Too Much Investment? A Problem of Endogenous Outside Options*

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Abstract

This paper shows that when agents on both sides of the market are heterogenous, varying in their costs of investment, ex ante investments by firms and workers (or buyers and sellers more generally) may be too high when followed by stochastic matching and bargaining over quasi-rents. The overinvestment is caused by the fact that low-cost agents, by investing more, can increase the value of their outside option and thus shift rent away from high-cost investors. Numerical simulations show that overinvestment can occur given parameter values calibrated to OECD labour markets.

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1. Introduction

According to Sicherman (1991), some 40% of US workers have more education than is needed for the job they do whereas 16% are underqualified (Vaizey (2006) reports similar figures).This finding might suggest that the private return to education is low. In fact, the opposite is the case. For example Carneiro and Heckman (2003) estimate that the average annual return to college for a randomly selected US citizen is 18.7%.

The standard reconciliation, following Spence (1974), is that education does not so much create human capital as signal otherwise unobservable productivity. This paper develops an alternative explanation. Since education widens employment opportunities, it increases bargaining power even in jobs for which it is unnecessary. For example, a firm may hire an available MBA graduate because finding a suitable candidate without an MBA involves delay. To make such an appointment the pay must reflect what the MBA is worth to firms for which it does have value. Acquiring an MBA is therefore privately beneficial, even in occupations for which it is unnecessary. As with signalling, there is a tendency to overinvestment from a social perspective, but the mechanism and specific implications differ.

Our focus is on the extensive human and physical capital investments that workers and firms make before they are matched. Becker (1964) provides the benchmark result that in a competitive economy investment in the acquisition of general skills will be socially optimal. More recently, search frictions have been introduced in models with *ex ante* investment. Acemoglu (1996, 1997), Acemoglu and Shimer (1999) and Masters (1998)[1] study stochastic matching with the quasi-rent from the match divided by *ex-post* bargaining The bargaining protocols studied in these papers allow for agents to have outside options, either in the form of a return to the search process or in switching between partners. Nevertheless, outside options are ineffective in "protecting" investors from the hold-up problem because they do not bind in equilibrium, or indeed for small deviations from equilibrium investments. So, any agent increasing their investment by a small enough increment only receives half the gross economic return, implying under-investment from a social viewpoint.[2]

In all these contributions, agents are assumed homogenous.[3] In this paper, we show that homogeneity is important for the under-investment result. If agents differ in investment costs, outside options may

[1]Masters (1998) claims that when the outside option takes the form of switching between partners, the outcome is efficient. de Meza and Lockwood (2006) show that there is an error in this argument: if switching is costless, as in Masters, there are a continuum of investment equilibria between the hold-up and efficient levels, and if there is any cost $\varepsilon > 0$ of switching, only the hold-up level of investment can be an equilibrium.

[2]A related partial-equilibrium literature initiated by Grossman and Hart (1986) and Hart and Moore (1990) studies investments that are to some degree relationship specific and undertaken post matching. In Hart (1995, p37), the marginal return to investment is plausibly assumed higher in the relationship for which it is designed than outside it. The result is under-investment for familiar hold up reasons. A further modification is to assume the nature of the investment is discretionary. For an agent with a binding outside option, partner-specific investment has no payoff whereas investment that enhances productivity with outsiders yields a private return but no social benefit. So if firms can choose the type of investment, as, for example, in Watson (2002, p178 Q2), they underinvest in partner-specific capital but overinvest in capital that will never be productively used. As the gross output of the economy is below the socially optimal level it seems reasonable to chracterise the outcome as underinvestment. In the model of this paper the gross output of the economy exceeds the socially optimal level.

[3]In Acemoglu(1996), one side of the market - the workers - are allowed to be heterogenous, but only as part of a comparative statics exercise in order to demonstrate social increasing returns. Specifically, all workers are initially assumed identical, then a fraction are assumed to have a reduction in their cost of investment in human capital. This induces firms to invest more, and thus with random matching, even workers whose cost of investment has not changed benefit from the greater investment by firms. But both at the initial and new equilibria, there is underinvestment.

become binding in equilibrium thereby doing more than "protecting" investors against hold-up: they may provide incentives for *over-investment*. This works as follows. We assume that a fraction of firms and workers have a low cost of investment (low-cost agents) and the complementary fraction have a high cost of investment (high-cost agents). Suppose further for simplicity that high-cost agents do not invest in equilibrium, because the cost of investment is too high. With sufficient search frictions, a low-cost agent will match with a high-cost agent (non-assortative matching). Assume further, for simplicity, that investments are perfectly complementary: so, an investment by the low-cost agent in a non-assortative match is completely unproductive. Nevertheless, an investment by the low-cost agent in such a match enhances his bargaining power by creating - or increasing the value of - a binding outside option, because the investment increases the value of a match with another low-cost agent who has also invested. This rent-transfer opportunity is privately profitable but not socially beneficial.

