Local Network Externalities and Market Segmentation

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Abstract

This paper models interaction between groups of agents by means of a graph where each node represents a group of agents and an arc represents bilateral interaction. It departs from the standard Katz-Shapiro framework by assuming that network benefits are restricted only amongst groups of linked agents. It shows that even if rival firms engage in Bertrand competition, this form of network externalities permits strong market segmentation in which firms divide up the market and earn positive profits. The analysis also shows that some graphs or network structures do not permit such segmentation, while for others, there are easy to interpret conditions under which market segmentation obtains in equilibrium.

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1 Introduction

It has long been known that some goods and services (for example, telecommunications, computer software and hardware) generate network effects or externalities. The seminal paper by Katz and Shapiro (1985) defines a network effect to exist when the utility that a user derives from consuming a product depends on the number of other agents who consume either the same brand of the product, or another brand which is compatible. This way of modelling the network effect is found throughout the large theoretical and empirical literature that has developed.[1] While this is reasonable in many contexts, in other instances it overlooks the fact that such positive externalities arise from the *specific* patterns of interaction between groups of users.

For instance, consider software packages with specific functions such as word processing, accounting, data analysis and so on. The use of such packages has *local* network effects. Thus the utility to a user (say, a researcher in a University) of a word processing or data analysis package depends at least partly on the number of her research *collaborators* who use the same package, rather than on the total number of users of the package. A main advantage to two collaborators using the same package is sharing files. For many of these products, there is a degree of incompatibility between brands. Two users using incompatible brands find it difficult if not impossible to share files; a program written on one software package cannot be read, or worked on, using a competing brand.

These two elements - a user's utility from a product depending on the number of other users who *interact* with her, and of some degree of incompatibility between competing brands, are present in other contexts as well. Thus many people using instant messaging typically communicate only with their friends or coworkers; and there are incompatibilities between the leading competing brands provided by AOL, MSN and Yahoo.[2] In interactions

[1] There is by now a large literature analyzing important issues in markets subject to network effects. See, for instance Katz and Shapiro (1985, 1986), Farrell and Saloner (1985, 1986), Economides and Salop (1992), Farrell and Katz (2000), Matutes and Regibeau (1992), Choi (1994), Ellison and Fudenberg (2000), Waldman (1993). Economides (1996) provides an insightful overview. Gandal, Kende and Rob (2000) and Saloner and Shepard (1994)are two interesting papers from the empirical literature.

[2] There is software available, such as Trillian, that provides interfaces between these products, but it involves costs (all the competing brands have to be installed in one's computer, for instance), and firms such as AOL constantly change their software to maintain incompatibility.

between businesses, it helps if software systems are compatible.

We use the formal network structure proposed in the important recent paper of Jackson and Wolinsky (1996) to model the interaction between groups of users. In particular, the set of all consumers is partitioned into different groups or *nodes*, and two nodes are connected to each other if they "interact".[3] Our main interest is in analysing whether the precise pattern of interactions - that is, the *specific* network structure- has any influence on market outcomes. For instance, suppose the overall "market" is the academic market for software. Does the fact that economists typically do not collaborate with physicists (that is, economists are not "linked" to physicists) matter in this market?

A typical feature of information goods such as software is that firms incur possibly high fixed costs to develop essentially unlimited capacity, and their marginal costs are negligible. As a first step towards understanding competition in markets with local network effects, we maintain the assumption of unlimited capacity and study price competition. The issue of pricing and competition is interesting when we study information goods for a variety of reasons. Local network interactions is one of them, for which price competition has not been analyzed so far. If firms produce competing, incompatible brands of the same *intrinsic* quality, and have the same constant marginal cost of production, existing models of network externalities would yield the Bertrand zero profit outcome. This is so for the Katz and Shapiro (1985) model as well, if it is modified to analyze price, rather than quantity competition.[4] A main result in this paper is that if network effects are generated from patterns of interaction among users, then there exist outcomes in which firms do make *positive* profits, and there is *market segmentation* in the sense that rival firms divide or partition the overall market into separate segments, with each firm selling to different segments.

Market segmentation accords well with casual observation, which suggests (a) positive profit outcomes arise even when firms compete in prices and capacity is essentially unlimited; and (b) a group of users often uses a single brand overwhelmingly, when several similar brands are available. For

[3]Although this kind of modelling has only very recently been used in the literature on network externalities, the use of such network structures in other areas of economics is becoming increasingly popular. Dutta and Jackson (2003) contains several interesting papers in this genre. See also Goyal(2007), Jackson (2008).

[4]The zero profit outcome obtains under a restriction (Assumption 1 below) on consumers' expectations that requires the demand for a brand to be non-increasing in its price. See the discussion following Theorem 1.

example, law firms in the U.S. continued to use WordPerfect when the rest of the world was switching over to Microsoft Word in the 1990s (Porter, 2000). The positive *local* network benefits to lawyers from using the same word processor other lawyers (and some clients) used explains this pattern.

Perhaps the most interesting contribution of the paper is that the specific interaction structure *matters*; for some networks, market segmentation can be ruled out in equilibrium, while other networks permit market segmentation. Thus, one way of interpreting our results is to say that there are interaction structures which convert the industry into a differentiated goods industry. However, there are other interaction structures - for instance, the complete graph where all users are linked to each other - where the goods remain homogeneous, and so firms do not earn positive profits. The discussion also shows that when positive profit equilibria exist, if firms could choose whether or not to make their brands mutually compatible, they would choose not to do so.

Related Work

Very recently, work has begun on understanding markets for products that exhibit local network effects, using an explicit model of the network structure (Jullien (2003), Sundararajan (2006), Tucker (2006)). In an interesting paper, Jullien (2003)[5] develops a model of oligopoly in an industry in which network effects can be local or group-specific, while the other two papers analyze the adoption of a single good in the presence of local network effects. The present paper analyzes competition and is therefore closer to Jullien (2003). Jullien analyzes a setting in which price discrimination across different groups of consumers is possible.[6] In his model, one firm (the Strong firm) has a reputational advantage. However, the ability of the Weak firm to price discriminate (by cross subsidizing some groups of consumers, inducing them to buy, thereby creating a strong network-effect inducement for other groups of customers) creates strong competition for the Strong firm. This keeps equilibrium profits low. In contrast with Jullien (2003), the present paper studies competition in situations where price discrimination is not possible; a major difference in emphasis is also the attempt to study how the structure of interactions affects market segmentation.

Sundararajan (2006) studies a model (with incomplete information) in which agents must simultaneously and independently decide whether or not

[5]We became aware of this paper after writing a draft of the present paper. We thank Bruno Jullien for pointing us to this paper.

[6]This is especially reasonable in the context of two-sided markets and competition among intermediaries. See Caillaud and Jullien (2003).

to adopt such a product. Each agent is located at a node of a graph, knows the nodes that he/she is connected with, but is not informed about the rest of the network structure. Sundararajan finds that the symmetric Bayesian equilibria can be Pareto-ranked, and that the greatest of these is the unique coalition-proof equilibrium. Tucker (2006) analyzes a rich data set describing the adoption of a video-messaging technology by employees of a financial firm. Among other interesting findings, the data strongly support the hypothesis that the network effect to an employee of adopting the technology is limited to people that she communicates with. While these two papers study the adoption of a single good, the present paper analyzes an oligopoly model in which firms compete for customers who are linked over a network whose structure is common knowledge.

