to $s^1$ (legislators 2 and 3), $s^1$ is preferred by a majority to $s^3$ (legislators 1 and 2), but $s^2$ and $s^3$ tie because of legislator 1’s indifference. This means that there is no Condorcet winner (though $s^2$ is technically a core point as it is not defeated by any other alternative). For $N \geq 5$ the situation is even worse; there is no Condorcet winner and the core is empty.

![Figure 2: An Example without a Condorcet Winner](image2)
We can secure single-peakedness with the following assumption to break the indifference:

**Assumption 1 (Juniors’ Preference for Dispersion of Power).** A legislator, if junior, strictly prefers a legislature with $S$ seniors to one with $S'$ seniors where $S > S'$.

Note that $S > S'$ implies that when there are $S$ seniors the seniority cut-off $s^*$ is lower than the seniority cut-off $s^{*'}$ when there are $S'$ seniors. The assumption covers two cases: (a) the legislator who is junior when there are $S'$ seniors might become senior if there are $S > S'$ seniors; or (b) the legislator who is junior when there are $S'$ seniors might still remain junior when there are $S$ seniors. In the first case, it is clear from our earlier analysis of $x_S$ and $x_J$ that the legislator prefers the world in which she is senior to one in which she is junior, i.e., the claim in the Assumption already holds. So we only need to worry about the case in which the status of the junior is unaffected by reducing the cut-off. This is the second case, where we assume she prefers, even though she will not be senior, a seniority system with power more dispersed among those who are senior. Note, from the expression for $x_J$ in (2), that in this scenario her equilibrium expected share of the dollar is the same irrespective of the number of seniors.$^{23}$ Assumption 1 plays a tie-breaking role in helping ensure strict single-peakedness over the entire domain of choice.$^{24}$

To make this assumption plausible, consider the following perturbation. Instead of allocating all initial recognition probability in the Stage 2 bargaining game to senior legislators, we reserve $\epsilon > 0$ recognition probability to be shared amongst junior legislators. This implies a more general form of equation 2:

$$x_J = \frac{\epsilon}{N - S} \left[ 1 - \left( \frac{N - 1}{2} \right) \frac{\delta}{N} \right] + \left( 1 - \frac{\epsilon}{N - S} \right) \left[ \left( \frac{1}{2} \right) \frac{\delta}{N} \right].$$

That is, with probability $\epsilon/(N - S)$, an arbitrary junior legislator obtains the right to make a proposal yielding the payoff in the first bracketed term and with complementary probability she obtains the second bracketed payoff. This simplifies to:

$$x_J = \frac{\delta}{2N} + \frac{\epsilon}{N - S} \left[ 1 - \frac{\delta}{2} \right].$$  \hspace{1cm} (4)

Note that as $\epsilon \to 0$, the junior payoff in (4) tends to that in (2). More importantly, Assumption 1 directly follows from the fact that $x_J$ is strictly increasing in $S$.\footnote{It may be noted that even though $i$’s status doesn’t change in this situation when the size of the senior set is expanded, with the lower threshold, $s^*$, she is “closer” to that threshold than to the higher threshold, $s^{*'}$. In our equilibrium, in which there is no turnover, this argument is irrelevant. But perturb this situation (as we do below in subsection 3.3) with a small probability of turnover, and “distance” from a cutoff now is more compelling.}

\footnote{This assumption is sufficient but not necessary. In some circumstances, for example when senior recognition power is permanent not transitory (cf. section 3.4), individuals when junior will prefer a smaller junior cohort to a larger one. This is because a smaller junior cohort means each junior is more likely to be included in a proposal.}

\footnote{The careful reader will have noted that we have assumed a particular functional form where a junior
We are now ready to state a preliminary result key to our main result:

**Lemma 1** (Single-Peaked Preferences). Given the state \( s \), let the set \( X = \{s^1, s^2, \ldots, s^N\} \) be the domain of choice of the cut-off, \( s^* \). If Assumption 1 holds, then the preferences of the legislator from district \( i \) \((i = 1, 2, \ldots, N)\) over the cut-off, \( s^* \), are single peaked on \( X \), with peak at \( s^* = s^i \).

