How Should Financial Intermediation Services be Taxed?*

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Abstract

This paper considers the optimal taxation of savings intermediation services in a dynamic general equilibrium setting, when the government can also use consumption, income and profit taxes. When 100% taxation of profit is available, taxes on services supplied to firms should be deductible from profit, implying the optimality of a VAT-type tax. As for the *rate* of tax, in the steady state, an optimal arrangement is to set it equal to the rate of tax on capital income, *not* consumption. In turn, the capital income tax is zero when the when an unrestricted profit tax is available, but in the more realistic case when such a tax is not available, this rate can be positive or negative, but generally different to the optimal rate of tax on consumption.

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1. Introduction

Financial intermediation services include such important services as intermediation between borrowers and lenders, insurance, and payment services (e.g. credit and debit card services). These services comprise a significant and growing part of the national economy. For example, financial intermediation services as conventionally defined in the national accounts includes activities such as the taking of deposits and the granting of credit, financial leasing, investment in securities and properties by financial intermediaries, insurance and pension funding, and services ancillary to financial intermediation[1]. The EU KLEMS database shows that this sector comprised 6.5% of national output in the UK in 1997, increasing to 7.9% by 2007. The figures for the US, using the same definition of financial intermediation services, are 7.3% in 1997, rising to 8.6% and for the Eurozone, 4.8%, rising to 5.3%[2]. Even excluding insurance - which is beyond the scope of this paper - financial intermediation is quantitatively important in OECD countries[3].

The question of whether, and how, financial intermediation services should be taxed is a contentious one[4]. For example, within European Union countries, most financial services are currently exempt from VAT, and there is considerable debate about the possible benefits from bringing them into the VAT system (de la Feria and Lockwood (2010)). Also, the recent IMF proposals for a "bank tax" to cover the cost of government interventions in the banking system include a Financial Activities Tax levied on bank profits and remuneration, one version of which - FAT1 - which would work very much like a VAT, levied using the addition method (IMF(2010)).

In the policy literature on this topic, it is largely assumed that within a consumption tax system, such as a VAT, it is desirable to tax financial services supplied at the standard rate of VAT, and allow providers of intermediation services to claim back VAT they pay on inputs: see e.g. Ebril, Keen, Bodin, and Summers(2001)). However, this policy prescription is at variance with a small academic literature on this topic (Grubert and Mackie(1999), Jack(1999), and Boadway and Keen(2003)), which suggests that while

[1] Financial intermediation comprises activities 65,66,67 in the ISIC/NACE system of national accounts. The definition of these activities can be found for example, in the handbook NACE: REV.1, published by Eurostat.

[2] Authors' calculations: financial intermediation comprises lines J65-67 in the EU KLEMS Growth and Productivity Accounts (http://www.euklems.net/index.html).

[3] Data showing the size of intermediation services relating to the taking of deposits and granting of credit *only* are presented for the UK in Section 1.2 below.

[4] There are technical difficulties in taxing financial intermediation services; however, these difficulties are not insurmountable - see Section 1.1 below.

payment services should be taxed at the same rate as consumption, intermediation between borrower and lender should not be taxed at all. However, this literature is based on first-best arguments i.e. finding the tax arrangement that does not drive a wedge between the household marginal rate of substitution and the marginal rate of transformation in production.

The objective of this paper is to take a fresh look at this question, from a tax design point of view. Our focus on the most important intermediation service - intermediation between borrowers and lenders[5]. We set up and solve the tax design problem in a dynamic general equilibrium model of the Chamley(1986) type, where the government chooses a tax on savings intermediation, as well as the usual taxes on consumption (or equivalently, wage income) and income from capital, to finance a public good, and where financial intermediaries, in the form of banks, are explicitly modelled. Realistically, we assume that savings intermediation is not explicitly priced, but charged for via a spread between borrowing and lending rates set by competitive banks. This spread can be taxed, at a rate that may be different from the tax on consumption, and the tax system is parameterized so that some fraction θ of the tax paid by firms on financial intermediation inputs can be credited against the consumption tax changed by firms. In the case of a VAT, $\theta = 1$.

The main results are as follows. First, the tax paid by firms on financial intermediation inputs should be fully credited i.e. the tax should be a VAT, when 100% taxation of profit is possible[6]. This is an example of the general Diamond-Mirrlees production efficiency result that intermediate inputs should not be taxed under these conditions. But, at what rate should the VAT be set? Here, there are two main findings. First, the optimal tax structure is indeterminate, because the government has two instruments, a capital income tax and a financial intermediation tax, to control the marginal rate of substitution between consumption in successive periods. However, it turns out that from an informational point of view, the simplest optimal tax structure is where capital income tax and a financial intermediation tax are equal.

In particular, when 100% taxation of pure profit is possible, the simplest optimal tax structure is to set both the tax rate on capital income, and the tax rate on financial intermediation services equal to zero. In the more realistic case when there is an upper bound on the rate of profit of less than 100%, we show that a simple optimal tax structure is again to set the two taxes at the same rate. The sign of this common rate then depends

[5]I study taxation of payment services in a companion paper, Lockwood and Yerushalmi(2013).

[6]Firms must make pure profit in equilibrium, because they must have decreasing returns, in turn because they possibly face different borrowing costs.

on the properties of the production function; it can be positive or negative. Moreover, this common rate is generally different from the optimal tax rate on consumption.

These results are quite different from the existing literature (see Section 1.2) which generally finds that financial intermediation services should be untaxed. However, these are first-best models, where there is (implicitly) no revenue requirement. There, the optimality of leaving financial intermediation services untaxed is derived just from the condition that the marginal rate of substitution in consumption is equal to the marginal rate of transformation. Our results also differ from Auerbach and Gordon(2002), which finds, using a rather different argument, that financial intermediation services should be taxed at the same rate as consumption.

