International Tax and Public Finance, 11, 1–10, 2004

Competition in Unit vs. Ad Valorem Taxes

BEN LOCKWOOD* B.Lockwood@warwick.ac.uk
Department of Economics, University of Warwick, Coventry CV4 7AL, UK

Abstract

This paper shows that in a standard model of tax competition, the Nash equilibrium in capital taxes depends on whether these taxes are unit (as assumed in the literature) or ad valorem (as in reality). In a symmetric version of the model, general results are established: taxes and public good provision are both higher, and residents in all countries are better off, when countries compete in unit taxes, as opposed to ad valorem taxes. However, the difference in equilibrium outcomes is negligible when the number of countries is large.

Keywords: tax competition, unit taxes, ad valorem taxes

JEL Code: H20, H21, H77

1. Introduction

There is now a substantial theoretical literature on tax competition, covering virtually every kind of tax where the tax base may possibly be mobile between jurisdictions[1] (capital income taxes, commodity taxes, corporate taxes, etc.). However, one aspect of tax competition that has received virtually no attention so far is whether it matters if the taxes are unit or ad valorem. Indeed, in the leading model of competition over taxes on capital, the Zodrow-Mieszkowski-Wilson (ZMW) model, it is always assumed[2] that the tax is levied per unit of capital (i.e. is a unit tax), whereas in reality, taxes are on capital income and are therefore always ad valorem (i.e. a proportion of gross income from capital).

It is the purpose of this paper to show that this unrealistic simplification is not without loss of generality. Specifically, we show, in the context of the ZMW model, that the Nash equilibrium in unit taxes is generally different than the Nash equilibrium in ad valorem taxes, in the sense that at the two equilibria, public good provision and private consumption are generally different. In particular, we establish the following quite general results. If countries are symmetric, and both private and public goods are normal, then (i) the symmetric Nash equilibrium in taxes exists and is unique in each case; and (ii) equilibrium taxes and public good provision are *always* lower when countries compete with ad valorem taxes. In other words, tax competition is more intense with ad valorem taxation. Moreover, under the same conditions, we also have a welfare result: residents in all countries are worse off

*I would like to thank an editor of this journal and also Nobuo Akai and Chuck Blackorby for very helpful comments.

when competition is in ad valorem taxes. Finally, we show that as the number of countries becomes large, the difference between the equilibrium taxes becomes negligible.

Perhaps one reason why these points have been missed in the literature is that in the original ZMW model, and many subsequent applications, the number of countries is assumed large enough so that each individual country takes the after-tax return on capital fixed when setting its own unit tax (the small open economy assumption). In this case, as our results indicate, assuming unit capital taxes is without loss of generality.

However, one of the lessons of game theory is that instruments that are equivalent for a single decision-maker, or for many small decision-makers, may not be equivalent when a few decision-makers interact.[3] Our results confirm that this kind of non-equivalence applies to unit and ad valorem taxes in a tax competition setting. Indeed, this is not the first paper to note this kind of non-equivalence in a public finance setting. For example, Wildasin (1988, 1991) shows, also in the context of the ZMW model, that under certain conditions, taxes are lower when jurisdictions compete in expenditures than when they compete in taxes.[4] So, our results can be interpreted as extending and refining his basic insight. Our results also relate to a literature comparing unit and ad valorem indirect taxes set by a single government, with imperfect competition in the product market; this literature is discussed in more detail in Section 3.

The difference in Nash equilibrium outcomes with unit and ad valorem taxation can be explained as follows. First, in symmetric equilibrium, the tax in any country is determined by the condition that the marginal rate of substitution in consumption between the public and the private good is equal to the marginal cost of public funds, (MCPF), setting the resource cost of the public good equal to one for convenience. In the case where all countries set unit taxes, the MCPF takes a very simple form; it is the reciprocal of one plus the elasticity of the tax base (the capital stock) with respect to the "home" country's tax i.e. $1/(1+\varepsilon^u)$.

