NEGATIVE VOTERS? ELECTORAL COMPETITION WITH LOSS-AVERSION∗

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This paper studies the effect of voter loss aversion in preferences over both candidate policy platforms and candidate valence on electoral competition. Loss-aversion over platforms leads to both platform rigidity and reduced platform polarisation, whereas loss-aversion over valence results in increased polarisation and the possibility of asymmetric equilibria with a self-fulfilling (dis)-advantage for the incumbent. The results are robust to a stochastic link between platforms and outcomes; they hold approximately for a small amount of noise. A testable implication of loss-aversion over platforms is that incumbents adjust less than challengers to shifts in voter preferences. We find some empirical support for this using data for elections to the US House of Representatives.

There is now considerable evidence that citizens place greater weight on negative news than on positive when evaluating candidates for office or the track records of incumbents. In the psychology literature, this is known as negativity bias.1 For example, several studies find that US presidents are penalised electorally for negative economic performance but reap fewer electoral benefits from positive performance (Bloom and Price, 1975; Lau, 1985; Klein, 1991).

Similar asymmetries have also been identified in the UK and other countries. For example, for the UK, Soroka (2006) finds that citizen pessimism about the economy, as measured by a Gallup poll, is much more responsive to increases in unemployment than falls. Kappe (2018) uses similar data to explicitly estimate a threshold or reference point value below which news is ‘negative’, and finds similar results. Nannestad and Paldam (1997) find using individual-level data for Denmark that support for the government is approximately three times more sensitive to a deterioration in the economy than an improvement.

Here, to further motivate our study, we present new US evidence that there is voter negativity bias, by showing that support for US state governors varies asymmetrically with improvements and declines in economic conditions.2 An illustration of our findings is given in Figure 1. One can observe that reductions in unemployment, the region to the left of the dashed red vertical line, have at best a weak impact on incumbents’ fortunes at the next election. Increases in unemployment,
Fig. 1. Incumbents’ Vote Share and Unemployment.
Notes: The vote share is the county vote share of the incumbent in the gubernatorial election. The change in the county unemployment rate is the change over the two years preceding the year of election. The underlying distribution of both variables is displayed as a binned scatter plot, with each circle representing a quintile of the joint distribution. The solid blue lines describe the estimated regression coefficients above and below the reference point, and the dotted lines the associated confidence intervals.

to the right of the red line, are however associated with a marked reduction in the expected vote share.

In this paper, we think of this negativity bias as arising from loss-aversion over either the policies of parties or the quality of politicians. We then explore the implications of voter loss-aversion for electoral competition. Specifically, we study a simple Downsian model where voters care both about parties’ policy choices and their competence in office (valence). Moreover, they are loss-averse either in the policy or valence dimension. There are two parties that choose policy platforms and that care about both policy outcomes and holding office. One of the parties is the incumbent, and their winning platform from the previous period, taken as fixed, is the voter’s policy reference point. The valence of the incumbent party is common knowledge, but the valence of the challenger is determined by random draw. Each of these valence levels is evaluated relative to a fixed reference level by the voter. Our assumption that in the policy dimension, the reference point is the status quo is widely made in the literature on loss-aversion applied to economic situations, and seems realistic as benefits and costs of political reforms are normally assessed relative to existing policies.3

Without loss-aversion, this setting is similar to the well-known one of Wittman (1983), where in equilibrium, parties set platforms by trading off the probability of winning the election against the benefits of being closer to their ideal points. Our model differs from Wittman’s in that in his model, this trade-off is generated by parties being uncertain about the position of the median voter, whereas in our model, it is generated by probabilistic voting, due to the challenger’s valence being unknown. As explained subsequently, the latter is required for loss-aversion to have any bite.

3 For example, de Meza and Webb (2007) for a principal–agent problem, Freund and Özden (2008) in the context of lobbying on trade policy, and Alesina and Passarelli (2015) for direct democracy; all assume a status quo reference point. We have investigated the case of a forward-looking reference point as in Kőszegi and Rabin (2006) and the results are available upon request.
We begin by showing that loss-aversion in the policy dimension has a number of implications for electoral competition. First, there is platform rigidity; for a range of values of the status quo, one party will choose the status quo, and the other will choose a platform on the other side of the median voter’s ideal point to the status quo, and equidistant from the ideal point of the median voter, regardless of other parameters. In this case, the election outcome is insensitive to small changes in other parameters, such as the weight that political parties place on office, the level of uncertainty about the challenger’s competence, or shifts in the ideal points of the political parties. Note, however, that platform rigidity is not the same as status quo bias, as the election outcome may be a long way from the status quo. Second, there is a moderation effect of loss-aversion; generally, the gap between equilibrium party platforms is smaller than in the absence of loss-aversion.

With loss-aversion over valence, the results are rather different. If the reference valence is low, loss-aversion has no effect on the symmetric equilibrium. If, on the other hand, the reference valence is high, platforms are more polarised than without loss-aversion. Further, even though the structure of the model is symmetric, depending on the value of the reference point, there are asymmetric equilibria where either the incumbent or the challenger has a self-fulfilling advantage and can set a more extreme platform without sacrificing the probability of re-election.

We also explore the robustness of these results to noise in the mapping from party platform choices to voter payoffs. For example, a party may propose a tax, but due to changes in political support or the state of the macroeconomy, is only able to set that tax plus some noise. In our setting with loss-aversion in the policy dimension, these shocks may matter because it is possible that the uncertainty might smooth the kink in the election probability as a function of platforms. We show that as long as the support of the noise is small, our main results will apply in an approximate sense.

The question then arises as to whether our model can generate distinctive empirical predictions. We show that with loss-aversion in the policy dimension, but not the valence dimension, shifts in voter preferences have a particular effect on platforms. Specifically, with this kind of loss-aversion, there is asymmetric adjustment — the incumbent’s platform will adjust by less than the challenger’s platform. In other words, loss-aversion generates a particular kind of asymmetry; incumbents adjust less than challengers to voter preference shifts. This prediction is potentially testable, if we can measure preference shifts and shifts in the ideological positions of candidates.

We then take this prediction to a setting where both of these things can be measured, namely elections to the US House of Representatives for 1980–2012. To measure preference shifts, we use a standard measure, namely, the change in the Democrat vote share in the presidential election in that district. To measure the ideological positions of candidates, we use a new data set introduced by Bonica (2014b) that contains estimates of the platforms of all candidates, winners and losers, in elections to the US Congress based on the campaign donations they received. Crucially, compared to the more common DW-NOMINATE data of Poole and Rosenthal (2006), the Bonica data provide time-varying estimates of both winning and losing candidates’ platforms.

Employing these data, we find robust evidence that incumbent parties are significantly less responsive to shifts, as predicted by the theory. In particular, we control for a variety of district

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4 It is of course possible that other models could generate asymmetric adjustment. This is discussed further following Proposition 4 in Section 6, where we rule out several other explanations, such as a simple version of incumbency advantage or loss-aversion in the competence dimension.
and candidate fixed effects. We do not claim that our theory is the only possible explanation of this finding, but the results are certainly consistent with voter loss-aversion.

The remainder of the paper is organised as follows. Section 1 reviews related literature, Section 2 lays out the model and Section 3 derives the results for loss-aversion over policy platforms. Section 4 studies the case of loss-aversion over valence. Section 5 then explains how the results are robust to introducing noise, as defined above, into the model. Section 6 explains how loss-aversion offers a distinctive prediction about how incumbents and challengers respond to preference shifts. Section 7 discusses the data we use to test our main hypotheses, our empirical strategy, and results. Finally, Section 8 concludes.

1. Related Literature

The closest paper to ours is Alesina and Passarelli (2015), henceforth AP. This paper studies loss-aversion in a direct democracy setting, where citizens vote directly in a referendum on the size of a public project or policy. However, to our knowledge, ours is the first work to study the effect of loss-aversion in a representative democracy setting. In AP, citizens vote directly on a one-dimensional policy describing the scale of a project, which generates both costs and benefits to voters. In this setting, for loss-aversion to play a role, the benefits and costs of the project must be evaluated relative to separate reference points. This is because if loss-aversion applies to the net benefit from the project, the status quo cannot affect the ideal point of any voter. We do not need this construction because in our setting, voters compare the utility from policy positions to party valences. So, loss-aversion over platforms has ‘bite’ in our model via an entirely different mechanism to theirs — that is, via the voters’ comparison of utility from policy and party valence rather than via multiple reference points.

In their setting, AP find a number of features of the equilibrium which are similar to ours, for example, status quo bias (for a range of values of the median voter’s ideal point, the policy outcome is equal to the status quo), and a form of policy moderation (an increase in loss-aversion increases the number of voters who prefer the status quo). For the case of loss-aversion over platforms, several of our results are similar in spirit to these, although the details differ substantially. In addition, we study the case of loss-aversion over valence which obviously does not arise with direct democracy. Finally, our main empirical prediction, that incumbents adjust less than challengers to voter preference shifts has no counterpart in their analysis.

A small number of other papers study electoral competition with voter behavioural biases. Razin and Levy (2015) study a model of electoral competition in which the source of the polarisation in voters’ opinions is ‘correlation neglect’, that is, voters neglect the correlation in

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5 Their paper is contemporaneous with the working paper version of our paper (Lockwood and Rockey, 2015).
7 One way of seeing this is to note that if we introduce political parties and electoral competition into the AP model, then, absent any other changes, the classic Downsian result would emerge, i.e., parties would converge to the median voter’s ideal point.
8 The relationship between our notions of platform rigidity and platform moderation and theirs is discussed in more detail below.
9 There are also a number of recent papers that consider the effects of voter biases in non-Downsian settings, either where party positions are fixed, or where policy can be set ex post, e.g., political agency settings. However, these papers are clearly less closely related to what we do. For example, Ashworth and Bueno De Mesquita (2014) and Lockwood (2015) consider deviations from the full rationality of the voter in a political agency setting. Ortoleva and Snowberg (2013) show theoretically that the cognitive bias of correlation neglect can explain both voter overconfidence and ideological polarisation. Levy and Razin (2015) find that the cognitive bias of correlation neglect can improve outcomes for voters. Gould and Rablen (2019) finds evidence of loss-aversion in politicians, rather than voters.

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their information sources. Their main finding is that polarisation in opinions does not necessarily translate into platform polarisation by political parties compared with rational electorates. This can be compared to our finding that loss-aversion might reduce or magnify polarisation in platforms, depending on the dimension in which loss-aversion occurs.

Matějka and Tabellini (2015), study how voters optimally allocate costly attention in a model of probabilistic voting. They show that in equilibrium, extremist voters are more influential and public goods are under-provided, and policy divergence is possible, even when parties have no policy preferences. Bisin et al. (2015) consider Downsian competition between two candidates in a setting where voters have self-control problems and attempt to commit using illiquid assets.\(^\text{10}\)

Our empirical work in Section 7 is related to that of Adams et al. (2004) and Fowler (2005). In particular, both study party platform responses to changes in the position of the median voter. Adams et al. (2004) is a purely empirical study, which uses data on party positions and voter preferences in eight West European countries over the period 1976–98, from the Comparative Manifesto Project and Eurobarometer respectively. They find that a party only responds to disadvantageous moves in public opinion for that party.