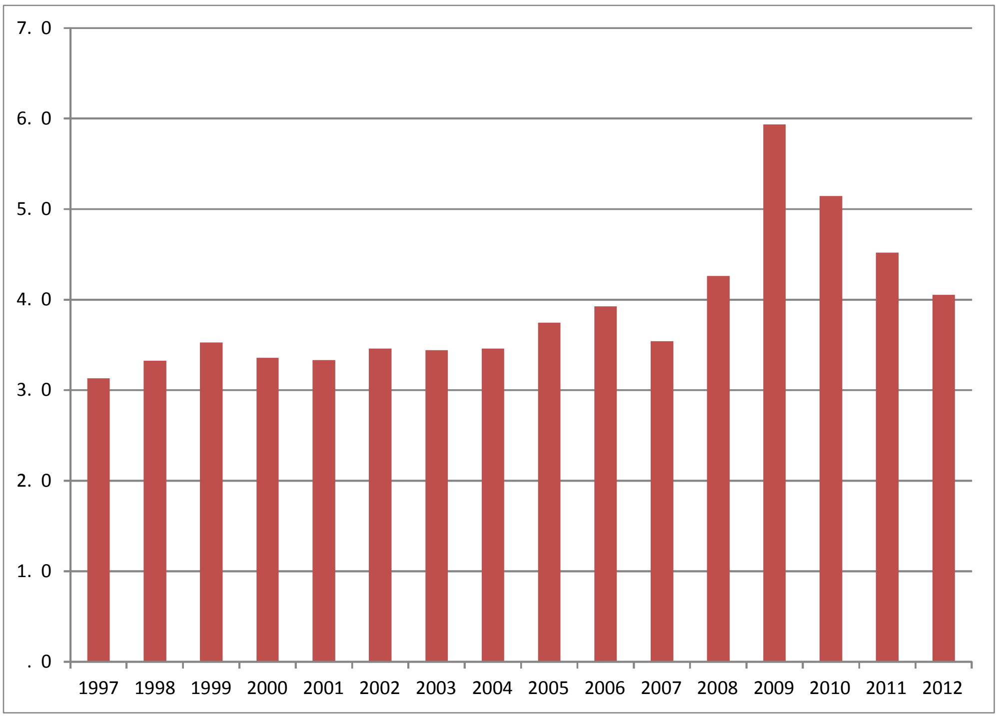
The remainder of the paper is organized as follows. Section 1.1 discusses some basic facts about the taxation of financial intermediation, and Section 1.2 discusses related literature. Section 2 outlines the model and Section 3 presents the main results. Section 4 considers the case without 100% profit taxation, Section 5 considers other extensions, and Section 6 concludes.

1.1. The Size and Tax Treatment of Financial Intermediation Services

The figures quoted at the beginning of the paper measure the overall size of the financial intermediation sector. To get an idea of the value of financial intermediation associated with the taking of deposits and the grant of credit only, we can look at FISIM[7]. FISIM is computed from the transactions between the banking sector and other sectors of the economy (for example, non-financial firms and households). For each of these sectors, the loans from, and deposits with, the banking sector, are measured and the margins made by the banking sector on these activities are calculated. Specifically, the margin per currency unit deposited is a reference rate minus the average rate of interest on deposits, and the margin per currency unit lent is the average rate of interest on loans, minus a reference rate (Akritidis(2007)). So, FISIM can be calculated by sector, and also on loans and deposits separately. As our focus is primarily on taxation of the household sector, we show consumption of FISIM by households and non-profit institutions serving households for the UK over the period 1997 to 2012.

[7] This is an acronym for "financial intermediation services indirectly measured".

Figure 1: FISIM Consumed by Households as a
Percentage of Household Consumption Expenditure, UK



Source: Office of National Statistics, UK. The chart shows FISIM on loans (series IV8X) and deposits (series IV8W) consumed by households and non-profit institutions serving households as a percentage of aggregate consumption (series RPQM)

This shows that consumption of FISIM by households is between 3% and 4% of total household expenditure in the UK over this period; not large, but not a negligible fraction, either. Also the financial crisis has had as positive impact on this figure, as banks have increased their spreads on loans to repair their balance sheets. So, overall, it can be seen that financial intermediation services are a significant and growing part of the economy in the UK. The picture is the same for other OECD countries with a developed financial sector.

As regards the taxation of financial intermediation services, in our theoretical analysis below, we assume that intermediation services can be taxed; specifically, that banks can charge taxes to households and firms separately on their consumption of intermediation services. It is recognized that in practice there are technical difficulties when those services are not explicitly priced (so-called margin-based services), because it is not straightforward to divide the "value-added" between borrower and lender. In particular, this raises a problem for the use of a VAT via the usual invoice-credit method (Ebril, Keen, Bodin and Summers(2001)). As a result, the status quo in most countries is that a wide range of financial intermediation services are not taxed. For example, in the EU, many such services are exempt as a result of the 6th VAT directive[8].

However, conceptually, the problems can be solved in several different ways. One administratively straightforward system would be to zero-rate sales to VAT-registered entities and tax sales to non-registered entities e.g. households on an aggregate basis (Huizinga(2002)). Alternatively, Poddar and English(1997) have proposed a cash-flow VAT with tax calculation accounts; this is administratively more complex, but the increasing sophistication of banks' IT systems means that this solution is becoming practical. A recent study by the European Commission calculates that EU-27 tax revenue might rise by around £15 billion Euro if intermediation services were brought into the VAT system, and taxed at the standard rate (European Commission(2011)).

1.2. Related Literature

There is a small literature directly addressing the optimal taxation of borrower-lender intermediation and payment services, Grubert and Mackie(1999), Jack(1999), and Boadway and Keen(2003). Using a simple two-period consumption-savings model, these papers agree on a policy prescription[9]. Given a consumption tax that is uniform over time, payment services should be taxed at this uniform rate, but savings intermediation should be left untaxed. The argument used to establish this is simple; in a two-period consumption-savings model with the same, exogenously fixed, tax on consumption in both periods, this arrangement leaves the marginal rate of substitution between current and future consumption undistorted i.e. equal to the marginal rate of transformation in production. Using a different approach, Auerbach and Gordon (2002) do not make a sharp distinction between payment services and savings intermediation, and argue that both activities should be taxed at the same rate as consumption[10]. More precisely, they show that a wage tax is equivalent to a uniform tax on consumption and intermediation services.

[8] The Sixth VAT Directive and subsequent legislation exempts a wide range of financial services from VAT, including insurance and reinsurance transactions, the granting and the negotiation of credit, transactions concerning deposit and current accounts, payments, transfers, debts, cheques, currency, bank notes and coins used as legal tender etc. (Council Directive 2006/112/EC of 28 November 2006, Article 135).

[9] Chia and Whalley(1999), using a computational approach, reach the rather different conclusion that payment services should be untaxed, but but their model is not directly comparable to these others, as the intermediation costs are assemed to be proportional to the *price* of the goods being transacted.

[10] Auerbach and Gordon(2002) state: "transactions costs can include the real resources ...lost when investing these funds so that they will be available in a later period" (Auerbach and Gordon(2002), p412).