Note that over-investment requires "intermediate" match frictions.[4] For rent transfer to be relevant, match frictions must be high enough to ensure that non-assortative matches occur in equilibrium, but not so high that the outside option of a low-cost agent does not bind in these matches. For the case of a CES revenue function, we provide a complete characterization of the set of parameter values for which overinvestment can occur. Interestingly, overinvestment is possible for a range of match friction values consistent with average unemployment durations in OECD countries.[5] Numerical simulations also indicate that this set tends to be larger, the higher the proportion of low-cost agents, and the lower the returns to scale in the revenue function.

The empirical implications of the matching model and signalling theory overlap to some extent. According to both, education burns up real resources in the process of redistributing income, implying the private return is positive but the social return is negative. This is consistent with the well-known empirical finding that higher education yields substantial private returns, yet in cross-country studies, the effect on GNP is weak.[6]. In Section 4, it is argued that our model, in some respects, fits the facts better than does signalling theory.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the overinvestment results when the outside option principle applies. Section 4 discusses the empirical implications of the model, Section 5 discusses related literature, and Section 6 concludes.

2. The Model

2.1. Preliminaries

There are two types of agents: firms and workers. Both are infinitely lived. Time is discrete, with a period length of Δ, and runs infinitely forward and back, and all agents have a discount factor $\delta = e^{-r\Delta}$. The

[4] Friction must be intermediate only in the precise sense that for fixed values of the other parameters, and normalizing match frictions between zero and one, overinvestment occurs only for frictions in an open interval strictly in $[0, 1]$. Thus, overinvestment can occur when match friction is quite close to 1, as is the case for OECD labour markets.

[5] Specifically, we calibrate the match arrival rate over a month as in the inverse of unemployment duration for a range of OECD countries. Combined with an annual discount rate of 5%, this gives a range of values of the match friction parameter between 0.9 and 1.

[6] For example, Ashenfelter and Rouse (1998) find the private return to higher education in the US is of the order of 10% whereas Barro and Lee (1994) find that, unlike secondary education, higher education is not significant in explaining the variation in international growth rates.

following events occur in each period t. First, a measure π_i of both workers and firms of type $i = h, l$ enter the pool of unmatched, with $\pi_h + \pi_l = 1$. On entering each agent chooses how much to invest. A type i agent has investment cost $c_i(e) = c_i e$, $i = h, l$ and $c_h > c_l$, all e. So, $h-$types have a higher cost of investment than $l-$types.

Then, a fraction $0 < \Delta a < 1$ of the measure of as yet unmatched firms and workers, are randomly matched with each other.[7] That is, every worker is matched with a firm (and vice versa) with probability Δa.[8] A firm which has invested e_w and a worker which has invested e_f can, once matched, produce present value of revenue of $y(e_w, e_f)$. If both firm and worker are matched, they decide simultaneously and independently whether to accept or reject the match. Should one or both reject, then nothing further happens to these agents until the next period.

If they both accept, both permanently exit the pool of unmatched, and start production. Rather than model bargaining over revenue explicitly, we just assume the outside option principle (Osborne and Rubinstein(1990)) i.e. the $y(e_w, e_f)$ is equally divided unless half of this quantity is strictly less than the payoff to continued search (say v) for one of the two parties, in which case that party gets v and the other the residual $y(e_w, e_f) - v$. Such a division can be shown to be an equilibrium of an explicit alternating offers bargaining game between the worker and the firm, where the responder has the option of returning to the pool of the unmatched[9].

2.2. The Revenue Function

We assume that $y(e_w, e_f)$ is non-negative, symmetric, and strictly increasing and strictly concave and twice differentiable for all $(e_w, e_f) \in \Re^2_+$, with y_i denoting the derivative with resect to the *ith* argument. Moreover, we will assume that y is supermodular: given differentiability, this is just the condition $y_{12} > 0$. That is, the inputs are *strict complements*. One class of functions that satisfies all of these assumptions is the symmetric CES revenue function

$$y(e_w, e_f) = (0.5e_w^\rho + 0.5e_f^\rho)^{\alpha/\rho} + y_0, \ \rho < 1, \ 1 > \alpha > \rho \tag{2.1}$$

Note that if $\rho \leq 0$, inputs are essential i.e. $y(0, e) = y(e, 0) = y_0$, all e. Specifically, $\rho = 0$ is the Cobb-Douglas case $y(e_w, e_f) = e_w^{\alpha/2} e_f^{\alpha/2} + y_0$.

