Decentralization via Federal and Unitary Referenda

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Abstract

This paper studies a model where the power to set policy (a choice of project) may be assigned to central or regional government via either a federal or unitary referendum. The benefit of central provision is an economy of scale, while the cost is political inefficiency. The relationship between federal and unitary referenda is characterized in the asymptotic case as the number of regions becomes large, under the assumption that the median project benefit in any region is a random draw from a fixed distribution, G. Under some symmetry assumptions, the relationship depends *only* on the shape of G, not on how willingness to pay is distributed *within* regions. The relationship to Cremer and Palfrey's "principle of aggregation" is established. Asymptotic results on the efficiency of the two referenda are also proved.

1. Introduction

The issue of assignment of tax and spending powers between different levels of government is receiving increasing attention among economists, perhaps because many countries are moving in the direction[1] of greater decentralization (Bird 1993). All countries have constitutional rules or procedures, explicit or implicit, for choosing the level of decentralization of a tax or spending power. These rules differ significantly between federal and unitary states. In federal states, the allocation of powers is usually specified in the constitution and

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[1]For example, Bird cites the "new federalism" in the United States, and moves to federalism in Spain and Belgium. In the UK, recent referenda on the devolution of powers to Scotland and Wales will result in the establishment of Scottish and Welsh parliaments.

may require[2] a constitutional amendment. In all major federal states, rules for constitutional amendment require that at least a *majority of sub-central governments* must approve the amendment (Wheare 1963). For example, in the United States, any amendment must be approved by at least three-quarters of all state legislatures. By contrast, in a unitary state, such as the UK, reallocation of powers is achieved either by legislation in the national parliament, or by national referendum:[3] the agreement of any sub-central level of government per se is not required.

While there is now a large and growing theoretical literature on decentralization, remarkably, there is only one[4] paper that addresses directly the different decision-making procedures of federal and unitary states (Cremer and Palfrey 1996). In their model, regional or central governments choose some value of a policy variable (a real number) by majority voting.[5] In this setting, Cremer and Palfrey study two referenda, which we call *unitary* and *federal* referenda respectively. Under the unitary referendum, a choice between centralization and decentralization is made by a single vote by of all citizens, with the alternative that attracts more votes being chosen. Under the federal referendum, every region chooses between centralization and decentralization using a referendum, and then the arrangement preferred by a majority of regions is chosen. These referenda capture in a simplified way the distinguishing features of federal and unitary states mentioned above. They obtain a remarkable result:[6] as the number of (equal-sized) regions become large, whenever the unitary referendum selects centralization, the federal referendum also selects centralization (but not necessarily vice versa), so federal

[2]However, the degree to which reallocation of powers leads to constitutional amendment varies considerably across federal countries. In the United States, there has only been one constitutional amendment for this purpose (in 1913, to allow a Federal income tax), whereas in Switzerland there have been a large number of amendments over the last 100 years, enhancing the tax powers of central government (Wheare 1963, Chapter 6).

[3]Again, if the reallocation of powers requires a constitutional amendment, a national referendum is sometimes required; e.g., in France, it sometimes happens that a constitutional amendment is put to a referendum after it has been approved by parliament (Curtis 1997). In countries without a well-defined constitution, such as the UK, reallocation of powers requires only a majority in parliament.

[4]Cremer and Palfrey (1999) use the same model to study the implications of unit- and population-proportional representation.

[5]In their model, the cost of centralization is *policy uniformity:* the value of the policy variable must be the same for all regions. Moreover, they assume that voters are incompletely informed about the preferences of other voters, both in their regions and in other regions. It turns out in this set-up that the benefit of centralization is *policy moderation*. That is, when the number of regions becomes large, the subjective probability for any particular voter that the policy variable will, in voting equilibrium, take on an extreme value (i.e., far from that voter's most preferred value) is lower with centralization.

[6]This follows from Figure 1 in their paper, where it is clear that if the proportion of voters preferring centralization is greater than 0.5, then the proportion of regions preferring centralization must also be greater than 0.5.

referenda unambiguously lead to more centralization. They call this result the *principle of aggregation*.

This paper addresses the same question in a different setting, where the costs and benefits of centralization are somewhat different. First, the policy space in our model is n-dimensional (each region has a discrete project). Second, in our model, the benefit of centralization is a reduction in project costs (economies of scale), so it may be efficient to choose centralization: this allows a discussion of the welfare properties of the two referenda.[7] Third, the cost of centralization is endogenously derived ex ante policy uniformity (explained in more detail below). Finally, we are able to avoid imposing very strong assumptions on the distribution of preferences for projects within regions and between regions, and this is important, as the shape of the distribution across regions of the median willingness to pay for the project turns out to be crucial for our results.[8]

In this setting, we then ask which of the two referenda is more *decentralizing* i.e., will choose fiscal decentralization whenever the other one does. In Section 3, we show that with a fixed and finite number of regions, and no restrictions on the distribution of project benefits, either within or across regions, there is no particular reason to think that the federal referendum will be systematically more decentralizing than the unitary referendum or vice versa.

In Section 4, we establish the main (asymptotic) results of the paper, under the following assumptions: (i) regional median project benefits are random draws from a fixed distribution; (ii) conditional on the regional median, the distribution of tastes within any region is the same; (iii) the number of regions is large. We first have a key benchmark result. Say that the federal and unitary referenda are *asymptotically equivalent* if, in the limit as the number of regions becomes large, the federal referendum will choose decentralization if and only if the unitary referendum does. Then, under some symmetry assumptions on preferences, we show that the federal and unitary referenda are asymptotically equivalent if the distribution of median project benefits across regions is uniform. This result holds *irrespective of how preferences are distributed within regions.*

So, the uniform distribution is obviously the borderline case. We then have two more results. First, if the distribution of median project benefits

[7]Cremer and Palfrey, as they say themselves, do not model any efficiency gains from centralization; in their setting policies are costless (or equivalently, equally costly). This means that decentralization is always the efficient choice, as it allows for policy diversity. More precisely, as is shown in an earlier version of this paper (available on request from B.Lockwood@warwick.ac.uk), in their model, the sum of utilities across all voters is always strictly greater with decentralization than with centralization.

[8]Due to the information structure in Cremer and Palfrey (1996), their model is only tractable if very specific assumptions on the distribution of preferences within regions and between regions are made, and indeed, they assume for the most part that both these distributions are normal.

across regions is positively single-peaked (i.e., has a quasi-concave density) then the federal referendum is *asymptotically more centralizing* than the unitary referendum—i.e., in the limit as the number of regions becomes large it chooses centralization whenever the unitary referendum does. Second, if the distribution of median project benefits across regions medians is negatively single-peaked (i.e., has a quasi-convex density) then the federal referendum is *asymptotically less centralizing* than the unitary referendum. The intuition for these results is that the federal referendum is *more sensitive* to changes in the distribution of regional medians away from the uniform than is the unitary referendum. For example, a mean-preserving spread of the distribution of regional medians may convert a uniform distribution into a negatively single-peaked one. In this case, the proportion of median voters preferring decentralization rises by more than the proportion of voters in total preferring decentralization.[9]

As argued above, our set-up also allows us to analyze the efficiency of referenda. Buchanan (1975, 1978, 1987) argues that while policy acts (conditional on constitutional rules) may well be inefficient in particular cases, we should expect society as a whole to choose constitutional rules that are in some sense efficient. Constitutional decisions are long-run ones, and so the performance of any constitution should be evaluated from behind a Rawlsian veil of ignorance, where citizens are not sure about what their position in "society" will be. In Section 5, we study the efficiency of federal and unitary referenda in this sense.

In general, both referenda will be inefficient, for the usual reason that majority voting does not take account of intensity of preferences. However, in the asymptotic case, under the same assumptions as before, a number of results can be proved. Again, the benchmark case is where the distribution of median project benefits across regions is uniform. In this case, both federal and unitary referenda are fully efficient. Also, deviations of both rules from full efficiency can again be characterized when the distribution of median project benefits across regions is either positively or negatively single-peaked.

2. The Costs and Benefits of Decentralization

2.1. Preliminaries

We develop the simplest possible model for our purpose. There is an odd number $i = 1, \ldots, n$ of regions, with equal populations of measure 1. The assumption of equal populations is made because when regional populations differ, differences between federal and unitary referenda may arise because

[9]These findings relate to Cremer and Palfrey's "principle of aggregation" as follows. The two cases analyzed in their model were when preferences were normal. But the normal distribution is single peaked, in which case our result is that the federal referendum is more centralized, consistent with their principle of aggregation.

of the distribution of population across regions,[10] and we do not wish to complicate the analysis in this way. In each region there is a discrete project $x_i \in \{0, 1\} = X$. The payoff of a resident of region i is

$$u_i = b_i x_i + y_i \tag{1}$$

where b_i is a benefit parameter, and y_i is the consumption of a numeraire private good. In region i, b_i is a continuously distributed random variable with median b_{mi}, support $[\underline{b}_i, \bar{b}_i]$, and distribution function F_i.

The project in region i may be provided by regional government i (decentralization), or by central government (centralization). In either case, the relevant government is assumed to finance the public good by levying a proportional income tax. Every citizen in region i has an endowment of 1 unit of the private good, and so, as consumption is equal to after-tax income, $y_i = 1 - t_i$, where t_i is the income tax rate.