However, one can make three criticisms of this literature. First, taxes are taken as given. In particular, consumption taxes are assumed equal in both periods, and capital income taxes are set to zero. Combined with the (implicit) assumption of fixed labour supply in those models[11], the tax system (other than taxes on financial services) amounts to a non-distortionary tax on labour income. In this setting, it is of course, optimal for marginal rate of substitution in consumption to be equal to the marginal rate of transformation in production. It is then not very surprising that the tax on borrower-lender intermediation should be zero. Second, as taxes are not distortionary, there is no second-best tax design problem, so the question of trading off distortions generated by a tax on financial services against other distortionary taxes does not arise. Third, the production side of the economy is not explicitly modelled, so that questions of distortions in input prices caused by taxes on financial services cannot be addressed.

Second, there is also a less closely related literature on the use of taxation to control "bad banks". The idea here is that while banks may engage in socially undesirable activities on both lending and deposit-taking margins, these should be corrected by Pigouvian taxes (or regulations) that apply directly to these decision margins. There has recently been surge of literature on such Pigouvian taxes; see e.g. Acharya et. al.(2010), Bianchi and Mendoza(2010), Keen(2010), Perrotti and Suarez(2011), Coulter, Mayer, and Vickers(2012). In our model, banks are merely producers that price intermediation services at marginal cost, so there is no role for Pigouvian taxes.

2. The Model

2.1. Households

The model is a version of Atkeson, Chari and Kehoe(1999) with savings intermediation by banks added to the basic structure. There is a single infinitely lived household with preferences over levels of a single consumption-capital good, leisure, and a public good in each period $t = 0, ..\infty$ of the form

$$\sum_{t=0}^{\infty} \beta^t (u(c_t, l_t) + v(g_t)) \tag{2.1}$$

where c_t is the level of consumption in period t, $l_t \in [0, 1]$ is the supply of labour hours, and g_t is public good provision. Utility $u(c, l)$ is strictly increasing in c, strictly decreasing

[11]The exception here is Auerbach and Gordon(2002), where labour supply is variable. However, in their model, the consumption tax is just assumed to be uniform, not optimised.

in l, and strictly concave, and $v(g)$ is strictly increasing and strictly concave in g. Finally, $0 < \beta < 1$ is a discount factor.

In any period t, the household is assumed to pay an ad valorem tax τ_t^c on c_t, and also pays proportional taxes on labour and capital income. Using the well-known fact that a consumption tax is equivalent to a wage tax, we assume w.l.o.g. that the wage tax is zero. Finally, for the moment, we suppose that the household has no profit income in any period: firms generate pure profits (for reasons explained in Section 3.2 below), but these are taxed at 100%. So, in any period, the budget constraint is

$$c_t(1+\tau_t^c) + k_{t+1} = w_t l_t + (1+\rho_t)k_t$$

where ρ_t is the post-tax return to the household on savings, and w_t is the wage, and k_{t+1} is savings. Finally, $\rho_t = (1-\tau_t^r)r_t^h$, where r_t^h is the pre-tax return on savings for the household, determined below, and τ_t^r is the capital income tax.

So, following Atkeson, Chari and Kehoe(1999), the present value budget constraint of the household can be obtained by aggregating over per-period budget constraints:

$$\sum_{t=0}^{\infty} p_t(c_t(1+\tau_t^c) + k_{t+1}) = \sum_{t=0}^{\infty} p_t(w_t l_t + (1+\rho_t)k_t) \tag{2.2}$$

where p_t is the price of output in period t. We normalize by setting $p_0 = 1$ and assume for convenience that $k_0 = 0$ i.e. initial capital is zero[12]. The first-order conditions for a maximum of (2.1) subject to (2.2) with respect to $c_t,\ l_t, k_{t+1}$ respectively are:

$$\beta^t u_{ct} = \lambda p_t(1+\tau_t^c) \tag{2.3}$$

$$-\beta^t u_{lt} = \lambda p_t w_t \tag{2.4}$$

$$p_t = (1+\rho_{t+1})p_{t+1} \tag{2.5}$$

where λ is the multiplier on (2.2), and we use (here and below) the notation that for any any function $f(x_t\,, y_t)$, the partial derivative of f with respect to x_t is f_{xt}, the cross-derivative is f_{xyt} etc.

2.2. Banks

For simplicity, we assume 100% depreciation of the capital good. So, in the standard version of this model, without financial intermediation, the household provides k_t units of

[12] This implies that the government cannot set a first-period capital levy on fixed capital k_0 and thus simplifies the analysis (see Atkeson, Chari and Kehoe(1999)).

the consumption-capital good to the firms in period $t-1$, and firms (in aggregate) repay the k_t units of consumption-capital good to the household in period t, plus interest. In our version of the model, the k_t units of the consumption-capital good are deposited with a bank at $t-1$, who can then provide this stock k_t to firms as an input to production. The firms then repay k_t to the bank at t, plus interest, and finally, the bank repays k_t to the households, plus interest.

The cost of intermediating one unit of savings between the household and firm i is $\tilde{s}^i$ units of labour. Note that we take $\tilde{s}^i$ as fixed, but possibly varying between firms. This is realistic; lending is a complex process involving initial assessment of the borrower via e.g. credit scoring, structuring and pricing the loan, and monitoring compliance with loan covenants (Gup and Kolari(2005, chapter 9).

We also suppose that the total intermediation cost $\tilde{s}^i$ can be divided by the bank between the cost of services provided to the household, s^h, (e.g. safekeeping of deposits, liquidity) and cost of services provided to firm i (e.g. monitoring), s^i i.e. $\tilde{s}^i = s^h + s^i$. This is a realistic assumption, because in practice, any scheme (such as the tax calculation accounts of Poddar and English(1997)) which implements a VAT on margin-based financial intermediation services must split the "value-added" between borrower and lender. We assume that banks can borrow and lend at a "pure" rate of interest r_t which will eventually be determined in equilibrium in the capital market. Finally, we assume that banks are perfectly competitive, which combined with the constant returns intermediation technology, implies that they make zero profit in equilibrium.

Finally, we assume that a tax on intermediation services, or spread tax, is in operation at rate τ_t^s, which can be different from the rate τ_t^c on final consumption. Banks must break even on each of the two activities of providing services to households, and to firms separately, otherwise a competitor bank could profitably undercut them. So, in equilibrium, the costs of intermediation $w_t s^i, w_t s^h$ plus the tax paid, are equal to the spreads $r_t^i - r_t,\ r_t - r_t^h$ respectively, where r_t^i is the rate at which firm i borrows, and r_t^h is the rate of return on savings for the household. That is:

$$r_t^i - r_t = w_t s^i (1+\tau_t^s),\ r_t - r_t^h = w_t s^h (1+\tau_t^s) \tag{2.6}$$

This is intuitive: the spread on both borrowing and lending is equal to the real resource cost plus by the tax.