In the ad valorem case, it is shown below that the MCPF can be written $1/(1+\varepsilon^{u,a})$, where $\varepsilon^{u,a} < 0$ is the elasticity of any country's capital stock with respect to its own tax when *that country chooses a unit tax and all other countries choose an ad valorem tax.* The rationale for writing the MCPF in this way is that it allows easy comparison of the two cases, and it is legitimate to do this as any particular country's choice between unit and ad valorem taxes as control variables is a matter of indifference for that country—for any unit rate, it can find an equivalent unit rate, and visa versa. The main result of this paper is that $\varepsilon^{u,a}$ is greater in absolute value than ε^u, implying in turn that the MCPF is higher with ad valorem taxation, and so public good provision is lower.

The intuition for this is the following. Assume two countries for simplicity, and consider an initial equilibrium where both countries set unit taxes T^u. A small increase in expenditure on the public good—and thus increase in tax by country 1—will cause a given capital outflow of size Δ from country 1, and thus an increase Δ in the capital stock employed in country 2, before equilibrium in the capital market is restored, assuming that country 2 maintains its unit tax at T^u. This capital outflow is measured by ε^u.

Now suppose country 2 switches to an ad valorem tax that is equivalent to the equilibrium unit tax i.e. raises the same revenue at the initial equilibrium. Consider the same small increase in expenditure on the public good, and thus increase in tax, by country 1. As the ad valorem tax in country 2 is now fixed, any outflow of capital from country 1 will now

P1: LOP
International Tax and Public Finance KL3048-04/1007-02 May 8, 2004 21:44

lower the effective unit tax in country 2, because the latter is the fixed ad valorem tax times the marginal product of capital in that country, and the latter falls with an inflow of capital to country 2. So, following the initial capital outflow Δ, the effective unit tax on capital in country 2 is now lower than in the case where country 2 sets a unit tax. Thus the outflow Δ of capital from country 1 is no longer sufficient to restore equilibrium to the capital market, as the net return to investing in country 2 is now higher than in the unit case. An additional outflow from country 1 is required. This additional outflow is the reason that $\varepsilon^{u,a}$ is greater in absolute value than ε^{u}.

The remainder of the paper is organized as follows. Section 2 sets out the model, Section 3 states and proves the main results, and Section 4 concludes.

2. The Model

We consider a symmetric version of the ZMW model. Each country $i = 1, ..n$ is populated by L_i identical residents each of whom supplies one unit of labour, and owns K units of capital. Capital is mobile between countries, while labour is not. In both countries competitive firms combine capital and labour to produce output. Output in country i is $F(K_i, L_i)$, where L_i, K_i are the amounts of labour and capital employed in that country, where F has the usual properties: it has constant returns to scale, and is concave and twice differentiable. We can therefore write the production function for any country in intensive form as $f(k_i)$, where $k_i = K_i/L_i$. We assume that all countries are the same size i.e. $L_i = L$, $i = 1, ..n$, in which case w.l.o.g., $L = 1$. Finally, let T_i be a unit tax on the number of units of capital employed in country i, and let t_i be an ad valorem tax on the income from capital employed in country i. By definition, both kinds of taxes are assumed to be source-based.

Given taxes, capital market equilibrium in the model is described as follows. Since capital is mobile, the post-tax rate of return, r, for the investor in any country $i = 1, ..n$ must be the same whether taxes are unit or ad valorem i.e.

$$f'(k_i) - T_i = r \tag{2.1}$$

and

$$f'(k_i)(1 - t_i) = r \tag{2.2}$$

These conditions give the demand for capital in any country i in the unit and ad valorem cases as $k_i = \phi(r + T_i)$, $k_i = \phi(r/(1 - t_i))$, where ϕ is the inverse of f'. Equilibrium on the capital market requires world supply, nk, equals world demand

$$nk = \begin{cases} \sum_{i=1}^{n} \phi(r + T_i) & \text{(unit)} \\ \sum_{i=1}^{n} \phi(r/(1 - t_i)) & \text{(ad valorem)} \end{cases} \tag{2.3}$$

So, (2.3) determines r as a function of the n tax rates in the unit and ad valorem cases.

Each of the residents of country i has a utility function $v(c_i, g_i)$ defined over private consumption $c_i \in \Re_+$ and consumption of a public good $g_i \in \Re_+$, where u is strictly

P1: LOP
International Tax and Public Finance KL3048-04/1007-02 May 8, 2004 21:44

increasing in both arguments, strictly quasi-concave, and twice continuously differentiable. Private consumption is equal to the sum of income from labour and from capital:

$$c_i = f(k_i) - f'(k_i)k_i + rk \tag{2.4}$$

where $f'(k_i) - f(k_i)k_i$ is the income from the fixed factor, labour.