Fowler (2005) considers elections to the US Senate over the period 1936–2010. His theoretical model shows that parties learn about voter preferences from election results, and consequently predicts that Republican (Democratic) victories in past elections yield candidates who are more (less) conservative in subsequent elections, and the effect is proportional to the margin of victory. This is a rather different hypothesis to the one we test, which concerns the effects of shifts in voter preferences before elections.

Also related is the substantial empirical literature on incumbency advantage. This is related because in our empirical work, we control for incumbency directly. This is somewhat different to the conventional regression discontinuity (RD) design to identify incumbency advantage (Lee, 2008). This is because we are not concerned with explaining the probability that the incumbent wins, but how incumbents change their platforms relative to non-incumbents.

2. The Model

2.1. The Environment

There are two parties, \(L\) and \(R\), and a finite set of voters who interact over two periods, \(t = 0, 1\). The number of voters, \(n\), is odd. We take the interaction in the first period as predetermined. Specifically, we suppose that at \(t = 0\), one of the parties, \(I \in \{L, R\}\), won the election and set a platform \(x_0\) in the policy space, \(X \equiv [-1, 1]\), where \(I, x_0\) are exogenously fixed. Thus, party \(I\) is the incumbent at \(t = 1\). At \(t = 1\), the two parties, \(L\) and \(R\), choose platforms \(x_L, x_R\) in the policy space, \(X\). They are assumed to be able to commit to implement these platforms. Thus, the basic framework is Downsian competition. However, parties are also described by a party valence characteristic, \(v\). Our primary interpretation of \(v\) will be as competence, although it could capture other things such as the charisma of the candidate, etc.

\(^\text{10}\) Passarelli and Tabellini (2017) is also somewhat related; there, citizens belonging to a particular interest group protest if government policy provides them with utility that is below a reference point that is deemed fair for that interest group. In equilibrium, policy is distorted to favour interest groups who are more likely to protest or who do more harm when they riot. However, in their setting, there is no voting, so the main shared feature between that paper and ours is that we both consider the role of reference points in social choice.
2.2. Order of Events and Information Structure

The valence of the incumbent party is assumed to be common knowledge at the beginning of period 1 and is normalised to zero. The idea is that all agents have had a chance to observe the incumbent party’s performance in office in the previous period.11

Within period 1, the order of events is as follows. First, parties $L, R$ simultaneously choose their platforms. Then, $v_C$, the valence of the challenger, is drawn from a uniform mean zero distribution, $F$, with support $[-\frac{1}{2\rho}, \frac{1}{2\rho}]$. As we will see, the parameter $\rho$ measures the responsiveness of the median voter to policy changes by the parties. In Online Appendix C, we show how our analysis generalises to other symmetric mean-zero distributions of $v_C$. Then, having observed $x_L, x_R, v_C$, all voters vote simultaneously for one party or the other. We will assume that voters do not play with weakly dominated strategies; with only two alternatives, this implies that they vote sincerely.

There are two aspects of this timing that deserve comment. First, voters are assumed to observe the challenger’s valence before voting. The idea here is that an election campaign and scrutiny by the media give voters additional information about the competence or fitness for office of the challenger before the election. This assumption could be relaxed without changing the results by allowing the voter to observe some informative signal of $v_C$ before voting.

Second, we are assuming that the valence of the challenger party is not known to either party at the point when platforms are chosen. The purpose of this assumption is to create a smooth trade-off between the probability of winning the election and the closeness of the party’s platform to its ideal point. In this respect, it plays the same role as imperfect knowledge about the position of the median voter in the Wittman model.

2.3. Voter Payoffs

Payoffs over policy. Following Osborne (1995), we assume that utility over platforms $x \in X$ for voter $i$ is given by $u_i(x) = -\ell(|x - x_i|)$ where $\ell$ is twice continuously differentiable with $\ell' > 0, \ell'' \geq 0$. So, $x_i$ is the ideal point of voter $i$. We rank voters by their ideal points, i.e., $-1 < x_1 < x_2 < \ldots < x_n < 1$. We assume that voter $m = \frac{n+1}{2}$ has an ideal point $x_m = 0$. As we shall see shortly, this voter will be the median voter in the usual sense, i.e., will be decisive in any election.

Following Kőszegi and Rabin (2006; 2007; 2009), we specify the utility over platforms for voter $i$ as:

$$u_i(x; x_0) = \begin{cases} u_i(x) - u_i(x_0), & u_i(x) \geq u_i(x_0) \\ \lambda(u_i(x) - u_i(x_0)), & u_i(x) < u_i(x_0). \end{cases}$$

That is, the parameter $\lambda > 1$ measures the degree of loss-aversion, and the previous period’s platform $x_0$ is the reference point. The assumption that $\lambda$ is the same for all voters is made just for convenience and could be relaxed.

Note that we have assumed that voters are ‘backward-looking’ in that the reference point is the status quo, $x_0$. The main reason for this is to ensure that voter behaviour is consistent with the evidence of Figure 1 and the Online Appendix A, i.e., that voters evaluate positive and negative changes from the status quo asymmetrically. However, there are also other reasons why this is a case of interest. For example, in a recent experiment, Heffetz and List (2014) finds there is little evidence for a forward-looking reference point.

11 This assumption is also made, for example, by Bernhardt et al. (2011).
Payoffs over valence. We will assume that all voters have a common reference point, \( v_0 \), for valence, which need not be equal to the incumbent’s observed valence. This seems plausible because voters might form their idea of what an ‘acceptable’ level of valence is from a variety of sources. In addition, as we shall see, restricting \( v_0 \) to be equal to the incumbent’s valence of zero gives rise to non-existence of equilibrium.

If a candidate for election has valence \( v \), we assume that all voters have a valence payoff from the candidate, if elected, of

\[
\phi(v; v_0) = \begin{cases} 
    v - v_0, & v \geq v_0, \\
    \beta(v - v_0), & v < v_0,
\end{cases}
\]  

(2)

where \( v_0 \in \left(-\frac{1}{2\rho}, \frac{1}{2\rho}\right) \) is the reference level of valence and \( \beta \geq 1 \), with a strict inequality if there is loss-aversion. In particular, the valence payoff from the incumbent is \( \phi(0; v_0) \).

Overall payoffs. The overall payoff to voter, \( i \), from a party with platform, \( x \), and valence, \( v \), is

\[
u_i(x; x_0) + \phi(v; v_0).
\]

(3)

We see from (3) that the trade-off between the two dimensions changes discontinuously if the outcome in either the policy or valence dimension passes the relevant reference point. This change in the trade-off ultimately drives all of our results.

2.4. Party Payoffs

As is standard, parties have a payoff to holding office, denoted by \( M \). Parties are also assumed to have policy preferences, with the \( L \) party having an ideal point of \(-1\), and party \( R \) an ideal point of \( 1 \). Payoffs of the \( L \) and \( R \) party members are then \( u_L(x) \equiv -\tilde{\ell}(|x + 1|) \), \( u_R(x) \equiv -\tilde{\ell}(|x - 1|) \), respectively, where \( \tilde{\ell} \) is twice differentiable, strictly increasing, symmetric and convex in \( |x - x_i| \) and \( \tilde{\ell}(0) = \tilde{\ell}'(0) = 0 \).

Note that we allow the loss function of the parties, \( \tilde{\ell}(.) \) to be different from that of the voters, \( \ell(.) \). This specification allows for parties to be risk-neutral (\( \tilde{\ell}'' = 0 \)) or strictly risk-averse (\( \tilde{\ell}'' > 0 \)) over policy outcomes, separate to any assumptions about risk attitudes of voters. Note also that parties are assumed not to be loss-averse; party loss-aversion raises a number of new issues which are not addressed in this paper.

So, expected payoffs for the parties are calculated in the usual way as the probability of winning, times the policy payoff plus \( M \), plus the probability of losing times the resulting policy payoff. For parties \( R, L \) respectively, this gives

\[
\pi_R = p(u_R(x_R) + M) + (1 - p)u_R(x_L),
\]

\[
\pi_L = (1 - p)(u_L(x_L) + M) + pu_L(x_R),
\]

(4)

where \( p \) is the probability that party \( R \) wins the election and is defined below. As we shall see, \( p \) depends not only on the platforms \( x_L, x_R \), but also on the voter reference point, \( x_0 \).

2.5. Win Probabilities

From now on, without loss of generality, we assume that the incumbent party is party \( R \). Here, we characterise the probability, \( p \), that party \( R \) wins the election. Also, from now on, set \( v_C \equiv v \).
We have assumed that all voters do not use weakly dominated strategies, implying that they vote sincerely. So, from (3), any voter \( i \) will vote for party \( R \), given platforms \( x_L, x_R \), if and only if

\[
u_i(x_R; x_0) + \phi(0; v_0) \geq u_i(x_L; x_0) + \phi(v; v_0).
\]

Now, note from (1) that even with loss-aversion, the policy payoffs, \( u_i(x; x_0) \) are single-peaked in \( x \) for a fixed \( x_0 \). It follows immediately that the median voter is decisive.\(^{12}\) So, the probability that party \( R \) wins the election is the probability that the median voter votes for \( R \).

From (5), this is the probability that the utility gain to the median voter in the policy dimension from voting for the incumbent exceeds any valence advantage of the challenger i.e.

\[
p = \Pr(\phi(v; v_0) - \phi(0; v_0) \leq u_m(x_R; x_0) - u_m(x_L; x_0)).
\]

From now on, we can focus only on the median voter, and we can therefore drop the ‘m’ subscripts, so we write \( u_m(x) \equiv u(x) \) and \( u_n(x; x_0) \equiv u(x; x_0) \) for the intrinsic and gain–loss utility of the median voter respectively. Then, given (6), we can explicitly calculate the win probabilities as required.

2.6. Assumptions

So far, we have allowed for a wide class of voter and party loss functions. To proceed further, we need these elements to satisfy some technical assumptions.

The first assumption is that party \( R \)'s election probability \( p \) is strictly between 0 and 1 for all \( x_R, -x_L \in [0, 1], x_0 \in [-1, 1] \). For this, we require that for the median voter, the highest possible utility gain in the policy dimension from re-electing party \( R \) is smaller than the highest possible value of the valence loss from electing party \( R \). The latter is \( \frac{\partial}{\partial x} \). The former is the gain when \( x_R = 0, x_L = -1 \). This is largest when the status quo policy is \( x_0 \), giving a gain to re-election of \( \lambda \) times zero minus \( 1 \), or simply \( \lambda \ell(1) \). So, our first assumption is:

\[\text{A1. } \frac{\partial}{\partial p} > \lambda \ell(1).\]

Next, we will characterise equilibrium by first-order conditions for the choice of \( x_L, x_R \) by the parties. For this to be valid, we require that the party payoffs, \( \pi_L, \pi_R \), defined above in (4), are strictly concave in \( x_L, x_R \), respectively. Sufficient conditions for this are derived in Online Appendix C, for the cases of loss-aversion over either policy and valence separately, as we analyse these cases separately below. In both cases, we allow \( v_C \) to have a general mean zero symmetric distribution, \( F \), and density, \( f \). In the case of loss-aversion over policy, we develop a sufficient condition for concavity which says that the rate of change of the density, \( f' / f \), not be too large. If \( F \) is uniform, as assumed here, this is automatically satisfied. In the case of loss-aversion over valence, we also require that \( \beta \) not be too large.