2.3. Strategies and Equilibrium

Section 2.2 above describes a simple stochastic game played at periods $t = -2, 1, 0, 1, 2...$ by a continuum of players. We focus on *symmetric steady-state Markov-perfect equilibrium.* By symmetry, we mean an agent of a given cost type behaves in the same way, irrespective of whether he is firm or worker. The Markov property is simply that agents condition their actions only on payoff-relevant state variables. The steady-state assumption says that in equilibrium, inflows to the pool of unmatched equal outflows.

[7] As any firm must exit matched with a worker, in any period, the measures of firms and workers in the unmatched state are the same.

[8] For concreteness, think of a two-stage matching process where measure Δa agents on either side of the market are randomly selected from the pool of the unmatched, and then these Δa workers and firms are randomly matched with each other. The existence of such procedure even with a continuum on each side of the market is guaranteed by the arguments of Alos-Ferrer (2002).

[9] See an earlier version of this paper, De Meza and Lockwood(2004).

Under these assumptions, if a firm (or worker) of cost type $i = l, h$ enters the market at t, he invests e_i^* in equilibrium, independently of t. So, if a firm f and a worker w are matched at the beginning of period t, the only payoff-relevant variables for this pair are (i) their two investment levels e_w, e_f : (ii) the distribution of equilibrium investments across all as yet unmatched agents, which is characterized by (e_h^*, e_l^*). Perfection, or sequential rationality, implies that an agent accepts a match at any date iff doing so gives a higher payoff than continued search.

3. Overinvestment Results

To generate conditions under which overinvestment occurs, we construct a (symmetric steady-state Markov) equilibrium where (i) matching is *non-assortative* (NAM) i.e. where an $l-$type firm accepts a match with an $h-$type worker and vice versa and (ii) where, when an $h-$type matches with an $l-$type, the outside option of the $l-$type binds. Call such an equilibrium a *N-B equilibrium*. Non-assortative matching is required in equilibrium for over-investment to occur, for reasons discussed in the introduction.

3.1. The N-B Equilibrium

In an N-B equilibrium, the present value expected payoffs to continued search for the two types, denoted v_h, v_l, satisfy the following dynamic programming equations in the limit as $\Delta \to 0$:

$$
\begin{aligned}
rv_l &= a\pi_h(v_l - v_l) + a\pi_l(\frac{y(e_l, e_l)}{2} - v_l) && (3.1)\\
rv_h &= a\pi_h(\frac{y(e_h, e_h)}{2} - v_h) + a\pi_l(y(e_h, e_l) - v_l - v_h) && (3.2)
\end{aligned}
$$

where π_l, π_h are the shares of low- cost and high-cost agents in the pool of unmatched. Note that due to non-assortative matching, these are the same as the shares of the two types that enter the pool of the unmatched over any interval.

The first equation (3.1) follows because when matched with an $h-$type (which occurs with probability $\Delta a\pi_h$) the $l-$type's outside option binds, so he gets payoff v_l, just the value of being in the unmatched pool. The second equation (3.2) follows because when matched with an $l-$type (which occurs with probability $\Delta a\pi_l$) the $h-$type is residual claimant. Solving (3.1), (3.2), we get

$$
\begin{aligned}
v_l &= \phi_l \frac{y(e_l, e_l)}{2} && (3.3)\\
v_h &= \phi[\pi_h \frac{y(e_h, e_h)}{2} + \pi_l(y(e_h, e_l) - \phi_l \frac{y(e_l, e_l)}{2})] && (3.4)
\end{aligned}
$$

where $\phi = \frac{a}{r+a}$, $\phi_l = \frac{a\pi_l}{r+a\pi_l}$. Finally, a binding outside option and NAM respectively require:

$$
\begin{aligned}
v_l &> \frac{y(e_h, e_l)}{2} && (3.5)\\
y(e_h, e_l) &\geq v_h + v_l && (3.6)
\end{aligned}
$$

So, given investments, e_h, e_l, (3.3)-(3.6) fully characterize the N-B equilibrium.

It remains to find the equilibrium investments. Suppose that an individual l agent deviates by a small amount from equilibrium investment e_l^* to e'. Then, as his outside option continues to bind in a match with an $h-$type (for a small enough deviation), his payoff net of investment costs is

$$\phi_l \frac{y(e', e_l^*)}{2} - c_l e' \tag{3.7}$$

So, the equilibrium investment must maximize this expression i.e.

$$\frac{\phi_l}{2} y_1(e_l^*, e_l^*) \ = c_l \tag{3.8}$$

where y_1 denotes the first derivative of y. By the same argument, if an individual h agent deviates by a small amount from equilibrium investment e_h^* to e', he is still residual claimant in a match with an $h-$type (for a small enough deviation), so his payoff net of investment costs is

$$\phi[\pi_h \frac{y(e', e_h^*)}{2} + \pi_l(y(e', e_l^*) - \phi_l \frac{y(e_l^*, e_l^*)}{2})] - c_h e' \tag{3.9}$$

So, the equilibrium investment must maximize this expression i.e.

$$\frac{\phi \pi_h}{2} y_1(e_h^*, e_h^*) + \phi \pi_l y_1(e_h^*, e_l^*) = c_h \tag{3.10}$$

Equations (3.8),(3.10) are thus the first-order necessary conditions for equilibrium investments.