Following the standard exposition of the ZMW model (e.g. Wilson, 1999) we assume that the public good is financed entirely from the tax on capital income. Also, we assume w.l.o.g. that production of one unit of the public good requires one unit of the private good. So, the government budget constraint is

$$g_i = \begin{cases} T_i k_i & \text{(unit)} \\ t_i f'(k_i)k_i & \text{(ad valorem)} \end{cases} \tag{2.5}$$

Finally, welfare of the typical resident in country i, conditional just on T_i or t_i, and k_i can be written by substituting (2.4), (2.5) into $u(c_i, g_i)$, and using (2.1), (2.2), to get:

$$u(k_i, r, T_i) = u(f(k_i) - T_i k_i + r(k - k_i),\ T_i k_i) \tag{2.6}$$

$$u(k_i, r, t_i) = u(f(k_i) - t_i f'(k_i)k_i + r(k - k_i),\ t_i f'(k_i)k_i) \tag{2.7}$$

This completes the description of the model.

3. Competition in Unit and Ad Valorem Taxes

First, consider competition in unit taxes. We will make the standard assumption that governments are welfare-maximising (it is simple to show that our results extend to the case where governments maximise tax revenue). The government in country i chooses T_i to maximise $u(k_i, r,\ T_i)$ taking $T_j,\ j \neq i$ as fixed, but taking into account the dependence of k_i and r on T_i though capital market equilibrium conditions (2.1)–(2.3). We impose the restriction that $0 \leq T_i \leq f'(k)$; the upper bound on T ensures that in symmetric equilibrium, r is non-negative, and the lower bound is required as public good supply is non-negative.

Assuming an interior solution,[5] the first-order condition for this choice of T_i is:

$$(-u_c + u_g)k_i + [u_c(f'(k_i) - T_i - r) + u_g T_i]\frac{\partial k_i}{\partial T_i} + u_c(k - k_i)\frac{\partial r}{\partial T_i} = 0 \tag{3.1}$$

These three effects are familiar. An increase in T_i transfers consumption from the private to the public good (term 1), induces a capital outflow (term 2), and causes a change in the world price of capital, r (term 3, the "terms of trade" effect).

Now we evaluate (3.1) at a symmetric equilibrium where $T_i = T^u$, $k_i = k$. So, the terms of trade effect vanishes, and also $f'(k_i) - T_i - r = 0$ from (2.1), an envelope result. So, from (3.1), after simple rearrangement, we have:

$$\frac{u_g}{u_c} = \frac{1}{1 + \varepsilon^u}, \quad \varepsilon^u = \frac{T_i}{k_i}\frac{\partial k_i}{\partial T_i} \tag{3.2}$$

where it is understood that u_g, u_c are evaluated at $c = f(k) - T^u k,\ g^u = T^u k$. This is of course, a modified Samuelson condition: the marginal rate of substitution between the private and the public good, u_g/u_c, is equal to is equal to the marginal rate of transformation, (set to unity by assumption), times the marginal cost of public funds (MCPF). Note that ε^u is the elasticity of the tax base in the unit case. From (2.3) and (2.1) we have,[6] at a symmetric equilibrium:

$$\varepsilon^u = \frac{T_i}{k_i}\frac{\partial k_i}{\partial T_i} = \frac{n-1}{n}\frac{T^u}{kf''(k)} \tag{3.3}$$

So, the MCPF is greater than unity as long as $T^u > 0$, as $f'' < 0$.

Next, consider competition in ad valorem taxes. The government in country i chooses t_i to maximise $u(k_i, r, t_i)$ taking t_j, $j \neq i$ as fixed, but taking into account the dependence of r and k_i on t_i though capital market equilibrium conditions (2.2), (2.3). We impose the restriction[7] that $0 \leq t_i \leq 1$. Assuming an interior solution,[8] from (2.6), the first-order condition for this choice of t_i is:

$$(-u_c + u_g)f'(k_i)k_i + \left[u_c(f'(k_i)(1-t_i) - r) + u_g t_i f'(k_i) + (u_g - u_c)t_i k_i f''(k_i)\right]\frac{\partial k_i}{\partial t_i}$$
$$+ u_c(k - k_i)\frac{\partial r}{\partial t_i} = 0 \tag{3.4}$$