2.2. Decentralization

In this case, the cost to any regional government of funding its project is c. So, the regional budget constraint is $t_i = c$ where t_i is the regional income tax rate. Substituting personal and regional budget constraints into the utility function (1), we get

$$u_i = x_i(b_i - c) + 1.$$

Then, in region i, x_i is determined by majority voting over the space of alternatives, X. This implies that

$$x_i^d = \begin{cases} 1 & \text{if } b_{mi} \geq c \\ 0 & \text{otherwise.} \end{cases}$$

So, the utility from decentralized provision for a citizen of i with benefit parameter b_i is

$$u^d(b_i) = \begin{cases} b_i - c + 1 & \text{if } b_{mi} \geq c \\ 1 & \text{otherwise.} \end{cases} \tag{2}$$

2.3. Centralization

We assume that there are economies of scale with centralized provision, i.e., central government can produce a vector of projects more cheaply than can regional governments, reflecting the assumption that regional governments

[10]For example, suppose that there are three regions, and no intra-regional variation in tastes, and that regions 1, 2 prefer decentralization, while 3 prefers centralization. If region 3 has more than 50% of the total population, the federal referendum will select decentralization, and the unitary referendum will select centralization.

cannot cooperate to exploit economies of scale in such activities as research and development. We model economies of scale in the simplest possible way by supposing that with centralization, the cost per project in any region is $c - k$, $k > 0$. So, k measures the degree of economies of scale. This captures in a crude way the benefits of centralized R&D or procurement.[11]

Following the large literature on distributive politics, (see e.g., Persson 1998), we assume that it is a constitutional restriction that the cost of public good provision is financed out of a proportional income tax levied nationally at rate t. So, the national budget constraint is

$$nt = q(c - k)$$

where $q = \#\{i \in N \mid x_i = 1\}$, and $N = \{1, \ldots, n\}$. Substituting personal and national budget constraints into the utility function (1), we see that the utility from project vector $\mathbf{x} = (x_1, \ldots, x_n)$ for a type-b_i individual in region i is

$$u_i(\mathbf{x}; b_i) = b_i x_i - \frac{q}{n}(c - k) + 1. \tag{3}$$

So, in choosing $\mathbf{x}$, the central government faces a problem of distributive, or "pork barrel" politics: expenditures are specific to particular regions, whereas the tax is national. The simplest form of social choice in this case would be to have a national referendum over pairs of alternatives in X^n. The problem with this procedure is that it is well known that in this setting, there is no policy $\mathbf{x}^*$ which is a Condorcet winner (Ferejohn, Fiorina, and McKelvey 1987), and so a voting cycle would emerge.

Several resolutions of this problem have been suggested, by placing some structure on the that agenda-setting of the legislature. As our model is so simple, it turns out that the outcome is not very sensitive to the type of agenda-setting restriction we impose. Specifically, our key assumption is that the cost of the project is the same in every region. It turns out that this assumption implies that all regions get projects with the same probability under several different kinds of agenda-setting restrictions. We refer to this outcome as *ex ante policy uniformity*: it is the cost of centralization, which must be weighed by voters against the benefit of economies of scale.

We demonstrate ex ante policy uniformity using one of the most influential models of agenda-setting and voting, the legislative bargaining model[12] of Baron and Ferejohn (1989). As our model is one where the identities of

[11]A more sophisticated approach would allow the economies of scale to depend on the number of regions in which projects are provided, i.e., $k = k(q)$, where $k(\cdot)$ is decreasing in q. This would not change the outcome with centralization, as described in Proposition 1, provided that k is not decreasing so fast that the agenda setter wishes to offer projects to more than just a minimum winning coalition of regions (i.e., $m - 1$ regions) in order to reduce costs. If k is not decreasing too fast, then, all the analysis of this paper goes though, with k replaced by $k(m)$.

[12]The legislative bargaining model has been widely used in recent contributions to political economy and public finance (see e.g., Persson 1998).

the legislators are not exogenously given, we must also specify a procedure by which legislators are selected from regions, and here, we make use of the citizen-candidate model of Besley and Coate (1997) and Osborne and Slivinski (1996). The order of events is as follows.

1. *Election of delegates.* (i) Any citizen in a region can stand for election (at some small positive cost, $\sigma > 0$); (ii) those citizens who stand are voted on; (iii) the winner is selected by plurality rule[13] and is that region's *delegate* in the national legislature; (iv) if no delegate stands for election, the region is not represented in the legislative process.
2. *The legislative process.* Suppose that a set $K \subseteq N$ of delegates are elected. In the first session, each delegate is selected with probability $1/\#K$ to make a *proposal*. A proposal from $i \in K$ is a vector $\mathbf{x}^i \in X^n$ of projects to be funded. It is then voted on. If it is accepted by a strict majority of delegates, it is implemented, but if it is not accepted by a strict majority, then the legislature continues to the next session in which a member is selected to make another proposal and so on. Sessions take time, and delegates have a per session discount factor of $\delta < 1$.

A *political equilibrium* is (i) a subgame perfect-equilibrium to the legislative game, conditional on the set of delegates; (ii) a voting equilibrium in each region, conditional on the set of candidates in that region, and the delegates elected by other regions; (iii) a candidate set for each region, where in every region, every candidate who stands for election does so only if the benefit of doing so is at least σ. A more formal description of each of these three stages, plus a proof of the Proposition 1,[14] is given in Appendix A. Let $m = (n + 1)/2$. Then we have:

PROPOSITION 1: *Assume $n\sigma/m < \bar{b}_i$, $i \in N$. Then, there is a political equilibrium*[15] *where (i) exactly one resident of region i with $b_i \geq n\sigma/m$, stands for election; (ii) this resident is unanimously elected as the delegate from region i; (iii) when selected as proposer, the delegate from region i proposes an $\mathbf{x}^i$ consisting of a project for region i and $m - 1$ regions in $N/\{i\}$, selected at random; (iv) the first proposal is accepted by the legislature.*

[13]If k candidates get equal numbers of votes, then each candidate is selected with equal probability $1/k$.

[14]All subsequent propositions are proven in Appendix B if a proof is required.

[15]The political equilibrium described above is not unique, nor are the equilibrium payoffs described in (4) unique, as there exist multiple equilibria to the legislative subgame. For example, with three regions, there is an asymmetric equilibrium where the delegate from 1 always makes a proposal to the delegate from 2, and vice versa, and where region 3 never gets a project, even if it elects a delegate, and consequently does not even bother to elect a delegate. However, the equilibrium generating payoffs (4) seems an excellent candidate for a "focal" equilibrium; any region is chosen with equal probability to join the "minimum winning coalition" with the agenda setter.

Then from (3) and Proposition 1, the expected payoff to any citizen of region i in this equilibrium is

$$u^c(b_i) = \frac{m}{n}\left[b_i - \frac{m}{n}(c-k)\right] + \left(1-\frac{m}{n}\right)\left[-\frac{m}{n}(c-k)\right] + 1$$
$$= \frac{m}{n}[b_i - c + k] + 1. \tag{4}$$

The first term on the right-hand side is the expected payoff in the event that region i's legislator is either selected as proposer, or is randomly selected to be "bribed" to vote for the proposal. The second term is the expected payoff otherwise.

One way to interpret (4) is that under centralization,[16] any region gets a project with probability $\frac{m}{n}$, *regardless* of the region's willingness to pay for the project, as measured by b_{mi}. This is precisely what we mean by *ex ante* policy uniformity; every region has the same probability of gaining a project.[17] *Ex post*, of course, projects are not uniformly distributed across regions, but concentrated in about half the regions. This is broadly consistent with evidence from the United States, where "pork-barrel" projects are concentrated in certain states (Besley and Coate 1998).

However, we assume below that the choice of centralization or decentralization is *prior* to the legislative bargaining process (which seems a reasonable assumption: in most countries, constitutional changes are very infrequent). In this case, it does not matter that *ex post*, the allocation of projects is not uniform with centralization. Rather, what is important is that *ex ante*, regions are comparing a "free" choice of project under decentralization with the fixed probability m/n of a project under centralization.

Note that some other alternative agenda-setting models will also generate the outcome[18] described in Proposition 1. So, it is not really the legislative bargaining model per se that is restrictive, but the assumption of equal costs, which is driving the policy " uniformity" result. Intuitively, because all regions share the cost of any project equally through the tax system, a majority of

[16]In other words, with centralization, provision of projects is entirely insensitive to regions' willingness to pay.

[17]This is in the spirit of the original definition of policy uniformity, due to Oates (1972). Oates' insight was that centralized fiscal policy was less sensitive to local preferences than decentralized fiscal policy, and he modeled this in a rather *ad hoc* way by assuming equal per capita expenditures in every region with centralization.

[18]For example, consider the model of Ferejohn, Fiorina, and McKelvey (1977) as extended by Lockwood(2002) to include a proposal stage. In this set-up, delegates are elected as above, i.e., stage 1 is as above. Then, all delegates can propose any alterative in X^n, and all proposals are randomly ordered into an amendment agenda. An alternative is then selected by voting on successive pairs of alternatives on this agenda. It is easy to show that there is an equilibrium of this model where each of the $n!/m!(n-m)!$ possible proposals that allocate projects to exactly m regions is chosen with equal probability. In this equilibrium, therefore, every region gets a project with probability m/n.

regions will always prefer projects only in the lowest-cost regions. But if all regions have the same project costs, any agenda setter is indifferent about the identity of his coalition partners, and so there is always an equilibrium where every coalition is equally likely.