However, some discussion of sufficient conditions is required. By assumption, y is strictly concave in investments, so this might appear to ensure that (3.8),(3.10) are also sufficient. But, there is the additional complication that large deviations in e away from the equilibrium level can cause the "regime" facing the deviant to change e.g. whether or not he faces a binding outside option in a given kind of match. For example, if the $l-$type chooses an e' sufficiently below e_l^*, he will face first a binding outside option in a match with another l, then as e' falls further, he will face a binding outside option in a match with an h-type, $l-$types will reject a match with the deviant, etc. But, as all these changes make the deviant worse off, his payoff to (downward) deviation must be bounded above by (3.7). So, if e_l^* maximizes (3.7), it must certainly be a global maximum for the $l-$type. A similar argument implies that an $h-$type's payoff to (upward) deviation must be bounded above by (3.9). So, if e_h^* maximizes (3.9), it must certainly be a global maximum for the $h-$type[10]. So, to conclude, the N-B equilibrium is fully characterized by (3.3),(3.4),(3.5),(3.6), (3.8),(3.10).

3.2. Overinvestment

As payoffs are linear in consumption, the natural efficiency criterion is the sum of the payoffs to search net of investment costs at some levels of investment for each type e_l, e_h (aggregate surplus). In N-B equilibrium, aggregate surplus can be written as:

$$\begin{aligned} W(e_h, e_l) &= \pi_h v_h + \pi_l v_l - \pi_l c_l e_l^* - \pi_h c_h e_h^* \qquad (3.11) \\ &= \pi_h \phi \left[\pi_h \frac{y(e_h^*, e_h^*)}{2} + \pi_l y(e_h^*, e_l^*) \right] + \pi_l \phi_l (1 - \pi_h \phi) \frac{y(e_l^*, e_l^*)}{2} - \pi_l c_l e_l^* - \pi_h c_h e_h^* \end{aligned}$$

where in the second line, we have used (3.3),(3.4). Differentiating (3.11), and collecting terms:

$$\begin{aligned} \frac{1}{\pi_l} \frac{\partial W(e_h^*, e_l^*)}{\partial e_l} &= \pi_h \phi y_2(e_h^*, e_l^*) + (1 - \pi_h \phi) \phi_l y_1(e_l^*, e_l^*) - c_l \qquad (3.12) \\ &= \pi_h \phi y_2(e_h^*, e_l^*) + \phi_l [0.5 - \pi_h \phi] y_1(e_l^*, e_l^*) \end{aligned}$$

[10] These claims are more formally proved in an online Appendix available at http://www2.warwick.ac.uk/fac/soc/economics/staff/academic/lockwood.

where in the second line, we have used (3.8). So, investment of the $l-$types is locally too high if the term on the RHS of the second line of (3.12) is negative. Inspection of this term, using $y_1(e_l^*, e_h^*) = y_2(e_h^*, e_l^*)$ from symmetry of the revenue function, gives:

Proposition 1. *In N-B equilibrium, e_l^* is locally too high i.e. $\frac{\partial W(e_h^*, e_l^*)}{\partial e_l} < 0$, iff (i) $\pi_h \phi > 0.5$; (ii) investments are sufficiently complementary i.e.*

$$\frac{y_1(e_l^*, e_l^*)}{y_1(e_l^*, e_h^*)} > \frac{\pi_h \phi}{\phi_l[\pi_h \phi - 0.5]} \tag{3.13}$$

Condition $\pi_h \phi > 0.5$ is intuitive. First, it is more likely to hold, the higher π_h, as the higher π_h, the higher the negative fiscal externality imposed on the $h-$types. Second, it is more likely to hold the higher ϕ, as the higher ϕ, the more likely are *hl* matches.

Condition (3.13) can be explained and interpreted as follows. First, the RHS of the inequality in (3.13) is by definition, strictly greater than 1 so (3.13) requires $y_1(e_l^*, e_l^*) > y_1(e_l^*, e_h^*)$. Second, it is easy to show that $e_l^* > e_h^*$ in equilibrium[11]. These two facts imply that some complementarity in investments is necessary for overinvestment. The reason is that we know that for the standard holdup reason there is too little investment in an *ll* match so overinvestment requires that the marginal product of e_l is lower in an *hl* match. As the only difference between the two types of match is that e_h is lower, the complementarity requirement follows.