**Proof.** Fix an arbitrary legislator, say from district \( i \). It follows from the expressions for \( x_S \) and \( x_J \) in (1) and (2) that his equilibrium expected share is maximal when \( s^* = s^i \). The intuition for this is then the number of seniors is minimized but includes him or her in that group. For any \( s^* < s^i \), he or she continues to be a senior, but, since \( x_S \) is strictly decreasing in \( S \), it follows that his expected utility is strictly increasing over the subset \( \{s^1, s^2, \ldots, s^{i-1}\} \). Now turn to \( s^* \in \{s^{i+1}, s^{i+2}, \ldots, s^N\} \). In that case he is always a junior, receiving \( x_J \) which is independent of the number \( S \) of seniors. Assumption 1 breaks that indifference to give strict monotonicity. Hence, \( i \)'s preferences over the cut-off, \( s^* \), are single-peaked on the choice set \( X \), with peak at \( s^* = s^i \). \( \square \)

Given Lemma 1, it is straightforward to establish our main result:

**Proposition 2** (Median Tenure Length as Equilibrium Cut-Off). If Assumption 1 holds, then there exist a Markov subgame perfect equilibrium (MSPE) in the generalized model, with the following properties. For any vector of tenure lengths \( s \) (i.e., the value of the state variable), the equilibrium cut-off \( s^* \) selected at stage 1 is the median tenure length. That is, for any state \( s \):

\[
s^*(s) = s_M,
\]

where \( s_M \) denotes the median of the \( N \) tenure length components of \( s \). Furthermore, all seniors (i.e., those with tenure greater than or equal to \( s_M \)) are re-elected.

**Proof.** The equilibrium analysis of stage 2 is the same as that in the proof of Proposition 1. With respect to the analysis of stage 1, the desired result follows immediately from an application of the MVT, given Lemma 1. Turning to stage 3, incentives of voters in districts with incumbents who are seniors (those with tenure lengths greater than or equal to \( s_M \)) are in line with the argument presented in the proof of Proposition 1, and hence such legislators will be re-elected. But the same is not the case for voters in districts with incumbents who are legislator’s recognition probability is \( \epsilon/(N - S) \). Suppose we instead consider general initial recognition probabilities, and let \( p_J(S) \) denote the initial recognition probability of an arbitrary junior legislator when there are \( S \in \{1, 2, \ldots, N - 1\} \) seniors. It is straightforward to verify that in this case the expected payoff to a junior is as stated in equation (4) but with the expression \( \epsilon/(N - S) \) replaced by \( p_J(S) \). Hence, \( x_J \) is strictly increasing in \( S \) (on the set \( \{1, 2, \ldots, N - 1\} \)) if and only if the (exogenously given) initial recognition probability to an arbitrary junior \( p_J(S) \) is strictly increasing in \( S \) (or, equivalently, strictly decreasing in the number, \( N - S \), of juniors).
juniors. The pivotal voter in that district is indifferent between re-electing and defeating his or her (junior) incumbent since, with cut-off seniority, that district’s legislator will be junior next period whether he is re-elected or newly minted.

Proposition 2 is obvious once it is stated. A legislator is indifferent between instituting a seniority system and not doing so if everyone is senior or everyone is junior, since expected payoffs in these cases under a seniority system are equivalent to those without a seniority system. If a seniority system is to be established, however, then a legislator would rather be senior than junior and prefers fewer co-seniors to more. The former follows directly from (3). The latter follows from the fact that recognition probability for seniors is strictly decreasing in \( S \). From these two facts and Lemma 1, it follows that a legislator, \( i \), would most like to set the seniority cut-off at his or her tenure length, \( s^i \), i.e., he or she would like to be senior but would not prefer a lower bar permitting more seniors. With these preferences the MVT implies that the bar is set at \( s_M \).

Cut-off seniority entails somewhat different incentives for pivotal voters to re-elect incumbents than in MR92. Districts with senior legislators have a powerful incentive to re-elect. Districts represented by junior legislators are indifferent between re-electing and defeating. Consequently, an MSPE in which all incumbents are re-elected still exists, but there are other MSPEs in which some juniors are not re-elected. This follows from the rigidity of the partition of juniors and seniors in a cut-off seniority system. Consequently, there now may be turnover despite the imposition of a seniority system. But this turnover will not involve senior legislators. In contrast, in MR92 the composition of the legislature is unchanging. In our generalization the composition of districts represented by juniors and seniors is unchanging, but there may be some churning of junior membership.

### 3.3 Robustness to Legislative Turnover

Suppose, now, that a legislator dies or retires with an exogenous probability \( \rho > 0 \). It is straightforward to verify that Proposition 2 is strengthened because in this context all voters have incentives to re-elect incumbents, juniors as well as seniors. Why? In a world of exogenous turnover, queue position is valuable: a re-elected junior incumbent has a positive probability of rising above the median queue position because of turnover amongst seniors.

