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Imperfect competition, the marginal cost of public funds and public goods supply

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Abstract

This paper analyses the impact of changes in product market competition on the marginal cost of public funds (MCPF) and public goods supply when distortionary commodity taxes are used to raise revenue. First, less competition (measured by a switch from Bertrand to Cournot conduct, or a decrease in the elasticity of demand) does not necessarily raise the MPCF. Second, even if it does, optimal public good supply does not necessarily fall. The paper also presents a method for modelling Bertrand and Cournot competition in general equilibrium that may be of independent interest.

Keywords: Imperfect competition; Marginal cost; Public funds; Public goods

1. Introduction

Although many product markets are clearly imperfectly competitive, the implications of imperfect competition for the design of taxation and the level of public good provision have received relatively little attention. Atkinson and Stiglitz devote only one page of their textbook to this issue (Atkinson and Stiglitz, 1980, p. 468), and remark; ‘‘further development of the (tax structure) requires a more soundly based general equilibrium theory of imperfect competition, and . . . this is at present at a rather early stage’’. Subsequently, Kay and Keen

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(1983), Delipalla and Keen (1992), and Myles (1989a,b), have derived modified Ramsey tax rules characterizing the optimal tax structure in an economy with imperfect competition in the product market. However, relatively few general results are available from such an approach, although some robust results have been established about the relative merits of specific versus ad valorem taxes,[1] and the desirability of taxes on intermediate goods.[2]

This paper attempts a somewhat different approach, by abstracting from question of tax *structure*, focusing instead on the impact of imperfect competition on the *level* of taxation, or equivalently the level of provision of a public good. Thus, this paper is similar in approach to Atkinson and Stern (1974), Wilson (1991a,b), and Gaube (2000). These papers compare the level of provision of a public good when taxation is lump-sum and when it is distortionary, assuming the economy is competitive. In this paper, we are concerned with how the level of provision of public goods financed by distortionary taxation changes as the level of product market competition changes.[3]

The initial economic intuition that many might have about the effect of decreasing product market competition on optimal public goods supply is the following. With imperfect competition, there is an initial distortion in the economy:production of all goods is too low, as firms are pricing above marginal cost. Other things equal, this distortion will raise the marginal cost of public funds (MCPF), as reallocation of labor from the production of private to public goods is more costly when production of private goods is already suboptimally low. In turn, this will imply a lower optimal public good supply.

The main message of this paper is that this economic intuition is not always correct, as the interaction between government and firms is more complex than this simple story would indicate. Our first point is that the connection between changes in the MCPF and public good supply is not straightforward with imperfect competition. This is because the profit income of the household is now endogenous. Specifically, a change in a parameter typically will affect both the MCPF and household disposable income, and thus the marginal utility of income, and these effects may go in different directions. For example, suppose that the number of

[1] One set of results that seem quite robust concern the relative merits of specific and ad valorem taxes with imperfect competition. With a monopoly or homogeneous-products oligopoly with identical firms, it has been shown that an ad valorem product tax dominates a specific product tax in the sense that a specific product tax can always be replaced with an ad valorem tax that yields the same revenue and greater consumer plus producer surplus (Suits and Musgrave, 1953; Delipalla and Keen, 1992).

[2] Myles (1989b) has shown that non-zero taxes on intermediate goods may be desirable in the presence of imperfect competition, implying that the Diamond–Mirlees aggregate production efficiency result does not generalise to economies with imperfect product markets.

[3] Similar issues have been addressed in a series of papers by Heijdra and co-authors on the Kenyesian multiplier in general equilibrium models of imperfect competition (Heijdra and van der Ploeg, 1996; Heijdra and Ligthart, 1997; and especially Heijdra et al., 1998). The relationship of this paper to those just cited is discussed in the last section.

firms in the industry increases. This will reduce price–cost margins, and may reduce the MCPF. But it will also reduce household disposable income, and thus raise the marginal utility of income. Thus, the willingness to pay for additional units of the public good will fall. We show that the second effect can outweigh the first, so that the MCPF and public good supply may *both* fall.

Second, an increase in the degree of imperfect competition need not always raise the MCPF. To understand why, note that the model we use for our analysis is a relatively standard one in the literature on general equilibrium models of imperfect competition (Dixon, 1987; Mankiw, 1988; Heijdra and van der Ploeg, 1996). A representative consumer has preferences over leisure, a public good, and a CES index of n varieties of a produced good. Each variety is produced by a single firm from a labour input. Government can levy commodity and profit taxes.[4] In our model, we measure 'degrees' of imperfect competition in two ways. First, in terms of the underlying elasticity of demand for the firm's product (i.e., the elasticity of substitution between varieties), and second, in terms of conduct of firms (i.e., either Bertrand or Cournot behaviour). In our base model (see Section 6), the elasticity of demand for the goods index is fixed,[5] as are the number of firms; in this case, the MCPF is increasing with the degree of imperfect competition, measured in either way.

However, suppose we allow the elasticity of demand for the goods aggregate to vary positively with price, rather than be constant (see Section 8). In this case, the MCPF may be *lower* with Cournot than with Bertrand behaviour. The reason is that mark-up is now *endogenous*. In particular, an increase in government expenditure raises the commodity tax, which raises the price of all goods, which increases the elasticity of demand and lowers both Cournot and Bertrand mark-ups. The last effect lowers the MCPF, other things being equal. If the sensitivity of the Cournot mark-up to the tax is sufficiently greater than the Bertrand (which is possible for some parameter values), then this effect can dominate, making the Cournot MCPF less than the Bertrand MCPF.

Also, with free entry, is shown that the formula for the MCPF is very different than it is the case with a fixed number of firms. In the latter case, the MCPF is increasing in the price-cost margin (or any other measure of the mark-up of price over cost). In the former, it is *decreasing* in the price-cost margin, and also depends on the *elasticity* of the price-cost margin with respect to the number of firms. In this case, it turns out that with Bertrand competition, the MCPF is independent of the degree of imperfect competition as measured by the elasticity of substitution between varieties, and with Cournot competition, the MCPF is decreasing in the degree of imperfect competition measured in this way!

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[4]As is well-known, a general income tax is redundant in this environment, as it is equivalent to a uniform commodity tax.

[5]Technically, the representative household has Cobb–Douglas preferences over the goods index and leisure.

An additional contribution of this paper, which may be of independent interest, is in our modelling of imperfect competition. The recent general equilibrium literature on 'macroeconomic' models of imperfect competition invariably makes the assumption of monopolistic competition, i.e., the number of varieties is large, and so a change in any firm's price (or quantity) has negligible effect on demand for the varieties produced by other firms. This 'monopolistic competition' assumption has two undesirable consequences for our purposes. First, it implies that the *conduct* of firms (i.e., whether they choose prices or quantities) makes no difference in equilibrium, so the standard model cannot be used to study the effects of changes in conduct on the MCPF and public goods supply. Second, with free entry, a large number of varieties in equilibrium requires negligible set-up costs, in which case the economy is approximately competitive. We avoid these problems by allowing the number of varieties to be small: we show that, as well as allowing us to model firm conduct in a non-trivial way, this extension makes a real qualitative difference to the results in the case of free entry.

2. The model

2.1. Households

There is a single household (or a number, normalized to unity, of identical households) which has preferences of the form

$$U = u(X, l) + h(g) \tag{1}$$

where X, l, g are levels of consumption of an aggregate consumption index, leisure, and the public good, respectively.[6] We assume that the quantity index X is a symmetric CES function of levels of consumption $x_1, \ldots x_n$ of n different varieties;

$$X(x_1, \ldots x_n) = n^{1(1-\sigma)} \left[\sum_{i=1}^{n} x_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \tag{2}$$

with elasticity of substitution $\sigma > 1$. Here, the constant $n^{1/(1-\sigma)}$ ensures[7] that there is no preference for diversity (PFD) per se, i.e., if $x_i = x$, $i = 1, \ldots n$ then $X = nx$. PFD only makes a qualitative difference in the case of free entry, and we wish to focus on the case of an endogenous number of firms without the additional complication of PFD. Also, we assume preferences over aggregate consumption and leisure to be Cobb–Douglas:

[6] In assuming additive separability between the public good on the one hand, and X, l on the other, we are following Atkinson and Stern (1974), and Wilson (1991a).