2.3. Firms

There are firms $i = 1, ..n$ which produce the homogenous consumption good in each period[13]. Firm i produces output from labour and capital via the strictly concave production function $F^i(k_t^i, l_t^i)$, where k_t^i, l_t^i are capital and labour inputs[14]. Because firms may differ in intermediation costs s^i, firms face differences in the cost of capital i.e. firm i must repay $1 + r_t^i$ per unit of capital borrowed from the bank. So, the profit of firm i is

$$F^i(k_t^i, l_t^i) - l_t^i w_t - (1 + r_t^i)k_i + \theta^i \tau_t^s w_t s^i k_i$$

where $\tau_t^s w_t s^i k_i$ is the total tax paid on intermediation services by the firm, and θ^i is the fraction of the tax paid on intermediation services that the firm can claim against the tax paid on output. Of course, in the usual VAT system, $\theta^i = 1$. So, substituting r_t^i from (2.6) profit can be written

$$F^i(k_t^i, l_t^i) - l_t^i w_t - (1 + r_t + w_t s^i(1 + (1 - \theta^i)\tau_t^s))k_i \tag{2.7}$$

Maximizing (2.7) with respect to k_t^i, l_t^i implies the first-order conditions:

$$F_l^i(k_t^i, l_t^i) = w_t, \; F_k^i(k_t^i, l_t^i) = 1 + \tilde{r}_t^i \tag{2.8}$$

where $\tilde{r}_t^i = r_t + w_t s^i(1 + (1 - \theta^i)\tau_t^s)$ is the cost of capital for firm i. Finally, the capital and labour market clearing conditions are:

$$\sum_{i=1}^{n} k_t^i = k_t, \; \sum_{i=1}^{n} l_t^i + k_t s^h + \sum_{i=1}^{n} k_t^i s^i = l_t \tag{2.9}$$

These conditions (2.8),(2.9) jointly determine w_t and r_t, given household savings and labour supply decisions.

2.4. Discussion

The above model provides a general framework which encompasses some aspects of the specific models of taxation of financial services (Auerbach and Gordon(2002), Boadway and Keen(2003)), Jack(1999), Grubert and Mackie(1999)) that have been developed so

[13] We also assume for convenience that one unit of the consumption good can be transformed into one unit of the public good. This fixes the relative pre-tax price of c_t and g_t at unity.

[14] We assume that firms face decreasing returns, because with different costs of capital, and the same wage, with constant returns, only the one firm with the lowest unit cost would operate, and this case is of limited interest.

far. Specifically, ignoring payment services, which are not dealt with in this paper, Boadway and Keen(2003)), Jack(1999), Grubert and Mackie(1999) are two-period versions of the above model[15], with fixed taxes and (implicitly) fixed labour supply. Auerbach and Gordon(2002) is a finite-horizon version of the model, with the additional feature[16] that there are n consumption goods in each period. Payment services are dealt with in a companion paper, Lockwood and Yerushalmi(2013).

3. Tax Design

We take a primal approach to the tax design problem. In this approach, an optimal policy for the government is a choice of all the primal variables in the model, in this case $\left\{c_t, l_t, k_{t+1}, g_t, (k_t^i, l_t^i)_{i=1}^n\right\}_{t=0}^{\infty}$ to maximize utility (2.1) subject to the capital and labour market clearing conditions (2.9), aggregate resource, and implementability constraints. We are thus assuming, following Chamley(1986), that the government can pre-commit to a policy at $t = 0$. The aggregate resource constraint says that total production must equal to the sum of the uses to which that production is put:

$$c_t + k_{t+1} + g_t = \sum_{i=1}^{n} F^i(k_t^i, l_t^i), \quad t = 0, 1, .. \tag{3.1}$$

The implementability constraint ensures that the government's choices also solve the household optimization problem, and is in our model, quite standard. We obtain it by substituting the household's first-order conditions (2.3)-(2.5) in (2.2). After some rearrangement, this gives the condition:

$$\sum_{t=0}^{\infty} \beta^t (u_{ct}c_t + u_{lt}l_t) = 0 \tag{3.2}$$

As is standard in the primal approach to tax design, we can incorporate the implementability constraint (3.2) into the government's maximand by writing

$$W_t = u(c_t, l_t) + v(g_t) + \mu(u_{ct}c_t + u_{lt}l_t) \tag{3.3}$$

where μ is the Lagrange multiplier on (3.2). If $u_{cl} \leq 0$, i.e. consumption and leisure are complements, it is possible to show that $\mu \geq 0$ at the solution to this tax design problem

[15] A minor qualification here is that Boadway and Keen allow for a fixed cost of savings intermediation e.g. fixed costs of opening a savings account. These introduce a non-convexity into household decision-making, which greatly complicates the optimal tax problem, and so we abstract from these in this paper.

[16] It also has labour supply in only one period.

(see Appendix). If $\mu = 0$, the revenue from profit taxation is sufficient to fund the public good, g. We will rule out this uninteresting case, and so will assume that $\mu > 0$ at the optimum in what follows.

The government's choice of primal variables must maximize $\sum_{t=0}^{\infty} \beta^t W_t$ subject to (3.1) and (2.9). The first-order conditions with respect to $c_t, l_t, k_t, g_t, k_{it}, l_{it}$ are, respectively;

$$\begin{aligned} \beta^t W_{ct} &= \zeta_t && (3.4) \\ -\beta^t W_{lt} &= \zeta_t^l && (3.5) \\ \zeta_t^k &= \zeta_{t-1} + \zeta_t^l s^h && (3.6) \\ \beta^t v_{gt} &= \zeta_t && (3.7) \\ \zeta_t F_{kt}^i &= \zeta_t^k + \zeta_t^l s^i, \ i = 1, ..n && (3.8) \\ \zeta_t F_{lt}^i &= \zeta_t^l, \ i = 1, ..n && (3.9) \end{aligned}$$

where ζ_t, ζ_t^k, ζ_t^l are the multipliers on the resource constraint and the capital and labour market clearing conditions at time t respectively.