There is a recent interesting literature on two-sided markets (e.g. Armstrong (2006), Caillaud and Jullien (2003), Rochet and Tirole (2003)) such as markets for payment cards, intermediation services, and mobile telephony; Jullien (2003, 2008) provides more general, multi-sided market analyses. Ambrus and Argenziano (2008) show that endogenous market segmentation can result in a two-sided market if consumers differ in their valuation of the network externality. Gabrielsen and Vagstad (2008)explain observed differences in on- and off-network call termination charges between mobile telecommunication service providers in terms of switching costs and local network externalities that operate within 'calling clubs' of friends. The specific patterns of interactions between agents in two-sided markets provide further motivation for looking at models with local network effects; however as noted earlier, price discrimination is a key element in these markets and the models above differ from ours in this respect.[7]

2 A Model of Network Externalities

Our model of network externalities in the context of a partial equilibrium duopoly is very similar to that of Katz and Shapiro (1985). A major difference is in the way in which we model network externalities. Another difference is that in our model firms compete in prices, in contrast to Katz and Shapiro (1985) who assumed that firms behaved a la Cournot.

Our model has the following components and structure. There are two profit-maximizing firms 1 and 2, firm j producing network good j. To bring out the main points simply, the two goods are assumed to be functionally

[7]Patterns of price discrimination in these models share some similarity with price discrimination over time in dynamic models of network competition (Cabral (2008)).

identical. The two firms simultaneously announce prices p_1, p_2. Given these prices, consumers simultaneously decide which good to buy. A consumer buys one unit from either firm 1 or firm 2, or abstains from consumption. Consumers benefit from own consumption, as well as from interaction with others who consume the same good.[8] The presence of network externalities generates a coordination problem for the consumers. We assume that for each vector of prices, consumers coordinate on a rational expectations equilibrium allocation. Both firms correctly anticipate which allocation will be chosen by the consumers. So, an overall equilibrium is a vector of prices which are best responses to each other given the firms' common (and correct) anticipations of the rational expectations equilibrium allocation chosen by the consumers.

We now describe each component of the model in greater detail.

Consumers

Consumers are partitioned into groups, and each group "interacts" with *some* but not necessarily all groups. For instance, consider the set of all faculty members in a university. Each department then constitutes a group. Economists may collaborate with political scientists and mathematicians, but perhaps not with physicists or other scientists. Similarly, members of the science departments may interact with each other, but not with sociologists. The pattern of such interactions is modeled as an *undirected graph* or *network* (I, g) where I is a set of n *nodes* and $g \subset I \times I$ is a set of *arcs* or links. Each group of consumers is located at a different node $i \in I$, and $ij \in g$ (that is, nodes i and j are linked) if consumers located at node i interact with consumers located at j. We assume that consumers within each group interact with each other and that if *some* consumers at node i interact with *some* consumers at j, then *all* consumers located at i interact with *all* consumers at j.[9]

Given a graph (I, g), the *degree* of node $i \in I$ is the number of other nodes that it is linked to. Given any graph g, $N(g)$ will denote the set of nodes which have degree at least one. A sequence $(i_1, i_2, ..., i_n)$ of distinct nodes is a *path* connecting i_1 and i_n if $\{i_1 i_2, i_2 i_3, ..., i_{n-1} i_n\} \subset g$. A graph is *connected* if there is a path connecting every pair of nodes.

For each node i, let $N(i, g) = \{j \in I | ij \in g\} \cup \{i\}$. That is, $N(i, g)$ is the set of nodes that are linked to node i, with the convention that i is linked to itself.

[8]If the goods are partially compatible, then consumers also derive some benefit from interaction with other consumers who consume the *other* good.

[9]This is without loss of generality since we can define the set of nodes appropriately in order to represent any pattern of interaction.

The network (I, g) is *complete* if $g = \{ij | ij \in I \times I\}$. That is, all groups interact with *all* other groups in a complete network - this will correspond to the original Katz-Shapiro model of network externalities.

We will also refer to some specific network structures later on. These are defined below.

A *circle* on a set of nodes I is a connected graph in which every node has degree two.

A *star* on a set of nodes I is a connected graph g such that $g = \{i^*j | j \in I \setminus \{i^*\}\}$, where i^* is a distinguished node called the hub of the star.

A *line* on a set of nodes I is a connected graph g such that exactly two nodes have degree one, while all the other nodes have degree two.

Let α_i denote the measure of consumers located at node i.[10] Each consumer wishes to consume at most one unit of a good. There are *two* brands of the good - for example, the good itself may be a type of software. The two brands differ in inessential ways in the sense that each brand is functionally identical as far as consumers are concerned. Let r_i denote the *basic* willingness to pay for the good of a consumer who is located at node i. However, the total utility or surplus that a consumer gets from a particular brand of the good also depends on the number of other consumers with whom she interacts. If the two brands are *incompatible*, then the consumer derives network benefits from others she interacts with only if they consume the same brand. More generally, the brands could be partially compatible. Then, if consumer A, using brand j, interacts with consumers who use brand k, she gets *some* network benefit, but not as much as she would have got had these consumers also used brand j. The examples that motivate this paper show that some degree of incompatibility is a realistic assumption.

Following Caillaud and Jullien (2003), Jullien (2003), Choi (1994), Farrell and Saloner (1985, 2000), we model partial compatibility by letting a parameter $\theta \in [0, 1]$ denote the *degree of compatibility* between the two brands. Thus, $\theta = 0$ and $\theta = 1$ will denote, respectively, incompatibility and full compatibility.

Let p_j be the price of a unit of brand j, and α_{sj}, α_{sk} be the measure of consumers at node s who consume brands j and k. Then the utility of a consumer at node i from buying a unit of brand j is

$$u_i(j, p_j) = r_i - p_j + \sum_{s \in N(i,g)} (\alpha_{sj} + \theta \alpha_{sk}).$$

So, by consuming brand j, a consumer at node i gets a gross benefit r_i and a *network benefit* of $\sum_{s \in N(i,g)} (\alpha_{sj} + \theta \alpha_{sk})$. The network benefit to a

[10] Any single consumer has zero measure.

consumer at node i consuming brand j, from neighbors who consume brand k is a proportion θ of the benefit from neighbors consuming brand j.

Notice that the network externality is *local* since the externality at node i is restricted to only the neighboring nodes.

To simplify the analysis, we will henceforth set $r_i = 0$ for all i. This simplification does not alter the qualitative nature of our subsequent results.

Following Katz and Shapiro(1985), we will refer to $p_j - \sum_{s \in N(i,g)} (\alpha_{sj} + \theta \alpha_{sk})$ as the *hedonic price* of brand j at node i.

It is well known that in making consumption decisions in the presence of network effects, consumers face a coordination problem. We model this in a way quite similar to the notion of "fulfilled expectations" used in the literature (Katz and Shapiro (1985)). Given a vector of prices (p_1, p_2), consumers form an expectation $a_{sj}(p_1, p_2)$, which is the measure of consumers at node s expected to purchase brand j, for all $s \in I, j \in \{1, 2\}$. Given this expectation, each consumer at each node purchases the brand whose hedonic price is lower, or abstains from buying either brand if both hedonic prices are *positive*.[11] Expectations are fulfilled, in that for every node s and brand j, consumers who purchase brand j on the basis of the expectations $\{a_{sj}(p_1, p_2)\}$ aggregate to exactly $\{a_{sj}(p_1, p_2)\}$ for each node and brand. We use the notion of an *allocation*, satisfying certain conditions, to model these rational expectations.

Allocations

An allocation describes the pattern of consumption at each node corresponding to each vector of prices. More formally,

Definition 1 *An allocation a is a function $a : \Re^2_+ \rightarrow \Re^{2n}_+$, such that for all (p_1, p_2) and for all $i \in I$, $a_{i1}(p_1, p_2) + a_{i2}(p_1, p_2) \leq \alpha_i$.*

Here, $a_{ij}(p_1, p_2)$ is the amount of brand j consumed at node i corresponding to prices (p_1, p_2).