[7] To see this, suppose that $x_i = x, i = 1, \ldots n$, then $X = n^{1/(1-\sigma)} n^{\sigma/(\sigma-1)} x = nx$.

$$u(X, l) = \frac{(X^{\alpha} l^{1-\alpha})^{1-\eta}}{1-\eta}, \quad 0 < \alpha < 1 \tag{3}$$

where $\eta \geq 0$ is the elasticity of the marginal utility of income. Finally, h is assumed strictly increasing and concave.

The household is assumed to have a time endowment of unity, and we take labour to be the numeraire good, so income from labour is $1 - l$. Also, pre-tax profit is denoted π. We will allow the government to tax consumption, and income from labour and from profit. Then, the budget constraint of the household is

$$(1+t)\sum_{i=1}^{n} p_i x_i = (1-l)(1-\tau_w) + \pi(1-\tau_\pi) \tag{4}$$

where p_i is the price of good i, t is a tax on consumption, and τ_w, τ_π are taxes on wage and profit income, respectively. It is clear from (4) that one of these tax instruments is redundant, so that our analysis applies if the government has access to any pair of these taxes. Without loss of generality, we suppose that $\tau_w = 0$, $\tau_\pi = \tau$, i.e., the government uses a consumption tax and a profit tax. Then, from (4), the budget constraint is:

$$(1+t)\sum_{i=1}^{n} p_i x_i + l = 1 + \pi(1-\tau) \equiv y$$

where y is *full disposable* (*post-tax*) *income*.

In this set-up, the behavior of the household is described by two stage budgeting (Dixit and Stiglitz, 1977). Let $q_i = p_i(1+t)$ be the consumer price of good i, and

$$Q = n^{1/(\sigma-1)}\left[\sum_{i=1}^{n} q_i^{1-\sigma}\right]^{1/(1-\sigma)}$$

be the ideal price index associated with X. Note that if $q_i = q$, $Q = q$. As preferences over X, l are Cobb–Douglas, at the first stage, demands for the consumption aggregate and leisure are:

$$X = \frac{\alpha y}{Q}, \quad l = (1-\alpha)y \tag{5}$$

At the second stage, individual commodity demands are:

$$x_i = \frac{1}{n}\left(\frac{q_i}{Q}\right)^{-\sigma} X \tag{6}$$

Combining (6), (5), we get

$$x_i = \frac{\alpha}{n}(q_i)^{-\sigma} Q^{\sigma-1} y \tag{7}$$

From (7) it is easy to see that goods $i = 1, \ldots n$ are substitutes iff $\sigma > 1$ and we assume this in what follows.

2.2. Firms

Every good is produced by a single firm, so that there are n firms. Each firm can produce one unit of output with one unit of labour input. The first case is where firms set prices (Bertrand). In this case, the demand curve facing firm i is just (7). With Bertrand competition, firm i takes other prices q_j, $j \neq i$ as given. We will assume that firms recognize the effect of q_i on Q, which is of the order $1/n$, i.e., we are not restricting ourselves to the limiting case to the case when n is large. As explained in the introduction, this is important in the case of free entry. On the other hand, following most of the literature on imperfect competition, we will assume[8] that firms take full disposable income y as given when choosing q_i. It is then easily checked from (7) that when all prices are the same ($q_i = q$), the elasticity of demand for any firm's product (the elasticity of Chamberlain's dd curve) is

$$-\left[\frac{q_i}{x_i}\frac{\mathrm{d}x_i}{\mathrm{d}q_i}\right]_{q_{-i}\ \text{const}} = \theta_B = \sigma + \frac{1}{n}(1-\sigma) \tag{8}$$

Note also that as goods become perfect substitutes ($\sigma \to \infty$), the elasticity becomes infinite.

Now suppose that firms set quantities, rather than prices (Cournot). Then, the demand functions (7) can be inverted straightforwardly[9] to yield

$$q_i = \frac{\alpha}{n^6}(x_i)^{-1/\sigma}X^{(1-\sigma)/\sigma}y \tag{9}$$

where x_i is now the output of firm $i = 1, \ldots n$, and X is the aggregate quantity index (2). Then from (9), it is straightforward to calculate that, when all quantities are the same, that the elasticity of demand for any firm's output, taking the outputs of other firms as given is

[8] When firm i changes q_i, it will have an effect of order $1/n$ on Q, and thus on y. So, one interpretation of our assumption is simply that the firm irrationally ignores this feedback effect. On the other hand, this assumption can be justified as follows, using arguments of Hart (1985). Assume that the economy is divided into s identical 'islands'. Each island has the economic structure of the model just outlined. Islands are only connected in that the representative consumer on island i owns equal fractions of the shares on *all* islands, so his profit income comes from ns firms, only n of which are operating on island i. So, when a firm on island i changes its price, this only has an effect of order $1/ns$ on that consumer's profit income. If s is assumed large, then, whatever n, the firm rationally ignores this feedback.

[9] From (6), we have $q_i = \left(\frac{x}{nx_i}\right)^{1/\sigma}Q$ and also, from (5), $Q = \alpha y/X$. Combining these two equations gives (9).

$$-\left[\frac{q_i}{x_i}\frac{\mathrm{d}x_i}{\mathrm{d}q_i}\right]_{x_{-i}\ \mathrm{const}} = \theta_C = \frac{n\sigma}{n+\sigma-1} \tag{10}$$

Note that as goods become perfect substitutes ($\sigma \to \infty$), the elasticity tends to n. Note also that with price-setting, the demand curve is more elastic than with quantity-setting,[10] i.e., $\theta_C < \theta_B$.

Now, recalling that marginal cost is constant at unity, the profit of firm i can be written

$$\pi_i = x_i p_i - x_i = \frac{1}{1+t}(q_i x_i - (1+t)x_i) \tag{11}$$

implying that the firm has an effective marginal cost of $1+t$. Then, in the Bertrand (resp. Cournot) case, the firm chooses price (resp. quantity) to maximise (11). All firms behave alike, so profit maximization results in a common price $q_i = q$ which is a mark-up over effective marginal cost, i.e.,

$$q_k = \left(\frac{1}{1-1/\theta_k}\right)(1+t), \quad k = B, C. \tag{12}$$

Also, we denote equilibrium producer prices by $p_k = 1/(1-1/\theta_k)$, $k = B, C$. We measure the degree of imperfect competition in the usual way via the price-cost margin

$$\mu_k = \frac{q_k - (1+t)}{q_k} = \frac{p_k - 1}{p_k} = \frac{1}{\theta_k}, \quad k = B, C \tag{13}$$

so the margin μ_k is inversely related to the elasticity of demand. It is clear that the price cost margin depends both on σ and firms' conduct, and moreover, σ does not affect any of the equilibrium conditions of the model except through μ. So, we have two possible measures of the degree of imperfect competition: the *elasticity of demand* measure σ, and the *conduct* measure $k = B, C$.

3. Equilibrium and tax design

3.1. Equilibrium with a fixed number of firms

Given tax parameters t, τ, define a *symmetric equilibrium* to be one where all goods are priced the same ($q_i = q$, $i = 1, \ldots n$), and are produced in the same quantities ($x_i = x$, $i = 1, \ldots n$). In symmetric equilibrium, we take the endogenous

[10] This inequality reduces to $0 < 1/n(1-1/n)(\sigma-1)^2$, which certainly holds.

variables to be q, and full post-tax income y. First, q is determined[11] by the mark-up Eq. (12). Next, note that π is the sum of π_i defined in (11), so

$$\begin{aligned}\pi &= n(p-1)x \\ &= (p-1)X \\ &= \frac{(p-1)\alpha y}{p(1+t)} \\ &= \frac{\mu \alpha y}{(1+t)}\end{aligned} \tag{14}$$

where in the second line we have used $X = nx$, and in the third $Q = p(1+t)$ and (5), and in the fourth, the definition of μ. So, from (14), household full income y satisfies:

$$y = 1 + (1-\tau)\frac{\mu \alpha y}{1+t} \tag{15}$$

Solving (15), we get;

$$y = \frac{1}{1-(1-\tau)\mu\alpha/(1+t)} \tag{16}$$

Eq. (16) makes it very clear that a 'profit multiplier' is at work: an exogenous increase in income increases spending, which increases profit, which increases income (Dixon, 1987; Mankiw, 1988; Heijdra and van der Ploeg, 1996).