Finally, we want to rule out the uninteresting case where the incentives to converge to the median voter’s ideal point, zero, are so large that parties set \( x_R = x_L = 0 \) in equilibrium. To rule this out, note from (4), the derivative of (for example) \( \pi_R \) with respect to \( x_R \) at this point is\(^{13}\)

\[
\frac{\partial \pi_R}{\partial x_R} = 0.5u_R'(0) + \frac{\partial p}{\partial x_R} M.
\]

\(^{12}\) To see this, let \( v_m \) be such that \( m \) is indifferent between voting for \( L \) and \( R \), i.e., \( u_m(x_R; x_0) - u_m(x_L; x_0) = v_m \). So, assuming \( x_R > x_L \), single-peakedness implies immediately that (i) \( v < v_m \), all \( i > m \) will vote for \( R \); and (ii) if \( v > v_m \), all \( i < m \) will vote for \( L \). So, when \( v < v_m \), a majority vote for party \( R \), and when \( v > v_m \), a majority vote for party \( L \).

\(^{13}\) It is easily checked from (1) and (6) that \( p \) is differentiable at \( x_R = x_L = 0 \), no matter what the value of \( x_0 \).

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So, as $u_R'(0) > 0$, we see that from (7), to rule out an equilibrium with complete convergence, it is sufficient to ensure that $\frac{\partial p}{\partial x_R} = 0$ at $x_R = x_L = 0$. Intuitively, we need to assume that parties are not penalised by a lower election probability following a small move away from $x_R = x_L = 0$. In turn, for this, it is sufficient to assume that the payoff of the median voter $u(x)$ is differentiable at zero.\textsuperscript{14} So, we will assume:

A2. $u(x)$ is differentiable at zero.

This assumption is satisfied by, for example, the quadratic loss function $u = -x^2$ and many others. One important exception is where the median voter has an absolute value loss function, $u = -|x|$. In this case, $u$ is not differentiable at zero, and so we need to assume that $M$ must also be ‘small enough’ to ensure divergence. It is difficult to write down a general condition for this, but we present an example below where we derive the required condition on $M$.

3. Electoral Competition with Loss-Aversion over Platforms

Here, we assume that $\beta = 1$, ruling out loss-aversion over valence. When $\beta = 1$, the valence difference between the $R$ and $L$ parties simplifies to $\phi(v; v_0) - \phi(0; v_0) = v$, and we know that $v$ is uniformly distributed. So, from (6), we can calculate

$$p = \frac{1}{2} + \rho (u(x_R; x_0) - u(x_L; x_0)).$$

(8)

We also know that the median voter prefers a platform to the reference platform if and only if it is smaller in absolute value than $x_0$. Using this fact, from (8) and (1), we can explicitly calculate the right-hand side of (8) to obtain:

$$p = \frac{1}{2} + \rho \Delta, \quad \Delta = \begin{cases} 
-u(x_R) - u(x_L), & x_R \leq |x_0|, \\
u(x_R) - \lambda u(x_L) + (\lambda - 1)u(x_0), & -x_L \geq |x_0| > x_R, \\
\lambda u(x_R) - u(x_L) - (\lambda - 1)u(x_0), & x_R \geq |x_0| > -x_L, \\
\lambda (u(x_R) - u(x_L)), & -x_L, x_R > |x_0|. 
\end{cases}$$

(9)

Formula (9) tells us that the mapping from platforms to $p$ has four different ‘regimes’. The first and fourth are where both platforms are in the gain or loss domains respectively for the median voter. The second and third are asymmetric cases: for example, the second case is where $x_L$ is large in absolute value, and $x_R$ is small, so that these platforms are in the loss and gain domains respectively.

The key implication of (9) is the following. For a fixed platform of party $L$, loss-aversion induces a kink in the slope of $p$ as a function of $x_R$ at $|x_0|$ and vice versa. To illustrate, Figure 2 shows $p$ as $x_R$ rises from 0 to 1 for a fixed $x_L = 0$, and assuming also $\rho = 1$, $u(x) = -|x|$, so the median voter has absolute value preference values. We see that to the left of this point, a small increase $\Delta$ in $x_R$ decreases $p$ by $\Delta$, and to the right, a small increase in $x_R$ decreases $p$ by $\Delta \lambda$. The intuition is that to the right of the kink point, the median voter’s payoff from $x_R$ is now in the loss domain, so the effect of changes in policy on voting behaviour are now magnified by $\lambda > 1$.

This kink in the win probability drives our results on the effect of loss-aversion. It is also broadly consistent with the empirical findings regarding asymmetric voter responses to macroeconomic

\textsuperscript{14} This is because given that $u(.)$ has a maximum at zero, it must be that $u'(0) = 0$. In turn, if $u'(0) = 0$, from (6), $\frac{\partial p}{\partial x_R} = 0$.

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shifts; in our model, where an economic policy platform yields the voter a lower utility than the status quo, they respond by ‘punishing’ that party.

We begin with the following intermediate result, proved in the Appendix.

**Lemma 1.** Given A2, there exist unique solutions $x^+ > x^- > 0$ to the equations

\begin{align}
0.5u_R'(x^+) + \rho u'(x^+) (u_R(x^+) + M - u_R(-x^+)) &= 0, \\
0.5u_R'(x^-) + \lambda \rho u'(x^-) (u_R(x^-) + M - u_R(-x^-)) &= 0.
\end{align}

It is easily verified that these solutions $x^+, x^-$ describe the symmetric Nash equilibria in the games where the median voter’s payoffs from the platforms are always in the gain or loss domain respectively. For example, $x_R = -x_L = x^+$ is the Nash equilibrium in the first case, which is the benchmark case without loss-aversion. To see this, note that in (10), $0.5u_R'(x) > 0$ is the utility gain for party $R$ from moving away from the median voter’s ideal point, 0. In equilibrium, this is offset by the second term in (10), which is negative as $u'(x) < 0$, and measures party $R$’s loss from a lower probability of winning. Specifically, if party $R$ loses, it loses the office benefit $M$ and suffers a further loss because the opponent’s platform, not its own, is implemented.

Equation (11) has a similar interpretation; the only difference is that the reduction in the probability of winning caused by moving away from the median voter’s ideal point is now larger by a factor of $\lambda$, as the median voter’s policy payoffs are in the loss domain and are thus more heavily weighted relative to valence.

We are now in a position to characterise the equilibrium with loss-aversion. We will focus on symmetric equilibria, which are defined in the usual way. That is, $x_R = -x_L = x^*$ is a symmetric equilibrium if: (a) $x_R = x^*$ maximises $\pi_R$ given $x_L = -x^*$ fixed and $p$ given by (9); (b) $x_L = -x^*$ maximises $\pi_L$ given $x_R = x^*$ fixed and $p$ given by (9).

**Proposition 1.** Assume A1, A2. Then, there always exists a unique symmetric equilibrium. If $x^+ < |x_0|$, then $x_R = -x_L = x^+$ is the unique symmetric equilibrium. If $x^- > |x_0|$, then $x_R = -x_L = x^-$ is the unique symmetric equilibrium. If $x^+ \geq |x_0| \geq x^-$, then $x_R = -x_L = |x_0|$ is the unique symmetric equilibrium. The value $x^-$ is decreasing in $\lambda$, so the interval $[x^-, x^+]$ is increasing in voter loss-aversion, $\lambda$. 

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This baseline result is best understood graphically. Figure 3 shows how the initial status quo maps onto the equilibrium platforms. For convenience of exposition, the figure portrays how the absolute value of the status quo, which is also minus the median voter’s utility from the status quo, maps onto the absolute value of the equilibrium policy platforms. The latter is, of course, the equilibrium platform of the R party and minus the equilibrium platform of the L party.

Note from Proposition 1 and Lemma 1 that in the absence of loss-aversion, the equilibrium platforms are simply \( x_R = -x_L = x^+ \). So, bearing this in mind, Proposition 1 demonstrates that there are two important impacts of loss-aversion, both of which are clear from Figure 3. First, there is platform rigidity; for a range of absolute values of the status quo in the interval \( [x^-, x^+] \), the outcome is insensitive to changes in other parameters, such as the weight \( M \) that political parties place on office or the responsiveness of the median voter to policy, \( \rho \). However, note that platform rigidity is not the same as simple status quo bias; at a given \( x_0 \) in the interval \( [x^-, x^+] \), the election outcome can either be \( x_0 \) or \(-x_0 \). Second, there is a reduced polarization effect of loss-aversion; the equilibrium platforms are both closer to the median voter’s ideal point than in the absence of loss-aversion.

The intuition for these results is the following. First, if \( |x_0| \) is large, i.e., greater than \( x^+ \), then electoral competition effectively takes place in the ‘gain’ domain for the median voter, i.e., where platforms are closer to zero in absolute value than the status quo platform. As a result, the equilibrium outcome is always \( x^+ \), the outcome without loss-aversion. Conversely, if \( |x_0| \) is small, i.e., less than \( x^- \), then electoral competition takes place in the ‘loss’ domain for the median voter. Here, the median voter is more sensitive to platform changes as they evaluate them as losses, so now electoral competition will be more intense, and so the equilibrium involves greater convergence to the median voter’s preferred point of zero, i.e., \( x^- < x^+ \).

Finally, if \( |x_0| \) is intermediate, i.e., between \( x^- \) and \( x^+ \), then political competition must be at the margin between the gain and loss domains. This is easy to see. Suppose for example that competition takes place in the gain domain. Then, equilibrium will be \( x^+ \). But this gives the median voter a payoff lower than the median voter’s reference payoff, because \( x^+ > x_0 \), contradicting the assumption that competition is in the gain domain. So, competition between
the two parties forces them to locate at the point where the election probability is kinked, i.e., at \(|x_0|\). The implication of being at the margin between the two domains is, of course, that the equilibrium platform is exactly at the status quo, i.e., platform rigidity.

The following example shows these effects more explicitly. Assume both the median voter and political parties have absolute value preferences, i.e., \(u(x) = -|x|\), \(u_R(x) = -|1 - x|\), \(u_L(x) = -|1 + x|\). Then, it is easily checked that (10), (11) solve to give

\[
x^+ = \frac{1}{4\rho} - \frac{M}{2}, \quad x^- = \frac{1}{4\lambda\rho} - \frac{M}{2}.
\]

Note that polarisation of platforms (the size of \(x^+\)) is increasing in the variance of the valence shock, and decreasing in the payoff to office, \(M\), as expected. We assume that \(M < \frac{1}{2\lambda\rho}\), so \(x^+, x^-\) are strictly positive. So, for

\[
|x_0| \in \left[ \frac{1}{4\lambda\rho} - \frac{M}{2}, \frac{1}{4\rho} - \frac{M}{2} \right]
\]

(12)

there is platform rigidity, i.e., \(x^* = |x_0|\). Note that as claimed in Proposition 1, the length of the interval in (12) is increasing in \(\lambda\).