Given a vector of prices (p_1, p_2), a network g and an allocation a, the hedonic price of brand j at node i will depend upon the vector (p_j, g, a, θ, i). Since the network structure g and θ are exogenous, we will simplify notation whenever possible and denote the hedonic price as $h(p_j, a, i)$. It is given by

$$h(p_j, a, i) = p_j - \sum_{s \in N(i,g)} (a_{sj}(p_1, p_2) + \theta a_{sk}(p_1, p_2))$$

Consumers' decisions about which brand to purchase will determine which allocation is "observed" in the market. Since such allocations are the outcome

[11] This follows from our assumption that each $r_i = 0$.

of utility-maximizing behavior, it makes sense to impose some restrictions on "permissible" allocations.

Definition 2 *An allocation a is Rational if for all nodes i and non-negative prices (p_1, p_2), the following are satisfied*

(i) For $j = 1, 2$, $a_{ij}(p_1, p_2) > 0$ implies that $h(p_j, a, i) \leq 0$.

(ii) For $j = 1, 2$, $a_{ij}(p_1, p_2) > 0$ implies that $h(p_j, a, i) \leq h(p_k, a, i)$ where $k \neq j$.

Thus, Rationality imposes the requirements that no individual consumes a brand whose hedonic price is positive, and also consumes that brand whose hedonic price is lower. These are minimal requirements which arise straightaway from utility-maximizing behavior.

Since the pattern of consumption also depends on consumers' expectations, it may be possible to justify or rationalize allocations which satisfy these restrictions, but are nevertheless non-intuitive simply because of the self-fulfilling nature of expectations. Suppose, for instance that "initial" prices of the two brands are p_1 and p_2. Now, let there be an increase in the price of brand 1, with p_2 remaining constant. If all consumers now expect everyone to switch to brand 1, then this may turn out to be self-fulfilling because the network externalities associated with brand 1 are now much larger and so the hedonic price of brand 1 is correspondingly lower at all nodes. The following assumption[12] is imposed to bring about some regularity on how the pattern of consumption changes with changes in prices.

Assumption 1: An allocation a is monotone in prices if for all $i \in I$ and $j = 1, 2$, $a_{ij}(p_j, p_k)$ is non-increasing in p_j.

By itself, Assumption 1 imposes a very weak restriction on how allocations change with respect to a change in prices. In particular, Assumption 1 still allows for allocations which seem somewhat counterintuitive. For example, suppose that the two brands are incompatible ($\theta = 0$). Consider a network structure in which nodes i and j are linked, and such that at prices (p_1, p_2), all consumers at node i are consuming say brand 1 because the hedonic price of brand 1 is smaller than the hedonic price of brand 2 by α_i. Suppose there is an arbitrarily small reduction in the price of p_2. Then, Assumption 1 allows for the possibility that *all* consumers at node i will switch brands and

[12]Caillaud and Jullien(2003) also make the same assumption.

consume only brand 2. Of course, if all consumers expect this to happen, then the self-fulfilling nature of expectations guarantees that the allocation will satisfy Rationality and Assumption 1. In order to rule out such changes, we impose the following assumption.

Assumption 2: For every $i \in I$, the component a_i of an allocation a is continuous except possibly at any (p_1, p_2) where the hedonic prices are equal.

Definition 3 *An allocation is admissible if it satisfies Rationality, and Assumptions 1 and 2.*

Notice that since an allocation is *endogenous*, it is more appropriate to impose the restrictions embodied in Assumptions 1 and 2 on the primitive concept of consumer expectations. However, the preceding discussion (hopefully) clarifies the kind of restrictions required to be imposed on expectations so as to ensure that the resulting allocations satisfy admissibility. We have taken the shorter route so as to economize on notation.

Throughout this paper, we will only consider *admissible* allocations.

Since an individual's net utility depends on the actions of other consumers, the optimal decisions of consumers may depend on whether consumers can coordinate their actions. Consider, for example, a situation where node i is not linked to any other node, and $p_1 - \alpha_i < 0 < p_1 < p_2$. Then, consumers at node i can derive some net utility if *all* consumers consume brand 1. On the other hand, *no* consumer on her own will want to consume either brand. In one subsequent result, we will assume that consumers at each node can coordinate their actions when this is mutually profitable.

Assumption C: At any node i and prices (p_1, p_2), if $\min_{j\in\{1,2\}}(p_j - \alpha_i - \sum_{s\in(N(i,g)-\{i\})} (a_{sj}(p_1, p_2) + \theta a_{sk}(p_1, p_2)) < 0$, then $a_{i1}(p_1, p_2) + a_{i2}(p_1, p_2) = \alpha_i$.

Assumption C states that if consumers at any node can coordinate their consumption decisions and attain strictly positive utility, then no consumer will abstain from consumption.

Firms

There are two firms, each producing a different brand. For simplicity, we assume that firms have zero cost of production.

Both firms anticipate the same allocation, and choose prices simultaneously to maximize profits. An important difference from Jullien (2003) is that the firms in our model cannot practice price discrimination - consumers at *all* nodes face the same prices. Given any allocation a, firm j's profit corresponding to prices (p_j, p_k) is

$$\pi_j(p_j, p_k; a) = p_j \sum_{i \in I} a_{ij}(p_j, p_k)$$

Equilibrium

An equilibrium will be a set of prices (p_1, p_2) and an admissible allocation such that each firm j maximizes profit given the other firm's price and the allocation rule. Notice that the restrictions on a ensure that consumers' expectations are fulfilled in equilibrium.

Definition 4 *A vector (p_1^*, p_2^*, a^*) constitutes an equilibrium if*

(i) The allocation a^* is admissible.

(ii) For each $j = 1, 2$, and $k \neq j$, $\pi_j(p_j^*, p_k^*; a^*) \geq \pi_j(p_j, p_k^*; a^*)$ for all p_j.

It is easy to see that an equilibrium always exists in this model. For consider prices $p_1^* = p_2^* = 0$, and an admissible allocation a^* such that at each node i, $a_{i1}^*(p_1, p_2) = a_{i2}^*(p_1, p_2) = \frac{\alpha_i}{2}$ whenever $p_1 = p_2$. Since $p_1^* = p_2^*$, and the allocation divides consumers equally between the two brands, the two hedonic prices must be equal at each node. Since the hedonic prices are also negative, the allocation a^* is admissible. Neither firm can raise price and earn positive profit. Indeed, suppose firm 1 charges $p_1^{'} > p_1^* = 0$. Then, for all $i \in I$,

$$a_{i1}^*(p_1^{'}, p_2^*) + \theta a_{i2}^*(p_1^{'}, p_2^*) \leq a_{i1}^*(p_1^*, p_2^*) + \theta a_{i2}^*(p_1^*, p_2^*)$$

(This follows as by Assumption 1, it must be that $a_{i1}(p_1^{'}, p_2^*) \leq \frac{\alpha_i}{2}$). So, at node i, $h(p_1^{'}, a^*, i) > h(p_2^*, a^*, i)$. Therefore in fact we get $a_{i1}(p_1^{'}, p_2^*) = 0$. A higher price results in zero market share and zero profits.

Notice that in the equilibrium described above, the two hedonic prices are equal at each node. The pair of prices remain in equilibrium because neither firm wants to deviate by quoting a lower price since the "current" level is already zero. The lemma below shows that this is the only case when hedonic prices can be equal at any node. That is, if hedonic prices are equal at any node i, and brand j is consumed at this node, then the price of brand k ($k \neq j$) must be zero - the latter condition ensures that firm k has no incentive to lower price any further in order to capture a larger share of the market.

Lemma 1 *Suppose* (p_1, p_2, a) *is an equilibrium. Then, at all nodes* $i \in I$*, for* $j = 1, 2$ *and* $k \neq j$*, if* $h(p_j, a, i) = h(p_k, a, i)$*, either* $a_{ij}(p_1, p_2) = 0$ *or* $p_k = 0$.

The proof of all lemmas and theorems is contained in the Appendix.