Moreover, from (3.3),

$$W_{lt} = u_{lt}(1 + \mu(1 + H_{lt})), \ H_{lt} = \frac{u_{lct}c_t + u_{llt}l_t}{u_{lt}} \tag{3.10}$$

and

$$W_{ct} = u_{ct}\left(1 + \mu(1 + H_{ct})\right), \ H_{ct} = \frac{u_{cct}c_t + u_{clt}l_t}{u_{ct}} \tag{3.11}$$

Here, H_{lt}, H_{ct} are given by standard formulae found, for example, in the primal approach to the static tax design problem (Atkinson and Stiglitz(1980)). In particular, $-H_{ct} > 0$ under our assumption $u_{clt} \leq 0$, and its magnitude measures the degree of complementarity between consumption and leisure.

We begin by characterizing the rate of tax on the consumption good via the following result, which is proved in the Appendix:

Proposition 1. *At any date* $t = 0, 1, 2..$ *the optimal tax on final consumption in ad valorem form is*

$$\frac{\tau_t^c}{1 + \tau_t^c} = \left(\frac{v_{gt} - \alpha_t}{v_{gt}}\right)\left(\frac{H_{lt} - H_{ct}}{1 + H_{lt}}\right), \ \alpha_t = -\frac{u_{lt}}{w_t} \tag{3.12}$$

Note that (3.12) is a formula for an optimal consumption tax that also occurs in the static optimal tax problem, when the primal approach is used (Atkinson and Stiglitz(1980, p377). In particular, v_{gt} is the marginal benefit of \$1 to the government, and α_t is a measure of the marginal utility of \$1 to the household, so $\frac{v_{gt} - \alpha_t}{v_{gt}}$ is a measure of the social gain from additional taxation at the margin. It says that other things equal, the higher the degree of complementarity between consumption and leisure, $-H_{ct}$, the higher is τ_t^c. Note also that by our assumption that $u_{clt} \leq 0,\ H_{lt} > 0$.

Now we turn to consider the question of whether tax paid on financial intermediation should be deductible by firms i.e. the choice of θ^i. From (3.8),(3.9), we see that the marginal rate of substitution between labour and capital is

$$\frac{F_{kt}^i}{F_{lt}^i} = \frac{\zeta_t^k}{\zeta_t^l} + s^i \tag{3.13}$$

which implies that

$$\frac{F_{kt}^i}{F_{lt}^i} - \frac{F_{kt}^j}{F_{lt}^j} = s^i - s^j \tag{3.14}$$

However, from the first-order conditions for the firm, (2.8), we see that

$$\frac{F_{kt}^i}{F_{lt}^i} = \frac{1 + r_t}{w_t} + s^i(1 + (1 - \theta^i)\tau_t^s) \tag{3.15}$$

which implies that

$$\frac{F_{kt}^i}{F_{lt}^i} - \frac{F_{kt}^j}{F_{lt}^j} = (s^i - s^j)(1 + (1 - \theta^i)\tau_t^s) \tag{3.16}$$

If $s^i \neq s^j$, and $\tau_t^s \neq 0$ equations (3.14),(3.16) can only hold simultaneously if $\theta^i = 1$. So, we have shown:

Proposition 2. *If there is heterogeneity in intermediation costs, ($s^i \neq s^j$, some i, j) and the rate of tax on financial services $\tau_t^s \neq 0$, then any date t, efficiency requires $\theta^i = 1$ i.e. full deductibility of τ_t^s by firms.*

The intuition for this result is clear. Equation (3.14) says that the marginal product of capital net of true intermediation costs should be equal across firms, which of course is just the condition for capital to be allocated efficiently across firms. But, condition (3.14) is generally not consistent with a non-zero τ_t^s when firms are heterogenous. This is just an instance of the Diamond-Mirrlees production efficiency theorem. A tax on the bank margin is an intermediate tax on the allocation of capital, and given our assumptions (a full set of tax instruments, and no pure profits), this tax should be set to zero. Note also that when there is only one firm, this argument has no bite, and thus θ^i is left indeterminate.

We now turn to the question of how the taxes on capital income, τ_t^r and on intermediation services, τ_t^s, should be set. Generally, it can be shown, by straightforward manipulation of the first-order conditions to the optimal tax problem and the household and firm problems, that:

Proposition 3. *At all dates $t = 1, 2.., \tau_t^r, \tau_t^s$ solve*

$$A_t \left(1 + r_t - w_t s^h\right) = 1 + (1 - \tau_t^r)(r_t - w_t s^h (1 + \tau_t^s)) \tag{3.17}$$

where $A_t = \frac{1+\mu(1+H_{ct})}{1+\mu(1+H_{c,t-1})} \frac{1+\tau_t^c}{1+\tau_{t-1}^c}$.

The proof of this is in the Appendix. Clearly, τ_t^r, τ_t^s are not uniquely determined from this single condition. Ultimately, this is because the planner has two instruments, τ_t^r, τ_t^s, for controlling the marginal rate of substitution between consumption at t and $t+1$. However, we can use the following criterion to choose between solutions. Say that a solution τ_t^r, τ_t^s to (3.17) is *simple* if it solves (3.17) independently of the precise values of the economic data $H_{ct},\ w_t, r_t, s^h$. As the tax authority is unlikely to know these values, or at least to set taxes conditional on them, a simple solution is administratively convenient.

Now consider the steady state. Then, $A_t = 1$, and therefore (3.17) becomes

$$\left(1 + r - w s^h\right) = 1 + (1 - \tau^r)(r - w s^h (1 + \tau^s)) \tag{3.18}$$

By inspection, the only simple solution to (3.18) is $\tau^r = \tau^s = 0$. The result that the interest income tax is zero is of course, the classic result of Chamley(1986) and Judd(1985); our new result is that the rate of tax on financial services should be zero. So, we have proved:

Proposition 4. *At the steady state, the only simple tax system is where the tax on interest income, τ^r, and the tax on financial intermediation services, τ^s are both zero.*

Two comments can be made here. First, away from the steady state, $\tau_t^r = \tau_t^s = 0$ is generally not optimal. So, Proposition 4 - along with Proposition 6 below - makes precise the conditions under which the result of the existing literature that savings intermediation should not be taxed generalizes to a second-best environment; zero taxation of intermediation services requires (a) unrestricted taxation of profit, and (b) a steady state. Second, the celebrated result of Chamley(1986) and Judd(1985) that in the steady state, the tax on capital income is zero does not hold precisely in our model, as the planner has an additional tax instrument, τ_t^s. However, zero taxation of capital emerges if we also impose administrative simplicity.