3.2. Equilibrium with free entry

Here, we suppose that the number of firms, n is determined by free entry. We follow the usual convention of treating n as a continuous variable, and we also assume a fixed set-up cost $f > 0$. We will also allow government to tax entry. Now, with free entry, there is no net profit, and so no profit multiplier: consequently, $y = 1$. Then, using (14), the free-entry condition can be written;

$$\frac{\pi}{n} = \frac{(p-1)\alpha}{pn(1+t)} = f + T \tag{17}$$

where T is the entry tax. The other equilibrium condition[12] of the model is, as in the case of a fixed number of firms, the mark-up Eq. (12) determining q. Note also that μ depends on n from (8), (10), and (13).

[11] From now on, we drop the 'B' and 'C' subscripts in cases where the relevant variable or equation applies to both Bertrand and Cournot cases.

[12] Here, there is no equilibrium condition determining y, as here is no profit so $y = 1$.

3.3. Government

The activity of the government in this model is as follows. Tax revenue is used to purchase the numeraire labour input from which the public good is produced. One unit of labour produces one unit of the public good. The government chooses the level of public good provision and the tax structure to maximise the welfare of the representative household.[13]

This problem can be formalized as follows. First, in the case of a fixed number of firms, the government budget constraint may be written

$$g = tpX + \tau\pi \tag{18}$$

where pX is the tax base of the ad valorem commodity tax. Using (14) and (5), we can write (18) in terms of tax parameters t, τ and endogenous variables q, y;

$$g = (tp + \tau(p-1))\frac{\alpha y}{q} \tag{19}$$

So, equilibrium conditions (12), (16) and (19) uniquely determine[14] q, y and public good supply g given tax instruments t, τ.

Next, the utility of the household in equilibrium is

$$w(q, y, g) = v(q, y) + h(g) \tag{20}$$

where $v(q, y)$ is the indirect utility function of the household over goods and leisure,[15] which we call *private indirect utility* in what follows. So, the *expenditure and tax design problem* of the government is to choose $t > -1$, $\tau \le \bar{\tau}$ to maximise $w(q, y, g)$ subject to (12), (16) and (19), where $\bar{\tau}$ is an upper bound on the profit tax. By the argument in Footnote 14, this problem is well-defined.

In the case of free entry, the expenditure and tax design problem of the government can be reformulated as follows. The government budget constraint is as in (19), except that profit income is now zero, i.e., $y = 1$, so we have;

$$g = tpX + nT \tag{21}$$

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[13] Our results would go through essentially unchanged if we allowed the government to maximise the weighted sum of tax revenue and welfare, i.e., be a partial Leviathan.

[14] Note that this equilibrium mapping from t, τ is to q, y, g is unique. First (12) and (16) have a separable structure; q is determined uniquely by t, and then y is determined uniquely by t, τ. So, for all $t > -1$, $0 \le \tau \le \bar{\tau}$, there must be a unique solution (q, y) to (12) and (16). It then follows that g is uniquely determined via (19).

[15] That is, $v(q, y) = \max_{X,l} u(X, l)$, s.t. $Xq + l = y$.

The problem of tax design is thus to choose $t > -1$, T to maximize $w(q,1,g)$ subject to equilibrium conditions (12), (17) and budget constraint (21).

4. Benchmark results

It is first helpful to ask what the solution to the government's expenditure and tax design problem would be if the government had access to a lump-sum tax on the households, as well as commodity taxes.[16] Here, the answer is well-known. The government can use the revenue raised by the lump-sum tax to (i) finance the first-best level of public good provision; and (ii) finance a Pigouvian consumption subsidy to the household (or a production subsidy to the firm) that would 'undo' the effects of imperfect competition by reducing the consumer prices of goods down to their marginal cost of unity. This can be achieved by setting a negative consumption tax equal to minus the price-cost markup, i.e., $t = -\mu$. So, with a lump-sum tax, imperfect competition is irrelevant to the government.

Now suppose that a lump-sum tax is not available, but 100% profit taxation is available. The profit tax is similar to a lump-sum tax: the crucial difference is that the revenue from the profit tax will not be enough to fund the provision of the public good at its first-best level, *and* finance the Pigouvian subsidies to consumers. However, we can show that in this case, imperfect competition is also 'irrelevant' in the following sense.

Suppose that firms were perfectly competitive, i.e., $\mu = 0$. Then, as $q = 1 + t$, $y = 1$, the tax design problem for the government would be:

$$\textit{Choose } t \textit{ to maximise } v(1+t,1) + h(g) \textit{ subject to } g = tX(1+t,1) \qquad (\mathrm{P}_0)$$

where $X(q,y)$ is given in (5). Let the solution to P_0 be t_0, with corresponding public good supply g_0. Also, in the government's expenditure and tax design problem of Section 3.3, let the solution values of the tax instruments be (t_k,τ_k), $k = B,C$, in the cases with Bertrand and Cournot conduct. We have:[17]

Proposition 1. *If* $\bar{\tau} = 1$ (100% *profit taxation is available*), *then at the solution to the government's expenditure and tax design problem*, *the optimal supply of the public good is* g_0, *is independent of whether firms set prices or quantities, and of* σ. *In both cases, the maximum possible profit tax is set* ($\tau_B = \tau_C = 1$). *Also, in both cases, commodity taxes are set to ensure that the consumer price is the same as*

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[16]Note that commodity taxes are not redundant here when lump-sum taxes are available, as they are used to deliver Pigouvian subsidies. On the other hand, the profit tax *is* redundant in this case.

[17]The proof of Proposition 1, and proofs of all subsequent propositions (where proof is required) is in Appendix A.

with perfect competition (i.e., $q_0 = 1 + t_0$), whatever the conduct of firms, i.e., $t_B = (q_0 - p_B)/p_B$, $t_C = (q_0 - p_C)/p_C$.

Proposition 1 is a generalization[18] (to the case of an environment with distortionary taxes) of the well-known result that a monopolist can be induced to behave competitively by offering him an output subsidy financed out of profit taxation. It is important to note that Proposition 1 is *not* claiming that a 100% profit tax is the same as a lump-sum tax: in fact, under our assumptions, the outcomes are quite different.

A similar equivalence result holds in the case with free entry, if the government can tax entry. As before, suppose that firms are price-takers, i.e., $\mu = 0$. Now, with a cost of entry, the government must subsidise entry by amount f to induce any firm to enter. Given this subsidy, all firms are indifferent about entering. As entry is costly, and there is no preference for diversity (PFD), the government will allow only one firm to enter. Then, the tax design problem for the government will be:[19]

$$\text{Choose } t \text{ to maximise } v(1+t,1) + h(g) \text{ subject to } g + f = tX(1+t,1). \qquad (\mathrm{P}_1)$$

Let the solution to P_1 be t_1, with corresponding public good supply g_1. Also, in the tax design problem of Section 3.3, in the cases with Bertrand and Cournot conduct, let the solution values of the tax instruments be (t_k, T_k), $k = B,C$. Then, we have:

Proposition 2. *If entry taxes are available, then at the solution to the government's expenditure and tax design problem, the optimal supply of the public good is g_1, independent of whether firms set prices or quantities, and of σ. In both cases, commodity taxes are set to ensure that the consumer price is the same as with perfect competition (i.e., $q_1 = 1 + t_1$), whatever the conduct of firms, i.e., $t_B = (q_1 - p_B)/p_B$, $t_C = (q_1 - p_C)/p_C$. Also, in both cases, T_B, T_C are set so that only one firm enters.*

So, the conclusion is that when 100% profit taxation is available, imperfect competition is irrelevant; whatever form it takes (i.e., Cournot or Bertrand) the government can always replicate the allocation of resources that it can achieve with price-taking behaviour by firms.