Finally, there is a possibility (pointed out by a referee) that a delegate from region i may have an incentive to reject the offer of a project by the agenda setter. This will occur if the delegate's project benefit (say b_i) is less than his share of the project cost, i.e., $(c - k)/n$. We rule out this possibility by assuming that that $\underline{b}_i \geq (c - k)/n$ in what follows.

2.4. Voters' Preferences for Decentralization

In region i, a citizen with taste parameter $\hat{b}_i$ will be indifferent between centralization and decentralization if $u^c(\hat{b}_i) = u^d(\hat{b}_i)$. Writing this out in full using (2) and (4) and solving for $\hat{b}_i$, we get

$$\hat{b}_i = \begin{cases} c + \frac{n+1}{n-1}k & \text{if } x_i^d = 1 \\ c - k & \text{if } x_i^d = 0. \end{cases} \tag{5}$$

In the event that $\hat{b}_i$ is not in $[\underline{b}_i, \bar{b}_i]$, i.e., when there is no citizen that is indifferent between centralization and decentralization, we define $\hat{b}_i$ as follows. If $x_i^d = 1$, then if $c + \frac{(n+1)k}{n-1} \leq \underline{b}_i$, then $\hat{b}_i = \underline{b}_i$, and if $c + \frac{(n+1)k}{n-1} \geq \bar{b}_i$, then $\hat{b}_i = \bar{b}_i$. If $x_i^d = 0$, then if $c - k \leq \underline{b}_i$, then $\hat{b}_i = \underline{b}_i$, and if $c - k \geq \bar{b}_i$, then $\hat{b}_i = \bar{b}_i$. The importance of the critical value $\hat{b}_i$ is that it characterizes voters' preferences over centralization versus decentralization:

LEMMA 1: *If* $x_i^d = 1$, *then all residents of region i with* $b_i > \hat{b}_i$ *strictly prefer decentralization, and all residents of region i with* $b_i < \hat{b}_i$ *strictly prefer centralization. If* $x_i^d = 0$, *the reverse is true.*

This is intuitive. As all residents in a region bear the same share of cost of provision, those who value the project more than (respectively, less than) $\hat{b}_i$ will prefer the arrangement that gives the higher probability (lower probability) of the project taking place.

3. Referenda for the Assignment of Powers

The two referenda that we wish to study are the following:

The Unitary Referendum: Centralization or decentralization is selected by national referendum.

The Federal Referendum: Centralization or decentralization is selected by a two-stage referendum. All citizens within a region vote on centralization or decentralization, and the alternative that has the support of the majority of regions is selected.

The unitary referendum captures the idea that a vote in the national parliament, or national referenda, are used in unitary states to (re)assign powers. The federal referendum is intended to capture the idea that (re) assignment of powers in a federal state usually requires the approval of at least a simple majority of the regions.[19]

First, consider the federal referendum. Note[20] from (5) and Lemma 1 that the median voter in region i strictly prefers centralization to decentralization iff

$$b_{mi} \in \left(c - k,\, c + \frac{n+1}{n-1}k\right) = B_C. \tag{6}$$

So, as the number of regions becomes large, the interval B_C becomes symmetric around c, with length approximately $2k$. To simplify the statement and proof of subsequent results, we assume in what follows that no median voter is indifferent between centralization and decentralization, i.e., $b_{mi} \neq c - k,\, c + \frac{n+1}{n-1}k$.

Now with the federal referendum, the region votes for the median voter's most preferred alternative. So, the above analysis implies that under the federal referendum, the fraction of votes in favor of centralization is

$$\pi_F = \frac{\#\{i \in N \mid b_{mi} \in B_C)\}}{n}. \tag{7}$$

Then the federal referendum selects centralization iff $\pi_F > 0.5$, and decentralization otherwise.

If the unitary referendum is used,[21] from Lemma 1, the fraction of votes in favor of centralization is

$$\pi_U = \frac{1}{n} \sum_{\{i \mid b_{mi} \geq c\}} F_i\left(c + \frac{n+1}{n-1}k\right) + \frac{1}{n} \sum_{\{i \mid b_{mi} < c\}} [1 - F_i(c - k)] \tag{8}$$

and the unitary referendum selects centralization iff $\pi_U > 0.5$, and decentralization otherwise.

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[19]An important caveat here is that in practice, rules for constitutional amendment are more complex than this (Wheare 1963). For example, the approval of a super-majority of regions may be required, as in the United States, where 3/4 of states must approve. Or, as in the case of Switzerland and Australia, a majority of voters, as well as regions, must approve. These amendment rules are difficult to analyze, as they give a privileged position to the status quo. Study of such rules is a topic for future work.

[20]The proof of this is simple. If $c \leq b_{mi} < c + \frac{n+1}{n-1}k = \hat{b}_i$, then clearly $x_i^d = 1$, and so from Lemma 1, the median voter strictly prefers centralization. Again, if $\hat{b}_i = c - k < b_{mi} < c$, then clearly $x_i^d = 0$, and so from Lemma 1, the median voter again strictly prefers centralization.

[21]This follows from the fact that if $b_{mi} \geq c$, $x_i^d = 1$, and so from Lemma 1, all citizens in region i with taste parameters $b_i < c + \frac{n+1}{n-1}k$ will vote for centralization, and there is a measure $F_i(c + \frac{n+1}{n-1}k)$ of these citizens. Summing over all regions with $b_{mi} \geq c$, we get the first term in (8). The second term is derived in a similar way.

How do the two referenda compare? Generally, there will be a minority of voters in a region who disagree with the decision of the median voter of the region. We call the voters who disagree *dissenting voters*. It is clear that the way in which dissenting voters are distributed across regions determines whether or not the unitary referendum conflicts with the federal referendum. For example, if a majority of regional median voters prefer decentralization, but in those regions, there are large numbers of dissenting voters who prefer centralization, then the unitary referendum may choose centralization. Of course, this argument works in reverse, so there is no presumption that the federal referendum will choose decentralization more often, or indeed less often, than the unitary referendum.

To understand how dissenting voters may determine the difference between the two rules, it is very helpful to start with benchmark conditions under which the rules are equivalent. Say that federal and unitary referenda are *equivalent* if the federal referendum selects decentralization iff the unitary referendum selects decentralization. Two simple sufficient conditions for equivalence are the following.

LEMMA 2: *If there is either (i) no intra-regional variation in tastes* ($\underline{b}_i = \bar{b}_i \equiv \{b_{mi}\}, \forall i \in N$), *or (ii) no inter-regional variation in tastes* ($F_i \equiv F_j, \forall i, j \in N$), *or both, federal and unitary referenda are equivalent.*

The intuition for this result is clear. First, condition (i) implies that there is no dissenting vote in any region. Condition (ii) implies that if (de)centralization is chosen by the federal referendum, *all* regions must vote for this option. So, as at least 50% of the electorate in each region prefers the option, so must at least 50% of the electorate overall.

The implication of Lemma 2 is that any difference between the two referenda only will appear when *both* intra-regional and inter-regional variances in tastes are present. The following example illustrates this point.

Example 1: There are three regions, where regional medians are $b_{m1} = b_{m2} - \varepsilon$, $b_{m2} = c + \frac{k}{2}$, $b_{m3} = b_{m2} + \varepsilon$. Also, F_1 is uniform with support of length 2ϕ, and F_2, F_3 are uniform with support of length 2θ. So, θ, ϕ measure *intra-regional variation* in tastes, and ε measures *inter-regional variation* in tastes.

We then have the following fact proved in the Appendix:

Fact 1: Assume $\phi = 0$ in Example 1. Then, when $\varepsilon < \frac{3}{2}k$, centralization is chosen by the federal referendum, while decentralization is chosen otherwise. When $\theta \geq 2\varepsilon - 3k$, centralization is chosen by the unitary referendum, while decentralization is chosen otherwise.

So, when $\phi = 0$, there exist parameter values where decentralization is chosen by the federal referendum, and centralization by the unitary referendum (but not vice versa), as shown in panel (a) of Figure 1. The intuition is as follows. When there is no intra-regional variance ($\theta = 0$), federal and unitary