4. Less than Full Taxation of Pure Profit

Recall that we have to assume that firms have decreasing returns, because they face different costs of capital. Therefore, they generate pure profit. So far, we have made the strong assumption that 100% taxation of this profit is possible for the government. However, is well-known that this is a key assumption behind the classical Diamond-Mirrlees result that inputs are not taxed at the second-best optimum. Here, we investigate to what extent our results generalize to the more realistic case where pure profit cannot be taxed at 100%. Ideally, we should model this via some kind of incentive constraint for managers or entrepreneurs that constrains a profit tax. However, that is beyond the scope of this paper, and following a large literature in tax design, we just assume that the profit tax is $\overline{\tau}$ is fixed at some $\overline{\tau} < 1$.

The main change to the tax design problem is that now post-tax profit appears in the budget constraint of the household. This post-tax profit can be written as $(1-\overline{\tau})\pi_t$, where π_t is aggregate pre-tax profit:

$$\pi_t = \sum_{i=1}^{n}(F^i - F^i_{lt}l^i_t - F^i_{kt}k^i_t) \tag{4.1}$$

and so income in period t is now $w_t l_t + (1+\rho_t)k_t + (1-\overline{\tau})\pi_t$. It can then be checked that π_t also appears in the implementability constraint as follows:

$$\sum_{t=0}^{\infty}\beta^t(u_{ct}c_t + u_{lt}\left(l_t + (1-\overline{\tau})\pi_t/w_t\right)) = 0 \tag{4.2}$$

From (4.1), and the fact that $w_t = F^i_{lt}$, we see that π_t/w_t only depends on $l^i_t, k^i_t,\ i = 1,..n$. So, the only first-order conditions to the tax design problem that change are (3.8), (3.9). They change to:

$$\zeta_t F^i_{kt} + (1-\overline{\tau})\beta^t\mu u_{lt}\frac{\partial(\pi_t/w_t)}{\partial k^i_t} = \zeta^k_t + \zeta^l_t s^i,\ i = 1,..n \tag{4.3}$$

$$\zeta_t F^i_{lt} + (1-\overline{\tau})\beta^t\mu u_{lt}\frac{\partial(\pi_t/w_t)}{\partial l^i_t} = \zeta^l_t,\ i = 1,..n \tag{4.4}$$

The first question is whether aggregate production efficiency holds i.e. whether $\theta^i = 1$. It is easy to check[17] that $\frac{\partial(\pi_t/w_t)}{\partial k^i_t}, \frac{\partial(\pi_t/w_t)}{\partial l^i_t}$ cannot be written $\kappa_i F^i_{kt},\ \kappa_i F^i_{lt}$ respectively for some common constant κ_i. This implies that we cannot conclude that it is optimal to have (3.13) holding at the tax optimum. This implies in turn, that we can no longer show that τ^s_t should be fully deductible i.e. $\theta^i = 1$. This is not surprising: generally, without 100% taxation of profit, and heterogenous firms, it is well-known that aggregate production efficiency does not hold.

[17] This can be seen from the fact that π_t/w_t depends on k^i_t, l^i_t via the term $(F^i - F^i_k.k^i_t))/F^i_l - l^i_t$; differentiation of the latter with respect to k^i_t, l^i_t generate expressions that are not commonly proportional to F^i_k, F^i_l, even for special cases such as the Cobb-Douglas.

But, with one firm, the question of whether the efficient allocation of capital across firms does not arise. So, to look at the key question of the optimal rate for τ_t^s, we assume just one firm, so that we can abstract from the question of whether τ_t^s can be deductible from firm costs. In fact, without loss of generality we can assume full deductibility, so the cost of capital for the single firm is $\tilde{r}_t = r_t + w_t s^f$, where we replace s^i by s^f to emphasize that there is now a single firm. In this case, we can prove the following analogue of Proposition 3. The proof follows the proof of Proposition 3 closely, except that conditions (4.3),(4.4) replace (3.8),(3.9), and is thus omitted[18].

Proposition 5. *At all dates $t = 1, 2.., \tau_t^r, \tau_t^s$ solve*

$$A_t \left(1 + r_t - w_t s^h + B_t\right) = 1 + (1 - \tau_t^r)(r_t - w_t s^h(1 + \tau_t^s)) \tag{4.5}$$

where A_t is defined in Proposition 3, and

$$B_t = w_t(1 - \overline{\tau})\frac{1}{1 + H_{lt}}\left(\frac{v_{gt} - \alpha_t}{v_{gt}}\right)\left((s^h + s^f)\frac{\partial(\pi_t/w_t)}{\partial l_t} - \frac{\partial(\pi_t/w_t)}{\partial k_t}\right)$$

Note that if 100% profit taxation is available i.e. $\overline{\tau} = 1$, then (4.5) reduces to (3.17). As before, τ_t^r, τ_t^s are not uniquely determined from this single condition. However, we can proceed as above, by focussing on a solution where the interest income tax and the spread tax are equal. Assume a steady state, so that $A_t = 1$. Also, assume that the two taxes are equal[19] i.e. $\tau^r = \tau^s/(1 + \tau^s)$. Substituting this into (4.5) gives

$$\frac{\tau^s}{1 + \tau^s} = \tau^r = (1 - \overline{\tau})\frac{1}{1 + H_l}\left(\frac{v_g - \alpha}{v_g}\right)\frac{w}{r}\left(\frac{\partial(\pi/w)}{\partial k} - (s^h + s^f)\frac{\partial(\pi/w)}{\partial l}\right) \tag{4.6}$$

This is quite an intuitive formula. First, when 100% profit taxation is available i.e. $\overline{\tau} = 1$, it reduces to $\tau^r = \tau^s = 0$, consistently with Proposition 4. Second, τ^r is non-zero only when it is socially desirable to tax more i.e. $\frac{v_g - \alpha}{v_g} > 0$. Third, without transactions costs, the sign of τ_t^r is the same as the sign of $\frac{\partial(\pi/w)}{\partial k}$.

The intuition is as follows. If taxation is distortionary at the margin the government would like to tax profit (more), as it is a non-distortionary source of tax revenue; more precisely, it would like to reduce π_t/w_t, as this relaxes the implementability constraint. It cannot do this directly. However, if a reduction in k_t reduces π_t/w_t, this can be done indirectly via taxing capital income. This is in line with results in Stiglitz and Dasgupta(1971) for the case of a static (one-period) economy. It is also similar to Correia

[18] It is available on request.