So, generally, an overinvestment equilibrium will exist if (i) all the conditions for N-B equilibrium exist, *and* (ii) the conditions in Proposition 1 hold. We now wish to obtain conditions in terms of underlying model parameters for which these conditions hold simultaneously. To do this, some simplifying assumptions are required. We make two such assumptions.

A1. $h-$types face a cost of investment that is prohibitively costly i.e. $c_h = \infty$.

A2. (2.1) holds with $\rho \leq 0$.

The main simplification with A1, A2 is the following. First, A1 implies $e_h^* = 0$, and A2 then implies that $y_1(e_l^*, e_h^*) = y_1(e_l^*, 0) = 0$, as $y(e, 0) \equiv y_0$, all e. Thus, (3.13) in Proposition 1 automatically holds as long as $\pi_h \phi > 0.5$. Moreover, conditions (3.5)-(3.6) reduce to the condition that y_0 in (2.1) lie in a certain interval. We can thus prove (see Appendix):

Proposition 2. *Assume A1, A2. Then, if $\pi_h \phi > 0.5$, and*

$$b^+ = \frac{\kappa \phi_l^{1/(1-\alpha)}}{(1-\phi_l)} > y_0 c_l^{\alpha/(1-\alpha)} > \frac{\kappa 0.5 \theta \phi_l^{1/(1-\alpha)}}{1 - 0.5\theta\phi_l} = b^- \tag{3.14}$$

where $\theta = \frac{1-\phi+\phi\pi_h}{1-\phi+0.5\phi\pi_h}$, $\kappa = \left(\frac{\alpha}{2}\right)^{\alpha/(1-\alpha)} > 0$, then there exists an overinvestment equilibrium. There is always a non-empty set of parameters for which there exists an overinvestment equilibrium. For any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are "intermediate" i.e. ϕ lies in an interval strictly in $[0, 1]$.

[11] For suppose not i.e. $e_l^* \leq e_h^*$, but continue to assume that $l-$types have a binding outside option i.e. (3.5) holds. Then $y(e_l^*, e_l^*) < y(e_h^*, e_l^*)$, and moreover, v_l must be less than (due to match frictions) a weighted average of $0.5y(e_l^*, e_l^*)$ and v_l itself. Thus, $v_l < 0.5y(e_h^*, e_l^*)$, contradicting (3.5).

Note that the conditions for existence of an overinvestment equilibrium are given entirely in terms of parameters ϕ, π_h(or π_l), α, c_l; in particular, the precise elasticity of substitution between inputs, ρ, does not matter, as long as it is non-positive. The concavity of the production function, as measured by α, does matter, however.

To prove that there are always parameters for which an overinvestment equilibrium exists, we turn to numerical simulations, which also give us a feel for how big this set of parameters is. Our numerical simulations, reported in Figure 1 below, proceed as follows.

First, we wish to choose ranges of parameter values which are "realistic" for our main application, the labour market. We begin with the match friction parameter, $\phi = a/(a+r)$. In the model, in steady-state equilibrium, a is by definition equal to the flow (over time period Δ) of new entrants to the pool of unmatched, divided by the stock of unmatched. So, $1/a$ is equal to the average time to find a match. Empirically, this corresponds to average unemployment duration. Typical unemployment durations measured in months for OECD countries range between 14 months (France) and 3 months (the US), with an average for Europe of about 6 months (Pissarides (2007)). This suggests a range of values for a of 1/3 to 1/14.

Next, following Pissarides (2007), we choose an annual discount rate of 5%, implying a monthly discount rate of $\ln(1.05)/12 = 0.0041$. Overall, this gives a range of values of ϕ , 0.934-0.987, i.e. indicating that according to this measure, the labour market is close to frictionless. So, we shall let ϕ range between 0.9 and 1. Nevertheless, as we shall see, for reasonable values of the other parameters, it is possible to find overinvestment.

The other parameters here are α, the returns to scale of the production functions, and π_h. These are much more difficult to calibrate from labour market data. First, on α, even if there are constant returns to *all* inputs, the investments considered here may be only small subset of inputs (e.g. investment in IT training by workers, and capital investments complementary to IT training, for example, computer hardware, by firms) so α could be quite small. We let α take on the values 0.1, 0.5, 0.9. Finally, for π_h, we need $\pi_h > 0.5/\phi$ for $\phi \geq 0.9$, i.e. $\pi_h > 0.55$. So, we let $\pi_l = 1 - \pi_h$ range between 0.04 and 0.44.