3 Market Segmentation

Both firms had positive market share at each node in the equilibrium described in the preceding section. However, this is not surprising since neither firm had any incentive to cut into the other firm's market share as prices were driven down to unit cost (zero). The main purpose of this paper is to show that some network structure(s) representing interactions between consumer groups may result in *segmented* markets with both firms earning *strictly positive profits although firms are competing in prices.* A formal definition of market segmentation follows.

Definition 5 *: An equilibrium* (p_1, p_2, a) *exhibits strong market segmentation if there are nodes* i *and* j *such that* $a_{i1}(p_1, p_2) = \alpha_i$*,* $a_{j2}(p_1, p_2) = \alpha_j$ *and* $p_k > 0$ *for* $k = 1, 2$.

We construct an equilibrium which exhibits strong market segmentation.

Example 1 *Let the two brands be incompatible* ($\theta = 0$)*. Let* $I = \{1, \ldots, 4\}$*, and let* g *be a circle on* I*. Suppose* $\alpha_i = \alpha$ *for each* i*. Suppose* $a^*_{11}(p_1, p_2) = a^*_{21}(p_1, p_2) = \alpha$ *whenever* $p_1 - p_2 < \alpha$ *and* $p_1 - \alpha \leq 0$*. Similarly, let* $a^*_{32}(p_1, p_2) = a^*_{42}(p_1, p_2) = \alpha$ *whenever* $p_2 - p_1 < \alpha$ *and* $p_2 - \alpha \leq 0$*. Let* $p^*_1 = p^*_2 = 2\alpha$.

Then

$$h(p^*_1, a^*, 1) = h(p^*_1, a^*, 2) = h(p^*_2, a^*, 3) = h(p^*_2, a^*, 4) = 0$$

Also,

$$h(p^*_1, a^*, 3) = h(p^*_1, a^*, 4) = h(p^*_2, a^*, 1) = h(p^*_2, a^*, 2) = \alpha$$

So for example, all consumers at node 1 consume brand 1 at prices (p_1^*, p_2^*) because $0 = h(p_1^*, a^*, 1) < h(p_2^*, a^*, 1) = \alpha$. It follows more generally that the allocation a^* satisfies the requirements imposed by Rationality at (p_1^*, p_2^*). Similarly, it can be checked that a^* is admissible at *all* price profiles. Now, suppose the producer of brand 2 wants to "steal" consumers located at node 1. Since $h(p_1^*, a^*, 1) - h(p_2^*, a^*, 1) = -\alpha$, firm 2 must reduce its price by at least α to make this happen. If this enables firm 2 to capture the entire market, its profit will be $4\alpha^2$. But, this is its profit at $p_2^* = 2\alpha$. For exactly the same reason, firm 1 does not have a profitable deviation either. Hence, (p_1^*, p_2^*, a^*) constitutes an equilibrium exhibiting strong market segmentation.

How is it that both firms are earning positive profits despite being Bertrand duopolists? If prices are strictly positive, then lemma 1 implies that at each node, all consumers buy the same brand; the one whose hedonic price at that node is strictly less than that of the other brand. So, each firm i will have to lower its price by an amount ϵ_i strictly bounded away from zero in order to eat into its rival's market share. So, strong market segmentation can be sustained if ϵ_i is sufficiently large so as to make the revenue loss from its existing customers larger than the gain in revenue from new customers. It is worth noting also the role of the continuity restriction imposed on expectations (hence on the allocation) by Assumption 2. In its absence (i.e. for a discontinuous allocation), an arbitrarily small price cut by firm i could potentially increase the firm's profit, by causing a large enough expected switch from brand j to brand i.

There is nothing pathological about the network structure used in the above example. So, this suggests that market segmentation of this kind can arise quite generally.

Notice though that if the brands are fully compatible ($\theta = 1$), then strong market segmentation is not possible in equilibrium. The reason is that at every node i, the magnitude of the network effect from consuming either good then equals $\sum_{s \in N(i,g)} (a_{s1}(p_1, p_2) + a_{s2}(p_1, p_2))$. So, the difference in hedonic prices at all nodes is simply the difference between p_1 and p_2. Thus a firm can undercut the price charged by its rival by an arbitrarily small amount and capture the entire market. So, the standard logic of Bertrand price competition holds and results in zero prices in equilibrium. We record this fact in the following proposition.

Proposition 1 *If $\theta = 1$, then there cannot be any strong market segmentation.*

In what follows, we analyze strong market segmentation when $\theta < 1$.

First, we show that there are types of network structures which *cannot* give rise to market segmentation. One such structure is when all customers are linked to each other, while a second is when the network structure is a *circle* with all nodes having the same mass of consumers, and $\theta \in (0, 1)$. Second, we derive sufficient conditions for the *star* and the *circle* to permit market segmentation.

Theorem 1 *: If (I, g) is a complete network, then there cannot be strong market segmentation.*

The idea behind this result is that in a complete network, the hedonic price of a brand is the same at every node. So both brands are purchased only if their hedonic prices are equal. This cannot be an equilibrium if brand prices are positive, because a firm can attract all customers of the rival brand with an arbitrarily small price cut. Since the complete network corresponds to the standard way of modeling network externalities a la Katz and Shapiro, Theorem 1 shows that price competition in such a model yields the standard Bertrand result. It may be useful to reiterate here the role of Assumption 1 in restricting the allocation: arbitrarily small price undercutting works because the allocation (or expectations) is nondecreasing in own price.

The following lemma will be used in the proof of the next theorem. It shows that in an equilibrium with strong market segmentation, for each brand j there is some node i where j is consumed and consumers get zero utility.

Lemma 2 *Suppose (p_1, p_2, a) is an equilibrium with strong market segmentation for some g. Then for each brand j, there exists a node i such that $a_{ij}(p_1, p_2) > 0$ and $h(p_j, a, i) = 0$.*

Lemmas 1 and 2 together display some of the structure that any equilibrium with strong market segmentation must have: both prices are positive; each brand is consumed at least at one node; at any node where a brand is purchased by consumers, the hedonic prices of the two brands are unequal (so that all consumers there buy the same brand); finally, for each brand, there is a node where consumers consume that brand and get zero utility.

The next theorem shows that under Assumption C, strong market segmentation cannot exist if the network is a circle in which all nodes have an equal mass of consumers *and* when the two brands are not completely incompatible. Recall that Example 1 demonstrated the possibility of market segmentation when the two brands are incompatible.

Theorem 2 *Suppose (I, g) is a circle such that all nodes have the same measure of consumers α, and $\theta \in (0, 1)$. Then there cannot be strong market segmentation if Assumption C is satisfied.*

Assumption C plays a crucial role in the theorem. If Assumption C does not hold, then even when the network is a symmetric circle and $\theta \in (0, 1)$, one can have market segmentation of the following kind. Let $p_1 = p_2 = \alpha$, and consumers at *odd* nodes abstain from consumption, consumers at nodes which are multiples of 4 consume brand 2, while consumers at all other even nodes consume brand 1.

The proof of the theorem illustrates an instance in which firm 1 finds *network tipping* to be profitable. Cutting its price to p'_1 attracts one node, and therefore the next, and the next, and so on; due to a network effect, the entire network switches to brand 1.

Note that even if $\theta > 0$, there can be market segmentation on the circle if masses of consumers differ across nodes. Example 2, which is presented after Theorem 3, illustrates this possibility.

In the remainder of this section, we derive sufficient conditions which ensure market segmentation for the circle and the star (or hub-spoke network). These and other simple networks have arisen as descriptions of economic and social relationships in a variety of contexts. The analysis below illustrates that strong market segmentation can arise quite easily in such contexts.