In equilibrium, \( s_t \) is related to \( s_{t-1} \) as follows. For each \( i \),

\[
s^i_t = \begin{cases} 
  s^i_{t-1} + 1 & \text{with probability } 1 - \rho, \\
  0 & \text{with probability } \rho.
\end{cases}
\]

If an \( i < M \) in period \( t - 1 \) departs the legislature, then the replacement legislator for \( i \) and the remaining set of period \( t - 1 \) juniors, \( \{ j < M : j \neq i \} \), will all be period \( t \) juniors (with \( i \) moving to the bottom of the queue). If, however, \( i > M \) departs the legislature in period \( t - 1 \), then the period \( t - 1 \) junior with seniority rank \( M - 1 \) will cross the cut-off threshold and become a senior in period \( t \).
We now describe some properties of the steady state (ergodic) distribution of tenure lengths. Let \( \pi_s \) denote the probability in a steady state that the legislator from an arbitrary district has tenure length \( s \), where \( s \in X = \{0, 1, 2, \ldots\} \). One may show that \( \pi_s = \rho(1 - \rho)^s \) (for \( s \in X \)).

The intuition for this expression is this. Conditional on starting his term, a legislator survives \( s \) periods with probability \( (1 - \rho)^s \). The condition that he starts a term of incumbency depends on his predecessor departing, an event that occurs with probability \( \rho \).

Given this steady state distribution, it follows that the probability a legislator from an arbitrary district is a senior in any period is

\[
P_S = \sum_{s=s_M}^{\infty} \pi_s = 1 - \frac{\rho}{1 - \rho} \left[ 1 - (1 - \rho)^{s_M} \right],
\]

where \( s_M \) is the median tenure length in the period in question. For simplicity, assume that in every period the median is unique. This implies there are exactly \((N + 1)/2\) seniors in every period (Proposition 2). Hence,

\[
P_S = \frac{1}{N} \left[ \frac{N + 1}{2} \right].
\]

The fact of exogenous legislative turnover strengthens electoral incentives to re-elect incumbents. For those legislators above the seniority cut-off, the advantage to constituents of reelecting them continues to follow from equation (3). For legislators who have not yet acquired a sufficient number of terms of service to qualify as senior, the fact that there is some prospect encourages constituents to maintain them in office.

The main effect of exogenous turnover is churning. The composition of seniors changes stochastically, both from juniors whose persistent reelects take them above the seniority cut-off and from random departures. The composition of juniors changes as well, with some “graduating” to the senior ranks and others departing stochastically.

We should also mention the idea of state-contingent exit. In the analysis above we assumed \( \rho \) was constant across \( i \) and \( t \). Yet it is plausible for \( \rho \) to vary both in anticipated and unanticipated ways across individuals and time periods. For example, there may be an anticipated career effect with retirement looking more attractive to a legislator, or death more
likely, as $s_i$ grows (or passes some threshold). There may also be anticipated temporal ebbs and flows of legislative exit — for example, differing in off-year elections from on-year elections (possibly also conditioned by the legislators partisan type or connection to the party of the president). Finally, there may be unanticipated shocks, both temporal (so that $\rho$ varies with, say, unexpected economic or foreign policy events in period $t$) and legislator-specific (unexpected physical ailments or scandals, for example). Again, these are technical elaborations calling for further research.

### 3.4 Persistence of Senior Power

In the seniority systems characterized in MR92 and our generalization, the agenda power accruing to seniors is assumed to be transitory, not persistent. Seniors have a higher initial probability of being recognized to make a proposal on how to divide the dollar. But, if that proposal should not garner the necessary majority support and a new legislator must be recognized to make a proposal, all legislators, seniors and juniors alike, are assumed equally likely to be recognized. In this section we relax this assumption.

The motivation for assuming transitory recognition advantage for seniors is that otherwise a senior would have a higher continuation value, which in turn means that a senior legislator would require a higher current payment to join a winning coalition as a coalition partner, leaving less residual for the proposer. The proposer would prefer to turn to cheaper partners and, with recognition probability disadvantage and hence lower continuation value, juniors would be more attractive. That is, while in the “transitory” world the coalition partners are perfect substitutes (in that all of the $N-1$ possibilities are identical), that is not the case in the “permanent” world (in that the $S-1$ seniors are distinguishable from, and less desirable than, the $N-S$ juniors).