However, dynamic allocative distortions associated with taxation, or the presence of institutional or political constraints (e.g., lobbying by producers) may

[18] In an earlier version of this paper, Lockwood (1996), I showed that this result also holds in a much more general setting where households are heterogeneous (in income and preferences), so that the government may have redistributive, as well as revenue-raising goals.

[19] Note that in contrast to the previous case, the cost of entry must be subtracted from the tax revenue available for the public good.

place constraints on profit taxation.[20] Also, entry taxes are rarely observed unless the industry in question is a natural monopoly, in which case a licence fee may be charged. So, it is desirable to relax these assumptions. We do so by assuming that $\bar{\tau} < 1$ and $T = 0$ the case of n fixed and free entry, respectively.

5. The MCPF and public good provision

When moving away from the benchmark case, it is helpful to think of the government solving a two-stage problem. At the first stage, t and τ are chosen to maximize private indirect utility $v(q,y)$, given a revenue requirement g, and this gives rise to private indirect utility $v(g)$, with $v_g < 0$. Then, at the second stage, g is chosen to maximize $v(g) + h(g)$, which implies first-order condition $-v_g = h_g$. Dividing though by the marginal utility of income for the household, $v_y = \lambda$, we get:

$$\frac{-v_g}{\lambda} = \frac{h'}{\lambda} \tag{22}$$

This is a *modified Samuelson rule* for public good provision. The right-hand side, h'/λ, is simply the marginal rate of substitution between the public good and leisure (as $u_l = \lambda$ at the solution to the household's optimization problem). The left-hand side, $-v_g/\lambda$ is the money value of private utility foregone from one dollar of revenue raised, i.e., the *marginal cost of public funds, or MCPF*. Moreover, by assumption, the marginal rate of transformation between leisure and the public good is unity, as one unit of labour is needed to make one unit of the public good. So (22) is of the standard form: $MRT.MCPF = \Sigma MRS$, bearing in mind that there is only one household in the economy.

Let z be any parameter of the model, e.g., σ or τ (or μ in the short run, when n is fixed) that affects $v(g)$ but not $h(g)$. How will a change in z affect the MCPF and optimal supply of g? First, from (22), the condition determining g can be written

$$\lambda MCPF = h'(g) \tag{23}$$

Second, from (3), private indirect utility is easily calculated to be

$$v(q,y) = \frac{\kappa}{1-\eta} q^{-\alpha(1-\eta)} y^{1-\eta}, \quad \kappa = \alpha^{\alpha(1-n)}(1-\alpha)^{(1-\alpha)(1-n)}$$

so $\lambda = \kappa q^{-\alpha(1-\eta)} y^{-\eta}$. Consequently, g is determined by

[20]Indeed, rates of taxation income from profit (including both personal and corporate taxes) often low in the OECD countries. For example, the top personal rate of tax on interest and dividend income in the EU in 1991 was The Netherlands, with a rate of 60%. The corresponding rate in the US is 36% (Commission of the European Communities, 1992).

$$\kappa q^{-\alpha(1-\eta)} y^{-\eta} MCPF = h'(g) \tag{24}$$

From (24), we have the following. Let εx_y denote the elasticity of variable x with respect to variable y. Then:

Proposition 3.

$$sgn\ \frac{\partial g}{\partial z} = sgn\ \Delta,\ \Delta = \alpha(1-\eta)\varepsilon q_z + \eta \varepsilon y_z - \varepsilon MCPF_z \tag{25}$$

This result says that the effect of z on public good supply is determined not only by how z affects the MCPF, but also how it affects endogenous variables q and y. The intuition for this result is clear from (24): if the parameter z affects endogenous variables q, y, as well as the MCPF, it will affect the marginal utility of income λ, which measures the household's willingness to pay for increases in g. To apply Proposition 3, our first task is to develop a general formula for the MCPF. We begin with the case of a fixed number of firms.

6. The base case: a fixed number of firms

Consider the tax design problem facing the government given a fixed revenue requirement g. This is to choose τ,t to maximize $v(q,y)$ subject to the equilibrium conditions (12), (16), the revenue constraint (19), and the constraint $\tau \leq \bar{\tau}$. As a profit tax is non-distortionary, and the revenue from the tax can be used to reduce commodity taxes, it is always[21] optimal to set $\tau = \bar{\tau}$, whatever g. The Lagrangian for this problem is therefore:

$$\mathscr{L} = v(p(1+t),y) + \rho[(tp + \bar{\tau}(p-1))X(p(1+t),y) - g]$$

where y is given by (16). The first-order condition for the optimal choice of t is:

$$\frac{\partial \mathscr{L}}{\partial t} = \lambda(-pX + y_t) + \rho[pX + (tp + (p-1)\bar{\tau})(pX_q + X_y y_t)] = 0 \tag{26}$$

where subscripts denote derivatives.

After some manipulation, and using (13), we can rewrite (26) in elasticity form:

$$(1+t)(-1 + \frac{\varepsilon y_{(1+t)}}{\alpha}) + \frac{\rho}{\lambda}[1 + t + (t + \mu\bar{\tau})(\varepsilon X_q + \varepsilon X_y \varepsilon y_{(1+t)})] = 0 \tag{27}$$

As the MCPF is equal to ρ/λ by definition, (27) gives a general formula for the MCPF, which in fact applies to the case where $u(X,l)$ is any homothetic utility function:

[21] To see this, note that even when $g = 0$, the government would like to set $\tau = 1$ in order to subsidise production so firms produce at marginal cost.

$$MCPF = \frac{\rho}{\lambda} = \frac{(1+t)(1-\varepsilon y_{(1+t)}/\alpha)}{1+t+(t+\mu\bar{\tau})(\varepsilon X_q + \varepsilon X_y \varepsilon y_{(1+t)})} \tag{28}$$

When $\mu = 1$, $y \equiv 1$ from (16), and so Eq. (28) reduces to

$$MCPF = \frac{1+t}{1+t+t\varepsilon X_q} \tag{29}$$

which is the standard formula for the MCPF with one taxed (non-numeraire) good (see, e.g., Triest 1990). Now note from (16) that

$$\varepsilon y_{(1+t)} = 1 - y < 0 \tag{30}$$

So, an increase in t reduces income through the mechanism of increasing q and thus reducing demand and profit. Using (30), and $\varepsilon X_q = -1$, $\varepsilon X_y = 1$ from the properties of Cobb–Douglas demand, we see from (28) that a more specific formula for the MCPF is;

$$MCPF(t,y) = \frac{(1+t)(1+(y-1)/\alpha)}{1+t-(t+\mu\bar{\tau})y} \tag{31}$$

So, by inspection of (28) or (31), we see that at a given tax rate t, the effect of imperfect competition on the MCPF comes from two[22] sources. *First*, the term in the price-cost margin μ captures the fact that in the imperfectly competitive case, there is an initial distortion in the economy: production of all goods is too low, as firms are pricing above marginal cost. Other things equal, this distortion will raise the MCPF, as reallocation of labor from the production of private to public goods is more costly when production of private goods is already suboptimally low.

Second, from (16), an increase in μ will increase y. Then, an increase in y will raise $\varepsilon y_{(1+t)}$ (in absolute value), and in turn, the increase in $\varepsilon y_{(1+t)}$ will raise the *MCPF* from (28). So, a change in the price-cost margin μ also impacts on the MCPF though its effect on income y.