[19] The spread tax is expressed as a percentage of the cost of intermediation services, gross of the tax, and thus must be divided through by $1 + \tau^s$ to make it comparable to τ^r.

(1996), who shows, in the context of the Chamley model, that if an untaxed (or incompletely taxed) third factor of production is complementary to capital in the production function, then capital income should be taxed positively[20]. With transactions costs, an additional term $-(s^h + s^f)\frac{\partial(\pi/w)}{\partial l}$ is added. Fourth, taking the ratio of $\frac{\tau^s}{1+\tau^s}$ in (4.6), and $\frac{\tau^c}{1+\tau^c}$ in (3.12), and cancelling the common factor $\frac{1}{1+H_l}\left(\frac{v_g-\alpha}{v_g}\right)$, we can see that τ^s and τ^c will differ, although we cannot say generally which will be larger.

To get a feel for when τ^r will be positive, consider two cases. First, suppose that the production function is linear in labour, i.e. $F(k,l) = F(k) + l$. Then, in this case, it is easily checked that $\pi/w = F(k) - F'(k)k$, so

$$\frac{\partial(\pi/w)}{\partial k} - (s^h + s^f)\frac{\partial(\pi/w)}{\partial l} = -F''(k)k > 0$$

and so the interest income tax and spread tax are both positive. On the other hand, if the production function is Cobb-Douglas i.e. $F(k,l) = k^\alpha l^\beta$, $\alpha + \beta < 1$, then $\pi/w = (1-\alpha-\beta)\beta l$, so

$$\frac{\partial(\pi/w)}{\partial k} - (s^h + s^f)\frac{\partial(\pi/w)}{\partial l} = -(s^h + s^f)(1-\alpha-\beta)\beta < 0$$

and so the interest income tax and spread tax are both negative. To summarize:

Proposition 6. *Assume just one firm, and that the economy is in the steady state. Then, an optimal tax scheme is to tax interest income and financial intermediation services at the same rate given by (4.6). This common tax can be positive or negative, depending on the properties of the production function.*

5. Other Extensions

5.1. Unitary Taxation of Wage and Capital Income

Another interesting special case is where there is unitary taxation of wage and capital income. This is relevant because in practice, many countries tax wage and non-wage income in a unitary way, according to a single progressive schedule[21]. In this simple

[20] The exact conditions for $\tau_t^r > 0$ are somewhat different here to Correia(1996), as we assume that the third factor of production, which gives rise to pure profit, it is fixed supply, whereas in Correia(1996), it is in elastic supply. The latter assumption imposes an additional implementability contrsaint on the optimal tax problem.

[21] A well-known exception here is the the dual income tax system which levies a proportional tax rate on all net income (capital, wage and pension income less deductions) combined with progressive tax rates on gross labour and pension income. The dual income tax was first implemented in the four Nordic countries (Denmark, Finland, Norway and Sweden) through a number of tax reforms from 1987 to 1993.

model, with linear taxes, unitary taxation of income simply means taxing both wage and capital income at the same rate. Here, there is no explicit wage income tax; it is implicitly defined via the budget constraint via $\tau_t^w = \frac{\tau_t^c}{1+\tau_t^c}$. That is, if the government replaced a consumption tax at rate τ_t^c by a wage income tax at rate $\frac{\tau_t^c}{1+\tau_t^c}$ the real equilibrium in the model would be unchanged. So, with unitary taxation ($\tau_t^w = \tau_t^r$), we can replace $1 - \tau_t^r$ by $\frac{1}{1+\tau_t^c}$ in (3.17) to get:

$$(1 + \tau_t^c)A_t \left(1 + r_t - w_t s^h\right) = 1 + \tau_t^c + (r_t - w_t s^h(1 + \tau_t^s)) \tag{5.1}$$

This is most easily analyzed in the steady state when $A_t = 1$. Then, (5.1) can be easily solved for τ_t^s to give the following result:

Proposition 7. *Suppose that there is unitary taxation of wage and non-wage income. Then, at the steady state, optimal taxes τ^c, τ^s satisfy*

$$\tau^s = \left[1 - \frac{r}{ws^h}\right] \tau^c < \tau^c$$

So, we see that with unitary income taxation, the tax rate on intermediation services is proportional to the tax on consumption, but is at a lower rate, and could be negative.

5.2. Endogenizing Savings Intermediation Services

We have, so far, treated the service of savings intermediation by banks in rather "black box" fashion. In particular, we have treated s^i, the amount of intermediation services per unit of capital supplied to firm i, as exogenous. However, it is clear that banks supply several different kinds of intermediation services, notably liquidity services (Diamond and Dybvig(1983)), and monitoring services (Diamond (1991), Besanko and Kanatas(1993), Holmstrom and Tirole(1997)).

In this version of the paper, we do not attempt provide a fully microfounded version of these kinds of intermediation services, for several reasons. First, it is technically difficult to embed some explicit models of intermediation services into the dynamic optimal tax framework. Second, the payoff from doing so in terms of increased insights is not really proportionate to the increased complexity. In the end, bank intermediation activity, when explicitly modelled, may (or may not) have spillovers on the rest of the economy. If there are spillovers, then the optimal tax is a Piguovian one to internalise these spillovers. Ultimately, this is because the government can use the interest income tax to control the household's marginal rate of substitution between present and future consumption, and so any tax on intermediation services is a free instrument which can be used to internalize externalities arising from bank activity.

These general points are illustrated in a previous version of the paper, Lockwood(2010), where s^i is interpreted as the level of bank monitoring, along the lines of Holmstrom and Tirole(1997). In their framework, without monitoring, bank lending to firms is impossible, because the informational rent they demand is so high that the residual return to the bank does not cover the cost of capital. So, as monitoring is costly, the socially efficient level of monitoring is that level which just induces to bank to lend. In the case where the bank is competitive, i.e. where firm chooses the terms of the loan contract subject to a break-even constraint for the bank, an assumption commonly made in the finance literature, this is also the equilibrium level of monitoring. In this case, savings intermediation should *not* be taxed, because doing to will violate production efficiency, as in the case with heterogenous firms and a fixed amount of intermediation services per unit of savings. But, in the case where the bank is a monopolist i.e. it chooses the contract, it will generally choose a *higher* level of monitoring than this, in order to reduce the firm's informational rent. So, in this case, the optimal tax is a positive Pigouvian tax, set to internalize this negative externality.