The question remains, however, of whether a senior legislator is better served by transitory or permanent recognition advantage. Put differently, a seniority differential throughout increases the likelihood that a senior will be selected to make a proposal in each round required in that session (recognition effect), but it decreases the prospect that a senior will be included in a winning coalition if he or she is not recognized to make a proposal (inclusion effect). In this section we seek to establish which effect is larger.

#### 3.4.1 Maintaining Zero Junior Proposer Power

Formally, we analyze an amended version of our model (described in section 3.1) by assuming that senior recognition advantage is permanent. This means that in any round of any period, the recognition probability of a senior persists. A key novel element of the analysis of the MSPE of this amended dynamic game concerns the equilibrium expected shares of the dollar respectively secured by a senior and a junior in the stage 2, divide-the-dollar bargaining game. Fix an arbitrary period and suppose that at stage 1 a seniority system is instituted and that there are $S$ seniors and $N-S$ juniors, where $1 \leq S \leq N-1$. We divide the characterization of
the unique MSPE in the stage 2, Baron-Ferejohn divide-the-dollar bargaining game according to whether or not there are at least \((N - 1)/2\) juniors present.

Suppose, first, that \(N - S \geq (N - 1)/2\), i.e., \(S \leq (N + 1)/2\). In this case, when an arbitrary senior is recognized in any bargaining round (with probability \(1/S\)), there are enough juniors present for only them to be part of the minimum winning coalition (i.e., all other seniors are excluded). This means that the expected shares (values) of a senior, \(V_S\), and a junior \(V_J\), are respectively

\[
V_S = \frac{1}{S} \left[ 1 - \left( \frac{N - 1}{2} \right) \delta V_J \right] + \left( \frac{S - 1}{S} \right)(0) \tag{6}
\]

\[
V_J = \left( \frac{\left(\frac{(N - 1)/2}{N - S}\right)}{\delta V_J} \right) \tag{7}
\]

Hence, when in any period a seniority system is instituted at stage 1 and there are \(S\) seniors where \(1 \leq S \leq (N + 1)/2\), the equilibrium expected shares of the dollar to a senior and a junior are respectively \(1/S\) and zero. A senior’s payoff cannot be greater than \(1/S\), and indeed this payoff strictly exceeds a senior’s payoff when recognition advantage is transitory (cf. equation (1)). We emphasize that in the equilibrium just derived, a senior is never selected, when another senior is recognized, to join a minimum winning coalition. Thus, in the scenario just analyzed, the recognition effect dominates the inclusion effect. Not surprisingly, then, in the other case where there aren’t enough juniors (i.e., when \(N - S < (N - 1)/2\)), and a senior is invited by another senior to join his minimum winning coalitions, it is also the case that \(V_S = 1/S\). The two recursive equations in this case, when \(N - S < (N - 1)/2\), are as follows.

\[
V_S = \frac{1}{S} \left[ 1 - (N - S)\delta V_J \right] - \left( \frac{(N - 1)/2}{N - S} \right) \delta V_S + \left( \frac{S - 1}{S} \right) \left[ \frac{(N - 1)/2 - (N - S)}{S - 1} \right] \delta V_S \tag{8}
\]

\[
V_J = \delta V_J. \tag{9}
\]

Solving gives \(V_S = 1/S\) and \(V_J = 0\). We summarize these results in the following lemma:

**Lemma 2.** If senior recognition advantage persists in every round in any period, then in any MSPE and in any period, if a seniority system is instituted in stage 1 and there is at least one senior and at least one junior present (i.e., \(1 \leq S \leq N - 1\)), then the equilibrium expected shares of the dollar to a senior and a junior are respectively \(1/S\) and zero.

Given Lemma 2, one can now show that the results in Proposition 2 carry over:

**Proposition 3 (Equilibrium with Persistent Senior Recognition Advantage).** If a senior recognition advantage persists in every round of a period, then the results stated in Proposition 2 continue hold.
We have thus shown that even in a world with persistent seniority advantage, a cut-off seniority system emerges in equilibrium with the property that all seniors are always re-elected, but juniors need not be re-elected. There is however an important difference between the equilibria. In the case of transitory advantage equal probability is assigned to all legislators after the initial round; therefore, in equilibrium, any of the $N - 1$ legislators not proposing has an equal probability of being included in the winning coalition. In contrast, with a persistent recognition advantage, seniors are disadvantaged in being included in winning coalitions. Indeed, in equilibrium, and in the non-degenerate case, a senior will propose a coalition of herself and $(N - 1)/2$ juniors.

The following corollary shows that a persistent recognition advantage is preferred to a transitory advantage, and is an immediate consequence of Lemma 2 and equation (1).