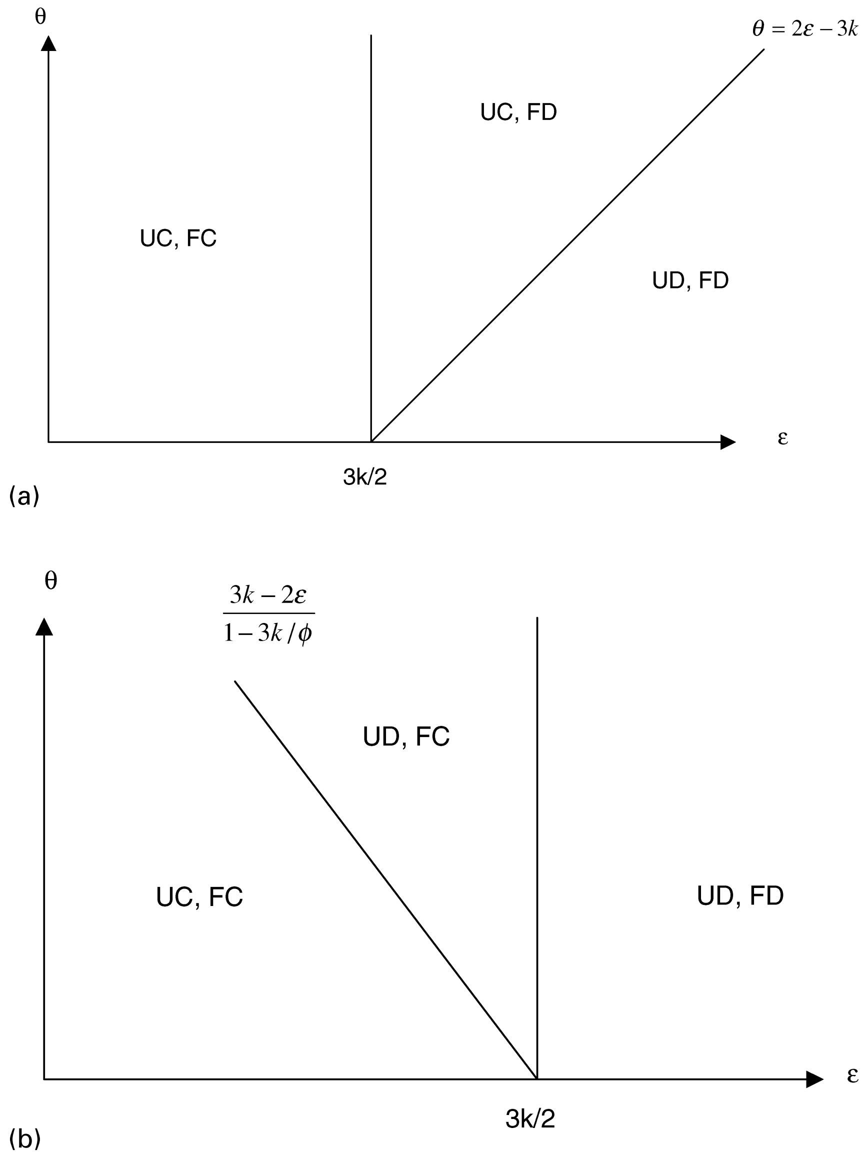

Figure 1 **Q1**

referenda agree, as predicted by Lemma 2. Now suppose that $\varepsilon > 3k/2$, so decentralization is chosen by both. As θ rises from zero, a dissenting votes in favor of centralization develop in both high-taste region 3, and low-taste region 1 (namely those residents with high $b's$ in the low-taste regions, and low $b's$ in the high-taste regions). With a federal referendum, these dissenters are ignored (the tyranny of the majority), but with a unitary referendum, these voters' preferences count. When there are enough of these dissenting voters (when θ is high enough), the unitary referendum chooses centralization when the federal referendum chooses decentralization. (The disagreement is in the region *RD*, *UC*).

However, we can also establish the opposite, using a different variant of Example 1.

Fact 2: Assume $\phi > 3k$ in Example 1. Then, as before, when $\varepsilon < \frac{3}{2}k$, centralization is chosen by the federal referendum, while decentralization is chosen otherwise. When $\theta < \frac{3k-2\varepsilon}{1-3k/\phi}$, centralization is chosen by the unitary referendum, while decentralization is otherwise.

So, in this variant of the example, there exist parameter values where centralization is chosen by the federal referendum, and decentralization by the unitary referendum. This is shown in panel (b) of Figure 1, where the federal and unitary decisions are compared. When there is no intra-regional variance ($\theta = 0$), federal and unitary referenda agree, as predicted by Lemma 2. As θ rises, dissenting votes accumulate in regions 2 and 4, and so the unitary referendum eventually chooses decentralization when the federal referendum chooses centralization. (The disagreement is in the region *RC*, *UD*).

4. Some Asymptotic Results

Example 1 indicates that without imposing some more structure on the problem, we are unlikely to be able to make general statements comparing unitary and federal referenda. In this section, we study the asymptotic behavior of the two rules as that when the number of regions is "large," under some symmetry assumptions on the distribution of preferences both within and across regions.

In this case, it turns out, somewhat surprisingly, that a comparison of the two referenda can be based *only* on the distribution of regional median project benefits, i.e., $\{b_{mi}\}_{i\in N}$. In particular, if this distribution is uniform (in the limit, as defined below), then the two referenda are equivalent, *no matter how* the project benefits are distributed within regions. Starting from this benchmark, we can then develop simple conditions on the limiting distribution of regional median benefits for the federal referendum to be either "more" or "less" centralized than the unitary referendum.

To conduct an asymptotic analysis as the number of regions becomes large, we assume the following structure: (i) regional median project benefits are random draws from a known distribution; (ii) conditional on the regional median, the distribution of tastes within any region is the same. Specifically, we assume:

A0. Every b_{mi} is a random draw from a common distribution G, where G is absolutely continuous with support $[\underline{b}_m, \bar{b}_m]$.

A1. The distribution of project benefit b in any region with median b_m, net of the median, i.e., $y = b - b_m$, is given by $F(y)$ on $[\underline{y}, \bar{y}]$, with $F(0) = 0.5$ by definition.

Armed with A0 and A1, we can now derive relatively simple asymptotic formulas for the proportions of regions and citizens that prefer centralization. Let π_F^n be the proportion of n regions that prefer centralization, given A0 and A1. From A0, for fixed n, π_F^n is a random variable. Moreover, from (6), region i chooses centralization iff $b_{mi} \in (c - k, c + \frac{n+1}{n-1}k) = B_n$. So, as $n \to \infty$, the probability limit of the proportion of regions choosing centralization is

$$\begin{aligned} \underset{n\to\infty}{plim}\, \pi_F^n &= \underset{n\to\infty}{plim} \frac{\#\{i \in N \mid b_{mi} \in B_n\}}{n} \\ &= G(c+k) - G(c-k) = \pi_F(k) \qquad (9) \end{aligned}$$

where $\pi_F(k)$ is strictly increasing in k. This is intuitive; the higher the cost saving from centralization, then ceteris paribus, the larger the fraction of regional median voters who will be in favor of centralization.

Now consider the unitary referendum. From Lemma 1, in all regions with a median project benefit $b_m \geq c$, the proportion of residents who prefer centralization is $F(c + k - b_m)$, and in all regions with a median $b_m < c$, the proportion of residents who prefer centralization is $1 - F(c - k - b_m)$. Now let π_U^n be the proportion of citizens in n regions that prefer centralization, given A0 and A1. Again, from A0, for fixed n, π_U^n is a random variable. Its probability limit is

$$\begin{aligned} \underset{n\to\infty}{plim}\, \pi_U^n = &\int_c^{\bar{b}_m} F(c + k - b_m) g(b_m)\, db_m \\ &+ \int_{\underline{b}_m}^{c} [1 - F(c - k - b_m)] g(b_m)\, db_m = \pi_U(k) \qquad (10) \end{aligned}$$

Armed with formulas (9) and (10), we have a very simple way of comparing federal and unitary referenda. Recall that $\pi_F(.)$ is increasing in k, and let the unique solution to $\pi_F(k) = 0.5$ be k_F. Then, the federal referendum selects centralization iff the cost saving from centralization is sufficiently high, i.e., $k > k_F$. Again, note that $\pi_U(k)$ is increasing in k, and let the unique solution to $\pi_U(k) = 0.5$ be k_U. Then, the federal referendum selects centralization iff $k > k_U$.

Now, we say that *the federal referendum is more centralized (decentralized) than the unitary referendum* if, when centralization is chosen by the unitary referendum, it is also chosen by the federal referendum (vice versa). Using the above arguments, these two cases can be expressed as

$$k_U > k_F, \; k_U < k_F, \qquad (11)$$

respectively. For example, if the federal referendum is more centralized, the cost advantage to centralization has to be higher (ceteris paribus) for centralization to be chosen under the unitary referendum. Finally, the *federal and unitary referenda are equivalent* when $k_U = k_F$.

We can now move to the main results of this section. They show that when certain symmetry assumptions are made about the distributions F, G, then whether the federal referendum is more or less decentralized than the unitary referendum depends *only* on the shape of G. These assumptions are the following:

A2. F, G are symmetric around their means, i.e., $F(-x) = 1 - F(x)$, $G(c - x) = 1 - G(c + x)$, all $x \in \Re$.

A3. In the limit, half the regions choose projects with decentralization, i.e., $G(c) = 0.5$.

These assumptions impose two forms of symmetry. A2 requires that the within-region project benefits and median benefits across regions are both symmetrically distributed. A3 ensures that the choices of regions under decentralization are symmetric.

Assumptions A2 and A3 imply the following very useful simplifications. First, A3 plus A2 imply that b_m has a mean value of c. Therefore, it follows that $G(b_m) \equiv \Gamma(b_m - c)$, where Γ is a symmetric mean-zero distribution. So, from (9),

$$\pi_F(k) = \Gamma(k) - \Gamma(-k) = 2\Gamma(k) - 1 \tag{12}$$

where the second equality follows from the symmetry of Γ. Second, defining $x = b_m - c$, $\bar{x} = \bar{b}_m - c$, $\gamma(x) = \Gamma'(x)$, we get

$$\begin{aligned}
\pi_U(k) &= \int_c^{\bar{b}_m} F(c + k - b_m) g(b_m)\, db_m + \int_{\underline{b}_m}^{c} [1 - F(c - k - b_m)] g(b_m)\, db_m \\
&= \int_c^{\bar{b}_m} F(c + k - b_m) \gamma(b_m - c)\, db_m \\
&\quad + \int_{\underline{b}_m}^{c} [1 - F(c - k - b_m)] \gamma(b_m - c)\, db_m \\
&= \int_0^{\bar{x}} F(k - x)\gamma(x)\, dx + \int_{-\bar{x}}^{0} [1 - F(-k - x)]\gamma(x)\, dx \\
&= \int_0^{\bar{x}} F(k - x)\gamma(x)\, dx + \int_{-\bar{x}}^{0} F(k + x)\gamma(x)\, dx \\
&= \int_0^{\bar{x}} F(k - x)\gamma(x)\, dx + \int_0^{\bar{x}} F(k - z)\gamma(-z)\, dz \\
&= 2\int_0^{\bar{x}} F(k - x)\gamma(x)\, dx.
\end{aligned} \tag{13}$$

Here, we have used the definition of Γ in the second line, a change of variables in the third, the symmetry of F (around zero, by definition) in the fourth, a change of variables in the second integral in the fifth, and finally the symmetry of γ around zero in the sixth.