4. Electoral Competition with Loss-Aversion over Valence

Here, we explore the consequences of allowing for loss-aversion in the valence dimension. As before, we assume that the \(R\) party is the incumbent. Throughout, we rule out loss-aversion in the policy dimension by assuming that \(\lambda = 1\). It is helpful to define the valence of the challenger relative to the reference point as \(w \equiv v - v_0\). Then, writing out the valence payoffs of the median voter from the challenger and the incumbent explicitly, we get:

\[
\phi_L = \begin{cases} w, & w \geq 0, \\ \beta w, & w < 0. \end{cases} \quad \phi_R = \begin{cases} -v_0, & v_0 \leq 0, \\ -\beta v_0, & v_0 > 0. \end{cases}
\]

(13)

respectively. Moreover, \(\beta \geq 1\), with a strict inequality if there is loss-aversion. So, defining the valence advantage of the challenger as \(\phi_L - \phi_R\), we see from (13) that

\[
\phi_L - \phi_R = \begin{cases} w + \beta v_0, & v_0 > 0, w \geq 0, \\ \beta(w + v_0), & v_0 > 0, w < 0, \\ w + v_0, & v_0 \leq 0, w \geq 0, \\ \beta w + v_0, & v_0 \leq 0, w < 0. \end{cases}
\]

(14)

This is analogous to equation (9); there are four different regimes, depending on \(v_0, w\). For example, if \(v_0 \leq 0, w \geq 0\), both valences are weakly better than the reference value and so \(\beta\) does not appear in the expression. On the other hand, if \(v_0 > 0, w < 0\), both valences are strictly worse than the reference value and so each of \(w, v_0\) is weighted by \(\beta\). There are also two asymmetric cases.
Using (14) above, we can then compute the probability that the incumbent wins. It is helpful to look at this separately for \( v_0 \) positive and negative. If \( v_0 > 0 \), it is shown in the Appendix that

\[
p = \frac{1}{2} + \rho v_0 + \rho \max\left\{ \frac{\Delta u - \beta v_0}{\beta}, \Delta u \geq \beta v_0, \frac{1}{\beta}(\Delta u - \beta v_0), \Delta u < \beta v_0 \right\},
\]

where \( \Delta u \equiv u(x_R) - u(x_L) \) is the policy-related advantage for the incumbent. The key feature of (15) is that the effect of a small policy change by either party on the election probability of the incumbent, as measured by a change in \( \Delta u \), varies with \( v_0 \). For example, when the reference valence is small (\( v_0 < \Delta u/\beta \)), the impact on the election probability on \( p \) is relatively large for \( \rho \), whereas if the reference valence is large, (\( v_0 \geq \Delta u/\beta \)), the impact on \( p \) is relatively small at \( \rho/\beta \).

The intuition is that when the reference valence is small (respectively large), the valence of the challenger, \( w \), is quite likely to be in the gain (loss) domain for the median voter, so the marginal increase in the policy payoff needed to compensate for a reduction in challenger valence is small (large).

Similarly, if \( v_0 \leq 0 \), it is shown in the Appendix that

\[
p = \frac{1}{2} + \rho v_0 + \rho \max\left\{ \frac{\Delta u - \beta v_0}{\beta}, \Delta u \geq \beta v_0, \frac{1}{\beta}(\Delta u - \beta v_0), \Delta u < \beta v_0 \right\}.
\]

This has a similar interpretation to (15).

The parties then maximise their payoffs (4), subject to either (15), (16), depending on the value of \( v_0 \). We start by looking at symmetric equilibrium, which is defined exactly as in the case with loss-aversion over policy. To characterise the symmetric equilibrium, consider the equation

\[
\frac{1}{2} u'_R(x) + \frac{\rho u'(x)}{\beta}(u_R(x) - u_R(-x) + M) = 0.
\]

This is equation (11) above with \( \lambda \) replaced by \( \frac{1}{\beta} \). Also, we will continue to assume A1 and A2. Then, by Lemma 1, we can be sure that (17) has a unique solution, \( x(\beta) \), and following the proof of Proposition 1, we can show that \( x(\beta) \) is strictly increasing in \( \beta \), as \( 1/\beta \) replaces \( \lambda \) in (11). Moreover, note that if \( \beta = 1 \), (17) reduces to (10), so \( x(1) = x^+ \), which is the symmetric equilibrium without any kind of loss-aversion. We can then state:

**Proposition 2.** Assume A1, A2. If \( v_0 > 0 \), there exists a unique symmetric equilibrium \( x_R = -x_L = x^+ \). If \( v_0 < 0 \), there exists a unique symmetric equilibrium \( x_R = -x_L = x^+ \). So, when the reference valence is high, equilibrium platforms are more polarised than in the case without loss-aversion. If \( v_0 = 0 \), there is no pure-strategy symmetric equilibrium.

Overall, this is clearly in contrast to the case with loss-aversion in the policy dimension, where equilibrium platforms are less polarised than in the baseline case. Also, there is no counterpart to the platform rigidity that we found in the case of loss-aversion in the policy dimension.

The intuition for increased polarisation is the following. If \( v_0 > 0 \), the valence of the challenger is more likely to be in the loss domain than the gain domain, i.e., \( E[w] = -v_0 < 0 \), meaning that the median voter weights changes in valence relatively high relative to changes in policy. This means that parties have a relatively strong incentive to push their platforms out to their ideal points because the electoral consequences of doing so, i.e., the effect on the election probabilities, are

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relatively minor. For example, starting at symmetric equilibrium, an increase in (say) \( x_R \), moving \( x_R \) towards party R’s ideal point, has a relatively small negative effect on \( p \), proportional to \( \frac{\rho}{\beta} \). So, when \( \beta \) is high, this is low. On the other hand, if \( v_0 < 0 \), the argument works in reverse, implying less polarisation.\(^{15}\)

So far, we have concentrated on symmetric equilibria. However, because the ‘regime’ that determines \( p \) is endogenous to \( \Delta u \), it is also possible to find asymmetric equilibria, even though the model is symmetric. To make the point as simply as possible, we assume that both the median voter and two political parties have absolute value preferences, as in the example in Section 3. Assume first that \( v_0 > 0 \). We will look for an equilibrium where \( \Delta u < v_0 \). In this case, from (16), we get:

\[
p = \frac{1}{2} + \frac{v_0}{\beta} (\beta - 1) + \rho \Delta u. \tag{18}
\]

To interpret this, suppose hypothetically that platforms give the median voter the same payoff, i.e., \( \Delta u = 0 \); then, from (18), and the fact that \( \beta > 1 \), we see that \( p > 0.5 \). That is, party R has a self-fulfilling advantage relative to party L.

Given the assumptions on utility functions and (18), it is easy to compute that in equilibrium,

\[
x_R = \frac{1}{4\rho} - \frac{M}{2} + \rho v_0 \frac{(\beta - 1)}{\beta}, \quad x_L = -\left( \frac{1}{4\rho} - \frac{M}{2} - \rho v_0 \frac{(\beta - 1)}{\beta} \right), \quad p = 0.5. \tag{19}
\]

In this case, the incumbent party, R, chooses to take all of their ‘advantage’ by moving towards their ideal point, up to where the win probabilities are equal for the two parties. Moreover,

\[
\Delta u = -(x_R + x_L) = -2\rho v_0 \frac{(\beta - 1)}{\beta}.
\]

So, we see that if \( 2\rho > \frac{\beta}{\beta - 1} \), \( \Delta u < v_0 \), and thus such an equilibrium exists.

If \( v_0 < 0 \), by the same kind of argument, we can find an equilibrium where party R now has a self-fulfilling disadvantage relative to party L. The required condition is the same as in the first case, i.e., \( 2\rho > \frac{\beta}{\beta - 1} \), and in this equilibrium, party R is forced closer to the median voter’s position than party L, but both win with a probability of one-half.

5. Noise in Setting Policies

So far, we have made the standard assumption that given a platform, \( x \), there is no uncertainty about either the policy actually implemented or the utility outcome for the voter. This is in fact a strong assumption. For example, a party may propose a tax, \( x \), but due to changes in political support or the state of the macroeconomy, is only able to set tax, \( x + \epsilon \), once in government, where \( \epsilon \) is a random shock. Or, the tax rate, \( x \), might actually be set as promised, but the payoff to a voter given \( x \) at the time of voting may be uncertain because the voter may not know exactly what their wage will be. We will call these sources of uncertainty implementation shocks and voter outcome shocks, respectively.

If loss-aversion is in the valence dimension, this kind of uncertainty does not make a qualitative difference to the results. However, with loss-aversion in the policy dimension, these shocks may

\(^{15}\) Specifically, the valence of the challenger is more likely to be in the gain domain than the loss domain, i.e., \( E[w] = -v_0 > 0 \), meaning that the median voter weighs changes in valence relatively low relative to changes in policy. This means that parties have a relatively weak incentive to push their platforms out to their ideal points, implying less polarisation in equilibrium.
matter because it is possible that the uncertainty might smooth the kink in the election probability as a function of platforms. In this section, we will show that as long as the support of the shock is small, our main result with policy loss-aversion, Proposition 1, will apply in an approximate sense, so our results are robust to this kind of uncertainty.

To keep the exposition simple, we will focus on implementation shocks. Specifically, we will assume that for either party, the policy platform, $x$, if promised, leads to an actual implemented policy $y \in \mathbb{R}$ of $y = x + \varepsilon$, where $\varepsilon$ is mean zero and symmetric with a continuous distribution, $G$, and support, $[-\sigma, \sigma]$. It is also natural to suppose that the utility of a voter is defined on the actual implemented policy, $y$. So, we define voter $i$’s utility as $\omega_i(y) = -\ell(|y - y_i|)$, where as before, $\ell$ is a loss function with the properties assumed above and $y_i$ is the ideal point of the voter.

We also assume that voters are loss-averse over implemented policy, $y$, with the reference point being the policy implemented in the previous period by the incumbent, $y_0$. So, if $\omega_i(y) \geq \omega_i(y_0)$, the voter’s payoff is $\omega_i(y) - \omega_i(y_0)$, but if $\omega_i(y) < \omega_i(y_0)$, the voter’s payoff is $\lambda (\omega_i(y) - \omega_i(y_0))$, $\lambda > 1$.

Given this structure, the median voter, i.e., the voter with the median ideal point, $y_m$, is still decisive. We can then compute an expression for the median voter’s expected utility from platform $x$, $u(x; y_0)$, taking the expectation over values of $\varepsilon$. The actual formula is cumbersome and is given in equation (D.1) of the Online Appendix, where it is discussed further. Given $u(x; y_0)$, we can then compute the incumbent’s win probability as $p = \frac{1}{2} + \rho(u(x_R; y_0) - u(x_L; y_0))$ much as before.