6. Conclusions

This paper has considered the optimal taxation of financial intermediation services in a dynamic economy, when the government can also use wage and capital income taxes. The objective of this paper has been to take a fresh look at this question, from a tax design point of view. We set up and solve the tax design problem in a dynamic general equilibrium model of the Chamley(1986) type, where the government chooses taxes savings intermediation, as well as the usual taxes on consumption (or equivalently, wage income) and income from capital, to finance a public good, and where financial intermediaries, in the form of banks, are explicitly modelled.

Our main finding is that at the steady state, the only administratively simple optimal tax structure is to set the tax rate on financial intermediation services equal to the tax rate on capital income. When 100% profit taxation is available, this common rate is zero; in the more realistic case of less than 100% profit taxation, this common rate can be positive or negative, and is generally different from the optimal tax on consumption.

There are several obvious limitations of the analysis. The first and most fundamental, is that the role of banks is not microfounded. This is clearly a topic for future work: some preliminary results in this direction are described in Section 5.2. The second is the restriction to linear income taxation. The classic result of Atkinson and Stiglitz tells us that with non-linear income taxation, commodity taxation is redundant, and more recently, Golosov et. al. (2003) has recently shown that this result generalizes to a dynamic economy. Their result would apply, for example, in a version of our model where households differ in skill levels, and without any financial intermediation. It is a topic for future work to introduce financial intermediation in this environment.

7. References

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A. Appendix

Proof that $\mu \geq 0$. Suppose to the contrary that $\mu < 0$ at the optimum. Then, as $u_{llt} < 0, u_{clt} \leq 0,\ H_{lt} > 0$ from (3.10). So, from (3.10),

$$W_{lt} = u_{lt}(1 + \mu(1 + H_{lt})) > u_{lt} \tag{A.1}$$

But from (3.5),(3.7), (3.9), (2.8):

$$-\beta^t W_{lt} = \zeta_t^l = \zeta_t F_{lt}^i = \beta^t v_{gt} w_t \tag{A.2}$$

So, combining (A.1), (A.2), we see that

$$v_{gt} < -u_{lt}/w_t \tag{A.3}$$

But, (A.3) says that utility could be increased if 1$ of spending on the public good were returned to the household as a lump-sum, contradicting the optimality of the policy. □
Proof of Proposition 1. From (3.4), (3.5), (3.9),(2.8), we have

$$-\frac{W_{ct}}{W_{lt}} = -\frac{u_{ct}}{u_{lt}}\frac{1 + \mu(1 + H_{ct})}{1 + \mu(1 + H_{lt})} = \frac{\zeta_t}{\zeta_t^l} = \frac{1}{F_{lt}^i} = \frac{1}{w_t} \tag{A.4}$$

And, from (2.3),(2.4):

$$-\frac{u_{ct}}{u_{lt}} = \frac{1 + \tau_t^c}{w_t} \tag{A.5}$$

So, combining (A.4), (A.5), we get, after some rearrangement:

$$\frac{\tau_t^c}{1 + \tau_t^c} = \frac{\mu(H_{lt} - H_{ct})}{1 + \mu(1 + H_{lt})} \tag{A.6}$$

Also, from (A.2), we have:

$$-W_{lt} = -u_{lt}(1 + \mu(1 + H_{lt})) = w_t v_{gt}$$

which implies

$$\mu = \frac{1}{1 + H_{lt}}\frac{v_{gt} - \alpha_t}{\alpha_t}, \quad \alpha_t = -\frac{u_{lt}}{w_t} \tag{A.7}$$

Combining (A.6),(A.7) to eliminate μ, and rearranging, we get (3.12) as required. □
Proof of Proposition 3. From (3.4), (3.11), we get

$$\frac{\beta^{t-1} W_{c,t-1}}{\beta^t W_{ct}} = \frac{1}{\beta \tilde{A}_t}\frac{u_{c,t-1}}{u_{ct}} = \frac{\zeta_{t-1}}{\zeta_t} \tag{A.8}$$

where $\tilde{A}_t = \frac{(1+\mu(1+H_{ct}))}{(1+\mu(1+H_{ct-1}))}$. Next, using (4.3),(4.4) to eliminate ζ_t^k, ζ_t^l in (3.6), and also using (3.7), we get

$$\zeta_t F_{kt}^i - s^i \zeta_t F_{lt}^i = \zeta_{t-1} + s^h \zeta_t F_{lt}^i \tag{A.9}$$

So, then from (A.9),

$$\begin{aligned} \frac{\zeta_{t-1}}{\zeta_t} &= (F_{kt}^i - F_{lt}^i(s^i + s^h)) \qquad \text{(A.10)} \\ &= w_t(\frac{F_{kt}^i}{F_{lt}^i} - (s^i + s^h)) \end{aligned}$$

But now, from (2.8) and (3.15) with $\theta^i = 1$, we also have

$$\frac{F_{kt}^i}{F_{lt}^i} = \frac{1 + r_t}{w_t} + s^i \tag{A.11}$$

Combining (A.10) and (A.11) gives

$$\frac{\zeta_{t-1}}{\zeta_t} = w_t(\frac{1 + r_t}{w_t} - s^h) \tag{A.12}$$

Next, combining (A.12) and (A.8), we get:

$$\frac{u_{c,t-1}}{u_{ct}} = \beta \tilde{A}_t w_t(\frac{1 + r_t}{w_t} - s^h) \tag{A.13}$$

Finally, from (2.3),(2.5), (2.8), we get:

$$\begin{aligned} \frac{u_{c,t-1}}{u_{ct}} &= \beta(1 + \rho_t)\frac{1 + \tau_{t-1}^c}{1 + \tau_t^c} \qquad \text{(A.14)} \\ &= \beta\left(1 + (1 - \tau_t^r)(r_t - w_t s^h(1 + \tau_t^s)\right)\frac{1 + \tau_{t-1}^c}{1 + \tau_t^c} \end{aligned}$$

Combining (A.13), (A.14), and eliminating $\frac{u_{c,t-1}}{u_{ct}}$, we get

$$A_t w_t(\frac{1 + r_t}{w_t} - s^h) = 1 + (1 - \tau_t^r)(r_t - w_t s^h(1 + \tau_t^s)) \tag{A.15}$$

where $A_t = \tilde{A}_t \frac{1+\tau_t^c}{1+\tau_{t-1}^c} = \frac{(1+\mu(1+H_{ct}))}{(1+\mu(1+H_{ct-1}))}\frac{1+\tau_t^c}{1+\tau_{t-1}^c}$ as required. $\square$