**Corollary 1.** The equilibrium with persistent recognition advantage yields a higher expected payoff for each senior than his or her payoff in the equilibrium with transitory advantage.

In our generalized setting, with senior recognition advantage transitory or persistent, the cut-off is set at $s^* = s_M$, yielding $(N + 1)/2$ seniors and $(N - 1)/2$ juniors. Each senior is recognized with probability $2/(N + 1)$ and each junior with probability zero. If this recognition advantage is persistent, then the senior recognized in any period will optimally build a winning coalition with the $(N - 1)/2$ juniors, since they are cheaper coalition partners. She needs only to offer each of them a vanishingly small portion of the dollar (since, with no probability of recognition, a junior’s continuation value is zero) and keep essentially the entire dollar for herself. Hence, a senior’s ex ante expectation is $2/(N + 1)$ — she receives positive compensation only if she is the proposer since, if she is not, she will not be included as a coalition partner. If, on the other hand, the seniority recognition advantage is suspended after an initial offer, then a senior’s ex ante expectation is given by (1). Comparing these, seniors prefer a seniority system in which initial recognition advantage persists to one in which it suspended in subsequent rounds.

It thus appears that persistent seniority constitutes the equilibrium regime. In fact, it may be shown that a senior is indifferent between persistent advantage in subsequent recognition and mild disadvantage. To see this, suppose the probability of recognition for a senior in a subsequent round is set to $\beta/N$, where $\beta < 1$. Thus a senior is less likely to be recognized than a junior if a proposal fails. This means that some senior is certain to be recognized initially and, if not recognized, a senior will certainly be included in the winning coalition. That is, the proposer will build a winning coalition by including only other seniors, since their continuation values are just less than that of any junior, and there are just enough remaining seniors to constitute a majority. From (1) it may be seen that the inclusion effect (now positive) just balances the payments the proposer must make to senior coalition partners as part of the recognition effect, yielding $x_S = 2/(N + 1)$, the same as in the persistent advantage case. Thus, either a persistent advantage for seniors or a transitory advantage followed by a subsequent
disadvantage following a defeated proposal is preferred by seniors to a temporary advantage followed by equality between juniors and seniors.

3.4.2 Endowing Juniors with Proposal Power

We now show that the main result established above, Corollary 1, does not necessarily carry over to a setting in which juniors are allocated positive proposal power. We consider a simple formulation in which the total probability of recognition is shared amongst seniors and juniors in the ratio \(1 - \epsilon\) and \(\epsilon > 0\). This means a senior legislator is recognized in each round with probability \(\frac{1 - \epsilon}{S}\) and a junior legislator is recognized in each round with probability \(\frac{\epsilon}{(N - S)}\), where \(S\) is the number of seniors.

To make our point on this matter in a succinct manner, we restrict attention to the equilibrium path in which the cut-off is set at \(s^* = s_{M}\), yielding \((N + 1)/2\) seniors and \((N - 1)/2\) juniors. When \(S = (N + 1)/2\) the equilibrium expected values to a senior and a junior are respectively:

\[
V_S = \frac{2(1 - \epsilon)}{N + 1} \left[ 1 - \frac{N - 1}{2} \right] \delta V_J + \epsilon \left[ \frac{2}{N + 1} \right] \delta V_S \quad \text{and}
\]

\[
V_J = \frac{2 \epsilon}{N - 1} \left[ 1 - \frac{N - 3}{2} \right] \delta V_J - \delta V_S + \left[ 1 - \frac{2 \epsilon}{N - 1} \right] \delta V_J.
\]

Solving these equations for \(V_S\) and \(V_J\), we obtain that the equilibrium payoff to a senior is:

\[
V_P^S(\epsilon) = \frac{2(1 - \epsilon)(1 - \delta)}{(1 - \delta + \epsilon \delta)(N + 1 - 2\delta \epsilon) - 2\delta^2 \epsilon (1 - \delta)}.
\]

It is straightforward to verify that \(V_P^S(0) = 2/(N + 1)\) (as expected), \(V_P^S(1) = 0\) and that if \(N > 5\) then \(V_P^S(\epsilon)\) is strictly decreasing in \(\epsilon\).

Now, to address the issue under consideration, we note that the expected payoff to a senior in this same setting (in which juniors are allocated proposal power in the manner specified above) but when the recognition probability advantage is transitory is as follows:

\[
V_T^S(\epsilon) = \frac{\delta}{2N} + \frac{2(1 - \epsilon)}{N + 1} \left( 1 - \frac{\delta}{2} \right).
\]

It can be verified that that \(V_P^S(0) > V_T^S(0), V_T^S(1) > V_P^S(1)\) and that \(V_T^S(\epsilon)\) is strictly decreasing in \(\epsilon\).