To state a result comparable to Proposition 1, we need to restate assumption A1, replacing $u(x; x_0)$ by $u(x; y_0)$. This new assumption $A1’$, is found in the Online Appendix D. Given these, it is then shown formally in the Online Appendix D that the unique symmetric equilibrium without loss-aversion will be some $x_R = -x_L = x^+$, where the median voter’s expected utility from a platform, $x$ is just the expectation, $E[\omega(x + \varepsilon)]$. We can also define $x_R = -x_L = x^-$ to be the symmetric equilibrium when the median voter has the expected policy payoff, $\lambda E[\omega(x + \varepsilon)]$, and thus puts more weight on policy relative to valence. Both $x^+$, $x^-$ are defined formally in the Online Appendix. As in the case without noise, $x^+ > x^-$. Taking all these elements together, we can show:

**Proposition 3.** Assume $A1’$, $A2$. (i) If $x^+ + \sigma < |y_0|$, then $x_R = -x_L = x^+$ is the unique symmetric equilibrium; (ii) if $x^- - \sigma > |y_0|$, then $x_R = -x_L = x^-$ is the unique symmetric equilibrium; (iii) if $x^+ + \sigma \geq |y_0| \geq x^- - \sigma$, then there is a unique symmetric equilibrium $x^- < \lambda^0(|y_0|) \leq x^+$ and $\lambda^0(.)$ is strictly increasing in $|y_0|$.

We can make the following observations at this point. First, for a fixed $\sigma$, we again have the moderation effect—the equilibrium platform is always less than or equal to $x^+$ in absolute value. We also have platform rigidity if $|y_0|$ takes on an intermediate value; that is, if model parameters, e.g., $M$ or $\rho$ change, then equilibrium platforms are bounded in a narrow range and hardly respond to the parameter change. Also, because the noise vanishes in the sense that $\sigma$ approaches zero,

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16 The case of voter outcome shocks is complex as it requires some microfoundations; the results for this case are similar and are available on request.

17 Intuitively, if the policy, $x$, is close enough to zero that it ensures that the outcome is in the gain domain with a probability of 1 (i.e., that $x + \sigma \leq |y_0|$), the payoff is a simple expectation, $E[\omega(x + \varepsilon)] - \omega(y_0)$. Alternatively, if the policy, $x$, is close enough to one that it ensures that the outcome is in the loss domain with a probability of 1 (i.e., that $x - \sigma \geq |y_0|$), the payoff is a simple expectation, $\lambda (E[\omega(x + \varepsilon)] - \omega(y_0))$. In the intermediate case, the utility is a weighted average of the two elements.

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we recover Proposition 1 as a special case. So, in this sense, our main results are robust to a ‘small’ amount of noise as measured by the support of the shock to platforms, \( \varepsilon \).

6. Empirical Predictions

We would like to be able to test our theory. The most straightforward way would be to empirically measure changes in loss-aversion over either policy or valence, and then ask whether this leads to changes in polarisation as predicted. The obvious problem here is there are no measures of voter loss-aversion in these dimensions, and indeed, no accepted way of measuring loss-aversion in this setting.

In this section, we take a more indirect approach, asking how shifts in voter preferences affect equilibrium. Shifts in voter preferences have the advantage that there is a well-accepted way of measuring them, at least for two-party electoral systems such as in the USA. It turns out that when we perform this comparative statics exercise on the model, we arrive at an empirical prediction that distinguishes loss-aversion over policy from either loss-aversion over valence or no loss-aversion at all.

The timing is now as follows. At period 0, the two parties compete as described in Proposition 1. They set platforms, \( x_{R,0} = x_0, x_{L,0} = -x_0 \). One of these parties wins the election and is thus the incumbent at the beginning of period 1. At the beginning of period 1, there is a shift in the ideal points of both the median voter and of the two parties. We allow the preference shift to affect both voters and parties equally. That is, the ideal points of both the median voter and the parties shift by \( \Delta \). This shift is common knowledge. Without loss of generality, we assume that the shift is positive, i.e., \( \Delta > 0 \). When it has occurred, the parties then set new equilibrium platforms, \( x_{R,1}, x_{L,1} \).

The question of interest is how the two platforms change with \( \Delta \). Specifically, let \( I \in \{ R, L \} \) be the incumbent at period 0. We are interested in the size of \( \Delta_I = x_{I,1} - x_{I,0} \) relative to \( \Delta_C = x_{C,1} - x_{C,0} \). Before proceeding, we note that there are several reasons for allowing the ideal points of political parties to shift, not just voters’ ideal points. First, it is plausible that preference shifts will affect the views of party members as well as uncommitted voters. Second, without this assumption, we obtain the same kind of result at the cost of considerable additional complexity.

Without loss-aversion of any kind, i.e., \( \lambda = \beta = 1 \), it is clear that the period 0 equilibrium platform has no effect on the period 1 equilibrium, because in period 1, the parties play the same game before the shift, but the point of origin is moved from 0 to \( \Delta \). So, it is obvious that the new equilibrium will be the same, but with all variables translated by \( \Delta \). In other words, there is symmetric adjustment in platforms; that is, party platforms both move to the right by \( \Delta \). The same argument applies if we only have loss-aversion over valence.

With loss-aversion over policy, i.e., \( \lambda > 1 \), the effect of the preference shift will be very different. To obtain clean results, we will assume that either (i) the preference shift is unanticipated at time zero or (ii) parties have absolute value preferences, i.e., \( u_R = x - 1, u_L = -(x + 1) \). This assumption is required because if the preference shift is anticipated, and parties care about the degree of polarisation of the two future equilibrium platforms (i.e., the gap between \( x_R \) and \( x_L \)), there may be dynamic incentives to choose the current platform in order to affect the future status quo. In an earlier version (Lockwood and Rockey, 2015), we show that dynamic incentives are absent when parties have absolute value preferences.\(^{18}\)

\(^{18}\) This incentive works through the following mechanism. As the model is symmetric, the equilibrium is always symmetric about the median voter’s ideal point. So, in equilibrium \( x_R = -x_L = x^* \), a party faces a lottery, \((x^*, -x^*)\).
With this assumption, Figure 4 shows what will happen, assuming (without loss of generality) that that party $R$ is the incumbent. The top horizontal line in Figure 4 shows the initial equilibrium outcome, which will be at some $x_0 \in [x^-, x^+]$ because $R$ won the last election. The top line also shows the platform $-x_0$ set by party $L$ who, by definition, lost the last election. The bottom line indicates the new ideal points; in particular, the ideal points of the median voter and the two parties all move rightward by $\Delta$, as shown. We assume for purposes of illustration that this rightward shift is sufficiently small so that $x_0 \in (x^- + \Delta, x^+ + \Delta)$. Then, the new equilibrium must be as shown in the bottom line of the figure.\footnote{The argument is reversed when party $L$ is the incumbent.}

The reason is the following. First, as shown, the status quo policy, $x_0$, has effectively moved inwards towards the new ideal point of the median voter, $\Delta$. Moreover, as $x_0 \in (x^- + \Delta, x^+ + \Delta)$, from Proposition 1, the new platform for party $R$ must be equal to the status quo policy, i.e., $x_{R,1} = x_0$. Also, the new platforms must be centred around $\Delta$, meaning that party $L$’s new equilibrium platform is $x_{L,1} = -x_0 + 2\Delta$. In other words, the incumbent’s platform does not move at all, whereas the challenger’s platform moves by double the amount of the preference shift, $\Delta$, i.e., $2\Delta$, as shown by the red dashed lines.

In the same way, we can compute what happens to equilibrium platforms for all shifts, not just small ones. Define the platform shift to be the change in the platform of a party in response to $\Delta$. Given that initial platforms are $x_L = -x_0$, $x_R = x_0$, formally, platform shifts are $\Delta x_R = x_{R,1} - x_0$, $\Delta x_L = x_{L,1} - (-x_0) = x_L + x_0$ for parties $R$, $L$, respectively. Platform shifts can be defined in an analogous way if the incumbent is party $L$. We can then prove:

**Proposition 4.** (Asymmetric Adjustment) Assume that the status quo is $x_0$ if the incumbent is party $R$ ($I = R$) and $-x_0$ if $I = L$, where $x_0 \in [x^-, x^+]$. Following a preference shift, $\Delta > 0$, the equilibrium party platform shift is smaller for the incumbent than the challenger, i.e., $\Delta_I \leq \Delta_C$, with $\Delta_I < \Delta_C$ if $x_0 \neq x^-, x^+$.

where each outcome occurs with probability 0.5. So, generally risk-averse parties dislike polarised platforms, i.e., a higher $x^*$. If by manipulating the status quo, they can reduce future polarisation a little, they will do so. However, clearly, this incentive is absent when parties are risk-neutral, i.e., their payoffs are linear in the policy outcome.
This result, combined with our observation that there is symmetric adjustment to the shift without loss-aversion in policies, shows that loss-aversion in the policy dimension generates a particular kind of asymmetry, which is testable; with this kind of loss-aversion, incumbents adjust less than challengers.

Of course, there may be other explanations for this pattern of behaviour. For example, another possible explanation for asymmetric adjustment is some kind of incumbency advantage. However, if we model incumbency advantage in a standard way, by supposing that incumbency advantage is because of higher valence (e.g., Peskowitz, 2019), then it is easy to see that following a preference shift, both the incumbent and challenger will adjust symmetrically.

Specifically, we can capture incumbency advantage in the version of the model without loss-aversion, i.e., \( \lambda = 1 \), by assuming that the valence of the incumbent is \( v_I > 0 \), and is thus higher than the expected valence of the challenger. This version of the model is studied in the Online Appendix D.2. It is easily verified that the equilibrium platforms will generally be asymmetric, with the incumbent party \( R \)’s platform further from the median voter’s ideal point than party \( L \)’s platform is, i.e., \( x_R > -x_L > 0 \). Yet, it is also clear that because of the absence of loss-aversion, the preferences of the median voter are independent of the initial platform, and so following a preference shift, \( \Delta \), both \( x_R, -x_L \) shift by \( \Delta \). So, in this case, there is asymmetry in initial platforms, not in adjustment, the reverse to the case of loss-aversion. Therefore, to explain asymmetric adjustment, some more sophisticated incumbency advantage story must be developed.20

7. Some Empirical Evidence

The previous section makes the prediction that with voter loss-aversion over policy, incumbents adjust less than challengers to changes in voter preferences. In this section, we present some suggestive evidence that is consistent with this prediction.

We look at elections to the US House of Representatives. These elections are a good test bed for our theory for a number of reasons. First, these are high-information elections, so voters are likely to pay attention to campaign promises and voting records. Second, each citizen only gets to vote for one representative, as in the theory. Third, for these elections, there is a standard measure of shifts in voter preferences, which is the change in the Democratic vote share between the current and previous presidential elections, as used, for example, by Kernell (2009); Jacobson (2013); Baker et al. (2014); Jacobson (2015); Cayton (2017).