Again, the three terms in (3.4) have the same general interpretation as in the unit case. Next, we evaluate (3.4) at a symmetric equilibrium where $t_i = t^a$, $k_i = k$, $i = 1, ..n$. So, the terms of trade effect vanishes, and also $f'(k_i)(1-t_i) - r = 0$ from (2.1), an envelope result. Rearranging, this yields a modified Samuelson rule:

$$\frac{u_g}{u_c} = \frac{1+\theta\eta}{1+\eta(1+\theta)}, \quad \eta = \frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i}, \quad \theta = \frac{kf''(k)}{f'(k)} \tag{3.5}$$

where the right-hand side is the MCPF with ad valorem taxes, η is the elasticity of the capital stock with respect to the ad valorem tax, and θ is the elasticity of the marginal product of capital.

At first sight,[9] formula (3.5) for the MCPF seems complicated. However, it is possible to give a very simple interpretation to the formula, using the following argument. Although a single country—say i—cares about whether *its rivals* use ad valorem or unit tax rates, its *own* choice between these instruments as control variables is a matter of indifference: for any unit rate, it can find an equivalent unit rate, and visa versa. Thus, it must be the case that the MCPF with ad valorem taxes (3.5) is unchanged if country i chooses a unit tax when all other countries choose ad valorem taxes.

Define $T_i = t_i f'(k_i)$ to be the *equivalent unit tax rate for* i. The base of this tax is just k_i, so the elasticity of the tax base when i chooses T_i and all other countries choose ad valorem taxes is therefore

$$\varepsilon^{u,a} = \frac{T_i}{k_i}\frac{\partial k_i}{\partial T_i} = \frac{t_i f'(k_i)}{k_i}\frac{\partial k_i}{\partial(t_i f'(k_i))} = \frac{\frac{t_i f'(k_i)}{k_i}\frac{\partial k_i}{\partial t_i}}{\frac{\partial(t_i f'(k_i))}{\partial t_i}} = \frac{\frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i}}{1 + \frac{t_i}{f'(k_i)}\frac{\partial f'(k_i)}{\partial t_i}} = \frac{\eta}{1+\theta\eta} \tag{3.6}$$

where the notation $\varepsilon^{u,a}$ indicates that the "home" country is choosing a unit tax, and all other countries are choosing an ad valorem tax.

So, we see from (3.5), (3.6) that the MCPF in the ad valorem case is simply

$$\frac{1+\theta\eta}{1+\eta(1+\theta)} = \frac{1}{1+\frac{\eta}{1+\theta\eta}} = \frac{1}{1+\varepsilon^{u,a}} \tag{3.7}$$

i.e. the reciprocal of one plus the elasticity of the tax base, appropriately defined. Using (3.7) we may therefore write the modified Samuelson rule (3.5) in the usual form

$$\frac{u_g}{u_c} = \frac{1}{1+\varepsilon^{u,a}} \tag{3.8}$$

Comparison of (3.2) and (3.8) is then straightforward. If the "home" country's tax instrument is a unit one, and all other countries switch from unit to ad valorem, the marginal cost of public funds and public good supply changes because the tax base elasticity changes from ε^u to $\varepsilon^{u,a}$.

Inspection of (3.2) and (3.8) does not immediately reveal whether (at symmetric equilibrium) $\varepsilon^{u,a}$ is greater or less than ε^u. However, as the following Proposition indicates, we can show quite generally that $\varepsilon^{u,a}$ is greater in absolute value than ε^u, implying a higher MCPF, and thus lower taxation and public good provision, in the ad valorem case. The intuition for this result has already been given in the last paragraph of the introduction.