The fact that both these effects work in the same direction indicates that we should find that an increase in the mark-up μ will raise the MCPF. Of course, from Proposition 3, we are interested in the effect of changes in μ when the *tax revenue* requirement g is fixed, not when the *tax rate* is fixed. Note that the MCPF in (31) depends on g though the endogenous variables t,y. But, as shown in Appendix A.3, we can solve for t, y from Eqs. (16) and (19) to get:

[22]Note that a *third* difference arises from the fact that the elasticity of demand for the consumption goods aggregate X will generally vary with q, the price of consumption relative to leisure. We have closed this third effect down by assuming unit-elastic aggregate demand ($\varepsilon X_q = -1$), but we investigate this further in Section 8 below.

$$t = \frac{g - \alpha((1-\bar{\tau})g - \bar{\tau})\mu}{\alpha - g} \tag{32}$$

$$y = \frac{1-\bar{\tau}\mu - (1-\bar{\tau})\mu g}{1-\bar{\tau}\mu - (1-\bar{\tau})\mu\alpha} \tag{33}$$

Note that if $g = 0$, $\bar{\tau} = 1$, then $t = -\mu < 0$, which is the Pigouvian subsidy needed to ensure that the non-competitive firms price at marginal cost. Now, it is straightforward to substitute (32) and (33) into (31) and rearrange to get:

$$MCPF(g,\mu) = \frac{\alpha}{(\alpha - g)} + \frac{(1-\bar{\tau})\mu}{1-(\bar{\tau} + (1-\bar{\tau})\alpha)\mu} \tag{34}$$

This is our final formula for the MCPF. It is clear from (34) that when $\bar{\tau} < 1$, MCPF is increasing in μ, as expected. Also, note that the MCPF at the solution to the benchmark problem with perfect competition, P_0, is $MCPF(g,0) = \alpha/(\alpha - g)$. So we have proved:

Proposition 4. *If* $\bar{\tau} < 1$, $MCPF(g,\mu_C) > MCPF(g,\mu_B) > MCPF(g,0)$, *i.e., the MCPF is higher with Cournot Competition than with Bertrand competition, and higher with Bertrand competition than in the competitive benchmark case. It is also decreasing in* σ, *the demand elasticity measure of imperfect competition.*

To get a feel for the effect of imperfect competition on the MCPF in practice, first note that there are a number of estimates of the price-cost margin μ at the industry level for the UK and other countries (e.g., Haskel and Martin, 1992; Lukacs, 2000). The average (across industries) mark-up that emerges from this kind of study is in the range 0.2–0.4. Second, note that statutory rates of corporation tax are for OECD countries are in the range of 0.3 to 0.6 (excluding Ireland with a rate of 0.1), so we might take $0.3 \leq \bar{\tau} \leq 0.6$ (Devereux et al., 2001). Finally, α is the marginal propensity to consume out of full disposable income, and a simple calibration exercise in a companion paper, Lockwood (2001) gives a range of 0.3–0.4 for α. These ranges combine to give a range of values for the second term in (34) of 0.094–0.337. So, for g small, as the competitive MCPF is approximately unity, imperfect competition (in this model) may plausibly raise the MCPF by 10–30% or so.

We now turn to consider the effects of an increase in the degree of imperfect competition. This can be thought of as an increase in the exogenous producer price p. Recalling that $q = p(1 + t)$, and taking $z = p$, we see from the general formula (25) that

$$\Delta = \alpha(1-\eta)(1 + \varepsilon(1+t)_p) + \eta\varepsilon y_p - \varepsilon MCPF_p$$

Now, we know that as $p = 1/(1-\mu)$, from (34), $\varepsilon MCPF_p < 0$. So, if the effect of a change in p though the MCPF dominates, $\Delta < 0$ and then g will be decreasing in

Table 1
Values of Δ

η	0	0.5	1	1.5
$g=0.1$	0.026	−0.179	−0.386	−0.592
$g=0.2$	0.147	−0.078	−0.303	−0.529
$g=0.3$	0.277	0.032	−0.213	−0.458
$g=0.4$	0.420	0.154	−0.112	−0.378

Values of the other parameters: $\alpha=0.5$, $\tau=0.5$, $\mu=1/3$.

p. It is a routine exercise to use (33)–(34) to calculate the elasticities in Δ, which results in a formula[23] for Δ in terms of parameters $\alpha, p, g, \bar{\tau}$. The above simulation results in Table 1 show that in fact $\Delta > 0$ when the household is not too risk-averse (i.e., indirect utility is not too concave in income). So, for these parameter values, $\partial g/\partial p > 0$, i.e., public good supply is *higher* when the degree of imperfect competition is higher.

7. Free entry

To keep things simple, we will assume that the government cannot levy entry taxes at all, i.e., $T=0$. We proceed by first deriving a formula for the MCPF. Recall that the tax design problem, given fixed revenue requirement g, is to choose t to maximize $v(q,y)$ subject to equilibrium conditions (12), (17) and revenue requirement (21). Recalling that $y=1$, the Lagrangian for this problem is:

$$\mathscr{L} = v(p(1+t),1) + \rho[tpX(p(1+t),1) - g]$$

Bear in mind that n is now endogenous and depends on t through the free-entry condition (17), and that consequently p, which depends on n though the elasticity of demand θ, is also endogenous. Then, the first-order condition for t in this tax design problem is:

$$\frac{\partial \mathscr{L}}{\partial t} = -\lambda X(p + (1+t)p_n n_t) + \rho[pX + tp_n n_t X + tpX_q(p + (1+t)p_n n_t)] = 0 \tag{35}$$

where as before, subscripts denote derivatives. Manipulation of (35) gives the first-order condition in elasticity form:

$$-(1+t)(1+\varepsilon p_n \varepsilon n_{(1+t)}) + \frac{\rho}{\lambda}[1 + t + t\varepsilon p_n \varepsilon n_{(1+t)} + t\varepsilon X_q(1+\varepsilon p_n \varepsilon n_{(1+t)})] = 0 \tag{36}$$

[23] This formula is available on request. It is not included as it is rather long.

So, as $MCPF = \rho/\lambda$, and of course $\varepsilon X_q = -1$, we have from (36) that

$$MCPF = (1+t)(1 + \varepsilon p_n \varepsilon n_{(1+t)}) \tag{37}$$

To proceed, note that the relationship between n and t is given by (17). Totally differentiating this condition, and recalling that p depends on n, we obtain (as shown in Appendix A.2):

$$\varepsilon n_{(1+t)} = \frac{p-1}{p-1+\varepsilon p_n} \tag{38}$$

Note that it is possible to show that $p - 1 + \varepsilon p_n < 0$, so there is a negative relationship between n and t in equilibrium.[24] Next, note that $1 + \varepsilon p_n \varepsilon n_{(1+t)}$ is in fact the elasticity of the consumer price with respect to the tax, $\varepsilon q_{(1+t)}$, taking into account the endogeneity of n. So, using this fact, plus (38), we can reformulate (37) is a more intuitive way as

$$MCPF = (1+t)\varepsilon q_{(1+t)} = (1+t)\left(\frac{1-p+p|\varepsilon p_n|}{1-p+|\varepsilon p_n|}\right) \tag{39}$$

where we now express εp_n in absolute value form to avoid carrying negative variables. Note that as $p > 1$, it is clear from (39) that $\varepsilon q_{(1+t)} > 1$, i.e., we have more than 100% shifting of the tax to the consumer (overshifting[25]). So, we conclude that at a given tax *rate*, t, the MCPF is higher than it would be in a competitive economy, i.e., $1 + t$, due to the overshifting of taxes.

It is convenient, for purposes of comparison to the case of n fixed, to have the formula (39) in terms of the price-cost margin μ, rather than the price, p. From the definition of the price-cost margin, $\varepsilon p_n = \varepsilon \mu_n (p-1)$. Substituting in (39), and rearranging, we have;

$$MCPF(g,n) = (1+t)\left(\frac{\mu - 1 + |\varepsilon\mu_n|}{\mu - 1 + (\mu - 1)|\varepsilon\mu_n|}\right) \tag{40}$$

Note two crucial facts from (40) as compared to (31). *First*, unlike in the case of fixed n, the MCPF depends not only in the *level* of the price–cost margin μ, but also on the (absolute value of) the *elasticity* of μ with respect to n. Specifically, treating μ, $\varepsilon\mu_n$ as parameters, as $1 > \mu > 0$, it is easy to check from (40) that the *MCPF* is increasing in $|\varepsilon\mu_n|$. The intuition is clear: the higher is $|\varepsilon\mu_n|$, the bigger is the degree of overshifting of the price in response to the tax, and hence the bigger the burden of a given tax. *Second*, the MCPF is now *decreasing* in the price–cost margin μ, in contrast to the case of fixed n.

[24] This fact follows from the formulae (42), (43) derived below.

[25] The intuition for overshifting is simple. An increase in t causes the price q to rise one-for-one given p fixed, i.e., 100% shifting. But then there is a second-round effect: a higher t lowers profit per firm, so firms exit the industry. This raises the producer price p as $\varepsilon p_n < 0$, and this further raises q.