However, one issue is that these are legislative elections, so the outcome affects not only policies that are local to the district (such as the type of pork-barrel spending), but also affect the probability of one party or the other having a majority in Congress and therefore the choice of national policy. In turn, this will affect calculations of the median voter in any district. This ‘national policy’ channel has been analysed formally by Krasa and Polborn (2018).

The question then arises as to whether our results, particularly Proposition 4, are robust to this complication. In Online Appendix D, we extend the model of Section 3 to account for this, following Krasa and Polborn (2018). As in their model, we assume that the median voter cares about both local and national policies, but takes both national policies and the probability that the elected representative is pivotal in Congress as fixed.

20 One would be where the incumbent is lobbied by a special interest group, whose preferences do not shift.
In this setting, we show that formally, the national policy channel boils down to one or other of the two parties having a valence advantage in the model of Section 3. Moreover, we show that: (i) if the probability that the elected representative is pivotal in Congress is small, the valence advantage is small; and (ii) if the valence advantage is small, the equilibrium is close to that described by Proposition 1.

With 435 members of Congress, it seems likely that empirically the pivot probability is small. In Online Appendix D, we analyse a stylised example based on Krasa and Polborn (2018) with both ‘centrist’ and ‘leaning’ districts. There, we also document the fact that around one-third of the districts in the elections to the US House of Representatives could be described as centrist. This implies that the probability that the member of Congress from either type of district is pivotal is about 0.07. So, overall, we think it is reasonable to use Proposition 4 to predict the equilibrium candidate positions in this setting.

7.1. Data Description

Our data are for elections to the US House of Representatives for the period 1980–2012. Elections are every two years, and are indexed by \( t \in \{1980, 1982, \ldots, 2012\} \). We construct two key variables, which we denote \( \Delta \text{Preference}_{dt} \) and \( \Delta \text{Position}_{pdt} \) respectively. The variable \( \Delta \text{Preference}_{dt} \) is the change in ideological preference of the median voter in congressional district \( d \) between two elections, \( t \) and \( t - 1 \), and is measured as the change in the Democrat vote share between the current and previous presidential election. As already noted, this is a standard measure of preference shifts in this setting.

The variable \( \Delta \text{Position}_{pdt} \) is the change in the ideological position of the candidate from party \( p \) in congressional district \( d \) between year \( t \) and \( t - 1 \). Because we are comparing the shifts in position of both incumbents and challengers for the congressional seat, we cannot use the usual measure of the ideological position of members of Congress, DW-NOMINATE, as this measure does not include challengers. Instead, we use the DIME database (Bonica, 2014a) that accompanies Bonica (2014b). These data construct ideological positions of all candidates in US congressional elections, using publicly available campaign finance information, collated by the National Institute on Money in State Politics and the Sunlight Foundation.\(^{21}\) Critically, because donors donate to losing candidates, we observe the ideological position of all candidates. Bonica (2014a) shows that the correlation between his measure and the standard Poole and Rosenthal (2006) DW-NOMINATE measure is very high. For the specific sample we use, it is 0.92, and 0.91 with the time-varying Nokken and Poole (2004) measure.

Table 1 contains summary statistics for the key variables, \( \Delta \text{Position}_{pdt} \), \( \Delta \text{Preference}_{dt} \), for all US states by party. We also show \( \text{Position}_{pdt} \), the absolute position of party \( p \)’s candidate in district \( d \) in elections \( t \). The table shows, as expected, that \( \text{Position}_{pdt} \) for the Republicans is to the right of that for Democrats. Note, however, that the difference between the Democrat and Republican mean values on the \([-1, 1]\) scale are small—only 0.23—because the endpoints of this scale are determined by the most ideologically extreme candidates in the sample.

Looking now at the values for \( \Delta \text{Position}_{pdt} \) over the sample period, we see, not surprisingly, that there has been polarisation; the Republicans have moved to the right and the Democrats to

\(^{21}\) Bonica (2014b) uses a correspondence analysis procedure that exploits the fact that many politicians receive funds from multiple sources and many sources donate to multiple politicians to recover estimates for the positions of both politicians and donors. As this procedure is applied simultaneously at the federal and state levels, estimates for candidates in state-level elections are in a common space and comparable over time and between states.

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the left. Reflecting this, there are also relatively few large party moves with the 90th percentile of $\Delta \text{Position}_{pdt}$ also being less than 0.04 for both parties. Comparison of the 1st and 99th percentiles suggests shifts are approximately symmetrically distributed.

We can also see that, consistent with the literature (Erikson et al., 1993), that voter preferences are relatively stable; for example, both the mean and median of the $\Delta \text{Preference}_{dt}$ distribution are less than 0.01 and the 90th (99th) percentile is 0.065 (0.184) compared to a theoretical maximum move of 2. Table 2 suggests that the change in incumbents’ positions has a lower correlation with voter preferences than non-incumbents’.

### 7.2. Empirical Strategy

To test Proposition 4, we can compare the change in a party’s candidates’ positions for a given change in voter positions by regressing $\Delta \text{Position}$ on an incumbency dummy, $\text{Inc}$, $\Delta \text{Preference}$, and the interaction of the two explanatory variables. In other words, we estimate an equation of the form:

$$
\Delta \text{Position}_{pdt} = \psi \Delta \text{Preference}_{st} + \gamma \text{Inc}_{pdt} + \beta \text{Inc}_{pdt} \times \Delta \text{Preference}_{st} + \varepsilon_{pdt}.
$$

Here, $\text{Inc}_{pdt}$ is a dummy variable recording whether the candidate of party $p$ in district $d$ at date $t$ was the incumbent. Our key prediction from Proposition 4 is that the incumbent party shifts less, i.e., $\beta < 0$, while $\psi > 0$. We also test for a non-linear impact on the effect of incumbency on the response to the shift, by adding quadratic terms, as described below.

Given the data at hand, a key challenge in estimating (20) is to adequately control for any common factor, captured by $\varepsilon_{pdt}$, that may be jointly driving changes in parties’ platforms and changes in voters’ preferences. These are likely myriad and will include both local political and economic factors in the districts of individual representatives (Healy and Lenz, 2014), the spillover effects of other elections (Campbell, 1986), the characteristics of the representatives themselves (Buttice and Stone, 2012; Kam and Kinder, 2012) or media bias (Chiang and Knight, 2011). As
well as endogeneity because of external events, there is also the possibility of simultaneity due to the campaigning efforts or persuasive powers of state parties or individual politicians.

Our identification strategy is simple. First, as is conventional, we take the change in the presidential partisan vote share as conditionally exogenous; that is, that the vote share of the Democrat presidential candidate in a given year in a given district is determined by the candidates in that election’s national platforms and the preferences of voters in that district and does not reflect any particular impact of the platform of the congressional candidates in that district at that election, other things being equal. This is plausible because there are many districts and thus the incentive of a presidential candidate to target their message to any given one will be limited.

Second, we partial out idiosyncratic common factors to deal with the issues discussed above using candidate, party and district fixed effects and time trends as follows:

\[ \varepsilon_{pdt} = \phi_c + \phi_{dp} + \xi_c(t) + \xi_{dp}(t) + \varepsilon_{pdt}. \] (21)

Here, \( \phi_c, \phi_{dp} \) are candidate and district-party fixed effects, and \( \xi_c(t), \xi_{dp}(t) \) are candidate-specific and district-party specific linear time trends, respectively. The idea is as follows. Our baseline interpretation is that candidates in congressional elections behave in a manner similar to the parties in the theory, so that a specification where \( \varepsilon_{pdt} = \phi_c + \varepsilon_{pdt} \) will estimate the effect of incumbency on responsiveness of candidates to shocks, holding the identity of the candidate fixed.\(^{22}\) By replacing \( \phi_c \) by \( \xi_c(t) \), we further allow the behaviour of the candidate to vary over time, independent of incumbency. This allows, for example, a stronger effect of incumbency over time or local political trends.\(^{23}\)

We also want to allow for the fact that parties, rather than candidates, might influence the setting of platforms within a district. So, as an alternative, we have a specification where \( \varepsilon_{pdt} = \phi_{dp} + \varepsilon_{pdt} \). This will estimate the effect of incumbency on responsiveness of candidates to shocks, holding the party affiliation/district pair of the candidate fixed. By replacing \( \phi_c \) by \( \xi_c(t) \), we further permit the behaviour of the candidate in a particular party/district pair to vary over time, independently of incumbency. This again allows for the effects of political parties and other factors to vary over time.

In both cases, \( \varepsilon_{pdt} \sim N(0, \Sigma) \), where \( \Sigma \) is allowed to be clustered by both state \( \times \) year to capture state-level political events affecting all districts and spillovers across them, as well as district \( \times \) party to allow for arbitrary auto-correlation in district \( \times \) party behaviour. To avoid bias, we estimate \( \Sigma \) using the bootstrap.

One complication of using District Presidential Vote shares as our measure of \( \Delta Preference \) is that they only change every four years whereas elections to the US House are every two years. Our preferred specification includes all years as this retains a direct correspondence with the actual pattern of elections and will provide conservative estimates.\(^{24}\) Tables B.2 and B.3

\(^{22}\) So, we are using variation within representatives: that is, it compares responses to shocks for the same politician in and out of office. This will capture potential differences in the responsiveness of individual politicians to shocks, and how they impact local elections, and given that candidates only normally contest one district, the non-random matching of these traits to districts. In addition, given again that a candidate may only contest one election at a time, it will also capture contemporaneous political effects.

\(^{23}\) In our preferred specifications, we do not include year effects because they absorb much of the variation in the partisan vote share. Table B.1 in the Online Appendix reports alternative results demonstrating our results are robust to including them.

\(^{24}\) To see this, it is useful to consider the implications of recording \( \Delta Preference = 0 \) for years in which presidential elections are not taking place. In this case, (20) simplifies to \( \Delta Position_{pdt} = \gamma Inc_{pdt} + \varepsilon_{pdt} \). There are two concerns. First, there may be systematic changes in \( Position \) in off-cycle elections in the absence of any change in district partisanship.