Putting these results together, it follows (as shown in Figure 3) that there exists an \(0 < \epsilon^* < 1\) such for any \(\epsilon < \epsilon^*\), the result of Corollary 1 carries over. But for \(\epsilon > \epsilon^*\), the opposite result to that stated in Corollary 1 holds. In conclusion, then, we have shown that in a setting with persistent seniority, the “recognition” effect dominates the “inclusion” effect if and only if the recognition probability for a senior is sufficiently greater than that for a junior (i.e., for sufficiently small values of \(\epsilon\)).
All of the analysis done in this paper thus far has been conducted for an arbitrary but fixed value of the discount factor, $\delta$. Notice from the expression for $V^P_S$ in (10) that for any fixed $\epsilon > 0$, $V^P_S$ tends to zero as $\delta$ tends to one. But when juniors have no proposal power ($\epsilon = 0$) then $V^P_S = 2/(N + 1)$, i.e., $V^P_S$ is independent of $\delta$. There is thus a discontinuity in the limiting function $\lim_{\delta \to 1} V^P_S(\epsilon; \delta)$ at $\epsilon = 0$. Given these observations, it can be verified that for any $\epsilon > 0$ there exists a $\delta(\epsilon)$ such that for any $\delta > \delta(\epsilon)$, $V^T_S > V^P_S$. This means (as shown in Figure 4) that when juniors have some positive proposal power, no matter how small, then, provided legislators are sufficiently patient, it will be the case that (like in the McKelvey and Reizman (1993) set-up) a senior legislator prefers a seniority institution with transitory recognition advantage over one with persistent recognition advantage.

This result illustrates the the importance of the discount factor in determining the relative importance to a senior legislator of the “recognition” and “inclusion” effects in a world when juniors have positive proposal power. With an increase in $\delta$ (patience), the value of proposal power declines (relative to that of inclusion) because the continuation value of potential coalition partners increases, i.e., anyone who “wins” the right to make proposals has to pay out more to coalition partners when $\delta$ is high than when it is low.
Figure 4: Limiting expected payoffs as a function of junior proposal power.

4 Conclusion and Future Directions

In this paper we have sought to elaborate on a strategic view of institutional features. Our focus has been on seniority, though this general approach may be deployed to understand other institutional features. We have taken the initial game-theoretic model of seniority of McKelvey and Riezman (1992), simplified it in order to characterize its fundamental implications, and then generalized it in several ways.

It is worth emphasizing that MR92 provides a strategic rather than a functional rationale for seniority institutions. Much of the social science literature on seniority emphasizes internal functions and objectives it serves as the reasons for its institutionalization — avoidance of conflict over leadership, channeling of experience-based expertise to positions of agenda power, prevention of the diversion of internal resources into leadership struggles (e.g., favor-giving and pork-barreling side payments), etc.28 These functional purposes surely have some explanatory

28The classic in anthropology is Simmons (1945). Labor economics discussions of seniority focus on labor-management contract negotiations. See Mater (1940) and Burda (1990). In the context of advanced industrial societies, “generational” seniority as in pay-as-you-go pension schemes is developed by, among others, Davis (1990), Peterson (1999), Kotlikoff (2003), and Kotlikoff and Burns (2004). The classic in political science is Goodwin (1959). A theoretical piece similar in spirit to our model is Epstein et. al. (1997). Major statistical studies of seniority practice in the U.S. House include Abram and Cooper (1968), Polsby, Gallagher, and Rundquist (1969), and Krehbiel and Wiseman (2001).
power. But MR92’s strategic explanation is an important complement that ties the endogenous creation of institutional arrangements directly and explicitly to legislator self-interest.

We conclude by laying out an agenda of additional opportunities for generalization and extension. Some of these we hope to tackle in a sequel.

4.1 Generalizing the Game Form

In stage 1 of MR92 and our generalization, the decision on whether to institute a seniority system is effectively taken under a closed rule. The only alternatives on offer are whether to have a seniority system or not. There is no opportunity to institute recognition advantage other than according to strict seniority. However, empirically, most legislative organizations adopt their rules at the beginning of a period under “general parliamentary procedure” in which amendments and other alternatives are normally in order. For example, it would be in order for a junior legislator to move an arrangement in which some juniors join some seniors in having recognition advantage.