Next, from Proposition 3, we are of course interested in the MCPF at a given level of tax *revenue*, g, and how for fixed g, the MCPF changes following a change in firm conduct. Also, εp_n is endogenous, as it depends on n. To compute εp_n, we proceed as follows. First, we can write the producer prices in the Bertrand and Cournot cases, i.e., p_B, p_C in terms of n, σ using (8) and (10) as follows:

$$p_B = \frac{n\sigma + 1 - \sigma}{(n-1)(\sigma-1)}, \; p_C = \frac{n\sigma}{(n-1)(\sigma-1)} \tag{41}$$

Then, from (41) it is straightforward to calculate that

$$\varepsilon p_n^B = -\left(\frac{1}{n-1}\right)\left(\frac{n}{(n-1)\sigma+1}\right) \tag{42}$$

$$\varepsilon p_n^C = -\left(\frac{1}{n-1}\right) \tag{43}$$

both of which are clearly negative. This is intuitive: the larger the number of firms, the smaller the producer price. Also, note that $0 > \varepsilon p_n^B > \varepsilon p_n^C$, as $\sigma > 1$. So, at this stage, we can substitute (41)–(43) into (39) as appropriate, to obtain the MCPF in terms of n, t and the parameters.

To proceed further, we next substitute out t. From the government budget constraint (21), and using (5), we have $g = tpX = t\alpha/(1+t)$, so solving for t, we obtain

$$1 + t = \frac{\alpha}{(\alpha - g)} \tag{44}$$

Then, from (41)–(44) and (39), we have:

$$MCPF^B(g,n) = \frac{\alpha}{\alpha - g}\left(\frac{n\sigma + 1 - \sigma}{n\sigma - \sigma}\right) \tag{45}$$

$$MCPF^C(g,n) = \frac{\alpha}{\alpha - g}\left(\frac{2n\sigma + n^2 - 2n - \sigma + 1}{(2\sigma - 2 + n)(n-1)}\right) \tag{46}$$

Finally (45) and (46) still depend on the endogenous variable n. But, combining (41) and the free entry condition (21), and eliminating t using (44), we see that in the Bertrand and Cournot cases, the free entry conditions determining n are, respectively;

$$n\sigma + 1 - \sigma = \frac{\alpha - g}{f}, \; \frac{n^2\sigma}{n + \sigma - 1} = \frac{\alpha - g}{f} \tag{47}$$

Let the values of n that solve (47) in the Bertrand and Cournot cases be

$n_B(g)$, $n_C(g)$, respectively.[26] Note that in the Cournot case, there are multiple (two) solutions, but only one is positive, so there is a unique economically meaningful solution $n_C(g)$ in (47). We see that in either case, there is a negative relationship between g and n. This is intuitive; an increase in g requires an increase in t from the government budget constraint, which increases q and lowers demand, so p must rise to allow firms to cover their fixed costs, so n must fall. Finally, our MCPFs in the two cases are $MCPF_k(g) \equiv MCPF_k(g, n_k(g))$, $k = B, C$.

Now let $MCPF_1(g)$ be the MCPF in problem P_1 above, i.e., where there is only one perfectly competitive firm in the economy. The MCPF in problem P_1 at a given tax rate t is $MCPF_1 = 1 + t$. But from (P_1), the government budget constraint in this case is $t\alpha(1+t)^{-1} = g + f$, so $1 + t = \alpha/(\alpha - g - f)^{-1}$. So, we see that

$$MCPF_1(g) = \frac{\alpha}{\alpha - g - f} \tag{48}$$

We now have some results on how the MCPF may vary with the degree of imperfect competition:

Proposition 5. *For all* $0 \leq g < 1$, $MCPF_C(g) > MCPF_B(g) = MCPF_1(g)$. *Consequently at any given level of expenditure on public goods, the MCPF is higher with Cournot conduct than with Bertrand conduct. Also,* $MCPF_B(g)$ *is independent of* σ, *and* $MCPF_C(g)$ *may be increasing in* σ.

The first result, that $MCPF_C(g) > MCPF_B(g)$, partially extends Proposition 4 to the case of free entry, i.e., we see that according to the conduct measure, more imperfect competition raises the MCPF, in line with simple economic intuition. However, we now see that the MCPF behaves in a very counter-intuitive way in response to an increase in the degree of imperfect competition, as measured by a fall in σ; it may either be unchanged or actually *fall*, depending on whether firm conduct is Bertrand or Cournot. This is counter-intuitive as the higher σ, the lower the firm-specific demand elasticity θ_C and so the lower the mark-ups over marginal cost (and from the case of fixed number of firms, we know from Proposition 4 that a lower mark-up reduces the MCPF). This is explained by the fact that the MCPF depends not just on the mark-up, but also on the *elasticity* of the mark-up with respect to n.

Second, from Proposition 5 a striking finding is that *no matter what f*, and therefore no matter what n, the Bertrand MCPF is equal to the MCPF of the benchmark competitive economy. This is the case even though in Bertrand

[26] So, the two solutions are:

$$n_B(g) = \frac{\alpha - g}{f\sigma} + \frac{\sigma - 1}{\sigma}, \; n_C(g) = \frac{1}{2\sigma f}\left(\alpha - g + \sqrt{(\alpha - g)^2 + 4(\alpha - g)\sigma(\sigma - 1)f}\right).$$

equilibrium, the number of firms will usually be greater than one, whereas there is always one firm operating in the benchmark competitive economy with free entry costs.

All of Proposition 5 is proved in the Appendix A, except for the result that $MCPF_C(g)$ may be increasing in σ. The latter is established by the simulation results reported in Table 2, which also indicate that as expected, both MCPFs rise with g and f.

A final remark can be made at this stage. First, the standard 'monopolistic competition' modelling of imperfect competition in general equilibrium, as discussed in Section 1, effectively assumes a limit case where $f \to 0$, and $n \to \infty$. It is easily established from (40) and (48) that as $f \to 0$, all of $MCPF_B(g)$, $MCPF_C(g)$, $MCPF_1(g)$ converge to $\alpha/(\alpha - g)$. That is, under standard monopolistic competition assumptions,[27] the MCPF is just equal to its competitive level, whatever is (i) firm conduct, or (ii) the elasticity of substitution, σ. So, making the limit assumption (as do Heijdra et al. (1998)) effectively rules out a priori any analysis of the MCPF under imperfect competition!

We now turn to consider the effects of a change in the degree of imperfect competition on public good supply with free entry. This is difficult, as in the case of a fixed number of firms, for reasons explained in Section 5. However, there is a special case in which we can make a statement. Recall from Section 5 that generally, optimal public goods supply satisfies (24), $\lambda MCPF = h'(g)$, and that $\lambda = \kappa q^{-\alpha(1-\eta)} y^{-\eta}$. With free entry, as $y = 1$, so $\lambda = \kappa q^{-\alpha(1-\eta)}$. So, if $\eta = 1$, the marginal utility of income is constant at κ. But then, from Proposition 3, the effect

Table 2
The MCPF and number of firms with free entry

σ	$g=0.1$ $f=0.005$	$g=0.1$ $f=0.01$	$g=0.2$ $f=0.005$	$g=0.2$ $f=0.01$
2	1.265(40)	1.282(20)	1.694(30)	1.724(15)
	1.281(40)	1.312(20)	1.722(30)	1.778(15)
5	1.265(16)	1.282(8)	1.694(12)	1.724(6)
	1.309(19)	1.358(10)	1.765(15)	1.845(8)
10	1.265(8)	1.282(4)	1.694(6)	1.724(3)
	1.323(13)	1.366(8)	1.792(10)	1.875(6)

Note: $\alpha = 0.5$ throughout. For each combination of parameters, the number in the upper (lower) panel of the box is the MCPF with Bertrand (Cournot) competition. The integers in brackets are the equilibrium numbers of firms with Bertrand and Cournot competition. In each case, the number of firms is the largest integer smaller than the real number which solves (47), as this is the largest number of firms that can make non-negative profit, given the parameters.