The first step is to ensure that symmetric equilibrium taxes exist and are unique in each case. Equations (3.2) and (3.5) implicitly define the symmetric equilibrium taxes T^u and t^a respectively, and so it is sufficient to prove that each of these equations has a unique solution. In the Appendix, it is shown that the following condition is sufficient for a unique interior solutions $T^u \in (0, f'(k))$ and $t^a \in (0, 1)$ to (3.2) and (3.5) respectively:

(E) *Both c and g are normal goods, and*

$$\frac{u_g(f(k),0)}{u_c(f(k),0)} > 1, \quad \frac{u_g(f(k)-f'(k)k,\, f'(k)k)}{u_c(f(k)-f'(k)k,\, f'(k)k)} < 1$$

The first (second) limit condition on the marginal rate of substitution between public and private goods simply says that if capital is taxed at the minimum (maximum) rate, the marginal rate of substitution is above (below) unity.[10]

Now define the *ad valorem equivalent* to the equilibrium unit tax as $t^u = T^u/f'(k)$. Then, by inspection of the budget constraints (2.5), it is clear that if $t^u > t^a$ (resp. $t^u < t^a$) then public good provision will be higher (lower) with unit taxes than with ad valorem taxes. We can now state our main result:

Proposition 1. *Assume that condition (E) holds. Then $t^u > t^a$, so public good provision will be higher with unit taxes than with ad valorem taxes. However, as the number of countries becomes large, ($n \to \infty$), the difference between the two levels of public good provision becomes small ($t^u \longrightarrow t^a$).*

Proof: First, from (2.3) and (2.2) we see that:[11]

$$\eta = \frac{t_i}{k_i}\frac{\partial k_i}{\partial t_i} = \frac{n-1}{n}\frac{f'(k)}{kf''(k)}\frac{t^a}{(1-t^a)} \tag{3.9}$$

Next, note from (3.5) and (3.9) that for any equal ad valorem taxes $t_i = t$, the ad valorem MCPF is

$$\frac{1+\theta\eta}{1+\eta(1+\theta)} = \frac{1+\left(\frac{n-1}{n}\right)\frac{t}{1-t}}{1+\left(\frac{n-1}{n}\right)\frac{t}{1-t}+\frac{1}{\theta}\left(\frac{n-1}{n}\right)\frac{t}{1-t}} = \frac{1-\frac{t}{n}}{1-\frac{t}{n}+\frac{1}{\theta}\left(\frac{n-1}{n}\right)t} = b^a(t) \qquad (3.10)$$

Also, from (3.2) and (3.3), for any equal unit taxes $T_i = T$, the unit MCPF at the ad valorem equivalent $t = T/f'(k)$ is

$$\frac{1}{1+\frac{1}{\theta}\left(\frac{n-1}{n}\right)t} = b^u(t) \qquad (3.11)$$

Next, in the Appendix, it is shown that $t^a \in (0, \bar{t}^a)$, $t^u \in (0, \bar{t}^u)$, where $t^u = T^u/f'(k)$ is the ad valorem equivalent of the equilibrium unit tax, and $\bar{t}^u = \min\{-\frac{n}{n-1}\theta, 1\}$, $\bar{t}^a = \min\{-\frac{n}{\theta(n-1)-1}, 1\}$. Note that $\bar{t}^a \leq \bar{t}^u$.

Now, suppose contrary to the claim in the proposition, $\bar{t}^a > t^a \geq t^u > 0$. Then by inspection of (3.10) and (3.11), it is clear that (i) $b^u(t)$ is strictly increasing in t on $[0, \bar{t}^u]$, and $b^a(t)$ is strictly increasing in t on $[0, \bar{t}^a]$, and (ii) $b^a(t) > b^u(t)$, all $\bar{t}^a > t > 0$. So, certainly $b^a(t^a) > b^u(t^u)$. But then, by (3.2) and (3.5),

$$\frac{u_g(f(k)-g^a, g^a)}{u_c(f(k)-g^a, g^a)} = b^a(t^a) > b^a(t^u) = \frac{u_g(f(k)-g^u, g^u)}{u_c(f(k)-g^u, g^u)}$$

where $g^a = t^a f'(k)k$, $g^u = t^u f'(k)k$ from (2.5). As shown in the Appendix, given condition (E), $\frac{u_g(f(k)-g,g)}{u_c(f(k)-g,g)}$ is decreasing in g. But then $g^a < g^u$, implying $t^a < t^u$, a contradiction.