To fix ideas, note that in stage 1 of the MR92 model, the choice between a seniority system and no seniority is formally equivalent to the choice between two recognition vectors, \( \mathbf{p}^S \) and \( \mathbf{p}^N \), where \( \mathbf{p}^S = \{p^S_1, \ldots, p^S_N\} \) such that \( p^S_i \) is zero if \( i \) is junior and \( 1/S \) if \( i \) is senior (where \( S \) is the number of seniors), and \( \mathbf{p}^N = \{p^N_1, \ldots, p^N_N\} \) such that \( p^N_i \) is \( 1/N \) for all \( i \). But now suppose the stage 1 game form is modified as follows. The seniority vector \( \mathbf{p}^S \) is proposed and some amendment \( \mathbf{p}^A \), which reallocates recognition probability, is in order. In this extensive form the seniority vector is pitted against the amended vector, the winner is then pitted against the status quo of no seniority (\( \mathbf{p}^N \)). And then stages 2 and 3 proceed as in MR92. In effect, this is one way forward in more fully endogenizing proposal power. We suspect that the MR92 seniority-incumbency equilibrium (Proposition 1) will be weakened in this circumstance, but this needs further exploration.

4.2 Valence

In MR92 and our generalization, the voting subgame is dominated by voter expectations of future payoffs. A senior is a more attractive representative than a junior because the former’s recognition advantage translates into higher expected returns for voters. Voter preferences are characterized entirely in terms of these expected returns. There may, however, be other criteria that matter to voters — either unmodeled preferences for particular types of representatives unrelated to concrete payoffs (e.g., gender, race, age, ethnicity) or for payoff-relevant characteristics in a richer institutional setting (e.g., competence, energy, honesty, committee assignments). One way forward is to think of the “type space” as multidimensional, including the seniority attribute but not limited to it. In the voting subgame, voters compare the incumbent’s type, \( t^I = (\tau^I, t^I_2, \ldots, t^I_n) \), with the challenger’s type, \( t^C = (0, t^C_2, \ldots, t^C_n) \). The first argument is the number of terms of previous service — the challenger has zero terms and the incumbent \( \tau^I > 0 \). Seniority is but one among many valence characteristics about which
voters care. What is distinctive about it in comparison to other valence characteristics is the unchallengeable advantage that goes to the incumbent. A challenger may possess gender, race, age, and ethnicity advantages; he or she may be more competent, energetic, or honest than the incumbent. But he or she cannot trump the incumbent on seniority. If voters anticipate an important role for seniority in the next period, and if the incumbent will benefit substantially from that arrangement, then this institutional advantage may compensate for deficiencies on other type dimensions.

In short, enriching the type space may moderate the incumbency advantage, but the advantage does not attenuate altogether. It does suggest, however, that those incumbents who do not immediately benefit from seniority are more vulnerable to challenge than their more senior colleagues (or compared to circumstances later in their own careers when they have accumulated more terms of service). They may be paired against a challenger whose domination on other type dimensions more than compensates for a (slight) disadvantage on the seniority dimension. This is consistent with patterns in U.S. House elections in which an incumbent is most vulnerable to defeat in his or her first reelection campaign where the seniority advantage over a challenger is minimal.

4.3 Corruption and Legislator Incentives

We have had very little to say in this paper about legislator incentives. Taking our cue from MR92, legislators share with their respective constituents a preference monotonic in the amount of the dollar they obtain in the stage 2 bargaining. Legislator and constituents divide this amount in the proportions \( \theta \) and \( 1 - \theta \), respectively. This performance-based form of compensation for legislators eliminates the moral hazard that often infects principal-agent relationships. It permits constituents to focus on future benefit flows in their stage 3 decision-making, rather than engaging in retrospective assessment as a disciplining device if moral hazard had been an issue. This redounds, as our results demonstrate, to the advantage of incumbents by inducing constituents to stick with their current representative in order to maintain the constituency’s recognition advantage in legislative bargaining. In a type-space context, it raises the bar on other valence advantages a challenger must enjoy to be a viable threat to the incumbent.