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error is correlated with support we do not capture. This then becomes a measurement error problem and the concern is whether this measurement we might be concerned that there are other preference shocks, say connected with local politics, that lead to changes in

\[ \psi / \Delta_1 \]

\[ /\Delta_1 \]

\[ \text{Inc}_{pdt} \times \text{Shift}_{st} \]

\[ -0.0344^\ast -0.0669^\ast -0.0430^\ast -0.0669^\ast -0.0430^\ast -0.0622^\ast -0.0447^\ast \]

\[ (0.0190) (0.0374) (0.0197) (0.0333) (0.0175) (0.0317) (0.0199) \]

\[ (20) \]

\[ \text{Shift}^2_{st} \]

\[ 0.00230 0.00165 \]

\[ (0.0220) (0.0173) \]

\[ R^2 \]

\[ 0.00 0.29 0.05 0.29 0.05 0.29 0.05 \]

\[ 10,656 7,723 10,650 7,723 10,650 7,723 10,650 \]

\[ \text{Trends} \times \text{Party} \times \text{State} \times \text{Year} \]

\[ \text{Trends} \times \text{Party} \times \text{State} \times \text{Year} \]

Notes: This table presents estimates of (20), OLS regressions that test for an asymmetric response of incumbent compared to non-incumbent district parties to shocks. The specification estimated is:

\[
\Delta \text{Position}_{pdt} = \psi \Delta \text{Preference}_{st} + \gamma \text{Inc}_{pdt} + \beta \text{Inc}_{pdt} \times \Delta \text{Preference}_{st} + \varepsilon_{pdt},
\]

where \( \text{Position}_{pdt} \) is the position of the candidate for the US House of party \( p \in \{D, R\} \) in congressional district \( d \) at election \( t \) while \( \Delta \text{Preference}_{st} \) is the change in the district presidential partisan vote share in district \( d \) at election \( t \). \( \text{Inc}_{pdt} \) is the dummy variable recording if a party is incumbent in district \( d \) at election \( t \). Column 1 reports results excluding all fixed effects. Column 2 includes candidate-specific fixed effects such that the estimated coefficients now describe the asymmetric responses of candidates allowing for candidates’ average responses to vary in an unrestricted way. Column 3 replaces candidate fixed effects with district \( \times \) party, fixed effects which partial out local heterogeneity while maximising the available sample. Columns 4 and 5 replace these two fixed effects with linear trends while columns 6 and 7 augment (20) to allow for potential non-linearities in shock size. Column 3 includes standard deviation changes in \( \text{Preference}_{st} \) so that results describe the change across both cycles. The results are qualitatively and quantitatively similar to those from our preferred specification.

7.2.1. Results

We now report estimates of (20). To facilitate inference, all variables are standardised such that coefficients may be interpreted in terms of standard deviation changes in \( \Delta \text{Position}_{pdt} \) and \( \Delta \text{Preference}_{st} \), etc. As a first step, column 1 of Table 3 reports results from a simplified version of (20) where \( \beta_2 = 0 \), and in which there are no fixed effects. We see that, as expected, parties react to movements in the median voter, with the coefficient on \( \Delta \text{Preference}_{st} \) positive and significant. We also find, as the theory suggests, that the incumbent party’s candidate reacts less.

This is a concern that \( \Delta \text{Position}_{pdt} \) may be auto-correlated and is addressed by clustering by district \( \times \) party. Second, we might be concerned that there are other preference shocks, say connected with local politics, that lead to changes in support we do not capture. This then becomes a measurement error problem and the concern is whether this measurement error is correlated with \( \Delta \text{Preference} \). It will not be, since in off-cycle elections, \( \Delta \text{Preference} = 0 \), and thus the correlation must be 0 and for presidential years, we need only know that the correlation is 0 conditional on our district or candidate fixed effects or time trends, which is plausible for the reasons articulated in the discussion of our identification strategy. Although, any measurement error will lead to attenuation bias, it will bias both \( \gamma \) and \( \beta \) by the same amount, thus not affecting the quantitative interpretation of the model, but making our inference conservative given it will inflate the estimated standard errors.

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Column 2 reports our preferred specification, and is as column 1, but now including candidate fixed effects. The coefficients are again in line with the theory, with $\psi$ again positive and $\beta$ negative. This coefficient is negative and significant and approximately two-thirds as large as for $\Delta Preference_{st}$. Thus, a one standard deviation move rightwards would move the incumbent party only 0.05 standard deviations rightwards, but a party not in power nearly 0.15 standard deviations to the right, or three times as far. This is consistent with our theoretical predictions. Given that we include $Inc_{pdt} \times \Delta Preference_{st}$, $\gamma$ gives the effect of $Inc_{pdt}$ given no shift. Perhaps unsurprisingly, given the shift will almost always be non-zero, the estimated effect is small, although positive.

Column 3 includes our alternative district-party fixed effects. We see the same pattern in the coefficients as in the previous two columns, with the $\beta$ around half as large as in magnitude as $\psi$. $\gamma$ is now significant, but this reflects the fact that the alternative fixed effects do not capture diverging long-term trends in the average position of each party’s representatives. The advantage of this specification is that it is less data intensive as the candidate fixed-effect model necessitates dropping all candidates only observed once.

Columns 4 and 5 replace the fixed effects in columns 2 and 3, $\phi_c$, $\phi_{dp}$ with candidate and district-party specific linear time trends, $\xi_c(t), \xi_{dp}(t)$. The coefficients are almost identical to those in the fixed-effects specifications. However, the inferences are now different as the coefficients describe the different deviations of incumbents and non-incumbents around the candidate’s or district party’s long-run trend in response to a given shock. The similarity of the results suggest that they are not being driven by local trends.

The effects of any shock may not be linear however: parties’ candidates may respond disproportionately to smaller or larger shifts. In columns 6 and 7, we therefore augment (20) with the quadratic terms, $\Delta Preference_{st}^2$ and $Inc_{pdt} \times \Delta Preference_{st}^2$, and the respective associated coefficients, $\psi_2$ and $\beta_2$. Column 6 reports results including candidate fixed effects, whereas column 7 includes district-party fixed effects. In both cases, $\beta_2$ and $\psi_2$ are imprecisely measured and not significant at any conventional level, meaning we cannot reject a linear relationship.

8. Conclusions

This paper studied how voter loss-aversion affects electoral competition in a Downsian setting. We provided some evidence that US voters may be loss-averse over policy outcomes with a reference point at the status quo. We then showed theoretically that such loss-aversion has a number of effects on electoral competition. First, for some values of the status quo, there is policy rigidity such that both parties choose platforms equal to the status quo, regardless of other parameters.

Second, there is a moderation effect when there is policy rigidity; the equilibrium policy outcome is closer to the median voter’s ideal point than in the absence of loss-aversion. We also considered the effect of loss-aversion in the valence dimension; here, loss-aversion has rather different effects on electoral competition.

Finally, we tested an empirical prediction that with loss-aversion, incumbents adjust less than challengers to changes in voter preferences. Using data on elections to the US House of Representatives, we found evidence consistent with the predictions of the theory. Specifically, the data suggest that incumbent parties respond less than challengers to shifts in the preferences of the median voter.

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Appendix. Proofs of Propositions and Other Results

Proof of Lemma 1. (a) First we define
\[ g(x; \phi) = 0.5u'_R(x) + \rho \phi u'(x)(u_R(x) + M - u_R(-x)), \quad \phi \in [1, \lambda]. \]  
(A1)

Then clearly (10), (11) are defined compactly as \( g(x; 1) = 0 \), \( g(x; \lambda) = 0 \). So, for both existence and uniqueness of a solution strictly between zero and one, it is sufficient to show \( g(0; \phi) > 0, \ g(1; \phi) < 0, \ g_x < 0, \ x \in [0, 1] \).

(b) To prove that \( g(0; \phi) > 0 \), note that
\[ g(0; \phi) = 0.5u'_R(0) - \rho \phi u'(0)(u_R(0) + M - u_R(-0)) \]
\[ = 0.5u'_R(0) - \rho \phi u'(0)M \]
\[ = 0.5u'_R(0) > 0, \]
where the last line follows as \( u'(0) = 0 \) from Assumption A2.

(c) To prove \( g(1; \phi) < 0 \), note
\[ g(1; \phi) = 0.5u'_R(1) + \rho \phi u'(1)(u_R(1) + M - u_R(-1)) \]
\[ = \rho \phi u'(1)(u_R(1) + M - u_R(-1)) \]
\[ < 0, \]
where the second line follows as \( u'_R(1) = 0 \), as 1 is party R’s ideal point and the third follows because \( u'(1) < 0 \), and, of course, \( u_R(1) > u_R(-1), M > 0 \).

(d) To prove \( g_x(x; \phi) < 0, x \in [0, 1] \), first differentiate (A1):
\[ g_x(x; \phi) = 0.5u''_R(x) + \rho \phi u''(x)(u_R(x) + M - u_R(-x)) \]
\[ + \rho \phi u'(x)u'_R(x) + u'_R(-x). \]  
(A2)

Now, the first and second terms are negative by the concavity of \( u(x) \), \( u_R(x) \) in \( x \), and the third term is negative as \( u' < 0 \).

(e) Finally, to prove \( x^+ > x^- \), we just need to show that \( \frac{dx}{d\phi} < 0 \). Totally differentiating (A1), we get
\[ \frac{dx}{d\phi} = \frac{g_\phi(x; \lambda)}{-g_x(x; \lambda)} = \frac{\rho u'(x)(u_R(x) + M - u_R(-x))}{-g_x(x; \lambda)}. \]  
(A3)

The denominator of (A3) is positive as \( g_x < 0 \). Moreover, the numerator is negative as \( u'(x) < 0 \) and \( u_R(x) \) is increasing in \( x \). So, from (A3), \( \frac{dx}{d\phi} < 0 \) as required. \( \square \)

Proof of Proposition 1. (a) Generally, any symmetric equilibrium \( x_R = -x_L = x^* \) is characterised by the FOC for a maximum of \( \pi_R \), evaluated at equilibrium (by symmetry, we do not need to consider the FOC for party L). Assume first that \( x^* \neq |x_0| \). Then from (4), this FOC is
\[ \frac{\partial \pi_R(x^*, x^*)}{\partial x^*} = 0.5u'_R(x^*) + \frac{\partial p(x^*, x^*)}{\partial x^*} (u_R(x^*) + M - u_R(-x^*)) = 0, \]  
(A4)

where from (9):
\[ \frac{\partial p(x^*, x^*)}{\partial x^*} = \left\{ \begin{array}{ll} \rho u'(x^*), & x^* < |x_0|, \\
\rho \lambda u'(x^*), & x^* > |x_0|, \end{array} \right. \]  
(A5)

So, using (A5), we can rewrite (A4) as
Finally, from (9), if the equilibrium $x^* = |x_0|$, the FOC must be stated differently as $p$ is not differentiable at this point. Note from (A5) that $p$ has left-hand and right-hand derivatives in $x_R$ with the right hand being smaller than the left, as $\lambda > 1, u'(x^*) < 0$. So, if $x^* = |x_0|$, equilibrium must satisfy the condition that no small deviation in $x_R$ from $x^*$ is profitable for party $R$, i.e.,

$$0.5u'_R(x^*) + \rho u'(x^*) (u_R(x^*) + M - u_R(-x^*)) = 0, \quad x^* < |x_0|,$$

$$0.5u'_R(x^*) + \rho \lambda u'(x^*) (u_R(x^*) + M - u_R(-x^*)) = 0, \quad x^* > |x_0|.$$  

(A6)  

(A7)

So, (A6), (A7), (A8), fully characterise all possible symmetric equilibria.

(b) First, assume $x^+ < |x_0|$. We show that $x_R = -x_L = x^+$ is the unique symmetric equilibrium. From Lemma 1, $x_R = -x_L = x^+$ solves (A6) and is thus an equilibrium.