The last part of the proposition follows directly from taking the limit as $n \longrightarrow \infty$ in (3.10) and (3.11), noting that the MCPF with ad valorem taxation converges to the MCPF with unit taxation at any fixed t. □

Now we turn to welfare comparisons. Let v^u, v^a be the payoffs to residents of each country in Nash tax equilibrium with unit and ad valorem taxes respectively. Then we have:

Proposition 2. *Assume that there is a unique symmetric Nash equilibrium to the tax competition game in both unit and ad valorem taxes. Then, given that taxes are set non-cooperatively, a switch from ad valorem to unit taxation is Pareto-improving i.e.* $v^u > v^a$.

Proof: From the definition of u in (2.6) and (2.7) plus the budget constraint (2.5), we see that at the symmetric Nash equilibrium in taxes, the payoff to any agent is

$$v^i = v(g^i) = u(f(k) - g^i,\ g^i), \quad i = u, a$$

where $g^a = t^a f'(k)k$, $g^u = t^u f'(k)k$ from (2.5). Note that $v(g)$ is a concave function of g with a maximum at g^*, the first-best level of the public good. As the MCPF with unit taxation is greater than unity, $g^u < g^*$. Moreover, as $t^u > t^a$, then $g^a < g^u$. So $v^a = v(g^a) < v(g^u) = v^u$. □

Given the results on tax levels, the intuition for Proposition 2 is straightforward. It is already well-known that public good provision is inefficiently low in competition with unit taxes (e.g. Wilson, 1999). As competition in ad valorem taxes drives taxes (and thus public good provision) even lower, country welfare must fall in symmetric equilibrium. Of course, Proposition 1 implies that if the number of countries is large, the welfare gain from unit taxation will be small.

This welfare result is of course, opposite to that found in the literature which studies unit and ad valorem taxes in a single tax jurisdiction with imperfect product market competition. In that setting, under quite general conditions, a revenue-neutral switch from unit to ad valorem taxation causes firms to produce more and thus raises welfare (Wicksell, 1896; Suits and Musgrave, 1953; Delipalla and Keen, 1992; Anderson, de Palma, and Kreider, 2001a, b). Such a switch may even be Pareto-improving i.e. raise both consumer surplus, and profit simultaneously (Skeath and Trandel, 1994; Anderson, de Palma, and Kreider, 2001a, b). The reason for the difference is just that very different effects are at work: in the literature just cited, a switch to ad valorem taxation effectively increases the elasticity of the demand schedule facing the firm, and thus allows the government to extract a given amount of revenue at a lower consumer price. The mechanism at work in our model is by contrast, the change in the elasticity of the tax base.

Au: Pls. verify year 1959 or 1896.

4. Related Literature and Conclusions

We have shown in this paper that in tax competition, it matters whether taxes on capital income are unit or ad valorem: in the latter case, taxes, public good provision are all lower than they are with unit taxes. This raises the issue of why we do not observe unit taxes on capital in practice. The answer is that they are most probably infeasible. The heterogeneity of machinery, for example, would make such a tax difficult to implement.

The other question that is prompted by the above analysis is whether any of the existing results in the existing tax competition literature depend qualitatively on the usual assumption of unit taxes. Our results already indicate that the basic qualitative result of the literature, that equilibrium taxes are too low, is unaffected by the alterative assumption of ad valorem taxes. However, as is clear e.g. from the comprehensive survey of Wilson (1999), many of the interesting results in the tax competition literature are generated by allowing *asymmetry* between countries: for example, the conclusion of Bucovetsky (1991), and Wilson (1991), that small countries may be better off with tax competition than without. To test the robustness of such results to the use of ad valorem taxes, one would have to solve for the asymmetric ad valorem taxes: given the additional complexity of dealing with ad valorem taxes even in the symmetric case, this is really beyond the scope of the current paper.

Finally, we briefly mention related work. The papers of Wildasin (1988, 1991), and the literature on unit vs. ad valorem commodity taxes in a closed economy with imperfect competition has already been discussed. An additional closely related paper is Lockwood and Wong (2000), where it is shown that unit and ad valorem tariffs give rise to different tariff equilibria in a simple model of international trade. However, there it is shown, at least for the case of symmetric countries, that ad valorem tariffs lead to *less* trade distortion in Nash equilibrium and thus make both countries better off. So, when considering tax or tariff competition there are clearly no general results on the relative merits of specific and ad valorem taxes.