A generalization of this approach, one that maintains the spirit of MR92 but moderates the advantage of incumbents, is to treat \( \theta \) in a more sophisticated way. Suppose, for example, that \( \theta \) is not constant across legislators or legislative careers, but rather is a function of experience — more experienced legislators develop a facility for extracting a larger share of whatever she delivered to the folks back home than less experienced colleagues. To put it slightly differently, experienced legislators lack the capacity to commit not to extract a larger share, and constituents, therefore, harbor suspicions that experienced politicians are corrupted by their incumbency. Constituents represented by a senior still enjoy the advantage of their

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29 We thank Andrew Eggers for this insight.
30 On retrospective voting see Fiorina (1979) and Muthoo and Shepsle (2008, 2010).
representative’s recognition differential, but at a growing cost. At some point the expected net returns to the constituency represented by a senior falls below that which they could secure from a junior, \((1 - \theta)x_J\) (the latter given in (2)). It would appear, then, though the detailed development is beyond the scope of the present paper, that the incumbency advantage weakens as the prospect of corruption grows.

As a second, even subtler elaboration of \(\theta\), one may examine how it emerges endogenously. Instead of treating \(\theta\) as fixed and exogenous, as in MR92, or as varying across a legislative career, as in the previous paragraph, suppose \(\theta\) (which might be thought of as the level of corruption) is an endogenous consequence of legislators taking as much as they can at the end of stage 2, restrained only by electoral judgments in stage 3. This is an extremely interesting direction for future research. It appears to us that \(\theta\) will depend intimately on specific features of seniority (permanent v. transitory, categorical v. ordinal, etc.).

4.4 Multiple Cut-offs and Fully Ordinal Seniority

McKelvey and Riezman devote all of their attention to categorical seniority: the set of legislators is partitioned into junior and senior subsets. Our contribution generalizes the cut-off, but not the two-class partitioning of legislators. Suppose instead that we sought a three-class seniority arrangement defined by two cut-offs. The lowest class — rookies — had no (or very limited) proposal power, the middle class — experienced — had some, and the upper class — veterans — had the most proposal power. We have demonstrated in Proposition 2 that there exists a cutoff \(s^* = s_M\) that is supported by a majority of legislators. Is there an ordered pair \(s^* = (s_{RE}, s_{EV})\), with \(s_{RE} \leq s_{EV}\), that separates rookies from experienced legislators and experienced legislators from veterans?

This is a harder problem in three respects. First, by its very nature it is a multidimensional collective choice problem for which a tool like the MVT is no longer applicable. Second, as in the single cut-off problem, legislators will be indifferent to choices for at least one of the components of \(s^*\). Very senior legislators, for example, will have strict preferences over the \(s_{EV}\) cut-off, but will be indifferent about where to set the \(s_{RE}\) cut-off. A condition even stronger than Assumption 1 will be required to break ties produced by pervasive indifference. Third, the problem of assigning recognition probabilities to each class in the partition of legislators must be faced head on.

To give some indication of the difficulties accompanying multidimensionality, imagine a two-dimensional space where the horizontal axis measures \(s_{RE}\) and the vertical axis \(s_{EV}\). The 45° line is the locus of points in which \(s_{RE} = s_{EV}\). It constitutes the set of two-class partitions; the off-diagonal points represent non-degenerate three-class partitions. It may be shown that, with something along the lines of Assumption 1, the MVT implies that \(s_M\), the median length of service among the \(N\) legislators, is preferred by a majority to any other point on the diagonal. (An argument along the lines establishing Proposition 2 gives this result.) With off-diagonal elements available, however, it may not remain a Condorcet winner. In particular, it may be beaten by a three-class partition which, in turn, is beaten by another three-class partition (or
a non-median two-class partition). This analysis may be extended to the $K$ cut-point problem and, in particular, $K = N - 1$ — the case of full ordinality. Certainly many seniority systems are ordinal not categorical, so it would be theoretically appropriate to extend our argument to this case. We hope to develop this line of analysis further in a subsequent paper.

The broad messages of our paper, articulated by McKelvey and Riezman as well, are two. First, the endogenous choice of institutional features like seniority by self-governing groups is a *strategic* endeavor. While the fine-grained ways of doing things in an institutional context surely serve internal functional objectives, these are not the only objectives. Agents making choices on how to govern themselves have private motivations — in the case of elected politicians they often revolve around reelection. The establishment of a seniority system may be considered in just this light. McKelvey and Riezman establish the tight connection between seniority and electoral advantage accruing to incumbents. We generalize this conclusion, and show how it may extend to other features of legislative life.

This leads to our second broad message. The institutions through which self-governing groups conduct their business do not exist in a vacuum. They are embedded in a broader context. Those offering functional explanations for various institutional features overlook this. Particular institutional arrangements have effects *outside* the governance institution itself. These effects, in principle, could be accidental by-products. Our strategic approach, however, argues that they may well be the primary reasons for a practice being instituted.

References