[27] It is also worth noting that if we assumed n large at the stage of defining the price-cost margin, i.e., if we made the approximation $\mu \simeq 1/\sigma$, then we would obtain misleading results. With this approximation, μ is independent of n, so $\varepsilon\mu_n = 0$, and so from (40), we would obtain an MCPF with imperfect competition of $\alpha/(\alpha - g)$, *below* what it is in the benchmark case!

of any parameter change on g is signed by its effect on the MCPF. So, as $MCPF_C(g) > MCPF_B(g)$, when $\eta = 1$, a switch from Bertrand to Cournot will always lower the supply of public goods. Also, the supply of public goods will be independent of σ, our other measure of imperfect competition, when conduct is Bertrand, and may be decreasing in σ when conduct is Cournot.

8. A variable elasticity of demand

By assuming $u(X,l)$ to be Cobb–Douglas, we have implicitly assumed εX_q, the elasticity of demand for the aggregate consumption index, to be constant at unity, i.e., $\varepsilon X_q = -1$. Suppose that $|\varepsilon X_q|$ were instead (say) increasing in price. Then, as an increase in g requires an increase in t, which from (12) increases q, an increase in g would now increase $|\varepsilon X_q|$. Then from (8) and (10), the firm specific elasticities of demand θ_B, θ_C would be increased, and thus the margins μ_B, μ_C reduced. It turns out that this endogeneity of the margins as g varies may, under some conditions, make the MCPF *lower* with Cournot than with Bertrand. This implies, of course, that Proposition 4 above does not generally extend to the case where εX_q is endogenous.

To demonstrate this, we will assume that

$$u(X,l) = \frac{1}{a}(X - \frac{X^2}{2}) + l$$

i.e., quasi-linear utility: it is convenient to abstract from income effects to simplify the analysis. Then, it is easily checked that demand for X is linear, i.e., $X(q) = 1 - aq$. We will denote the absolute value of εX_q by e, so we have;

$$e = -\frac{qX_q}{X} = \frac{aq}{1-aq} = \frac{a(1+t)p}{1-a(1+t)p} \tag{49}$$

i.e., e is increasing in price as required.

Now, in this new setting, firm behaviour is as before: the only difference is that εX_q is now endogenous, rather than fixed at -1. We will consider the limiting case where $\sigma \to \infty$. In this case, from (8) and (12), the Bertrand producer price is fixed at the perfectly competitive level $p = 1$. So, the Bertrand MCPF is the perfectly competitive MCPF defined in (29). Using (49) in (29), we see that;

$$MCPF_B = \frac{1+t}{1+t-ta(1+t)/(1-a(1+t))} \tag{50}$$

So, in the Bertrand case, given a government revenue requirement g, $MCPF_B$, t, are simultaneously determined by (50), and the government budget constraint (18) which in this case, is $g = t(1 - a(1+t))$. Table 3 reports values of $MCPF_B$ for different values of parameters g. We see that $MCPF_B$ is increasing in g.

Table 3
The MCPF when εX_q is endogenous

g		0.1	0.2	0.3	0.4
$n=2$	mc	0.1820	0.197	0.21399	0.23333
	mb	1.0128	1.0267	1.0417	1.0582
$n=5$	mc	0.3743	0.41175	0.45292	0.49788
	mb	1.0128	0.0128	1.0417	1.0582

Note: mb, mc = MCPF with Bertrand, Cournot competition. Other parameters are: $a=0.1$, $\bar{\tau}=0.5$. The Cournot system of equations has three solutions, and the Bertrand system two, for the parameter values chosen. We eliminate solutions that are not economically feasible, i.e., we require $X(q)=1-aq>0$, and $t>-1$. If more than one such solution is economically feasible, the one corresponding to the true MCPF is the one for which private consumption $X(q)$ is highest, given g.

In the Cournot case, as $\sigma \to \infty$, the elasticity of demand tends to $\theta_C \to ne$ in the general case when $e \neq 1$. So, from (12), the producer price is

$$p=\frac{ne}{ne-1}=\frac{naq}{(n+1)aq-1}=\frac{nap(1+t)}{(n+1)ap(1+t)-1} \tag{51}$$

using (49). Solving (51) for p, we get

$$p=\frac{an(1+t)+1}{a(n+1)(1+t)} \tag{52}$$

From (52), it is important to note that an increase in tax will *reduce* the Cournot producer price:

$$\varepsilon p_{(1+t)}=-\frac{1}{an(1+t)+1}<0 \tag{53}$$

It is this effect that may reduce the Cournot MCPF below the Bertrand.

Now, the MCPF can be derived exactly as in Section 6, subject to the additional complication that p now depends on the tax through (52). First, note that now full disposable income is simply $y=1+(1-\bar{\tau})(p-1)X$ as there are no income effects in demand. The Lagrangian for the government's tax design problem is therefore

$$\mathscr{L}=v(p(1+t),\ 1+(1-\bar{\tau})(p-1)X)+\rho[(tp+\bar{\tau}(p-1))X-g]$$

Recalling that p is now endogenous, the first-order condition for the optimal choice of t in this problem is therefore

$$(-X+(1-\bar{\tau})(p-1)X_q)q_t+\rho[pX+(tp+\bar{\tau}(p-1))X_q q_t+p_t X(t+\tau)]=0 \tag{54}$$

where $q_t=p+(1+t)p_t$. So, after some rearrangement of (54), noting that the *MCPF* is just ρ, we get:

$$MCPF_C = \frac{[1 + t + e\mu(1 - \bar{\tau})](1 + \varepsilon p_{(1+t)})}{1 + t - (t + \mu\bar{\tau})e(1 + \varepsilon p_{(1+t)}) + \varepsilon p_{(1+t)}(t + \bar{\tau})} \tag{55}$$

So, in the Cournot case, given a government revenue requirement g, $MCPF_C, t, e, p$ are simultaneously determined by (55), (52) and (49) and the government budget constraint (18) which in this case, is

$$g = [tp + \bar{\tau}(p - 1)](1 - aq).$$

The simulations in Table 3 show that for a range of g, when n is low, it is possible that $MCPF_C < MCPF_B$, even though the Cournot mark-up $\mu_C > 1$. The mechanism by which this occurs is clear from (53): an increase in g requires an increase in t, which from (53), lowers the Cournot mark-up.

So, for the parameter values in Table 3, the MCPF is lower with imperfect (Cournot) competition than it is with perfect competition. In fact, it can be below unity. Note also that in the Cournot case, the MCPF is *increasing* in the number of firms in the industry! Finally, note that in this case, as utility is quasi-linear, $\lambda = 1$ and so following the analysis of Section 5, it is possible to show that an increase (decrease) in the MCPF must always increase (decrease) public good provision. So, we can conclude from the above simulation results that optimal public good supply can also be *higher* with imperfect competition than with perfect competition.

9. Conclusions and related literature

9.1. Related literature

Broadly related literature has been discussed in the introduction. The papers closest to this one are a series by Heijdra and co-authors (Heijdra and van der Ploeg, 1996; Heijdra and Ligthart, 1997; Heijdra et al., 1998). These papers analyse a general equilibrium model of imperfect competition similar to that of ours, except that utility $u(X,l)$ is CES. Their main focus is on the Keynesian multiplier, but they also calculate formulae for the MCPF. Heijdra and van der Ploeg (1996) and Heijdra and Ligthart (1997) only consider the case of lump-sum taxation, but Heijdra et al. (1998) allow for distortionary taxation in the form of an a uniform income tax, i.e., profit and labour income taxed at the same rate. In terms of our model, this is equivalent to a commodity tax and a zero profit tax ($\tau = 0$). They derive formulae for the MCPF with a fixed number of firms and with free entry, but these formulae are conditional on the (endogenous) tax rate and wage. Because their MCPF formulae are not evaluated at fixed g, they cannot be used to make inferences about levels of public good provision. Moreover, as they make the 'monopolistic competition' assumption, with free entry, the MCPF and public good supply is only different from the competitive case when there is PFD. This is in contrast to this paper, where imperfect competition can have complex effects on the MCPF though the endogeneity of n, even in the absence of PFD.