We are now in a position to state and prove our first, benchmark, result.

PROPOSITION 2: *If A0–A3 hold, and in addition, the regional medians are uniformly distributed across regions, i.e.,* Γ *is uniform, then the federal and unitary referenda are equivalent, i.e.,* $k_F = k_U$.

Therefore, we see that the borderline case is where the distribution of regional median project benefits is uniform, *irrespective* of how project benefits are distributed within regions. What happens when we move away from the uniform? Let H be any absolutely continuous distribution function with support $[a, b]$. Then we have the following definition:

DEFINITION: *H is strictly positively (negatively) single-peaked on* $[a, b]$ *if the density* $h(\cdot)$ *is strictly quasi-concave (quasi-convex) on* $[a, b]$.

Note that if a density function is positively (negatively) single-peaked and is symmetric around zero, then it must have a global maximum (minimum) at zero. Given these definitions, we now have:

PROPOSITION 3: *Assume that A0–A3 hold, that* Γ *is strictly positively single-peaked, and in addition, that* $\Pr(|b_m - c| \leq \bar{y}) \geq 0.5$. *Then, the federal referendum is more centralized than the unitary referendum, i.e.,* $k_F < k_U$.

The assumptions required for this result are easy to interpret, with the possible exception of the requirement that $\Pr(|b_m - c| \leq \bar{y}) \geq 0.5$. This says essentially that the dispersion of the regional medians b_m around c must not be too large relative to the dispersion of project benefits *within* regions, as measured by $\bar{y}$.

The intuition for the result is that the proportion of median voters in each region preferring centralization under the federal referendum, π_F, is *more responsive to changes in* Γ *away from the uniform distribution* than the proportion of all voters preferring centralization under the unitary referendum, π_U. In turn, this is because π_F does not take account of the views of dissenting voters. The following example may help clarify this argument. Suppose that Γ is initially uniform on $[-1, 1]$ and it is changed to a positively single-peaked distribution Γ^+ by moving some probability weight δ from the tails to the center, i.e., so that Γ^+ has a mass point of δ at zero, and the remaining fraction $1 - \delta$ of regional means are distributed uniformly on $[-(1 - \delta/2), 1 - \delta/2]$. Then, as long as the median voters in the regions in the tails of the distribution initially preferred decentralization ($1 - \delta/2 > c + k$), a fraction δ more median voters will prefer centralization with Γ^+. So, π_F rises by δ.

Now consider a region i whose median voter was initially in the positive tail of the distribution, i.e., where $b_{mi} \simeq 1$, assuming δ small. Initially, a fraction $F(c + k - b_{mi}) \simeq F(c + k - 1) < 0.5$ of the voters in i *already* prefer centralization (the dissenting voters). After the switch, $F(k) > 0.5$ of the voters now prefer centralization. So, in this region, the net increase in the number of voters preferring centralization is $F(k) - F(c + k - 1) < 1$. So, π_U rises by approximately $\delta[F(k) - F(c + k - 1)] < \delta$ following the switch.

The same intuition explains our next result.

PROPOSITION 4: *Assume that A0–A3 hold, that Γ is strictly negatively single-peaked, and in addition $0.5 \le \Pr(|b_m - c| \le \bar{b}_m - \bar{y})$. Then, the federal referendum is more decentralized than the unitary referendum, i.e., $k_F > k_U$.*

Again, the assumption $0.5 \le \Pr(|b_m - c| \le \bar{b}_m - \bar{y})$ requires that regional medians not be too dispersed around the mean value c. The condition obviously also requires $c < \bar{y} < \bar{b}_m$.

This strong characterization of the relationship between federal and unitary referenda has used the symmetry assumptions A2 and A3. We now present two examples that show that these assumptions cannot be relaxed.

Example 2: First, $x = b_m - c$ is distributed according to Γ, with density

$$\gamma(x) = \begin{cases} \frac{1}{2\gamma} & 0 \le x \le \gamma \\ \frac{1}{2\pi} & -\pi \le x < 0. \end{cases}$$

Also, F is given by the density[22]

$$f = \frac{1}{2\phi}, -\phi \le y \le \phi.$$

Note that if $\gamma \ne \pi$, Γ is asymmetric, violating assumption A2, but that A3 is always satisfied. So, from (10), the proportion of regions preferring centralization is

$$\pi_F(k) = \Gamma(k) - \Gamma(-k) = \frac{k}{2\gamma} + \frac{k}{2\pi} = \frac{k}{2}\left(\frac{\pi\gamma}{(\pi + \gamma)}\right).$$

So, as $\pi_F(k_F) = 0.5$,

$$k_F = \frac{\pi\gamma}{(\pi + \gamma)}.$$

Now, from (11), and changing the variable of integration, see that the proportion of voters preferring centralization is

$$\begin{aligned} \pi_U(k) &= \int_0^\gamma \frac{k - x + \phi}{2\phi}\frac{1}{2\gamma}dx + \int_{-\pi}^0 \left[1 - \left(\frac{-k - x + \phi}{2\phi}\right)\right]\frac{1}{2\gamma}dx \\ &= 0.5 + \frac{1}{8\phi}\left[(2k - \gamma + 2\phi) + (2k - \pi - 2\phi)\right]. \end{aligned}$$

[22]To ensure that $b \ge 0$, we need $c \ge \pi + \phi$.

So, as $\pi_U(k_U) = 0.5$,

$$k_U = \frac{\gamma + \pi}{4}.$$

Therefore, the two referenda are only equivalent if $\frac{\gamma+\pi}{4} = \frac{\pi\gamma}{(\pi+\gamma)}$, which holds iff $\pi = \gamma$. And, as long as the distribution of Γ asymmetric, i.e., $\pi \neq \gamma$, Proposition 2 no longer holds.

Now we present an example that shows that the benchmark result does not hold either when A3 does not hold.

Example 3: First, $x = b_m - c$ is distributed uniformly on $[-\gamma + \beta, \gamma + \beta]$ with $|\beta| < \gamma$, i.e., $\Gamma(x) = \frac{x+\gamma-\beta}{2\gamma}$. So, as long as $\beta \neq 0$, the mean of b_m is no longer c and consequently, A3 is violated. It is easily checked that $k_F = 0.5\gamma$. Now, also assume that F is distributed uniformly on $[-0.5, 0.5]$, i.e., $F(y) = y + 0.5$. Then

$$\begin{aligned}\pi_U(k) &= \frac{1}{2\gamma}\left[\int_0^{\beta+\gamma} F(k-x)\,dx + \int_{\beta-\gamma}^{0} (1 - F(-k-x))\,dx\right]\\ &= \frac{1}{2\gamma}\left[\int_0^{\beta+\gamma} (0.5 + k - x)\,dx + \int_{\beta-\gamma}^{0} (0.5 + k + x)\,dx\right]\\ &= \frac{1}{2\gamma}\left[2\gamma(0.5+k) - 0.5(\beta+\gamma)^2 - 0.5(\beta-\gamma)^2\right]\\ &= 0.5 + k - 0.5\beta^2/\gamma - 0.5\gamma.\end{aligned}$$

Now, as $\pi_U(k_U) = 0.5$, we see that

$$k_U = 0.5\beta^2/\gamma + 0.5\gamma.$$

So, as long as A3 is not satisfied, i.e., $\beta \neq 0$, then $k_U > k_F$, i.e., the unitary referendum is more decentralized than the federal referendum, and consequently Proposition 2 fails. Interestingly, the unitary referendum is more decentralized whether Eb_m is greater or less than c.

Finally, we can state some comparative statics results that describe how k_F, k_U vary as the dispersion of preferences for project benefits increases, both across and within regions. We model an increase in the dispersion of median project benefits across (within) regions as a mean-preserving spread in the distribution of G (F). It is fairly obvious from (12) that (i) following a mean-preserving spread in the distribution of G, median voters in more regions will prefer decentralization, and so the cost saving from centralization at which half the median voters prefer centralization, namely k_F, will rise, and (ii) following a mean-preserving spread in the distribution of F, k_F is unchanged. The following theorem also establishes some less obvious results about what happens to k_U.

PROPOSITION 5: *If A0–A3 hold, then following a symmetric mean-preserving spread in G, both k_F, k_U rise. If A0–A3 hold, then following a symmetric mean-preserving spread in F, (i) k_F is left unchanged; (ii) if, in addition, the hypotheses of Proposition 3 (Proposition 4) hold, k_U rises (falls).* **Q2**

These results establish that an increase in the dispersion of median project benefits across and within regions affects our two referenda in quite different ways. An increase in the dispersion of median project benefits *across* regions makes both referenda unambiguously more "likely" to choose decentralization, whereas an increase in the dispersion of project benefits *within* regions has an ambiguous effect on the unitary referendum—it may make centralization more likely.