Now suppose that there is another equilibrium, $x' \neq x^+$. If $x' < |x_0|$, then $x'$ must also solve (A6), and so from Lemma 1 must be equal to $x^+$, a contradiction. If $x' > |x_0|$, then $x'$ must solve (A7). However, from Lemma 1, $x' = x^+ < x^+ < |x_0|$, contradicting the assumption that $x' > |x_0|$.

Finally, if there is another equilibrium, $x' = |x_0|$, (A8) must be satisfied at $x^* = |x_0|$. Yet we know from the proof of Lemma 1 that $\frac{\partial \pi_R(x^*, x^*)}{\partial x^*} = g(x; \phi)$ is strictly decreasing in $x^*$. So, as $x^+ < |x_0|$, we must have

$$0.5u'_R(|x_0|) + \rho u'(\{|x_0|\}) (u_R(|x_0|) + M - u_R(-|x_0|))$$

$$< 0.5u'_R(x^+) + \rho u'(x^+) (u_R(x^+) + M - u_R(-x^*)) = 0.$$  

(A9)

Though this is clearly inconsistent with (A8) holding at $x^* = |x_0|$, as the term on the left of the inequality in (A9) is negative, not positive.

(c) Second, assume $x^- > |x_0|$. We show that $x_R = -x_L = x^-$ is the unique symmetric equilibrium. From Lemma 1, $x_R = -x_L = x^-$ solves (A7) and is thus an equilibrium.

Now suppose that there is another equilibrium $x' \neq x^-$. If $x' > |x_0|$, then $x'$ must also solve (A7), and so by Lemma 1 must be equal to $x^-$, a contradiction. If $x' < |x_0|$, then $x'$ must solve (A6). Yet, then, from Lemma 1, $x' = x^+ > x^- > |x_0|$, contradicting the assumption that $x' > |x_0|$.

Finally, if there is another equilibrium, $x' = |x_0|$, (A8) must be satisfied at $x^* = |x_0|$. As $\frac{\partial \pi_R(x^*, x^*)}{\partial x^*}$ is strictly decreasing in $x^*$, and as $x^- > |x_0|$, we must have

$$0.5u'_R(|x_0|) + \rho \lambda u'(\{|x_0|\}) (u_R(|x_0|) + M - u_R(-|x_0|)) >$$

$$0.5u'_R(x^-) + \rho \lambda u'(x^-) (u_R(x^-) + M - u_R(-x^-)) = 0.$$  

(A10)

Though this is clearly inconsistent with (A8) holding at $x^* = |x_0|$, as the first term in (A10) is positive, not negative.

(d) Assume $x^- \leq |x_0| \leq x^+$. Then, it is easy to check that (A8) holds at $x^* = |x_0|$, so this is certainly an equilibrium. Now, suppose that there is another equilibrium, $x' < |x_0|$. Then, this equilibrium must satisfy (A6) and thus $x' = x^+$ so $x^+ < |x_0|$. However, this contradicts the assumption $|x_0| \leq x^+$. Finally, suppose that there is another equilibrium, $x' > |x_0|$. Then, this equilibrium must satisfy (A7) and thus $x' = x^- \leq |x_0|$. Yet, this contradicts the assumption, $|x_0| \geq x^-$. 

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Derivation of Equations (15), (16). First, suppose that \( v_0 > 0 \). Then, from (14):
\[
p = \Pr(\phi_L - \phi_R \leq \Delta u)
\]
\[
= \Pr(w \leq \Delta u - \beta v_0 | w \geq 0) \Pr(w \geq 0)
+ \Pr(w \leq (\Delta u - \beta v_0)/\beta | w < 0) \Pr(w < 0).
\]
Now, denoting the cumulative distribution of \( w \) by \( G \), it is easy to calculate that
\[
\Pr(w \leq \Delta u - \beta v_0 | w \geq 0) \Pr(w \geq 0) = \begin{cases} G(\Delta u - \beta v_0), & \Delta u - \beta v_0 \geq 0, \\ 0, & \Delta u - \beta v_0 < 0. \end{cases}
\]
\[
\Pr(w \leq (\Delta u - \beta v_0)/\beta | w < 0) \Pr(w < 0) = \begin{cases} G(0), & \Delta u - \beta v_0 \geq 0, \\ G((\Delta u - \beta v_0)/\beta), & \Delta u - \beta v_0 < 0. \end{cases}
\]
So, from (A12), (A13):
\[
p = \begin{cases} G(\Delta u - \beta v_0), & \Delta u \geq \beta v_0, \\ G((\Delta u - \beta v_0)/\beta), & \Delta u < \beta v_0. \end{cases}
\]
Also, by assumption,
\[
G(x) = \frac{1}{2} + \rho(x + v_0).
\]
So,
\[
p = \frac{1}{2} + \rho v_0 + \rho \begin{cases} \Delta u - \beta v_0, & \Delta u \geq \beta v_0, \\ \frac{1}{\beta}(\Delta u - \beta v_0), & \Delta u < \beta v_0, \end{cases}
\]
as required. If \( v_0 < 0 \), then from (14):
\[
p = \Pr(\Phi(v) \leq \Delta u)
= \Pr(w \leq \Delta u - v_0 | w \geq 0) \Pr(w \geq 0)
+ \Pr(w \leq (\Delta u - v_0)/\beta | w < 0) \Pr(w < 0),
\]
Then, following the same argument as in part (a), we get
\[
p = \frac{1}{2} + \rho v_0 + \rho \begin{cases} \Delta u - v_0, & \Delta u \geq v_0, \\ \frac{1}{\beta}(\Delta u - v_0), & \Delta u < v_0, \end{cases}
\]
as required.

Proof of Proposition 2. (a) As in the case of loss-aversion over policy, the symmetric equilibrium is characterised by (A4) whenever \( p \) is differentiable in \( x_R \). But now, at a symmetric equilibrium \( x_R = -x_L = x \), we have \( \Delta u = 0 \), so as long as \( v_0 \neq 0 \), \( \Delta u \neq \beta v_0 \). Consequently, from (15), (16), \( p \) is always differentiable as long as \( v_0 \neq 0 \). There are then three cases to consider.
(b) $v_0 > 0$. At a symmetric equilibrium $x_R = -x_L = x$, $\Delta u = 0 < v_0$ and so from (15):

$$p = 0.5, \quad \frac{\partial p}{\partial x_R} = \frac{\rho}{\beta} u'(x).$$  \hfill (A16)$$
Substituting terms in (A16) into (A4) and using the fact that $u_R'(x) = -u'_L(-x)$, $u'(x) = -u'(-x)$, implies that (A4) reduces to:

$$\frac{1}{2}u_R'(x) + \frac{\rho u'(x)}{\beta}(u_R(x) - u_R(-x) + M) = 0.$$  
By Lemma 1, this has a unique solution $x(\beta)$, so $x_R = -x_L = x(\beta)$ must be the unique symmetric equilibrium.

(c) $v_0 < 0$. At a symmetric equilibrium $x_R = -x_L = x$, $\Delta u = 0 > v_0$ and so from (16):

$$p = 0.5, \quad \frac{\partial p}{\partial x_R} = \rho u'(x).$$ \hfill (A17)$$
Substituting terms in (A17) into (A4), we again see that (A4) reduces to:

$$\frac{1}{2}u_R'(x) + \rho u'(x)(u_R(x) - u_R(-x) + M) = 0.$$  
By Lemma 1, this has a unique solution $x^+$, so so $x_R = -x_L = x^+$ must be the unique symmetric equilibrium.

(d) Finally, suppose $v_0 = 0$. Then, it is clear from (15), (16) that at a symmetric equilibrium where $\Delta u = 0$, $p$ is not differentiable. Given this, a necessary condition for a symmetric equilibrium is that starting at $x_R = -x_L = x$, party $R$ cannot raise $\pi_R$ by either raising $x_R$ or lowering it. If $x_R$ is lowered, the marginal effect on $\pi_R$ is:

$$\Delta \pi_R = \frac{1}{2}u_R'(x) + \rho u'(x)(u_R(x) - u_R(-x) + M).$$ \hfill (A18)$$
If $x_R$ is raised, the marginal effect on $\pi_R$ is:

$$\Delta \pi_R^+ = \frac{1}{2}u_R'(x) + \frac{\rho}{\beta} u'(x)(u_R(x) - u_R(-x) + M).$$ \hfill (A19)$$
For this to be an equilibrium, we require that it does not pay to adjust $x_R$ in either direction, i.e., $\Delta \pi_R \geq 0 \geq \Delta \pi_R^+$. However, from (A18) and (A19), and using $\beta > 1$, $u'(x) < 0$, we get

$$\Delta \pi_R^+ - \Delta \pi_R = \rho u'(x)(u_R(x) - u_R(-x) + M)(1 - \frac{1}{\beta}) < 0,$$
a contradiction. \hfill $\square$

**Proof of Proposition 4.** Assume w.l.o.g. that the incumbent is party $R$. We establish the result for three different mutually exclusive and exhaustive cases.

(a) $\Delta + x^\pm \leq x_0 \leq \Delta + x^-. This is the case considered in Figure 4, so no proof is needed.

(b) $x_0 > \Delta + x^+$. In this case, the equilibrium is as in Proposition 1, with: (i) a status quo $x_0 - \Delta$; and (ii) all equilibrium variables shifted right by $\Delta$. So, as the effective status quo, $x_0 - \Delta$, is greater than $x^+$, the (unshifted) equilibrium outcome is $x_R' = x^+$, $x_L' = -x^+$. So, the actual equilibrium outcome is $x_{R,1} = \Delta + x^+$, $x_{L,1} = \Delta - x^+$. So, party platform shifts following the shock are

$$\Delta_R = \Delta + x^+ - x_0, \Delta_L = \Delta - x^+ + x_0.$$

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So, as $R = I, L = C$ we see

$$\Delta C - \Delta I = 2(x_0 - x^+) > 0,$$

where the last inequality follows as $x_0 > \Delta + x^+$ by assumption, and $\Delta > 0$.

(c) $x_0 < \Delta + x^-$. Again, in this case, the equilibrium is as in Proposition 1, with: (i) a status quo $x_0 - \Delta$; and (ii) all equilibrium variables shifted right by $\Delta$. So, as the effective status quo, $x_0 - \Delta$, is less than $x^-$, the (unshifted) equilibrium outcome is $x'^R = x^-, x'^L = -x^-$. So, the actual equilibrium outcome is $x^R,1 = \Delta + x^-, x^L,1 = \Delta - x^-$. So, party platform shifts following the shock are

$$\Delta R = \Delta + x^- - x_0, \quad \Delta L = \Delta - x^- + x_0.$$ 

So, as $R = I, L = C$ we see

$$\Delta C - \Delta I = 2(x_0 - x^-) \geq 0,$$

where the last inequality follows as $x_0 \geq x^-$ by assumption. □

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Additional Supporting Information may be found in the online version of this article:

Online Appendix
Replication Package

References

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