9.2. Conclusions

In this paper, we have examined the impact of imperfect competition on the marginal cost of public funds and optimal public good provision. The conventional wisdom — that more imperfect competition will raise the MCPF and lower the optimal supply of public goods — is shown to be only partially true. Except in the special case when there the marginal utility of income is constant, the connection between changes in the MCPF and public good supply is not straightforward, due to the endogeneity of the marginal utility of income. with imperfect competition. Also, non-isoelastic demand for consumption, and endogenous firm numbers through free entry can both (for different reasons) cause an increase in some plausible measure of the degree of imperfect competition to lead to a *fall* in the MCPF. So, the main message of this paper is that simple economic intuition about the effect of imperfect competition on the MCPF and public good supply is not an infallible guide, and each case must be judged on its merits.

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Appendix A

A.1. Proofs of propositions

Proof of Proposition 1. Let (q_0,y_0, g_0) be the be the consumer price, income, and level of public good at the solution to problem P_0. Note that $y_0 = 1$. It is sufficient to show that we can find (τ_k,t_k) which will replicate the solution to problem P_0. For then, the government can achieve the second-best efficient outcome via choice of (τ_k,t_k), so this choice would surely be optimal in its design problem.

Let (q_k,y_k, g_k) be the consumer price, income, and level of public good at the solution to the tax and expenditure design problem with Bertrand or Cournot competition ($k = B,C$). First, to ensure $q_k = q_0$, $p_k(1 + t_k) = q_0$ is required, implying

$$t_k = \frac{q_0 - p_k}{p_k},\ k = B,C \tag{A.1}$$

Next, as $y_0 = 1$, to achieve $y_k = y = 1 + (1 - \tau_k)(p_k - 1)X(q_k, y) = 1$, clearly $\tau_k = 1$ is needed. The final step is to show that this choice of (τ_k, t_k) will yield tax revenue g_0, i.e., that $g_k = g_0$. From (19), we have

$$\begin{aligned} g_k &= (t_k p_k + p_k - 1)X(q_k, 1) \\ &= (q_0 - 1)nX(q_0, 1) \\ &= g_0 \end{aligned}$$

where the second line follows from (A.1) and $q_k = q_0$, and the third from the fact that $q_0 = 1 + t_0$. □

Proof of Proposition 2. Let (q_1, y_1, g_1) be the be the price index, income, and level of public good at the solution to the problem P_1. It is sufficient to show that we can find (t_k, T_k) which will replicate the solution to the problem P_1. For then, the government can achieve the second-best efficient outcome via choice of (τ_k, T_k).

Let $(q_k, y_k,\ g_k)$ be the price index, income, and level of public good at the solution to the tax and expenditure design problem with Bertrand or Cournot competition $(k = B,C)$. Denote by $p_k(1)$ the equilibrium producer price with one firm. Given that only one firm enters in problem P_1 to ensure $q_k = q_1$, $p_k(1)(1 + t_k) = q_1$ is required, implying

$$t_k = \frac{q_0 - p_k(1)}{p_k(1)}\ \ k = B,C \tag{A.2}$$

Next, we require that only one firm does enter, i.e., the free-entry condition (17) holds with $n = 1$ at q_1, i.e.,

$$(p_k(1) - 1)X(q_1, 1) = f + T_k \tag{A.3}$$

which determines T_k. The final step is to show that this choice of (t_k, T_k) will yield tax revenue g_1, i.e., that $g_k = g_1$. From (21) we have:

$$\begin{aligned} g_k &= t_k p_k X(q_k, 1) + n_k T_k \\ &= t_k p_k(1)X(q_k, 1) + [(p_k(1) - 1)X(q_k, 1) - f] \\ &= (t_k p_k(1) + p_k(1) - 1)X(q_k, 1) - f \\ &= (q_1 - 1)X(q_1, 1) - f \\ &= g_1 \end{aligned}$$

where the second line follows from (A.3) and the fact that $n_k = 1$, and the fourth by (A.2), and the fifth by $q_1 = 1 + t_1$. □

Proof of Proposition 3. Total differentiation of (23) gives

$$\frac{\partial g}{\partial z}=\frac{\partial(\lambda MCPF)/\partial z}{D} \tag{A.4}$$

where $D<0$ by the second-order condition for the optimal choice of g. Moreover,

$$\frac{\partial(\lambda MCPF)}{\partial z}=\frac{\lambda MCPF}{z}\left[\frac{z}{\lambda}\frac{\partial\lambda}{\partial z}+\frac{z}{MCPF}\frac{\partial MCPF}{\partial z}\right] \tag{A.5}$$

Now, as $\lambda=\kappa q^{-\alpha(1-\eta)}y^{-\eta}$, we have:

$$\frac{z}{\lambda}\frac{\partial\lambda}{\partial z}=-\alpha(1-\eta)\left(\frac{z}{q}\frac{\partial q}{\partial z}\right)-\eta\left(\frac{z}{y}\frac{\partial y}{\partial z}\right) \tag{A.6}$$

So, combining (A.4)–(A.6) we see that

$$\frac{\partial g}{\partial z}=\frac{\lambda MCPF}{z(-D)}\Delta$$

where Δ is defined in the Proposition. As $\lambda MCPF/z(-D)>0$, we have the result. □

Proof of Proposition 5. From (45), using (47), we get,

$$\begin{aligned}MCPF_B(g)&=\frac{\alpha}{\alpha-g}\left(\frac{n_B(g)\sigma+1-\sigma}{\sigma(n_B(g)-1)}\right)\\&=\frac{\alpha}{\alpha-g}\left(\frac{(\alpha-g)/f}{(\alpha-g)/f-1}\right)\\&=\frac{\alpha}{\alpha-g-f}\end{aligned} \tag{A.7}$$

and from (48), (A.7), the result $MCPF_B(g)=MCPF_1(g)$ follows. Next, from (A.7) and (46), we can rewrite

$$MCPF_B(g)=\frac{\alpha}{\alpha-g}\left(1+\frac{f}{1-g-f}\right)$$

$$MCPF_C(g)=\frac{\alpha}{\alpha-g}\left(1+\frac{n+\sigma-1}{(2\sigma-2+n)(n-1)}\right)$$

so that we have $MCPF_C(g)>MCPF_B(g)$ as required if

$$\frac{n+\sigma-1}{(2\sigma-2+n)(n-1)}>\frac{f}{1-g-f} \tag{A.8}$$

But from (47), we have

$$\frac{f}{1-g-f}=\frac{n+\sigma-1}{n^2\sigma-n-\sigma+1} \tag{A.9}$$

So from (A.8) and (A.9), we require only $(2\sigma-2+n)(n-1)<n^2\sigma-n-\sigma+1$, which in turn is equivalent to $2n-1<n^2$. But this last inequality holds for all $n\geq 2$, and $n=1$ is impossible in equilibrium as then from (10), $\theta_C=1$, so the mark-up is not defined. □

A.2. Calculation of $\epsilon n_{(1+t)}$

Totally differentiating (17), and rearranging, we get

$$\frac{\mathrm{d}n}{n}\left(1+\frac{\varepsilon p_n}{p-1}\right)-\frac{\mathrm{d}t}{1+t}=0 \tag{A.10}$$

where εp_n is the elasticity of the mark-up with respect to the number of firms. So, from (A.10) we get;

$$\epsilon n_{(1+t)}=\frac{\mathrm{d}n}{\mathrm{d}t}\frac{1+t}{n}=\frac{p-1}{p-1+\varepsilon p_n} \tag{A.11}$$

But this is the formula in (38).

A.3. Calculation of t,y in the Cobb–Douglas Case

From (19), using $q=p(1+t)$, and the definition of μ, we get;

$$g=\frac{(t+\bar{\tau}\mu)\alpha y}{(1+t)} \tag{A.12}$$

Solving (A.12) for t, we get

$$t=\frac{g-\bar{\tau}\mu\alpha y}{\alpha y-g} \tag{A.13}$$

Substituting (A.13) in (16), we get after some rearrangement

$$y=\frac{1-\bar{\tau}\mu-(1-\bar{\tau})\mu g}{1-\bar{\tau}\mu-(1-\bar{\tau})\mu\alpha} \tag{A.14}$$

which is (33). To get (32), substitute (A.14) back in (A.13) and rearrange. □

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