5. Efficiency of Referenda

We are now in a position to assess the relative efficiency of federal and unitary referenda. As utility is linear in income, the model is one of transferable utility, and so the natural measure of efficiency is the aggregate surplus, or sum of utilities.[23] The aggregate surplus is

$$W^k = E\sum_{i=1}^{n} u^k(b_i)$$

in the case of centralization ($k = c$) and decentralization ($k = d$), and where the expectation is taken with respect to variables $(b_1,.b_n)$. An alternative way of justifying the use of aggregate surplus as a measure of efficiency is to suppose, following Buchanan (1975, 1978, 1987) and Dixit (1996), that choice between constitutions—here, understood as referenda—should be thought of as taking place behind a Rawlsian "veil of ignorance," as constitutions are changed infrequently. If we suppose that the veil is complete, i.e., every citizen, *ex ante*, believes that it is equally likely that he will be resident in any region and if resident in region i, will have characteristics b_i drawn at random from the distribution F_i. In this case, the expected utility of the citizen behind the veil of ignorance can be calculated as W^k/n.

Now, using (4) and (3), we see that the gain from a move to decentralization for a resident of region i with taste parameter b_i is **Q3**

$$u^d(b_i) - u^c(b_i) = \Delta_i = \phi(b_i, b_{mi}) - \frac{m}{n}(b_i - c) - \frac{m}{n}k$$

with

$$\phi(b_i, b_{mi}) = \begin{cases} b_i - c & \text{if } b_{mi} \geq c \\ 0 & \text{otherwise.} \end{cases}$$

[23] If the aggregate surplus is greater under the federal referendum, then the federal referendum is unambiguously potentially Pareto-preferred. Of course, this is only of interest if lump-sum transfers between regions are possible at the point where the choice between centralization and decentralization is made.

For simplicity, we assume from now on that the distribution of b_i is symmetric, which implies $Eb_i = b_{mi}$. Then, taking expectations over the b_i, it is easy to show that

$$E\Delta_i = \left[\max\{b_{mi} - c, 0\} - \frac{m}{n}(b_{mi} - c)\right] - \frac{m}{n}k.$$

That is, the expected gain from decentralization across all residents of region i is just the gain to decentralization for the median voter in that region. The first term of $E\Delta_i$ in the square brackets captures the gain of being able to respond more flexibly to regional preferences. The term $-mk/n$ is the loss implied by the inability to exploit economies of scale. So the gain to decentralization, as measured by aggregate surplus is

$$W^d - W^c = \Delta W = \sum_{i=1}^{n} E\Delta_i.$$

Therefore, the social planner selects decentralization iff $\Delta W \geq 0$. Note that with b_i symmetrically distributed, the social planner's choice is *independent* of the distribution of the taste parameter b_i within each region, as is the federal referendum.

We now turn to discuss efficiency of federal and unitary referenda against this benchmark. Define a referendum to be *efficient* if it makes the same selection of centralization or decentralization as the social planner. Say that the federal referendum is *more efficient* than the unitary referendum, if whenever the unitary referendum leads to an efficient decision, the federal referendum does also, and vice versa. Also, say that a referendum is *inefficiently (de)centralized* if when it makes an inefficient choice, it chooses (de)centralization.

Both referenda may be inefficient, for the usual reason that majority voting does not take into account intensity of preference. However, in general, neither referendum is biased in any particular direction; i.e. neither is inefficiently centralized or decentralized. The following example illustrates this.

Example 4: Suppose that there are three regions, with $b_{m1} = b_{m2} = c = 1$, $k = 0.5$, and $b_{m3} = 9$. Then, it is easy to calculate that $E\Delta_1 = E\Delta_2 = -1$, but $E\Delta_3 = 3$. Then, as $\Delta W = \sum_{i=1}^{3} E\Delta_i = 1$, the social planner will choose decentralization, but as a majority of regions have $E\Delta_i < 0$, the federal referendum will choose centralization.

On the other hand, suppose that $b_{m3} = 0$, $b_{m1} = b_{m2} = 3$, with the other parameters as before. Then, $E\Delta_1 = E\Delta_2 = 1/3$, but $E\Delta_3 = -1$. Then, as $\Delta W = \sum_{i=1}^{3} E\Delta_i = -1/3$, the social planner will choose centralization, but as a majority of regions have $E\Delta_i > 0$, the federal referendum will choose decentralization.

However, in the asymptotic case as n becomes large, as shown above, there are a number of conditions (A0–A3), under which we can compare the federal and unitary referenda in quite a straightforward manner. It turns out that under the same assumptions, we can obtain a quite tight characterization of how both rules compare to the efficient benchmark.

First, we can calculate the probability limit of the welfare gain from decentralization under conditions A0–A3 when the principle of aggregation holds;

$$\begin{aligned} \underset{n\to\infty}{plim} \frac{\Delta W}{n} &= \int_c^{\bar{b}_m} (b_m - c) g(b_m)\, db_m - 0.5(Eb_m - c) - 0.5k \\ &= \int_c^{\bar{b}_m} (b_m - c) g(b_m)\, db_m - 0.5k \\ &= \int_0^{\bar{x}} x g(x)\, dx - 0.5k. \end{aligned} \tag{14}$$

In (14), we have used the fact that $Eb_m = c$ from A3 in the second line, and a change of variable in the last line. Clearly, from (14), centralization is strictly more efficient iff $\int_0^{\bar{x}} xg(x)\, dx - 0.5k > 0$, or

$$k > 2 \int_0^{\bar{x}} xg(x)\, dx \equiv k_E.$$

So, k_E characterises the efficient allocation of fiscal power. If $k > k_E$, then the gains from economies of scale outweigh the losses from *ex ante* policy uniformity, and centralization is efficient, and the converse is true if $k < k_E$. We now have the following result.

PROPOSITION 6: *Assume A0–A3. If G is uniform, then both rules are efficient* $(k_F = k_U = k_E)$. *If G is strictly positively single-peaked, then the federal referendum is inefficiently centralized* $(k_F < k_E)$. *If G is strictly negatively single-peaked, then the federal referendum is inefficiently decentralized* $(k_F > k_E)$.

The intuition for this result is the following. When Γ deviates from the uniform by (say) becoming strictly positively single-peaked, a proportion $x\%$ of median voters will change their preference from decentralization to centralization. But some of these "switchers" will only gain a very small amount, as they were nearly indifferent, so that the increase in expected benefit from centralization relative to decentralization (i.e., the percentage change in ΔW) will be less than $x\%$.

Something remains to be said about the unitary referendum in the nonuniform case. Let $\mathcal{F}$ be the set of symmetric single-peaked zero-mean distributions on $[-\bar{y}, \bar{y}]$, and let $\mathcal{A} \subset \mathcal{F}$ have the property that for any two distributions F, F' in $\mathcal{A}$, one distribution is a mean-preserving spread of another (Rothschild and Stiglitz 1970). Then, due to the symmetry of members of $\mathcal{A}$, if F' is a mean-preserving spread of F, the variance of F' is greater than that of F. Suppose we denote the variance of F by σ_F^2; this number[24] uniquely defines any F in $\mathcal{A}$. Then we can state the following:

PROPOSITION 7: *Assume $F \in \mathcal{A}$. Under the assumptions of Proposition 3, there exists a $\hat{\sigma}^2$ such that (i) for any $\sigma_F^2 < \hat{\sigma}^2$, the unitary referendum is more efficient than the federal referendum, but is inefficiently centralized; (ii) when $\sigma_F^2 = \hat{\sigma}^2$, the unitary referendum is efficient: (iii) when $\sigma_F^2 > \hat{\sigma}^2$, the unitary referendum is inefficiently decentralized.*

Under the assumptions of Proposition 4, there exists an $\hat{\sigma}^2$ such that (i) for any $\sigma_F^2 < \hat{\sigma}^2$, the unitary referendum is more efficient than the federal referendum, but is inefficiently decentralized; (ii) when $\sigma_F^2 = \hat{\sigma}^2$, the unitary referendum is efficient: (iii) when $\sigma_F^2 > \hat{\sigma}^2$, the unitary referendum is inefficiently centralized.

So, not surprisingly, when σ_F^2 is small, the unitary referendum behaves in a similar way to the federal referendum, but less obviously, when σ_F^2 is large enough, the unitary referendum may exhibit a different direction of inefficiency than the federal referendum.

6. Related Literature and Conclusions

This paper has attempted both a positive and normative analysis of two "constitutional rules" for choosing the degree of decentralization, in the setting of a particular model of the costs and benefits of decentralization. In the asymptotic case, we have obtained a number of results about how these rules differ; the key determinant of the difference seems to be how median voter preferences are distributed across regions. The efficiency of both referenda has also been investigated.

The results of this paper can be compared to Cremer and Palfrey (1996). In the basic model of their paper, where taste parameters were normally distributed, they showed[25] that—in our terminology—whenever a unitary referendum chooses centralization, the federal referendum does also. Cremer and Palfrey call this result "the principle of aggregation." They also argued that it was robust to several extensions, including other possible statistical distributions for tastes, such as the uniform, but do not provide any general

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[24]Note that $\sigma_F^2 \leq \bar{y}^2$, as $\bar{y}^2$ is the maximum possible variance of all distributions in $\mathcal{A}$.