Both sufficient conditions (C^* and S^* below) have the following interpretation. Each firm i sells to a distinct set of submarkets or nodes I_i. The price p_i is such that for some submarket or node $k \in I_i$, the hedonic price for brand i is zero.[13] But firm i does not find it profitable to increase its price (thereby excluding some submarkets or nodes in I_i from consumption). Given these prices and the graph structure, firm i has some minimal, strictly positive price cut at which it can attract customers at nodes that consume the rival good, or abstain from consumption. Under the sufficient conditions, a firm's loss from its existing market due to this price cut exceeds the gain from attracting its rival's customers.

Consider a circle g on I. Let g' be any *nonempty* subgraph of the circle g. Nodes in $N(g')$ which have degree one in g' will be called *extremal* nodes. All other nodes in $N(g')$ are called *internal.* Notice that if g' is connected,

[13] If the hedonic price were negative at all nodes consuming brand i, firm i has a profitable deviation: it could increase price p_i slightly without changing the hedonic price inequalities at any node, and so without changing the quantity it sells.

then it must be a line, and will have exactly two extremal nodes. If k is an extremal node of g', then $n(k, g')$ refers to the neighbour of k which is in $N(g')$ while $e(k, g')$ refers to the node j such that $kj \in g \setminus g'$.

Suppose k is an extremal node of g'. Then, define

$$\Gamma_k(g') \equiv \alpha_k + \alpha_{n(k,g')} + \theta\alpha_{e(k,g')}$$

$\Gamma_k(g')$ equals the network benefit at extremal node k if all consumers in $N(g')$ use the same brand that is used at node k, and consumers at $e(k, g')$ use the rival brand.

If k is an internal node of g', then define

$$\Gamma_k(g') \equiv \sum_{j\in N(k,g')} \alpha_j$$

Let

$$\Gamma_m(g') \equiv \min_{k\in N(g')} \Gamma_k(g').$$

Notice that again for each internal node k, $\Gamma_k(g')$ represents the size of the network benefit under the expectation that consumers in $N(g')$ consume brand i. So, $\Gamma_m(g')$ is a *feasible* price in the sense that the corresponding hedonic price at every node in $N(g')$ is non-positive.

Then, Rev $(\Gamma_m(g')) = \Gamma_m(g') \sum_{k\in N(g')} \alpha_k$ is the revenue that any producer can derive from customers in $N(g')$ when the price is $\Gamma_m(g')$.

We will say that $\{L_1, L_2\}$ is a *line partition* of g if $\{L_1, L_2\}$ is a partition of g and each L_i is a line.

The sufficient condition for market segmentation when the network structure g is a circle results in firms 1 and 2 servicing nodes in $N(L_1), N(L_2)$, where $\{L_1, L_2\}$ is a line partition of g. The prices charged will be $\Gamma_m(L_1)$ and $\Gamma_m(L_2)$. Notice that at these prices, the hedonic price of each of the brands is zero at some node. These prices will also be "maximal" in the sense that neither firm will want to *raise* prices - the loss of revenue from a lower market size will be at least as high as the gain in revenue due to the higher price. The sufficient condition will additionally ensure that neither firm will want to lower price in order to steal customers away from its rival.

In order to define the sufficient condition, we also need to derive the largest price at (or alternatively, the smallest price cut by) which one firm can steal customers from its rival's market.

For each node k in $N(L_i)$, define

$$\begin{aligned} \rho_k &= \alpha_k + \alpha_{n(k,L_i)} - \alpha_{e(k,L_i)}, \text{ if } k \text{ is an extremal node of } L_i \\ \rho_k &= \Gamma_k(L_i), \text{ if } k \text{ is an internal node of } L_i \end{aligned}$$

We will show that the highest price at which firm j can attract some node in L_i is given by

$$\bar{p}_j = \Gamma_m(L_i) - (1-\theta) \min_{k \in N(L_i)} \rho_k$$

Intuitively, if node k consumes brand i, then to attract a consumer there, the price p_j must be sufficiently lower than $\Gamma_m(L_i)$ to compensate for the loss of network benefit. This loss equals $(1-\theta)\rho_k$, for node k. At price $\bar{p}_j$, there is compensation for the loss of network benefit at a node that has the smallest such benefit.

Definition 6 *A circle g on I satisfies Condition C^* if there exists a line partition $\{L_1, L_2\}$ of g such that for each $i = 1, 2$, and all $g' \subset L_i$,*

(i) Rev $(\Gamma_m(L_i)) \geq$ *Rev* $(\Gamma_m(g')) + \Gamma_m(g') \sum_{k \in K(L_i, g')} \alpha_k$, *where* $K(L_i, g') = \{k \in N(L_i) \setminus N(g') | \Gamma_m(g') \leq \alpha_k + \theta(\alpha_{k-1} + \alpha_{k+1})\}$

(ii) $\bar{p}_j \sum_{k \in N(g)} \alpha_k \leq \Gamma_m(L_j) \sum_{k \in N(L_j)} \alpha_k, \quad j = 1, 2.$

Part (i) ensures that firm i does not want to raise price beyond $\Gamma_m(L_i)$. Doing so would imply serving a smaller market (corresponding to some subgraph g' of L_i, and possibly other nodes in L_i where the hedonic price of brand i is non-positive). The two terms on the right hand side of Part (i) of Condition C^* correspond to revenue or profit from these, with price equal to $\Gamma_m(g')$. Part (ii) ensures that price cutting to attract consumers of the rival brand is not profitable relative to charging the price $\Gamma_m(L_j)$.

Theorem 3 *Suppose a circle g satisfies Condition C^*. Then, there can be strong market segmentation under some admissible allocation a.*

The theorem yields the following corollary.

Corollary 1 *Suppose $\theta = 0$, I contains an even number of nodes, with $\alpha_i = \alpha$. Then, every circle g on I can give rise to strong market segmentation.*

The next example is another application of Theorem 3 - it shows that market segmentation can prevail on the circle even when $\theta > 0$.

Example 2 *Let $I = \{1, ..., 5\}$, and consider the circle $g = \{12, 23, 34, 45, 51\}$. Let $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5) = (3, 4, 2, 4, 1)$.*

Let $\{L_1, L_2\}$ be the line partition in which $N(L_1) = \{1, 2\}$ and $N(L_2) = \{3, 4, 5\}$. Then, $p_1^* = \Gamma_m(L_1) = 7 + \theta$, $p_2^* = \Gamma_m(L_2) = 5 + 3\theta$, $\bar{p_1} = 3 + 5\theta$, $\bar{p}_2 = 2 + 6\theta$. Let a^* be an admissible allocation such that at prices (p_1^*, p_2^*), all consumers at nodes 1 and 2 consume brand 1, while all consumers at nodes 3, 4 and 5 consume brand 2. It is straightforward to check that if $\theta \in [0, \frac{1}{9}]$, then Condition C^* holds. Thus, strong market segmentation can hold.

We now derive a sufficient condition for strong market segmentation when the network structure is a star. This structure arises naturally in many applications. In the present context, one example of a star network is that of lawyers and client firms: lawyers are located at the center, their clients at the peripheral nodes. Lawyers interact with themselves and all clients, while each client firm interacts within itself and with its law firm.

Without loss of generality, we consider a star g with n as the hub or center. Let $\{M_1, M_2\}$ denote a partition of the peripheral nodes $I - \{n\}$. For each $j \in M_i$, denote $A_j = \sum_{\{k \in M_i | \alpha_k \geq \alpha_j\}} \alpha_k$. Let

$$\alpha_{1^*} \in argmax\{(\alpha_n + \alpha_i)(\alpha_n + A_i) | i \in M_1\}$$

and

$$\alpha_{2^*} \in argmax\{(\theta\alpha_n + \alpha_i)A_i | i \in M_2\}$$

The idea is to look for prices and allocation (p_1^*, p_2^*, a^*) such that the Center will consumer brand 1, and nodes in set M_j will either consume brand j or abstain from consumption. The network benefit at a node i from consuming brand 1 is thus $\alpha_n + \alpha_i$, while from consuming brand 2 it is $\theta\alpha_n + \alpha_i$. We require 2 peripheral nodes, 1^* and 2^*, at which the hedonic prices of the 2 brands are zero. (This will also pin down the 2 prices, $p_1^* = \alpha_n + \alpha_{1^*}$, $p_2^* = \theta\alpha_n + \alpha_{2^*}$).Node i^* is chosen to maximize profits of firm i, subject to the constraint that firm i services nodes in the set M_i (and the Center, if $i = 1$). The choice of 1^* and 2^* will ensure that no firm has an incentive to raise prices.