P1: LOP
International Tax and Public Finance KL3048-04/1007-02 May 8, 2004 21:44

Appendix A: Existence and Uniqueness of Symmetric Nash Equilibrium

First, consider the unit case. It is convenient to work with the *ad valorem equivalent* of $T,\ t = T/f'(k)$. Let

$$a(t) = \frac{u_g(f(k) - tf'(k)k, tf'(k)k)}{u_c(f(k) - tf'(k)k, tf'(k)k)}, \quad b^u(t) = \frac{1}{1 + \frac{n-1}{n}\frac{t}{\theta}} \tag{A.1}$$

Note that a is the marginal rate of substitution between c and g, and b is the unit MCPF. Differentiating $a(t)$, after a little rearrangement:

$$a'(t) = \frac{kf'(k)}{u_c}\left[\left(u_{gg} - \frac{u_g}{u_c}u_{cg}\right) + \frac{u_g}{u_c}\left(u_{cc} - \frac{u_c}{u_g}u_{gc}\right)\right] \tag{A.2}$$

Now, the condition that g be a normal good is that $u_{gg} - \frac{u_g}{u_c}u_{cg} < 0$, and similarly, the condition that c be a normal good is that $u_{cc} - \frac{u_c}{u_g}u_{gc} < 0$. So, from (A.2), $a'(t) < 0$. Also, by inspection, $b^u(t) \geq 1$, and is strictly increasing in t for all $t \in [0, \bar{t}^u]$, where $\bar{t}^u = \min\{-\frac{n}{n-1}\theta, 1\}$. Moreover, by the conditions in (E) above on $a(.)$, and inspection of (A.1), $a(0) > 1 = b(0),\ a(1) < 1 < b^u(1)$, and certainly $a(-\frac{n}{n-1}\theta) < b^u(-\frac{n}{n-1}\theta) = \infty$. So, $a(\bar{t}^u) < b^u(\bar{t}^u)$. We conclude that there is exactly one $t^u \in [0, \bar{t}^u]$ for which $a(t^u) = b(t^u)$, and moreover, $0 < t^u < \bar{t}^u$. So, there is a unique symmetric unit tax is $T^u = t^u f'(k)$.

The proof in the ad valorem case is identical, except that

$$b^a(t) = \frac{1 - \frac{t}{n}}{1 - \frac{t}{n} + \frac{n-1}{n}\frac{t}{\theta}}$$

replaces $b^u(.)$, and so the upper bound becomes $\bar{t}^a = \min\{-\frac{n}{\theta(n-1)-1}, 1\}$.

Notes

1. See for example, Wilson (1999) for a general survey, and Lockwood (2001) for a synthesis of some models of commodity tax competition.
2. For example, Wilson (1986), Zodrow and Miezowski (1986) assume a unit tax, as do well-known subsequent developments of the model by Wildasin (1988), Bucovetsky (1991), and others.
3. The classic example is of course, price and quantity setting by firms: for a monopolist, these two instruments are equivalent, but for two duopolists, they are not.
4. He requires the following assumptions: all jurisdictions are identical, residents of jurisdictions receive no income from capital, the elasticity of demand for capital is constant, and utilities are linear in private consumption. The conditions required for our results are considerably weaker.
5. The condition (E) below guarantees that in symmetric equilibrium, $f'(k) > T > 0$.
6. From (2.1), we have at the symmetric equilibrium, where $k_i = k$ that $\frac{\partial k_i}{\partial T_i} = \frac{1}{f''(k)}(1 + \frac{\partial r}{\partial T_i})$, and from (2.3), we have $\frac{\partial r}{\partial T_i} = -\frac{1}{n}$. Combining the two gives $\frac{\partial k_i}{\partial T_i} = \frac{n-1}{n}\frac{1}{f''(k)}$.
7. Again, $t_i \leq 1$ to ensure that the capital market equilibrium condition (2.2) is satisfied at a non-negative interest rate, and $t_i \geq 0$ is required as public good supply is non-negative, and the only tax is one on capital.
8. The condition (E) below guarantees that in symmetric equilibrium, $1 > t > 0$.
9. In particular, as the tax base with ad valorem taxes is $f'(k_i)k_i$, it is easily calculated that the elasticity of the tax base with respect to t_i is $\varepsilon^a = (1 + \theta)\eta$, so, the MCPF can be re-written as $\frac{1+\theta\varepsilon^a/(1+\theta)}{1+\varepsilon^a}$, which appears to depends on the elasticity of the tax base, ε^a, in a different and more complex way than in unit case.