[25]Cremer and Palfrey obtained formulas for the proportion of regions, and the proportion of voters, who prefer centralization, as a function of only one parameter σ^2, the ratio of the *inter*-regional variance in tastes to the *intra*-regional variance in tastes. They established two main facts in their paper, both in the limiting case as the number of regions went to infinity. First, when this ratio was below some $\hat{\sigma}^2$, the proportion of *regions* preferring centralization was greater than the proportion of individual *voters* preferring centralization. Second, for all values of the ratio below $\hat{\sigma}^2$, the proportion of individual *voters* preferring centralization was greater than 0.5 (and so the unitary referendum chose centralization).

conditions under which it holds.[26] Bearing in mind that the two papers build quite different models of the costs and benefits of decentralization, we can note the following. If Γ has a normal distribution with mean zero, (truncated so that b_m is positive), it is symmetric and positively single-peaked, so Proposition 3 applies, and Proposition 3 states the "principle of aggregation" for this model.

There are several other recent papers that are more loosely related to this one. Besley and Coate (1998) and Lockwood (2002) build "political economy" models of the choice between centralization and decentralization, but with a different focus, namely to study the political inefficiency of centralization. Our model has elements of both models, but is deliberately stylized, so as to enable an analysis of federal and unitary referenda. Bolton and Roland (1997) study a model quite similar to the one of this paper,[27] but the purpose of their analysis is rather different, namely, to analyze possible secession by one of the regions, and how the threat of secession changes the tax policy of central government.

Appendix A: Political Equilibrium with Centralization

A.1. Legislative Equilibrium

Some details of the legislative game are as follows. Let K be the set of regions that elect delegates. Assume for the moment that $K = N$. A time index $t = 0, 1, 2, \ldots$ tracks the number of "rounds" of legislative bargaining. The strategies of the delegates are as follows. If at time t delegate i is proposer, he must choose a proposal $\mathbf{x}^i = (x_1^i, \ldots, x_n^i) \in X^n$ conditional on the history of play up to that point, H_t. If at time t delegate i is a responder, he must choose a response $r_i \in \{yes, no\}$ conditional on the history of play up to that point (H_t, a_j) where j is proposer. If a strict majority of delegates choose $r_i = yes$, the proposal is approved and the game terminates. Otherwise, the game continues to the next round.

Following Baron (1991), we restrict attention to stationary equilibria. A *stationary equilibrium* has the property that whenever g and h are structurally identical subgames, the continuation values of any player i (denoted V_i) are the same in both subgames, no matter what the time period, i.e., $V_i(\tau, g) = V_i(\tau', h)$ when $g = h$.

[26]In Section 4.3 of their paper, they present a uniform distribution example where 100% of regions prefer centralization, in which case the principle of aggregation certainly holds, but this example is not general.

[27]In their model, there are two, rather than n regions, agents differ in incomes, rather than preferences for the public good (in their model, the public good can be interpreted as a lump-sum transfer). Also, the economies of scale from centralized public good provision are modeled rather differently.

Now let V_i denote the payoff in the subgame beginning with the random selection of candidates. In a stationary equilibrium, delegate i will vote for proposal j iff

$$x_i^j b_i^d - \frac{k(\mathbf{x}^i)}{n}(c-k) \geq \delta V_i. \tag{A1}$$

where $\delta < 1$ is the discount factor, and b_i^d is the taste parameter of the delegate. Say that *j offers i a project* if $x_i^j = 1$. Conjecture then that if delegate i is proposer, he chooses $m-1$ delegates at random and offers them projects, (as well as choosing a project for himself) and these delegates accept. Given these strategies, delegate $i's$ continuation payoff at the proposer selection stage is

$$V_i = \frac{m}{n}\big(b_i^d - c + k\big). \tag{A2}$$

It is then clear from (A2) that if i is offered a project, (A1) reduces to

$$b_i^d - \frac{m}{n}(c-k) \geq \delta\frac{m}{n}(b_i^d - c + k),$$

which certainly holds. So, all delegates will accept projects if offered. It remains to show that this is the best strategy for the proposer. This follows from two observations.

First, if delegate i is proposer, he will never offer projects to more than $m-1$ delegates, as only m delegates are needed to approve a proposal, and additional offers raise the cost to the proposer through the tax rule. Second, if delegate i is not offered a project, he will never accept a proposal as $-\frac{m}{n}(c-k) < \delta\frac{m}{n}(b_i^d - c + k)$ as long as $\delta < 1$, $b_i^d \geq 0$.

Now consider the case where region i does not elect a delegate i.e., $K = N/(i\}$. If no delegate from region i is elected, then there is a legislative game with $n-1$ delegates excluding region i, but the residents of region i continue to pay tax. There is a stationary equilibrium of the game with $n-1$ delegates with the same structure as that described above, where at each round the proposer j selects at random $m-1$ regions in $N/\{i, j\}$ and offers them projects, and the first proposal is accepted. Consequently, the payoff to any resident of i in this case is

$$W = -\frac{m}{n}(c-k). \tag{A3}$$

A.2. Voting Equilibrium

At the voting stage, every resident of i can vote for one of the candidates, or abstain. First, assume that only one candidate stands. As $V_i - W > 0$, for all $b_i \in B_i$, *all* voters strictly prefer to be represented rather than not, so a single candidate is always elected.

Second, assume that $l > 1$ candidates are standing for election. We have established that *whatever* the b_i^d of the delegate from region i, he will adopt the same strategy at the legislative stage. So, all delegates yield any citizen the same payoff. This means that all voters are indifferent between candidates. Assuming w.l.o.g. that indifferent voters randomize over candidates with equal probabilities, all candidates are elected with equal probability.

A.3. Candidate Entry

It is clear from (A2), (A3) that as long as

$$V_i - W = \frac{m}{n} b_i \geq \sigma$$

it pays a resident of i with taste parameter σ to stand for election, given that no other resident is standing. So, there is always a one-candidate equilibrium where one candidate stands with probability 1, as claimed.

Appendix B: Proofs of Results

Proof of Lemma 1: We only give the proof for the case where $\hat{b}_i \in [\underline{b}_i, \bar{b}_i]$; the proof in the "corner" case is similar. It is clear that if $u^c(\hat{b}_i) = u^d(\hat{b}_i)$ and $x_i^d = 1$ then $u^c(b) < u^d(b)$, $b > \hat{b}_i$, and $u^c(b) < u^d(b)$, $b < \hat{b}_i$. The argument is the same for a region where $x_i^d = 0$. ■

Proof of Lemma 2: (i) In this case, all citizens in a given region have identical preferences, so again there are no dissenting voters. It follows immediately from (3.3) and (3.3) that the federal and unitary referenda are equivalent.

(ii) Let centralization $= C$, decentralization $= D$. It is clear that with $b_i \equiv b_j$, $i, j \in N$, regions are unanimous in their choice of some $A \subset \{C, D\}$. As the total net dissenting vote is bounded below 0.5, if the federal referendum chooses A, then so must the unitary referendum. ■

Proof of Fact 1: (a) First consider the choice of a federal referendum between C and D. In the example, b_{m2} is constructed to be in the center of the interval $B_C = (c - k, c + 2k)$, so that b_{m1}, b_{m3} are in this interval iff $\varepsilon < \frac{3}{2}k$.

(b) Now the unitary referendum. As $\phi = 0$, all voters in region 2 choose C. So, the proportion of all voters choosing C is

$$\pi = \frac{1}{3}\left[1 + (1 - F_1(c - k)) + F_3(c + 2k)\right], \tag{B1}$$

where

$$F_i(x) = \frac{x - b_{mi} + \theta}{2\theta}, \quad i = 1, 3, \quad b_{mi} - \theta \leq x \leq b_{mi} + \theta. \tag{B2}$$

So, it is easy to compute

$$1 - F_1(c - k) = F_3(c + 2k) = 0.5 + \frac{\frac{3}{2}k - \varepsilon}{2\theta}. \tag{B3}$$

Combining (B1) and (B3), the proportion of voters preferring C is greater than one half ($\pi > 0.5$) iff

$$\theta > 2\varepsilon - 3k.$$

As $\theta \geq 0$, the Fact follows. ■

Proof of Fact 2: In this case, we suppose that $\phi > \frac{3}{2}k$, so that some voters in region 2 prefer D. Then, the proportion of voters preferring C is then

$$\pi = \frac{1}{3}\left[F_2(c+2k) - F_2(c-k) + (1 - F_1(c-k)) + F_3(c+2k)\right].$$

Also, using the fact that F_2 is defined as in (B2) with θ replacing ϕ, it is easy to compute that

$$F_2(c+2k) - F_2(c-k) = \frac{3k}{2\phi}.$$

So, after substitution,

$$\pi = \frac{1}{3}\left[\frac{3k}{2\phi} + 1 + \frac{\frac{3}{2}k - \varepsilon}{\theta}\right].$$

Now assume that $\phi > 3k$. Then $\pi > 0.5$ iff $\theta < \frac{3k - 2\varepsilon}{1 - 3k/\phi}$, as required. ■

Proof of Proposition 2: First, we calculate an explicit formula for k_F. By assumption, $\Gamma(x) = \frac{x + \bar{x}}{2\bar{x}}$. So, from (12), $\pi_F(k) = k/\bar{x}$, so $k_F = 0.5\bar{x}$. To prove the result, it is sufficient to show that $\pi_U(k_F) = 0.5$. Now note that as Γ is uniform, from (13), we have;

$$\pi_U(k) = \frac{1}{\bar{x}}\int_0^{\bar{x}} F(k - x)\,dx.$$

So,

$$\pi_U(k_F) = \frac{1}{\bar{x}}\int_0^{\bar{x}} F(0.5\bar{x} - x)\,dx = \frac{1}{\bar{x}}\int_{-0.5\bar{x}}^{0.5\bar{x}} F(y)\,dy, \quad y = 0.5\bar{x} - x.$$

Now, as $F(0) = 0.5$, and F is symmetric around zero, it is easy to see that $\int_{-0.5\bar{x}}^{0.5\bar{x}} F(y)\,dy = 0.5\bar{x}$. So, $\pi_U(k_F) = 0.5$ as required. ■

Proof of Proposition 3: (i) Write $\pi_U(k, F)$ in (13) with the dependence of π_U on F made explicit. Define $k_U(F)$ implicitly by

$$\pi_U(k_U(F), F) \equiv 0.5. \tag{B4}$$

Let $\mathcal{F}$ be the class of symmetric, single-peaked zero mean distributions on $[-\bar{y}, \bar{y}]$. Then, then it is sufficient to show that

$$k_U(F) > k_F, \quad \text{all } F \in \mathcal{F}. \tag{B5}$$

Note also that $k_U(F_0) = k_F$, where F_0 is the degenerate distribution[28] with all the probability mass at $y = 0$. Also, any nondegenerate $F \in \mathcal{F}$ is a symmetric MPS of F_0. Then, to establish (B5), all we need to prove is that

$$k_U(F') > k_U(F), \tag{B6}$$

where F' is a symmetric MPS of F. For then from (B6), $k_U(F) > k_U(F_0) = k_F$, any $F \in \mathcal{F}$ as required.