Definition 7 *The star g satisfies Condition S^* if the following are satisfied for some partition $\{M_1, M_2\}$ of $I - \{n\}$*

(i) $(A_{2^*} - A_{1^*})(1-\theta) + \alpha_n + \alpha_{1^*} \leq 0$

(ii) $(\alpha_{1^*} + \alpha_n)(\alpha_n + A_{1^*}) \geq (\alpha_n + \theta\alpha_{2^*}) \sum_{i \in I} \alpha_i$

Part (i) of Condition S^* will ensure that it is rational for the Center to consumer brand 1 at the equilibrium price vector, and in fact, that we can find a rational allocation under which a consumer at the Center will not switch to brand 2 at any positive price p_2. Part (ii) of Condition S^* will ensure that it is rational for nodes consuming brand 2 to do so at the equilibrium price vector, and also that firm 1 cannot attract nodes consuming brand 2 without reducing its equilibrium profits.

Theorem 4 *Suppose the star g with hub n satisfies Condition S^*. Then, there exists an equilibrium with strong market segmentation.*

An easy corollary is the following.

Corollary 2 *Suppose $\theta = 0$, I has at least 4 nodes, and $\alpha_i = \alpha$ for all i. Then, for every star on I, there is some equilibrium with strong market segmentation.*

In the lawyers-clients example, this result can be interpreted to say that there are equilibria with market segmentation in which lawyers (at the center of the star) and some clients use WordPerfect, even if a large number of other clients are using Word.

4 Discussion

In many instances, network effects can be *local* in nature. The present paper is one of the first to analyze competition in the presence of local network effects. We have shown that even under Bertrand competition with unlimited capacity, firms can make positive profits in many scenarios, and different groups of consumers can specialize in the consumption of one specific brand. However, the interaction structure matters; some network structures rule out such equilibria with strong market segmentation.

The degree of compatibility also matters. We have shown that when the brands are *perfectly compatible*, the only possible equilibrium is the standard Bertrand equilibrium with both firms earning zero profit, *irrespective* of the network structure. However, *partial compatibility* can generate market segmentation with both firms earning positive profits.

We have taken the degree of compatibility between brands to be exogenously given. There is an obvious implication, though, if the choice of compatibility is *endogenous* and restricted to being either 0 (incompatibility) or 1 (full compatibility). Consider a network structure that permits equilibria with strong market segmentation when brands are incompatible. Suppose that before the firms compete in prices, they decide whether or not to make their brands compatible with each other, say, by providing a two way converter. Assume that if both play "Yes", then the brands are compatible, whereas if at least one plays "No", they are incompatible. Following this, there is price competition. If both play "Yes", price competition leads to zero profits. This is not an equilibrium, since if even a single firm plays "No", the firms can then coordinate on a positive profit, strong market segmentation equilibrium. This provides a justification for observing the existence of incompatible brands, even under price competition with unlimited capacities, and no differences in intrinsic product quality.

What will happen if consumers become "more connected"? The addition of a link or edge to a graph increases the network effect at least on the nodes that are incident on the new edge. However, the greater scope for network externalities does not necessarily result in higher consumer surplus. The following example illustrates that consumer surplus may actually go down.

Example 3 *Let I contain more than 5 nodes and $\alpha_i = \alpha$ for all $i \in I$. Let $\theta = 0$. Consider a star, where the hub and $(|I|-2)$ peripheral nodes consume brand 1, while 1 peripheral node consumes 2. Consider the equilibrium with market segmentation where $p_1 = 2\alpha$, while $p_2 = \alpha$. Then, only consumers at the hub enjoy positive utility. Suppose now that one of the peripheral nodes where brand 1 was being consumed "merges" with the hub -this is equivalent to this node being connected to all other nodes. Then, there is a new equilibrium where $p_1 = 3\alpha, p_2 = \alpha$. Then, aggregate consumer surplus goes down, while producer 1's profit goes up.*

It is difficult, however, to draw any general conclusions about the direction of changes in consumer and producer surpluses even for small changes in network structure.

The model in this paper is static. Purchases or adoption of goods across consumers usually happens over time, and this feature can be particularly important when there are potential bandwagon effects (such as in the setting of this paper). While there is a literature studying dynamic pricing issues, we know of no such work that studies situations with local network externalities. This would be an interesting topic for future research.

Lastly, we have assumed that the network structure is exogenous, in contrast to a strand of the recent literature on networks which models the endogenous formation of networks. The endogenous formation of networks does not appear to be an appropriate issue in the present context. One's choice of coauthors, for example, usually does not depend on what software they use.

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5 Appendix

We gather all proofs in this section.

Proof of Lemma 1: Suppose (p_1, p_2, a) is an equilibrium, and the two hedonic prices are equal at node i. Without loss of generality, let $a_{i1}(p_1, p_2) > 0$ and $p_2 > 0$. Suppose firm 2 lowers its price to $p_2' = p_2 - \epsilon$. Since a is admissible, $a_{i1}(p_1, p_2') \leq a_{i1}(p_1, p_2)$ and $a_{i2}(p_1, p_2') \geq a_{i2}(p_1, p_2)$. Moreover, for all nodes $s \in I$ at which consumers consume either good, $a_{s2}(p_1, p_2') + \theta a_{s1}(p_1, p_2') \geq a_{s2}(p_1, p_2) + \theta a_{s1}(p_1, p_2)$. Since $p_2' < p_2$, the hedonic price of brand 2 is lower than that of brand 1 at node i for all permissible values of $a_{i1}(p_1, p_2')$. Since a is admissible and hence rational, it must be the case that $a_{i1}(p_1, p_2) = 0$ and $a_{i2}(p_1, p_2') = \alpha_i$.[14] So, firm 2 can capture the entire market at node i by a small reduction in price. This increases profit by $a_{i1}(p_1, p_2)(p_2 - \epsilon)$. The loss of profit at other nodes can be made arbitrarily small by choosing an appropriately small ϵ.

Hence, firm 2 cannot be maximizing profit at (p_1, p_2). This contradiction establishes the result.

Proof of Theorem 1: Suppose to the contrary that an equilibrium with strong market segmentation exists. Let (p_1, p_2) be the equilibrium prices. Since (I, g) is complete, the hedonic price of each brand is the same at all nodes. So, let (h_1, h_2) denote the hedonic prices corresponding to (p_1, p_2). Consider any node i where consumers buy brand 1. Rationality requires that $h_1 \leq h_2$. Similarly, by considering any node j where consumers buy only brand 2, we get $h_2 \leq h_1$.

Hence, $h_1 = h_2$. But,this contradicts Lemma 1.

Proof of Lemma 2: Suppose that for every node i with $a_{ij}(p_1, p_2) > 0$, we have $h(p_j, a, i) < 0$. Since there is strong market segmentation, $p_j > 0$. Therefore, by Lemma 1, $h(p_j, a, i) < h(p_k, a, i)$.

By continuity of a, firm j can raise price p_j slightly - admissibility of a ensure that consumption of brand j at each node remains as before. So, firm $j's$ profit is higher. This contradicts the assumption that (p_1, p_2, a) is an equilibrium.