Figure 1 in here

These Figures graph b^+, b^- as functions of the match friction parameter $\phi = \frac{a}{a+r}$, which must lie between 0.9 and 1. Specifically, Figures 1(a)-(c) show ϕ along the horizontal axis, and b^+, b^- on the vertical axis. The set of parameter values satisfying (3.14) is shown by the shaded area in each case. So, from Proposition 2, the set of parameter values for which overinvestment equilibrium exists is just the shaded area. Generally, we see that for every configuration of parameter values illustrated, this set is non-empty. In several cases, this region is quite large, verifying our claim that overinvestment is a realistic possibility in the labour market.

Figures (a)-(c) also show what happens as the fraction of investors π_l increases. This clearly increases the size of the shaded area. This is because the higher π_l, the higher the value of the outside option of an $l-$type when matched with an $h-$type, and so the more likely it is that the outside option of an $l-$type will bind in a match with an $h-$type i.e. that equilibrium condition (3.5) holds. Other things equal, this makes an overinvestment equilibrium more likely.

Finally, we turn to the important fact, stated in Proposition 2, that for any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are intermediate i.e. ϕ is in an interval strictly in $[0,1]$. First, how can that be reconciled with Figure 1, where ϕ can take on very high values? Simply fix a point on the vertical axis and there is always an interval on the horizontal axis that generates coordinates in the shaded area. So, "intermediate" in Proposition 2 has a very precise meaning; it does *not* mean, for example, that values of ϕ in the middle of the feasible range $[0,1]$ i.e. around 0.5 always generate overinvestment.

Second, *why* are "intermediate" frictions required for overinvestment? The reason is that an N-B equilibrium only occurs with intermediate match frictions. Match frictions must be low enough to ensure that the l-type's outside option is binding, but must be high enough to ensure that matching is non-assortative. Moreover, the *only* way in which overinvestment can arise is in N-B equilibrium, and thus a necessary condition for overinvestment is that match frictions are intermediate.[12]

4. Discussion

Here we discuss some possible extensions and empirical implications of the model. One concern is that in the model, the matching process is entirely random, whereas in reality, labour market search is at least partially "directed" i.e. workers can apply to particular firms, and firms can accept applications only from particular workers.

Here, we sketch how our model can be extended to allow for directed search[13], and argue that our results are robust to a certain amount of "direction" in the search process. Another interpretation of our matching technology is that over a time interval Δ, a fraction of Δa of firms and workers are drawn at random from the pool of unmatched, and then matched randomly with each other via an employment agency of some kind. We now modify this as follows.

Continue to assume A1,A2. We suppose that low-cost types can express a preference to the agency for the type they wish to be matched with. An $l-$type will always wish to be matched with another $l-$type, as strict complementarity implies that total revenue from the match will be higher, and thus half the revenue from that match exceeds what the l-type could get in a match with an $h-$type, whether or not his outside option binds. We also suppose that the agency only meets the $l-$type$'$s request with probability p, and matches him randomly with probability $1-p$, with p thus measuring the efficeincy of the agency.

So, the probability, conditional on being matched at all, that an $l-$type is matched with another $l-$type is $\lambda_l = p + (1-p)\pi_l = \pi_l + p\pi_h$, and the probability, conditional on being matched at all, that an $h-$type is matched with another $h-$type is $\lambda_h = \pi_h + p\pi_l$. So, when $p=0$, we have random matching, and when $p=1$, we have $\lambda_l = \lambda_h = 1$ i.e. perfectly directed matching. Then , in (3.1), we replace π_l, π_h by $\lambda_l,\ 1-\lambda_l$ respectively and in (3.2), we replace π_l, π_h by $\lambda_h,\ 1-\lambda_h$ respectively. Then, the analysis proceeds as before, and it can be shown (details on request) that if p is not too high, an overinvestment equilibrium can exist as before. In this sense, our results are robust to the introduction of directed search.

[12] A formal proof that underinvestment occurs with any other kind of equilibrium is provided in de Meza and Lockwood (2004). The idea is that there are only two other possibilities; (i) assortative matching, where by definition, the outside option cannot bind, and (ii) non-assortative matching, with non-binding outside options. In either case, there is no rent-shifting incentive for investment.In addition, if the return to the unmatched state occurs randomly via stochastic match break-up in equilibrium, there is always underinvestment.

[13] For other models of directed search, see e.g. Moen(1997), Albrect, Gautier, and Vroman(2006).

Now we turn to empirical implications. In other respects, our matching model fits the facts better than does signalling theory. Sicherman (1991, p114) reports that *"Workers who are working in occupations that demand less schooling than they actually have (overeducated) get higher wages than their co- workers (holding other characteristics constant) but lower wages than workers with similar levels of schooling who work in jobs in which their schooling equals that which is required"* This is exactly the pattern predicted by the (mis)matching approach. Moreover, signalling theory is based on the proposition that education sorts, whereas the dispersion in matches indicates that to a considerable extent it fails to do so.