(ii) To prove (B6), as π_U is increasing in k, it suffices to show that if F' is a mean-preserving spread of F, then $\pi_U(F', k_F) < \pi_U(F, k_F)$. But from (13), this is equivalent to

$$\Delta = \int_0^{\bar{x}} [F(k_F - x)]\gamma(x)\,dx - \int_0^{\bar{x}} [F'(k_F - x)]\gamma(x)\,dx > 0$$

Now by assumption, $\Pr(|b_m - c| \le \bar{y}) \ge 0.5$. But as the distribution of $b_m - c$ is symmetric around zero, this implies $\Pr(b_m \le c + \bar{y}) \le 0.75$, which implies $\bar{y} \le \Gamma^{-1}(0.75)$. Also, $\pi_F(k_F) = 2\Gamma\,(k_F) - 1 = 0.5$, implying $\Gamma(k_F) = 0.75$. But then, $\bar{y} \le k_F$. Moreover, $F(k_F - x) = 0$ if $x > k_F + \bar{y}$. So, we have

$$\int_0^{\bar{x}} F(k_F - x)\gamma(x)\,dx = \gamma(k_F - \bar{y}) - \gamma(0) + \int_{k_F -, y}^{\min\{\bar{x}, k_F + \bar{y}\}} F(k_F - x)\gamma(x)\,dz.$$

So,

$$\Delta = \int_{k_F - \bar{y}}^{\min\{\bar{x}, k_F + \bar{y}\}} [F(k_F - x) - F'(k_F - x)]\gamma(x)\,dx.$$

But now from Rothschild and Stiglitz (1971) if F' is a MPS of F, we must have

$$F'(z) > F(z),\ z < 0, F'(z) < F(z),\ z > 0, \quad \int_{-\bar{y}}^{\bar{y}} F(z)\,dz = \int_{-\bar{y}}^{\bar{y}} F'(z)\,dz. \tag{B7}$$

If the MPS is symmetric, then both F and F' must also be symmetric. These conditions in fact imply that $F' - F$ is a symmetric function round 0 in the sense that

$$F'(-z) - F(-z) = F(z) - F'(z) = \phi(z),\ z > 0. \tag{B8}$$

[28]That is, $F_0(y) = \begin{cases} 0 & y < 0 \\ 1 & y \ge 0 \end{cases}$.

So, we have

$$\begin{aligned}
\Delta &= \int_{k_F-\bar{y}}^{k_F} [F(k_F - x) - F'(k_F - x)]\gamma(x)\,dx \\
&\quad + \int_{k_F}^{\min\{\bar{x}, k_F+\bar{y}\}} [F(k_F - x) - F'(k_F - x)]\gamma(x)\,dx \\
&\geq \int_{k_F-\bar{y}}^{k_F} [F(k_F - x) - F'(k_F - x)]\gamma(x)\,dx \\
&\quad + \int_{k_F}^{k_F+\bar{y}} [F(k_F - x) - F'(k_F - x)]\gamma(x)\,dx \\
&= \int_0^{\bar{y}} [F(z) - F'(z)]\gamma(k_F - z)\,dx \\
&\quad - \int_0^{\bar{y}} [F'(z) - F(z)]\gamma(k_F + z)\,dx \\
&= \int_0^{\bar{y}} \phi(z)[\gamma(k_F - z) - \gamma(k_F + z)]\,dx \\
&> 0.
\end{aligned}$$

In this sequence of inequalities, we have used: (i) in the second line, the fact that $F(k_F - x) < F'(k_F - x)$ when $x > k_F$, from (B7); (ii) change of variables in the third line; (iii) (B8) in the fourth line; (iv) $\gamma(k_F - z) > \gamma(k_F + z)$ from single-peakedness of Γ and $k_F > \bar{y} > z$ in the final line. So, we have proved $\Delta > 0$, as required. ■

Proof of Proposition 5: (i) The results concerning k_F are obvious. (ii) Note that it is established in the proof of Proposition 3 that $k_U(F') > k_U(F)$, where $F'$is any symmetric MPS of F. It follows immediately that when the hypotheses of Proposition 3 hold, k_U rises following a symmetric MPS in F. A similar argument shows that the hypotheses of Proposition 4 hold, k_U falls following a symmetric MPS in F.

(iii) It remains to prove that following a symmetric mean-preserving spread in G, k_U rises. But recall that

$$\pi_U(k, F) = 2\int_0^{\bar{x}} [F(k - x)]\gamma(x)\,dx.$$

Now, consider a symmetric MPS in Γ. This is a sequence of simple symmetric MPSs. From Rothschild and Stiglitz (1970), each simple symmetric MPS in the sequence (from Γ to Γ') can be characterized as follows:

$$\begin{aligned}
\gamma'(x) &= \gamma(x) + s(x) \\
s(x) &= \alpha\big[I_{[-\delta-\varepsilon-t,-\delta-\varepsilon]} - I_{[-\delta-t,-\delta]} \\
&\qquad + I_{[\delta+\varepsilon,\delta+\varepsilon+t]} - I_{[\delta,\delta+t]},\ \varepsilon > t,\ \delta + \varepsilon + t < \bar{x}\big],
\end{aligned}$$

where $I_{[a,b]}$ is the indicator function on $[a, b]$. So, s is a function that moves a probability mass $t\alpha$ to the tails of the distribution of γ, while keeping the distribution symmetric.

But then

$$\begin{aligned}\pi'_U(k,F) - \pi_U(k,F) &= 2\int_0^{\bar{x}} [F(k-x)](\gamma'(x) - \gamma(x))\,dx \\ &= 2\int_0^{\bar{x}} [F(k-x)]\left(I_{[\delta+\varepsilon,\delta+\varepsilon+t]} - I_{[\delta,\delta+t]}\right)dx \\ &\leq 2t\alpha\,[F(k-\delta-\varepsilon) - F(k-\delta-t)] \\ &< 0.\end{aligned}$$

So, if $\pi_U(k,F) = 0.5, \pi'_U(k,F) < 0.5$. Consequently, k must rise to achieve $\pi'_U(k,F) = 0.5$, implying that the curve shifts outward. ■

Proof of Proposition 6: Denote by H the distribution of x, conditional on x being positive (i.e., $H(x) = 2\Gamma(x) - 1$, so h is given by $2g(x)$, $x \in [0, \bar{x}]$). Then, k_E is the mean of this distribution, i.e.,

$$k_E = 2\int_0^{\bar{x}} x\gamma(x)\,dx = E\,[x\,|\,x \geq 0]\,.$$

Also, note that k_F is the median of H, as $H(k_F) = 2G(k_F) - 1 = 0.5$, where the last inequality follows from the definition of k_F.

Now if G, and therefore Γ is uniform, then H is uniform also and therefore symmetric, so $k_F = k_E$. If Γ is positively single-peaked, then H is skewed to the right and so $k_E > k_F$. If Γ is negatively single-peaked, then H is skewed to the left and so $k_E < k_F$. ■

Proof of Proposition 7: First note that if $F', F \in \mathcal{A}$, then F' has higher variance than F iff it is a MPS of F. Now suppose that the assumptions of Proposition 3 hold. Then from the proof of Proposition 3, it is clear that (a) $k_F < k_U(F)$, and (b) $k_U(F)$ is increasing in σ_F^2. Then, as $k_E > k_F$ from Proposition 6, either; (i) there must be some critical variance $\tilde{\sigma}^2$ for which $k_U(F) = k_E$, or (ii) $k_U(F) < k_E$, all F. To over both these cases, recall that the maximum possible variance of F is $\hat{\sigma}^2 = \bar{y}^2$, and set $\hat{\sigma}^2 = \min\left\{\tilde{\sigma}^2, \bar{y}^2\right\}$. In the case where the assumptions of Proposition 3 hold, the proof is similar. ■

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Queries

Q1 Author: Please provide caption.

Q2 Author: Please Check the Statement of this proposition.

Q3 Author: can equation (4) and (3) be transposed without allowing meaning?

Q4 Author: Please supply initials for Palfrey in Cremer and Palfrey.

Q5 Author: Please update Lockwood (2002).

Q6 Author: Please provide volume number for Rothschild and Stiglitz (1970).