[14] The latter follows because consumers at node i were purchasing at prices (p_1, p_2), and so had non-negative utility. Hence, they must be purchasing at price p_2' since the hedonic price of brand 2 is now lower.

Proof of Theorem 2: Suppose to the contrary that (p_1, p_2, a) is an equilibrium with strong market segmentation. If $|I| \leq 3$, this is ruled out by Theorem 1. So let $|I| > 3$. We proceed in steps.

Step 1: At all nodes i, either $a_{i1}(p_1, p_2) = \alpha$ or $a_{i2}(p_1, p_2) = \alpha$. That is, all consumers at each node buy one of the two brands.

Proof of Step 1: Since both prices are positive, lemma 1 implies that the hedonic prices of the two brands at each node are unequal. So, consumers at each node will completely specialize in one brand if they buy at all. Hence, we only need to prove that no consumer abstains from consumption.

Since there is market segmentation, there must be some node i where all consumers buy say brand 1. We want to show that no consumer at node $(i-1)$ abstains from consumption. In an equilibrium with strong market segmentation, either all consumers at node $(i-1)$ buy the same brand, or no consumer at this node buys either brand. Suppose the latter is true. Now, at node $(i+1)$, either consumers purchase brand 1 or brand 2 or neither. So at node i, the hedonic price $h(p_1, a, i)$ of brand 1 is either $p_1 - 2\alpha$, or $p_1 - (1+\theta)\alpha$, or $p_1 - \alpha$. From rationality at node i, therefore,

$$p_1 - 2\alpha \leq 0$$

Notice that if firm 1 lowers price to $p_1^{'}$, an arbitrarily small reduction from p_1, then $p_1^{'} - 2\alpha < 0$. By Assumption C, no consumer at node $(i-1)$ will then abstain from consumption; indeed, given that at (p_1, p_2) no one at this node consumed, at $(p_1^{'}, p_2)$ they will all purchase brand 1. Because firm $1's$ price reduction can be arbitrarily small, it can increase its profit by capturing this node in this fashion. This contradicts the assumption that (p_1, p_2, a) is an equilibrium. We have thus shown that consumers at $(i-1)$ must be buying *some* brand.

Step 2: If brand j is consumed at node i, then it is consumed at either node $(i-1)$ or node $(i+1)$.

Proof of Step 2: From Step 1, we know that consumers at nodes $i-1$ and $i+1$ consume one of the two brands. If Step 2 is wrong, then brand k must be consumed at nodes $(i-1)$ and $(i+1)$. By rationality at i, we have

$$p_j - \alpha - 2\theta\alpha < p_k - 2\alpha - \theta\alpha \tag{1}$$

The smallest possible hedonic price of brand k at node $(i-1)$ is $p_k - 2\alpha - \theta\alpha$ - this happens when consumers at $(i-2)$ consume k. The largest possible hedonic price of brand j at $(i-1)$ is $p_j - \alpha - 2\theta\alpha$. Equation (1) shows that the hedonic price of brand k is higher than the hedonic price of brand j at node $i-1$. This implies that rationality is violated at node $(i-1)$.

Step 3: $p_1 = p_2 = (2+\theta)\alpha$.

Proof of Step 3: Since g is a circle, Steps 1 and 2 imply that there exist nodes i and $(i+1)$ such that consumers at nodes i and $(i-1)$ consume brand j, while consumers at nodes $(i+1)$ and $(i+2)$ consume brand k. We will call nodes i and $i+1$ *marginal* nodes.[15] So, the hedonic prices of brands j and k at nodes i and $(i+1)$ respectively are $p_j - 2\alpha - \theta\alpha$ and $p_k - 2\alpha - \theta\alpha$. Also, if brand j is consumed at some node q, then its hedonic price at q cannot exceed $p_j - 2\alpha - \theta\alpha$.[16] Lemma 2 now completes the proof of Step 3.

Step 4: Suppose N_1 is the set of nodes where brand 1 is consumed. Without loss of generality, let $\#N_1 = n_1 \leq \frac{n}{2}$. Firm 1's profit is

$$\pi_1(p_1, p_2) = p_1 n_1 \alpha = n_1(2+\theta)\alpha^2 \leq \frac{n}{2}(2+\theta)\alpha^2$$

Let firm 1 lower price to $p_1' = (1+2\theta)\alpha - \epsilon$. Let i be any marginal node for brand 2 at prices (p_1, p_2). The hedonic price of brand 1 at i corresponding to p_1' is now $-\epsilon$. Hence, all consumers at i switch over to brand 1. Now, suppose consumers at $i+1$ were consuming brand 2 at (p_1, p_2), and $i+1$ was not a marginal node for brand 2. Since $a_{i1}(p_1', p_2) = \alpha$, the hedonic price of 1 at $i+1$ corresponding to p_1' is also $-\epsilon$. So, all consumers at $i+1$ must also switch to brand 1.

Continuing in this way, it is clear that at (p_1', p_2), firm 1 captures the entire market. Its profit is now

$$\pi_1(p_1', p_2) = ((1+2\theta)\alpha - \epsilon)n\alpha = \frac{n}{2}(2+4\theta)\alpha^2 - n\alpha\epsilon$$

Firm 1 can choose ϵ sufficiently small so that $\pi_1(p_1', p_2) > \pi_1(p_1, p_2)$.

Hence, (p_1, p_2, a) cannot be an equilibrium. This completes the proof of the theorem.

Proof of Theorem 3: Let g satisfy Condition C^*. Consider the pair (L_1, L_2) figuring in the definition of Condition C^*. Let $p_i^* \equiv \Gamma_m(L_i)$ be the price of good i.

Also, let a^* be an allocation which satisfies the following restrictions.

For $i = 1, 2$, $j \neq i$, for all *internal* nodes k of L_i,

$$\Gamma_m(L_i) - p_j \leq (1-\theta)\Gamma_k(L_i) \rightarrow a_{ki}^*(\Gamma_m(L_i), p_j) = \alpha_k \tag{2}$$

[15] That is, a node is marginal if consumers at one of its neighbors consume a different brand.

[16] It could be $p_j - 3\alpha$ if j is consumed at both nodes $(q-1)$ and $(q+1)$.

For $i = 1, 2$, $j \neq i$, for all *external* nodes k of L_i,

$$\Gamma_m(L_i) - p_j \leq (1-\theta)(\alpha_k + \alpha_{n(k,L_i)} - \alpha_{e(k,L_i)}) \rightarrow a^*_{ki}(\Gamma_m(L_i), p_j) = \alpha_k \quad (3)$$

These restrictions are easily interpreted. Notice that if k is an internal node of L_i, then $\theta\Gamma_k(L_i)$ is the network benefit that an individual located at node k will get from consuming brand j if all other individuals located at nodes in $N(L_i)$ consume brand i. So, $(1-\theta)\Gamma_k(L_i)$ is the gain in network benefit from consuming brand i instead of brand j. Equation 2 states that if the difference between p^*_i and p_j is smaller than the gain in network benefit, then all consumers at node k buy brand i. Equation 3 has an exactly similar interpretation for external nodes.

We first show that a^* is an admissible allocation. By definition, p^*_i ensures that $h(p^*_i, a^*, k)$ is non-positive at all nodes k. We first show that $h(p^*_i, a^*, k) < h(p^*_j, a^*, k)$ for all nodes $k \in N(L_i)$.