The theories also have different implications for how the growth in the numbers of graduates impacts on the level of wages. Acemoglu and Shimer (1999) document that in the US and elsewhere that an upsurge in the number of graduates has been associated with falls in the absolute wage of non graduates.[14] If we interpret the ex ante investment in our model as the acquisition of higher education, our rent-transfer effect provides an explanation. of the facts that does not rely on asymmetric information. An increase in the number of low-cost investors (those willing and able to invest in higher education) increases the outside option of these investors and thus reduce the wages of non-investors (non-graduates). So the absolute wage of non graduates will be lower in an equilibrium with high numbers of graduates, as the evidence suggests. The separating equilibrium of signalling theory implies that the least educated are paid their intrinsic productivity so an increase in the number of more educated workers would not depress their wages.

Sicherman does find that overqualified workers do eventually tend to move to better jobs, indicating that the phenomenon is due to mismatch rather than signalling. Reassignment may involve on-the-job search, a process that we do not formally model. Of course the opportunity to correct poor initial matches means that the incentive to overinvest is moderated, but the premium that the overeducated enjoy before any transition creates a socially excessive return.

5. Related Literature

We are aware of two papers that consider ex ante investments followed by competitive, or frictionless, mechanisms for pairing or matching agents (e.g. firms and workers): Cole, Mailath and Postlewaite (2001) and Felli and Roberts (2002).[15] Cole, Mailath and Postlewaite (2001), consider a matching model in which buyers and sellers make investment decisions non-cooperatively prior to entering a frictionless matching and bargaining process that is modelled as a cooperative game. The outcome of this second stage is constrained to be "stable" i.e. there is no pair of agents that by rematching and appropriately sharing the resulting surplus can both be strictly better off than in the equilibrium. Felli and Roberts (2002) analyze a model with a fixed number of heterogeneous buyers and sellers, and investment only by one side of the market. Following investment, a Bertrand-style game is assumed where firms bid for workers (or vice versa). Both these papers find examples of equilibria with overinvestment.

But, we would argue, these examples have important limitations. In Felli and Roberts (2002, unlike in our model, the agents invest efficiently, *conditional* on the match that they anticipate. But, relative

[14] Acemoglu's explanation is that with more graduates available, firms find it worth creating jobs specifically for graduates. The implication is that there should be a strong positive correlation between the number of graduates and GNP.

[15] Cole, Mailath and Postlewaite (2001b) is less closely related to our work, as it assumes a finite number of agents. In this case, the decision of any two agents to match has an external effect on the opportunities available to other agents, so the set-up is a bit different.

to the first-best, overinvestment by - for example - a relatively low quality worker is possible, because he anticipates being hired by a very high-quality firm. There is no general tendency to over- or under-investment.

In Cole, Mailath and Postlewaite (2001), Proposition 5 states that equilibrium investments with stable matching are at a *local* maximum of net surplus (the revenue from a match minus investment costs, $S = y(e_w, e_f) - c(e_w) - c(e_f)$ in our notation). So, if S is concave, the unique equilibrium investments with stable matching are the efficient investments that maximize S. So, *their example of overinvestment with a continuum of agents relies on S being non-concave*[16]. In contrast, our model has a well-behaved concave revenue function; overinvestment is due to a different mechanism.

Moreover, in their model, typically there are multiple stable equilibria to the post-investment game, and thus multiple equilibria overall, which may generally involve underinvestment, efficient investment, or overinvestment. In their cooperative framework, without an explicit bargaining model, they have no criterion for selecting among these equilibria.

6. Conclusions

A recent literature examines agents' incentives to make investments prior to entering a stochastic matching process and bargaining over the surplus. In these models outside options do not binding, investors are held up and will under-invest in equilibrium. Investment subsidies are therefore appropriate. We have shown that this finding is not robust. If agents are heterogenous, outside options influence investment incentives. This effect mitigates underinvestment tendency and allows over-investment to arise. Simulation indicates that for OECD countries, parameters may be in the over-investment zone. Unlike the signalling model, the matching model is consistent with findings that workers often take jobs for which they are overqualified, earning more than less qualified peers but less than if they were they to hold jobs for which their qualifications are required.

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[16] In our notation, their example is

$$y(e_w, e_f) = \begin{cases} e_w e_f, & e_w e_f \leq 0.5 \\ 2(e_w e_f)^2 & e_w e_f < 0.5 \end{cases}$$

which is clearly not concave.