Au: Pls. verify the name Miezowski.

Au: Pls. verify the name Lausell in refs.

10. These conditions for existence are rather different than those in Lausell and Le Breton (1988), or Bucovetsky (2002), because we are only concerned with existence and uniqueness of *symmetric* equilibria, and also because of differences in modelling. For example, Bucovetsky (2002) assumes a continuum of countries, and Lausell and Le Breton (1988) assume two symmetric countries, a special maximand of the sum of income of the fixed factor and the revenue from the capital tax, plus several conditions on the production function.
11. From (2.1), we have at the symmetric equilibrium, where $k_i = k$, $t_i = t$ that $\frac{\partial k_i}{\partial t_i} = \frac{1}{f''(k)}\left(\frac{r}{(1-t)^2} + \frac{1}{1-t}\frac{\partial r}{\partial T_i}\right)$, and from (2.3), we have $\frac{\partial r}{\partial T_i} = -\frac{r}{1-t}\frac{1}{n}$. Combining the two gives $\frac{\partial k_i}{\partial T_i} = \frac{n-1}{n}\frac{r}{f''(k)}\frac{1}{(1-t)^2} = \frac{n-1}{n}\frac{f'(k)}{f''(k)}\frac{1}{(1-t)}$.

References

Au: Pls. provide vol no.
Au: Pls. provide vol no.
Au: Pls. update.

Anderson, S. P., A. de Palma and B. Kreider. (2001a). "The Efficiency of Indirect Taxes under Imperfect Competition," *Journal of Public Economics* 231–251.

Anderson, S. P., A. de Palma and B. Kreider. (2001b). "Tax Incidence in Differentiated Product Oligopoly," *Journal of Public Economics* 173–192.

Bucovetsky, S. (1991). "Asymmetric Tax Competition," *Journal of Urban Economics* 30, 67–81.

Bucovetsky, S. (2002). "Existence of Nash Equilibrium for Tax Competition among a Large Number of Jurisdictions," unpublished paper.

Delipalla, S. and M. Keen. (1992). "The Comparison between Ad Valorem and Specific Taxation under Imperfect Competition," *Journal of Public Economics* 49, 351–367.

Laussel, D. and M. Le Breton. (1998). "Existence of Nash Equilibria in Fiscal Competition Models," *Regional Science and Urban Economics* 28, 283–296.

Lockwood, B. and K. Wong. (2000). "Specific and Ad Valorem Tariffs are not Equivalent in Trade Wars," *Journal of International Economics* 52, 183–195.

Lockwood, B. (2001). "Tax Competition and Coordination with Destination and Origin Principles: A Synthesis," *Journal of Public Economics* 81, 279–319.

Skeath, S. E. and G. A. Trandel. (1994). "A Pareto Comparison of Ad Valorem and Unit Taxes in Noncompetitive Environments," *Journal of Public Economics* 53, 53–71.

Suits, D. B. and R. A. Musgrave. (1953). "Ad Valorem and Unit Taxes Compared," *Quarterly Journal of Economics* 67, 598–604.

Wildasin, D. E. (1988). "Nash Equilibria in Models of Fiscal Competition," *Journal of Public Economics* 35, 229–240.

Wildasin, D. E. (1991) "Some Rudimentary "Duopolity" Theory," *Regional Science and Urban Economics* 21, 393–421.

Wilson, J. D. (1986). "A Theory of Interregional Tax Competition," *Journal of Urban Economics* 19, 296–315.

Wilson, J. D. (1986). "Tax Competition with Inter-Regional Differences in Factor Endowments" *Regional Science and Urban Economics* 21, 423–452.

Wilson, J. D. (1999). "Theories of Tax Competition," *National tax Journal* 52, 269–304.

Wicksell, K. (1959). "Taxation in the Monopoly Case," translated and reprinted in Musgrave and Shoup (eds.), Readings in the Economics of Taxation (Irwin, Homewood, IL).

Zodrow, G. and P. Mieszkowski. (1986). "Pigou, Tiebout, Property Taxation, and the Underprovision of Local Public Goods," *Journal of Urban Economics* 19(3), 356–370.