Suppose k is an internal node of L_i. It follows straightaway from (ii) of Condition C^* that $\Gamma_m(L_j) > \bar{p}_j$. Then,

$$\begin{aligned} h(p^*_j, a^*, k) &= \Gamma_m(L_j) - \theta\Gamma_k \\ &> \bar{p}_j - \theta\Gamma_k \\ &\geq \Gamma_m(L_i) - (1-\theta)\Gamma_k - \theta\Gamma_k \\ &= h(\Gamma_m(L_i), a^*, k) \end{aligned}$$

Suppose k is an extremal node of L_i. Then,

$$\begin{aligned} h(p^*_j, a^*, k) &= \Gamma_m(L_j) - \alpha_{e(k,L_i)} - \theta(\alpha_k + \alpha_{n(k,L_i)}) \\ &> \bar{p}_j - \alpha_{e(k,L_i)} - \theta(\alpha_k + \alpha_{n(k,L_i)}) \\ &\geq \Gamma_m(L_i) - (1-\theta)(\alpha_k + \alpha_{n(k,L_i)} - \alpha_{e(k,L_i)}) - \alpha_{e(k,L_i)} - \theta(\alpha_k + \alpha_{k-1}) \\ &= h(\Gamma_m(L_i), a^*, k) \end{aligned}$$

That is, $h(p^*_i, a^*, k) < h(p^*_j, a^*, k)$ for all nodes $k \in N(L_i)$, and so no consumer located at nodes in $N(L_i)$ wants to switch to consumption of good j. So, a^* satisfies Rationality.

From Assumption 1, an increase in price of good i cannot attract consumers located at nodes in $N(L_j)$. Any $p_i > p^*_i$ will also imply that the resulting hedonic price of good i will be strictly positive at some node k in $N(L_i)$. Hence, a price increase will result in a smaller market size. Let p_i correspond to some $\Gamma_m(g')$ where g' is a subgraph of L_i. Then, $N(g') \cup K(L_i, g')$ is the set of nodes in $N(L_i)$ where the hedonic price is non-positive. Part (ii) of Condition C^* ensures that the revenue corresponding to p_i is not higher than the revenue corresponding to p^*_i.

Hence, neither firm has any incentive to raise the price.

Finally, we show that neither firm has an incentive to reduce price. We demonstrate this for firm j.

We now show that $\bar{p}_j$ is the *maximum* price at which firm j can steal some node in L_i. From equation 2, it is clear that if p_j exceeds $\bar{p}_j$, then all customers at any internal node k of $N(L_i)$ will continue to consume brand j. Equation 3 implies the same for external nodes of $N(L_i)$.

Suppose indeed that firm j can capture the entire market at price $\bar{p}_j$. Part (ii) of Condition C^* ensures that firm j's profit still does not exceed its current profit.

This completes the proof of the theorem.

Proof of Corollary 1: Consider a partition of g into lines $\{L_1, L_2\}$ such that each $N(L_i)$ consists of $\frac{n}{2}$ nodes. Then, $\Gamma_m(L_i) = 2\alpha$, and $\Gamma_m(L_i)$ is maximal for L_i. (Note that the network benefit at an extremal node is 2α, so the hedonic price of the brand i consumed there is zero. So, raising price p_i beyond $\Gamma_m(L_i) = 2\alpha$ will lose the extremal node. But then, the network benefit at the neighboring node consuming brand i reduces from 3α to 2α, and is less than any price p_i higher than $\Gamma_m(L_i)$. Carrying this argument further, we see that raising price p_i beyond $\Gamma_m(L_i)$ loses all nodes consuming brand i). Hence, (i) of Condition C^* is satisfied. To check (ii), let i^* be an extremal node of L_i. To attract a consumer at i^* to consume brand j, the price p_j must drop sufficiently to compensate for the loss of network benefit, which equals α. Thus $\bar{p_j} = \Gamma_m(L_i) - \alpha = \alpha$. So, the inequality of (ii) reduces to

$$n(\alpha)^2 \leq 2\alpha(\frac{n}{2}\alpha),$$

which holds.

Notice, however, that if $\theta = 0$, $\alpha_i = \alpha$ for all $i \in I$, and I contains an *odd* number of nodes, then Condition C^* is not satisfied. In fact it can be shown that in this case, there is *no* equilibrium with strong market segmentation.

Proof of Theorem 4: Let $p_1^* = \alpha_n + \alpha_{1^*}$, and $p_2^* = \alpha_{2^*} + \theta\alpha_n$.

Consider an allocation a^* such that

$$a_{i1}^*(p_1^*, p_2^*) = \alpha_i \text{ for all } i \in \{j \in M_1 | \alpha_j \geq \alpha_{1^*}\} \cup \{n\}$$

$$a_{i2}^*(p_1^*, p_2^*) = \alpha_i \text{ for all } i \in \{j \in M_2 | \alpha_j \geq \alpha_{2^*}\}$$

and

$$a_{ij}^*(p_1^*, p_2^*) = 0 \text{ for all other nodes } i, \text{ for } j = 1, 2$$

We first check that this specification does not violate rationality.

Suppose i is a peripheral node where consumers consume brand 1. Then,

$$h(p_1^*, a^*, i) = \alpha_{1^*} - \alpha_i \leq 0 < h(p_2^*, a^*, i) = p_2^*$$

If i is a peripheral node consuming brand 2, then

$$h(p_2^*, a^*, i) = \alpha_{2^*} - \alpha_i \leq 0 < h(p_1^*, a^*, i) = \alpha_{1^*} - \theta\alpha_i$$

The last inequality follows from the fact that if peripheral node i consumes brand 2, at prices (p_1^*, p_2^*), then $\alpha_{2^*} - \alpha_i \leq 0$; so for the inequality to hold for all such nodes i, it is sufficient that $\alpha_{1^*} - \theta\alpha_{2^*} > 0$. But this follows from part (ii) of Condition S^*.

Also, for the Center n,

$$h(p_1^*, a^*, n) = (\alpha_n + \alpha_{1^*}) - A_{1^*} - \theta A_{2^*}$$

and

$$h(p_2^*, a^*, n) = (\alpha_{2^*} + \theta\alpha_n) - A_{2^*} - \theta A_{1^*}$$

Part (i) of Condition S^* ensures that $h(p_1^*, a^*, n) < h(p_2^*, a^*, n)$.

So, the specification of a^* does not violate rationality.

It is easy to check that neither firm has an incentive to raise prices.

Suppose also that a^* satisfies the following

$$a_{n1}(p_1^*, p_2) = \alpha_n \text{ if } (A_{2^*} - A_{1^*})(1 - \theta) + \alpha_n + \alpha_{1^*} \leq 0$$

This is consistent with rationality if all consumers at node n expect other consumers at node n to consume good 1 so long as the inequality is satisfied.

Then, it follows from part (i) of Condition S^* that the producer of brand 2 cannot induce consumers at node n to switch consumption at any positive price.

Finally, note that producer 1 must reduce price by at least $\alpha_{1^*} - \theta\alpha_{2^*}$ to induce nodes in M_2 to switch to consumption of good 1. The loss in revenue from existing customers is $(\alpha_{1^*} - \theta\alpha_{2^*})(\alpha_n + A_{1^*})$. The maximum possible gain in revenue occurs if *all* consumers currently not consuming 1 switch to consumption of 1. Hence, the maximum gain in revenue is $[(\alpha_n + \alpha_{1^*}) - (\alpha_{1^*} - \theta\alpha_{2^*})](\sum_{i \in I} \alpha_i - \alpha_n - A_{1^*})$. Part (ii) of Condition S^* ensures that the loss in revenue from existing customers is at least as large as the gain in revenue from new customers.

Hence, no producer has an incentive to reduce prices. So, (p_1^*, p_2^*, a^*) constitutes an equilibrium with market segmentation.

Proof of Corollary 2: Partition the peripheral nodes into sets $\{M_1, M_2\}$ such that $(|M_1| - 1) \geq |M_2|$. This can always be done as long as there are at least 3 peripheral nodes. Since $\alpha_i = \alpha$ for all i, $A_{1^*} = |M_1|\alpha$, $A_{2^*} = |M_2|\alpha$. Then, (i) of Condition S^* is satisfied. It also follows easily that (ii) of Condition S^* is satisfied.