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A. Appendix: Proof of Proposition 2

Note that as $y(e_h, e_l) = y(0, e_l) = y_0$, conditions (3.5) reduces to $2v_l > y_0$. Moreover, using (3.3), (3.4), (3.6) can be written

$$y_0 \geq \left(\frac{1 - \phi + \phi\pi_h}{1 - \phi + 0.5\phi\pi_h} \right) v_l \equiv \theta v_l \tag{A.1}$$

So, from (A.1), the condition for the binding outside option and non-assortative matching conditions to be satisfied together become

$$2v_l > y_0 \geq \theta v_l \tag{A.2}$$

Now from the CES assumption (2.1), we have

$$y(e_l, e_l) = e_l^\alpha + y_0 = \Delta + y_0 \tag{A.3}$$

Combining this with (3.3) gives $v_l = 0.5\phi_l(y_0 + \Delta)$. But then, assuming $y_0 > 0$, (A.2) reduces to

$$\frac{\phi_l \Delta}{1 - \phi_l} > y_0 > \frac{0.5\theta\phi_l\Delta}{1 - 0.5\theta\phi_l} \tag{A.4}$$

Finally, e_l satisfies the FOC (3.8):

$$\frac{\alpha\phi_l}{2}(e_l^\rho)^{\alpha/\rho - 1} e_l^{\rho - 1} = c_l \Longrightarrow e_l^* = \left(\frac{\alpha\phi_l}{2c_l} \right)^{1/(1-\alpha)}$$

So,

$$\Delta = (e_l^*)^\alpha \ = \kappa \left(\frac{\phi_l}{c_l} \right)^{\alpha/(1-\alpha)}, \ \kappa = \left(\frac{\alpha}{2} \right)^{\alpha/1-\alpha)} > 0 \tag{A.5}$$

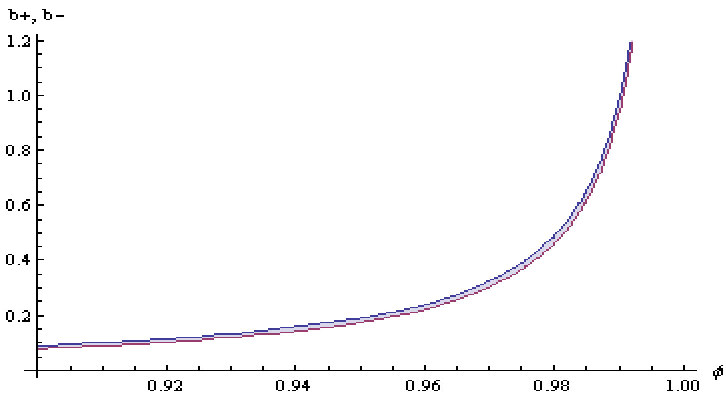
and thus, combining (A.4), (A.5), we need

$$\frac{\kappa\phi_l^{1/(1-\alpha)}}{(1 - \phi_l)c_l^{\alpha/(1-\alpha)}} > y_0 > \frac{\kappa 0.5\theta\phi_l^{1/(1-\alpha)}}{(1 - 0.5\theta\phi_l)c_l^{\alpha/(1-\alpha)}}$$

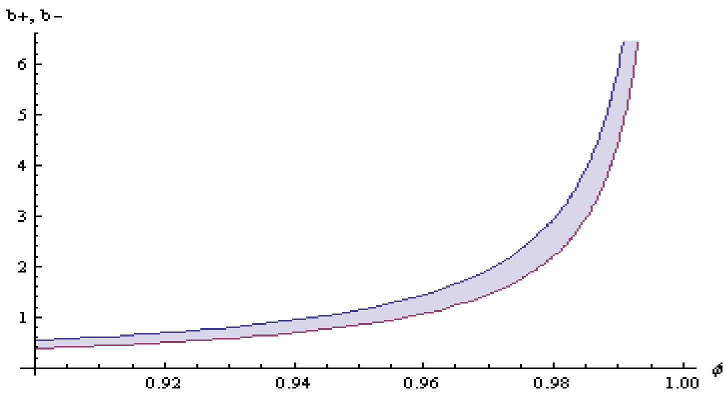
which gives (3.14). Finally, we need to prove that for any fixed values of the other parameters, an overinvestment equilibrium exists if match frictions are intermediate i.e. ϕ is in an interval strictly in $[0, 1]$. First, as $\phi \to 1$, $0.5\theta, \phi_l \to 1$ also, so that $b^+, b^- \to \infty$, so that for ϕ close enough to 1, any fixed $y_0 c_l^{\alpha/(1-\alpha)} < b^-$, so no overinvestment equilibrium can exist for ϕ close enough to 1. Second, as $\phi \to 0$, $\phi_l \to 0$, so $b^+, b^- \to 0$, so that for ϕ close enough to 1, any fixed $y_0 c_l^{\alpha/(1-\alpha)} > b^+$, so no overinvestment equilibrium can exist for ϕ close enough to 0. □

Figure 1: Parameter Values for which Overinvestment Equilibrium Exists

(a): λ_l=0.04, α=0.5



(b): λ_l=0.24, α=0.5



(c): λ_l=0.44, α=0.5